S.25.8

From Eq. (25.48)

$$GJ = 2 \times 16\,300 \times 50 \times \frac{1^3}{3} + 20\,900 \times 100 \times \frac{0.5^3}{3} = 6.3 \times 10^5\,\mathrm{N\,mm^2}$$

Then, from Eq. (25.49)

$$\frac{\mathrm{d}\theta}{\mathrm{d}z} = \frac{0.5 \times 10^3}{6.3 \times 10^5} = 0.8 \times 10^{-3}\,\mathrm{rad/mm}$$

From Eq. (25.50)

$$\tau_{\max}(\text{flanges}) = \pm 2 \times 16\,300 \times \frac{1.0}{2} \times 0.8 \times 10^{-3}$$

i.e.

$$\tau_{\max}(\text{flanges}) = \pm 13.0\,\mathrm{N/mm^2}$$

$$\tau_{\max}(\text{web}) = \pm 2 \times 20\,900 \times \frac{0.5}{2} \times 0.8 \times 10^{-3}$$

i.e.

$$\tau_{\max}(\text{web}) = \pm 8.4\,\mathrm{N/mm^2}$$

Therefore

$$\tau_{\max} = \pm 13.0\,\mathrm{N/mm^2}$$

The warping at 1 is, from Eq. (18.19)

$$W_1 = -2 \times \frac{1}{2} \times 50 \times 50 \times 0.8 \times 10^{-3} = -2.0\,\mathrm{mm}$$

Solutions to Chapter 26 Problems

S.26.1

In Fig. S.26.1 $\alpha = \tan^{-1}\ 127/305 = 22.6°$. Choose O as the origin of axes then, from Eq. (26.1), since all the walls of the section are straight, the shear flow in each wall is constant. Then

$$q_{12} = 1.625G(254\theta' - u') \qquad \text{(i)}$$

$$q_{23} = 1.625G(254\theta' \cos 22.6° - u' \cos 22.6° - v' \sin 22.6°)$$

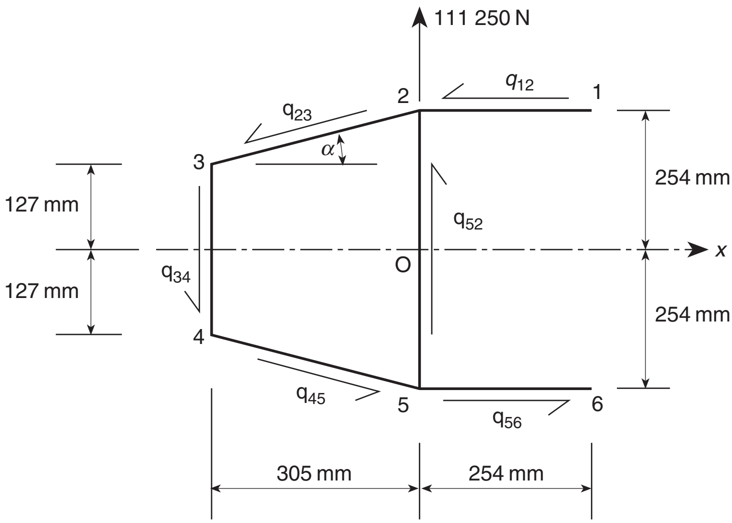

Fig. S.26.1

i.e.

$$q_{23} = 1.625G(234.5\theta' - 0.923u' - 0.384v') \tag{ii}$$

$$q_{34} = 2.03G(305\theta' - v') \tag{iii}$$

$$q_{52} = 2.54Gv' \tag{iv}$$

$$q_{45} = 1.625G(234.5\theta' + 0.923u' - 0.384v') \tag{v}$$

$$q_{56} = 1.625G(254\theta' + u') \tag{vi}$$

From symmetry $q_{12} = q_{56}$ and $q_{23} = q_{45}$ so that, from Eqs (i) and (vi) (or Eqs (ii) and (v)) $u' = 0$. Now resolving forces vertically

$$q_{52} \times 508 - q_{23} \times 127 - q_{34} \times 254 - q_{45} \times 127 = 111\,250$$

i.e.

$$508q_{52} - 2 \times 127q_{23} - 254q_{34} = 111\,250$$

Substituting for q_{52}, q_{23} and q_{34} from Eqs (iv), (ii) and (iii), respectively gives

$$v' - 129.3\theta' = \frac{56.63}{G} \tag{vii}$$

Now taking moments about O

$$2q_{12} \times 254 \times 254 + 2q_{23} \times 305 \times 254 + q_{34} \times 254 \times 305 = 0$$

Substituting for q_{12}, q_{23} and q_{34} from Eqs (i), (ii) and (iii), respectively gives

$$v' - 631.1\theta' = 0 \tag{viii}$$

Subtracting Eq. (viii) from (vii) gives

$$\theta' = \frac{0.113}{G} \quad \text{(ix)}$$

Hence, from Eq. (viii)

$$v' = \frac{71.2}{G} \quad \text{(x)}$$

Now substituting for θ' and v' from Eqs (ix) and (x) in Eqs (i)–(vi) gives

$$q_{12} = q_{56} = 46.6\,\text{N/mm} \quad q_{32} = q_{54} = 1.4\,\text{N/mm}$$
$$q_{43} = 74.6\,\text{N/mm} \quad q_{52} = 180.8\,\text{N/mm}$$

Finally, from Eq. (17.11)

$$x_R = -\frac{v'}{\theta'} = -\frac{71.2}{0.113} = -630.1\,\text{mm} \quad y_R = \frac{u'}{\theta'} = 0$$

S.26.2

In Fig. S.26.2, $\alpha = \tan^{-1} 125/300 = 22.6°$. Also, since the walls of the beam section are straight the shear flow in each wall, from Eq. (26.1), is constant. Choosing O, the mid-point of the wall 42, as the origin, then, from Eq. (26.1) and referring to Fig. S.26.2.

$$q_{51} = 1.6G(250\theta' + v') \quad \text{(i)}$$
$$q_{12} = 1.2G(125\theta' - u') \quad \text{(ii)}$$
$$q_{23} = 1.0G(125\theta' \cos 22.6° - u' \cos 22.6° - v' \sin 22.6°)$$

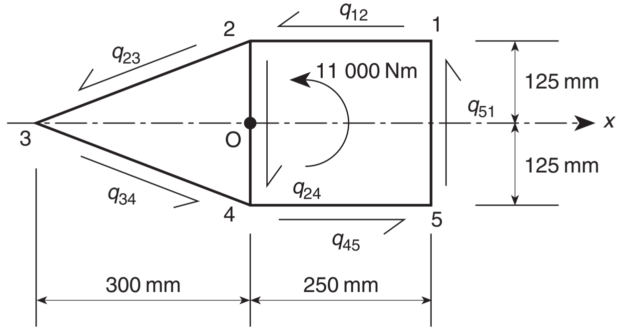

Fig. S.26.2

i.e.

$$q_{23} = 1.0G(115.4\theta' - 0.923u' - 0.384v') \qquad \text{(iii)}$$

$$q_{34} = 1.0G(115.4\theta' + 0.923u' - 0.384v') \qquad \text{(iv)}$$

$$q_{45} = 1.2G(125\theta' + u') \qquad \text{(v)}$$

$$q_{24} = 1.6G(-v') \qquad \text{(vi)}$$

From antisymmetry $q_{12} = q_{45}$ and $q_{23} = q_{34}$. Thus, from Eqs (ii) and (v) (or Eqs (iii) and (iv)), $u' = 0$. Resolving forces vertically

$$q_{51} \times 250 - q_{24} \times 250 - q_{23} \times 125 - q_{34} \times 125 = 0$$

i.e.

$$q_{51} - q_{24} - q_{23} = 0 \qquad \text{(vii)}$$

Substituting in Eq. (vii) for q_{51}, q_{24} and q_{23} from Eqs (i), (vi) and (iii), respectively gives

$$v' + 79.41\theta' = 0 \qquad \text{(viii)}$$

Now taking moments about O

$$2q_{12} \times 250 \times 125 + 2q_{23} \times 300 \times 125 + q_{51} \times 250 \times 250 = 11\,000 \times 10^3$$

i.e.

