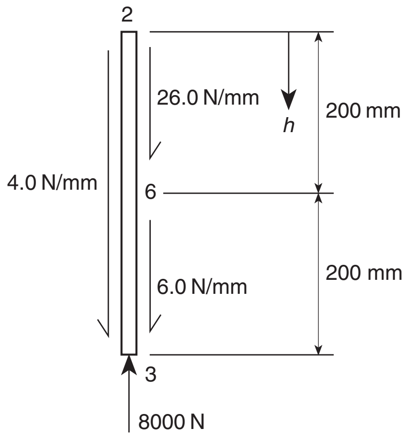

Fig. S.24.3(g)

i.e.

$$P_{63} = -4000 - 10h \tag{xiv}$$

Thus P_{63} varies linearly from $-6000\,\text{N}$ (compression) at 6 to $-8000\,\text{N}$ (compression) at 3.

Solutions to Chapter 25 Problems

S.25.1

From Eq. (25.5) the modulus of the bar is given by

$$E_1 = 140\,000 \times \frac{100 \times 10}{100 \times 55} + 3000 \times \frac{100 \times 45}{100 \times 55}$$

i.e.

$$E_1 = 27\,909.1\,\text{N/mm}^2$$

The overall direct stress in the longitudinal direction is given by

$$\sigma_1 = \frac{500 \times 10^3}{100 \times 55} = 90.9\,\text{N/mm}^2$$

Therefore, from Eq. (25.2), the longitudinal strain in the bar is

$$\varepsilon_1 = \frac{90.9}{27\,909.1} = 3.26 \times 10^{-3}$$

The shortening, Δ_1, of the bar is then

$$\Delta_1 = 3.26 \times 10^{-3} \times 1 \times 10^3 = 3.26\,\text{mm}$$

The major Poisson's ratio for the bar is obtained using Eq. (25.7). Thus

$$\nu_{\mathrm{lt}} = \frac{100 \times 45}{100 \times 55} \times 0.16 + \frac{100 \times 10}{100 \times 55} \times 0.28 = 0.18$$

Hence the strain across the thickness of the bar is

$$\varepsilon_{\mathrm{t}} = 0.18 \times 3.26 \times 10^{-3} = 5.87 \times 10^{-4}$$

so that the increase in thickness of the bar is

$$\Delta_{\mathrm{t}} = 5.87 \times 10^{-4} \times 55$$

i.e.

$$\Delta_{\mathrm{t}} = 0.032\,\mathrm{mm}$$

The stresses in the polyester and Kevlar are found from Eqs (25.3). Hence

$$\sigma_{\mathrm{m}}(\text{polyester}) = 3000 \times 3.26 \times 10^{-3} = 9.78\,\mathrm{N/mm^2}$$

$$\sigma_{\mathrm{f}}(\text{Kevlar}) = 140\,000 \times 3.26 \times 10^{-3} = 456.4\,\mathrm{N/mm^2}$$

S.25.2

For each cover

$$b_i t_i E_{Z,i} = 150 \times 1.0 \times 20\,000 = 3 \times 10^6$$

For each web

$$b_i t_i E_{Z,i} = 100 \times 2.0 \times 60\,000 = 12 \times 10^6$$

Then

$$\sum_{i=1}^{n} b_i t_i E_{Z,i} = 2 \times 3 \times 10^6 + 2 \times 12 \times 10^6 = 30 \times 10^6$$

From Eq. (25.37)

$$\varepsilon_Z = \frac{40 \times 10^3}{30 \times 10^6} = 1.33 \times 10^{-3}$$

Therefore

$$P(\text{covers}) = 1.33 \times 10^{-3} \times 3 \times 10^6 = 4000\,\mathrm{N} = 4\,\mathrm{kN}$$

$$P(\text{webs}) = 1.33 \times 10^{-3} \times 12 \times 10^6 = 16\,000\,\mathrm{N} = 16\,\mathrm{kN}$$

Check: $2 \times 4 + 2 \times 16 = 40\,\mathrm{kN}$

S.25.3

Since $I'_{xy} = 0$ and $M_y = 0$, Eq. (25.39) reduces to

$$\sigma_z = E_{z,i}\frac{M_x}{I'_{xx}}Y$$

where

$$I'_{xx} = 2 \times 60\,000 \times \frac{2.0 \times 100^3}{12} + 2 \times 20\,000 \times 1.0 \times 150 \times 50^2$$

i.e.

$$I'_{xx} = 3.5 \times 10^{10}\,\text{N mm}^2$$

Then

$$\sigma_Z = E_{Z,i} \times \frac{1 \times 10^6}{3.5 \times 10^{10}}Y = 2.86 \times 10^{-5}E_{z,i}Y \qquad \text{(i)}$$

