Problems and Solutions Section 5.1 (5.1 through 5.5)

5.1 Using the nomograph of Figure 5.1, determine the frequency range of vibration for which a machine oscillation remains at a satisfactory level under rms acceleration of 1*g*.

Solution:

An rms acceleration of 1 *g* is about 9.81 m/s^2. From Figure 5.1, a satisfactory level would occur at frequencies above 650 Hz.

5.2 Using the nomograph of Figure 5.1, determine the frequency range of vibration for which a structure's rms acceleration will not cause wall damage if vibrating with an rms displacement of 1 mm or less.

Solution:

From Figure 5.1, an rms displacement of 1 mm (1000 μm) would not cause wall damage at frequencies below 3.2 Hz.

5.3 What natural frequency must a hand drill have if its vibration must be limited to a minimum rms displacement of 10 μm and rms acceleration of 0.1 m/s^2? What rms velocity will the drill have?

Solution:

From Figure 5.1, the natural frequency would be about 15.8 Hz or 99.6 rad/s. The rms velocity would be 1 mm/s.

5.4 A machine of mass 500 kg is mounted on a support of stiffness 197,392,000 N/m. Is the vibration of this machine acceptable (Figure 5.1) for an rms amplitude of 10 μm? If not, suggest a way to make it acceptable.

Solution:

The frequency is $\omega_n = \sqrt{\dfrac{k}{m}} =$ 628.3 rad/s = 100 Hz.

For an rms displacement of 10 μm the vibration is unsatisfactory. To make the vibration satisfactory, the frequency should be reduced to 31.6 Hz. This can be accomplished by reducing the stiffness and/or increasing the mass of the machine.

5.5 Using the expression for the amplitude of the displacement, velocity and acceleration of an undamped single-degree-of-freedom system, calculate the velocity and acceleration amplitude of a system with a maximum displacement of 10 cm and a natural frequency of 10 Hz. If this corresponds to the vibration of the wall of a building under a wind load, is it an acceptable level?

Solution:

The velocity amplitude is

$$|v(t)| = A\omega_n = (0.1 \text{ m})\left(\frac{10}{2\pi}\right) = 0.159 \text{ m/s}$$

The acceleration amplitude is

$$|a(t)| = A\omega_n^2 = (0.1 \text{ m})\left(\frac{10}{2\pi}\right)^2 = 0.253 \text{ m/s}^2$$

The rms displacement is $\frac{A}{\sqrt{2}} = \frac{0.1}{\sqrt{2}} = 0.0707 \text{ m} = 70{,}700\ \mu\text{m}$ (from equation (1.21)). At 10 Hz and $70{,}700\ \mu\text{m}$, this could be destructive to a building.

Problems and Solutions Section 5.1 (5.6 through 5.26)

5.6 A 100-kg machine is supported on an isolator of stiffness 700×10^3 N/m. The machine causes a vertical disturbance force of 350 N at a revolution of 3000 rpm. The damping ratio of the isolator is ζ = 0.2. Calculate (a) the amplitude of motion caused by the unbalanced force, (b) the transmissibility ratio, and (c) the magnitude of the force transmitted to ground through the isolator.

Solution:

(a) From Window 5.2, the amplitude at steady-state is

$$X = \frac{F_o / m}{\left[\left(\omega_n^2 - \omega^2\right)^2 + \left(2\zeta\omega_n\omega\right)^2\right]^{1/2}}$$

Since $\omega_n = \sqrt{\frac{k}{m}}$ = 83.67 rad/s and $\omega = 3000\left(\frac{2\pi}{60}\right)$ = 314.2 rad/s,

(b) From equation (5.7), the transmissibility ratio is

$$\frac{F_T}{F_0} = \sqrt{\frac{1 + \left(2\zeta r\right)^2}{\left(1 - r^2\right)^2 + \left(2\zeta r\right)^2}}$$

Since $r = \frac{\omega}{\omega_n}$ = 3.755, this becomes

$$\frac{F_T}{F_0} = 0.1368$$

(c) The magnitude is

$$F_T = \left(\frac{F_T}{F_0}\right)F_0 = \left(0.1368\right)\left(350\right) = \left(47.9\right)$$

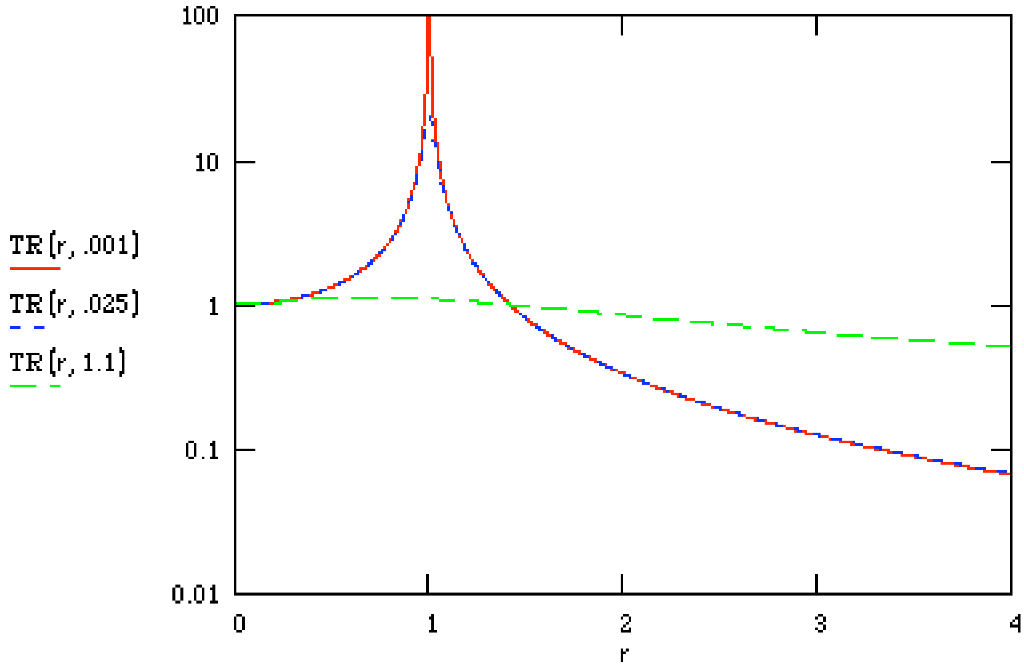
5.7 Plot the T.R. of Problem 5.6 for the cases ζ = 0.001, ζ = 0.025, and ζ = 1.1.

Solution:

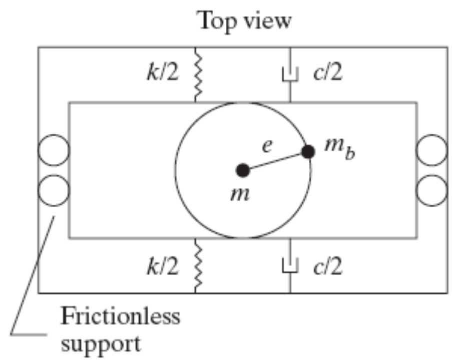
$$\text{T.R.} = \sqrt{\frac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}}$$

A plot of this is given for ζ = 0.001, ζ = 0.025, and ζ = 1.1. The plot is given here from Mathcad:

$$TR(r,\zeta) := \sqrt{\frac{1+(2\cdot\zeta\cdot r)^2}{(1-r^2)^2+(2\cdot\zeta\cdot r)^2}}$$



5.8 A simplified model of a washing machine is illustrated in Figure P5.8. A bundle of wet clothes forms a mass of 10 kg (m_b) in the machine and causes a rotating unbalance. The rotating mass is 20 kg (including m_b) and the diameter of the washer basket ($2e$) is 50 cm. Assume that the spin cycle rotates at 300 rpm. Let k be 1000 N/m and ζ = 0.01. Calculate the force transmitted to the sides of the washing machine. Discuss the assumptions made in your analysis in view of what you might know about washing machines.



Solution: The transmitted force is given by $F_T = \sqrt{k^2 + c^2\omega_r^2}$ where

$$c = 2\zeta\omega_n, \quad \omega_n = \sqrt{\frac{k}{m}} = 7.071 \text{ rad/s}, \; \omega_r = 300\frac{2\pi}{60} = 31.42 \text{ rad/s},$$

and X is given by equation (2.84) as

$$X = \frac{m_0 e}{m}\frac{r^2}{\sqrt{\left(1-r^2\right)^2 + \left(2\zeta r\right)^2}}$$

Since $r = \dfrac{\omega_r}{\omega_n} = 4.443$, then X = 0.1317 m and

$$F_T = (0.1317)\sqrt{(1000)^2 + [2(0.01)(20)(7.071)]^2(31.42)^2} = 132.2 \text{ N}$$

Two important assumptions have been made:

i) The out-of-balance mass is concentrated at a point and

ii) The mass is constant and distributed evenly (keep in mind that water enters and leaves) so that the mass actually changes.

5.9 Referring to Problem 5.8, let the spring constant and damping rate become variable. The quantities m, m_b, e and ω are all fixed by the previous design of the washing machine. Design the isolation system (i.e., decide on which value of k and c to use) so that the force transmitted to the side of the washing machine (considered as ground) is less than 100N.

Solution:

The force produced by the unbalance is $F_r = m_b a$ where a is given by the magnitude of equation (2.81):

$$F_r = m_0\left|\ddot{x}_r\right| = em_0\omega_r^2 = (0.25)(10)\left[300\left(\frac{2\pi}{60}\right)\right]^2 = 2467.4 \text{ N}$$

Since $F_T < 100$ N,

$$\text{T.R.} = \frac{F_T}{F_r} = \frac{100}{2467.4} = 0.0405$$

If the damping ratio is kept at 0.01, this becomes

$$\text{T.R.} = 0.0405 = \sqrt{\frac{1+\left[2(0.01)r\right]^2}{\left(1-r^2\right)^2+\left[2(0.01)r\right]^2}}$$

Solving for r yields $r = 5.079$.

Since $r = \dfrac{\omega_r}{\sqrt{k/m}}$,

$$k = \frac{m\omega_r^2}{r^2} = \frac{(20)\left[300\left(\frac{2\pi}{60}\right)\right]^2}{5.079^2} = 765 \text{ N/m}$$

and

$$c = 2\zeta\sqrt{km} = 2(0.01)\sqrt{(765)(20)} = 2.47 \text{ kg/s}$$

5.10 A harmonic force of maximum value of 25 N and frequency of 180 cycles/min acts on a machine of 25 kg mass. Design a support system for the machine (i.e., choose *c*, *k*) so that only 10% of the force applied to the machine is transmitted to the base supporting the machine.

Solution: From equation (5.7),

$$\text{T.R.} = 0.1 = \sqrt{\frac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}} \tag{1}$$

If we choose $\zeta = 0.1$, then solving the equation (1) numerically yields $r = 3.656$. Since $r = \dfrac{\omega}{\sqrt{k/m}}$ then:

$$k = \frac{m\omega^2}{r^2} = \frac{(25)\left[180\left(\frac{2\pi}{60}\right)\right]^2}{3.656^2} = 665 \text{ N/m}$$

and

$$c = 2\zeta\sqrt{km} = 2(0.1)\sqrt{(665)(25)} = 25.8 \text{ kg/s}$$

5.11 Consider a machine of mass 70 kg mounted to ground through an isolation system of total stiffness 30,000 N/m, with a measured damping ratio of 0.2. The machine produces a harmonic force of 450 N at 13 rad/s during steady-state operating conditions. Determine (a) the amplitude of motion of the machine, (b) the phase shift of the motion (with respect to a zero phase exciting force), (c) the transmissibility ratio, (d) the maximum dynamic force transmitted to the floor, and (e) the maximum velocity of the machine.

Solution:

(a) The amplitude of motion can be found from Window 5.2:

$$X = \frac{F_0 / m}{\left[\left(\omega_n^2 - \omega^2\right)^2 + \left(2\zeta\omega_n\omega\right)^2\right]^{1/2}}$$

where $\omega_n = \sqrt{\frac{k}{m}}$ = 20.7 rad/s. So,

$$X = 0.0229 \text{ m}$$

(b) The phase can also be found from Window 5.2:

$$\phi = \tan^{-1}\frac{2\zeta\omega_n\omega}{\omega_n^2 - \omega^2} = 22.5^\circ = 0.393 \text{ rad}$$

(c) From Eq. 5.7, with $r = \frac{\omega}{\omega_n}$ =0.628

$$\text{T.R.} = \sqrt{\frac{1 + \left(2\zeta r\right)^2}{\left(1 - r^2\right)^2 + \left(2\zeta r\right)^2}} = 1.57$$

(d) The magnitude of the force transmitted to the ground is

$$F_T = \left(\text{T.R.}\right)F_0 = \left(450\right)\left(1.57\right) = 707.6 \text{ N}$$

(e) The maximum velocity would be

$$\omega A_0 = \left(13\right)\left(0.0229\right) = 0.298 \text{ m/s}$$

5.12 A small compressor weighs about 70 lb and runs at 900 rpm. The compressor is mounted on four supports made of metal with negligible damping.
(a) Design the stiffness of these supports so that only 15% of the harmonic force produced by the compressor is transmitted to the foundation.
(b) Design a metal spring that provides the appropriate stiffness using Section 1.5 (refer to Table 1.2 for material properties).

Solution:

(a) From Figure 5.9, the lines of 85% reduction and 900 rpm meet at a static deflection of 0.35 in. The spring stiffness is then

$$k = \frac{mg}{\delta_s} = \frac{70 \text{ lb}}{0.35 \text{ in}} = 200 \text{ lb/in}$$

The stiffness of each support should be $k/4$ = 50 lb/in.

(b) Try a helical spring given by equation (1.67):

$$k = 50 \text{ lb/in} = 8756 \text{ N/m} = \frac{Gd^4}{64nR^3}$$

Using R = 0.1 m, n = 10, and $G = 8.0 \times 10^{10}$ N/m^2 (for steel) yields

$$d = \left[\frac{64(8756)(10)(0.1)^3}{8.0\times 10^{10}}\right]^{1/4} = 0.0163 \text{ m} = 1.63 \text{ cm}$$

5.13 Typically, in designing an isolation system, one cannot choose any continuous value of *k* and c but rather, works from a parts catalog wherein manufacturers list isolators available and their properties (and costs, details of which are ignored here). Table 5.3 lists several made up examples of available parts. Using this table, design an isolator for a 500-kg compressor running in steady state at 1500 rev/min. Keep in mind that as a rule of thumb compressors usually require a frequency ratio of r =3.

Solution:

Since $r = \dfrac{\omega}{\sqrt{k/m}}$, then

$$k = \frac{m\omega^2}{r^2} = \frac{\left(500\left[1500\left(\dfrac{2\pi}{60}\right)\right]^2\right)}{3^2} = 1371\times 10^3 \text{ N/m}$$

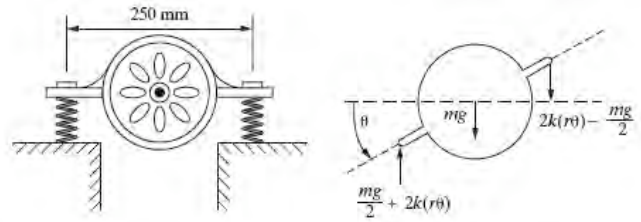
Choose isolator R-3 from Table 5.3. So, $k = 1000\times 10^3$ N/m and $c = 1500$ N·s/m.

Check the value of r:

$$r = \frac{1500\left(\dfrac{2\pi}{60}\right)}{\sqrt{1000\times 10^3/500}} = 3.51$$

This is reasonably close to $r = 3$.

5.14 An electric motor of mass 10 kg is mounted on four identical springs as indicated in Figure P5.14. The motor operates at a steady-state speed of 1750 rpm. The radius of gyration (see Example 1.4.6 for a definition) is 100 mm. Assume that the springs are undamped and choose a design (i.e., pick *k*) such that the transmissibility ratio in the vertical direction is 0.0194. With this value of *k*, determine the transmissibility ratio for the torsional vibration (i.e., using θ rather than *x* as the displacement coordinates).



