Aerospace Materials Formulae

AE1108 - Aerospace Materials & Structures

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by Juan G. de la Cierva

Introduction

This document contains a curated list of formulas for the AE1108-I Aerospace Materials' course based on Dr.S.J.García Espallargas' document on Brightspace, where the formulas from the Ashby book are listed in 3 categories:

1. "Must Know": These are the only formulas to be learnt by heart. The rest will be given.

2. "Others": These are the formulas whose meaning and the relation between the parameters must be understood, and be able to derive it if necessary from the "Must Know" equations. What I understand, from experience, is that if needed they will be given on the exam, but they will not tell you what each parameter represents.

3. "Know Existence": These are the formulas you must be aware of and be able to interpret. What I understand, from experience, is that if needed they will be given on the exam, and will let you know what each parameter represents.

For simplicity's sake, only the first two categories have been transcribed, being the most important. Further, if a formula is trivial, or is redundant because it is repeated, it will not be added.
If there are no formulas in a certain category, that category has not been added.
If a formula has been added for completeness but is NOT ON THE LIST, it will be indicated with an asterisk (*).

If the variables of an equation are not understood by the context, then their meaning can be checked in the Glossary. The terms are ordered in order or appearance.

Chapter 4: Elastic Stiffness & Weight

Must Know

Normal Stress (4.1)

$$\sigma = \frac{F}{A_0}$$

Shear Stress (4.2)

$$\tau = \frac{F_s}{A_0}$$

Volumetric Stress *

$$p = \frac{F}{A_0}$$

Normal Strain (4.3)

$$\varepsilon = \frac{\Delta L}{L_0}$$

Shear Strain (4.4)

$$\gamma = \frac{w}{L_0}$$

Vol. Strain (4.5)

$$\Delta = \frac{\Delta V}{V_0}$$

Elastic Modulus (4.6)

$$\sigma = E \cdot \varepsilon$$

Shear Modulus (4.7)

$$\tau = G \cdot \gamma$$

Bulk Modulus *

$$p = K \cdot \Delta$$

Poisson's Ratio (4.9)

$$\nu = -\frac{\varepsilon_t}{\varepsilon}$$

For METALS:
For POLYMERS: $\nu = \frac{1}{3}$
For CERAMICS:
For ELASTOMERS: $\nu = \frac{1}{2}$

E-K Relation (4.10/4.10a/4.10b)

$$K = \frac{E}{3(1 - 2\nu)}$$

For METALS:
For POLYMERS: $K \approx E$
For CERAMICS:
For ELASTOMERS: $K >> E$

E-G Relation (4.10/4.10a/4.10b)

$$G = \frac{E}{2(1 + \nu)}$$

For METALS:
For POLYMERS: $\quad G \approx \frac{3}{8}E$
For CERAMICS:
For ELASTOMERS: $\quad G \approx \frac{1}{3}E$

Upper Hybrid Modulus (4.19)

$$\widetilde{E_U} = \widetilde{E_\parallel} = f_r \cdot E_r + (1 - f_r) \cdot E_m$$

Lower Hybrid Modulus (4.20)

$$\widetilde{E_L} = \widetilde{E_\perp} = \frac{E_m \cdot E_r}{f_r \cdot E_m + (1 - f_r) \cdot E_r}$$

Hybrid Density (4.18)

$$\tilde{\rho} = f_r \cdot \rho_r + (1 - f_r) \cdot \rho_m$$

Thermal Expansion Coefficient (4.17)

$$\alpha = \frac{\varepsilon_T}{\Delta T}$$

Others

Hooke's Law (4.13)

Direction 1

$$\varepsilon_1 = \frac{1}{E} \cdot (\sigma_1 - \nu \cdot \sigma_2 - \nu \cdot \sigma_3)$$

Direction 2

$$\varepsilon_2 = \frac{1}{E} \cdot (-\nu \cdot \sigma_1 + \sigma_2 - \nu \cdot \sigma_3)$$

Direction 3

$$\varepsilon_3 = \frac{1}{E} \cdot (-\nu \cdot \sigma_1 - \nu \cdot \sigma_2 + \sigma_3)$$