$$q_{12} + 1.2q_{23} + q_{51} = 176 \qquad \text{(ix)}$$

Substituting in Eq. (ix) for q_{12}, q_{23} and q_{51} from Eqs (ii), (iii) and (i), respectively gives

$$v' + 604.4\theta' = \frac{154.5}{G} \qquad \text{(x)}$$

Subtracting Eq. (x) from (viii) gives

$$\theta' = \frac{0.294}{G} \qquad \text{(xi)}$$

whence, from Eq. (viii)

$$v' = -\frac{23.37}{G} \qquad \text{(xii)}$$

Substituting for θ' and v' from Eqs (xi) and (xii) in Eqs (i)–(vi) gives

$$q_{51} = 80\,\text{N/mm} \quad q_{12} = q_{45} = 44.1\,\text{N/mm}$$

$$q_{23} = q_{34} = 42.9\,\text{N/mm} \quad q_{24} = 37.4\,\text{N/mm}$$

The centre of twist referred to O has coordinates, from Eq. (17.11)

$$x_R = -\frac{v'}{\theta'} = \frac{23.37}{0.294} = 79.5\,\text{mm} \quad y_R = \frac{u'}{\theta'} = 0$$

S.26.3

Referring to Fig. S.26.3 the shear flows in the walls 12 and 23 are constant since the walls are straight (see Eq. (26.1)). Choosing O as the origin of axes, from Eq. (26.1)

$$q_{12} = Gt(\theta' R \cos 30^\circ + u' \cos 30^\circ + v' \sin 30^\circ)$$

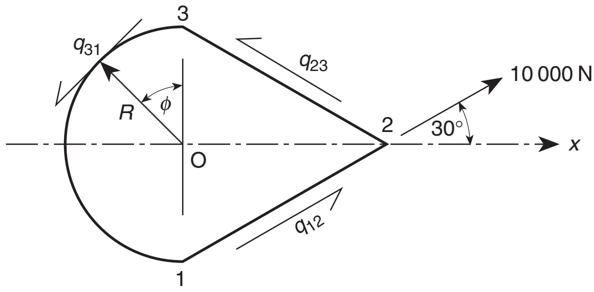

Fig. S.26.3

i.e.

$$q_{12} = Gt(0.866R\theta' + 0.866u' + 0.5v') \qquad \text{(i)}$$

$$q_{23} = Gt(0.866R\theta' - 0.866u' + 0.5v') \qquad \text{(ii)}$$

$$q_{31} = Gt(R\theta' - u' \cos\phi - v' \sin\phi) \qquad \text{(iii)}$$

Resolving forces vertically

$$q_{12}R + q_{23}R - \int_0^{\pi} q_{31} \sin\phi \, R \, \mathrm{d}\phi = 10\,000 \sin 30^\circ$$

i.e.

$$q_{12} + q_{23} - \int_0^{\pi} q_{31} \sin\phi \, \mathrm{d}\phi = \frac{5000}{R} \qquad \text{(iv)}$$

Substituting in Eq. (iv) for q_{12}, q_{23} and q_{31} from Eqs (i)–(iii), respectively gives

$$R\theta' - 9.59v' = -\frac{18\,656.7}{GtR} \qquad \text{(v)}$$

Resolving forces horizontally

$$q_{12}(R/\tan 30^\circ) - q_{23}(R/\tan 30^\circ) - \int_0^{\pi} q_{31} \cos\phi \, R \, \mathrm{d}\phi = 10\,000 \cos 30^\circ$$

i.e.

$$1.732q_{12} - 1.732q_{23} - \int_0^{\pi} q_{31} \cos\phi \, \mathrm{d}\phi = \frac{8660.3}{R} \qquad \text{(vi)}$$

Substituting in Eq. (vi) for q_{12}, q_{23} and q_{31} from Eqs (i)–(iii), respectively gives

$$u' = \frac{1894.7}{GtR} \tag{vii}$$

Taking moments about O

$$q_{12}(R/\tan 30°)R + q_{23}(R/\tan 30°)R + \int_0^{\pi} q_{31}R^2 \mathrm{d}\phi = 10\,000R\cos 30°$$

i.e.

$$1.732q_{12} + 1.732q_{23} + \int_0^{\pi} q_{31}\mathrm{d}\phi = \frac{8660.3}{R} \tag{viii}$$

Substituting in Eq. (viii) for q_{12}, q_{23} and q_{31} from Eqs (i)–(iii), respectively gives

$$R\theta' - 0.044v' = \frac{1410.0}{GtR} \tag{ix}$$

Now subtracting Eq. (ix) from (v)

$$-9.546v' = -\frac{20\,066.7}{GtR}$$

whence

$$v' = \frac{2102.1}{GtR} \tag{x}$$

Then, from Eq. (v)

$$R\theta' = \frac{1502.4}{GtR} \tag{xi}$$

Substituting for u', v' and θ' from Eqs (vii), (x) and (xi), respectively in Eqs (i)–(iii) gives

$$q_{12} = 3992.9/R\,\text{N/mm}, \quad q_{23} = 711.3/R\,\text{N/mm}$$
$$q_{31} = (1502.4 - 1894.7\cos\phi - 2102.1\,\sin\phi)/R\,\text{N/mm}$$

S.26.4

From Fig. P.26.4 the torque at any section of the beam is given by

$$T = 20 \times 10^3(2500 - z)\,\text{Nmm} \tag{i}$$

Eq. (26.16) for the warping distribution along boom 4 then becomes

$$w = C\cosh\mu z + D\sinh\mu z + \frac{20 \times 10^3(200 - z)}{8abG}\left(\frac{b}{t_b} - \frac{a}{t_a}\right) \tag{ii}$$

where

$$\mu^2 = \frac{8Gt_b t_a}{BE(bt_a + at_b)}$$

Comparing Figs 26.6 and P.26.4, $t_b = t_a = 1.0$ mm, $a = 500$ mm, $b = 200$ mm and $B = 800$ mm. Then

$$\mu^2 = \frac{8 \times 0.36 \times 1.0 \times 1.0}{800(200 \times 1.0 + 500 \times 1.0)}$$

from which

$$\mu = 2.27 \times 10^{-3}$$

Eq. (ii) then becomes

$$w = C \cosh 2.27 \times 10^{-3} z + D \sinh 2.27 \times 10^{-3} z - 3.75 \times 10^{-4}(2500 - z) \quad \text{(iii)}$$

When $z = 0$, $w = 0$, hence, from Eq. (iii), $C = 0.9375$. At the free end the direct stress in boom 4 is zero so that the direct strain $\partial w/\partial z = 0$ at the free end. Hence, from Eq. (iii), $D = -0.9386$ and the warping distribution along boom 4 is given by

$$w = 0.9375 \cosh 2.27 \times 10^{-3} z - 0.9386 \sinh 2.27 \times 10^{-3} z$$
$$- 3.75 \times 10^{-4}(2500 - z) \quad \text{(iv)}$$

Substituting for w from Eq. (iv) and T from Eq. (i) in Eq. (26.11)

$$\frac{\mathrm{d}\theta}{\mathrm{d}z} = -10^{-5}[1.6069 \cosh 2.27 \times 10^{-3} z - 1.6088 \sinh 2.27 \times 10^{-3} z$$
$$- 3.4998 \times 10^{-3}(2500 - z)] \quad \text{(v)}$$

Then

$$\theta = -10^{-5}\left[\frac{1.6069}{2.27 \times 10^{-3}} \sinh 2.27 \times 10^{-3} z - \frac{1.6088}{2.27 \times 10^{-3}} \cosh 2.27 \times 10^{-3} z\right.$$
$$\left. - 3.4998 \times 10^{-3}\left(2500z - \frac{z^2}{2}\right)\right] + F \quad \text{(vi)}$$

When $z = 0$, $\theta = 0$ so that, from Eq. (vi)

$$F = -10^{-5} \times \frac{1.6088}{2.27 \times 10^{-3}}$$

and

$$\theta = -10^{-5}\left[707.9 \sinh 2.27 \times 10^{-3} z - 708.7 \cosh 2.27 \times 10^{-3} z\right.$$
$$\left. - 3.4998 \times 10^{-3}\left(2500z - \frac{z^2}{2}\right) + 708.7\right] \quad \text{(vii)}$$

At the free and where $z = 2500$ mm Eq. (vii) gives

$$\theta = 0.1036 \text{ rad} = 5.9°$$

S.26.5

The warping distribution along the top right-hand corner boom is given by Eq. (26.16), i.e.