Solution:

TABLE 5.3 Catalog values of stiffness and damping properties of various off-the-shelf isolators

Part No.[a]	R-1	R-2	R-3	R-4	R-5	M-1	M-2	M-3	M-4	M-5
k(10^3N/m)	250	500	1000	1800	2500	75	150	250	500	750
c(N·s/m)	2000	1800	1500	1000	500	110	115	140	160	200

[a]The "R" in the part number designates that the isolator is made of rubber, and the "M" designates metal. In general, metal isolators are more expensive than rubber isolators.

With no damping, the transmissibility ratio is

$$\text{T.R.} = \frac{1}{r^2 - 1}$$

where

$$r = \frac{\omega}{\sqrt{4k/m}} = \frac{1750\left(\frac{2\pi}{60}\right)}{\sqrt{4k/10}} = \frac{579.5}{\sqrt{4k}}$$

$$0.0194 = \frac{1}{\frac{(579.5)^2}{4k} - 1}$$

$$4k = 6391 \text{ N/m}$$

For each spring, $k = 1598$ N/m.

For torsional vibration, the equation of motion is

$$I\ddot{\theta} = -\left[\frac{mg}{2} + 2kr\theta\right]r - \left[2kr\theta - \frac{mg}{2}\right]r$$

where $r = \dfrac{0.250 \text{ m}}{2}$ = 0.125 m and from the definition of the radius of gyration and the center of percussion (see Example 1.4.6):

$$I = mk_0^2 = (10)(0.1)^2 = 0.1\,\text{kg}\cdot\text{m}^2$$

So,

$$0.1\ddot{\theta} + 4(1598)(0.125)^2\,\theta = 0$$

$$\ddot{\theta} + 998.6\theta = 0$$

The frequency ratio, *r*, is now

$$r = \frac{1750\left(\dfrac{2\pi}{60}\right)}{\sqrt{998.6}} = 5.80$$

$$\text{T.R.} = \frac{1}{r^2 - 1} = 0.0306$$

5.15 A large industrial exhaust fan is mounted on a steel frame in a factory. The plant manager has decided to mount a storage bin on the same platform. Adding mass to a system can change its dynamics substantially and the plant manager wants to know if this is a safe change to make. The original design of the fan support system is not available. Hence measurements of the floor amplitude (horizontal motion) are made at several different motor speeds in an attempt to measure the system dynamics. No resonance is observed in running the fan from zero to 500 rpm. Deflection measurements are made and it is found that the amplitude is 10 mm at 500 rpm and 4.5 mm at 400 rpm. The mass of the fan is 50 kg and the plant manager would like to store up to 50 kg on the same platform. The best operating speed for the exhaust fan is between 400 and 500 rpm depending on environmental conditions in the plant.

Solution:

A steel frame would be very lightly damped, so

$$\frac{X}{Y} = \frac{1}{1 - r^2}$$

Since no resonance is observed between 0 and 500 rpm, $r < 1$.

When $\omega = 500\left(\frac{2\pi}{60}\right) = 52.36$ rad/s, $X = 10$ mm, so

$$10 = \frac{Y}{1 - \left(\frac{52.36}{\omega_n}\right)^2}$$

Also, at $\omega = 400\left(\frac{2\pi}{60}\right) = 41.89$ rad/s, $X = 4.5$ mm, so

$$4.5 = \frac{Y}{1 - \left(\frac{41.89}{\omega_n}\right)^2}$$

Solving for ω_n and Y yields

$\omega_n = 59.57$ rad/s
$Y = 2.275$ mm
The stiffness is $k = m\omega_n^2 = (50)(59.57)^2 = 177{,}453$ N/m. If an additional 50 kg is added so that $m = 100$ kg, the natural frequency becomes

$$\omega_n = \sqrt{\frac{177{,}453}{100}} = 42.13 \text{ rad/s} = 402.3 \text{ rpm}$$

This would not be advisable because the normal operating range is 400 rpm to 500 rpm, and resonance would occur at 402.3 rpm.

5.16 A 350-kg rotating machine operates at 800 cycles/min. It is desired to reduce the transmissibility ratio by one-fourth of its current value by adding a rubber vibration isolation pad. How much static deflection must the pad be able to withstand?

Solution:

From equation (5.12), with R = 0.25:

$$r = \sqrt{\frac{2-0.25}{1-0.25}} = 1.528 = \frac{\omega}{\sqrt{k/m}} = \frac{800\left(\frac{2\pi}{60}\right)}{\sqrt{k/350}}$$

$$k = 1.053\times10^{6} \text{ N/m}$$

The static deflection is

$$\delta_s = \frac{mg}{k} = \frac{(350)(9.81)}{1.053\times10^{6}} = 3.26 \text{ mm}$$

5.17 A 68-kg electric motor is mounted on an isolator of mass 1200 kg. The natural frequency of the entire system is 160 cycles/min and has a measured damping ratio of ζ = 1. Determine the amplitude of vibration and the force transmitted to the floor if the out-of-balance force produced by the motor is $F(t)$ = 100 sin (31.4t) in newtons.

Solution:

The amplitude of vibration is given in Window 5.2 as

$$A_0 = \frac{F_0/m}{\left[\left(\omega_n^2-\omega^2\right)^2+\left(2\zeta\omega_n\omega\right)^2\right]^{1/2}}$$

where F_0 = 100 N, m = 1268 kg, ω = 31.4 rad/s, and $\omega_n = 160\left(\frac{2\pi}{60}\right) = 16.76$ rad/s. So,

$$X = 6.226\times10^{-5} \text{ m}$$

The transmitted force is given by Eq. (5.6), with $r = \frac{31.4}{16.76} = 1.874$

$$F_T = F_0\sqrt{\frac{1+\left(2\zeta r\right)^2}{\left(1-r^2\right)^2+\left(2\zeta r\right)^2}} = 85.97 \text{ N}$$

5.18 The force exerted by an eccentric (e = 0.22 mm) flywheel of 1000 kg, is 600 cos(52.4t) in newtons. Design a mounting to reduce the amplitude of the force exerted on the floor to 1% of the force generated. Use this choice of damping to ensure that the maximum force transmitted is never greater than twice the generated force.

Solution:

Two conditions are given. The first is that T.R. = 2 at resonance (r = 1), and the second is that T.R. = 0.01 at the driving frequency. Use the first condition to solve for ζ. From equation (5.7),

$$T.R. = 2 = \left[\frac{1+4\zeta^2}{4\zeta^2}\right]^{1/2}$$

$$\zeta = 0.2887$$

At the frequency, $r = \dfrac{52.4}{\sqrt{k/1000}}$, so

$$T.R. = 0.01 = \left[\frac{1+\left[2\left(0.2887\right)r\right]^2}{\left(1-r^2\right)^2+\left[2\left(0.2887\right)r\right]^2}\right]$$

$$r = 57.78 = \frac{52.4}{\sqrt{k/1000}}$$

$$k = 822.6 \text{ N/m}$$

Also,

$$c = 2\zeta\sqrt{km} = 523.6 \text{ kg/s}$$

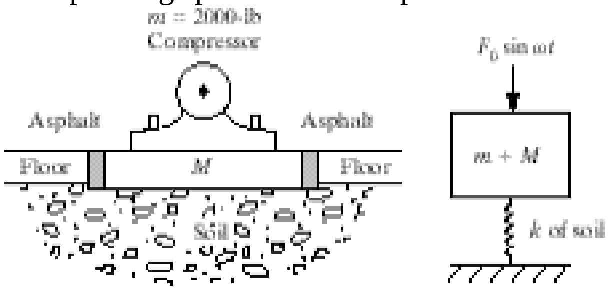
5.19 A rotating machine weighing 4000 lb has an operating speed of 2000 rpm. It is desired to reduce the amplitude of the transmitted force by 80% using isolation pads. Calculate the stiffness required of the isolation pads to accomplish this design goal.

Solution:

Using Figure 5.9, the lines of 2000 rpm and 80% reduction meet at $\delta_s = 0.053$ in. The spring stiffness should be

$$k = \frac{mg}{\delta_s} = \frac{4000 \text{ lb}}{0.053 \text{ in}} = 75{,}472 \text{ lb/in}$$

5.20 The mass of a system may be changed to improve the vibration isolation characteristics. Such isolation systems often occur when mounting heavy compressors on factory floors. This is illustrated in Figure P5.20. In this case the soil provides the stiffness of the isolation system (damping is neglected) and the design problem becomes that of choosing the value of the mass of the concrete block/compressor system. Assume that the stiffness of the soil is about $k = 2.0 \times 10^7$ N/m and design the size of the concrete block (i.e., choose m) such that the isolation system reduces the transmitted force by 75%. Assume that the density of concrete is $\rho = 23{,}000$ N/m^3. The surface area of the cement block is 4 m^2. The steady-state operating speed of the compressor is 1800 rpm.



Solution:

Using Figure 5.9, the lines of 75% reduction and 1800 rpm cross at δ_s = 0.053 in = 0.1346 cm. Thus the weight of the block should be

$$W_T = (m + M)g = k\delta_s = 2.0 \times 10^7 (0.1346 \times 10^{-2}) = 26{,}924 \text{ N}$$

The compressor weights mg = (2000 lb)(4.448222 N/lb) = 8896.4 N. The concrete block should weight $W = W_T - 8896.4 = 18{,}028$ N. The volume of the block needs to be

$$V = \frac{W}{\rho} = \frac{18{,}028}{23{,}000} = 0.7838 \text{ m}^2$$

Assume the surface area is part exposed to the surface. Let the top be a meters on each side (square) and b meters deep. The volume and surface area equations are

$$A = 4\text{m}^2 = a^2$$

$$V = 0.7838 \text{ m}^3 = a^2 b$$

Solving for a and b yields

$$a = 2 \text{ m}$$

$$b = 0.196 \text{ m}$$

5.21 The instrument board of an aircraft is mounted on an isolation pad to protect the panel from vibration of the aircraft frame. The dominant vibration in the aircraft is measured to be at 2000 rpm. Because of size limitation in the aircraft's cabin, the isolators are only allowed to deflect 1/8 in. Find the percent of motion transmitted to the instrument pane if it weights 50 lb.

Solution:

From equation (2.71), with negligible damping,

$$\frac{X}{Y} = \frac{1}{\sqrt{\left(1-r^2\right)^2}}$$

This is the same as the equation that yields Figure 5.9. The lines of 2000 rpm and δ_s = 0.125 in meet at 93%. So only 7% of the plane's motion is transmitted to the instrument panel.

5.22 Design a base isolation system for an electronic module of mass 5 kg so that only 10% of the displacement of the base is transmitted into displacement of the module at 50 Hz. What will the transmissibility be if the frequency of the base motion changes to 100 Hz? What if it reduces to 25 Hz?

Solution: Using Figure 5.9, the lines of 90% reduction and ω = (50 Hz)(60) = 3000 rpm meet at δ_s = 0.042 in = 0.1067 cm. The spring stiffness is then

$$k = \frac{mg}{\delta_s} = \frac{(5)(9.81)}{0.001067} = 45{,}979 \text{ N/m}$$

The natural frequency is $\omega = \sqrt{k/m}$ = 95.89 rad/s.

At ω = 100 Hz, $r = \dfrac{100(2\pi)}{95.89}$ = 6.552, so the transmissibility ratio is

$$T.R. = \frac{1}{r^2 - 1} = 0.0238$$

At ω = 25 Hz, $r = \dfrac{100(2\pi)}{95.89}$ = 1.638, so the transmissibility ratio is

$$T.R. = \frac{1}{r^2 - 1} = 0.594$$

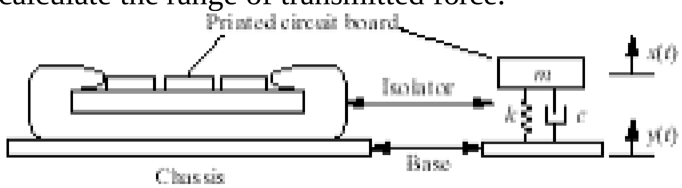
5.23 Redesign the system of Problem 5.22 such that the smallest transmissibility ratio possible is obtained over the range 50 to 75 Hz.

Solution:

If the deflection is limited, say 0.1 in, then the smallest transmissibility ratio in the frequency range of 50 to 75 Hz (3000 to 4500 rpm) would be 0.04 (96% reduction). The stiffness would be

$$k = \frac{mg}{\delta_s} = \frac{(5)(9.81)}{(0.1)(2.54)(0.01)} = 19{,}311 \text{ N/m}$$

5.24 A 2-kg printed circuit board for a computer is to be isolated from external vibration of frequency 3 rad/s at a maximum amplitude of 1 mm, as illustrated in Figure P5.24. Design an undamped isolator such that the transmitted displacement is 10% of the base motion. Also calculate the range of transmitted force.



Solution:

Using Figure 5.9, the lines of 90% reduction and $\omega = 3(2\pi)(60) = 1131$ rpm meet at δ_s = 0.3 in = 0.762 cm. The stiffness is

$$k = \frac{mg}{\delta_s} = \frac{(2)(9.81)}{0.00762} = 2574.8 \text{ N/m}$$

From Window 5.1, the transmitted force would be

$$F_T = kYr^2\left(\frac{1}{1-r^2}\right)$$

Since $Y = 0.001$ m and $r = \dfrac{3}{\sqrt{2574.8/2}} = 0.08361$

$$F_T = 0.0181 \text{ N}$$

5.25 Change the design of the isolator of Problem 5.24 by using a damping material with damping value ζ chosen such that the maximum T.R. at resonance is 2.

Solution:

At resonance, r = 1 and T.R. = 2, so

$$2 = \left[\frac{1+4\zeta^2}{4\zeta^2}\right]^{1/2}$$

Solving for ζ yields ζ = 0.2887. Also T.R. = 0.01 at ω = 3 rad/s, so

$$0.01 = \left[\frac{1+0.3333r^2}{\left(1-r^2\right)^2+0.3333r^2}\right]$$
$$r = 6.134$$

Solving for k,

$$k = \frac{m\omega^2}{r^2} = \frac{(2)(3)^2}{6.134^2} = 0478 \text{ N/m}$$

The damping constant is

$$c = 2\zeta\sqrt{km} = 0.565 \text{ kg/s}$$

5.26 Calculate the damping ratio required to limit the displacement transmissibility to 4 at resonance for any damped isolation system.

Solution:

At resonance r = 1, so

$$T.R. = 4 = \left[\frac{1+4\zeta^2}{4\zeta^2}\right]^{1/2}$$
$$\zeta = 0.129$$

Problems and Solutions Section 5.3 (5.27 through 5.36)

5.27 A motor is mounted on a platform that is observed to vibrate excessively at an operating speed of 6000 rpm producing a 250-N force. Design a vibration absorber (undamped) to add to the platform. Note that in this case the absorber mass will only be allowed to move 2 mm because of geometric and size constraints.

Solution:

The amplitude of the absorber mass can be found from equation (5.22) and used to solve for k_a:

$$X_a = 0.002 \text{ m} = \frac{F_0}{k_a} = \frac{250}{k_a}$$

$$k_a = 125{,}000 \text{ N/m}$$

From equation (5.21),

$$\omega^2 = \frac{k_a}{m_a}$$

$$m_a = \frac{k_a}{\omega^2} = \frac{125{,}000}{\left[6000\left(\frac{2\pi}{60}\right)\right]^2} = 0.317 \text{ kg}$$

5.28 Consider an undamped vibration absorber with β = 1 and μ = 0.2. Determine the operating range of frequencies for which $\left|Xk / F_0\right| \leq 0.5$.

