UWE Bristol

Thermodynamics & Fluids

UFMEQU-20-1

FLUIDS

Lecture 2: Turbulent Flow

University of the West of England
BRISTOL

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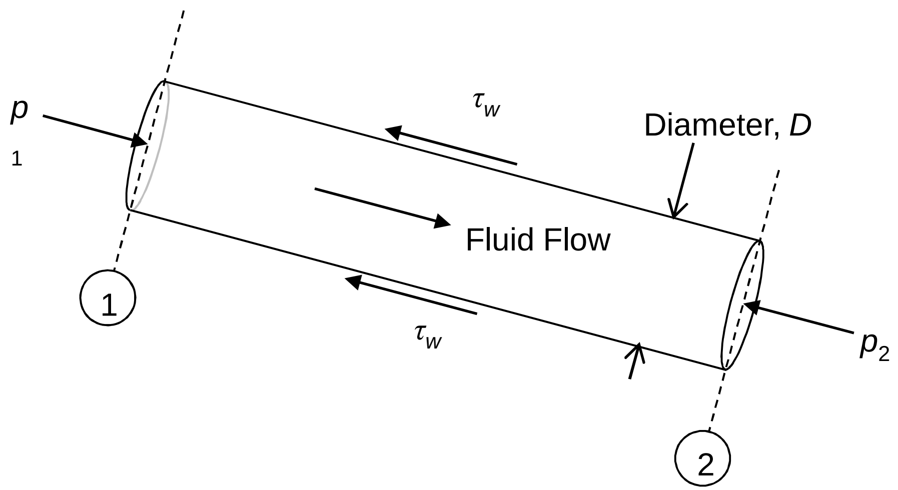
Today's Lecture

- Last week: Laminar Flow

- This week: Turbulent Flow
- Shear stress
- Pressure drop
- Friction factor
- Determining *f* (Moody Chart)
- Methods to solve problems

Fluid Flow with Friction

- Same situation:



- Bernoulli's equation:

$$p_1 + \frac{1}{2}\rho C_1^2 + \rho g z_1 = p_2 + \frac{1}{2}\rho C_2^2 + \rho g z_2 + \Delta p$$

Δp = pressure drop

- How do we determine Δp?

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Shear Stress

- Experimental results differ from laminar flow
- Dimension analysis to determine relationship

Variable	*Description*
τ_w	Wall shear stress (the variable we wish to investigate)
ρ	Density
μ	Viscosity
c	Average velocity
D	Pipe diameter (bore)
ε	Pipe roughness

$$\tau_w = \varphi(\rho, \mu, c, D, \varepsilon) \rightarrow \tau_w = K\rho^a \mu^b c^d D^e \varepsilon^f$$

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Shear Stress

- Go through dimensional analysis method:

$$\mathrm{MLT^{-2}} = \left(\mathrm{ML^{-3}}\right)^a \left(\mathrm{ML^{-1}T^{-1}}\right)^b \left(\mathrm{LT^{-1}}\right)^d \left(\mathrm{L}\right)^e \left(\mathrm{L}\right)^f$$

Equating powers of each dimension on both sides:

For M: $1 = a + b$

For L: $-1 = -3a - b + d + e + f$

For T: $-2 = -b - d$

$$a = 1 - b$$

$$d = 2 - b$$

$$e = 3a + b - d - f - 1 = 3(1 - b) + b - (2 - b) - f - 1$$

$$e = -b - f$$

Shear Stress

$$\tau_w = K\rho^a \mu^b c^d D^e \varepsilon^f = K\rho^{1-b} \mu^b c^{2-b} D^{-b-f} \varepsilon^f$$

$$\tau_w = K\rho c^2 \left(\frac{\mu}{\rho c D}\right)^b \left(\frac{\varepsilon}{D}\right)^f$$

- Rearranging:

$$\frac{\tau_w}{\frac{1}{2}\rho c^2} = 2K \left(\frac{\rho c D}{\mu}\right)^{-b} \left(\frac{\varepsilon}{D}\right)^f$$

Non-dimensional wall shear stress

Reynolds Number

Relative Roughness

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Relative Roughness

- Measure of how rough pipe wall is
- Measured in metres (or mm)
- Defined by material and manufacture method
- Represented by Epsilon ε
- Relative Roughness is ε/D
- Non-dimensional (m/m)

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Pressure Drop

- Shear stresses are related to
  - Reynolds Number
  - Relative Roughness

$$\frac{\tau_w}{\frac{1}{2}\rho c^2} = F\left(\frac{\rho c D}{\mu}, \frac{\varepsilon}{D}\right)$$

$$\tau_w = \frac{D}{4}\frac{\Delta p}{L} = F\frac{1}{2}\rho c^2$$

$$F \rightarrow \tau_w = F\frac{1}{2}\rho c^2$$

Pressure Drop

- We have:

$$\frac{D}{4}\frac{\Delta p}{L} = F\frac{1}{2}\rho c^2$$

- Rearranging:

