UWE Bristol

Thermodynamics & Fluids

UFMEQU-20-1

FLUIDS

Lecture 6: Fluids Revision

University of the West of England
BRISTOL

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Today's Lecture

- Manipulating Bernoulli's Equation
- Laminar Flow
- Turbulent Flow
- Minor Losses
- Fluid Machines
- Fluid Momentum

Bernoulli's Equation

- Always start with Bernoulli's Equation*

Static pressures

Hydrostatic pressures

$$p_1 + \frac{1}{2}\rho C_1^2 + \rho g z_1 + \Delta p_P = p_2 + \frac{1}{2}\rho C_2^2 + \rho g z_2 + \Delta p_L$$

Dynamic pressure

Pressure rise due to pump

Pressure loss due to pump

* Except with fluid momentum

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Bernoulli's Equation

- Most problems: certain terms can be neglected:
  - Closed reservoirs:
    - $C_1 = C_2 = 0 \rightarrow$ neglect dynamic pressures
  - Open reservoirs:
    - $p_1 = p_2 = p_{atm} \rightarrow$ neglect static pressures
    - $C_1 = C_2 = 0 \rightarrow$ neglect dynamic pressures
  - Horizontal system:
    - z terms can be neglected

Bernoulli's Equation

- Most problems: certain terms can be neglected:
  - With height difference:
    - Lowest point: $z = 0$
    - Heighest point: z = height *difference*
  - No pump: $\Delta p_P = 0$

Laminar Flows

- Laminar:

  - Layers of adjacent fluid slide over each other
  - Streamlines are straight
  - Flow near wall slower than centre
  - Example: honey falling off spoon
  - **Reynolds number < 2000**

Laminar Flow

- Flow rate

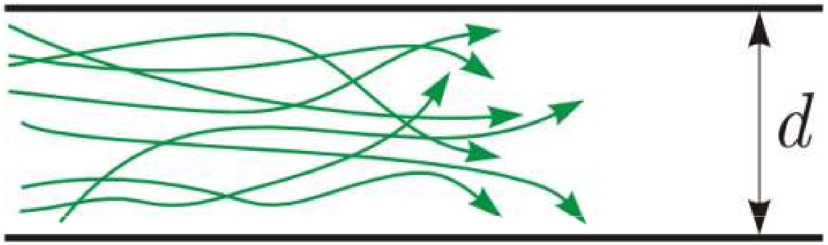
$$\dot{V} = \frac{\pi}{8\mu}\frac{\Delta p_L}{L}R^4$$

- Pressure drop

$$\Delta p_L = \frac{8\mu L\dot{V}}{\pi R^4}$$

Turbulent Flow

- Turbulent:



  - Particle paths irregular and chaotic
  - Large scale mixing
  - Flow in radial direction
  - Example: smoke billowing from chimney
  - **Reynolds Number > 3000**

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Turbulent Flow

- Pressure drop:

$$\Delta p = \frac{fL}{D}\frac{1}{2}\rho c^2$$

  - *L* = pipe length
  - *D* = pipe diameter
  - *C* = flow velocity
  - *f* **= friction factor**