The direct stress will be a maximum when Y is a maximum, i.e. at the top and bottom of the webs and in the covers. But $E_{Z,i}$ for the webs is greater than that for the covers, therefore

$$\sigma_Z(\text{max}) = \pm 2.86 \times 10^{-5} \times 60\,000 \times 50$$

i.e.

$$\sigma_z(\text{max}) = \pm 85.8\,\text{N/mm}^2 \quad \text{(at the top and bottom of the webs)}$$

S.25.4

The second moments of area are, from Example 25.5

$$I'_{xx} = 2.63 \times 10^{10}\,\text{N mm}^2$$
$$I'_{yy} = 0.83 \times 10^{10}\,\text{N mm}^2$$
$$I'_{xy} = 2.50 \times 10^{10}\,\text{N mm}^2$$

Also $M_X = 0$ and $M_Y = 0.5$ kN m so that Eq. (25.39) becomes

$$\sigma_Z = E_{Z,i}(-3.23 \times 10^{-5}X + 3.07 \times 10^{-5}Y) \qquad \text{(i)}$$

On the top flange, $E_{Z,i} = 50\,000\,\text{N/mm}^2$ and Y = 50 mm. Then, from Eq. (i)

$$\sigma_Z = -1.62X + 76.75$$

so that at 1 where $X = 50$ mm

$$\sigma_{Z,1} = -4.3\,\text{N/mm}^2$$

and at 2, $X = 0$

$$\sigma_{Z,2} = 76.8\,\text{N/mm}^2$$

In the web, $E_{Z,i} = 15\,000\,\text{N/mm}^2$, $X = 0$ so that

$$\sigma_Z = 0.46Y$$

and at 2

$$\sigma_{Z,2} = 0.46 \times 50 = 23.0\,\text{N/mm}^2$$

The maximum direct stress is therefore $76.8\,\text{N/mm}^2$

S.25.5

From Example 25.5 the second moments of area are

$$I'_{xx} = 2.63 \times 10^{10}\,\text{N mm}^2$$
$$I'_{yy} = 0.83 \times 10^{10}\,\text{N mm}^2$$
$$I'_{xy} = 2.50 \times 10^{10}\,\text{N mm}^2$$

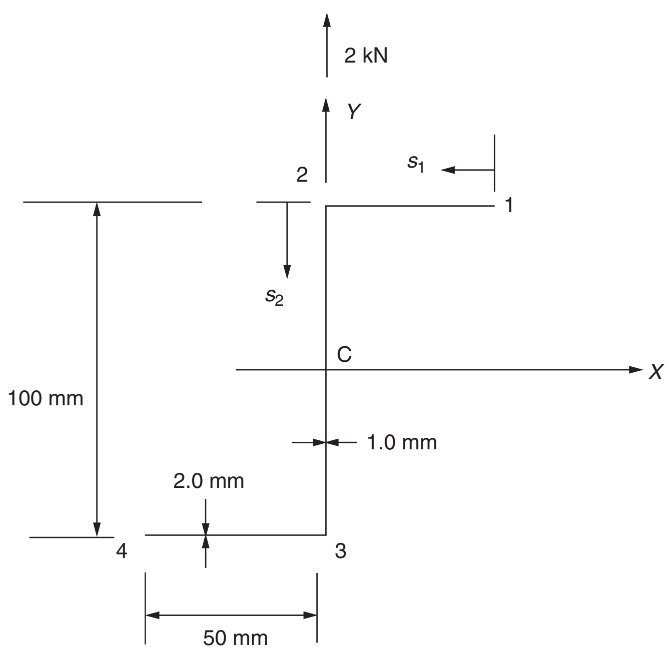

Fig. S.25.5

In this case $S_X = 0$, $S_Y = 2\,\text{kN}$ so that Eq. (25.40) becomes

$$q_s = -E_{Z,i}\left(1.15 \times 10^{-7}\int_0^s t_i X\,\text{d}s - 0.382 \times 10^{-7}\int_0^s t_i Y\,\text{d}s\right) \qquad \text{(i)}$$

On the top flange, $X = 50 - s_1$, $Y = 50\,\text{mm}$, $E_{Z,i} = 50\,000\,\text{N/mm}^2$. Eq. (i) then becomes

$$q_{12} = -11.5 \times 10^{-3}\int_0^{s_1}(50 - s_1)\text{d}s_1 + 190 \times 10^{-3}\int_0^{s_1}\text{d}s_1$$

which gives

$$q_{12} = 0.00575s_1^2 - 0.385s_1$$

when $s_1 = 50\,\text{mm}$ $\quad q_2 = -4.875\,\text{N/mm}$

In the web, $X = 0$, $Y = 50 - s_2$, $E_{Z,i} = 15\,000\,\text{N/mm}^2$. Eq. (i) then becomes

$$q_{23} = 5.73 \times 10^{-4}\int_0^{s_2}(50 - s_2)\text{d}s_2 - 4.875$$

so that

$$q_{23} = 0.0287s_2 - 2.865s_2^2 - 4.875$$

S.25.6

Referring to Fig. P.25.6, if the origin for s is chosen on the vertical axis of symmetry $q_{s,0}$, at 0, is zero.