Solution:

From equation (5.24), with β = $\frac{\omega_a}{\omega_p} = 1$(i.e., $\omega_a = \omega_p$) and $\mu = 0.2$,

$$\frac{Xk}{F_0} = \frac{1-\left(\frac{\omega}{\omega_a}\right)}{\left[1+0.2(1)^2-\left(\frac{\omega}{\omega_a}\right)^2\right]\left[1-\left(\frac{\omega}{\omega_a}\right)^2\right]-0.2(1)^2}$$

$$= \frac{1-\left(\frac{\omega}{\omega_a}\right)^2}{\left(\frac{\omega}{\omega_a}\right)^4-2.2\left(\frac{\omega}{\omega_a}\right)^2+1}$$

For $\frac{Xk}{F_0}$ = 0.5, this yields

$$0.5\left(\frac{\omega}{\omega_a}\right)^4-0.1\left(\frac{\omega}{\omega_a}\right)^2-0.5=0$$

Solving for the physical solution gives

$$\left(\frac{\omega}{\omega_a}\right)=1.051$$

Solving for $\left(\frac{\omega}{\omega_a}\right)$ gives

$$\left(\frac{\omega}{\omega_a}\right)=0.955,\ 1.813$$

Comparing this to the sketch in Figure 5.15, the values for which $\left|\frac{Xk}{F_0}\right| \leq 5$ are

$$0.955\omega_a \leq \omega \leq 1.051\omega_a \quad \text{and} \quad \omega \geq 1.813\omega_a$$

5.29 Consider an internal combustion engine that is modeled as a lumped inertia attached to ground through a spring. Assuming that the system has a measured resonance of 100 rad/s, design an absorber so that the amplitude is 0.01 m for a (measured) force input of 10^2 N.

Solution:

The amplitude of the absorber mass can be found from equation (5.22) and used to solve for k_a:

$$X_a = 0.01\text{m} = \frac{F_0}{k_a} = \frac{100}{k_a}$$

$$k_a = 10{,}000 \text{ N/m}$$

Choose ω = 2ω_n = 200 rad/s. From equation (5.21),

$$m_a = \frac{k_a}{\omega^2} = \frac{10{,}000}{200^2} = 0.25 \text{ kg}$$

5.30 A small rotating machine weighing 50 lb runs at a constant speed of 6000 rpm. The machine was installed in a building and it was discovered that the system was operating at resonance. Design a retrofit undamped absorber such that the nearest resonance is at least 20% away from the driving frequency.

Solution:

By observing Figure 5.15, the values of μ = 0.25 and β = 1 result in the combined system's natural frequencies being 28.1% above the driving frequency and 21.8% below the driving frequency (since $\beta = \frac{\omega_a}{\omega_p}$ = 1 and ω = ω_p). So the absorber should weigh

$$m_a = \mu m = (0.25)(50 \text{ lb}) = 12.5 \text{ lb}$$

and have stiffness

$$k_a = m_a\omega_a^2 = m_a\omega^2 = (12.5 \text{ lb})(4.448222 \text{ N/lb})\left(\frac{1}{9.81}\right)(6000)^2\left(\frac{2\pi}{60}\right)^2$$

$$k_a = 2.24\times10^6 \text{ N/m} = 12{,}800 \text{ lb/in}$$

5.31 A 3000-kg machine tool exhibits a large resonance at 120 Hz. The plant manager attaches an absorber to the machine of 600 kg tuned to 120 Hz. Calculate the range of frequencies at which the amplitude of the machine vibration is less with the absorber fitted than without the absorber.

Solution:

For $\frac{Xk}{F_0}$ = 1, equation (5.24) yields

$$\left[1+\mu\left(\frac{\omega_a}{\omega_p}\right)^2-\left(\frac{\omega}{\omega_a}\right)^2\right]\left[1-\left(\frac{\omega}{\omega_a}\right)^2\right]-\mu\left(\frac{\omega_a}{\omega_p}\right)^2=1-\left(\frac{\omega}{\omega_a}\right)^2$$

Since $\mu=\frac{m_a}{m}=\frac{600}{3000}=0.2,$ this becomes $\left(\frac{\omega}{\omega_a}\right)=0,\ 1.0954.$

For $\frac{Xk}{F_0}$ = -1, equation (5.24) yields

$$\left[1+\mu\left(\frac{\omega_a}{\omega_p}\right)^2-\left(\frac{\omega}{\omega_p}\right)^2\right]\left[1-\left(\frac{\omega}{\omega_a}\right)^2\right]-\mu\left(\frac{\omega_a}{\omega_p}\right)^2=\left(\frac{\omega}{\omega_a}\right)^2-1$$

$$\left(\frac{\omega_a}{\omega_p}\right)^2\left(\frac{\omega}{\omega_a}\right)^4-\left[2+(\mu+1)\left(\frac{\omega_a}{\omega_p}\right)^2\right]\left(\frac{\omega}{\omega_a}\right)^2+2=0$$

Since $\omega_a=\omega_p$,

$$\left(\frac{\omega}{\omega_a}\right)^4-3.2\left(\frac{\omega}{\omega_a}\right)^2+2=0$$

$$\left(\frac{\omega}{\omega_a}\right)=0.9229,1.5324$$

The range of frequencies at which $\left|\frac{Xk}{F_0}\right|$ > 1 is

$$0<\omega<0.9229\omega_a \text{ and } 1.0954\omega_a<\omega<1.5324\omega_a$$

Since $\omega_a = \omega_p$,

$$0 < \omega < 695.8 \text{ rad/s and } 825.9 < \omega < 1155.4 \text{ rad/s}$$

5.32 A motor-generator set is designed with steady-state operating speed between 2000 and 4000 rpm. Unfortunately, due to an imbalance in the machine, a large violent vibration occurs at around 3000 rpm. An initial absorber design is implemented with a mass of 2 kg tuned to 3000 rpm. This, however, causes the combined system natural frequencies that occur at 2500 and 3000 rpm. Redesign the absorber so that $\omega_1 < 2000$ rpm and $\omega_2 >$ 4000 rpm, rendering the system safe for operation.

Solution: The mass of the primary system can be computed from equation (5.25). Since $\beta = \dfrac{\omega_a}{\omega_p} = 1$ and $\left(\dfrac{\omega_1}{\omega_a}\right)^2 = \left(\dfrac{2500}{3000}\right)^2 = 0.6944$, then

$$(1)^2(0.6944)^2 - \left[1 + (1)^2(1+\mu)\right](0.6944) + 1 = 0$$

$$\mu = 0.1344$$

$$m = \frac{m_a}{\mu} = \frac{2}{0.1344} = 14.876 \text{ kg}$$

By increasing μ to 0.55 and decreasing β to 0.89, the design goal can be achieved. The mass and stiffness of the absorber should be

$$m_a = \mu m = (0.55)(14.876) = 8.18 \text{ kg}$$

$$k_a = m_a\omega_a^2 = m_a\beta^2\omega_p^2 = (8.18)(0.89)^2\left[3000\left(\frac{2\pi}{60}\right)\right]^2 = 639{,}600 \text{ N/m}$$

5.33 A rotating machine is mounted on the floor of a building. Together, the mass of the machines and the floor is 2000 lb. The machine operates in steady state at 600 rpm and causes the floor of the building to shake. The floor-machine system can be modeled as a spring-mass system similar to the optical table of Figure 5.14. Design an undamped absorber system to correct this problem. Make sure you consider the bandwidth.

Solution: To minimize the transmitted force, let $\omega_a = \omega = 600$ rpm. Also, since the floor shakes at 600 rpm, it is assumed that $\omega_p = 600$ rpm so that $\beta = 1$. Using equation (5.26) with $\mu = 0.1$ yields

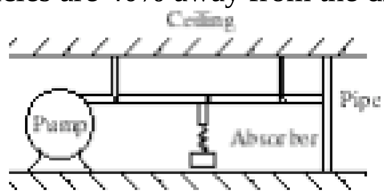
$$\frac{\omega_n}{\omega_a} = 0.8543,\ 1.1705$$

So the natural frequencies of the combined system are $\omega_1 = 512.6$ rpm and $\omega_2 = 702.3$ rpm. These are sufficiently enough away from 600 rpm to avoid problems. Therefore the mass and stiffness of the absorber are

$$m_a = \mu m = (0.1)(2000 \text{ lbm}) = 200 \text{ lbm}$$

$$k_a = m_a \omega_a^2 = (200 \text{ lbm})\left(\frac{\text{slug}}{32.1174 \text{ lbm}}\right)\left[600\left(\frac{2\pi}{60}\right)\right]^2 = 25{,}541 \text{ lb/ft}$$

5.34 A pipe carrying steam through a section of a factory vibrates violently when the driving pump hits a speed of 300 rpm (see Figure P5.34). In an attempt to design an absorber, a trial 9-kg absorber tuned to 300 rpm was attached. By changing the pump speed it was found that the pipe-absorber system has a resonance at 207 rpm. Redesign the absorber so that the natural frequencies are 40% away from the driving frequency.



Solution:

The driving frequency is 300 rpm. 40% above and below this frequency is 180 rpm and 420 rpm. This is the design goal.

The mass of the primary system can be computed from equation (5.25). Since $\beta = \frac{\omega_a}{\omega_p} = 1$ and $\left(\frac{\omega_1}{\omega_a}\right)^2 = \left(\frac{207}{300}\right)^2 = 0.4761,$ then

$$(1)^2(0.4761)^2 - \left[1+(1)^2(1+\mu)\right](0.4761)+1=0$$

$$\mu = 0.5765$$

$$m = \frac{m_a}{\mu} = \frac{9}{0.5765} = 15.611 \text{ kg}$$

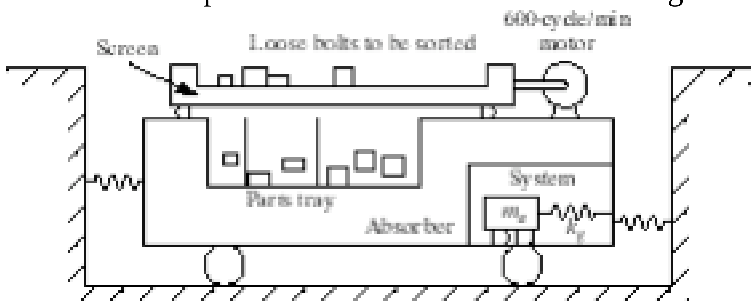
By increasing μ to 0.9 and decreasing β to 0.85, the design goal can be achieved. The mass and stiffness of the absorber should be

$$m_a = \mu m = (0.9)(15.611) = 14.05 \text{ kg}$$

$$\text{k}_\text{a} = m_a\omega_a^2 = m_a\beta^2\omega_p^2 = (14.05)(0.85)^2\left[300\left(\frac{2\pi}{60}\right)\right]^2 = 10{,}020 \text{ N/m}$$

Note that μ is very large, which means a poor design.

5.35 A machine sorts bolts according to their size by moving a screen back and forth using a primary system of 2500 kg with a natural frequency of 400 cycle/min. Design a vibration absorber so that the machine-absorber system has natural frequencies below 160 cycles/min and above 320 rpm. The machine is illustrated in Figure P5.35.



Solution:

Using Equation (5.26), and choose (by trial and error) $\beta = 0.4$ and $\mu = 0.01$, the design goal of $\omega_1 < 160$ rpm and $\omega_2 > 320$ rpm can be achieved. The actual values are $\omega_1 = 159.8$ rpm and $\omega_2 = 400.4$ rpm. The mass and stiffness of the absorber should be

$$m_a = \mu m = (0.01)(2500) = 25 \text{ kg}$$

$$k_a = m_a \omega_a^2 = m_a \beta^2 \omega_{[}^2 = (25)(0.2)^2 \left[400\left(\frac{2\pi}{60}\right)\right]^2 = 1754.6 \text{ N/m}$$

5.36 A dynamic absorber is designed with μ = 1/4 and $\omega_a = \omega_p$. Calculate the frequency range for which the ratio $\left| Xk / F_0 \right| < 1$.

Solution:

From Equation (5.24), with $\beta = \frac{\omega_a}{\omega_p}$ = 1 and μ = 0.25,

$$\frac{Xk}{F_0} = \frac{1-\left(\frac{\omega}{\omega_a}\right)^2}{\left[1+0.25\left(1^2\right)-\left(\frac{\omega}{\omega_a}\right)^2\right]\left[1-\left(\frac{\omega}{\omega_a}\right)^2\right]-0.25\left(1\right)^2}$$

$$= \frac{1-\left(\frac{\omega}{\omega_a}\right)^2}{\left(\frac{\omega}{\omega_a}\right)^4 - 2.25\left(\frac{\omega}{\omega_a}\right)^2 + 1}$$

For $\frac{Xk}{F_0} = 1$, this yields

$$\left(\frac{\omega}{\omega_a}\right)^4 - 1.25\left(\frac{\omega}{\omega_a}\right)^2 = 0$$

$$\left(\frac{\omega}{\omega_a}\right) = 0,\ 1.118$$

For $\frac{Xk}{F_0}$ = 1, this yields

$$-\left(\frac{\omega}{\omega_a}\right)^4 + 3.25\left(\frac{\omega}{\omega_a}\right)^2 - 2 = 0$$

$$\left(\frac{\omega}{\omega_a} = 0.9081,\ 1.557\right)$$

Comparing this to the sketch in Figure 5.15, the values for which $\left|\frac{Xk}{F_0}\right| < 1$ are

$$0.9081\omega_a < \omega < 1.118\omega_a \text{ and } \omega > 1.557\omega_a$$

Problems and Solutions Section 5.4 (5.37 through 5.52)

5.37 A machine, largely made of aluminum, is modeled as a simple mass (of 100 kg) attached to ground through a spring of 2000 N/m. The machine is subjected to a 100-N harmonic force at 20 rad/s. Design an undamped tuned absorber system (i.e., calculate m_a and k_a) so that the machine is stationary at steady state. Aluminum, of course, is not completely undamped and has internal damping that gives rise to a damping ratio of about $\zeta = 0.001$. Similarly, the steel spring for the absorber gives rise to internal damping of about $\zeta_a = 0.0015$. Calculate how much this spoils the absorber design by determining the magnitude X using equation (5.32).

Solution:

From equation (5.21), the steady-state vibration will be zero when

$$\omega^2 = \frac{k_a}{m_a}$$

Choosing $\mu = 0.2$ yields

$$m_a = \mu m = (0.2)(100) = 20 \text{ kg}$$
$$\mathrm{k_a} = m_a \omega_a^2 = (20)(20)^2 = 8000 \text{ N/m}$$

With damping of $\zeta = 0.001$ and $\zeta_a = 0.0015$, the values of c and c_a are

$$c = 2\zeta\sqrt{km} = 2(0.001)\sqrt{(2000)(100)} = 0.894 \text{ kg/s}$$
$$c_a = 2\zeta_a\sqrt{k_a m_a} = 2(0.0015)\sqrt{(8000)(20)} = 1.2 \text{ kg/s}$$

From equation (5.32),

$$X = \frac{(k_a - m_a\omega^2)F_0 + c_a\omega F_0 j}{\det(K - \omega^2 M + \omega jC)}$$

Since

$$M = \begin{bmatrix} 100 & 0 \\ 0 & 20 \end{bmatrix} \quad C = \begin{bmatrix} 2.0944 & -1.2 \\ -1.2 & 1.2 \end{bmatrix} \quad K = \begin{bmatrix} 10{,}000 & -8000 \\ -8000 & 8000 \end{bmatrix}$$

the denominator is $-6.4\times10^7 - 1.104\times10^6 j$, so the value of X is

$$X = \frac{\left(k_a m_a \omega^2\right)\left(F_0 + c_a \omega F_0 j\right)}{\det\left(K - \omega^2 M + \omega jC\right)}$$

Using Window 5.4, the magnitude is

$$\left|X\right| = 3.75 \times 10^{-5} \text{ m}$$

This is a very small displacement, so the addition of internal damping will not affect the design very much.

5.38 Plot the magnitude of the primary system calculated in Problem 5.37 with and without the internal damping. Discuss how the damping affects the bandwidth and performance of the absorber designed without knowledge of internal damping.

Solution: From Problem 5.37, the values are

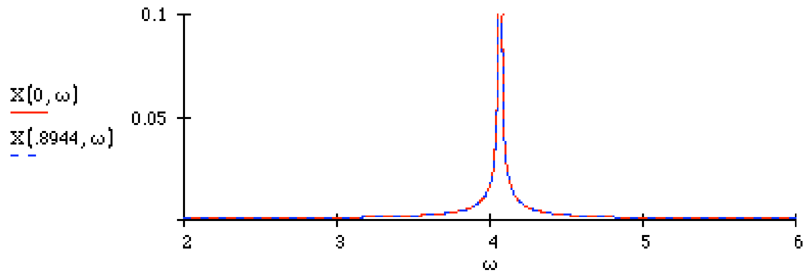
$$\begin{array}{ll} m = 100 \text{ kg} & m_a = 20 \text{ kg} \\ c = 0.8944 \text{ kg/s} & c_a = 1.2 \text{ kg/s} \\ k = 2000 \text{ N/m} & k_a = 8000 \text{ N/m} \\ F_0 = 100 \text{ N} & \omega = 20 \text{ rad/s} \end{array}$$

Using Equation (5.32), the magnitude of X is plotted versus ω with and without the internal damping (c). Note that X is reduced when $X < F_0/k = 0.05$ m and magnified when $X > 0.05$ m. The plots of the two values of X show that there is no observable difference when internal damping is added. In this case, knowledge of internal damping is not necessary.