$$\Delta p = \frac{4FL}{D}\frac{1}{2}\rho c^2$$

- *F* is **FANNING FRICTION FACTOR**
- *f* is **DARCY FRICTION FACTOR = 4F**

Pressure Drop

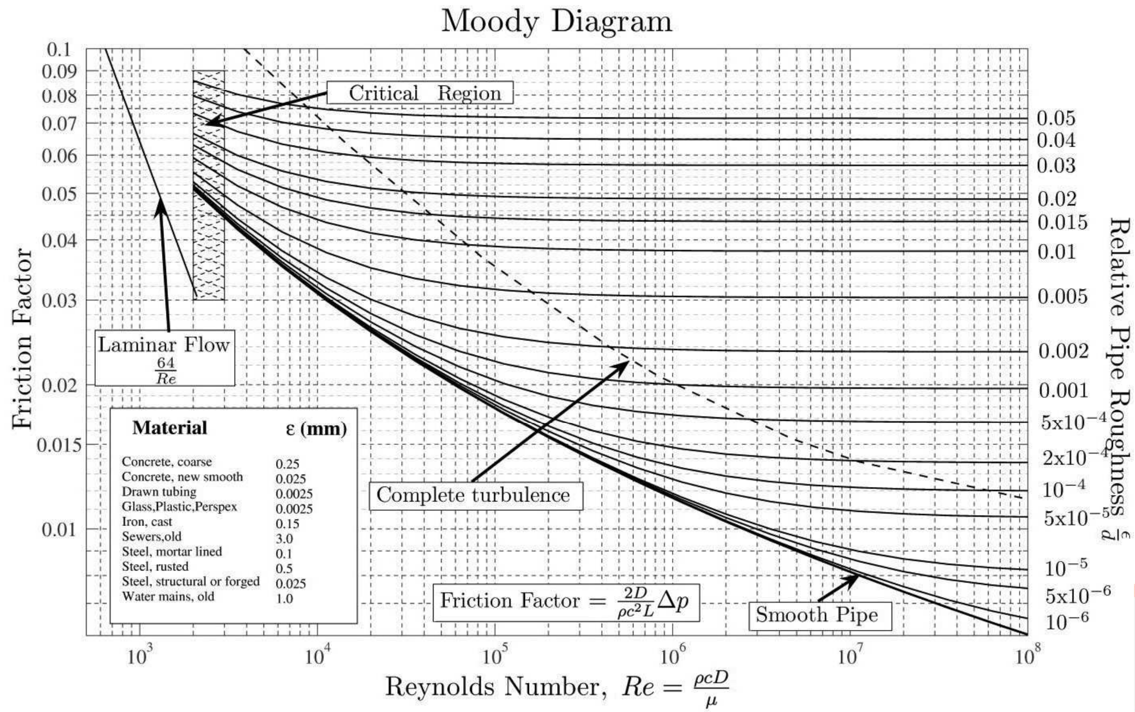
- Replacing Fanning friction factor with Darcy friction factor, *f*

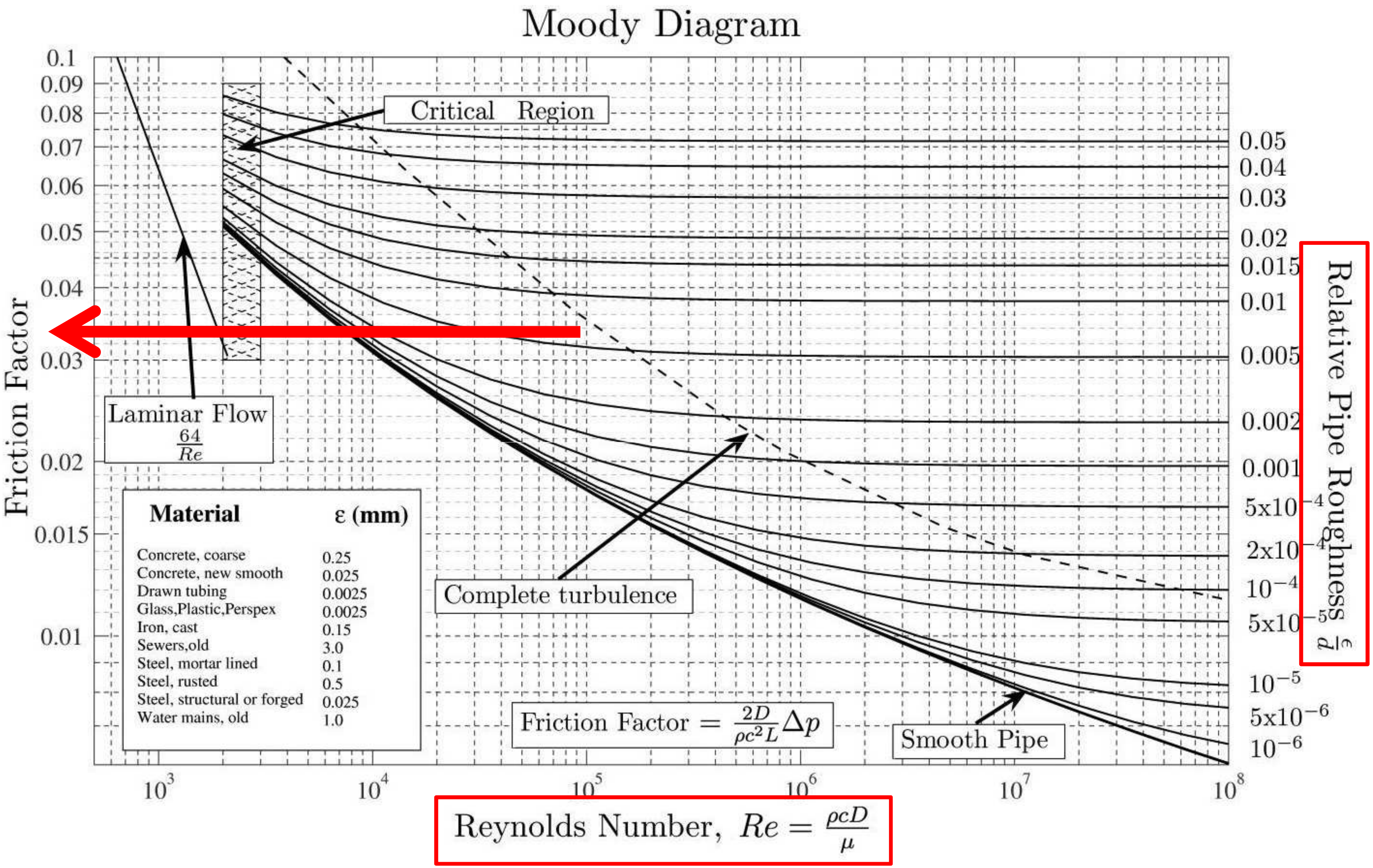
$$\Delta p = \frac{fL}{D}\frac{1}{2}\rho c^2$$

- If we have friction factor, we can calculate pressure drop to insert into Bernoulli's equation.