$$w = C\cosh\mu z + D\sinh\mu z + w_0 \tag{i}$$

where

$$\mu^2 = \frac{8Gt_2t_1}{BE(bt_1 + at_2)} \quad \text{and} \quad w_0 = \frac{T}{8abG}\left(\frac{b}{t_2} - \frac{a}{t_1}\right)$$

At each end of the beam the warping is completely suppressed, i.e. $w = 0$ at $z = 0$ and $z = l$. Thus, from Eq. (i)

$$0 = C + w_0$$

i.e.

$$C = -w_0$$

and

$$0 = C\cosh\mu l + D\sinh\mu l + w_0$$

which gives

$$D = \frac{w_0}{\sinh\mu l}(\cosh\mu l - 1)$$

Hence, Eq. (i) becomes

$$w = w_0\left[1 - \cosh\mu z + \frac{(\cosh\mu l - 1)}{\sinh\mu l}\sinh\mu z\right] \tag{ii}$$

The direct load, P, in the boom is then given by

$$P = \sigma_z B = BE\frac{\partial w}{\partial z}$$

Thus, from Eq. (ii)

$$P = \mu BEw_0\left[-\sinh\mu z + \frac{(\cosh\mu l - 1)}{\sinh\mu l}\cosh\mu z\right] \tag{iii}$$

or, substituting for w_0 from above

$$P = \frac{\mu BET}{8abGt_1t_2}(bt_1 - at_2)\left[-\sinh\mu z + \frac{(\cosh\mu l - 1)}{\sinh\mu l}\cosh\mu z\right] \tag{iv}$$

For a positive torque, i.e. T is anticlockwise when viewed along the z axis to the origin of z, the term in square brackets in Eq. (iv) becomes, when $z = 0$

$$\frac{\cosh\mu l - 1}{\sinh\mu l}$$

which is positive. Thus at $z = 0$ the load in the boom is tensile. At $z = l$ the term in square brackets in Eq. (iv) becomes

$$\frac{1 - \cosh \mu l}{\sinh \mu l}$$

which is negative. Thus at $z = l$ the load in the boom is compressive. Also, from Eq. (iv) $\partial P/\partial z = 0$ at $z = l/2$ and the distribution of boom load is that shown in Fig. S.26.5. The reverse situation occurs for a negative, i.e. a clockwise, torque.

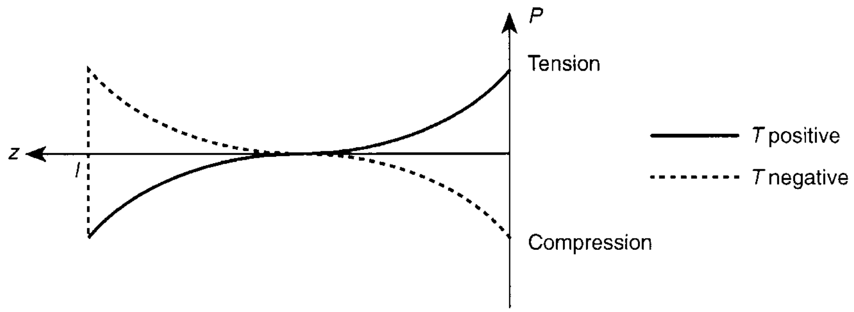

Fig. S.26.5

S.26.6

The warping distribution is given by Eq. (26.16), i.e.

$$w = C \cosh \mu z + D \sinh \mu z + \frac{T}{8abG}\left(\frac{b}{t_b} - \frac{a}{t_a}\right) \tag{i}$$

in which the last term is the free warping, w_0, of the section. Eq. (i) may therefore be written

$$w = C \cosh \mu z + D \sinh \mu z + w_0 \tag{ii}$$

When $z = 0$, $w = kw_0$ so that, from Eq. (ii)

$$C = w_0(k - 1)$$

When $z = L$, the direct stress is zero. Then, from Chapter 1

$$\sigma = E\frac{\partial w}{\partial z} = 0$$

so that

$$0 = \mu C \cosh \mu L + \mu D \sinh \mu L$$

which gives

$$D = -w_0(k - 1) \tanh \mu L$$

Eq. (ii) then becomes

$$w = w_0 \left[1 + (k-1) \frac{\cosh \mu(L-z)}{\cosh \mu L} \right] \qquad \text{(iii)}$$

Then

$$\sigma = E \frac{\partial w}{\partial z}$$

so that

$$\sigma = -\mu E w_0 (k-1) \frac{\sinh \mu(L-z)}{\cosh \mu L}$$

When $k = 0$,

$$\sigma = \mu E w_0 \frac{\sinh \mu(L-z)}{\cosh \mu L}$$

i.e. a rigid foundation.
When $k = 1$,

$$\sigma = 0$$

i.e. free warping (also from Eq. (iii)).

S.26.7

Initially directions for the shear flows, q, are chosen as shown in Fig. S.26.7(a). The panel is symmetrical about its horizontal centre line so that only half need be considered.

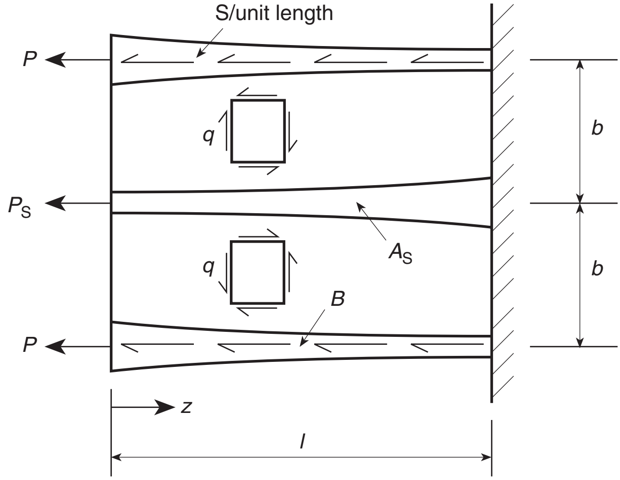

Fig. S.26.7(a)

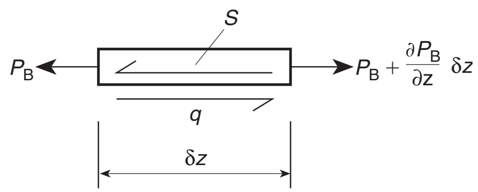

Fig. S.26.7(b)

For equilibrium of the element of the top boom shown in Fig. S.26.7(b)

$$P_{\rm B} + \frac{\partial P_{\rm B}}{\partial z}\delta z - P_{\rm B} - S\delta z + q\delta z = 0$$

i.e.

$$\frac{\partial P_{\rm B}}{\partial z} = S - q \tag{i}$$

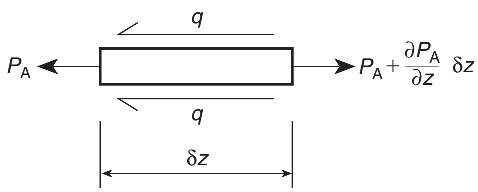

Fig. S.26.7(c)

Also, for equilibrium of the element of the central stringer shown in Fig. 26.7(c)

$$P_{\rm A} + \frac{\partial P_{\rm A}}{\partial z}\delta z - P_{\rm A} - 2q\delta z = 0$$

i.e.

$$\frac{\partial P_{\rm A}}{\partial z} = 2q \tag{ii}$$

For equilibrium of the length, z, of the panel shown in Fig. S.26.7(d)

$$2P_{\rm B} + P_{\rm A} - 2Sz - 2P - P_{\rm S} = 0$$

i.e.

$$P_{\rm A} = 2P + P_{\rm S} + 2Sz - 2P_{\rm B} \tag{iii}$$

The compatibility of displacement condition for the top boom and central stringer is shown in Fig. S.26.7(e). Thus

$$(1 + \varepsilon_{\rm A})\delta z = (1 + \varepsilon_{\rm B})\delta z + b\frac{{\rm d}\gamma}{{\rm d}z}\delta z$$

