Physics II

AE1240-II

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Preface

The summaries in this document have been completely stolen from the textbook you need to learn from. You can find them in the end of each chapter. I might later come back and add extra explanations to the summaries, but at the moment, none of this is my own work. As always you can find the most up to date version of this summary on my website: alanrh.com

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Summaries From the Book

1.1. Chapter 27 - Magnetism

A magnet has two poles, north and south. The north pole is that end which points toward geographic north when the magnet is freely suspended. Like poles of two magnets repel each other, whereas unlike poles attract.

We can picture that a magnetic field surrounds every magnet. The SI unit for magnetic field is the tesla (T). Electric currents produce magnetic fields. For example, the lines of magnetic field due to a current in a straight wire form circles around the wire, and the field exerts a force on magnets (or currents) near it.

A magnetic field exerts a force on an electric current. The force on an infinitesimal length of wire carrying a current I in a magnetic field $\vec{B}$ is:

$$d\vec{F} = Id\vec{l} \times \vec{B} \tag{1.1}$$

If the field $\vec{B}$ is uniform over a straight length $\vec{l}$of wire then the force is:

$$\vec{F} = I\vec{l} \times \vec{B} \tag{1.2}$$

which has a magnitude of

$$F = IlB \sin\theta \tag{1.3}$$

where θ is the angle between magnetic field $\vec{B}$ and the wire. The direction of the force is perpendicular to the wire and to the magnetic field, and is given by the right-hand rule. This relation serves as the definition of magnetic field $\vec{B}$. Similarly, a magnetic field $\vec{B}$ exerts a force on a charge q moving with velocity $\vec{v}$ given by

$$\vec{F} = q\vec{v} \times \vec{B} \tag{1.4}$$

which has a magnitude of

$$f = qvB \sin\theta \tag{1.5}$$

where θ is the angle between $\vec{v}$ and $\vec{B}$. The path of a charged particle moving perpendicular to a uniform magnetic field is a circle.

If both electric and magnetic fields ($\vec{E}$ and $\vec{B}$) are present, the force on a charge q moving with velocity is:

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B} \tag{1.6}$$

The torque on a current loop in a magnetic field $\vec{B}$ is

$$T\vec{a}u = \vec{\mu} \times \vec{B} \tag{1.7}$$

where $\vec{\mu}$ is the **magnetic dipole moment** of the loop:

$$\vec{\mu} = NI\vec{A} \tag{1.8}$$

Here N is the number of coils carrying current I in the loop and $\vec{A}$ is a vector perpendicular to the plane of the loop (use right-hand rule, fingers along current in loop) and has magnitude equal to the area of the loop.

The measurement of the charge-to-mass ratio (e/m) of the electron was done using magnetic and electric fields. The charge e on the electron was first measured in the Millikan oil-drop experiment and then its mass was obtained from the measured value of the e/m ratio.

In the Hall effect, moving charges in a conductor placed in a magnetic field are forced to one side, producing an emf between the two sides of the conductor. A mass spectrometer uses magnetic and electric fields to measure the mass of ions.

1.2. Chapter 28 - Sources of Magnetic Field

The magnetic field B at a distance r from a long straight wire is directly proportional to the current I in the wire and inversely proportional to r:

$$B = \frac{\mu_0}{2\pi}\frac{I}{r} \tag{1.9}$$

The magnetic field lines are circles centered at the wire.

The force that one long current-carrying wire exerts on a second parallel current-carrying wire 1 m away serves as the definition of the ampere unit, and ultimately of the coulomb as well.

Ampère's law states that the line integral of the magnetic field $\vec{B}$ around any closed loop is equal to μ_0 times the total net current I_{encl} enclosed by the loop.

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl} \tag{1.10}$$

The magnetic field inside a long tightly wound solenoid is

$$B = \mu_0 n I \tag{1.11}$$

where n is the number of coils per unit length and I is the current in each coil.

The **Biot-Savart law** is useful for determining the magnetic field due to a known arrangement of currents. It states that:

$$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{l} \times \hat{r}}{r^2} \tag{1.12}$$

where $d\vec{B}$ is the contribution to the total field at some point P due to a current I along an infinitesimal length of its path, and is the unit vector along the direction of the displacement vector from $d\vec{l}$ to P. The total field $\vec{B}$ will be the integral over all $d\vec{B}$

Iron and a few other materials can be made into strong permanent magnets. They are said to be ferromagnetic. Ferromagnetic materials are made up of tiny domains—each a tiny magnet—which are preferentially aligned in a permanent magnet, but randomly aligned in a nonmagnetized sample.

