Closed Systems

1 Boundary Work

A **closed system** is, as was mentioned earlier, a system which can not exchange mass with its surroundings. The only way in which energy interaction with the surroundings is possible, is through heat transfer and by work. Therefore we can derive the **energy balance**, stating that

$$\Delta E = E_{in} - E_{out} = Q_{in} + W_{in} - Q_{out} - W_{out}. \tag{1.1}$$

Usually it is assumed that the heat Q flows inward and the work W is the work performed by the system (thus "flowing" outward). This reduces the energy balance to

$$\Delta E = Q - W, \qquad \Leftrightarrow \qquad \Delta e = q - w. \tag{1.2}$$

If the values of Q or W will turn out to be negative, then the wrong direction has been assumed.

2 Boundary Work

Let's take a closer look at the work. Work is force times distance. The force comes from the pressure inside the system. But to have a distance present, the boundary of the system must move. So there can only be work done if there is compression/expansion of the system. The corresponding type of work is called **moving boundary work**. The boundary work W_b can be found using

$$\partial W_b = F\,ds = PA\,ds = P\,dV \qquad \Rightarrow \qquad W_b = \int_1^2 P\,dV, \tag{2.1}$$

where V is the volume of the system. Note that the boundary work performed during a process per unit mass is simply the area under a $P - v$ diagram.

During processes, pressure and volume are often related by $PV^n = C$, with V still the volume and n and C constants. Any process of this kind is called a **polytropic process**. Using this relation (or, to be more precise, $P = CV^{-n}$) the boundary work can be found. Sometimes the coefficient n first has to be derived from the **polytropic efficiency** $\eta_{polytropic}$. This can be done using

$$\eta_{polytropic} = \frac{\gamma/(\gamma-1)}{n/(n-1)} \quad \text{if } p \text{ decreases, and} \qquad \eta_{polytropic} = \frac{n/(n-1)}{\gamma/(\gamma-1)} \quad \text{if } p \text{ increases.} \tag{2.2}$$

The coefficient γ now does not denote the specific weight. It is the ratio of specific heats, which will be defined in the next paragraph.

3 Specific Heats

The **specific heat** is defined as the energy required to raise the temperature of a unit mass of a substance by one Kelvin. The amount of energy needed depends on the process. If the heating is done at constant volume, we find the **specific heat at constant volume** c_v. Identically, if the process is performed at constant pressure, we find the **specific heat at constant pressure** c_p. They can be found using

$$c_v = \left(\frac{\partial u}{\partial T}\right)_v \qquad \text{and} \qquad c_p = \left(\frac{\partial h}{\partial T}\right)_p. \tag{3.1}$$

For an ideal gas the internal energy u and the enthalpy h only depend on the temperature T. So the specific heats also depend on temperature only. Therefore we have

$$\Delta u = u_2 - u_1 = \int_1^2 c_v(T)\, dT \qquad \text{and} \qquad \Delta h = h_2 - h_1 = \int_1^2 c_P(T)\, dT. \tag{3.2}$$

Usually the functions for c_v and c_p are unknown. However, there are tables with their values for given temperatures. So, what we then do, is take the value of (for example) c_v at T_1 and T_2, take their average, and use that value to calculate Δu. In an equation this becomes

$$\Delta u = \frac{c_v(T_2) - c_v(T_1)}{2}\,(T_2 - T_1) = c_{v,avg}\,(T_2 - T_1) \qquad \text{and identically} \qquad \Delta h = c_{p,avg}\,(T_2 - T_1)\,. \tag{3.3}$$

There is an important relation between c_p and c_v. We know (from the definition of enthalpy and the perfect gas law) that $dh = du + R\,dT$. Differentiating with respect to temperature gives

$$c_p = c_v + R, \tag{3.4}$$

where R is, as we already know, the gas constant for ideal gases. We can also define the **specific heat ratio** as

$$\gamma = \frac{c_p}{c_v}. \tag{3.5}$$

Like the specific heats, also the specific heat ratio γ depends on the temperature T. The variation with temperature is very small though, so usually this ratio is assumed to be constant.

4 Incompressible Substances

An **incompressible substance** is a substance whose specific volume v is constant. Solids and liquids can be approximated as such substances. For such substances $R = 0$ and thus $c_p = c_v = c$. We now would like to know how these substances respond to changes. Or, to be more specifically, how does the enthalpy change during a process?

From the definition of enthalpy, we find that for incompressible substances

$$\Delta h = \Delta u + v\,\Delta P + P\,\Delta v = \Delta u + v\,\Delta P = c_{avg}\,\Delta T + v\,\Delta P. \tag{4.1}$$

Note that the term $P\,\Delta v$ has disappeared, since v is assumed to be constant. For liquids, we can distinguish two special cases, being

- **Constant pressure processes** ($\Delta P = 0$) where $\Delta h = \Delta u = c_{avg}\,\Delta T$.
- **Constant temperature processes** ($\Delta T = 0$) where $\Delta h = v\,\Delta P$.

For solids the term $v\,\Delta P$ is insignificant, so all processes for solids can be approximated as constant pressure processes.