Stiffness (4.25)

$$S = \frac{F}{\delta}$$

Atomic View of Stifness (4.26)

$$S = E \cdot a_0$$

Chapter 5: Stiffness-Limited Design

Must Know

Tie-Rod Elongation (5.1)

$$\delta = \frac{L_0 \cdot F}{A \cdot E}$$

Tie-Rod Stiffness (5.2)

$$S = \frac{A \cdot E}{L_0}$$

Beam Bending Flexural Rigidity (5.3)

$$\frac{\sigma}{y_m} = \frac{M}{I} = E \cdot k$$

Beam Bending Stiffness (5.5)

$$S = \frac{c_1 \cdot E \cdot I}{L^3}$$

Column/Plate Bucking Force (5.9)

$$F_{crit} = \frac{n^2 \cdot \pi^2 \cdot E \cdot I}{L^2}$$

Mass (5.13)

$$m = \rho \cdot A \cdot L_0$$

Stiffness Material Indices for Minimal Weight (5.16/5.22/5.27)

Tie-Rod

$$M_t = \frac{E}{\rho}$$

Plate Buckling

$$M_p = \frac{E^{1/3}}{\rho}$$

Beam Bending

$$M_b = \frac{E^{1/2}}{\rho}$$

Shape Factor (5.28)

$$\phi = \frac{I}{I_{square}}$$

Chapter 6: Beyond Elasticity

Must Know

Plastic Strain (6.1)

$$\varepsilon_{pl} = \varepsilon_{tot} - \frac{\sigma}{E}$$

Indentation Hardness (6.3)

$$H = \frac{F}{A}$$

Total Hardening (6.22)

$$\tau_y = \tau_i + \tau_{ss} + \tau_{ppt} + \tau_{wh} + \tau_{gb}$$

Others

True Stress (6.5)

$$\sigma_t = \frac{F}{A}$$

True/Nominal Stress (6.9)

$$\sigma_t = \sigma_n \cdot (1 + \varepsilon_n)$$

True/Nominal Strain (6.10)

$$\varepsilon_t = ln(1 + \varepsilon_n)$$

Plastic Work

$$W_{pl} = \int_0^{\varepsilon_f} \sigma d\varepsilon_{pl}$$

Dislocation Shear Force (6.11)

$$\tau = \alpha \cdot \frac{G \cdot b}{L}$$

Additional Dislocation Shear Resistance by:

Solid Solution Hardening (6.18)

$$\tau_{ss} = \alpha \cdot G \cdot \sqrt{c}$$

Work Hardening (6.20)

$$\tau_{wh} = \frac{\alpha \cdot G}{L} = \alpha \cdot G \cdot b \cdot \sqrt{\rho_d}$$

Particle Precipitation Hardening (6.19)

$$\tau_{ppt} = \frac{2 \cdot T}{b \cdot L} = \frac{G \cdot b}{L}$$

Grain Boundary Hardening (6.21)

$$\tau_{gb} = \frac{k_p}{\sqrt{D}}$$

Chapter 7: Strength-Limited Design

Must Know

Beam Yield Stress (7.2)

$$\sigma_{max,y} = \frac{M \cdot y_m}{I} = \frac{M}{Z_e}$$

Elastic Modulus Section (7.2)

$$Z_e = \frac{I}{y_m}$$

Beam Failure Moment (7.3)

$$M_f = Z_p \cdot \sigma_y$$

Plastic Section Modulus (7.3)

$$Z_p = 2 \cdot \int_0^{h/2} (b \cdot y) dy$$

Others

Stress Concentration Factor (7.12)

$$K_{SC} = \frac{\sigma_{max}}{\sigma_{nom}} = 1 + \alpha \cdot (\frac{c}{\rho_{sc}})^{1/2}$$

For TENSION: $\alpha = 2$
For BUCKLING: $\alpha = 0.5$
For BENDING: $\alpha = 0.5$

Permanent Deformation Resistance Material Indices for Minimal Weight *

Tie-Rod

$$M_t = \frac{\sigma_y}{\rho}$$

Plate

$$M_p = \frac{\sigma_y^{1/2}}{\rho}$$

Beam

$$M_b = \frac{\sigma_y^{2/3}}{\rho}$$

Spring

$$M_s = \frac{\sigma_y^2}{E}$$

Chapter 8: Fracture & Fracture Toughness

Must Know

Mode 1 Stress Intensity (8.3)

$$K_1 = Y \cdot \sigma \sqrt{\pi \cdot c}$$

Fracture Toughness (8.3/8.10)

$$K_{1c} = \sqrt{E \cdot G_c} = Y \cdot \sigma^* \sqrt{\pi \cdot c}$$

Critical Crack Length (8.13)