When a ferromagnetic material is placed in a magnetic field due to a current, say inside a solenoid or toroid, the material becomes magnetized. When the current is turned off, however, the material remains magnetized, and when the current is increased in the opposite direction (and then again reversed), a graph of the total field B versus B_0 is a hysteresis loop, and the fact that the curves do not retrace themselves is called hysteresis.

All materials exhibit some magnetic effects. Nonferromagnetic materials have much smaller paramagnetic or diamagnetic properties.

1.3. Chapter 29 - Electromagnetic Induction and Faraday's Law

The **magnetic flux** passing through a loop is equal to the product of the area of the loop times the perpendicular component of the (uniform) magnetic field: $\Phi_B = B_{perp}A = BA\cos\theta$ If $\vec{B}$ is not uniform, then

$$\Phi_B = \int \vec{B} \cdot d\vec{A} \tag{1.13}$$

If the magnetic flux through a coil of wire changes in time, an emf is induced in the coil. The magnitude of the induced emf equals the time rate of change of the magnetic flux through the loop times the number N of loops in the coil:

$$\varepsilon = -N\frac{d\Phi_B}{dt} \tag{1.14}$$

This is **Faraday's law of induction**.

The induced emf can produce a current whose magnetic field opposes the original change in flux (**Lenz's law**). We can also see from Faraday's law that a straight wire of length l moving with speed v perpendicular to a magnetic field of strength B has an emf induced between its ends equal to:

$$\varepsilon = Blv \tag{1.15}$$

Faraday's law also tells us that a changing magnetic field produces an electric field. The mathematical relation is:

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \tag{1.16}$$

and is the general form of Faraday's law. The integral on the left is taken around the loop through which the magnetic flux Φ_B is changing.

An electric generator changes mechanical energy into electrical energy. Its operation is based on Faraday's law: a coil of wire is made to rotate uniformly by mechanical means in a magnetic field, and the changing flux through the coil induces a sinusoidal current, which is the output of the generator.

A motor, which operates in the reverse of a generator, acts like a generator in that a **back emf** is induced in its rotating coil; since this counter emf opposes the input voltage, it can act to limit the current in a motor coil. Similarly, a generator acts somewhat like a motor in that a **counter torque** acts on its rotating coil.

A transformer, which is a device to change the magnitude of an ac voltage, consists of a primary coil and a secondary coil. The changing flux due to an ac voltage in the primary coil induces an ac voltage in the secondary coil. In a 100% efficient transformer, the ratio of output to input voltages (V_S/V_P) equals the ratio of the number of turns N_S in the secondary to the number N_P in the primary:

$$\frac{V_S}{V_P} = \frac{N_S}{N_P} \tag{1.17}$$

The ratio of secondary to primary current is in the inverse ratio of turns:

$$\frac{I_S}{I_P} = \frac{N_P}{N_S} \tag{1.18}$$

Microphones, ground fault circuit interrupters, seismographs, and read/write heads for computer drives and tape recorders are applications of electromagnetic induction.

1.4. Chapter 30 - Inductance, Electromagnetic Oscillations, and AC Circuits

A changing current in a coil of wire will induce an emf in a second coil placed nearby. The **mutual inductance**, M, is defined as the proportionality constant between the induced emf ε_2 in the second coil and the time rate of change of current in the first:

$$\varepsilon_2 = -M\frac{dI_1}{dt} \tag{1.19}$$

We can also write M as

$$M = \frac{N_2\Phi_{21}}{I_1} \tag{1.20}$$

where Φ_{21} is the magnetic flux through coil 2 with N_2loops, produced by the current in another coil (coil 1). Within a single coil, a changing current induces an opposing emf ε, so a coil has **self-inductance** L defined by:

$$\varepsilon = -L\frac{dI}{dt} \tag{1.21}$$

This induced emf acts as an impedance to the flow of an alternating current. We can also write L as:

$$L = N\frac{\Phi_B}{I} \tag{1.22}$$

where Φ_B is the flux through the inductance when a current I flows in its N loops. When the current in an inductance L is I, the energy stored in the inductance is given by

$$U = \frac{1}{2}LI^2 \tag{1.23}$$

This energy can be thought of as being stored in the magnetic field of the inductor. The energy density u in any magnetic field B is given by:

$$u = \frac{1}{2}\frac{B^2}{\mu_0} \tag{1.24}$$

where μ_0 is replaced by μ if a ferromagnetic material is present.

