Thermodynamics - Example Problems Problems and Solutions

1 Examining a Power Plant

Consider a power plant. At point 1 the working gas has a temperature of $T = 25°C$. The pressure is $1bar$ and the mass flow is $\dot{m} = 2kg/s$. After point 1, the air enters a compressor, at which the pressure is increased to 6 bar. (This is point 2.) After this, the flow is heated to $T_3 = 700°C$. (This is stage 3.) Afterwards, the flow goes through a turbine, at which the pressure is decreased to $1bar$ again. (This is stage 4.) The flow is then cooled to $T_1 = 25°C$. We're now back in stage 1.

We assume that:

1. air behaves like a thermally and calorically perfect gas with $C_p = 1004J/kg\,K$, $R = 287J/kg\,K$ and $\gamma = 1.4$.
2. the isentropic efficiency of the compressor is 0.88.
3. the polytropic efficiency of the turbine is 0.85.

Answer the following questions.

1. Depict the evolution of the cycle in the $T - s$ plane.
2. Determine the net power.
3. Determine the heat received from the hot source.
4. Determine the thermal efficiency of the power plant.
5. Determine the Carnot efficiency associated with the present power plant and compare it with the previous result.
6. Verify the first principle.

1 Solution

1. Let's draw the $T - s$ graph. We start by drawing two lines of constant pressure. (This can be seen in the graph below.)

   We now start at point 1. At this point the pressure and temperature are low. When we go to point 2, we don't go straight up, since the process is not isentropic. Instead, we go up and slightly right as well, until we reach the line $p = 6bar$. We then have found point 2.

   The heating takes place under constant pressure. So from point 2 to point 3 we stay on the isobaric line $p = 6bar$. This goes on until we reach $T = 973K$. This is the position of point 3.

   The turbine isn't isentropic either. So from point 3 we don't go straight down. Instead, we go down and a bit to the right as well, until we reach the line $p = 1bar$. This is point 4.

   From point 4 we return to point 1. Since the cooling occurs at constant pressure, we do this without leaving the line $p = 1bar$.

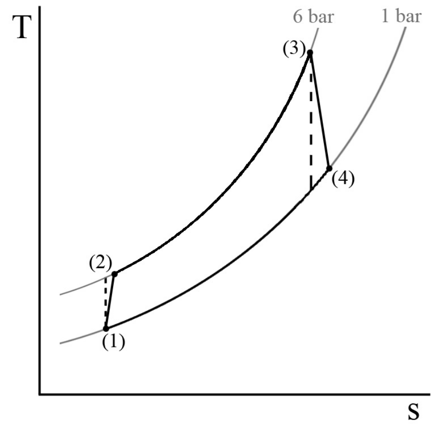

Figure 1: The $T-s$ graph of the power plant.

2. Let's just calculate all the temperatures in all the points. First we'll find T_2.

   We know that $p_2/p_1 = 6$. We also know that $T_1 = 298K$. To find T_2, we have to use the definition of the isentropic efficiency. For compression (and for constant c_p), this definition is

$$\eta_{is} = \frac{T_{2,is} - T_1}{T_2 - T_1}. \tag{1.1}$$

   Here the variable $T_{2,is}$ is the temperature in point 2 if the compression would be performed isentropically. If the compression would be performed isentropically, we would be able to find T_2 using the isentropic relations. So, we can find $T_{2,is}$, using

$$\frac{p_2}{p_1} = \left(\frac{T_{2,is}}{T_1}\right)^{\frac{\gamma}{\gamma-1}}. \tag{1.2}$$

   It follows that $T_{2,is} = 497.2K$. We can then find that $T_2 = 524.4K$.

   We already know that $T_3 = 973K$. To find T_4, we have to look at the turbine. The only thing given about the turbine is the polytropic efficiency. If the pressure decreases (which is the case in a turbine), the polytropic efficiency is defined as

$$\eta_{poly} = \frac{n-1}{n}\frac{\gamma}{\gamma-1}. \tag{1.3}$$

   We know γ and η_{poly}. We just need to solve for n. If we do this, we find that

$$n = \frac{\gamma}{\gamma - (\gamma-1)\,\eta_{poly}} = 1.321. \tag{1.4}$$

   Since a polytropic efficiency is given, we may assume that the process in the turbine is polytropic. So we have $pV^n = C$, with P the pressure and V the volume. Since the mass m in the turbine is constant (the process is steady), we also have $pv^n = C$, with v the specific volume. The perfect gas law states that $v = RT/p$. Using this, we find that also $T^n p^{1-n}$ is constant. This implies that

$$\frac{p_3}{p_4} = \left(\frac{T_3}{T_4}\right)^{\frac{n}{n-1}}. \tag{1.5}$$

   By inserting values, we find that $T_4 = 629.7K$.