$$c_{crit} = \frac{K_{1c}{}^2}{Y^2 \cdot \pi \cdot \sigma_y^2}$$

Others

Energy Release Rate (8.5)

$$G \geq 2\gamma$$

Surface Energy (p.215)

$$\gamma = \frac{1}{3} \cdot H_c \cdot r_0$$

Stored Elastic Energy (8.6)

$$U_V = \frac{\sigma^2}{2E}$$

Crack Released Energy (8.7)

$$U(c) = \frac{\sigma^2}{2E} \cdot \frac{\pi \cdot c^2}{2}$$

Toughness (8.9)/(8.10)

$$G_c = \frac{K_{1c}{}^2}{E}$$

Plastic Region Radius (8.11)

$$r_y = 2 \cdot \left(\frac{\sigma \cdot c}{2 \cdot \sigma_y{}^2} \right)$$

Chapters 9/10: Cyclic Loading & Fatigue Failure Design

Must Know

Stress Ampl. (9.1)

$$\sigma_A = \frac{\Delta\sigma}{2}$$

Mean Stress (9.2)

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

R-Value (9.3)

$$R = \frac{\sigma_{min}}{\sigma_{max}}$$

Others

Basquin's Law (9.4)

$$\Delta\sigma \cdot N_f{}^b = C_1 \quad [\sigma_A < \sigma_y]$$

Coffin's Law (9.6)

$$\Delta\varepsilon_{pl} \cdot N_f{}^c = C_2 \quad [\sigma_A > \sigma_y]$$

Goodman's Rule (9.7)

$$\Delta\sigma_{\sigma_m} = \Delta\sigma_{\sigma_0} \cdot (1 - \frac{\sigma_m}{\sigma_{ts}})$$

Miner's Rule (9.8)

$$\sum_{i=1}^{n} \frac{N_i}{N_{f,i}} = 1$$

Stress Intensity Range (9.9)

$$\Delta K = \Delta\sigma\sqrt{\pi \cdot c}$$

Paris Law (9.10)

$$\frac{dc}{dN} = A \cdot \Delta K^m$$

Chapter 12: Materials & Heat

Must Know

Units

Thermal Expansion Coefficient

$$\alpha[K^{-1}]$$

Specific Heat Capacity

$$C_p[J \cdot kg^{-1} \cdot K^{-1}]$$

Thermal Conductivity

$$\lambda[W \cdot m^{-1} \cdot K^{-1}]$$

Heat Flux

$$q[W \cdot m^{-2}]$$

Thermal Diffusivity

$$a[m^2 \cdot s^{-1}]$$

Others

Thermal Exp. Coefficient (12.1)

$$\alpha = \frac{\Delta L}{L_0 \cdot \Delta T}$$

Heat Flux (12.2)

$$q = -\lambda \cdot \frac{\Delta T}{\Delta x}$$

Thermal Diffusivity (12.3)

$$a = \frac{\lambda}{\rho \cdot C_p}$$

Thermal Conductivity (12.10)

$$\lambda = \frac{\rho \cdot C_p}{3} \cdot l_m \cdot C_0$$

Volumetric Heat Capacity (12.4/12.6/12.7)

$$\rho \cdot C_p = \frac{3k_b}{\Omega} \quad [2 \cdot 10^6 < \rho \cdot C_p < 3 \cdot 10^6]$$

Thermal Shock Resistance (12.14)

$$\sigma = \frac{E}{1-\nu} \cdot \alpha \cdot \Delta T$$

Chapter 13: Diffusion & Creep

Others

Property Approximation (13.1)

$$P_n \approx P_0 \cdot (1 + \beta \cdot \frac{T_0}{T_n})$$

Viscous Flow (13.2)

$$\dot{\varepsilon} = \frac{\sigma}{3\eta}$$

Steady-State Creep (13.3/13.6)

$$\dot{\varepsilon_{ss}} = C' \cdot \sigma^n \cdot e^{-\frac{Q_c}{R \cdot T}} = \dot{\varepsilon_0} \cdot (\frac{\sigma}{\sigma_0})^n \cdot e^{-\frac{Q_c}{R \cdot T}}$$