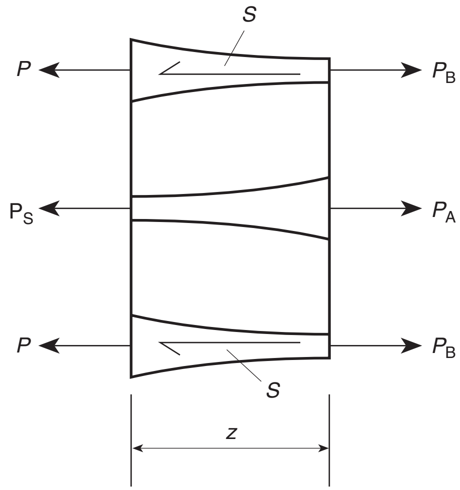

Fig. S.26.7(d)

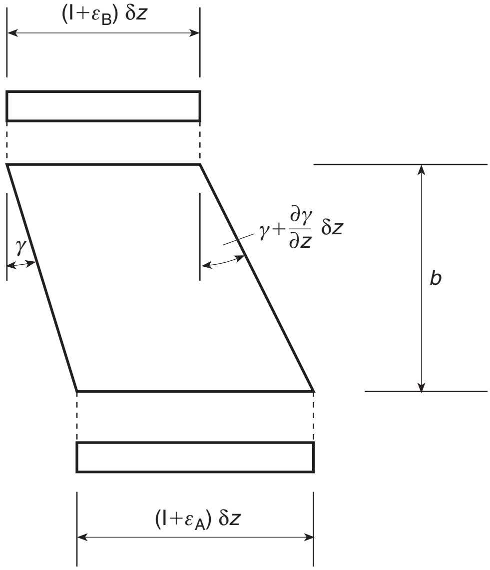

Fig. S.26.7(e)

i.e.

$$\frac{d\gamma}{dz} = \frac{1}{b}(\varepsilon_A - \varepsilon_B) \tag{iv}$$

Now

$$\varepsilon_A = \frac{\sigma_A}{E} \quad \text{and} \quad \varepsilon_B = \frac{\sigma_B}{E} = \frac{\sigma_e}{E} = \text{constant}$$

Also $\sigma_A = 0.8\sigma_e$ so that Eq. (iv) becomes

$$\frac{d\gamma}{dz} = -\frac{0.2\sigma_e}{bE} \tag{v}$$

In Eq. (v) $\gamma = q/Gt$, hence

$$\frac{dq}{dz} = -\frac{0.2Gt}{bE}\sigma_e \tag{vi}$$

Substituting for q in Eq. (vi) from (i) gives

$$\frac{\partial^2 P_B}{\partial z^2} = \frac{0.2Gt}{bE}\sigma_e$$

so that

$$P_B = \frac{0.1Gt\sigma_e}{bE}z^2 + Cz + D \tag{vii}$$

When $z = 0$, $P_B = P$ so that, from Eq. (vii), $D = P$. Also, when $z = l$, $q = 0$ so that, from Eq. (i), $\partial P_B/\partial z = S$ at $z = l$. Hence, from Eq. (vii)

$$C = -\frac{0.2Gt\sigma_e}{bE}l + S$$

and

$$P_B = \frac{0.1Gt\sigma_e}{bE}z^2 + \left(S - \frac{0.2Gt\sigma_e l}{bE}\right)z + P \tag{viii}$$

Now $P_B = \sigma_e B$ so that, from Eq. (viii)

$$B = \frac{0.1Gt}{bE}z^2 + \frac{1}{\sigma_e}\left[\left(S - \frac{0.2Gt\sigma_e l}{bE}\right)z + P\right] \tag{ix}$$

Substituting for P_B from Eq. (viii) in (iii) gives

$$P_A = \frac{0.4Gt\sigma_e}{bE}\left(lz - \frac{z^2}{2}\right) + P_S \tag{x}$$

But $P_A = A_S 0.8\sigma_e$ so that, from Eq. (x)

$$A_S = \frac{Gt}{2bE}\left(lz - \frac{z^2}{2}\right) + \frac{1.25P_S}{\sigma_e} \tag{xi}$$

Substituting the given values in Eqs (ix) and (xi) gives

$$B = 3.8 \times 10^{-4}z^2 + 0.3227z + 1636.4 \tag{xii}$$

and

$$A_S = 2.375z - 9.5 \times 10^{-4}z^2 + 659.1 \tag{xiii}$$

From Eq. (xiii) when $z=1250$ mm, $A_S=2143.5$ mm. Then

$$P_A = 0.8\sigma_e A_S = 0.8 \times 275 \times 2143.5 = 471\,570\,\text{N}$$

The total load, P_T, carried by the panel at the built-in end is

$$P_T = 2 \times 450\,000 + 145\,000 + 2 \times 350 \times 1250 = 1\,920\,000\,\text{N}$$

Therefore, the fraction of the load carried by the stringer is 471 570/1 920 000 = 0.25.

S.26.8

The panel is symmetrical about its vertical center line and therefore each half may be regarded as a panel with a built-in end as shown in Fig. S.26.8(a). Further, the panel is symmetrical about its horizontal centre line so that only the top half need be considered; the assumed directions of the shear flows are shown.

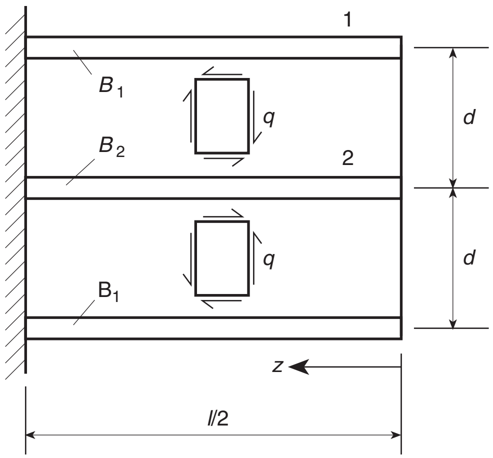

Fig. S.26.8(a)

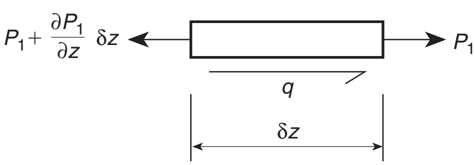

Fig. S.26.8(b)

Consider the equilibrium of the element of longeron 1 shown in Fig. S.26.8(b).

$$P_1 + \frac{\partial P_1}{\partial z}\delta_z - P_1 - q\delta z = 0$$

Hence

$$\frac{\partial P_1}{\partial z} = q \tag{i}$$

Now consider the equilibrium of the element of longeron 2 shown in Fig S.26.8(c).

$$P_2 + \frac{\partial P_2}{\partial z}\delta z - P_2 + 2q\delta z = 0$$

whence

$$\frac{\partial P_2}{\partial z} = -2q \tag{ii}$$

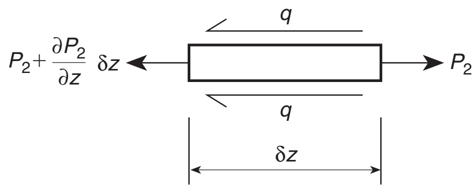

Fig. S.26.8(c)

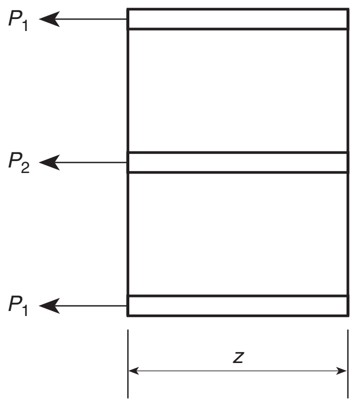

Fig. S.26.8(d)

From the overall equilibrium of the length z of the panel shown in Fig. S.26.8(d)

$$2P_1 + P_2 = 0 \tag{iii}$$

The compatibility condition for an element of the top half of the panel is shown in Fig. S.26.8(e). Thus

$$(1 + \varepsilon_1)\delta z = (1 + \varepsilon_2)\delta z + d\frac{\mathrm{d}\gamma}{\mathrm{d}z}\delta z$$

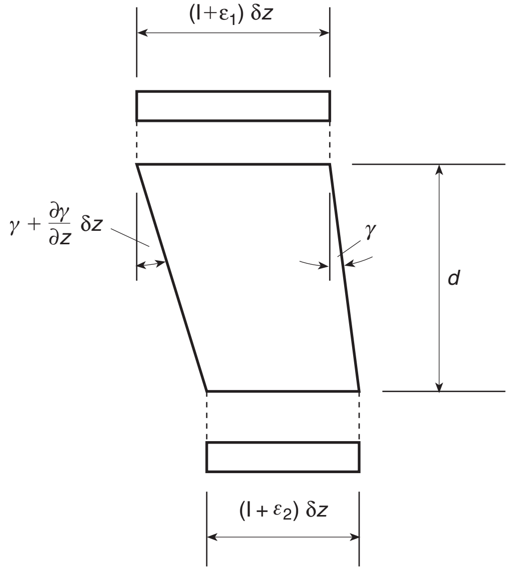

Fig. S.26.8(e)

i.e.

