UWE Bristol

Thermodynamics & Fluids

UFMEQU-20-1

FLUIDS

Lecture 1: Flow with Friction & Laminar Flow

University of the West of England

BRISTOL

bettertogether

Today's Lecture

- Fluid flow with Friction
- Nature of flows: Laminar and Turbulent
- Shear Stress and Fluid Viscosity
- Laminar Flow
  - Flow velocity
  - Flow Rate
  - Pressure Drop
  - Power
- Example

Fluid Flow with Friction

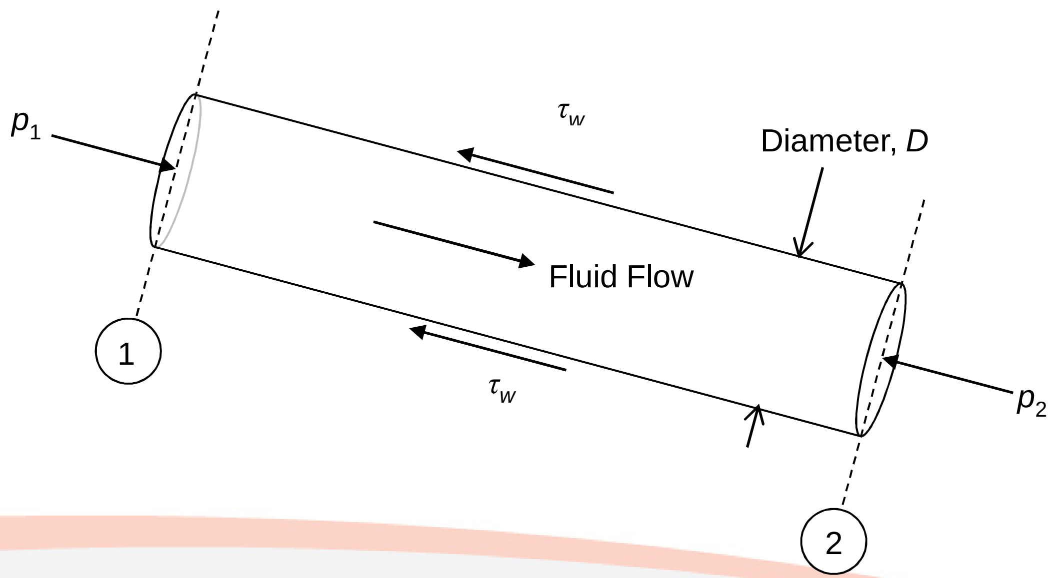
- Assumption has been no friction
- Bernoulli's Equation:

$$\text{Total pressure} = p + \frac{1}{2}\rho C^2 + \rho g z = \text{constant}$$

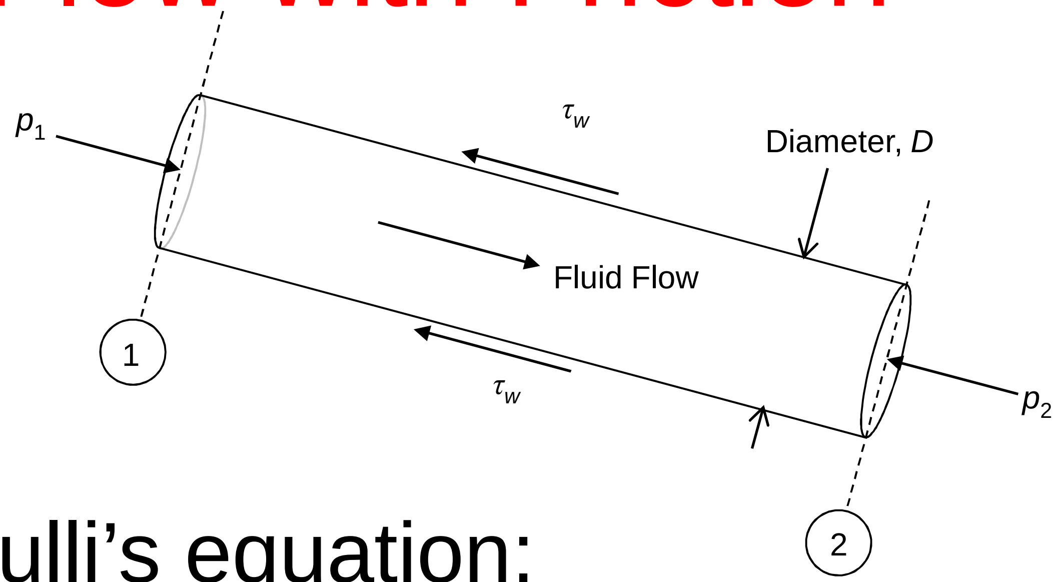
$$p_1 + \frac{1}{2}\rho C_1^2 + \rho g z_1 = p_2 + \frac{1}{2}\rho C_2^2 + \rho g z_2$$