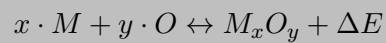**Fick's Law (13.11)**

$$J = -D \cdot \frac{\Delta c}{\Delta x}$$

Diffusion Coefficient (13.12)

$$D = D_0 \cdot e^{-(\frac{Q_D}{R \cdot T})}$$

Chapter 14: Durability

Must Know

Oxidation Reaction (14.1)

$$x \cdot M + y \cdot O \leftrightarrow M_x O_y + \Delta E$$

Oxidation Weight Gain (14.2/14.3)

$$\Delta m = k_l \cdot t \quad \text{or} \quad \Delta m^2 = k_p \cdot t$$

Oxidation Half-Reactions (p.384)

Metal

$$M = M^{2+} + 2e^-$$

Oxygen

$$O + 2e^- = O^{2-}$$
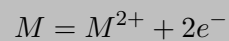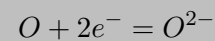
Water Dissociation (14.7)

$$H_2O \leftrightarrow H^+ + OH^-$$
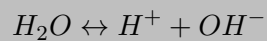
Law of Mass Action (14.8)

$$[H^+] \cdot [OH^-] = 10^{-14}$$

Hydrolisis Reaction (14.18)

$$H_2O + O + 2e^- \leftrightarrow 2OH^-$$
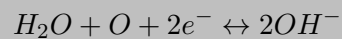
Acidity (14.9)

$$pH = -log[H^+]$$

Others

Nerst Equation (14.18)

$$E = E_0 + \frac{0.059}{z} \cdot log(C_{ion})$$

Glossary

Chapter 4

Symbol	Meaning	Symbol	Meaning
σ	Normal Stress	ε_t	Transversal Strain
F	Normal Force	$\widetilde{E_U}$	Upper-Bound Hybrid Elastic Modulus
A_0	Initial Cross-Sectional Area	$\widetilde{E_\parallel}$	Hybrid Elastic Modulus Parallel to Reinforcement
τ	Shear Stress	f_r	Volume of Reinforcement
F_s	Shear Force	E_r	Reinforcement Elastic Modulus
p	Pressure	E_m	Matrix Elastic Modulus
ε	Normal Strain	$\widetilde{E_L}$	Lower-Bound Hybrid Elastic Modulus
ΔL	Elongation	$\widetilde{E_\perp}$	Hybrid Elastic Modulus Perpendicular to Reinforcement
L_0	Initial Length	$\widetilde{rho}$	Hybrid Density
γ	Shear Strain	ρ_r	Reinforcement Density
w	Shear Displacement	ρ_m	Matrix Density
Δ	Volumetric Strain	α	Thermal Expansion Coefficient
ΔV	Change in Volume	ε_T	Thermal Strain
V_0	Initial Volume	ΔT	Change in Temperature
E	Young's (ELastic) Modulus	S	Stiffness
G	Shear Modulus	δ	Displacement/Deflection
K	Bulk Modulus	a_0	Atomic Radius
ν	Poisson's Ratio		

Chapter 5

Symbol	Meaning	Symbol	Meaning
δ	Displacement/Deflection	c_1	Configuration-Specific Bending Constant
L_0	Initial Length	L	Length
F	Force Applied	F_{crit}	Critical Buckling Force
A	Cross-Sectional Area	n	Configuration-Specific Buckling Constant
E	Young's (Elastic) Modulus	m	Mass
S	Stiffness	ρ	Density
σ	Stress	M_t	Tie-Rod Material Index
y_m	Vertical Distance to Neutral Axis	M_p	Plate Material Index
M	Bending Moment	M_b	Beam Material Index
I	Second Moment of Area	ϕ	Shape Factor
k	Beam Bending Curvature	I_{square}	Equivalent Square Cross-Section 2nd MM of Area

Chapter 6

Symbol	Meaning	Symbol	Meaning
ε_{pl}	Plastic Strain	τ_{wh}	Work Hardening DSR
ε_{tot}	Total Strain	τ_{gb}	Grain Boundary Hardening DSR
σ	Stress	σ_t	True Stress
E	Young's (Elastic) Modulus	σ_n	Nominal Stress
H	Indentation Hardness	ε_t	True Strain
F	Force Applied	ε_n	Nominal Strain
A	Indentation Area	W_{pl}	Work due to Plastic Deoformation
τ	Dislocation Shear Resistance (DSR)	ε_f	Final Strain
τ_y	Total DSR	α	Obstacle Strength Constant
τ_i	Initial DSR	G, b	Material-Specific Constants
τ_{ss}	Solid Solution Hardening DSR	L	Average Obstacle Spacing
τ_{ppl}	Particle Precipitation Hardening DSR	ρ_d	Dislocation Density