$$\frac{\mathrm{d}\gamma}{\mathrm{d}z} = \frac{1}{d}(\varepsilon_1 - \varepsilon_2) \tag{iv}$$

In Eq. (iv)

$$\varepsilon_1 = \frac{P_1}{B_1 E}$$

Also, an element, δz, of the central longeron would, without restraint, increase in length by an amount $\alpha T \delta z$. The element therefore suffers an effective strain equal to $(\varepsilon_2 - \alpha T)\delta z/\delta z$. Thus

$$\frac{P_2}{B_2 E} = \varepsilon_2 - \alpha T$$

so that Eq. (iv) becomes

$$\frac{\mathrm{d}\gamma}{\mathrm{d}z} = \frac{1}{dE}\left(\frac{P_1}{B_1} - \frac{P_2}{B_2} - \alpha TE\right) \tag{v}$$

Also $\gamma = q/Gt$ and from Eq. (ii) $q = -(\partial P_2/\partial z)/2$. Therefore, substituting for γ and then q in Eq. (v) and for P_1 from Eq. (iii) in (v)

$$-\frac{1}{2}\frac{\partial^2 P_2}{\partial z^2} = \frac{Gt}{dE}\left(-\frac{P_2}{2B_1} - \frac{P_2}{B_2} - \alpha TE\right)$$

or

$$\frac{\partial^2 P_2}{\partial z^2} - \frac{2GT}{dE}\left(\frac{1}{2B_1} + \frac{1}{B_2}\right) = \frac{2Gt\alpha T}{d} \qquad \text{(vi)}$$

The solution of Eq. (vi) is

$$P_2 = C\cosh\mu z + D\sinh\mu z - \frac{2Gt\alpha T}{\mu^2 d} \qquad \text{(vii)}$$

where

$$\mu^2 = \frac{2Gt}{dE}\left(\frac{1}{2B_1} + \frac{1}{B_2}\right)$$

When $z = 0$, $P_2 = 0$ so that, from Eq. (vii)

$$C = \frac{2Gt\alpha T}{\mu^2 d}$$

Also when $z = l/2$, $q = 0$ and, from Eq. (ii), $\partial P_2/\partial z = 0$. Hence, from Eq. (vii)

$$0 = \mu C\sinh\mu\frac{l}{2} + \mu D\cosh\mu\frac{l}{2}$$

from which

$$D = -C\tanh\frac{\mu l}{2} = -\frac{2Gt\alpha T}{\mu^2 d}\tanh\frac{\mu l}{2}$$

Thus,

$$P_2 = \frac{2Gt\alpha T}{\mu^2 d}\left(\cosh\mu z - \tanh\frac{\mu l}{2}\sinh\mu z - 1\right) \qquad \text{(viii)}$$

or, substituting for μ^2

$$P_2 = E\alpha T\frac{\left(\cosh\mu z - \tanh\frac{\mu l}{2}\sinh\mu z - 1\right)}{\left(\frac{1}{2B_1} + \frac{1}{B_2}\right)} \qquad \text{(ix)}$$

From Fig. S.26.8(e) the relative displacement of the central longeron at one end of the panel is $d(\gamma)_{z=0}$. Now

$$\gamma_{z=0} = \left(\frac{q}{Gt}\right)_{z=0} = -\frac{1}{2Gt}\left(\frac{\partial P_2}{\partial z}\right)_{z=0} \qquad \text{(from Eq. (ii))}$$

Hence, from Eq. (ix)

$$\text{relative displacement} = \frac{\alpha T}{\mu}\tanh\frac{\mu l}{2}$$

S.26.9

The panel is unsymmetrical so that the shear flows in the top and bottom halves will have different values as shown in Fig. S.26.9(a).

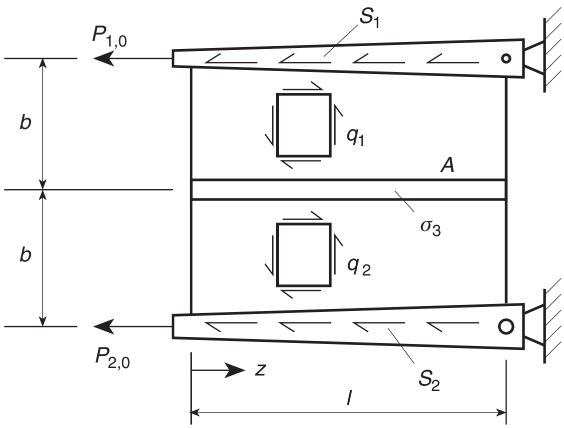

Fig. S.26.9(a)

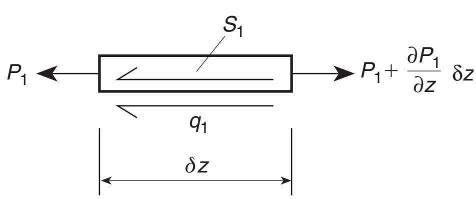

Fig. S.26.9(b)

For equilibrium of the element of the top member shown in Fig. 26.9(b)

$$P_1 + \frac{\partial P_1}{\partial z}\delta z - P_1 - S_1\delta z - q_1\delta z = 0$$

i.e.

$$\frac{\partial P_1}{\partial z} = S_1 + q_1 \tag{i}$$

Similarly, for the equilibrium of the element of the central stringer shown in Fig. S.26.9(c)

$$P_3 + \frac{\partial P_3}{\partial z}\delta z - P_3 - q_2\delta z + q_1\delta z = 0$$

i.e.

$$\frac{\partial P_3}{\partial z} = q_2 - q_1 \tag{ii}$$

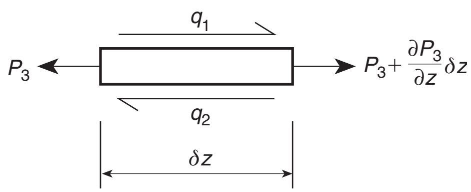

Fig. S.26.9(c)

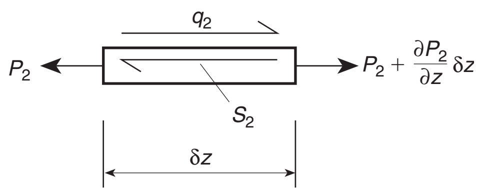

Fig. S.26.9(d)

Also, from Fig. S.26.9(d)

$$P_2 + \frac{\partial P_2}{\partial z}\delta z - P_2 - S_2\delta z + q_2\delta z = 0$$

whence

$$\frac{\partial P_2}{\partial z} = S_2 - q_2 \tag{iii}$$

Now, from the longitudinal equilibrium of a length z of the panel (Fig. S.26.9(e))

$$P_1 + P_3 + P_2 - P_{1,0} - P_{2,0} - S_1 z - S_2 z = 0$$

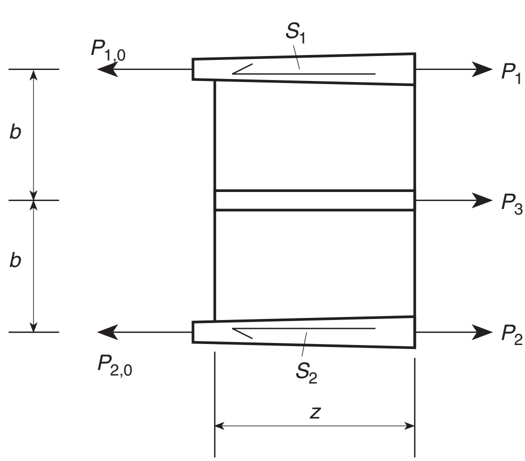

Fig. S.26.9(e)

i.e.

$$P_1 + P_3 + P_2 = P_{1,0} + P_{2,0} + (S_1 + S_2)z \qquad \text{(iv)}$$

and from its moment equilibrium about the bottom edge member

$$P_1 2b + P_3 b - P_{1,0} 2b - S_1 z 2b = 0$$

i.e.