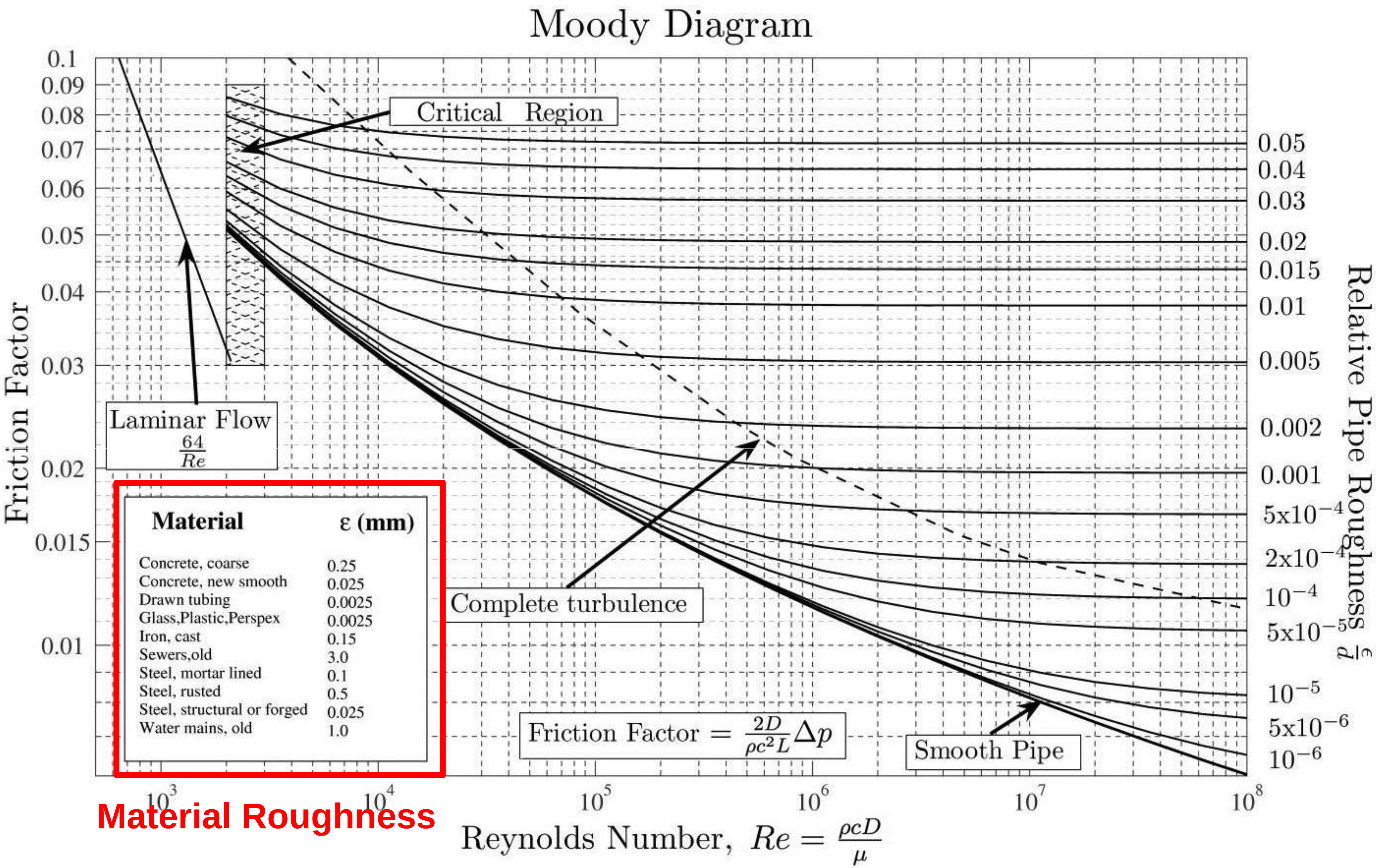
Friction Factor

- How do we determine Friction Factor?
- Moody Chart

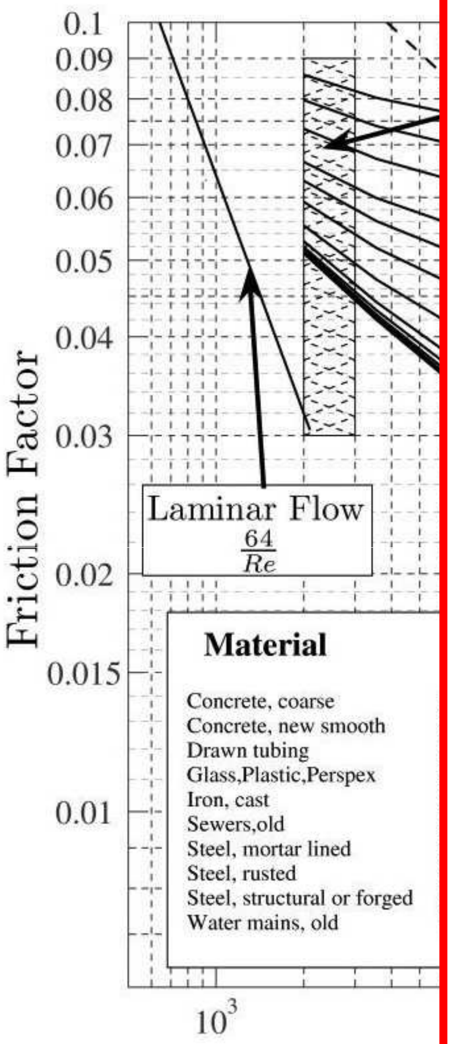




Moody Diagram
Critical Region
Laminar Flow
64/Re
Complete turbulence
Smooth Pipe
Friction Factor = 2D/(ρc²L) Δp
Friction Factor
Reynolds Number, Re = ρcD/μ
Relative Pipe Roughness ε/d
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.015
0.01
10^3
10^4
10^5
10^6
10^7
10^8
0.05
0.04
0.03
0.02
0.015
0.01
0.005
0.002
0.001
5x10^-4
2x10^-4
10^-4
5x10^-5
10^-5
5x10^-6
10^-6
Material
ε (mm)
Concrete, coarse 0.25
Concrete, new smooth 0.025
Drawn tubing 0.0025
Glass,Plastic,Perspex 0.0025
Iron, cast 0.15
Sewers,old 3.0
Steel, mortar lined 0.1
Steel, rusted 0.5
Steel, structural or forged 0.025
Water mains, old 1.0