$m := 100$ $ma := 20$ $ca := 1.2$ $k := 2000$

$ka := 8000$ $F0 := 100$

$$X(c,\omega) := \sqrt{\frac{\left(ka - ma\cdot\omega^2\right)\cdot F0^2 + \left[ca\cdot\left(\omega\cdot F0\right)\right]^2}{\left[\left(k - m\cdot\omega^2\right)\cdot\left(ka - ma\cdot\omega^2\right) - \left(ma\cdot ka + ca\cdot c\right)\cdot\omega^2\right]^2 + \left[ka\cdot c + k\,ca - \left[ca\cdot(m + ma) + c\cdot ma\right]\cdot\omega^2\right]^2\cdot\omega^2}}$$



5.39 Derive Equation (5.35) for the damped absorber from Eqs. (5.34) and (5.32) along with Window 5.4. Also derive the nondimensional form of Equation (5.37) from Equation (5.35). Note the definition of ζ given in Equation (5.36) is not the same as the ζ values used in Problems 5.37 and 5.38.

Solution:

Substituting Equation (5.34) into the denominator of Equation (5.32) yields

$$\frac{X}{F_0} = \frac{\left(k_a - m_a\omega^2\right) + c_a\omega j}{\left[\left(-m\omega^2 + k\right)\left(-m_a\omega^2 + k_a\right)\right] + \left[\left(k - \left(m + m_a\right)\omega^2\right)c_a\omega\right]j}$$

Referring to Window 5.4, the value of $\left|\frac{X}{F_0}\right|$ can be found by noting that

$$\begin{aligned} A_1 &= k_a - m_a\omega \\ B_1 &= c_a\omega \\ A_2 &= \left(-m\omega^2 + k\right)\left(-m_a\omega^2 + k_a\right) - m_a k_a\omega^2 \\ B_2 &= \left(k - \left(m + m_a\right)\omega^2\right)c_a\omega \end{aligned}$$

Since

$$\left|\frac{X}{F_0}\right| = \sqrt{\frac{A_1^2 + B_1^2}{A_2^2 + B_2^2}}$$

then

$$\frac{X^2}{F_0^2} = \frac{\left(k_a - m_a\omega^2\right)^2 + c_a^2\omega^2}{\left[\left(-m\omega^2 + k\right)\left(-m_a\omega^2 + k_a\right) - m_a k_a\omega^2\right]^2 + \left[k - \left(m + m_a\right)\omega^2\right]^2 c_a^2\omega 2}$$

which is Equation (5.35)

To derive Equation (5.37), substitute $c_a = 2\zeta m_a\omega_p, k_a = m_a\omega_a^2$, and $m_a = \mu m$, then multiply by k^2 to get

$$\frac{X^2k^2}{F_0^2} = \frac{k^2\left(\omega_a^2 - \omega^2\right)^2 + 4\zeta^2\omega_p^2\omega_{dr}^2 k^2}{\left[\left(k - m\omega^2\right)\left(\omega_a^2 - \omega^2\right) - \mu m_a^2\omega^2\right]^2 + \left[k - \left(1 - \mu\right)m\omega^2\right]^2\left(4\right)\zeta^2\omega_p^2\omega^2}$$

Substituting $k = m\omega_p^2, \omega = r\omega_p$, and $\omega_a = \beta\omega_p$ yields

$$\frac{X^2k^2}{F_0^2} = \frac{m^2\omega_p^4\left(\beta^2\omega_p^2 - r^2\omega_p^2\right) + 4\zeta\ \omega_p^2\omega_{dr}^2 k^2}{\left[\left(\omega_p^2 - r^2\omega_p^2\right)\left(\beta^2\omega_p^2 - r^2\omega_p^2\right)m - \mu m\beta^2 r^2\omega_p^4\right]^2 + \left[m\omega_p^2 - \left(1 - \mu\right)mr^2\omega_p^2\right]^2\left(4\right)\zeta^2 r^2\omega_p^2}$$

Canceling m^2 and ω_p^8 yields

$$\frac{X^2k^2}{F_0^2} = \frac{\left(\beta^2 - r^2\right)^2 + \left(2\zeta r\right)^2}{\left[\left(1 - r^2\right)\left(\beta^2 - r^2\right) - \mu r^2\beta^2\right]^2 + \left(2\zeta r\right)^2\left(1 - r^2 - \mu r^2\right)^2}$$

Rearranging and taking the square root gives the form of Equation (5.37):

$$\frac{Xk}{F_0} = \sqrt{\frac{\left(2\zeta r\right)^2 + \left(r^2 - \beta^2\right)^2}{\left(2\zeta r\right)^2\left(r^2 - 1 + \mu r^2\right)^2 + \left[\mu r^2\beta^2 - \left(r^2 - 1\right)\left(r^2 - \beta^2\right)\right]^2}}$$

5.40 (Project) If you have a three-dimensional graphics routine available, plot Equation (5.37) [i.e., plot (*X/Δ*) versus both *r* and ζ for 0 < ζ < 1 and 0 < *r* < 3, and a fixed μ and β.] Discuss the nature of your results. Does this plot indicate any obvious design choices? How does it compare to the information obtained by the series of plots given in Figures 5.19 to 5.21? (Three-dimensional plots such as these are becoming commonplace and have not yet been taken advantage of fully in vibration absorber design.)

Solution: To compare to Figure 5.18, the values μ = 0.25 and β = 0.8 in Equation (5.37) yield

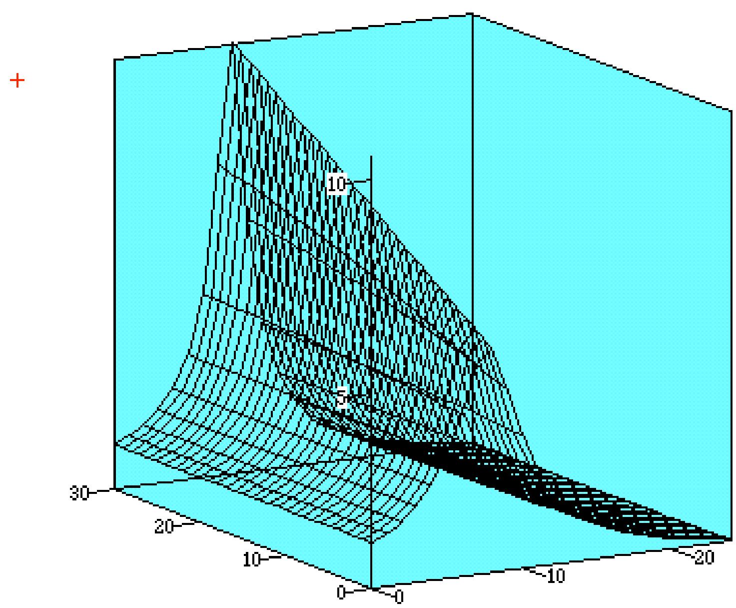
$$\frac{X}{\Delta} = \sqrt{\frac{(2\zeta r)^2 + (r^2 - 0.64)^2}{(2\zeta r)^2 (1.25r^2 - 1)^2 + \left[0.16r^2 - (r^2 - 1)(r^2 - 0.64)\right]^2}}$$

This is plotted for 0.5 < *r* < 2 and 0.5 < ζ < 1. A Mathcad plot is given.

$N := 30 \quad i := 0..N\cdot.8 \quad j := 0..N\cdot 1 \quad r_i := 0.5 + i\cdot 0.05 \quad \zeta_j := 0.5 + j\cdot 0.015$

$$X(r,\zeta) := \sqrt{\frac{(2\cdot\zeta\cdot r)^2 + (r^2 - 0.64)^2}{(2\cdot r\cdot\zeta)^2\cdot(1.25\cdot r^2 - 1^2)^2 + \left[0.16\cdot r^2 - (r^2 - 1)\cdot(r^2 - 0.64)\right]^2}}$$

$M_{[i,j]} := X(r_i, \zeta_j)$



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This supplies much more information than two-dimensional plots.

5.41 Repeat Problem 5.40 by plotting $\left|X/\Delta\right|$ versus r and β for a fixed ζ and μ.

Solution: Using Equation (5.37) with μ = 0.25 and ζ 0.1 yields

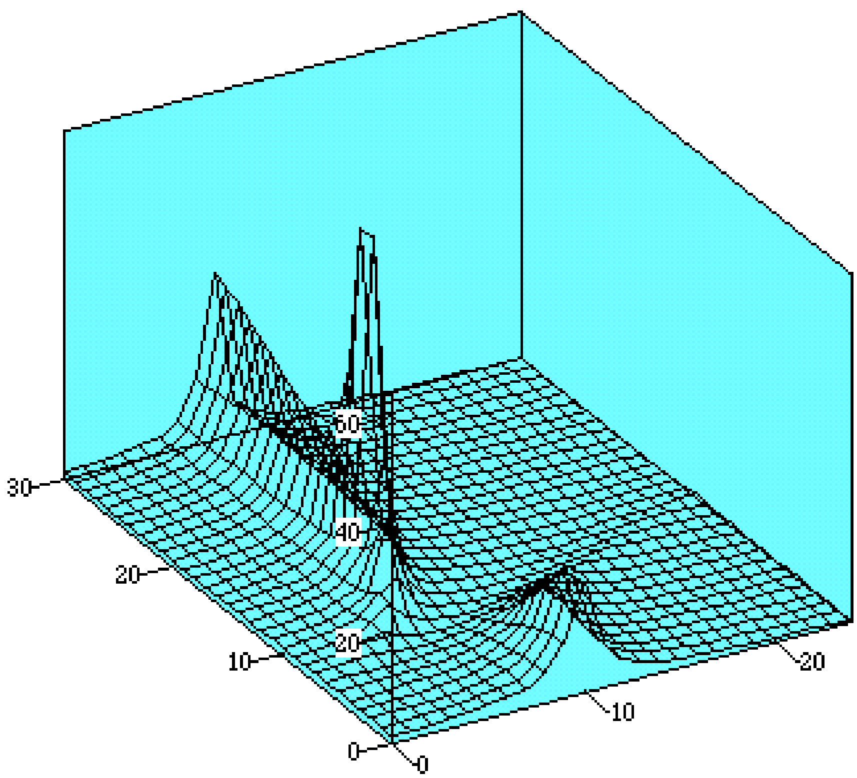
$$\frac{X}{\Delta} = \sqrt{\frac{0.04r^2 + \left(r^2 - \beta^2\right)^2}{0.04r^2\left(1.25r^2 - 1\right)^2 + \left[0.25r^2\beta^2 - \left(r^2 - 1\right)\left(r^2 - \beta^2\right)\right]^2}}$$

This is plotted for 0.5 < r < 1.25 and 0 < β < 3.

$N := 30 \quad i := 0..N\cdot .8 \quad j := 0..N\cdot 1 \quad r_i := 0.5 + i\cdot 0.05 \quad \beta_j := 0 + j\cdot 0.1$

$$X(r,\beta) := \sqrt{\frac{0.04\cdot r^2 + \left(r^2 - \beta^2\right)^2}{0.04\cdot r^2\cdot\left(1.25\cdot r^2 - 1^2\right)^2 + \left[0.25\cdot r^2\cdot\beta^2 - \left(r^2 - 1\right)\cdot\left(r^2 - \beta^2\right)\right]^2}}$$

$M_{[i,j]} := X\left(r_i, \beta_j\right)$



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5.42 (Project) The full damped vibration absorber equations (5.32) and (5.33) have not historically been used in absorber design because of the complicated nature of the complex arithmetic involved. However, if you have a symbolic manipulation code available to you, calculate an expression for the magnitude *X* by using the code to calculate the magnitude and phase of Equation (5.32). Apply your results to the absorber design indicated in Problem 5.37 by using m_a, k_a and ζ_a as design variables (i.e., design the absorber).

Solution:

Equation (5.32):

$$X = \frac{\left(k_a - m_a\omega^2\right)F_0 + c_a\omega F_0}{\det\left(K - \omega^2 M + \omega jC\right)}$$

where *M*, *C* and *K* are defined above Equation (5.32).

Using Equation (5.34) for the denominator, then calculating the magnitude yields

$$|X| = \sqrt{\frac{\left(k_a - m_a\omega^2\right)F_0^2 + c_a^2\omega^2F_0^2}{\left[\left(k - m\omega^2\right)\left(k_a - m_a\omega^2\right) - \left(m_ak_a + c_ac\right)\omega^2\right]^2 + \left[k_ac + kc_a - \left(c_a\left(m + m_a\right) + cm_a\right)\omega^2\right]^2\omega^2}}$$

The phase is

$$\phi = \tan^{-1}\left(\frac{\text{Im}}{\text{Re}}\right)$$

where the imaginary part, denoted *Im*, is

$$\text{Im} = -ck_a^2l + \left(2k_am_a - 2k_akm_a - k_a^2m_a\right)\omega^2$$

and the real part, denoted *Re*, is

$$\text{Re} = k_a^2k + \left(c_a^2 - k_a^2m - 2k_akm_a - k_a^2m_a\right)\omega^2$$
$$+\left(\left(k + k_a\right)m_a^2 + 2k_amm_a - c_a^2\left(m + m_a\right)\right)\omega^4 - mm_a^2\omega^6$$

From Problem 5.37 and its solution, the values are

m = 100 kg	m_a = 20 kg
c = 0.8944 kg/s	c_a = 1.2 kg/s
k = 2000 N/m	k_a = 8000 N/m
F_0 = 100 N	ω = 20 rad/s

Substituting these values into the magnitude equation yields

$$|X| = 3.75\times10^{-5}\text{ m}$$

This is the same result as given in Problem 5.37.

5.43 A machine of mass 200 kg is driven harmonically by a 100-N force at 10 rad/s. The stiffness of the machine is 20,000 N/m. Design a broadband vibration absorber [i.e., Equation (5.37)] to limit the machine's motion as much as possible over the frequency range 8 to 12 rad/s. Note that other physical constraints limit the added absorber mass to be at most 50 kg.

Solution:

Since $\omega_p = \sqrt{\dfrac{k}{m}}$ = 10 rad/s, then r ranges from

$$\frac{8}{10} \le r \le \frac{12}{10}$$

$$0.8 \le r \le 1.2$$

By observing Figure 5.21, the values of $\mu = 0.25$, $\beta = 0.8$, and $\zeta = 0.27$ yield a reasonable solution for the required range of r. So the values of m_a, c_a, and k_a are

$m_a = \mu m = (0.25)(200) = 50$ kg
$c_a = 2\zeta m_a \omega_a = 2(0.27)(50)(10) = 270$ kg/s
$k_a = m_a \omega_a \beta^2 \omega_p^2 = (50)(10)(0.8)^2(10)^2 = 32000$ N/m

Note that an extensive optimization could have been used to solve for μ, β, and ζ, but this is not covered until section 5.5.

5.44 Often absorber designs are afterthoughts such as indicated in example 5.3.1. Add a damper to the absorber design of Figure 5.17 to increase the useful bandwidth of operation of the absorber system in the event the driving frequency drifts beyond the range indicated in Example 5.3.2.