$$2P_1 + P_3 = 2P_{1,0} + 2S_1 z \qquad \text{(v)}$$

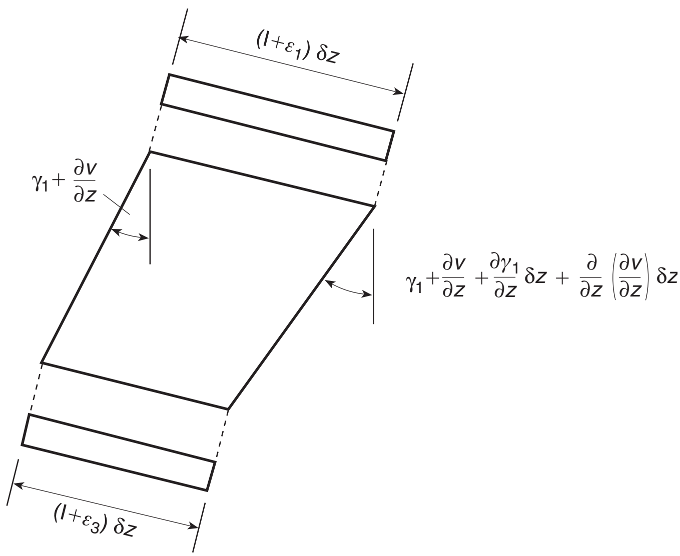

Fig. S.26.9(f)

From the compatibility condition between elements of the top edge member and the central stringer in Fig. S.26.9(f)

$$(1 + \varepsilon_1)\delta z = (1 + \varepsilon_3)\delta z + b\left(\frac{\mathrm{d}\gamma_1}{\mathrm{d}z} + \frac{\partial^2 v}{\partial z^2}\right)\delta z$$

or

$$\frac{\mathrm{d}\gamma_1}{\mathrm{d}z} = \frac{1}{b}(\varepsilon_1 - \varepsilon_3) - \frac{\partial^2 v}{\partial z^2} \qquad \text{(vi)}$$

Similarly for elements of the central stringer and the bottom edge member

$$\frac{\mathrm{d}\gamma_2}{\mathrm{d}z} = \frac{1}{b}(\varepsilon_3 - \varepsilon_2) - \frac{\partial^2 v}{\partial z^2} \qquad \text{(vii)}$$

Subtracting Eq. (vii) from (vi)

$$\frac{\mathrm{d}\gamma_1}{\mathrm{d}z} - \frac{\mathrm{d}\gamma_2}{\mathrm{d}z} = \frac{1}{b}(\varepsilon_1 - 2\varepsilon_3 + \varepsilon_2) \qquad \text{(viii)}$$

Now $\gamma = q/Gt, \varepsilon_1 = \sigma_1/E, \varepsilon_3 = \sigma_3/E$ and $\varepsilon_2 = \sigma_2/E$. Eq. (viii) may then be written

$$\frac{\mathrm{d}q_1}{\mathrm{d}z} - \frac{\mathrm{d}q_2}{\mathrm{d}z} = \frac{Gt}{bE}(\sigma_1 - 2\sigma_3 + \sigma_2)$$

or, from Eq. (i)

$$-\frac{\partial^2 P_3}{\partial z^2} = \frac{Gt}{bE}(\sigma_1 - 2\sigma_3 + \sigma_2)$$

Then, since $\sigma_3 = P_3/A$

$$\frac{\partial^2 \sigma_3}{\partial z^2} = \frac{Gt}{bEA}(2\sigma_3 - \sigma_1 - \sigma_2)$$

or

$$\frac{\partial^2 \sigma_3}{\partial z^2} - \frac{2Gt}{bEA}\sigma_3 = -\frac{Gt}{bEA}(\sigma_1 + \sigma_2) \qquad \text{(ix)}$$

The solution of Eq. (ix) is

$$\sigma_3 = C\cosh\mu z + D\sinh\mu z + (\sigma_1 + \sigma_2)/2$$

where $\mu^2 = 2Gt/bEA$.

When $z = 0, \sigma_3 = 0$ so that $C = -(\sigma_1 + \sigma_2)/2$. When $z = l, \sigma_3 = 0$ which gives

$$D = \frac{\sigma_1 + \sigma_2}{2\sinh\mu l}(\cosh\mu l - 1)$$

Thus

$$\sigma_3 = \left(\frac{\sigma_1 + \sigma_2}{2}\right)\left[1 - \cosh\mu z - \frac{(1 - \cosh\mu l)}{\sinh\mu l}\sinh\mu z\right] \qquad \text{(x)}$$

From Eq. (i)

$$q_1 = \frac{\partial P_1}{\partial z} - S_1 \qquad \text{(xi)}$$

Substituting for P_1 from Eq. (v) in (xi)

$$q_1 = -\frac{1}{2}\frac{\partial P_3}{\partial z} = -\frac{1}{2}A\frac{\partial \sigma_3}{\partial z}$$

Therefore, from Eq. (x)

$$q_1 = A\left(\frac{\sigma_1 + \sigma_2}{4}\right)\mu\left[\sinh\mu z + \frac{(1 - \cosh\mu l)}{\sinh\mu l}\cosh\mu z\right]$$

S.26.10

The assumed directions of shear flow are shown in Fig. S.26.10(a). For the equilibrium of an element of the top boom, Fig. S.26.10(b).

$$P_1 + \frac{\partial P_1}{\partial z}\delta z - P_1 + q_1 \delta z = 0$$

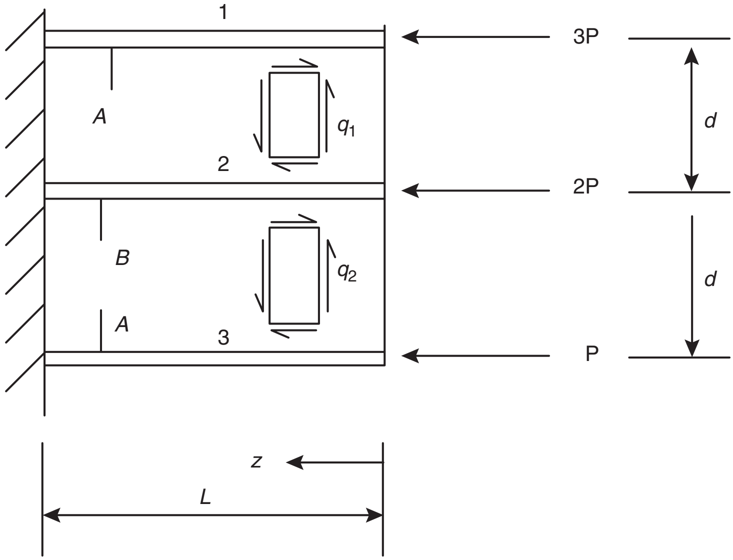

Fig. S.26.10(a)

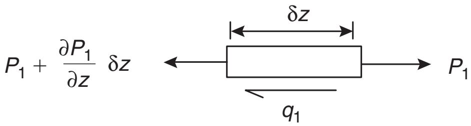

Fig. S.26.10(b)

from which

$$\frac{\partial P_1}{\partial z} = -q_1 \tag{i}$$

Similarly for an element of the central boom

$$\frac{\partial P_2}{\partial z} = q_1 - q_2 \tag{ii}$$

For overall equilibrium of the panel, at any section z

$$P_1 + P_2 + P_3 = -6P \tag{iii}$$

and taking moments about boom 3

$$P_1 2d + P_2 d + 3P2d + 2Pd = 0$$

so that

$$2P_1 + P_2 = -8P \tag{iv}$$

The compatibility condition is shown in Fig. S.26.10(c) for an element of the top panel.