Material Roughness



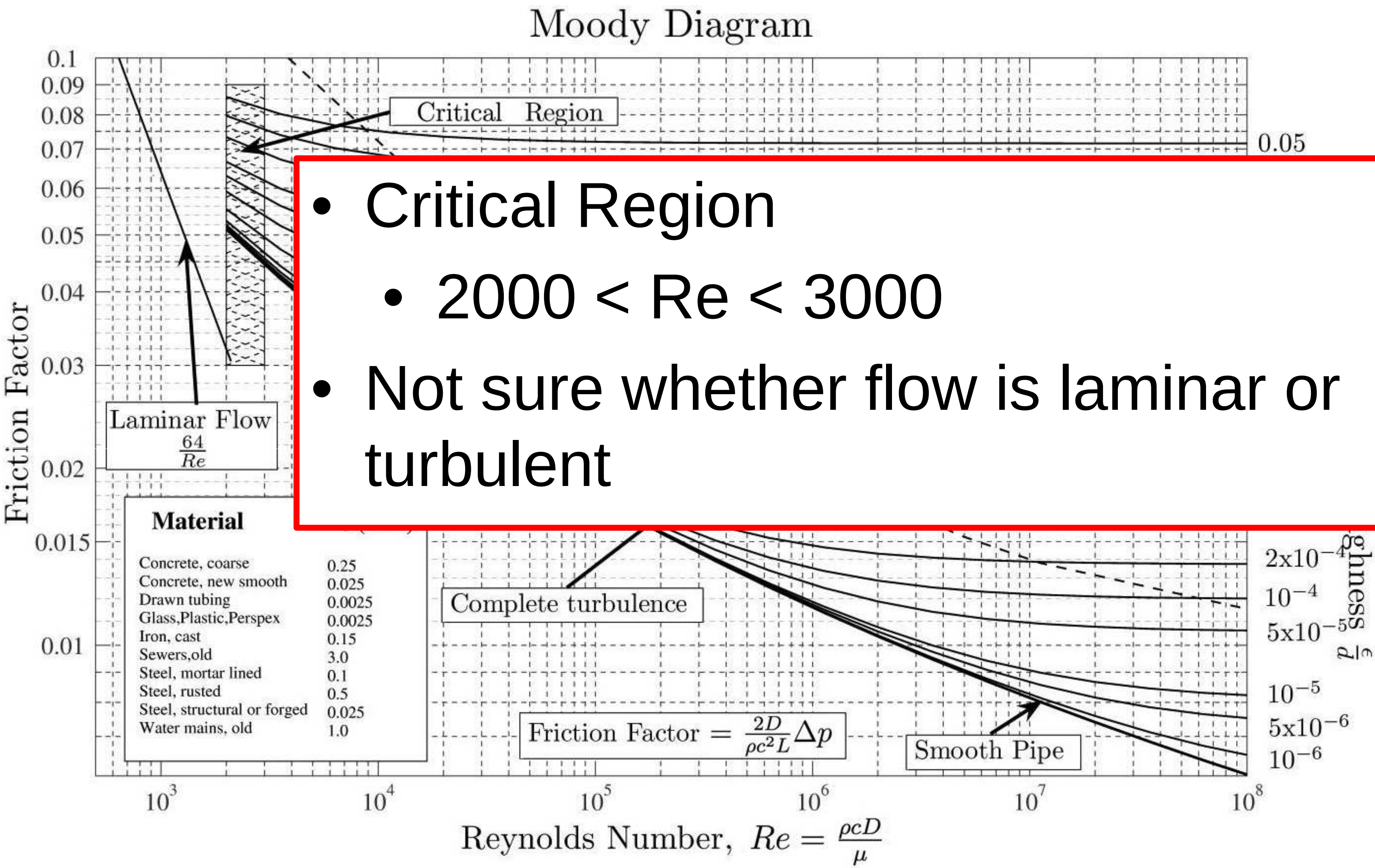
- Laminar Region (Re < 2000)