Solution:

From Examples 5.3.1 and 5.3.2,

$$m = 73.16 \text{ kg} \quad m_a = 18.29 \text{ kg}$$
$$k = 2600 \text{ N/m} \quad k_a = 6500 \text{ N/m}$$

$$7.4059 < \omega < 21.0821 \text{ rad/s}$$

The values μ and β are

$$\mu = \frac{m_a}{m} = 0.25$$

$$\beta = \frac{\omega_a}{\omega_p} = \frac{\sqrt{k_a / m_a}}{\sqrt{k / m}} = 3.1623$$

Choosing ζ = 0.2 (by trial and error) will allow ω to go beyond 21.0821 rad/s without $\frac{X_k}{F_0}$ going above 1. However, it will not prevent $\frac{Xk}{F_0}$ from going above 1 when ω < 7.4089 rad/s. The value of c_a is

$$c_a = 2\zeta m_a \omega_p = 2(0.2)(18.29)\sqrt{\frac{2600}{73.16}} = 43.61 \text{ kg/s}$$

5.45 Again consider the absorber design of Example 5.3.1. If the absorber spring is made of aluminum and introduces a damping ratio of $\zeta = 0.001$, calculate the effect of this on the deflection of the saw (primary system) with the design given in Example 5.3.1.

Solution:

From Examples 5.3.1 and 5.3.2,

$$X = \frac{\left(k_a - m_a\omega^2\right)F_0 + c_a\omega F_0 j}{\det\left(K - \omega^2 M + \omega jC\right)}$$

where $c_a = 2\zeta\sqrt{k_a m_a} = 2\left(0.001\right)\sqrt{\left(6500\right)\left(18.29\right)} = 0.6896$ kg/s

Since

$$M = \begin{bmatrix} 73.16 & 0 \\ 0 & 18.29 \end{bmatrix} \quad C = \begin{bmatrix} 0.6896 & -0.6896 \\ -0.6896 & 0.6896 \end{bmatrix} \quad K = \begin{bmatrix} 9100 & -6500 \\ -6500 & 6500 \end{bmatrix}$$

The denominator is $-1.4131\times10^7-12{,}363j$ when $\omega = 7.4089$ rad/s,

$$\left|X_1\right| = 0.00499 \text{ m}$$

and when $\omega = 21.0821$ rad/s,

$$\left|X_2\right| = 0.00512 \text{ m}$$

The nondimensional values become

$$\left|\frac{X_j k}{F_0}\right| = 0.999$$

$$\left|\frac{X_2 k}{F_0}\right| = 1.023$$

There is very little effect on the saw deflection since the values of $\left|\frac{Xk}{F_0}\right|$ are still approximately 1 at the endpoints of the driving frequency range.

5.46 Consider the undamped primary system with a viscous absorber as modeled in Figure 5.22 and the rotational counterpart of Figure 5.23. Calculate the magnification factor $\left|Xk / M_O\right|$ for a 400 kg compressor having a natural frequency of 16.2 Hz if driven at resonance, for an absorber system defined by μ = 0.133 and ζ = 0.025.

Solution:

From Eqs. (5.39), with μ = 0.133, ζ = 0.025, and *r* = 1:

$$\frac{Xk}{M_0} = \sqrt{\frac{4\zeta^2 + r^2}{4\zeta^2\left(r^2 + \mu r^2 - 1\right)^2 + \left(r^2 - 1\right)^2}} = 150.6$$

The design with ζ = 0.1 produces the smallest displacement.

5.47 Recalculate the magnification factor $\left|Xk / M_O\right|$ for the compressor of Problem 5.46 if the damping factor is changed to ζ = 0.1. Which absorber design produces the smallest displacement of the primary system ζ = 0.025 or ζ = 0.1?

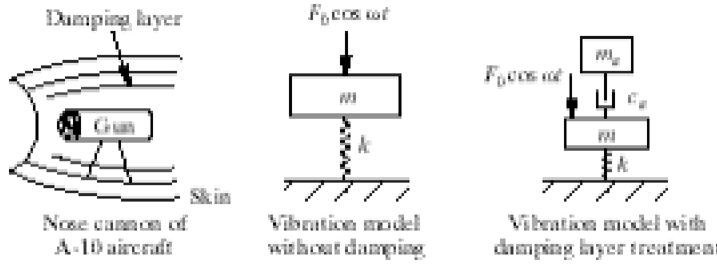
Solution:

From Equation (5.39), with μ = 0.133, ζ = 0.1, and *r* = 1:

$$\frac{Xk}{M_0} = \sqrt{\frac{4\zeta^2 + r^2}{4\zeta^2\left(r^2 + \mu r^2 - 1\right)^2 + \left(r^2 - 1\right)^2}} = 38.34$$

The design with ζ = 0.1 produces the smallest displacement.

5.48 Consider a one-degree-of-freedom model of the nose of an aircraft (A-10) as illustrated in Figure P5.48. The nose cracked under fatigue during battle conditions. This problem has been fixed by adding a viscoelastic material to the inside of the skin to act as a damped vibration absorber as illustrated in Figure P5.48. This fixed the problem and the vibration fatigue cracking disappeared in the A-10's after they were retrofitted with viscoelastic damping treatments. While the actual values remain classified, use the following data to calculate the required damping ratio given M = 100 kg, f_a = 3 Hz, and $k = 3.533 \times 10^6$ N/m, such that the maximum response is less than 0.25 mm. Note that since mass always needs to be limited in an aircraft, use μ = 0.1 in your design.



Solution:

From Equation (5.39), with μ = 0.1, and $r = \dfrac{30(2\pi)}{\sqrt{k/m}}$ = 1.885, and M_0 replaced by F_0,

$$\frac{Xk}{F_0} = \sqrt{\frac{4\zeta^2 + (1.885)^2}{4\zeta^2\left[(1.1)(1.885)^2 - 1\right]^2\left[(1.885)^2 - 1\right]^2(1.885)^2}}$$

$$= \sqrt{\frac{4\zeta^2 + 3.553}{33.834\zeta + 23.159}}$$

With no damping $\dfrac{Xk}{F_0} = 0.392$. This value must be reduced. Choose a "high" damping ratio of ζ = 0.7 so that

$$\frac{Xk}{F_0} = 0.372$$

The value of c_a is

$$c_a = 2\zeta\mu m\sqrt{\frac{k}{m}} = 2(0.7)(0.1)(100)\sqrt{\frac{10^6}{100}} = 1400 \text{ kg/s}$$

5.49 Plot an amplification curve such as Figure 5.24 by using Equation (5.39) for $\zeta = 0.02$ after several values of μ (μ = 0.1, 0.25, 0.5, and 1). Can you form any conclusions about the effect of the mass ratio on the response of the primary system? Note that as μ gets large $\left| Xk / M_o \right|$ gets very small. What is wrong with using very large μ in absorber design?

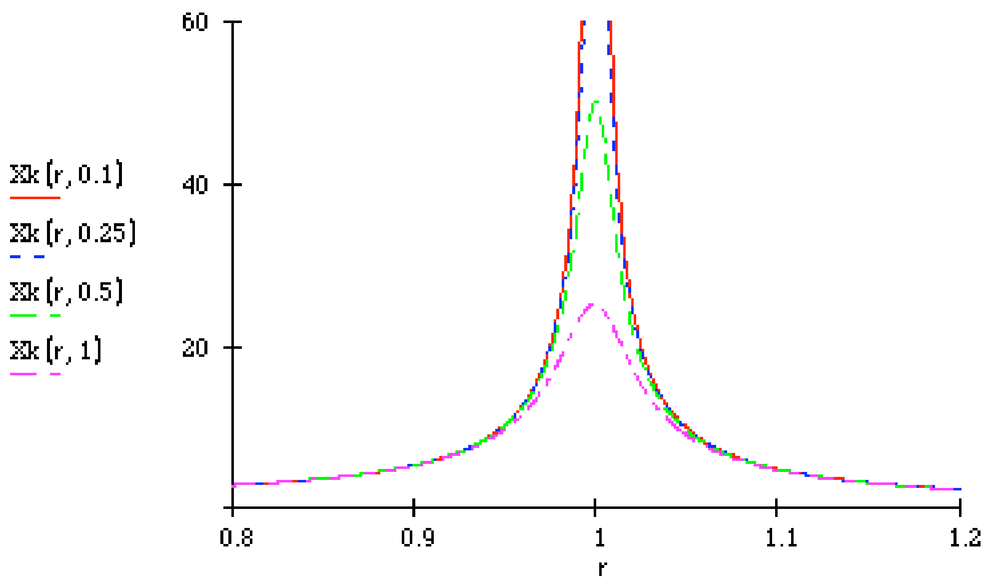
Solution:

From Equation (5.39), with $\zeta = 0.1$:

$$\frac{Xk}{M_0} = \sqrt{\frac{0.0016 + r^2}{0.0016\left(r^2 + \mu r^2 - 1\right)^2 + \left(r^2 - 1\right)^2 r^2}}$$

The following plot shows amplitude curves for μ = 0.1, 0.25, 0.5, and 1.

$$Xk(r, \mu) := \sqrt{\frac{0.0016 + r^2}{0.0016 \cdot \left(r^2 + \mu \cdot r^2 - 1\right)^2 + \left(r^2 - 1\right)^2 \cdot r^2}}$$



Note that as the mass ratio, μ, increases, the response of the primary system decreases, particularly in the region near resonance. A higher mass ratio, however, indicates a poor design (and can be quite expensive).

5.50 A Houdaille damper is to be designed for an automobile engine. Choose a value for ζ and μ if the magnification $\left|Xk/M_O\right|$ is to be limited to 4 at resonance. (One solution is μ = 1, ζ = 0.129.)

Solution:

From Equation (5.39), with $r = 1$:

$$\frac{Xk}{M_0} = \sqrt{\frac{4\zeta^2+1}{4\zeta^2\mu^2}}$$

For $\frac{Xk}{M_0} = 4$,

$$64\zeta^2\mu^2 = 4\zeta^2 + 1$$

If μ is limited to 0.3, then the value of ζ is

$$64\zeta^2\left(0.3\right)^2 = 4\zeta^2 + 1$$

$$\zeta = 0.754$$

5.51 Determine the amplitude of vibration for the various dampers of Problem 5.46 if ζ = 0.1, and F_0 = 100 N.

Solution:

From Problem 5.46,

$$k = m\omega_n^2 = \left(400\right)\left[\left(16.2\right)\left(2\pi\right)\right]^2 = 4.144\times10^6 \text{ N/m}$$

Also, μ = 0.1, r = 1, and F_0 = 100 N. So, from Equation (5.39), with M_0 replaced by F_0,

$$X = \frac{F_0}{k}\sqrt{\frac{4\zeta^2 + r^2}{4\zeta^2\left(r^2+\mu r^2-1\right)^2+\left(r^2-1\right)^2 r^2}} = 0.00123 \text{ m}$$

5.52 (Project) Use your knowledge of absorbers and isolation to design a device that will protect a mass from both shock inputs and harmonic inputs. It may help to have a particular device in mind such as the module discussed in Figure 5.6.

Solution:

One way to approach this problem would be to design an isolator to protect the mass from shock inputs, and an absorber to protect the mass from harmonic disturbances. An absorber would be particularly useful if the frequency of the harmonic disturbance(s) is well known.

This is a very general approach to such a problem, and solutions will vary greatly depending on the particular parameters involved in an actual system.

Problems and Solutions Section 5.5 (5.53 through 5.66)

5.53 Design a Houdaille damper for an engine modeled as having an inertia of 1.5 kg·m^2 and a natural frequency of 33 Hz. Choose a design such that the maximum dynamic magnification is less than 6:

$$\left|\frac{Xk}{M_0}\right| < 6$$

The design consists of choosing J_2 and c_a, the required optimal damping.

Solution:

From Equation (5.50),

$$\left(\frac{Xk}{M_0}\right)_{\max} = 1 + \frac{2}{\mu}$$

Since $\left|\frac{Xk}{M_0}\right| < 6,$ then

$$6 > 1 + \frac{2}{\mu}$$

$$\mu > 0.4$$

Choose μ = 0.4. From Equation (5.49), the optimal damping is

$$\zeta_{op} = \frac{1}{\sqrt{2(\mu+1)(\mu+2)}} = 0.3858$$

The values of J_2 and c_a are

$$J_2 = \mu J_1 = (0.4)(1.5 \text{ kg}\cdot\text{m}^2/\text{rad}) = 0.6 \text{ kg}\cdot\text{m}^2/rad$$
$$c_a = 2\zeta_{\text{op}} J_2 \omega_p = 2(0.3858)(0.6)(33)(2\pi) \; = 95.98 \text{ N}\cdot\text{m}\cdot\text{s/rad}$$

5.54 Consider the damped vibration absorber of equation (5.37) with β fixed at β = 1/2 and μ fixed at μ = 0.25. Calculate the value of ζ that minimizes $\left|X/\Delta\right|$. Plot this function for several values of 0 < ζ < 1 to check your design. If you cannot solve this analytically, consider using a three-dimensional plot of $\left|X/\Delta\right|$ versus r and ζ to determine your design.

Solution:

From equation (5.37), with β = 0.5 and μ = 0.25, let

$$f(r,\zeta) = \frac{X}{\Delta}\sqrt{\frac{4\zeta^2 r^2 + \left(r^2 - 0.25\right)^2}{4\zeta^2 r^2\left(1.25r^2 - 1\right)^2 + \left[0.065r^2 - \left(r^2 - 1\right)\left(r^2 - 0.25\right)\right]^2}}$$

From equataions (5.44) and (5.45), with $f = \dfrac{A^{1/2}}{B^{1/2}}$,

$$\frac{\partial f}{\partial \zeta} = 0$$

becomes

$$BdA - AdB$$

Since $B = 4\zeta^2 r^2\left(1.25r - 1\right)^2 + \left[0.0625r^2 - \left(r^2 - 1\right)\left(r^2 - 0.25\right)\right]^2$ and $A = 4\zeta^2 r^2 + \left(r^2 - 0.25\right)^2$, then

$$dA = \frac{\partial A}{\partial \zeta} = 8\zeta r^2$$

$$dB = \frac{\partial B}{\partial \zeta} = 8\zeta r^2\left(1.25r^2 - 1\right)^2$$

So,

$$\left\{4\zeta^2 r^2\left(1.25r^2 - 1\right)^2 + \left[0.0625r^2 - \left(4^2 - 1\right)\left(r^2 - 0.25\right)\right]^2\right\}\left(8\zeta r^2\right)$$

$$= \left\{4\zeta^2 r^2 + \left(r^2 - 0.25\right)^2\right\}\left(8\zeta r^2\right)\left(1.25r^2 - 1\right)^2$$

$$\left[0.0625r^2 - \left(r^2 - 1\right)\left(r^2 - 0.25\right)^2\right] = \left(r^2 - 0.25\right)^2\left(1.25r^2 - 1\right)^2$$

Taking the square root yields

$$0.625r^2 - \left(r^2 - 1\right)\left(r^2 - 0.25\right) = \pm\left(r^2 - 0.25\right)\left(1.25r^2 - 1\right)$$

Solving for r yields

$$r = 0.4896,\ 0.9628$$

Now take the derivative

$$\frac{\partial f}{\partial r} = 0$$

becomes

$$BdA = AdB$$

Since $B = 4\zeta^2 r^2\left(1.25r^2 - 1\right)^2 + \left[0.0625r^2 - \left(r^2 - 1\right)\left(r^2 - 0.25\right)\right]^2$ and $A = 4\zeta^2 r^2 + \left(r^2 - 0.25\right)^2$, then

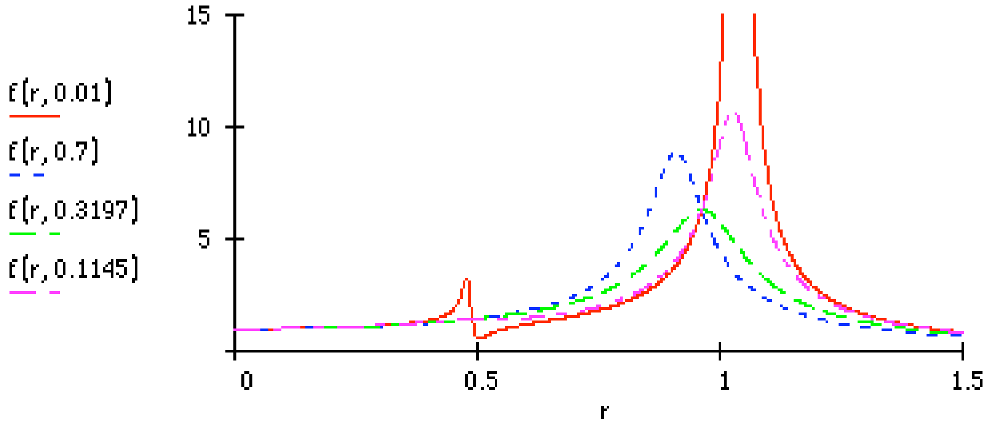
$$dA \equiv \frac{\partial A}{\partial \zeta} = 8\zeta^2 r + 2\left(r^2 - 0.25\right)\left(2r\right)$$
$$dB \equiv \frac{\partial B}{\partial \zeta} = 8\zeta^2 r\left(1.25r^2 - 1\right)^2 + 8\zeta^2 r^2\left(1.25r^2 - \left(2r\right)\right)\left(2.5r\right)$$
$$+2\left[0.0625r^2 - \left(4^2 - 1\right)\left(r^2 - 0.25\right)\right]\left[0.125r - \left(2r\right)\left(r^2 - 0.25\right) - \left(r^2 - 1\right)\left(2r\right)\right]$$

Solving $B\,dA = A\,dB$ for ζ yields

$$r = 0.4896 \rightarrow \zeta = 0.1145 \rightarrow \frac{X}{\delta_{st}} = 1.4279$$
$$r = 0.9628 \rightarrow \zeta = 0.3197 \rightarrow \frac{X}{\delta_{st}} = 6.3029$$

To determine the optimal damping ratio, make a plot of $\left|X / \Delta\right|$ versus r for ζ = 0.01, 0.1145, 0.3197, and 0.7.

$$f(r,\zeta) := \sqrt{\frac{(2\cdot\zeta\cdot r)^2 + (r^2 - 0.25)^2}{(2\cdot r\cdot\zeta)^2\cdot(1.25\cdot r^2 - 1^2)^2 + \left[0.0625\cdot r^2 - (r^2 - 1)\cdot(r^2 - 0.25)\right]^2}}$$



The value of ζ = 0.3197 yields the best overall response (i.e., the lowest maximum).

