Heat-Work Transformations

1 Reservoirs

In this chapter we'll take a look at how we can transform heat into work, and vice verse. But before we can go into details, we have to make some definitions.

We define a **thermal energy reservoir** (or just a **reservoir**) as a hypothetical body that can absorb/supply heat without any change in temperature. A reservoir that supplies energy in the form of heat is called a **source**. A reservoir that absorbs it is a **sink**.

2 Heat Engines

Work can be transformed entirely into heat. However, transforming heat to work is a bit troubling. So let's give that process a closer look. A device that converts heat to work is called a **heat engine**. A heat engine receives a heat Q_H from a high-temperature source. It converts part of the heat into work W_{out}. The remaining heat Q_L is dumped in a low-temperature sink. Using this data, we find that

$$W_{out} = Q_H - Q_L. \tag{2.1}$$

The **thermal efficiency** η_{th} of a heat engine can now be found using

$$\eta_{th} = \frac{\text{Desired output}}{\text{Required input}} = \frac{W_{out}}{Q_H} = 1 - \frac{Q_L}{Q_H}. \tag{2.2}$$

So what we need to do is make sure we need to dump as few heat as possible to the low-temperature sink. If possible, we would of course prefer to have $Q_L = 0$. But sadly this is not possible. This is because the **Kelvin-Plank statement** says that it is impossible for any device to receive heat from a single reservoir and produce work. Therefore the fraction Q_L/Q_H should simply be minimized. How this is done is something we will look at later.

3 Refrigerators and Heat Pumps

We saw that a heat engine uses temperature to create work. On the other hand, we can also use work to regulate temperature. A **refrigerator** is a device that transfers heat from a low-temperature region to a high-temperature region. The **Clausius statement** states that this transfer won't happen by itself. We therefore need to add work to get the desired effect.

We just saw that a heat engine produced work. A refrigerator, on the other hand, requires work W_{in}. Also, the direction of heat transfer is reversed. An amount of heat Q_L now comes from the low-temperature source, after which an amount Q_H goes to the high-temperature sink. So this time

$$W_{in} = Q_H - Q_L. \tag{3.1}$$

To check how well a refrigerator functions, we now don't use an efficiency. Instead, the **coefficient of performance** of a refrigerator is defined as

$$COP_R = \frac{\text{Desired output}}{\text{Required input}} = \frac{Q_L}{W_{in}} = \frac{Q_L}{Q_H - Q_L} = \frac{1}{\frac{Q_H}{Q_L} - 1}. \tag{3.2}$$

The reason for using an other term than the efficiency, is because the coefficient of performance can be bigger than 1, while an efficiency can not.

A device very similar to a refrigerator is a **heat pump**. In fact, refrigerators and heat pumps are the same, except for their goal. While refrigerators want to keep the low-temperature source cold, heat pumps want to keep the high-temperature sink warm. The coefficient of performance of a heat pump thus becomes

$$COP_{HP} = \frac{\text{Desired output}}{\text{Required input}} = \frac{Q_H}{W_{in}} = \frac{Q_H}{Q_H - Q_L} = \frac{1}{1 - \frac{Q_L}{Q_H}} = COP_R + 1. \tag{3.3}$$

4 Reversible Processes

A **reversible process** is a process that can be reversed without leaving any trace on the surroundings. Processes that are not reversible are called **irreversible processes**. Reversible processes actually do not occur in nature. They are simply idealizations of actual processes. Reversible processes are always more efficient than irreversible processes.

Factors that cause a process to be irreversible are called **irreversibilities**. The most well-known irreversibility is **friction**. Also **heat transfer** over a finite temperature difference causes an irreversible process. A process is called **internally reversible** if no irreversibilities occur within the boundaries of the system during the process. Identically, a process is called **externally reversible** if no irreversibilities occur within the surroundings of the system. A process is **totally reversible** (or simply **reversible**) if it is both internally and externally irreversible.

5 The Carnot Cycle

We know that reversible processes are the most efficient processes. One example of a reversible process is the **Carnot cycle**, using a so-called **Carnot heat engine**. This cycle consists of four reversible steps.

First the heat engine is connected to an energy source at temperature T_H, causing **isothermal expansion**. The energy source is then replaced by an insulation, causing **adiabatic expansion**. After that, the insulation is removed, and an energy sink at temperature T_L is connected to the heat engine. This causes **isothermal compression**. Finally, the energy sink is once more replaced by insulation, causing **adiabatic compression**. This completes the cycle. In this cycle, heat has flowed from the energy source (at T_H) to the energy sink (at T_L), producing work W_{out}.

The funny thing about the Carnot cycle is that it can be reversed. Now work is added to the system, to transport heat from the (new) energy source (at T_L) to the (new) energy sink (at T_H). We then would have a **Carnot refrigerator**.

6 Quality of Energy

We could ask ourselves: How does the efficiency/coefficient of performance depend on the temperatures of the source and the sink? We know that the efficiency strongly depends on the factor Q_H/Q_L. For reversible processes, it can be shown that

$$\frac{Q_H}{Q_L} = \frac{\phi(T_H)}{\phi(T_L)}, \tag{6.1}$$

where $\phi(T)$ is a certain unknown function. Usually this function is assumed to be $\phi(T) = T$, resulting in

$$\frac{Q_H}{Q_L} = \frac{T_H}{T_L}. \tag{6.2}$$

This makes the efficiency of a Carnot heat engine (the so-called **Carnot efficiency**)

$$\eta_{th} = 1 - \frac{T_L}{T_H}. \tag{6.3}$$

So if the source is much hotter than the sink, it's much easier to get work out of it. Identically, the coefficient of performance of the Carnot refrigerator now is

$$COP_R = \frac{1}{\frac{T_H}{T_L} - 1}. \tag{6.4}$$

So it gets increasingly hard to reduce the temperature of a sink, as it gets colder.

These conclusions give rise to an idea. It seems that very hot energy sources are much more valuable than less hot energy sources. Therefore, next to the quantity of energy, also the **quality** of energy matters. The higher the temperature, the higher its quality.