Fluid Flow with Friction

- Now, shear stresses in fluid and with pipe
- Shear stress works against flow



Fluid Flow with Friction



- Bernoulli's equation:

$$p_1 + \frac{1}{2}\rho C_1^2 + \rho g z_1 = p_2 + \frac{1}{2}\rho C_2^2 + \rho g z_2 + \Delta p$$

Δp = pressure drop

- How do we determine Δp?

Nature of Flows

- Δp due to shear stresses
- Vary depending on nature of flow
- Two types (in general)
  - Laminar
  - Turbulent

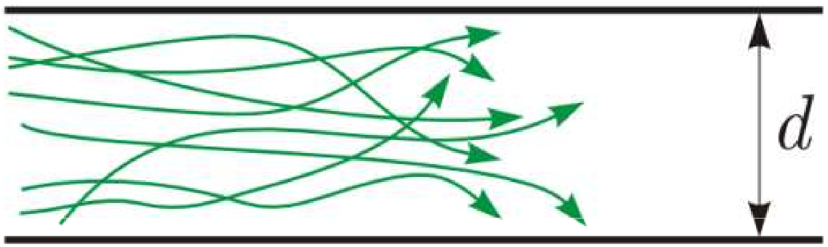
Laminar and Turbulent Flows

- Laminar:

  - Layers of adjacent fluid slide over each other
  - Streamlines are straight
  - Flow near wall slower than centre
  - Example: honey falling off spoon

Laminar and Turbulent Flow

- Turbulent:



  - Particle paths irregular and chaotic
  - Large scale mixing
  - Flow in radial direction
  - Example: smoke billowing from chimney

Laminar and Turbulent

- Transition between laminar and turbulent flow
- Related to fluid density, fluid viscosity, flow velocity and pipe diameter
- The Reynolds number

Flow velocity

Fluid density

Pipe diameter

$$\mathrm{Re} = \frac{\rho c D}{\mu}$$

Fluid viscosity

Laminar and Turbulent

- Reynolds number:

$$\mathrm{Re} = \frac{\rho c D}{\boxed{\mu}}$$

What is viscosity?

  – Non-dimensional number
  – Helps to determine nature of flow:

Reynolds Number	*Flow*
Less than approx. 2000	Laminar
2000 – 3000	Critical Region
Above 3000	Partial or fully turbulent

Shear Stress and Fluid Viscosity

- Relative movement of sliding fluids layers
- Shear stress:

$$\tau = \mu \times \frac{\text{difference in speed (between layers)}}{\text{distance between layers}}$$

- μ = dynamic viscosity
- As μ increases, shear stresses increase
  - High viscosity: honey, toothpaste
  - Low viscosity: water, alcohol
  - Generally, viscosity decreases with increase in temperature

Shear Stress and Fluid Viscosity

- μ = dynamic viscosity
- υ = kinematic viscosity
- Relationship:

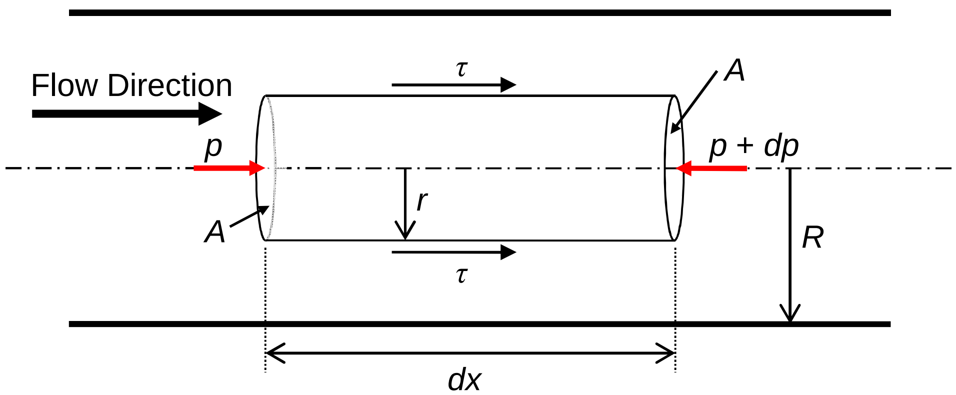
$$\upsilon = \frac{\mu}{\rho}$$

- Reynolds number:

$$\mathrm{Re} = \frac{\rho c D}{\mu} = \frac{c D}{\upsilon}$$

Laminar Flow – Flow Velocity

- Element of fluid
  - Pipe radius *R*
  - Element radius *r*
  - Length *dx*
  - Force on LHS: *pA*
  - Force RHS: –(*p* + *dp*)*A*
  - Shear stresses: τ
- Analysis on pp 6-7 shows that velocity in pipe is:



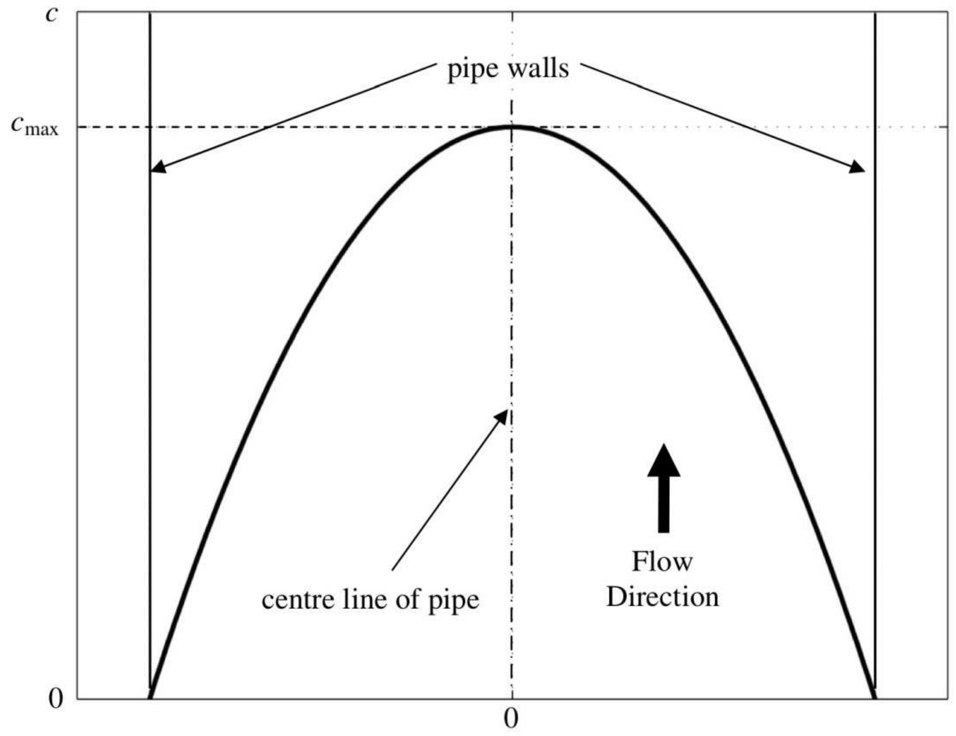
$$c = -\frac{R^2}{4\mu}\frac{dp}{dx}\left(1-\left(\frac{r}{R}\right)^2\right)$$

Laminar Flow – Flow Velocity

- Velocity:

$$c = -\frac{R^2}{4\mu}\frac{dp}{dx}\left(1-\left(\frac{r}{R}\right)^2\right)$$

  - Quadratic equation
  - Parabolic velocity profile
  - If $r = R$
    - velocity = 0
  - If $r = 0$
    - max velocity = c_{max}