Turbulent Flow

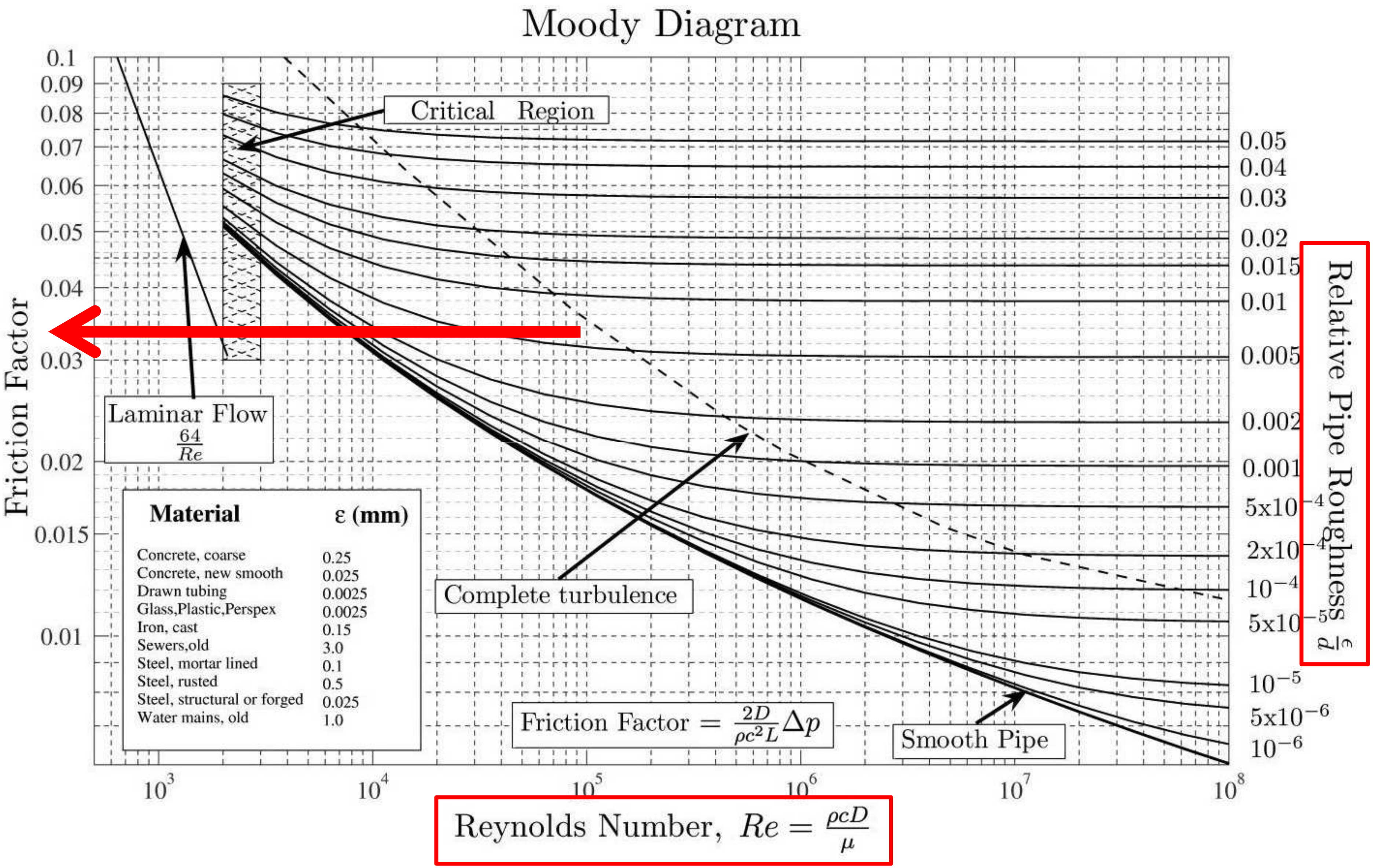
- Friction Factor:
  - Depends on
    - Reynolds number:

$$\mathrm{Re} = \frac{\rho C D}{\mu} = \frac{C D}{\upsilon}$$

    - Relative Roughness:

$$\text{Relative Roughness} = \frac{\varepsilon}{D}$$

  - Use ***Moody Chart*** to determine *f*



Moody Diagram
Friction Factor
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.015
0.01
Critical Region
Laminar Flow
$\frac{64}{Re}$
Material
ε (mm)
Concrete, coarse 0.25
Concrete, new smooth 0.025
Drawn tubing 0.0025
Glass,Plastic,Perspex 0.0025
Iron, cast 0.15
Sewers,old 3.0
Steel, mortar lined 0.1
Steel, rusted 0.5
Steel, structural or forged 0.025
Water mains, old 1.0
Complete turbulence
Friction Factor $= \frac{2D}{\rho c^2 L}\Delta p$
Smooth Pipe
Relative Pipe Roughness $\frac{\varepsilon}{d}$
0.05
0.04
0.03
0.02
0.015
0.01
0.005
0.002
0.001
5×10^{-4}
2×10^{-4}
10^{-4}
5×10^{-5}
10^{-5}
5×10^{-6}
10^{-6}
10^{3}
10^{4}
10^{5}
10^{6}
10^{7}
10^{8}
Reynolds Number, $Re = \frac{\rho c D}{\mu}$