5.55 For a Houdaille damper with mass ratio μ = 0.25, calculate the optimum damping ratio and the frequency at which the damper is most effective at reducing the amplitude of vibration of the primary system.

Solution:

From equation (5.49), with μ = 0.25,

$$\zeta_{op} = \frac{1}{\sqrt{2(\mu+1)(\mu+2)}} = 0.422$$

From equation (5.48),

$$r = \sqrt{\frac{2}{2+\mu}} = 0.943$$

The damper would be most effective at $\omega = r\omega_n = 0.943\omega_n$, i.e., where the amplitude is greatest:

5.56 Consider again the system of Problem 5.53. If the damping ratio is changed to ζ = 0.1, what happens to $|Xk\ /\ M_0|$?

Solution:

If ζ_{op} = 0.1, the value of μ becomes

$$0.1 = \frac{1}{\sqrt{2(\mu+1)(\mu+2)}}$$

$$0.02\mu^2 + 0.06\mu = 0.96 = 0$$

$$\mu = -8.589,\ 5.589$$

Clearly μ = 5.589 is the physical solution. The maximum value of $\left|\frac{Xk}{M_0}\right|$ would be

$$\left(\frac{Xk}{M_0}\right)_{\max} = 1 + \frac{2}{\mu} = 1.358$$

which is less than 6 (the requirement of Problem 5.53). Note that the value of μ is extremely large.

5.57 Derive Equation (5.42) from Equation (5.35) and derive Equation (5.49) for the optimal damping ratio.

Solution:

Equation (5.37) is derived from Equation (5.35) in Problem 5.39.

Start with Equation (5.37):

$$\frac{Xk}{F_0} = \sqrt{\frac{(2\zeta r)^2 + (r^2 - \beta^2)^2}{(2\zeta r)^2(r^2 - 1 + \mu r^2)^2 + \left[\mu r^2\beta^2 - (r^2 - 1)(r^2 - \beta^2)\right]^2}}$$

To derive Equation (5.42), which is the same as Equation (5.39), note that $c = k_a = \omega_a = 0$, which also means β = 0. Since this is a moment equation, F_0 is replaced by M_0. Therefore,

$$\frac{Xk}{F_0} = \sqrt{\frac{(2\zeta r)^2 + r^4}{(2\zeta r)^2(r^2 - 1 + \mu r^2)^2 + (r^2 - 1)^2 r^4}} = \sqrt{\frac{4\zeta^2 + r^4}{4\zeta^2(r^2 + \mu r^2 - 1)^2 + (r^2 - 1)^2 r^2}}$$

which is Equation (5.42) after canceling r^2.

To derive Equation (5.49), first let Equation (5.42) be $f(r,\zeta)$. Since $f = \frac{A^{1/2}}{B^{1/2}}$, where $A = 4\zeta^2 + r^2$ and $B = 4\zeta^2(r^2 + \mu r^2 - 1)^2 + (r^2 - 1)^2 r^2$, then

$$\frac{\partial f}{\partial \zeta} = 0$$

becomes

$$BdA = AdB$$

where

$$dA \equiv \frac{\partial A}{\partial \zeta} = 8\zeta$$

$$dB \equiv \frac{\partial B}{\partial \zeta} = 8\zeta(r^2 + \mu r^2 - 1)^2$$

So,

$$\left\{4\zeta^2\left(r^2+\mu r^2-1\right)^2+\left(r^2-1\right)^2 r^2\right\}\left(8\zeta\right)=\left\{4\zeta^2+r^2\right\}\left(8\zeta\right)\left(r^2+\mu r^2-1\right)^2$$
$$\left(r^2-1\right)^2=\left(r^2+\mu r^2-1\right)^2$$
$$\left(r^2-1\right)=\pm\left(r^2+\mu r^2-1\right)$$

Taking the minus sign (the plus sign yields $r = 0$).

$$\left(2+\mu\right)r^2-2=0$$
$$r=\sqrt{\frac{2}{\left(2+\mu\right)}}$$

Now take the other partial derivative $\frac{\partial f}{\partial r}=0$, which becomes

$$BdA = AdB$$
$$dA\equiv\frac{\partial A}{\partial r}=2r$$
$$dB\equiv\frac{\partial B}{\partial r}=16\zeta^2 r\left(1+\mu\right)\left(r^2+\mu r^2-1\right)+4r^3\left(r^2-1\right)+2r\left(r^2-1\right)^2$$

So,

$$\left\{4\zeta^2\left(r^2+\mu r^2-1\right)^2+\left(r^2-1\right)^2 r^2\right\}\left(2r\right)$$
$$=\left\{4\zeta^2+r^2\right\}\left[16\zeta^2 r\left(1+\mu\right)\left(r^2+\mu r^2-1\right)+4r^3(r^2-1)+2r\left(r^2-1\right)^2\right]$$

Substituting $r=\sqrt{\frac{2}{\left(2+\mu\right)}}$ yields, after rearranging

$$4\zeta^2\left[\frac{2}{2+\mu}+\frac{2\mu}{2+\mu}-1\right]^2+\left[\frac{2}{2+\mu}-1\right]^2\left(\frac{2}{2+\mu}\right)$$
$$=\left[4\zeta^2+\frac{2}{2+\mu}\right]\left[8\zeta^2\left(1-\mu\right)\left(\frac{2}{2+\mu}+\frac{2\mu}{2+\mu}-1\right)+2\left(\frac{2}{2+\mu}\right)\left(\frac{2}{2+\mu}-1\right)+\left(\frac{2}{2+\mu}-1\right)^2\right]$$

Expanding and canceling terms yields

$$4\zeta^4(1+\mu)(2+\mu)+2\zeta^2\mu-\frac{2}{2+\mu}=0$$

The physical solution for ζ is

$$\zeta=\frac{1}{\sqrt{2(1+\mu)(2+\mu)}}$$

which is Equation (5.49).

5.58 Consider the design suggested in Example 5.5.1. Calculate the percent change in the maximum deflection if the damping constant changes 10% from an optimal value. If the optimal damping is fixed but the mass of the absorber changes by 10%, what percent change in $\left|Xk / M_0\right|_{\max}$ results? Is the optimal absorber design more sensitive to changes in c_a or m_a?

Solution:

From Problems 5.51 and 5.46, F_0 = 100 N, $k = 4.144 \times 10^6$ N/m, and μ = 0.133. The optimal damping is

$$\zeta_{op} = \frac{1}{\sqrt{2(1+\mu)(2+\mu)}} = 0.4549$$

The deflection is given by Equation (5.42), and M_0 replaced by F_0,

$$X = \frac{F_0}{k}\sqrt{\frac{4\zeta^2 + r^2}{4\zeta^2\left(r^2 + \mu r^2 - 1\right)^2 + \left(r^2 - 1\right)^2 r^2}}$$

Also, the maximum displacement will occur at $r = \sqrt{\frac{2}{(2+\mu)}}$ = 0.9683. If the damping constant changes by 10%, ζ will also change by 10% since $\zeta = \frac{c_a}{2m\omega_p}$. The value of X for 0.9 ζ_{op}, ζ_{op}, and 1.1 ζ_{op} is

$$\begin{aligned}
\zeta = 0.9\zeta_{op} &\quad\rightarrow\quad X = 3.870\times10^{-4}\text{ m}\\
\zeta = \zeta_{op} &\quad\rightarrow\quad X = 3.870\times10^{-4}\text{ m}\\
\zeta = 1.1\zeta_{op} &\quad\rightarrow\quad X = 3.870\times10^{-4}\text{ m}
\end{aligned}$$

There is no change in X with a 10% change in ζ_{op}.

If m_a changes by 10%, μ will also change by 10% since $\mu = \frac{m_a}{m}$. The value of $\left(\frac{Xk}{F_0}\right)_{\max}$ for 0.9μ, μ, and 1.1μ is

$$0.9\mu \quad \rightarrow \quad r = 0.9714 \quad \rightarrow \quad \left(\frac{Xk}{F_0}\right)_{max} = 17.708(+10.4\%)$$

$$\mu \quad \rightarrow \quad r = 0.9683 \quad \rightarrow \quad \left(\frac{Xk}{F_0}\right)_{max} = 16.038$$

$$1.1\mu \quad \rightarrow \quad r = 0.9318 \quad \rightarrow \quad \left(\frac{Xk}{F_0}\right)_{max} = 14.671(-8.5\%)$$

The displacement is more sensitive to changes in m_a than c_a.

5.59 Consider the elastic isolation problem described in Figure 5.26. Derive equations (5.57) and (5.58) from equation (5.53).

Solution:

Rewrite equation (5.53) in matrix form as

$$\begin{bmatrix} k_1 - m\omega^2 + jc\omega & -jc\omega \\ -jc\omega & -\left(k_2 + jc\omega\right) \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} F_0 \\ 0 \end{bmatrix}$$

The inverse of the matrix on the left is

$$\frac{1}{-k_2\left(k_1 - m\omega^2\right) - jc\omega\left(k_1 + k_2 m\omega^2\right)} \begin{bmatrix} -\left(k_2 + jc\omega\right) & jc\omega \\ jc\omega & k_1 m\omega^2 + jc\omega \end{bmatrix}$$

Solving for X_1 and X_2 yields

$$X_1 = \frac{\left(k_2 + jc\omega\right)F_0}{k_2\left(k_1 - m\omega^2\right) + jc\omega\left(k_1 + k_2 - m\omega_{dr}^2\right)}$$

$$X_2 = \frac{c\omega_{dr} F_0 j}{k_2\left(k_1 - m\omega^2\right) + jc\omega\left(k_1 + k_2 - m\omega_{dr}^2\right)}$$

which are equations (5.54) and (5.55).

5.60 Use the derivative calculation for finding maximum and minimum to derive equations (5.57) and (5.58) for the elastic damper system.

Solution:

From equation (5.56)

$$T.R. = \frac{\sqrt{1+4(1+\gamma)^2\zeta^2 r^2}}{\sqrt{(1-r^2)^2+4\zeta^2 r^2(1+\gamma-r^2\gamma)^2}}$$

Equation (5.45) is applicable here, so that

$$BdA = AdB$$

where $A = 1+4(1+\gamma)^2\zeta^2 r^2$ and $B = (1-r^2)^2+4\zeta^2 r^2(1+\gamma-r^2\gamma)^2$ differentiating with respect to ζ yields

$$dA \equiv \frac{\partial A}{\partial \zeta} = 8(1+\gamma)^2\zeta r^2$$
$$dB \equiv \frac{\partial B}{\partial \zeta} = 8\zeta r^2(1+\gamma-r^2\gamma)^2$$

So,

$$\left\{(1-r^2)^2+4\zeta^2 r^2(1+\gamma-r^2\gamma)^2\right\}(8)(1+\gamma)^2\zeta r^2$$
$$=\left\{1+4(1+\gamma)^2\zeta^2 r^2\right\}(8\zeta r^2)(1+\gamma-r^2\gamma)^2$$
$$(1-r^2)^2(1+\gamma)^2 = (1+\gamma-r^2\gamma)^2$$
$$(1-r^2)(1+\gamma) = \pm(1+\gamma-r^2\gamma)$$

The minus sign yields the physical result

$$r^2(2\gamma+1) = 2(1+\gamma)$$
$$r = \sqrt{\frac{2(1+\gamma)}{1+2\gamma}}$$

which is equation (5.57)

Differentiating with respect to *r* yields

$$dA \equiv \frac{\partial A}{\partial r} = 8\left(1+\gamma\right)^2 \zeta^2 r$$
$$dB \equiv \frac{\partial B}{\partial r} = 2\left(1-r^2\right)\left(-2r\right) + 8\zeta^2 r\left(1+\gamma-r^2\gamma\right)^2 + 8\zeta^2 r^2\left(1+\gamma-r^2\gamma\right)\left(-2r\gamma\right)$$

So,

$$\left\{\left(1-r^2\right)^2 + 4\zeta^2 r^2\left(1+\gamma-r^2\gamma\right)^2\right\}\left(8\zeta^2 r\right)\left(1+\gamma\right)^2$$
$$=\left\{1+4\left(1+\gamma\right)^2\zeta^2 r^2\right\}\left[-4r\left(1-r^2\right)+8\zeta^2 r\left(1+\gamma-r^2\gamma\right)^2 - 16\gamma\zeta^2 r^3\left(1+\gamma-r^2\gamma\right)\right]$$

Substituting for *r* and manipulating yields

$$\left[64\gamma\left(1+\gamma\right)^5\left(\frac{1}{1+2\gamma}\right)\right]\zeta^4 + 8\left[\gamma\left(1+\gamma\right)^2 + \left(1+\gamma\right)^3\left(1+2\gamma\right) - 2\left(1+\gamma\right)^4\right]\zeta^2 - \left(1+2\gamma\right) = 0$$

Solving for ζ yields the physical result

$$\zeta = \frac{\sqrt{2\left(1+2\gamma\right)/\gamma}}{4\left(1+\gamma\right)}$$

which is Equation (5.58).

5.61 A 1000-kg mass is suspended from ground by a 40,000-N/m spring. A viscoelastic damper is added, as indicated in Figure 5.26. Design the isolator (choose k_2 and c) such that when a 70-N sinusoidal force is applied to the mass, no more than 100 N is transmitted to ground.

Solution:

From equation (5.59),

$$(T.R.)_{\max} = 1 + 2\gamma$$

$$\frac{F_T}{F_0} = \frac{100}{70} = 1.429 = 1 + 2\gamma$$

$$\gamma = 0.2143$$

The isolator stiffness should be

$$k_2 = \gamma k_1 = (0.2143)(40,000) = 8571\text{N/m}$$

From equation (5.58),

$$\zeta_{op} = \frac{\sqrt{2(1+2\gamma)/\gamma}}{4(1+\gamma)} = 0.7518$$

The isolator damping should be

$$c = 2\zeta_{op}\sqrt{\frac{k_1}{m}} = 2(0.7518)\sqrt{\frac{40,000}{1000}} = 9.51\text{kg/s}$$

5.62 Consider the isolation design of Example 5.5.2. If the value of the damping coefficient changes 10% from the optimal value (of 188.56 kg/s), what percent change occurs in $(T.R.)_{max}$? If c remains at its optimal value and k_2 changes by 10%, what percent change occurs in $(T.R.)_{max}$? Is the design of this type of isolation more sensitive to changes in damping or stiffness?