Now it's time to calculate the net power of the power plant. We know that the power used by the compressor is given by

$$P_{comp} = c_p \dot{m} (T_2 - T_1) = 455 KJ/s. \tag{1.6}$$

Similarly, the power created by the turbine is given by

$$P_{turb} = c_p \dot{m} (T_3 - T_4) = 689 KJ/s. \tag{1.7}$$

The net power produced by the entire power plant thus becomes

$$P_{net} = P_{turb} - P_{comp} = 234.8 KJ/s. \tag{1.8}$$

This power is produced by the power plant, and thus, in a way, "flows" outward.

3. Let's calculate the heat received from the hot source. This is simply given by

$$Q_{in} = c_p \dot{m} (T_3 - T_2) = 900.8 KJ/s. \tag{1.9}$$

4. The thermal efficiency is "useful output" divided by "total input". The input is the heat received from the hot source. The output is the net work done by the power plant. So the thermal efficiency is

$$\eta_{ther} = \frac{P_{net}}{Q_{in}} = \frac{234.8}{900.8} = 0.2606 = 26.06\%. \tag{1.10}$$

5. The Carnot efficiency (the maximally achievable efficiency) can be found using

$$\eta_{Carnot} = 1 - \frac{Q_L}{Q_H} = 1 - \frac{T_L}{T_H} = 1 - \frac{298}{973} = 0.6937 = 69.37\%. \tag{1.11}$$

6. The first principle states that energy is conserved. The energy going into the system is the heat entering the system Q_{in}. The energy leaving the system is the net work P_{in} and the heat leaving the system Q_{out}. This heat is given by

$$Q_{out} = c_p \dot{m} (T_4 - T_1) = 666.1 KJ/s. \tag{1.12}$$

Now we see that

$$Q_{in} - P_{net} - Q_{out} = 900.8 - 234.8 - 666.1 = 0 KJ/s. \tag{1.13}$$

The first principle still holds.

2 The Hirn cycle

Consider the Hirn cycle, described below. The working fluid is water/vapor.

1. From point 3 to point 4, the water/vapor passes through a pump. This pump delivers work W_{p2}.
2. From point 4 to point 5 the water/vapor moves through a heater.
3. From point 5 to point 6 the water/vapor moves through a high pressure turbine.
4. At point 6 the channel splits up. One part, with mass flow $\dot{m}_s$, goes to the mixer. The other part, with mass flow $\dot{m} - \dot{m}_s$, goes to a low pressure turbine. After this low pressure turbine we arrive at point 7. (By the way, $\dot{m}$ is the total mass flow in the points 3, 4 and 5.)
5. From point 7 to point 1 the water/vapor goes through a condenser.
6. From point 1 to point 2 the water/vapor goes through a pump. This pump delivers work W_{p1}.

7. From point 2 to point 3 the water/vapor goes through the mixer, where it is joined with (part of) the water/vapor coming from point 6.

Assuming that:

1. the steam enters the high pressure (HP) turbine at $p_5 = 3.5MPa$ and $T_5 = 400°C$. It exits the HP turbine at $p_6 = 0.4MPa$ and exits the LP turbine at $10kPa$.
2. The working fluid is under the state of saturated liquid at the inlet of both pumps.
3. The heating and cooling processes taking place within the heater and the condenser are isobaric.
4. The mixer is perfectly insulated.
5. The pumping work is negligible.
6. The pumps and the turbines are adiabatic and reversible.

Answer the following questions:

1. Depict the evolution of the cycle in the $T - s$ plane.
2. Find the ratio $\dot{m}_s/\dot{m}$.
3. Determine the work per unit mass produced by the HP turbine and the quality of the working fluid at the outlet of the turbine.
4. Determine the work per unit mass produced by the LP turbine and the quality of the working fluid at the outlet of the turbine.
5. Determine the heat per unit mass received by the working fluid in the heater.
6. Determine the heat per unit mass released to the cold source by the working fluid.
7. Determine the thermal efficiency of the power plant.
8. Verify that the first principle of thermodynamics is indeed satisfied (both numerically and theoretically. What's the relative influence of the pumps' work?).

2 Solution

To solve this question, you need tables with a lot of numbers concerning water. In these tables you can, for example, look up the saturation temperatures of water at different pressures. Without these tables, you can only qualitatively examine the Hirn cycle.

1. We start by drawing a $T-s$ graph, with the characteristic bell-curved shape of the saturation lines. We also draw the three isobaric lines (lines of constant pressure) for $p = 3.5MPa$, $p = 0.4MPa$ and $p = 0.010MPa$.

   Now we draw point 5. At point 5 the temperature is high, and the pressure is high as well. So the point must be somewhere to the right top of the graph.

