Control Volumes

1 Conservation of Mass

In the previous chapter we looked at closed systems. Now we look at control volumes. A **control volume** is a fixed volume in space, where mass can move through. Just like we have applied conservation of energy in closed systems, we can apply the **conservation of mass principle** in control volumes. So we have

$$\Delta m = m_{in} - m_{out}. \tag{1.1}$$

The **mass flow rate** $\dot{m}$ is the mass flowing through a cross-section per unit time. Suppose we have a pipe or duct. We can then determine the mass flow through this pipe using

$$\dot{m} = \int_{A_c} \rho \nu_n \, dA_c = \rho \nu_{n,avg} A_c, \tag{1.2}$$

where A_c is the cross-sectional area of the pipe and ν_n is the velocity of the flow normal to this area. Identically, we can define the **volume flow rate** $\dot{V}$ as

$$\dot{V} = \int_{A_c} \nu_n \, dA_c = \nu_{n,avg} A_c = \dot{m} v. \tag{1.3}$$

We can now rewrite the conservation of mass principle as

$$\frac{dm}{dt} = \dot{m}_{in} - \dot{m}_{out}. \tag{1.4}$$

2 Energy of a Flowing Fluid

The interesting thing about a control volume, is that mass can enter it. What we want to know is how much energy is added to the system when a piece of mass enters a control volume. First of all, we know that the specific energy e consists of specific internal energy u, specific kinetic energy $V^2/2$ and specific potential energy gz. But there is more.

When a mass enters a control volume, something pushes it in. This pushing performs so-called **flow work** (also called **flow energy**), having magnitude

$$W_{flow} = PV, \qquad \text{or, expressed per unit mass,} \qquad w_{flow} = Pv. \tag{2.1}$$

So we find that the **total energy of a flowing fluid** per unit mass is

$$\theta = w_{flow} + e = Pv + u + \frac{1}{2}V^2 + gz = h + \frac{1}{2}V^2 + gz. \tag{2.2}$$

3 Steady Flow Processes

During a **steady flow process** the properties inside the control volume do not change with time. Since the total mass then is constant, we have $dm/dt = 0$ and thus

$$\dot{m}_{in} = \dot{m}_{out}. \tag{3.1}$$

If the flow is also incompressible, then also

$$\dot{V}_{in} = \dot{V}_{out}. \tag{3.2}$$

For steady flow processes, also the energy stays constant. This implies that $\dot{E}_{in} = \dot{E}_{out}$, or equivalently,

$$\dot{Q}_{in} + \dot{W}_{in} + \sum_{in} \dot{m}\theta = \dot{Q}_{out} + \dot{W}_{out} + \sum_{out} \dot{m}\theta. \tag{3.3}$$

If we define $\dot{Q} = \dot{Q}_{in} - \dot{Q}_{out}$ as the heat that goes into the system and $\dot{W} = \dot{W}_{out} - \dot{W}_{in}$ as the work done by the system, we will get

$$\dot{Q} - \dot{W} = \dot{m}\left(h_{out} - h_{in} + \frac{V_{out}^2 - V_{in}^2}{2} + g\left(z_{out} - z_{in}\right)\right). \tag{3.4}$$

Dividing by $\dot{m}$ gives

$$q - w = h_{out} - h_{in} + \frac{V_{out}^2 - V_{in}^2}{2} + g\left(z_{out} - z_{in}\right), \tag{3.5}$$

where $q = q_{in} - q_{out}$ and $w = w_{out} - w_{in}$.

There are many examples of steady flow devices in real life. For example, a **nozzle** is a device that increases velocity by decreasing the pressure. A **diffuser** does exactly the opposite. For both devices, the amount of heat transferred ΔQ, the amount of work performed ΔW and the change in potential energy Δpe can be neglected. This reduces the energy balance equation to

$$h_{in} + \frac{1}{2}V_{in}^2 = h_{out} + \frac{1}{2}V_{out}^2. \tag{3.6}$$

A similar thing can be done for other devices with a steady flow processes, such as **turbines** and **compressors**.

4 Unsteady Flow Processes

For many processes there are properties inside the control volume that change over time. Such processes are called **unsteady flow processes**. The relations of the previous paragraph are not applicable to those processes.

Dealing with unsteady flow processes is a lot more difficult. We therefore often assume that such processes are uniform flow processes. A **uniform flow process** is a process for which the properties at the inlets and outlets do not change with time or position.

Now let's look at two situations in this flow: One initial situation (1) and one situation a bit later (2). A heat Q has been added. The system has done an amount of work W. Also incoming mass has brought an energy $(m\theta)_{in}$, and mass leaving the system has taken energy $(m\theta)_{out}$ with it. Using these data, we can set up the energy balance for uniform flow processes. We get

$$Q - W + (m\theta)_{in} - (m\theta)_{out} = m_2 e_2 - m_1 e_1. \tag{4.1}$$

Once more this equation can be simplified, based on various assumptions.