$$\Delta p = \frac{8\mu L\dot{V}}{\pi R^4} = \frac{fL}{D}\frac{1}{2}\rho c^2$$

$$\frac{32\mu Lc}{D^2} = \frac{fL}{D}\frac{1}{2}\rho c^2$$

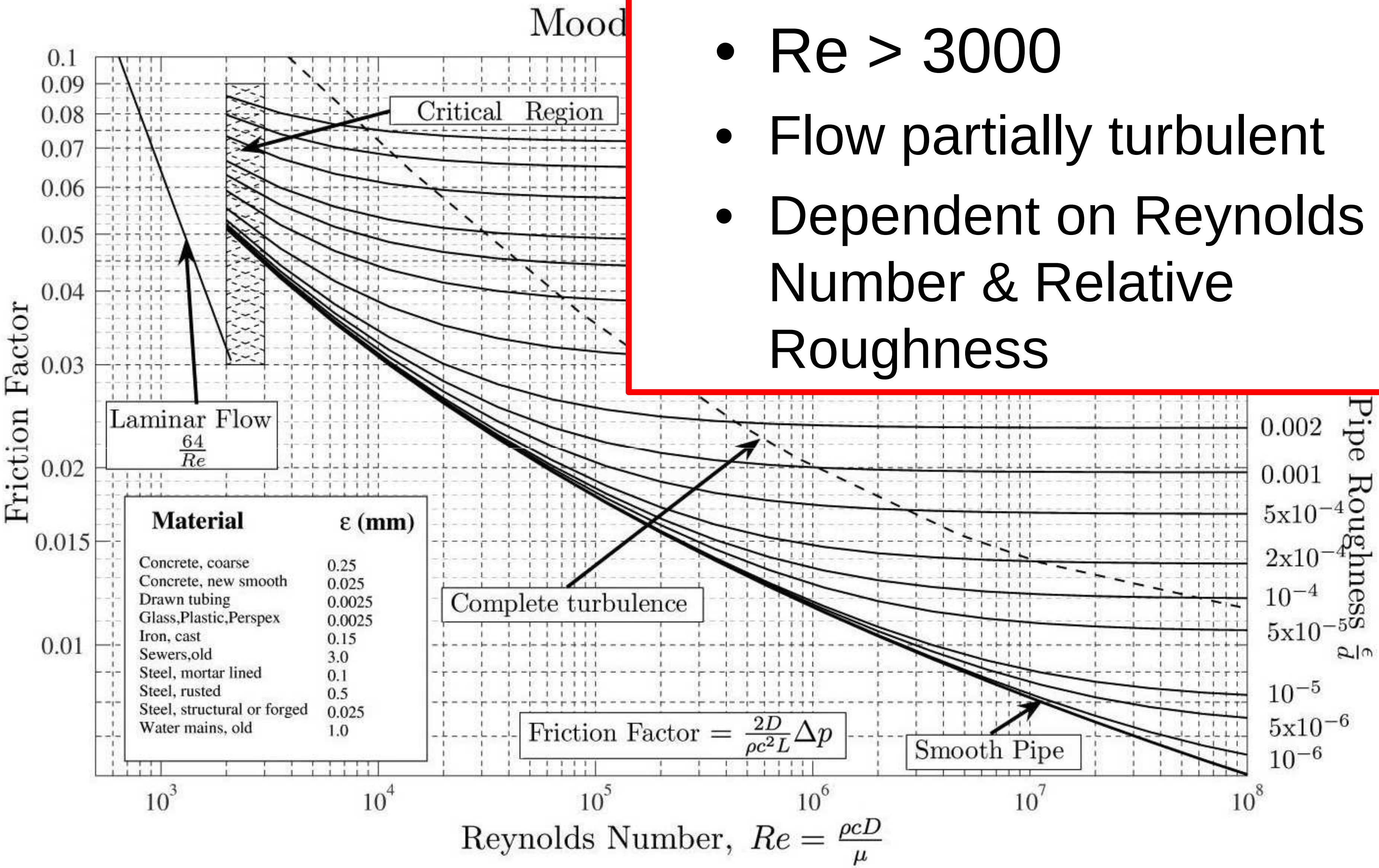
$$\boxed{f = \frac{64\mu}{\rho c D} = \frac{64}{\mathrm{Re}}}$$

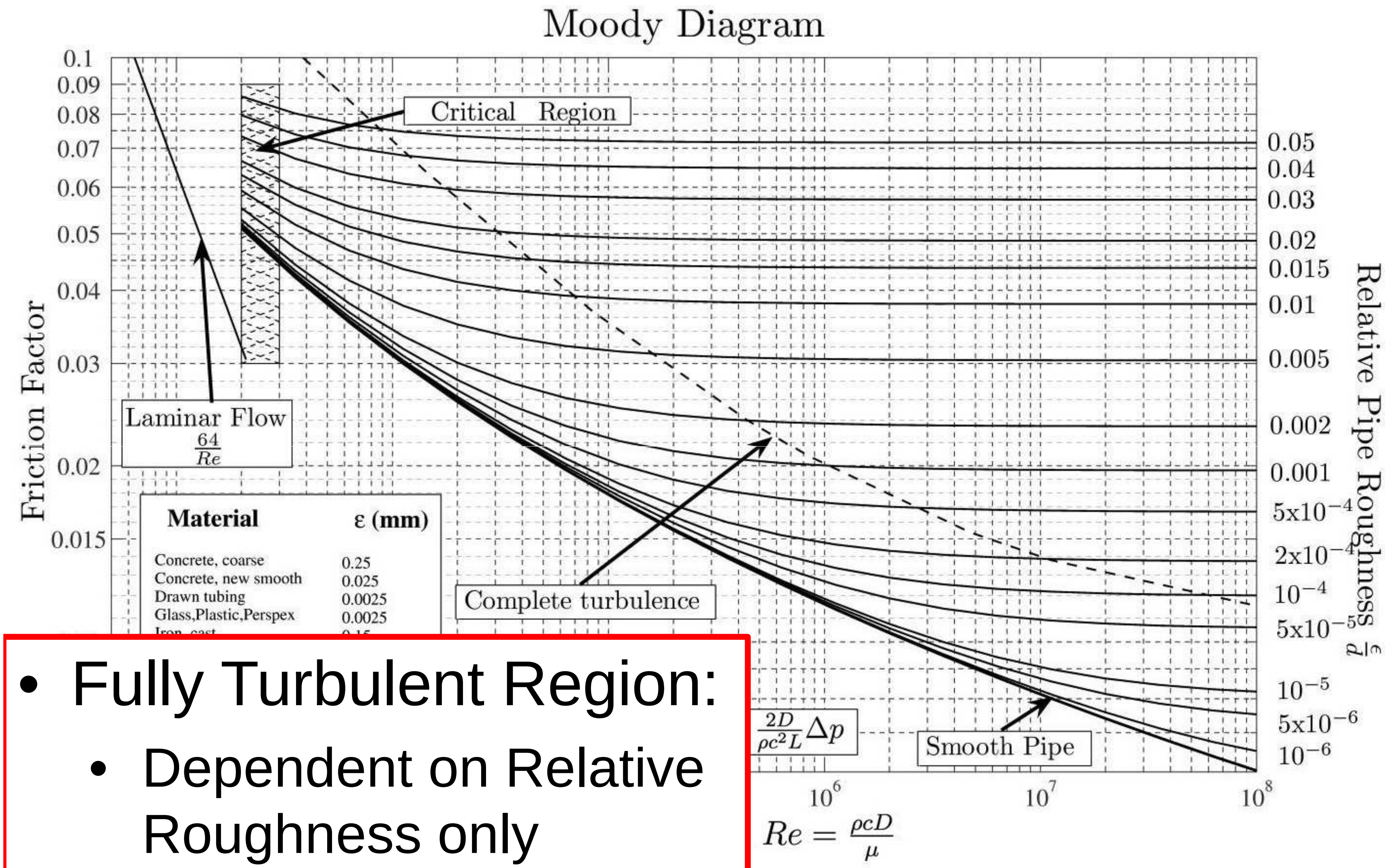
- So we can use equation from last lecture, or this one!