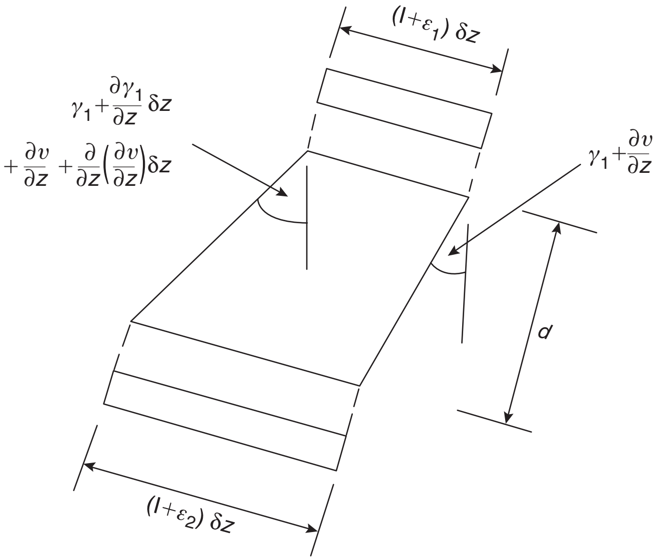

Fig. S.26.10(c)

Then

$$(1 + \varepsilon_2)\delta z = (1 + \varepsilon_1)\delta z + d\left(\frac{\partial \gamma_1}{\partial z} + \frac{\partial^2 v}{\partial z^2}\right)\delta z$$

i.e.

$$\frac{\partial \gamma_1}{\partial z} = \frac{1}{d}(\varepsilon_2 - \varepsilon_1) - \frac{\partial^2 v}{\partial z^2} \tag{v}$$

Similarly for an element of the lower panel

$$\frac{\partial \gamma_2}{\partial z} = \frac{1}{d}(\varepsilon_3 - \varepsilon_2) - \frac{\partial^2 v}{\partial z^2} \tag{vi}$$

Subtracting Eq. (vi) from (v)

$$\frac{\partial \gamma_1}{\partial z} - \frac{\partial \gamma_2}{\partial z} = \frac{1}{d}(2\varepsilon_2 - \varepsilon_1 - \varepsilon_3) \tag{vii}$$

But

$$\gamma_1 = \frac{q_1}{Gt} \quad \gamma_2 = \frac{q_2}{Gt} \quad \varepsilon_2 = \frac{P_2}{BE} \quad \varepsilon_1 = \frac{P_1}{AE} \quad \varepsilon_3 = \frac{P_3}{AE}$$

Then, from Eq. (vii)

$$\frac{\mathrm{d}q_1}{\mathrm{d}z} - \frac{\mathrm{d}q_2}{\mathrm{d}z} = \frac{Gt}{dE}\left(\frac{2P_2}{B} - \frac{P_1}{A} - \frac{P_3}{A}\right) \qquad \text{(viii)}$$

Substituting in Eq. (viii) for $q_1 - q_2$ from Eq. (ii) and for P_1 and P_3 from Eqs (iv) and (iii) and rearranging

$$\frac{\partial^2 P_2}{\partial z^2} - \mu^2 P_2 = \frac{6GtP}{dEA} \qquad \text{(ix)}$$

where

$$\mu^2 = \frac{Gt(2A+B)}{dEAB}$$

The solution of Eq. (ix) is

$$P_2 = C\cosh\mu z + D\sinh\mu z - \frac{6PB}{2A+B}$$

When $z = 0$, $P_2 = -2P$ and when $z = L$, $q_1 = q_2 = 0$ so that, from Eq. (ii) $\partial P_2/\partial z = 0$. These give

$$C = 4P\left(\frac{B-A}{2A+B}\right) \qquad D = -4P\left(\frac{B-A}{2A+B}\right)\tanh\mu L$$

Then

$$P_2 = \frac{6P}{2A+B}\left[-B + \frac{2}{3}(B-A)\frac{\cosh\mu(L-z)}{\cosh\mu L}\right]$$

From Eq. (iv)

$$P_1 = \frac{6P}{2A+B}\left[-\left(\frac{B+8A}{6}\right) - \frac{1}{3}(B-A)\frac{\cosh\mu(L-z)}{\cosh\mu L}\right]$$

and from Eq. (iii)

$$P_3 = \frac{6P}{2A+B}\left[-\left(\frac{4A-B}{6}\right) - \frac{1}{3}(B-A)\frac{\cosh\mu(L-z)}{\cosh\mu L}\right]$$

When $A = B$

$$P_1 = -3P \quad P_2 = -2P \quad P_3 = -P$$

and there is no shear lag effect.

S.26.11

This problem is similar to that of the six-boom beam analysed in Section 26.4 (Fig. 26.11) and thus the top cover of the beam is subjected to the loads shown in Fig. S.26.11(a). From symmetry the shear flow in the central panel of the cover is zero.

Considering the equilibrium of the element δz of the corner longeron (1) in Fig. S.26.11(b)

$$P_1 + \frac{\partial P_1}{\partial z}\delta z - P_1 + \frac{S}{2h}\delta z - q\delta z = 0$$

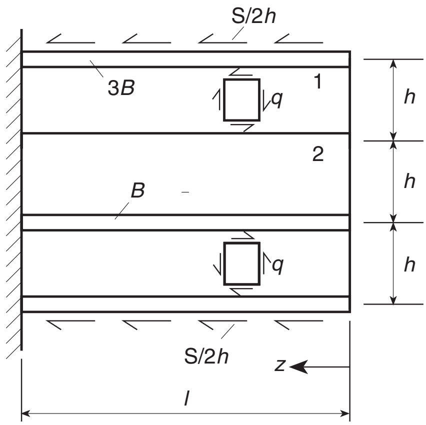

Fig. S.26.11(a)

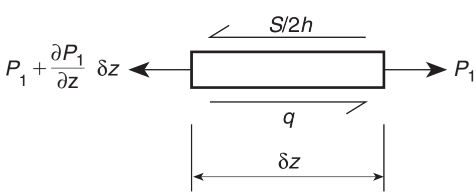

Fig. S.26.11(b)

i.e.

$$\frac{\partial P_1}{\partial z} = q - \frac{S}{2h} \tag{i}$$

Now considering the equilibrium of the element δz of longeron 2 in Fig. S.26.11(c)

$$P_2 + \frac{\partial P_2}{\partial z}\delta z - P_2 + q\delta z = 0$$

which gives

$$\frac{\partial P_2}{\partial z} = -q \tag{ii}$$

From the equilibrium of the length z of the panel shown in Fig. S.26.11(d)

$$2P_1 + 2P_2 + 2\frac{S}{2h}z = 0$$

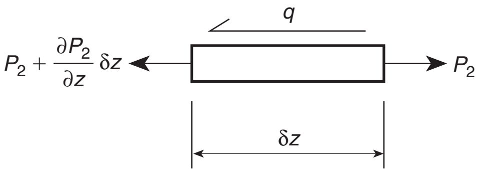

Fig. S.26.11(c)

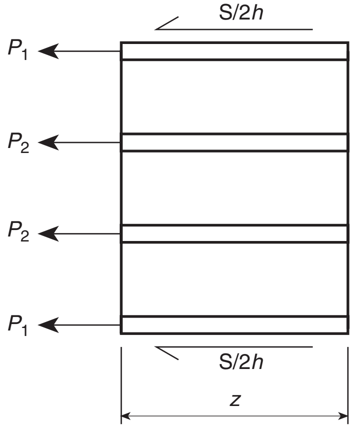

Fig. S.26.11(d)

or

$$P_1 + P_2 = -\frac{Sz}{2h} \tag{iii}$$

The compatibility of the displacement condition between longerons 1 and 2 is shown in Fig. S.26.11(e). Thus

$$(1 + \varepsilon_1)\delta z = (1 + \varepsilon_2)\delta z + h\frac{d\gamma}{dz}\delta z$$

from which

$$\frac{d\gamma}{dz} = \frac{1}{h}(\varepsilon_1 - \varepsilon_2) \tag{iv}$$

In Eq. (iv) $\gamma = q/Gt$, $\varepsilon_1 = P_1/3BE$, and $\varepsilon_2 = P_2/BE$. Equation (iv) then becomes

$$\frac{dq}{dz} = \frac{Gt}{hBE}\left(\frac{P_1}{3} - P_2\right) \tag{v}$$

From Eq. (i)

$$\frac{dq}{dz} = \frac{\partial^2 P_1}{\partial z^2}$$

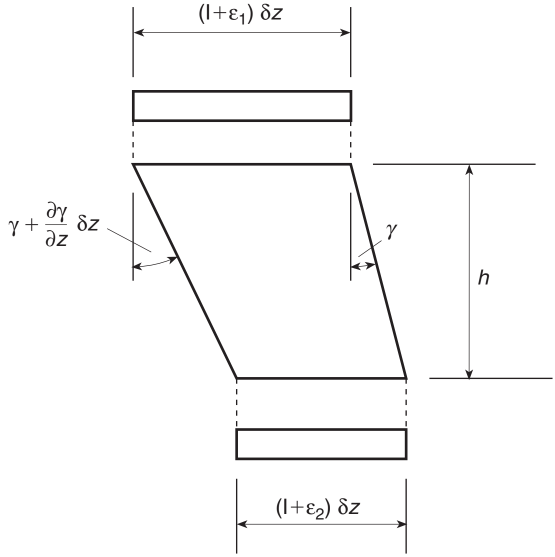

Fig. S.26.11(e)

and from Eq. (iii)

$$P_2 = -P_1 - \frac{Sz}{2h}$$

Substituting in Eq. (v)

$$\frac{\partial^2 P_1}{\partial z^2} = \frac{Gt}{hBE}\left(\frac{4P_1}{3} + \frac{Sz}{2h}\right)$$

or

$$\frac{\partial^2 P_1}{\partial z^2} - \frac{4Gt}{3hBE}P_1 = \frac{GtSz}{2h^2BE} \qquad \text{(vi)}$$