Minor Losses

- Pressure drop caused by
  - Frictional effects in straight pipes
  - What about other components?
    - Bends
    - Entrances
    - Exits
    - Section Changes
    - Junctions
    - Filters
    - Valves

All contribute to pressure drop

Minor Losses

- Systems with more than one loss:
  - Effective *k* is sum of *k* factors

$$k_e = k_1 + k_2 + \cdots + k_n = \sum_{i=1}^{n} k_n$$

$$\boxed{\Delta p = k_e \frac{1}{2}\rho c^2 = \sum_{i=1}^{n} k_n \frac{1}{2}\rho c^2}$$

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Minor Losses

- Total pressure drop in pipe due to minor losses *and* friction:

$$\Delta p = \text{drop due to friction} + \text{drop due to losses}$$

$$\Delta p = \frac{fL}{D}\frac{1}{2}\rho c^2 + k_e \frac{1}{2}\rho c^2$$

$$\boxed{\Delta p = \left(\frac{fL}{D} + k_e\right)\frac{1}{2}\rho c^2}$$

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Type 1 Problem

- Given:
  - Diameter – use to calculate Reynolds number and relative roughness → Friction factor
- Apply equation to determine Δ*p*

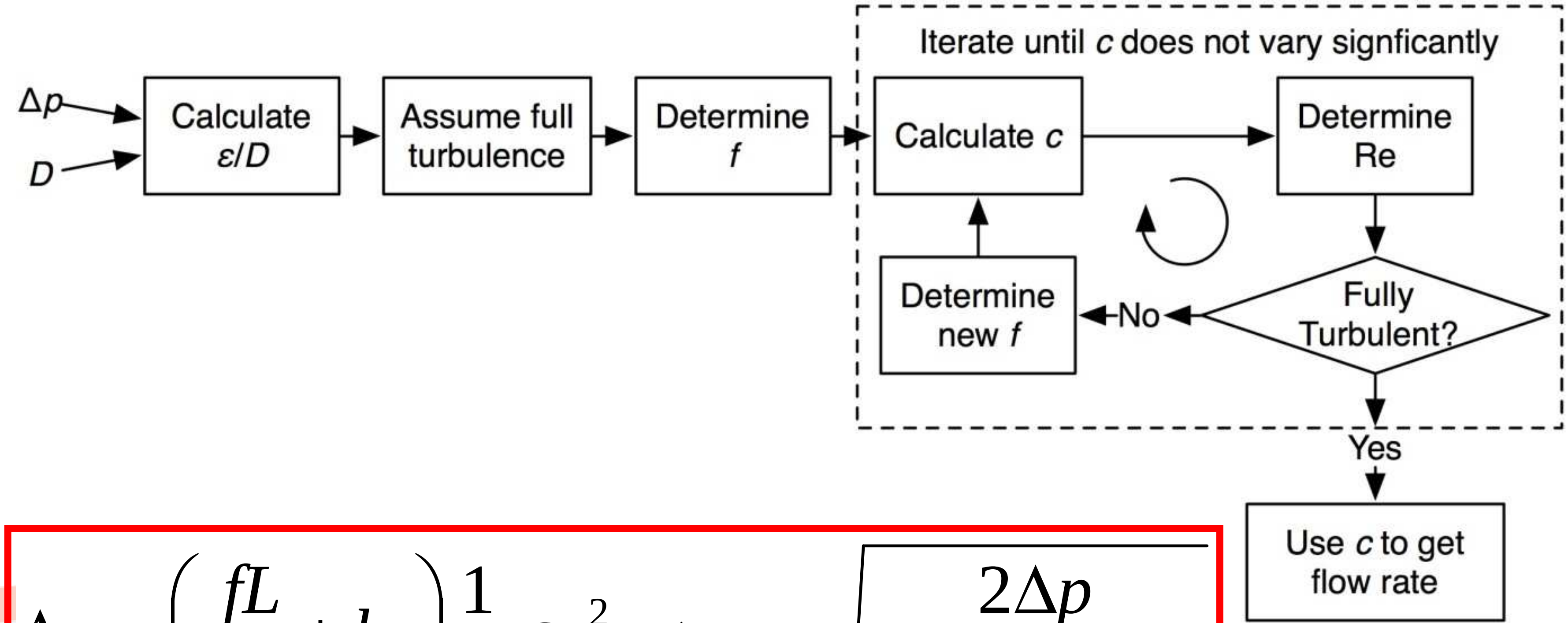
$$\Delta p = \left(\frac{fL}{D} + k_e \right) \frac{1}{2} \rho c^2$$