Solution:

From Example 5.5.2, c = 188.56 kg/s and k_2 = 200 N/m. If the value of c changes by 10%, the value of T.R. becomes (with $r = 5$ and $\gamma = 0.5$),

$$\begin{aligned} &0.9c && \rightarrow && \zeta_{op} = 0.4243 && \rightarrow && T.R. = 0.1228(-1.78\%) \\ &c && \rightarrow && \zeta_{op} = 0.4714 && \rightarrow && T.R. = 0.1250 \\ &1.1c && \rightarrow && \zeta_{op} = 0.5185 && \rightarrow && T.R. = 0.1267\left(+1.39\%\right) \end{aligned}$$

If the value of k_2 changes by 10%, the value of T.R. becomes (with $r = 5$ and $\zeta = 0.4714$),

$$\begin{aligned} &0.9k_2 && \rightarrow && \gamma = 0.45 && \rightarrow && T.R. = 0.1327\left(+6.17\%\right) \\ &k_2 && \rightarrow && \gamma = 0.5 && \rightarrow && T.R. = 0.1250 \\ &1.1k_2 && \rightarrow && \gamma = 0.55 && \rightarrow && T.R. = 0.1183\left(-5.31\%\right) \end{aligned}$$

This design is more sensitive to changes in stiffness.

5.63 A 3000-kg machine is mounted on an isolator with an elastically coupled viscous damper such as indicated in Figure 5.26. The machine stiffness (k_1) is 2.943×10^6 N/m, $\gamma = 0.5$, and $c = 56.4 \times 10^3$ N·s/m. The machine, a large compressor, develops a harmonic force of 1000 N at 7 Hz. Determine the amplitude of vibration of the machine.

Solution:

The amplitude of vibration is given by Equation (5.54) as

$$X_1 = \frac{\left(k_2 + jc\omega\right)F_0}{k_2\left(k_1 - m\omega^2\right) + jc\omega\left(k_1 + k_2 - m\omega_{dr}^2\right)}$$

Since F_0 = 1000 N, $\omega = 7(2\pi) = 43.98$ rad/s, m = 3000 kg, $c = 56.4 \times 10^3$ N·s/m, $k_1 = 2.943 \times 10^6$ N/m, and $k_2 = \gamma k_1 = 1.4715 \times 10^6$ N/m, then

$$X_1 = -4.982 \times 10^{-4} - 1.816 \times 10^{-4}\, j$$

The magnitude is

$$\left|X_1\right| = 5.303 \times 10^{-4} \text{ m}$$

5.64 Again consider the compressor isolation design given in Problem 5.63. If the isolation material is changed so that the damping in the isolator is changed to $\zeta = 0.15$, what is the force transmitted? Next determine the optimal value for the damping ratio and calculate the resulting transmitted force.

Solution:

From Problem 5.63, $\gamma = 0.5$, $F_0 = 1000$ N, and $r = \dfrac{\omega}{\sqrt{k_1/m}} = \dfrac{7(2\pi)}{\sqrt{2.943\times10^6/3000}} =$ 1.404. Since $\zeta = 0.15$, the transmitted force is [from Equation (5.56)],

$$F_T = F_0\sqrt{\frac{1+4(1+\gamma)^2\zeta^2r^2}{(1-r^2)^2+4\zeta^2r^2(1+\gamma-r^2\gamma)^2}} = 1188 \text{ N}$$

The optimal value for the damping ratio is found from equation (5.58):

$$\zeta_{op} = \frac{\sqrt{2(1+2\gamma)/\gamma}}{4(1+\gamma)} = 0.4714$$

The transmitted force is then

$$F_T = 1874 \text{ N}$$

5.65 Consider the optimal vibration isolation design of Problem 5.64. Calculate the optimal design if the compressor's steady-state driving frequency changes to 24.7 Hz. If the wrong optimal point is used (i.e., if the optimal damping for the 7-Hz driving frequency is used), what happens to the transmissibility ratio?

Solution:

From Problems 5.63 and 5.64, $\gamma = 0.5$, $F_0 = 1000$ N, $k_1 = 2.943 \times 10^6$ N, and $m = 3000$ kg.

The optimal damping is

$$\zeta_{op} = \frac{\sqrt{2(1+2\gamma)/\gamma}}{4(1+\gamma)} = 0.4714$$

The value of c and k_2 would be

$$c = 2\zeta_{op}\sqrt{k_1 m} = 88.589 \text{ kg/s}$$
$$k_2 = \gamma k_1 = 1.472 \times 10^6 \text{ N/m}$$

The isolation design is independent of the driving frequency in this problem, so the transmissibility ratio would not change.

5.66 Recall the optimal vibration absorber of Problem 5.53. This design is based on a steady-state response. Calculate the response of the primary system to an impulse of magnitude M_0 applied to the primary inertia J_1. How does the maximum amplitude of the transient compare to that in steady state?

Solution:

The response of the system given in Problem 5.53 cannot be solved by the means of modal analysis given in Chapter 4 because the system is not proportionally damped. However, the steady-state response of a damped system to an impulse is simply zero. Therefore, the maximum amplitude of the transient will be of interest. For a sinusoidal input, a numerical simulation might be necessary to determine the effects of the transient response.

Problems and Solutions Section 5.6 (5.67 through 5.73)

5.67 Compare the resonant amplitude at steady state (assume a driving frequency of 100 Hz) of a piece of nitrite rubber at 50°F versus the value at 75°F. Use the values for η from Table 5.2.

Solution:

From equation (5.63),

$$X = \frac{F_0}{k\left(1+\eta j\right) - m\omega^2}$$

At resonance $\omega = \sqrt{\frac{k}{m}}$ so

$$X = \frac{F_0}{k\left(1-\eta j\right) - 1} = \frac{F_0}{k\eta j}$$

The magnitude is

$$\left|X\right| = \frac{1}{\eta}\left(\frac{F_0}{k}\right)$$

At 50°, η = 0.5 and at 75°, η = 0.28, so

$$\left|X\right|_{50^\circ} = \frac{2F_0}{k}$$

$$\left|X\right|_{75^\circ} = \frac{3.57F_0}{k}$$

5.68 Using Equation (5.67), calculate the new modulus of a 0.05 × 0.01 × 1, piece of pinned-pinned aluminum covered with a 1-cm-thick piece of nitrite rubber at 75°F driven at 100 Hz.

Solution:

From Table 1.2, E_1 = 7.1 × 10^{10} N/m^2 for aluminum. From Table 5.2, $E_2 = 2.758 \times 10^7$ N/m^2 for nitrate rubber, Also,

$$I = I_1 = \frac{1}{3}(0.05)(1)^3 = 0.01667 \text{ m}^4$$

$$e_2 = \frac{E_2}{E_1} = \frac{2.758 \times 10^7}{7.1 \times 10^{10}} = 3.885 \times 10^{-4}$$

$$h_2 = \frac{H_2}{H_1} = \frac{0.01}{0.01} = 1$$

From Equation (5.67),

$$E = \frac{E_1 I_1}{I}\left[1 + e_2 h_2^2 + 3(1 + h_2)^2 \frac{e_2 h_2}{1 + e_2 j_2}\right] = 7.136 \times 10^{10} \text{ N/m}^2$$

5.69 Calculate Problem 5.68 again at 50°F. What percent effect does this change in temperature have on the modulus of the layered material?

Solution:

From Problem 5.68, with $E_2 = 4.137 \times 10^7$ N/m^2,

$$I = I_1 = 0.01667 \text{ m}^4$$
$$e_2 = \frac{E_2}{E_1} = \frac{4.137 \times 10^7}{7.1 \times 10^{10}} = 5.827 \times 10^{-4}$$
$$h_2 = \frac{H_2}{H_1} = \frac{0.01}{0.01} = 1$$

From Equation (5.67),

$$E = \frac{E_1 I_1}{I}\left[1 + e_2 h_2^2 + 3\left(1 + h_2\right)^2 \frac{e_2 h_2}{1 + e_2 h_2}\right] = 7.154 \times 10^{10}\ \text{N/m}^2$$

This is an increase of 0.25% of the layered material's modulus.

5.70 Repeat the design of Example 5.6.1 by
(a) changing the operating frequency to 1000 Hz, and
(b) changing the operating temperature to 50°F.
Discuss which of these designs yields the most favorable system.

Solution:

From Ex. 5.6.1, $E_1 = 7.1\times10^{10}$ N/m^2 and $h_2 = 1$.

(a) 75°, 1000 Hz

$$\eta_2 = 0.55$$
$$E_2 = 4.826\times10^{7}\ \text{N/m}^2$$
$$e_2 = \frac{E_2}{E_1} = 6.797\times10^{-4}$$

From Equation (5.68),

$$\eta = \frac{e_2h_2\left(3+6h_2+4h_2^2+2e_2h_2^2+e_2^2h_2^4\right)}{\left(1+e_2h_2\right)\left(1+4e_2h_2+6e_2h_2^2+4e_2h_2^3+e_2^2h_2^4\right)}\eta_2 = 0.00481$$

(b) 50°, 1000 Hz

$$\eta_2 = 0.5$$
$$E_2 = 4.137\times10^{7}\ \text{N/m}^2$$
$$e_2 = \frac{E_2}{E_1} = \frac{4.137\times10^{7}}{7.1\times10^{10}} = 5.827\times10^{-4}$$

From Equation (5.68),

$$\eta = 0.00375$$

Increasing the driving frequency results in a higher loss factor compared to the effects of lowering the temperature.

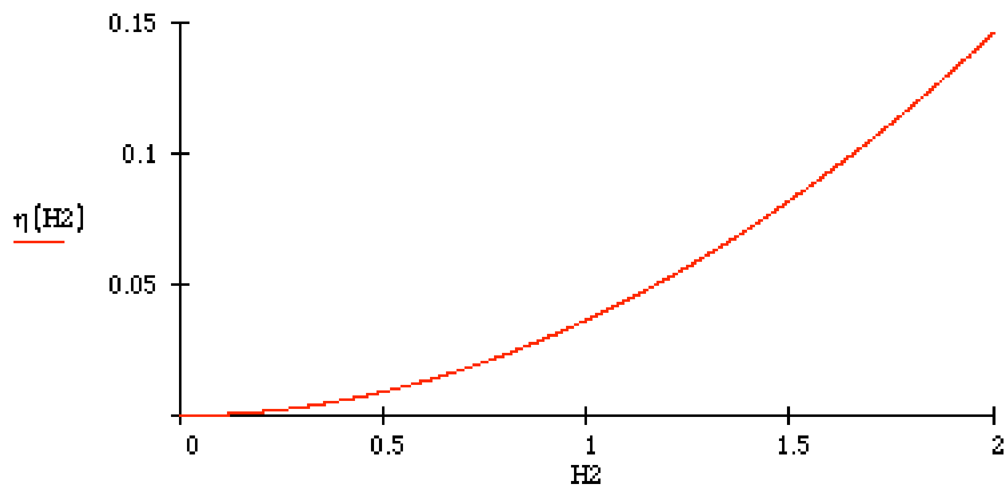
5.71 Reconsider Example 5.6.2. Make a plot of thickness of the damping treatment versus loss factor.

Solution:

From Ex. 5.6.2, $\eta_2 = 0.261$, $e_2 = 0.01$, and $H_1 = 1$ cm. So, from Equation (5.69),

$$\eta = 14 e_2 \frac{H_2^2}{H_1^2} \eta_2 = 0.03654 H_2^2 \qquad \left(H_2 \text{ in cm}\right)$$

$\eta(H2) := 0.03654 \cdot H2^2$



A plot of η versus H_2 in centimeters

5.72 Calculate the maximum transmissibility coefficient of the center of the shelf of Example 5.6.1. Make a plot of the maximum transmissibility ratio for this system frequency, using Table 5.2 for each temperature.

Solution: If the system is modeled as shown in Figure 5.18, then the maximum transmissibility occurs at (from Equation (5.50)),

$$\left(\frac{Xk}{F_0}\right)_{\max} = 1 + \frac{2}{\mu}$$

where μ is found from Equation (5.49) as the solution to

$$\zeta = \frac{1}{\sqrt{2(\mu+1)(\mu+2)}}$$

The value of ζ is $\frac{\eta}{2}$ at resonance. So, at 75° and 100 Hz,

$$\zeta = \frac{\eta}{2} = \frac{0.00151}{2} = 0.000755 = \frac{1}{\sqrt{2(\mu+1)(\mu+2)}}$$

$$\Rightarrow \mu = 935$$

$$\frac{Xk}{F_0} = 1 + \frac{2}{935} = 1.002$$

For 50° and 100 Hz, η = 0.00375 (from Problem 5.70), so

$$\zeta = \frac{\eta}{2} = \frac{0.00375}{2} = 0.001875 = \frac{1}{\sqrt{2(\mu+1)(\mu+2)}}$$

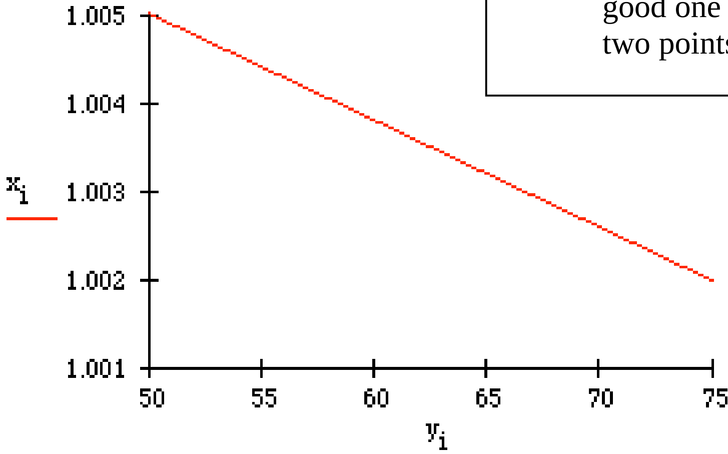
$$\mu = 375.6$$

$$\frac{Xk}{F_0} = 1 + \frac{2}{375.6} = 1.005$$

$i := 1 .. 2$ $x_1 := 1.002$ $x_2 := 1.005$

$y_1 := 75$ $y_2 := 50$

This gives some idea of the relationship, but not a very good one as it includes only two points



5.73 The damping ratio associated with steel is about $\zeta = 0.001$. Does it make any difference whether the shelf in Example 5.6.1 is made out of aluminum or steel? What percent improvement in damping ratio at resonance does the rubber layer provide the steel shelf?

Solution:

If the shelf in Ex. 5.6.1 is made out of steel, $E_1 = 2.0 \times 10^{11}$ N/m^2. Therefore,

$$e_2 = \frac{E_2}{E_1} = \frac{2.758 \times 10^7}{2.0 \times 10^{11}} = 0.0001379$$

Also, $\eta_2 = 0.55$ and $h_2 = 1$. From Equation (5.68),

$$\eta = \frac{e_2 h_2 \left(3 + 6h_2 + 4h_2^2 + 2e_2 h_2^2 + e_2^2 H_2^4\right)}{\left(1 + e_2 h_2\right)\left(1 + 4e_2 h_2 + 6e_2 h_2^3 + 4e_2 h_2^3 + e_2 h_2^4\right)} \eta_2 = 0.0005$$

At resonance, $\zeta = \dfrac{\eta}{2} = 0.00025.$ The rubber actually reduced the damping of the steel shelf by 75%.

Problems and Solution Section 5.7 (5.74 through 5.80)

5.74 A 100-kg compressor rotor has a shaft stiffness of 1.4×10^7 N/m. The compressor is designed to operate at a speed of 6000 rpm. The internal damping of the rotor shaft system is measured to be $\zeta = 0.01$.

(a) If the rotor has an eccentric radius of 1 cm, what is the rotor system's critical speed?

(b) Calculate the whirl amplitude at critical speed. Compare your results to those of Example 5.7.1.

Solution:

(a) The critical speed is the rotor's natural frequency, so

$$\omega_c = \sqrt{\frac{k}{m}} = \sqrt{\frac{1.4\times 10^7}{100}} = 374.2 \text{ rad/s} \; = \; 3573 \text{ rpm}$$

(b) At critical speed, $r = 1$, so from Equation (5.81),

$$X = \frac{\alpha}{2\zeta} = \frac{0.01}{2(0.01)} = 0.5 \text{ m}$$

So a system with higher eccentricity and lower damping has a greater whirl amplitude (see Example 5.7.1).