The solution of Eq. (vi) is

$$P_1 = C\cosh\mu z + D\sinh\mu z - \frac{3Sz}{8h} \qquad \text{(vii)}$$

where $\mu^2 = 4GT/3hBE$.

When $z = 0, P_1 = 0$ so that, from Eq. (vii), $C = 0$. When $z = l, q = 0$ so that, from Eq. (i), $\partial P_1/\partial z = -S/2h$. Hence from Eq. (vii)

$$D = -\frac{S}{8h\mu\cosh\mu l}$$

and Eq. (vii) becomes

$$P_1 = -\frac{S}{8h}\left(\frac{\sinh\mu z}{\mu\cosh\mu l} + 3z\right) \qquad \text{(viii)}$$

Substituting for P_1 in Eq. (i) gives

$$q = -\frac{S}{8h}\left(\frac{\cosh \mu z}{\cosh \mu l} - 1\right) \tag{ix}$$

If the effect of shear lag is neglected then Eq. (ix) reduces to

$$q = \frac{S}{8h}$$

and the shear flow distribution is that shown in Fig. S.26.11(f) in which $q_{12} = q_{43} = q_{65} = q_{78} = S/8h$ and $q_{81} = q_{54} = S/2h$. The deflection Δ due to bending and shear is given by Eqs (20.17) and (20.19) in which

$$M_0 = -Sz \quad \text{and} \quad M_1 = -1 \times z$$

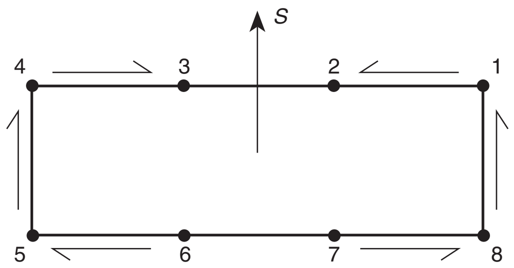

Fig. S.26.11(f)

Also $I_{xx} = 4 \times 3B \times (h/2)^2 + 4 \times B \times (h/2)^2 = 4Bh^2$ and $q_1 = q_0/S$. Thus

$$\Delta = \int_0^L \frac{Sz^2}{4Bh^2E}\mathrm{d}z + \int_0^l \left(\oint \frac{q_0 q_1}{Gt}\mathrm{d}s\right)\mathrm{d}z \tag{x}$$

In Eq. (x)

$$\oint \frac{q_0 q_1}{Gt}\mathrm{d}s = \frac{S}{G}\left(\frac{4h}{64h^2t} + \frac{2h}{4h^2 3t}\right) = \frac{11S}{48Ght}$$

Hence, substituting in Eq. (x)

$$\Delta = \frac{Sl}{12h}\left(\frac{l^2}{BhE} + \frac{11}{4Gt}\right)$$

S.26.12

The forces acting on the top cover of the box are shown in Fig. 26.12(a). Then for the equilibrium of the element δz of the edge boom shown in Fig. S.26.12(b).

$$\frac{\partial P_B}{\partial z} = -q + \frac{wz}{2h} \tag{i}$$

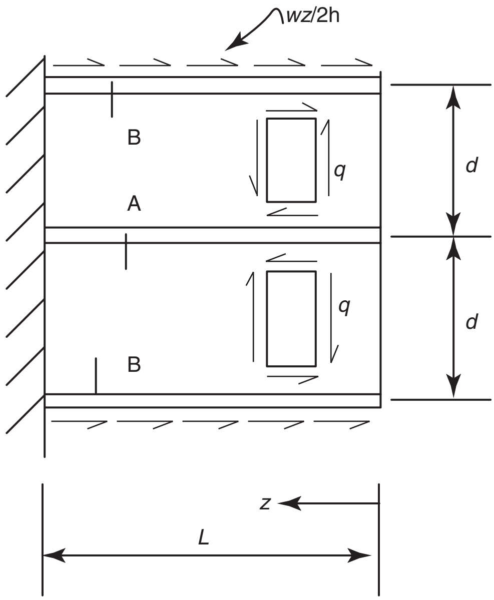

Fig. S.26.12(a)

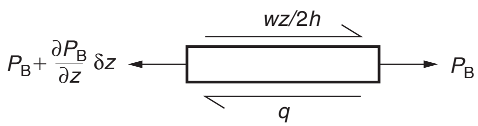

Fig. S.26.12(b)

Similarly, for the central boom

$$\frac{\partial P_A}{\partial z} = 2q \tag{ii}$$

For the equilibrium of a length z of the cover

$$2P_B + P_A - 2\frac{wz^2}{4h} = 0 \tag{iii}$$

The compatibility of displacement condition is shown in Fig. S.26.12(c).

Then

$$(1 + \varepsilon_A)\delta z = (1 + \varepsilon_B)\delta z + \frac{\partial \gamma}{\partial z}\delta z d$$

which gives

$$\frac{\partial \gamma}{\partial z} = \frac{1}{d}(\varepsilon_A - \varepsilon_B) \tag{iv}$$

But

$$\gamma = \frac{q}{Gt}, \quad \varepsilon_A = \frac{P_A}{AE}, \quad \varepsilon_B = \frac{P_B}{BE}$$

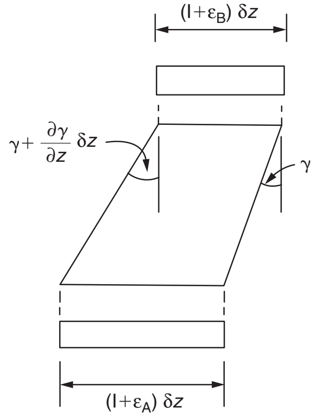

Fig. S.26.12(c)

Substituting in Eq. (iv)

$$\frac{\mathrm{d}q}{\mathrm{d}z} = \frac{Gt}{\mathrm{d}E}\left(\frac{P_{\mathrm{A}}}{A} - \frac{P_{\mathrm{B}}}{B}\right)$$

Substituting for q from Eq. (ii) and P_B from Eq. (iii) and rearranging

$$\frac{\partial^2 P_{\mathrm{A}}}{\partial z^2} - \mu^2 P_{\mathrm{A}} = -\frac{Gtw}{2dEhB}z^2 \tag{v}$$

where

$$\mu^2 = \frac{Gt(2B+A)}{dEAB}$$

The solution of Eq. (v) is

$$P_{\mathrm{A}} = C\cosh\mu z + D\sinh\mu z + \frac{w}{2h}\frac{A}{(2B+A)}\left(\frac{2}{\mu^2} + z^2\right)$$

When $z = 0$, $P_{\mathrm{A}} = 0$ which gives

$$C = -\frac{wA}{h(2B+A)\mu^2}$$

When $z = L$, $\partial P_{\mathrm{A}}/\partial z = 0$ since $q = 0$ at $z = L$. This gives

$$D = -\frac{wA}{\mu h(2B+A)\cosh\mu h}\left(L + \frac{\sinh\mu L}{\mu}\right)$$

Hence

$$P_{\mathrm{A}} = -\frac{wA}{h(2B+A)}\left[\frac{\cosh\mu z}{\mu^2} + \left(\frac{\mu L + \sinh\mu L}{\mu^2\cosh\mu L}\right)\sinh\mu z - \frac{l}{\mu^2} - \frac{z}{2}\right]$$