   Next we draw point 6. The HP turbine decreases the temperature, but it does this at constant entropy. (The pumps and the turbines are both adiabatic and reversible. They are thus isentropic.) So point 6 is directly below point 5.

   We also draw point 7. This one is (for the same reason) directly below point 6.

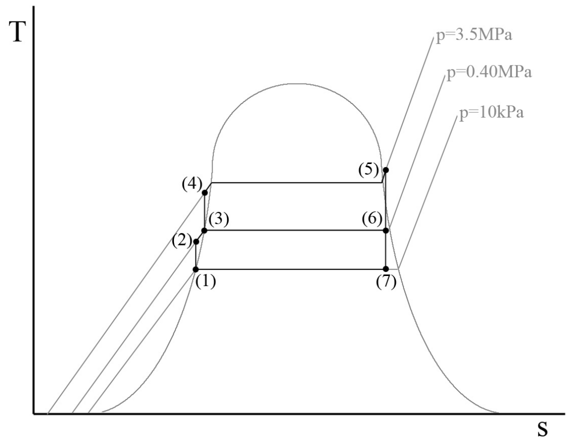

Figure 2: The $T - s$ graph of the Hirn cycle.

The reaction in the condenser is isobaric. So $p_7 = p_1$. We also know that the water at the start of the pump is a saturated liquid. So point 1 must lie at the intersection of the saturated liquid line and the $p = 0.010MPa$ line.

Between point 1 and point 2 the water passes through an isentropic pump. This pump causes the pressure to increase. Since points 2 and points 6 are about to be mixed together, they must have the same pressure. So $p_2 = p_6$. Therefore point 2 must lie directly above point 1 on the line $p = 0.40MPa$.

The mixer operates at a constant pressure. So $p_3 = 0.40MPa$. We also know that the water in point 3 (just before the second pump) is a saturated liquid. So point 3 must lie on the saturated liquid line. This is how we determine point 3.

Now all we need to do is determine point 4. The heater after point 4 operates at constant pressure, so $p_4 = p_5 = 3.5MPa$. To reach point 4 (from point 3), we once more pass through an isentropic pump. So from point 3, we go straight up (with constant entropy) until we reach the line $p = 3.5MPa$. That is where point 4 is.

So, all the points on the graph are known. Isn't it great? It's now time to calculate the values of the pressure and the temperature in every point. We also find the entropy s, the quality of the mixture x (if a mixture is present) and the enthalpy h. You never know when we might need it.

Luckily, we already know the pressures in every point. They are $p_7 = p_2 = 0.010MPa$, $p_2 = p_3 = p_6 = 0.40MPa$ and $p_4 = p_5 = 3.5MPa$. We also know that $T_5 = 400°C$. From this follows that the water/vapor in point 5 is actually entirely a vapor. The entropy in point 5 can be looked up to be $s_5 = 6.847kJ/kg\,K$. (We used $p_5 = 3.5MPa$ and $T_5 = 400°C$ to look up this value.) Also, we can look up the enthalpy. We find $h_5 = 3223kJ/kg$.

First we will find T_6. We know that the entropy in points 5 and 6 is equal. So we have $s_6 = s_5 = 6.847kJ/kg\,K$. We also have $p_6 = 0.40MPa$. If we try to look up the corresponding temperature T_6 we kind of run into a problem. We can't find the temperature, since the water/vapor mixture is in some kind of transition state. This implies that the temperature T_6 must be the saturation temperature at $p_6 = 0.40MPa$. This saturation temperature is $T_6 = 143.6°C$. Since we have a water/vapor mixture at point 6, we can also calculate the quality of the mixture at point 6. This quality is

$$x_6 = \frac{s - s_t}{s_g - s_t} = \frac{6.847 - 1.777}{6.897 - 1.777} = 0.99. \tag{2.1}$$

So most of the water/vapor mixture is still vapor. Finally, using this quality, we can calculate the enthalpy. We will get

$$h_6 = h_t + x h_{tg} = 604.8 + 0.99 \cdot 2134 = 2718 kJ/kg. \tag{2.2}$$

Now let's look at T_7. We can similarly find that the water/vapor at point 7 is a saturated water/vapor mixture. However, the saturation temperature at $p_7 = 0.010 MPa$ is $T_7 = 45.8°C$. The quality of the mixture is

$$x_7 = \frac{s - s_t}{s_g - s_t} = \frac{6.847 - 0.649}{8.151 - 0.649} = 0.826. \tag{2.3}$$

From this we can find the enthalpy. We will get

$$h_7 = h_t + x h_{th} = 191.9 + 0.826 \cdot 2393 = 2169 kJ/kg. \tag{2.4}$$

We move on to point 1. The water/vapor mixture has passed through a condenser. In point 1 the pressure is $p_1 = 0.010 MPa$. Also, the water is a saturated liquid, so the quality of the mixture $x_1 = 0$. Using these two data, we can find that $T_1 = T_7 = 45.8°C$. (The water is still saturated at $p_1 = p_7$.) We can also look up that the entropy in point 1 is equal to $s_1 = 0.649 kJ/kg\,K$. Finally, the enthalpy is $h_1 = 191.9$.