5.75 Redesign the rotor system of Problem 5.74 such that the whirl amplitude at critical speed is less than 1 cm by changing the mass of the rotor.

Solution: From Problem 5.74, $k = 1.4 \times 10^7$ N/m, $m = 100$ kg, $\zeta = 0.01$, and $\alpha = 0.01$m. Since the whirl amplitude at critical speed must be less than 0.01 m, the value of ζ that would satisfy this is, from equation (5.81),

$$X = \frac{\alpha}{2\zeta}$$

$$\zeta = \frac{\alpha}{2X} = \frac{0.01}{2(0.01)} = 0.5$$

The original damping ratio was 0.01, so the value of c is

$$c = 2\zeta m\omega = 2(0.01)(100)\sqrt{\frac{1.4\times 10^7}{100}} = 784.33 \text{ kg/s}$$

So, the new mass should be, with $\zeta = 0.5$,

$$748.33 = 2(0.5)m\sqrt{\frac{k}{m}} = \sqrt{km} = \sqrt{1.4\times 10^7\, m}$$

$$\Rightarrow m = 0.04 \text{ kg}$$

This is not practical.

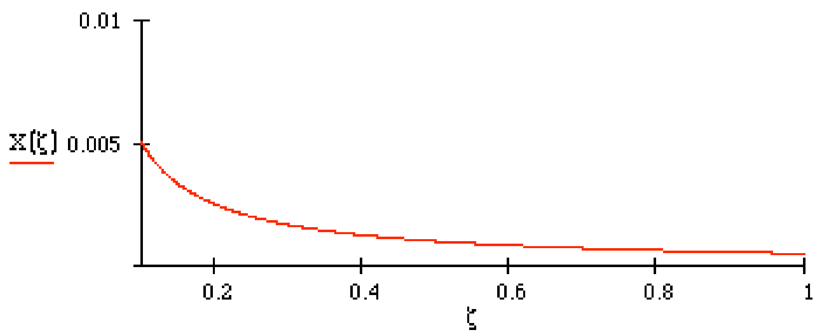
5.76 Determine the effect of the rotor system's damping ratio on the design of the whirl amplitude at critical speed for the system of Example 5.7.1 by plotting X at $r = 1$ for ζ between $0 < \zeta < 1$.

Solution:

From Example 5.7.1, with $r = 1$ and $\alpha = 0.001$ m,

$$X = \frac{0.001}{2\zeta} = \frac{0.0005}{\zeta}$$

$$x(\zeta) := \frac{0.0005}{\zeta}$$



5.77 The flywheel of an automobile engine has a mass of about 50 kg and an eccentricity of about 1 cm. The operating speed ranges from 1200 rpm (idle) to 5000 rpm (red line). Choose the remaining parameters so that whirling amplitude is never more than 1 mm.

Solution:

From Equation (5.81),

$$X = 0.001 = \frac{0.01r^2}{\sqrt{\left(1-r^2\right)^2 + \left(2\zeta r\right)^2}}$$

Choosing ζ = 0.1, the physical solution is

$$r = 0.3018$$

By observing Figure 5.34, r = 0.3018 is the maximum value of r. So at $\left(\omega_r\right)_{\max}$ = 500 rpm, the stiffness must be

$$r = 0.3018 = \frac{5000\left(\frac{2\pi}{60}\right)}{\sqrt{k/50}}$$

$$k = 1.505\times 10^8 \text{ N/m}$$

5.78 Consider the design of the compressor rotor system of Example 5.7.1. The amplitude of the whirling motion depends on the parameters α, ζ, m, k and the driving frequency. Which parameter has the greatest effect on the amplitude? Discuss your results.

Solution:

From Example 5.7.1, $\alpha = 0.001$ m, $\zeta = 0.05$, $m = 55$ kg, $\omega_r = 6000$ rpm, and $k = 1.4 \times 10^7$ N/m. To find out what effect each parameter has on this system, each value will be varied by 10%.

The original system has $r = 1.2454$ and $X = 0.002746$ m.

$$0.9a = 0.009\text{m} \quad \rightarrow \quad r = 1.2454 \quad \rightarrow \quad X = 0.002471 \text{ m } (\text{-}10.0\%)$$
$$1.1a = 0.0011 \text{ m} \quad \rightarrow \quad r = 1.2454 \quad \rightarrow \quad X = 0.003020 \text{ m } \left(+10.0\%\right)$$
$$1.9\zeta = 0.045 \quad \rightarrow \quad r = 1.2454 \quad \rightarrow \quad X = 0.002759 \text{ m } \left(+0.465\%\right)$$
$$1.1\zeta = 0.055 \quad \rightarrow \quad r = 1.2454 \quad \rightarrow \quad X = 0.002732 \text{ m } \left(\text{-}0.507\%\right)$$
$$0.9m = 49.5 \text{ kg} \quad \rightarrow \quad r = 1.1815 \quad \rightarrow \quad X = 0.003379 \text{ m } \left(+23.1\%\right)$$
$$1.1m = 60.5 \text{ kg} \quad \rightarrow \quad r = 1.3062 \quad \rightarrow \quad X = 0.002376 \text{ m } \left(\text{-}13.5\%\right)$$
$$0.9k = 1.26 \times 10^7 \text{ N/m} \quad \rightarrow \quad r = 1.3127 \quad \rightarrow \quad X = 0.002344 \text{ m } \left(\text{-}14.6\%\right)$$
$$1.1k = 1.54 \times 10^7 \text{ N/m} \quad \rightarrow \quad r = 1.1874 \quad \rightarrow \quad X = 0.003304 \text{ m } \left(+20.3\%\right)$$

The mass and stiffness values have the greatest effect on the amplitude, while the damping ratio has the smallest effect.

5.79 At critical speed the amplitude is determined entirely by the damping ratio and the eccentricity. If a rotor has an eccentricity of 1 cm, what value of damping ratio is required to limit the deflection to 1 cm?

Solution:

Since X = 0.01 m, a = 0.01 m, and at critical speed r = 1, then from Equation (5.81),

$$X = 0.01 \text{ m } = \frac{a}{2\zeta} = \frac{0.01}{2\zeta}$$

$$\zeta = 0.5$$

5.80 A rotor system has damping limited by ζ < 0.05. What is the maximum value of eccentricity allowable in the rotor design if the maximum amplitude at critical speed must be less than 1 cm?

Solution:

Since X = 0.01 m, ζ < 0.05, and at critical speed r = 1, then from Equation (5.81),

$$X = 0.01 \text{ m } = \frac{a}{2\zeta} = \frac{a}{2(0.05)}$$

$$a = 0.001 \text{ m } = \text{ 1 mm}$$

Problems and Solutions Section 5.8 (5.81 through 5.85)

5.81 Recall the definitions of settling time, time to peak, and overshoot given in Example 3.2.1 and illustrated in Figure 3.6. Consider a single-degree-of-freedom system with mass m = 2 kg, damping coefficient c = 0.8 N·s/m, and stiffness 8 N/m. Design a PD controller such that the settling time of the closed-loop system is less than 10 s.

Solution: The settling time is

$$t_s = \frac{3}{\zeta\omega}$$

Since t_s = 10 s,

$$\zeta\omega = 0.3$$

The equation of motion with a PD controller is

$$m\ddot{x} + \left(c + g_2\right)\dot{x} + \left(k + g_1\right)x = 0$$

So,

$$\omega = \sqrt{\frac{k + g_1}{m}} = \sqrt{\frac{8 + g_1}{2}}$$

$$\zeta = \frac{c + g_2}{2m\omega} = \frac{0.8 + g_2}{2\left(2\right)\omega}$$

Therefore,

$$\zeta\omega = \left(\frac{0.8 + g_2}{4\omega}\right)\omega = 0.3$$

$$g_2 = 0.4 \text{ N} \cdot \text{s/m}$$

The gain g_1 can take on any value (including 0).

5.82 Redesign the control system given in Example 5.8.1 if the available internal damping is reduced to 50 N·s/m.

Solution: If the value of c is limited to 50 N·s/m, then g_2 becomes

$$g_2 = 180 - c = 180 - 50 = 130 \text{ N} \cdot \text{s/m}$$

5.83 Consider the compressor rotor-shaft system discussed in Problem 5.74. Modern designers have considered using electromagnetic bearings in such rotor systems to improve their design. Use a derivative feedback control law on the design of this compressor to increase the effective damping ratio to $\zeta = 0.5$. Calculate the required gain. How does this affect the answer to parts (a) and (b) of Problem 5.74?

Solution: From Problem 5.74, $m = 100$ kg, $k = 1.4 \times 10^7$ N/m, $a = 0.01$, $\zeta_{old} = 0.01$. The value of c is

$$c = 2\zeta_{old}\sqrt{km} = 2(0.01)\sqrt{(1.4\times 10^7)(100)} = 748.3 \text{ kg/s}$$

With derivative feedback, the coefficient of $\dot{x}$ in the equation of motion is $c + g_2$. For $\zeta = 0.5$,

$$c + g_2 = 748.3 + g_2 = 2(0.5)\sqrt{(1.4\times 10^7)(100)} = 37{,}416.6$$

$$g_2 = 36{,}668.2 \text{ kg/s}$$

(a) The rotor's critical speed remains the same because it is only dependent upon the mass stiffness.

(b) The whirl amplitude becomes

$$X = \frac{a}{2\zeta} = \frac{0.01}{2(0.5)} = 0.01 \text{ m}$$

It is reduced by 80% because of the increased damping.

5.84 Calculate the magnitude of the force required of the actuator used in the feedback control system of Example 5.8.1. See if you can find a device that provides this much force.

Solution: The magnitude of the actuator force would be

$$F = g_2|\dot{x}| = g_2\omega_n X$$

where X is, from Equation (2.26), at steady-state,

$$X = \frac{F_0 / m}{\sqrt{(\omega_n^2 - \omega^2)^2 + (2\zeta\omega_n\omega\ )^2}}$$

A large value of X would occur at resonance, for example, where $\omega = \omega_{dr} = 10$ rad/s, so

$$F = (80)(10)\frac{F_0 / 10}{2(0.9)(10)(10)} = 0.444F_0$$

5.85 In some cases the force actuator used in a control system also introduces dynamics. In this case a system of the form given in Equation (5.27) may result where m_a, c_a and k_a are values associated with the actuator (rather than an absorber). In this case the absorber system indicated in Figure 5.18 can be considered as the control system and the motion of the mass *m* is the object of the control system. Let *m* = 10 kg, *k* = 100 N/m, and *c* = 0. Choose the feedback control law to be

$$u = -g_1 x - g_2 \dot{x}$$

and assume that c_a = 20 N·s/m, k_a = 100 N/m and m_a = 1 kg. Calculate g_1 and g_2 so that *x* is as small as possible for a driving frequency of 5 rad/s. [Hint: Replace *k* with *k* + *g*, and *c* with *c* + *g* in Equation (5.27)]

Solution:

Let the control law be called position feedback, applied to the mass m. The equation of motion then becomes Equation (5.27) with *k* replaced by $k + g_1$. Then the amplitude *X* can be expressed as Equation (5.35) with *k* replaced by $k + g_1$ and given values of *m*, m_a, k_a and c_a. This yields

$$\frac{X^2}{F_0^2} = \frac{(100-25)^2 + (25)(400)}{\left\{\left[100 + g_1 - (10)(25)\right]\left[100-25\right] - 2500\right\}^2 + \left[100 - (11)(25)\right]^2 (25)(400)}$$

$$\frac{X^2}{F_0^2} = \frac{2.78}{g_1^2 - 366.7 g_1 + 88{,}055.6}$$

Clearly *X* is a minimum if g_1^2 -366.7g_1 + 88,055.6 is a minimum. Thus consider the derivatives of the quadratic form with respect to g_1 to find the max value per the discussion on the top of page 265.

$$\frac{d}{dg_1}\left(g_1 - 366.7 g_1 + 88{,}055.6\right) = 2g_1 - 366.7 = 0$$

so that $g_1 = 183.35$

Note that $d^2 / dg_1^2 = 2 > 0$ so that this is a maximum and *X* is a minimum for this gain.

Problems Section 5.9 (5.86 through 5.88)

5.86 Reconsider Example 5.2.1, which describes the design of a vibration isolator to protect an electronic module. Recalculate the solution to this example using equation (5.92).

Solution: If data sheets are not available use $G'_\omega = G'/2$. One of many possible designs is given. From the example we have T.R. = 0.5, m = 3 kg and ω = 35 rad/s = 5.57 Hz. From equation (5.92):

$$T.R. = \frac{\sqrt{1+\eta^2}}{\sqrt{\left(1 - r^2\dfrac{G'}{G'_\omega}\right)^2 + \eta^2}} = 0.5$$

From Table 5.2 for 75°F and frequency of 10 Hz (the closest value listed), the value of E and η are:

$$E = 2.068 \times 10^7 \text{ N/m}^2 \qquad \text{and} \qquad \eta = 0.21$$

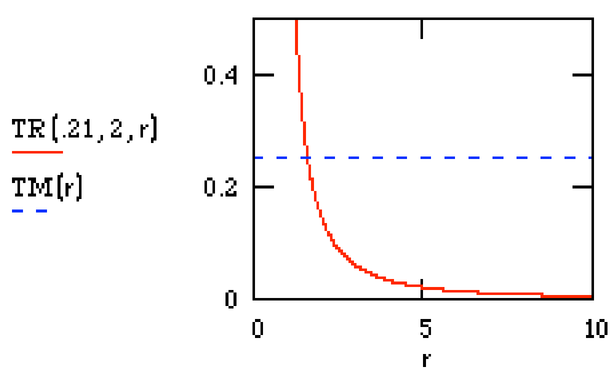
Thus $G' = E/3 = 6.89 \times 10^9$ N/m^2 using the approximation suggested after equation (5.86). They dynamic shear modulus is estimated from plots such as Figure 5.38 to be $G'_\omega = G'/2$. Thus equation 5.92 becomes

$$0.5^2 = \frac{1 + (0.21)^2}{\left(1 - r^2\dfrac{G'}{G'/2}\right)^2 + (0.21)^2}$$

This is solved numerically in the following Mathcad session:

$$TR(a, b, r) := \frac{\sqrt{1 + a^2}}{\sqrt{\left(1 - b \cdot r^2\right)^2 + a^2}}$$

$$TM(r) := .5^2$$



From the plot, any value of r greater then about 2.5 will do the trick. Choosing r = 2.5 yields $\omega_n = \dfrac{\omega}{3.5} = \dfrac{35}{3.5} \Rightarrow \sqrt{\dfrac{k}{m}} = 10 \Rightarrow k = 100(3) = 300$ N/m

5.87 A machine part is driven at 40 Hz at room temperature. The machine has a mass of 100 kg. Use Figure 5.42 to determine an appropriate isolator so that the transmissibility is less than 1.

Solution: Given f = 40 Hz, m = 100 kg or about 220 lbs. and *T.R.* <1. The maximum static load per mount is 3 lbs. Therefore the system would require a minimum of 73 mounts. Assume then that 75 mounts are used. Thus

$$\frac{220\#}{75} = 2.9\# \text{ per mount}$$

For the isolator, $f_n < 0.5 f = 0.5(40) = 20$ Hz. Therefore the f_n of the isolator must be less then 20 Hz. Referring to the performance characteristics of the table in Figure 5.42 yields 4 possible isolator choices:

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5.88 Make a comparison between the transmissibility ratio of Window 5.1 and that of equation (5.92).

Solution: Comparing equation (5.92) with Window 5.1 yields:

Window 5.1: $$T.R. = \frac{\sqrt{1+(2\zeta r)^2}}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}$$

Equation (5.92): $$T.R. = \frac{\sqrt{1+\eta^2}}{\sqrt{\left(1-r^2\dfrac{G'}{G'_\omega}\right)^2+\eta^2}}$$

Comparing the two equations yields

$$\eta = 2\zeta r \quad \text{and} \quad \frac{G'}{G'_\omega} \approx 1$$