Type 2 Problem

- Given:
  - Δp and Pipe diameter
- Need to find flow rate ($\dot{V} = Ac$)
  - We don't have *c*, so cannot determine Re
  - How to find *f*?
  - Need to assume full turbulence (no need for Reynolds number), then check answer
  - Iteration if necessary

Type 2 Problem

- Flow chart:



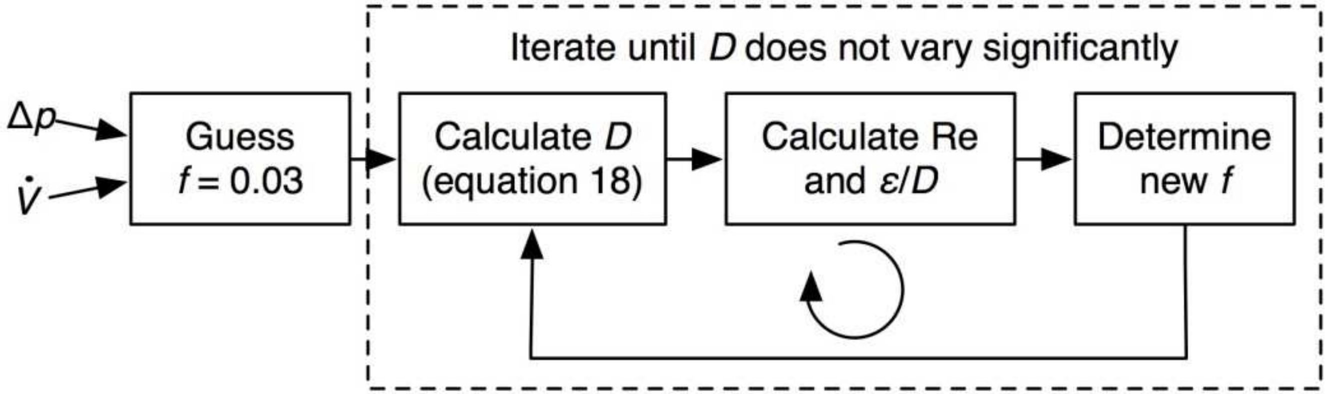
$$\Delta p = \left(\frac{fL}{D} + k_e\right)\frac{1}{2}\rho c^2 \rightarrow c = \sqrt{\frac{2\Delta p}{\rho\left(\frac{fL}{D} + k_e\right)}}$$

Type 3 Problem

- Given:
  - Δp and $\dot{V}$
- Need to find D
  - We can calculate neither Re nor ε/D
  - Assume $f = 0.03$ (middle of Moody)
  - Check solution and iterate if necessary