It's time to examine point 2. The water has now gone through an isentropic pump. So $p_2 = 0.40 MPa$ and $s_2 = s_1 = 0.649 kJ/kg\,K$. We were allowed to assume that the pumping work is negligible. So we don't need to take that into account. Based on the entropy and the pressure, we could use tables to extrapolate the temperature and the entropy. We will find that $T_2 = 46.2°C$ and $h_2 = 193.5 kJ/kg$. By the way, from our graph follows that we are now dealing with a compressed (subcooled) liquid.

We continue with point 3. We know that at this point the water/vapor mixture is, in fact, a saturated liquid. So the quality of the mixture $x_3 = 0$. We also know that the pressure is equal to $p_3 = 0.40 MPa$. This implies that the temperature in this point must be equal to the saturation temperature at $p = 0.40 MPa$. And this temperature can be looked up. We will find $T_3 = 143.6°C$. Also, the entropy in this point is $s_3 = 1.777 kJ/kg\,K$. The enthalpy is $h_3 = 604.8 kJ/kg$.

Our final point is point 4. Now $p_4 = 3.5 MPa$. The water has gone through an isentropic pump, so $s_4 = s_3 = 1.777 kJ/kg\,K$. Just like for point 2, we can find the temperature and enthalpy by extrapolating the table data. We will find that $T_4 = 144.3°C$ and $h_4 = 609.8 kJ/kg$. We are dealing with a compressed liquid, so there is no water/vapor mixture or anything of that kind.

2. Let's find the ratio $\dot{m}_s / \dot{m}$. Let's apply the conservation of energy principle to the mixer. The energy going in is $\dot{m}_s h_6 + (\dot{m} - \dot{m}_s) h_2$. The energy going out is $\dot{m} h_3$. It follows that

$$\frac{\dot{m}_s}{\dot{m}} = \frac{h_3 - h_2}{h_6 - h_2} = \frac{604.8 - 192}{2718 - 192} = 0.1634. \tag{2.5}$$

3. We already know the quality of the working fluid at the outlet of the turbine. This was $x_6 = 0.99$. The work produced per unit mass by the HP turbine is simply equal to

$$W_{HP} = h_5 - h_6 = 3223 - 2718 = 505 kJ/kg. \tag{2.6}$$

4. We already know the quality of the working fluid at the outlet of the turbine. This was $x_7 = 0.826$. Also, the work produced per unit mass by the LP turbine is equal to

$$W_{LP} = h_6 - h_7 = 2718 - 2169 = 549 kJ/kg. \tag{2.7}$$

5. To find the heat per unit mass received by the working fluid in the heater, we use

$$Q_{in} = h_5 - h_4 = 3223 - 606.9 = 2616 kJ/kg. \tag{2.8}$$

6. The heat per unit mass released to the cold source is given by

$$Q_{out} = h_7 - h_1 = 2169 - 191.9 = 1977 kJ/kg \tag{2.9}$$

7. The thermal efficiency is given by "energy in" divided by "useful output". The energy in is $Q_{in} = 2616 kJ/kg$. The "useful output" is the work done. However, this is NOT $W_{HP} + W_{LP}$. That is because not all the mass passes through the LP turbine. Instead, only $n = 1 - 0.1634 = 0.8366$ part of the mass passes through the LP turbine. So we should take that into account. This implies that the thermal efficiency is equal to

$$\eta_{thermal} = \frac{W_{HP} + nW_{LP}}{Q_{in}} = \frac{505 + 0.8366 \cdot 549}{2616} = 0.3686. \tag{2.10}$$

8. We need to verify that energy is maintained. The energy going into the system is $Q_{in,total} = Q_{in} = 2616 kJ/kg$. To find the energy leaving the system, we should take into account $n = 0.8366$ once more. So

$$Q_{out,total} = W_{HP} + nW_{LP} + nQ_{out} = 505 + 0.8366 \cdot 549 + 0.8366 \cdot 1977 = 2618 kJ/kg. \tag{2.11}$$

   There is a very small difference. This difference is partially caused by errors when reading data from tables. It is also partially caused by the work done by the pumps. Because we haven't taken into account the pumps' work, $Q_{in,total}$ seems to be lower than it actually should be.

   So, even the small difference has been explained. This means that the first principle still holds.