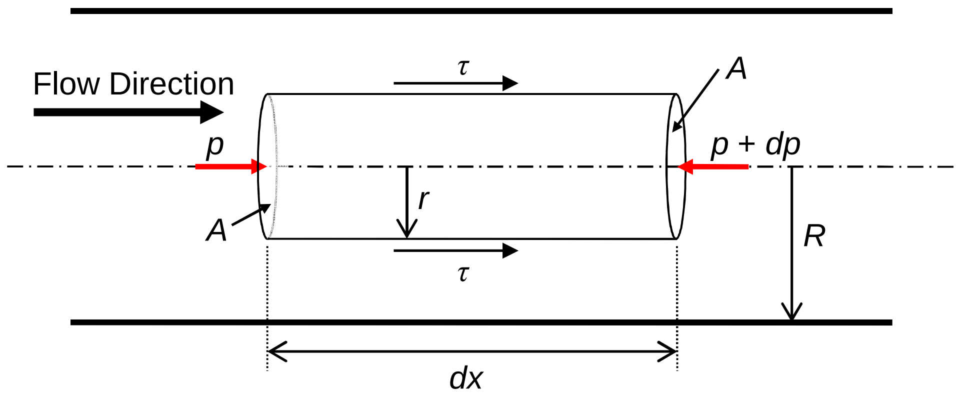
Laminar Flow – Volumetric Flow

- Volumetric Flow Rate: $\dot{V}$
  - Cannot just use $\dot{V} = Ac$
  - Velocity not constant across cross section
  - Analysis pp 8-9:

$$\dot{V} = -\frac{\pi}{8\mu}\frac{dp}{dx}R^4$$

Laminar Flow – Pressure Drop

- Fluid element:



$$p-(p+dp)=-dp$$

- Consider Pipe:

$$p_1-p_2=\Delta p \qquad dx=L$$

$$\frac{dp}{dx}=\frac{-\Delta p}{L}$$

Laminar Flow – Pressure Drop

- Using: $$\frac{dp}{dx} = \frac{-\Delta p}{L}$$
- Velocity:

$$c = -\frac{R^2}{4\mu}\frac{dp}{dx}\left(1-\left(\frac{r}{R}\right)^2\right) \Rightarrow c = \frac{R^2}{4\mu}\frac{\Delta p}{L}\left(1-\left(\frac{r}{R}\right)^2\right)$$

- Flow rate:

$$\dot{V} = -\frac{\pi}{8\mu}\frac{dp}{dx}R^4 \Rightarrow \dot{V} = \frac{\pi}{8\mu}\frac{\Delta p}{L}R^4$$

Laminar Flow – Pressure Drop

- From Flow Rate: $$\dot{V} = \frac{\pi}{8\mu}\frac{\Delta p}{L}R^4$$

- Rearranging:

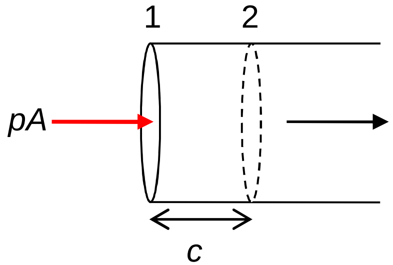
$$\Delta p = \frac{8\mu L\dot{V}}{\pi R^4}$$

- Bernoulli's Equation:

$$p_1 + \frac{1}{2}\rho C_1^2 + \rho g z_1 = p_2 + \frac{1}{2}\rho C_2^2 + \rho g z_2 + \Delta p$$

Laminar Flow – Power

- Consider force applied to fluid



- Power = force × velocity

$$\text{Force} = pA$$

$$\text{Velocity} = c$$

$$\therefore \text{Power} = pAc = p\dot{V} \rightarrow \boxed{\text{Power Loss} = \Delta p\dot{V}}$$

Example

Oil of density 900 kg/m^3 and viscosity 0.17 Pas is pumped through a 75 mm diameter pipe 750 m long at the rate of 2.7kg/s. If the critical Reynolds number is 2300, show that the critical velocity is not exceeded and calculate the pressure required at the pump and the power required. The pipe is horizontal.

Today's Lecture

- Fluid flow with friction:
  - Friction produces pressure drop
  - Bernoulli's equation: $p_1 + \frac{1}{2}\rho C_1^2 + \rho g z_1 = p_2 + \frac{1}{2}\rho C_2^2 + \rho g z_2 + \boxed{\Delta p}$
- Nature of flow determined by Reynolds number

$$\boxed{\mathrm{Re} = \frac{\rho c D}{\mu} = \frac{c D}{\upsilon}}$$

  - Re < 2000 → Laminar Flow
  - Re > 3000 → Turbulent Flow

Today's Lecture

- Flow velocity → $c = \frac{R^2}{4\mu}\frac{\Delta p}{L}\left(1-\left(\frac{r}{R}\right)^2\right)$
  - Parabolic profile
- Flow rate → $\dot{V} = \frac{\pi}{8\mu}\frac{\Delta p}{L}R^4$
- Pressure drop → $\Delta p = \frac{8\mu L\dot{V}}{\pi R^4}$
- Power Consumed → $\text{Power Consumed} = \Delta p\dot{V}$

Exercises on page 14