When an inductance L and resistor R are connected in series to a constant source of emf, the current rises according to an exponential of the form:

$$I = \frac{V_0}{R}(1 - e^{-t/\tau}) \tag{1.25}$$

where $\tau = L/R$ is the time constant. The current eventually levels out at $I = V_0/R$. If the battery is suddenly switched out of the ***LR* circuit**, and the circuit remains complete, the current drops exponentially, $I = I_0e^{-t/\tau}$ with the same time constant τ.

The current in a pure ***LC* circuit** (or charge on the capacitor) would oscillate sinusoidally. The energy too would oscillate back and forth between electric and magnetic, from the capacitor to the inductor, and back again. If such a circuit has resistance (LRC), and the capacitor at some instant is charged, it can undergo damped oscillations or exhibit critically damped or overdamped behavior.

Capacitance and inductance offer impedance to the flow of alternating current just as resistance does. This impedance is referred to as **reactance**, X, and is defined (as for resistors) as the proportionality constant between voltage and current (either the rms or peak values). Across an inductor,

$$V_0 = I_0 X_L \tag{1.26}$$

and across a capacitor

$$V_0 = I_0 X_C \tag{1.27}$$

The reactance of an inductor increases with frequency:

$$X_L = \omega L \tag{1.28}$$

where $\omega = 2\pi f$ and f is the frequency of the ac. The reactance of a capacitor decreases with frequency:

$$X_L = \frac{1}{\omega C} \tag{1.29}$$

Whereas the current through a resistor is always in phase with the voltage across it, this is not true for inductors and capacitors: in an inductor, the current lags the voltage by 90°, and in a capacitor the current leads the voltage by 90°.

In an ac ***LRC* series circuit**, the total impedance Z is defined by the equivalent of for resistance: namely or The impedance Z is related to R, C, and L by

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \tag{1.30}$$

The current in the circuit lags (or leads) the source voltage by an angle ϕ given by $\cos\phi = R/Z$. Only the resistor in an ac LRC circuit dissipates energy, and at a rate

$$\bar{P} = I_{rms}^2 Z \cos\phi \tag{1.31}$$

where the factor $\cos\phi$ is referred to as the **power factor**. An LRC series circuit **resonates** at a frequency given by:

$$\omega_0 = \sqrt{\frac{1}{LC}} \tag{1.32}$$

The rms current in the circuit is largest when the applied voltage has a frequency equal to $f_0 = \omega_0/2\pi$. The lower the resistance R, the higher and sharper the resonance peak

1.5. Chapter 31 - Maxwell's Equations and Electromagnetic Waves

James Clerk Maxwell synthesized an elegant theory in which all electric and magnetic phenomena could be described using four equations, now called Maxwell's equations. They are based on earlier ideas, but Maxwell added one more—that a changing electric field produces a magnetic field. Maxwell's equations are

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0} \tag{1.33}$$

$$\oint \vec{B} \cdot d\vec{A} = 0 \tag{1.34}$$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \tag{1.35}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt} \tag{1.36}$$

The first two are Gauss's laws for electricity and for magnetism; the other two are Faraday's law and Ampère's law (as extended by Maxwell), respectively.

Maxwell's theory predicted that transverse **electromagnetic (EM) waves** would be produced by accelerating electric charges, and these waves would propagate through space at the speed of light c, given by:

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 3.00 \times 10^8 ms^{-1} \tag{1.37}$$

The wavelength λ and frequency f of EM waves are related to their speed c by:

$$c = \lambda f \tag{1.38}$$

just as for other waves.

The oscillating electric and magnetic fields in an EM wave are perpendicular to each other and to the direction of propagation. EM waves are waves of fields, not matter, and can propagate in empty space.

After EM waves were experimentally detected in the late 1800s, the idea that light is an EM wave (although of much higher frequency than those detected directly) became generally accepted. The electromagnetic spectrum includes EM waves of a wide variety of wavelengths, from microwaves and radio waves to visible light to X-rays and gamma rays, all of which travel through space at a speed of light $c = 3.00 \times 10^8 ms^{-1}$.

The energy carried by EM waves can be described by the **Poynting vector**.

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B} \tag{1.39}$$

which gives the rate energy is carried across unit area per unit time when the electric and magnetic fields in an EM wave in free space are $\vec{E}$ and $\vec{B}$. EM waves carry momentum and exert a **radiation pressure** proportional to the intensity S of the wave