Solutions to Chapter 28 Problems

S.28.1

The solution is obtained directly from Eq. (28.9) in which $\partial c_1/\partial\alpha = a$, $\alpha = \alpha_0$ and $c_{m,0} = C_{M,0}$. Thus

$$\theta = \left(\frac{C_{M,0}}{ea} + \alpha_0\right)\left[\frac{\cos\lambda(s-y)}{\cos\lambda s} - 1\right]$$

which gives

$$\theta = \left(\frac{C_{M,0}}{ea} + \alpha_0\right)\frac{\cos\lambda(s-y)}{\cos\lambda s} - \frac{C_{M,0}}{ea} - \alpha_0$$

Thus

$$\theta + \alpha_0 = \left(\frac{C_{M,0}}{ea} + \alpha_0\right)\frac{\cos\lambda(s-y)}{\cos\lambda s} - \frac{C_{M,0}}{ea}$$

where

$$\lambda^2 = \frac{ea\frac{1}{2}\rho V^2 c^2}{GJ}$$

Also, from Eq. (28.11) the divergence speed V_d is given by

$$V_d = \sqrt{\frac{\pi^2 GJ}{2\rho ec^2 s^2 a}}$$

S.28.2

Since the additional lift due to operation of the aileron is at a distance hc aft of the flexural axis the moment equilibrium equation (28.25) for an elemental strip becomes

$$\frac{dT}{dy}\delta y - \Delta Lec - \Delta L_\xi hc = 0 \tag{i}$$

in which, from Eq. (28.23)

$$\Delta L = \frac{1}{2}\rho V^2 c\delta y\left[a_1\left(\theta - \frac{py}{V}\right) + a_2 f_a(y)\xi\right]$$

where $f_a(y) = 0$ for $0 \le y \le ks$ and $f_a(y) = 1$ for $ks \le y \le s$. Also

$$\Delta L_\xi = \tfrac{1}{2}\rho V^2 c\delta y a_2 f_a(y)\xi$$

Then, substituting for $T(= GJ\,d\theta/dy)$, ΔL and ΔL_ξ in Eq. (i) and writing $\lambda^2 = \rho V^2\, ec^2 a_1/2GJ$

$$\frac{d^2\theta}{dy^2} + \lambda^2\theta = \lambda^2\frac{py}{V} + \lambda^2\frac{h}{e}\frac{a_2}{a_1}f_a(y)\xi \tag{ii}$$

The solution of Eq. (ii) is obtained by comparison with Eq. (28.29). Thus

$$\theta_1(0 - ks) = \frac{p}{V}\left(y - \frac{\sin\lambda y}{\lambda\cos\lambda s}\right) - \frac{ha_2\xi}{ea_1}(\tan\lambda s\cos\lambda ks - \sin\lambda ks)\sin\lambda y \qquad \text{(iii)}$$

and

$$\theta_2(ks - s) = \frac{p}{V}\left(y - \frac{\sin\lambda y}{\lambda\cos\lambda s}\right) + \frac{ha_2\xi}{ea_1}(1 - \cos\lambda y\cos\lambda ks - \tan\lambda s\cos\lambda ks\sin\lambda y) \qquad \text{(iv)}$$

Then, from Eq. (28.32)

$$\int_0^{ks} a_1\left(\theta_1 - \frac{py}{V}\right)y\,\mathrm{d}y + \int_{ks}^{s} a_1\left(\theta_2 - \frac{py}{V}\right)y\,\mathrm{d}y = -a_2\xi\int_{ks}^{s} y\,\mathrm{d}y \qquad \text{(v)}$$

Substituting for θ_1 and θ_2 in Eq. (v) from Eqs (iii) and (iv) gives

$$-\tan\lambda s\int_0^s y\sin\lambda y\,\mathrm{d}y + \tan\lambda ks\int_0^{ks} y\sin\lambda y\,\mathrm{d}y - \int_{ks}^{s} y\cos\lambda y\,\mathrm{d}y + \frac{(e+h)}{h\cos\lambda ks}\int_{ks}^{s} y\,\mathrm{d}y$$
$$= \frac{pea_1}{ha_2\xi\lambda V\cos\lambda s\cos\lambda ks}\int_0^s y\sin\lambda y\,\mathrm{d}y$$

Hence the aileron effectiveness is given by

$$\frac{(ps/V)}{\xi} = \frac{-\tan\lambda s\int_0^s y\sin\lambda y\,\mathrm{d}y + \tan\lambda ks\int_0^{ks} y\sin\lambda y\,\mathrm{d}y - \int_{ks}^{s} y\cos\lambda y\,\mathrm{d}y + \frac{(e+h)}{2h\cos\lambda ks}[s^2 - (ks)^2]}{\frac{ea}{ha_2\lambda s\cos\lambda s\cos\lambda ks}\int_0^s y\sin\lambda y\,\mathrm{d}y} \qquad \text{(vi)}$$

The aileron effectiveness is zero, i.e. aileron reversal takes place, when the numerator on the right-hand side of Eq. (vi) is zero, i.e. when

$$\tan\lambda ks\int_0^{ks} y\sin\lambda y\,\mathrm{d}y - \tan\lambda s\int_0^s y\sin\lambda y\,\mathrm{d}y - \int_{ks}^{s} y\cos\lambda y\,\mathrm{d}y = \frac{(e+h)}{2h\cos\lambda ks}[(ks)^2 - s^2]$$