Also since $S_X = 0$ and $I'_{XY} = 0$, Eq. (25.41) reduces to

$$q_s = -E_{Z,i}\frac{S_Y}{I'_{XX}}\int_0^s tY\,\text{d}s$$

in which

$$I'_{XX} = 2(54\,100 \times 200 \times 25^2) + 2\left(17\,700 \times \frac{0.5 \times 50^3}{12}\right)$$

i.e.

$$I'_{XX} = 13.7 \times 10^9\,\text{N mm}^2$$

Then

$$q_{01} = -54\,100 \times \frac{20 \times 10^3}{13.7 \times 10^9}\int_0^{s_1} 1.0 \times 25\,\text{d}s_1$$

i.e.

$$q_{01} = -1.98s_1$$

so that

$$q_1 = -1.98 \times 100 = -198\,\text{N/mm}$$

Also

$$q_{12} = -17\,700 \times \frac{20 \times 10^3}{13.7 \times 10^9} \int_0^{s_2} 0.5(25 - s_2)\mathrm{d}s_2 - 198$$

which gives

$$q_{12} = 6.5 \times 10^{-3} s_2^2 - 0.325 s_2 - 198$$

The remaining distribution follows from symmetry

S.25.7

The shear flow is obtained from Eq. (25.42), i.e.

$$q = \frac{1 \times 10^6}{2 \times 200 \times 50} = 50\,\mathrm{N/mm}$$

The maximum shear stress will occur in the webs and is

$$\tau_{\max} = \frac{50}{0.5} = 100\,\mathrm{N/mm^2}$$

From Eq. (25.45)

$$GJ = 4 \times (50 \times 200)^2/[2 \times 200/(20\,700 \times 1.0) + 2 \times 50/(36\,400 \times 0.5)]$$

i.e.

$$GJ = 1.6 \times 10^{10}\,\mathrm{N\,mm^2}$$

Then

$$\frac{\mathrm{d}\theta}{\mathrm{d}z} = \frac{T}{GJ} = \frac{1 \times 10^6}{1.6 \times 10^{10}} = 6.25 \times 10^{-5}\,\mathrm{rad/mm}$$

Finally, from Eq. (25.47)

$$W_4 = \frac{1 \times 10^6}{2 \times 200 \times 50}\left[\frac{100}{20\,700 \times 1.0} - \frac{\left(\frac{1}{2} \times 100 \times 25\right)}{50 \times 200}\left(\frac{2 \times 200}{20\,700 \times 1.0} + \frac{2 \times 50}{36\,400 \times 0.5}\right)\right]$$

i.e.

$$W_4 = -0.086\,\mathrm{mm}$$

S.25.8

From Eq. (25.48)

$$GJ = 2 \times 16\,300 \times 50 \times \frac{1^3}{3} + 20\,900 \times 100 \times \frac{0.5^3}{3} = 6.3 \times 10^5\ \text{N mm}^2$$

Then, from Eq. (25.49)

$$\frac{d\theta}{dz} = \frac{0.5 \times 10^3}{6.3 \times 10^5} = 0.8 \times 10^{-3}\ \text{rad/mm}$$

From Eq. (25.50)

$$\tau_{\max}(\text{flanges}) = \pm 2 \times 16\,300 \times \frac{1.0}{2} \times 0.8 \times 10^{-3}$$

i.e.

$$\tau_{\max}(\text{flanges}) = \pm 13.0\ \text{N/mm}^2$$

$$\tau_{\max}(\text{web}) = \pm 2 \times 20\,900 \times \frac{0.5}{2} \times 0.8 \times 10^{-3}$$

i.e.

$$\tau_{\max}(\text{web}) = \pm 8.4\ \text{N/mm}^2$$

Therefore

$$\tau_{\max} = \pm 13.0\ \text{N/mm}^2$$

The warping at 1 is, from Eq. (18.19)

$$W_1 = -2 \times \frac{1}{2} \times 50 \times 50 \times 0.8 \times 10^{-3} = -2.0\ \text{mm}$$

Solutions to Chapter 26 Problems

S.26.1

In Fig. S.26.1 $\alpha = \tan^{-1} 127/305 = 22.6°$. Choose O as the origin of axes then, from Eq. (26.1), since all the walls of the section are straight, the shear flow in each wall is constant. Then

$$q_{12} = 1.625G(254\theta' - u') \qquad \text{(i)}$$

$$q_{23} = 1.625G(254\theta' \cos 22.6° - u' \cos 22.6° - v' \sin 22.6°)$$