

The aerodynamic center

In this chapter, we're going to focus on the aerodynamic center, and its effect on the moment coefficient C_m .

1 Force and moment coefficients

1.1 Aerodynamic forces

Let's investigate a wing. This wing is subject to a pressure distribution. We can sum up this entire pressure distribution. This gives us a resultant **aerodynamic force vector** \mathbf{C}_R .

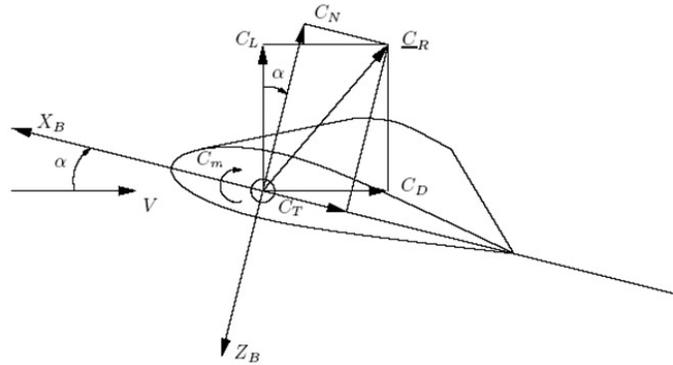


Figure 1: The forces and moments acting on a wing.

Let's split up the aerodynamic force vector \mathbf{C}_R . We can do this in multiple ways. We can split the force up into a (dimensionless) **normal force coefficient** C_N and a **tangential force coefficient** C_T . We can also split it up into a **lift force coefficient** C_L and a **drag force coefficient** C_D . Both methods are displayed in figure 1. The relation between the four force coefficients is given by

$$C_N = C_L \cos \alpha + C_D \sin \alpha, \quad (1.1)$$

$$C_T = C_D \cos \alpha - C_L \sin \alpha. \quad (1.2)$$

The coefficients C_L , C_N , C_T and C_D all vary with the angle of attack α . So if the angle of attack changes, so do those coefficients.

1.2 The aerodynamic moment

Next to the aerodynamic forces, we also have an **aerodynamic moment coefficient** C_m . This moment depends on the reference position. Let's suppose we know the moment coefficient $C_{m(x_1, z_1)}$ about some point (x_1, z_1) . The moment coefficient $C_{m(x_2, z_2)}$ about some other point (x_2, z_2) can now be found using

$$C_{m(x_2, z_2)} = C_{m(x_1, z_1)} + C_N \frac{x_2 - x_1}{\bar{c}} - C_T \frac{z_2 - z_1}{\bar{c}}. \quad (1.3)$$

Here, \bar{c} is the **mean aerodynamic chord length**. (The **mean aerodynamic chord** (MAC) can be seen as the 'average' chord of the entire 3D wing.) Also, x and z denote the position of the reference point in the vehicle reference frame F_r . We define (x_0, z_0) to be the position of the leading edge of the MAC.

2 Important points of the wing

2.1 The center of pressure

Let's put our reference point (x, z) (for calculating C_m) on the chord. (We thus set $z = z_0$.) There now is a certain value of x , for which $C_m = 0$. This point is called the **center of pressure** (CP). We denote its coordinates by (x_d, z_0) . (The CP is the point where the line of action of \mathbf{C}_R crosses the chord.)

Let's suppose that we know the moment coefficient $C_{m(x_0, z_0)}$ about the leading edge. We can then find x_d using

$$C_{m(x_d, z_0)} = 0 = C_{m(x_0, z_0)} + C_N \frac{x_d - x_0}{\bar{c}}. \quad (2.1)$$

Let's define $e = x_d - x_0$. We can then find that

$$\frac{e}{\bar{c}} = -\frac{C_{m(x_0, z_0)}}{C_N}. \quad (2.2)$$

2.2 Lines and metacenters

Let's examine a wing at a certain angle of attack α . This wing is subjected to a resultant force \mathbf{C}_R . For all points on the line of action of \mathbf{C}_R , we have $C_m = 0$.

Now let's examine all points for which $dC_m/d\alpha = 0$. These points also lie on one line. This line is called the **neutral line**. The point where this line crosses the MAC (and thus $z = z_0$) is called the **neutral point**. The crossing point of the neutral line and the line of action of \mathbf{C}_R is called the **first metacenter** M_1 . This point has both $C_m = 0$ and $dC_m/d\alpha = 0$.

Let's take another look at the neutral line. On this line is a point for which $d^2C_m/d\alpha^2 = 0$. This point is called the **second metacenter**.

It is important to remember that all the lines and points discussed above change as α changes. However, the second metacenter changes only very little. We therefore assume that its position is constant for different angles of attack α .