Type 3 Problem

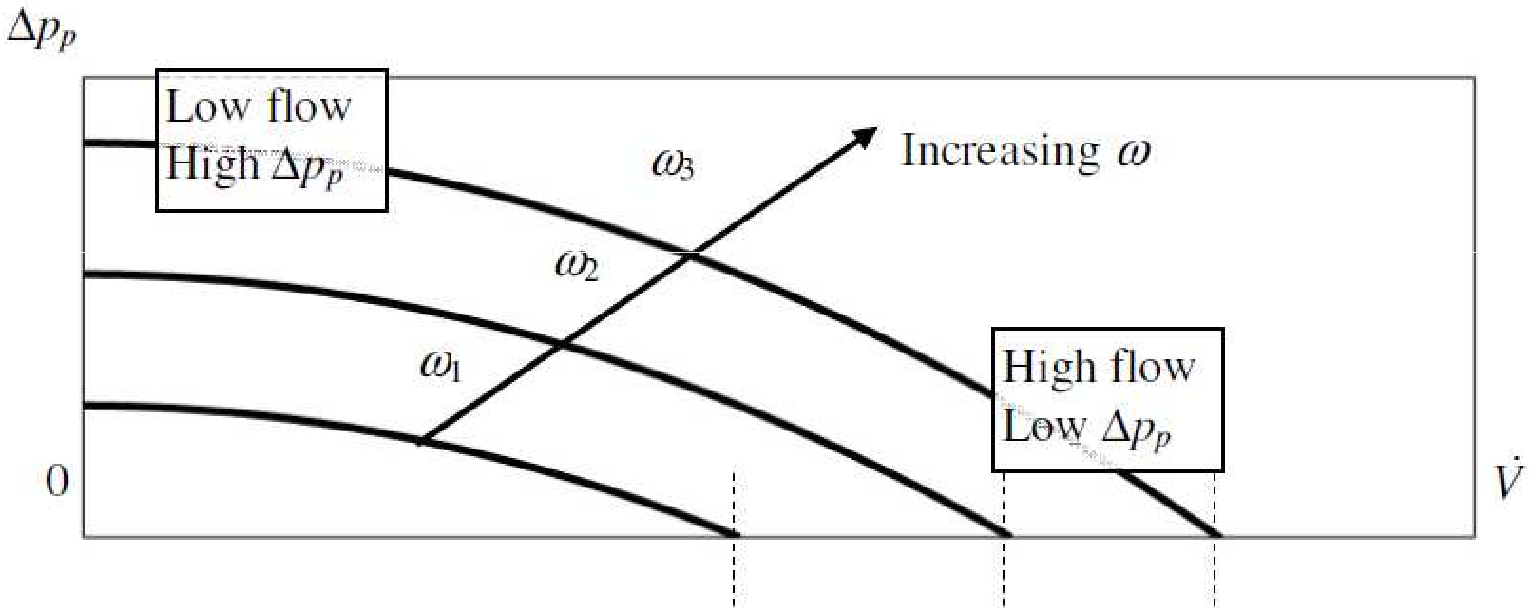
- Flow chart



$$\Delta p = \frac{fL}{D}\frac{1}{2}\rho C^2 = \frac{fL}{D^5}\frac{8\rho}{\pi^2}\dot{V}^2 \rightarrow D = \sqrt[5]{\frac{fL}{\Delta p}\frac{8\rho \dot{V}^2}{\pi^2}}$$

Fluid Machines: Pump

- **Pump Characteristic**



$$\Delta p_P = A_1 - A_2 \dot{V}^2$$

A_1 and A_2 are constants specific to pump

Fluid Machines: Pipe

- A and B are reservoirs, so c_A and c_B are zero:

$$p_A + \frac{1}{2}\rho c_A^2 + \rho g z_A + \Delta p_p = p_B + \frac{1}{2}\rho c_B^2 + \rho g z_B + \Delta p_L$$

$$p_A + \rho g z_A + \Delta p_p = p_B + \rho g z_B + \Delta p_L$$

$$\Delta p_p = \left(p_B - p_A\right) + \rho g\left(z_B - z_A\right) + \Delta p_L$$

$$\Delta p_p = \left(p_B - p_A\right) + \rho g h + \Delta p_L$$

- Pressure loss (excluding minor losses!):

$$\Delta p_L = \frac{fL}{D}\frac{1}{2}\rho c^2 = \frac{8 fL\rho}{\pi^2 D^5}\dot{V}^2$$

Fluid Machines: Pipe

- Substituting:

$$\Delta p_p = \left(\left(p_B - p_A\right) + \rho g h\right) + \left(\frac{8 fL\rho}{\pi^2 D^5}\right) \times \dot{V}^2$$