- Critical Region
  - 2000 < Re < 3000
- Not sure whether flow is laminar or turbulent

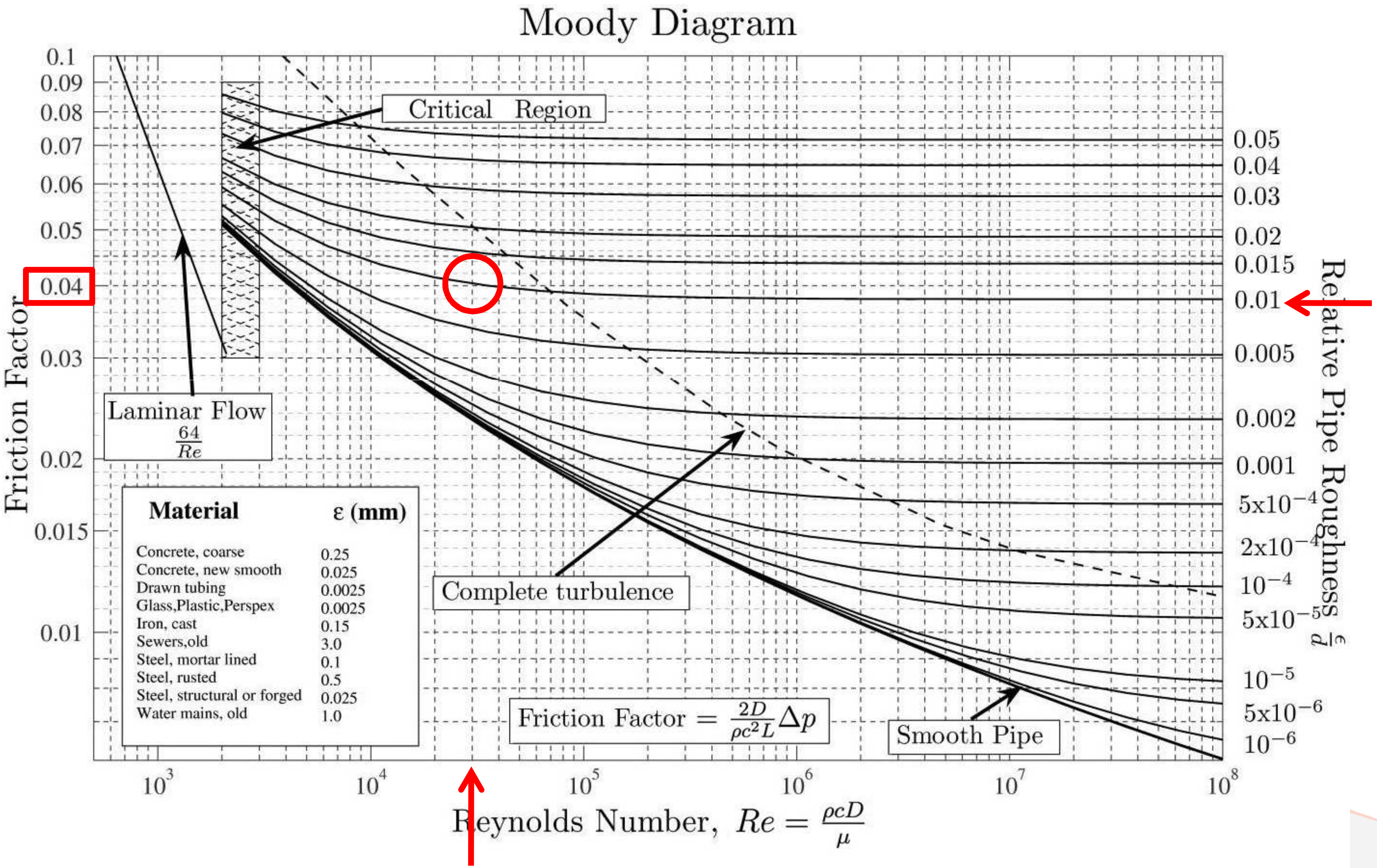
- Transition Region
  - Re > 3000
  - Flow partially turbulent
  - Dependent on Reynolds Number & Relative Roughness