2.3 The aerodynamic center

Previously, we have defined the second metacenter. However, in aerodynamics, we usually refer to this point as the **aerodynamic center** (AC). Its coordinates are denoted by (x_{ac}, z_{ac}) . The corresponding moment coefficient is written as $C_{m_{ac}}$. We know that we have $dC_{m_{ac}}/d\alpha = 0$ and $d^2C_{m_{ac}}/d\alpha^2 = 0$. We can use this to find x_{ac} and z_{ac} .

To find x_{ac} and z_{ac} , we have to differentiate equation (1.3) with respect to α . Differentiating it once gives

$$\frac{dC_{m_{ac}}}{d\alpha} = 0 = \frac{dC_{m(x_0, z_0)}}{d\alpha} + \frac{dC_N}{d\alpha} \frac{x_{ac} - x_0}{\bar{c}} - \frac{dC_T}{d\alpha} \frac{z_{ac} - z_0}{\bar{c}}. \quad (2.3)$$

(Note that we have used the fact that the position of the AC doesn't vary with α .) Differentiating it twice gives

$$\frac{d^2C_{m_{ac}}}{d\alpha^2} = 0 = \frac{d^2C_{m(x_0, z_0)}}{d\alpha^2} + \frac{d^2C_N}{d\alpha^2} \frac{x_{ac} - x_0}{\bar{c}} - \frac{d^2C_T}{d\alpha^2} \frac{z_{ac} - z_0}{\bar{c}}. \quad (2.4)$$

We now have two equations and two unknowns. We can thus solve for x_{ac} and z_{ac} . After this, it is easy to find the corresponding moment coefficient $C_{m_{ac}}$. And since $dC_{m_{ac}}/d\alpha = 0$, we know that this moment coefficient stays the same, even if α varies.

We have just described an analytical method to find the AC. There are also graphical methods to find the AC. We won't go into detail on those methods though.

2.4 Simplifications

We can make a couple of simplifications. Usually, $dC_T/d\alpha$ is rather small compared to $dC_N/d\alpha$. We therefore often neglect the effects of the tangential force coefficient C_T . If we do this, we find that the AC lies on the MAC ($z_{ac} = z_0$). In fact, the AC coincides with the neutral point.

Finding the position of the AC has now become a lot easier. We know that $z_{ac} = z_0$. We can use this to show that x_{ac} satisfies

$$\frac{x_{ac} - x_0}{\bar{c}} = -\frac{dC_{m(x_0, z_0)}}{dC_N}. \quad (2.5)$$

Once x_{ac} has been determined, we can find the moment coefficient about any other point on the wing. Based on our simplifications, we have

$$C_m(x) = C_{m_{ac}} + C_N \frac{x - x_{ac}}{\bar{c}}. \quad (2.6)$$

We can also see another interesting fact from this equation. If $C_N = 0$, the moment coefficient is constant along the wing. And the value of this moment coefficient is equal to $C_{m_{ac}}$. In other words, the value of $C_{m_{ac}}$ is the value of C_m when $C_N = 0$. (This rule holds for every reference point.)

3 Static stability

3.1 Stability types

Let's suppose that the aircraft is performing a steady flight. The aircraft is then in equilibrium. This means that the moment coefficient about the center of gravity (CG) must be 0. ($C_{m_{cg}} = 0$.) Now let's suppose that the aircraft gets a small deviation from this steady flight. For example, α increases to $\alpha + d\alpha$. What happens?

Due to the change in angle of attack, $C_{m_{cg}}$ is no longer zero. Instead, it will get a value of $dC_{m_{cg}}$. We can now distinguish three cases.

- The change in moment $dC_{m_{cg}}$ is in the same direction as $d\alpha$. We thus have $dC_{m_{cg}}/d\alpha > 0$. In this case, the moment causes α to diverge away from the equilibrium position. The aircraft is therefore **unstable**.
- The change in moment $dC_{m_{cg}}$ is directed oppositely to $d\alpha$. We now have $dC_{m_{cg}}/d\alpha < 0$. In this case, the moment causes α to get back to its equilibrium position. The aircraft is thus **stable**.
- The change in moment $dC_{m_{cg}} = 0$, and thus also $dC_{m_{cg}}/d\alpha = 0$. In this case, we are in a new equilibrium position. This situation is called **neutrally stable** or **indifferent**.

3.2 The position of the center of gravity

We just saw that, to have a stable aircraft, we should have $dC_{m_{cg}}/d\alpha < 0$. It turns out that the position of the CG is very important for this. To see why, we differentiate equation (2.6) with respect to α . We find that

$$\frac{dC_{m_{cg}}}{d\alpha} = \frac{dC_N}{d\alpha} \frac{x_{cg} - x_{ac}}{\bar{c}}. \quad (3.1)$$

In general, $dC_N/d\alpha > 0$. So, to have a stable aircraft, we must have $x_{cg} - x_{ac} < 0$. The aerodynamic center should thus be more to the rear of the aircraft than the CG. (This is also why airplanes have a stabilizing horizontal tailplane: It moves the aerodynamic center to the rear.)