Static lift (not dependent on flow rate) ↑ C_1

Flow rate dependent term ↑ C_2

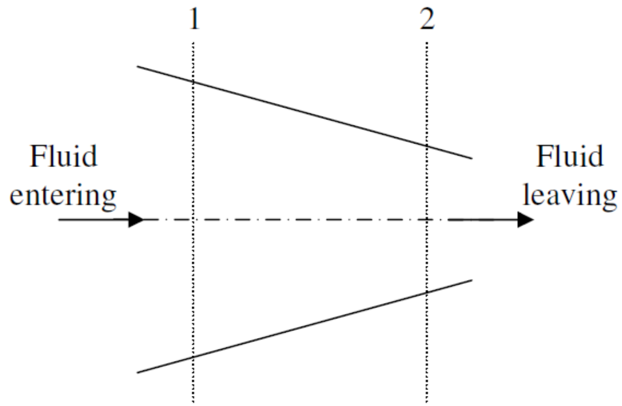
- **Pipe characteristic:** $\Delta p_P = C_1 + C_2 \dot{V}^2$

Fluid Machines: Operating Point

- Two equations for Δp_P:
  - For pump characteristic: $$\Delta p_P = A_1 - A_2 \dot{V}^2$$
  - For pipe characteristic: $$\Delta p_P = C_1 + C_2 \dot{V}^2$$
  - Operating point is where these two are equated:

$$A_1 - A_2 \dot{V}^2 = C_1 + C_2 \dot{V}^2$$

Fluid Momentum



- Force acting ***on*** the fluid is
  - Mass flow rate multiplied by
  - Change in velocity

$$F = \dot{m}\left(C_2 - C_1\right)$$

- So force ***on object*** by the fluid is:

$$-F = -\dot{m}\left(C_2 - C_1\right)$$

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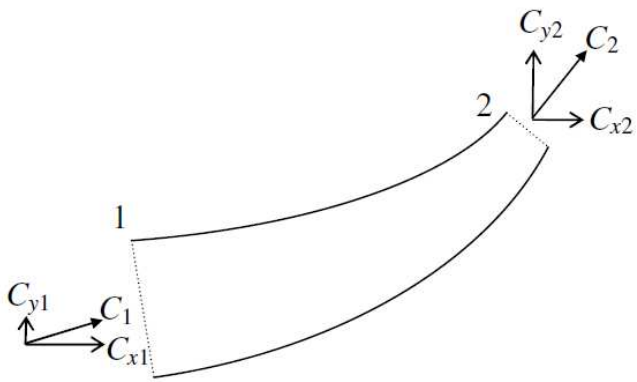
Fluid Momentum



- Use components:
  - C_{x1} and C_{x2} → $F_x = \dot{m}(C_{x2} - C_{x1})$
  - C_{y1} and C_{y2} → $F_y = \dot{m}(C_{y2} - C_{y1})$

Summary

- Bernoulli's Equation:
  - know how to manipulate it!

$$p_1 + \frac{1}{2}\rho C_1^2 + \rho g z_1 + \Delta p_P = p_2 + \frac{1}{2}\rho C_2^2 + \rho g z_2 + \Delta p_L$$

- Pressure Loss:

$$\Delta p = \left(\frac{fL}{D} + k_e\right)\frac{1}{2}\rho c^2$$

  - *f* determined from Reynolds number & *ε/D*
  - Know how to manipulate it!

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Summary

- Fluid Machines:
  - Pump characteristic has form:

$$\Delta p_P = A_1 - A_2 \dot{V}^2$$

  - Pipe characteristic has form:

$$\Delta p_P = C_1 + C_2 \dot{V}^2$$

  - Equate to find operating point and flow rate

$$A_1 - A_2 \dot{V}^2 = C_1 + C_2 \dot{V}^2$$

Summary

- Fluid Momentum:
  - allows us to calculate forces **on** fluid *and* **on object** by fluid

$$F = \dot{m}\left(C_2 - C_1\right)$$

  - Use components if there is a direction change:

$$F_x = \dot{m}\left(C_{x2} - C_{x1}\right)$$

$$F_y = \dot{m}\left(C_{y2} - C_{y1}\right)$$

Don’t Forget!

- Volumetric Flow Rate:

$$\dot{V} = AC$$

- Mass Flow Rate

$$\dot{m} = \rho AC$$

- Area of Flow

$$A = \frac{\pi D^2}{4}$$