**Frank-Wolfe Algorithm Demonstration**

1. **Introduction**

To illustrate the Frank-Wolfe algorithm, consider the demonstration example for the simplex method with a new objective function:



The feasible region is shown to the right. Without these constraints the maximum of   is easily found (by setting the partial derivatives equal to zero) to be . But what is the constrained maximum? Let us use as the initial trial solution   for the Frank-Wolfe algorithm.

1. **Objective Approximation**

**Initial trial solution:**





We begin the first iteration by developing a "linear approximation" for  near this trial solution. This is done by evaluating the partial derivatives at (0, 0):



So this approximation is  . Maximizing   subject to the original constraints yields the solution
   with . However since (2, 1) is not near (0, 0) the approximation may not be a good one at (2, 1). [Note that  is well under .] Rather than just accepting (2, 1) as the next trial solution, let us check the line segment between (0, 0) and (2, 1) and choose the  with the largest .

1. **Changing Solution**



The equation for the line segment between (0, 0) and (2, 1) is



Since  and , the values of  on the line are



The point  on this line segment having the largest  is found by maximizing  over  by the one-dimensional search procedure, which yields . Therefore the new trial solution is .

1. **Next Iteration**

**Second trial solution:**





We begin the second iteration by evaluating the partial derivatives of  at (2, 1):



So the approximated objective function is . Maximizing  subject to the original constraints yields the solution  with . [Note that .] The line between (2, 1) and (0, 7) is





which is maximized at . The resulting new trial solution is



1. **Final Iteration**

**Third trial solution:**





Evaluating the partial derivatives at (1.695, 1.914):



so the approximating objective function is . Maximizing , or equivalently (after dividing by 4.17) , results in **every** solution on the line between (2, 1) and (0, 7) being optimal for this linear programming problem. Regardless of which solution is used as the other endpoint of the line from (1.695, 1.914), we already know from the preceding iteration that  is maximized along this line at (1.695, 1.914). Since the trial solution did not move, this verifies that the optimal solution for our convex programming problem is