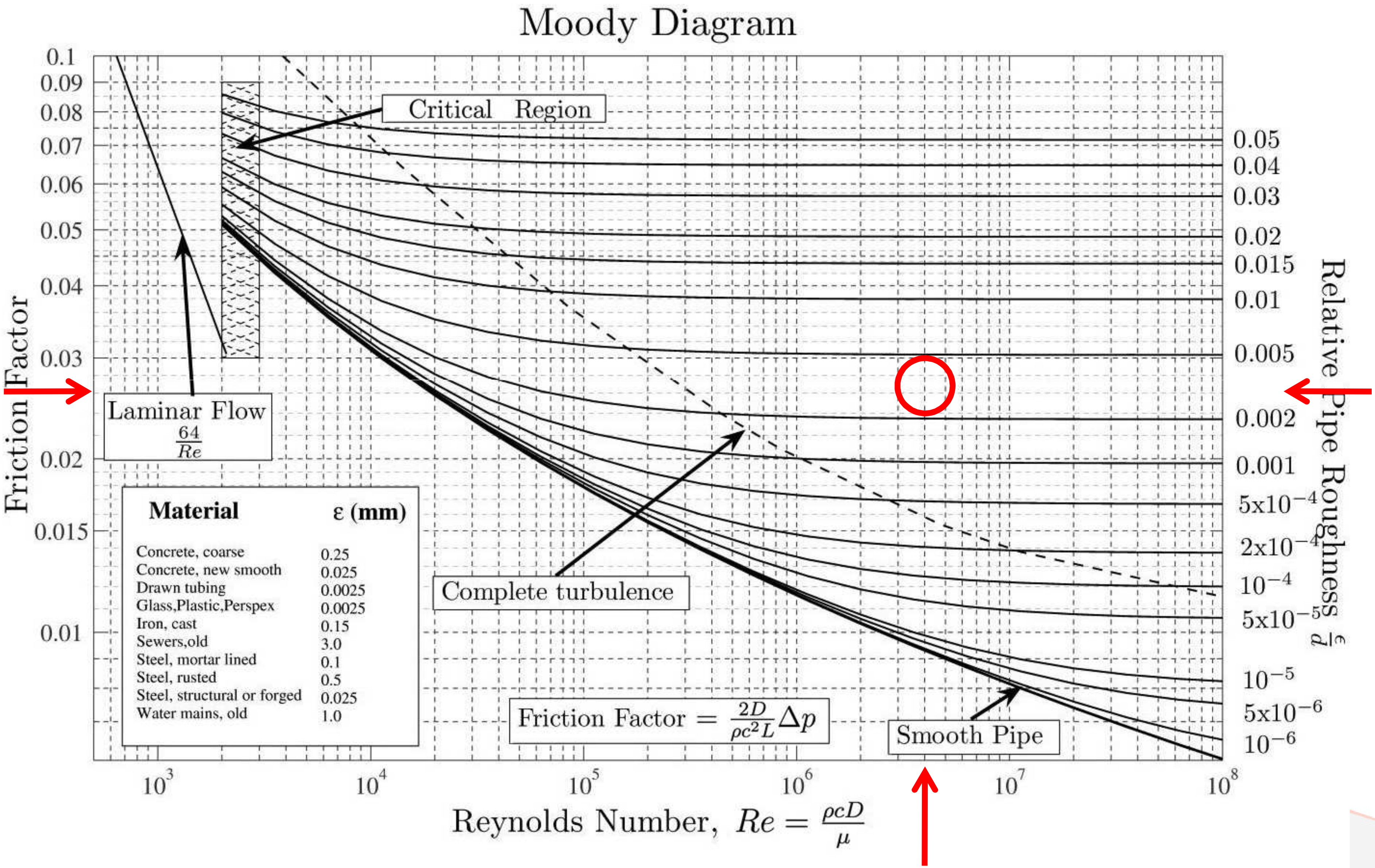


- Fully Turbulent Region:
  - Dependent on Relative Roughness only (straight lines)

- Example: Re = 3 x 10^4 and ε/D= 0.01



- Example: Re = 4 x 10^6 and ε/D= 0.003



Problems

- 3 Types of Problems
  - Type 1 – Pressure drop
    - (Given flow rate and diameter)
    - Know everything so apply equation
  - Type 2 – Flow rate
    - (Given pressure drop and diameter)
    - Don't know *c*, so cannot find Re!
  - Type 3 – Pipe Diameter
    - (Given pressure drop and flow rate)
    - Don't know *D*, so cannot find Re or ε/D!

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Type 1 Problem

- Given:
  - Diameter – use to calculate Reynolds number and relative roughness → Friction factor
- Apply equation to determine Δ*p*

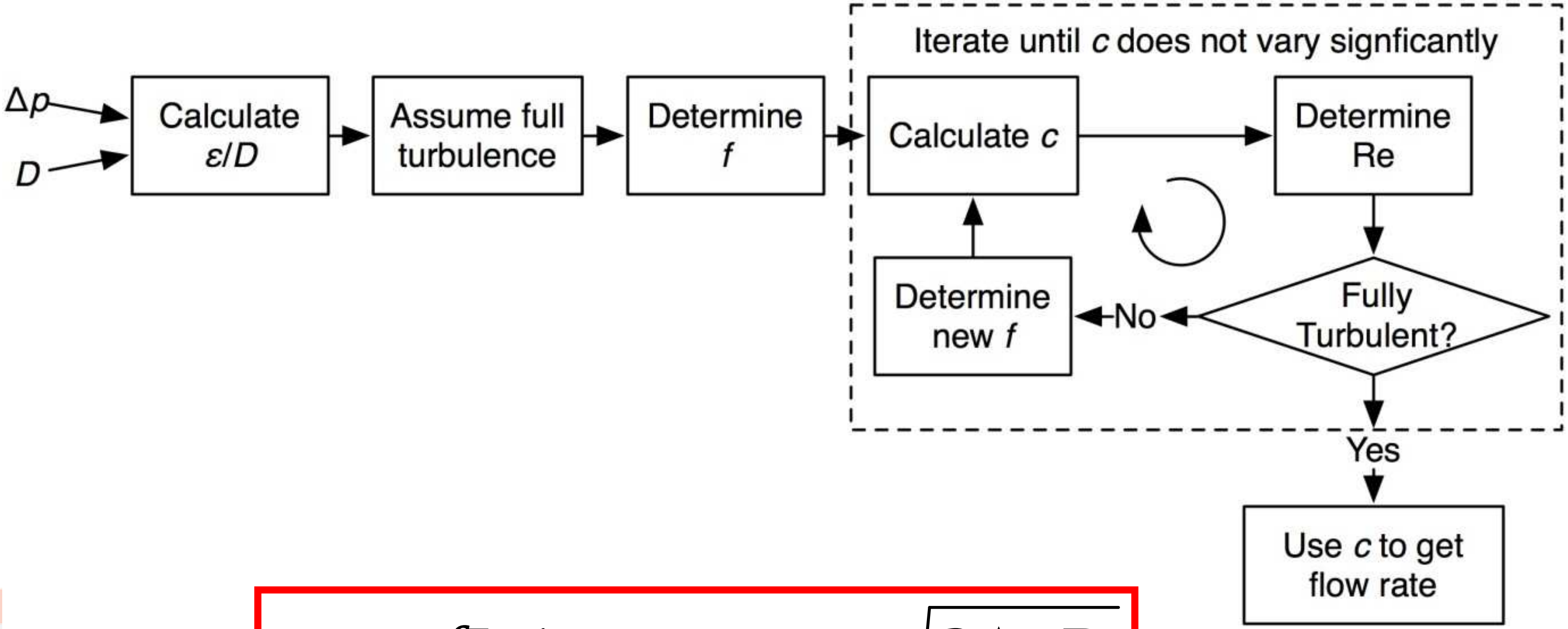
$$\Delta p = \frac{fL}{D}\frac{1}{2}\rho c^2$$