Chapter 7

Symbol	Meaning	Symbol	Meaning
$\sigma_{max,y}$	Beam Yield Stress	σ_{nom}	Nominal Stress
M	Bending Moment	α	Loading Mode Specific Constant
y_m	Vertical Distance to Neutral Axis	c	Characteristic Dimension
I	Second Moment of Area	ρ_{sc}	Minimum Radius of Feature
Z_e	Elastic Modulus Section	ρ	Density
M_f	Beam Failure Bending Moment	M_t	Tie-Rod Material Index
Z_p	Plastic Modulus Section	M_p	Plate Material Index
σ_y	Yield Stress	M_b	Beam Material Index
K_{SC}	Stress Concentration Factor	M_s	Spring Material Index
σ_{max}	Maximum Local Stress	E	Young's (Elastic) Modulus

Chapter 8

Symbol	Meaning	Symbol	Meaning
K_1	Mode 1 Stress Intensity Factor	G_c	Toughness
Y	Geometry-Specific Contant	c_{crit}	Critical Crack Length
σ	Stress	G	Energy Release Rate
c	Crack Length	γ	Surface Energy
K_{1c}	Fracture Toughness	U_V	Stored Elastic Energy
σ^*	Failure Stress	$U(c)$	Crack propagation Energy Release
E	Young's (Elastic) Modulus		

Chapters 9 & 10

Symbol	Meaning	Symbol	Meaning
σ_A	Stress Amplitude	$\Delta\sigma_{pl}$	Plastic Strain Increment
$\Delta\sigma$	Stress Increment	c, C_2	Coffin's Law Specific Constants
σ_m	Mean Stress	$\Delta\sigma_{\sigma_0}$	Stress Increment at 0 Mean Stress
σ_{max}	Maximum Stress	$\Delta\sigma_{\sigma_m}$	Equivalent Stress Increment
σ_{min}	Minimum Stress	σ_{ts}	Ultimate Tensile Stress
R	R-Value	N	Fatigue Cycles
σ_y	Yield Stress	ΔK	Stress Intensity Range
N_f	Fatigue Cycles until Failure	$\frac{dc}{dN}$	Crack Length Growth per Fatigue Cycle
b, C_1	Basquin's Law Specific Constants	A, m	Paris Law Specific Constants

Chapter 12

Symbol	Meaning	Symbol	Meaning
α	Thermal Expansion Coefficient	ΔT	Change in Temperature
C_p	Specific Heat Capacity	$\frac{\Delta T}{\Delta x}$	Uniform Temperature Gradient
λ	Thermal Conductivity	ρ	Density
q	Heat Flux	l_m	Mean Free Path
a	Thermal Diffusivity	C_0	Speed of Sound
ΔL	Elongation	k_b	Boltzmann Constant
L_0	Initial Length	Ω	Atom Volume

Chapter 13

Symbol	Meaning	Symbol	Meaning
P_n	New Property Value	$\dot{\varepsilon}_0$	Initial Creep
P_0	Initial Property Value	σ_0	Initial Stress
β	Specific Proportionality Constant	Q_c	Creep Activation Energy
T_0	Initial Temperature	R	Gas Constant
T_n	New Initial Temperature	T	Temperature
$\dot{\varepsilon}$	Creep	J	Net Flux of Diffused Atoms per Second
σ	Stress	D	Diffusion Coefficient
η	Viscosity	$\frac{\Delta c}{\Delta x}$	Atom Concentration Gradient
$\dot{\varepsilon}_{ss}$	Steady-State Creep	D_0	Diffusion Contant
B, n	Steady-State Creep Specific Constants	Q_D	Activation Energy per Mole

Chapter 14

Symbol	Meaning	Symbol	Meaning
x	Moles of Metal	H^+	Hydrogen Cation (Proton)
M	Any Metal	OH^-	Hydroxyl Group
y	Moles of Oxygen	$[H^+]$	Proton Concentration
O	Oxygen	$[OH^-]$	Hydroxyl Concentration
M_xO_y	Resulting Metal Oxide	e^-	Free Electron
ΔE	Energy	pH	Hydrogen Potential
Δm	Mass Increase	E	Corrected Reduction Potential
t	Time	E_0	Standard Reduction Potential
k_t	Linear Kinetic Constant	z	Ion Valence
k_p	Parabolic Kinetic Constant	C_{ion}	Ion Concentration
H_2O	Water		