4 Three-dimensional wings

4.1 Basic and additional lift distributions

Previously, we have only examined 2D wings. We will now examine a 3D wing. The wing has a wing span b . Also, at every point y , the 2D airfoil has its own chord $c(y)$ and lift coefficient $c_l(y, \alpha)$. It also has a contribution to the lift. By summing up all these lift contributions, we can find the total lift coefficient C_L of the wing. This goes according to

$$C_L \frac{1}{2} \rho V^2 S = 2 \int_0^{b/2} c_l(y) \frac{1}{2} \rho V^2 c(y) dy \quad \Rightarrow \quad C_L = 2 \int_0^{b/2} c_l(y) \frac{c(y)}{\bar{c}} dy. \quad (4.1)$$

Note that we have used that $S = b\bar{c}$. We also have used the assumption that the wing is symmetric, by integrating over only one half of the wing.

We can split the lift coefficient distribution $c_l(y, \alpha)$ up into two parts. First, there is the **basic lift distribution** $c_{l_b}(y)$. This is the lift distribution corresponding to the **zero-lift angle of attack** $\alpha_{C_L=0}$. (So $c_{l_b}(y) = c_l(y, \alpha_{C_L=0})$.) Per definition, we thus have

$$2 \int_0^{b/2} c_{l_b}(y) \frac{c(y)}{\bar{c}} dy = 0. \quad (4.2)$$

Second, there is the **additional lift distribution** $c_{l_a}(y, \alpha)$. This lift distribution takes into account changes in α . It is defined as $c_{l_a}(y, \alpha) = c_l(y, \alpha) - c_{l_b}(y)$. So, if we have $\alpha = \alpha_{C_L=0}$, then $c_{l_a}(y, \alpha_{C_L=0}) = 0$ for all y .

4.2 The aerodynamic center of a 3D wing

You may wonder, what is the use of splitting up the lift distribution? Well, it can be shown that the position of the aerodynamic center of the entire wing \bar{x}_{ac} only depends on c_{l_a} . In fact, we have

$$\frac{\bar{x}_{ac} - \bar{x}_0}{\bar{c}} = \frac{1}{C_L} \frac{2}{S\bar{c}} \int_0^{b/2} c_{l_a}(y, \alpha) c(y) (x_{ac}(y) - \bar{x}_0) dy. \quad (4.3)$$

It is important to note the difference between all the x 's. x_{ac} is the position of the AC of the 2D airfoil. \bar{x}_{ac} is the position of the AC of the entire 3D wing. Finally, \bar{x}_0 is the position of the leading edge of the MAC. By the way, the above equation only holds for reasonable taper and wing twist angles. For very tapered/twisted wings, the above equation loses its accuracy.

Now let's examine the moment coefficient of the entire wing. This moment coefficient only depends on the moment coefficients $c_{m_{ac}}$ and the basic lift distribution c_{l_b} of the individual airfoils. In fact, it can be shown that

$$C_{m_{ac}} = \frac{2}{S\bar{c}} \left(\int_0^{b/2} c_{m_{ac}}(y) c(y)^2 dy - \int_0^{b/2} c_{l_b}(y) c(y) (x_{ac}(y) - \bar{x}_0) dy \right). \quad (4.4)$$

4.3 Effects of the 3D wing shape

Let's investigate how the wing shape effects \bar{x}_{ac} and $C_{m_{ac}}$. There are several properties that we can give to our 3D wing.

- A cambered airfoil. **Camber** causes the value $c_{m_{ac}}$ of the individual airfoils to become more negative. So $C_{m_{ac}}$ also becomes more negative. \bar{x}_{ac} doesn't really change.

- A swept wing. When dealing with swept wings, the term $(x_{ac}(y) - \bar{x}_0)$ becomes important. Wings with high **sweep angles** Λ tend to have a shifting AC at high angles of attack. Whether this improves the stability or not depends on other parameters as well.
- A tapered wing. The **taper ratio** $\lambda = c_t/c_r$ (the ratio of the tip chord and the root chord) slightly influences stability. For swept back wings, a low taper ratio tends to have a stabilizing influence.
- A slender wing. The **aspect ratio** A has only little influence on the position of the AC. However, a slender wing (high A) with a large sweep angle Λ will become unstable at large angles of attack.
- A twisted wing. Applying a **wing twist angle** ε causes the basic lift distribution c_{l_b} to change. This causes $C_{m_{ac}}$ to change as well. In what way $C_{m_{ac}}$ changes, depends on the direction of the wing twist.

Predicting the exact behaviour of the wing is, however, rather difficult. A lot of parameters influence the wing behaviour. So don't be surprised if the above rules don't always hold.