Type 2 Problem

- Given:
  - Δp and Pipe diameter
- Need to find flow rate ($\dot{V} = Ac$)
  - We don't have *c*, so cannot determine Re
  - How to find *f*?
  - Need to assume full turbulence (no need for Reynolds number), then check answer
  - Iteration if necessary

Type 2 Problem

- Flow chart:



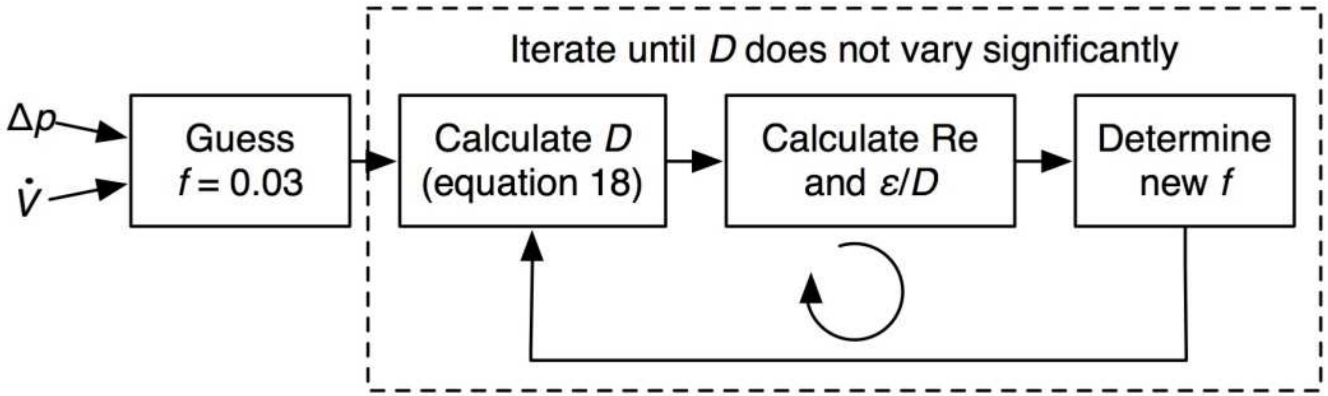
$$\Delta p = \frac{fL}{D}\frac{1}{2}\rho c^2 \rightarrow c = \sqrt{\frac{2\Delta p D}{\rho f L}}$$

Type 3 Problem

- Given:
  - Δp and $\dot{V}$
- Need to find *D*
  - We can calculate neither Re nor ε/D
  - Assume $f = 0.03$ (middle of Moody)
  - Check solution and iterate if necessary

Type 3 Problem

- Flow chart



$$\Delta p = \frac{fL}{D^5}\frac{8\rho}{\pi^2}\dot{V}^2 \rightarrow D = \sqrt[5]{\frac{fL}{\Delta p}\frac{8\rho\dot{V}^2}{\pi^2}}$$

Example

- Water flows through a 150 mm diameter pipe for which the relative roughness, ε/D is 0.0002 at a rate of 0.1 m^3/s. Calculate the pressure drop over a 100 m length of pipe. Take μ = 0.001 kg/ms.

Example

- Water flows through a 150 mm diameter pipe for which the relative roughness, ε/D is 0.0002 at a rate of 0.1 m^3/s. **Calculate the pressure drop** over a 100 m length of pipe. Take μ = 0.001 kg/ms.
  - **MUST BE TYPE 1 PROBLEM!**
- On Visualiser

Today's Lecture

- Turbulent flow
- Experimental analysis used to determine a 'friction factor', denoted by *f*
- Use Reynolds number and Relative Roughness to determine *f* from Moody Chart

Today's Lecture

- Three types of problems:
  - Type 1: use standard equation: $$\Delta p = \frac{fL}{D}\frac{1}{2}\rho c^2$$
  - Type 2: assume full turbulence then iterate to find c, and hence flow rate: $$\Delta p = \frac{fL}{D}\frac{1}{2}\rho c^2 \rightarrow c = \sqrt{\frac{2\Delta p D}{\rho f L}}$$
  - Type 3: assume *f* = 0.03, then iterate to find *D* $$\Delta p = \frac{fL}{D^5}\frac{8\rho}{\pi^2}\dot{V}^2 \rightarrow D = \sqrt[5]{\frac{fL}{\Delta p}\frac{8\rho\dot{V}^2}{\pi^2}}$$