The shear flows due to the combined action of the shear and torsional loads are then as follows:

Bay ①

$$\text{Spar webs:} \quad q = 12.5 - 5 = 7.5\,\text{N/mm}$$

Bay ②

$$\text{Spar webs:} \quad q = 5 - 3.125 = 1.875\,\text{N/mm}$$

$$\text{Skin panels:} \quad q = 9.375\,\text{N/mm}$$

The flange loads are:

Bay ①

$$\text{At the built-in end:} \quad P_1 = 5000 - 3125 = 1875\,\text{N} \quad \text{(tension)}$$

$$\text{At the central rib:} \quad P_1 = 2500 + 3125 = 5625\,\text{N} \quad \text{(tension)}$$

Bay ②

$$\text{At the central rib:} \quad P_1 = 3625\,\text{N} \quad \text{(tension)}$$

$$\text{At the free end:} \quad P_1 = 0$$

Finally the shear flows on the central rib are:

$$\text{On the horizontal edges:} \quad q = 9.375\,\text{N/mm}$$

$$\text{On the vertical edges:} \quad q = 7.5 + 1.875 = 9.375\,\text{N/mm}$$

Solutions to Chapter 24 Problems

S.24.1

From the overall equilibrium of the beam in Fig. S.24.1(a)

$$R_F = 4\,\text{kN} \quad R_D = 2\,\text{kN}$$

The shear load in the panel ABEF is therefore 4 kN and the shear flow q is given by

$$q_1 = \frac{4 \times 10^3}{1000} = 4\,\text{N/mm}$$

Similarly

$$q_2 = \frac{2 \times 10^3}{1000} = 2\,\text{N/mm}$$

Considering the vertical equilibrium of the length h of the stiffener BE in Fig. S.24.1(b)

$$P_{EB} + (q_1 + q_2)h = 6 \times 10^3$$

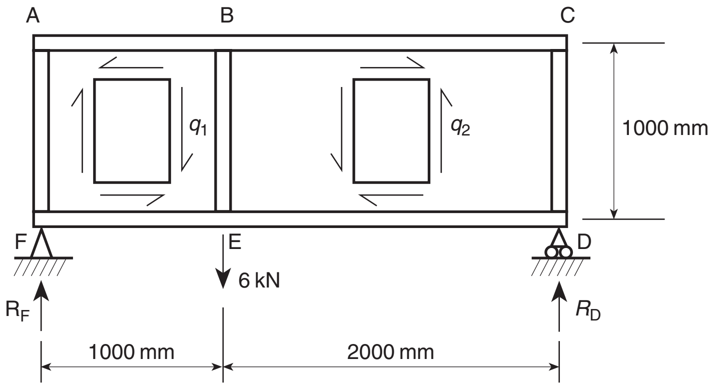

Fig. S.24.1(a)

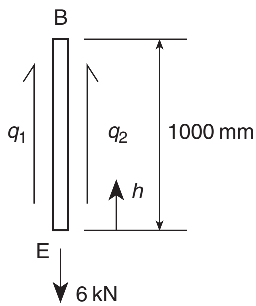

Fig. S.24.1(b)

where P_{EB} is the tensile load in the stiffener at the height h, i.e.

$$P_{EB} = 6 \times 10^3 - 6h \tag{i}$$

Then from Eq. (i), when $h = 0$, $P_{EB} = 6000\,\text{N}$ and when $h = 1000\,\text{mm}$. $P_{EB} = 0$. Therefore the stiffener load varies linearly from zero at B to 6000 N at E.

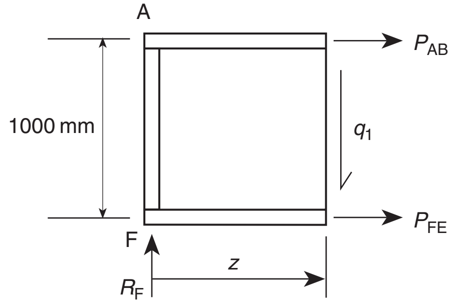

Fig. S.24.1(c)

Consider now the length z of the beam in Fig. S.24.1(c). Taking moments about the bottom flange at the section z

$$P_{AB} \times 1000 + R_F z = 0$$

whence

$$P_{AB} = -4z\,\text{N}$$

Thus P_{AB} varies linearly from zero at A to 4000 N (compression) at B. Similarly P_{CB} varies linearly from zero at C to 4000 N (compression) at B.

S.24.2

Referring to Fig. P.24.2 and considering the vertical equilibrium of the stiffener CDF

$$8000 \sin 30^\circ - q_1 \times 200 - q_2 \times 200 = 0$$

from which

$$q_1 + q_2 = 20 \quad \text{(i)}$$

Now considering the horizontal equilibrium of the stiffener ED

$$8000 \cos 30^\circ - q_1 \times 300 + q_2 \times 300 = 0$$

whence

$$q_1 - q_2 = 23.1 \quad \text{(ii)}$$

Adding Eqs (i) and (ii)

$$2q_1 = 43.1$$

i.e.

$$q_1 = 21.6\,\text{N/mm}$$

so that, from Eq. (i)

$$q_2 = -1.6\,\text{N/mm}$$

The vertical shear load at any section in the panel ABEGH is $8000 \sin 30^\circ = 4000$ N. Hence

$$400q_3 = 4000$$

i.e.

$$q_3 = 10\,\text{N/mm}$$

Now consider the equilibrium of the flange ABC in Fig. S.24.2(a). At any section z between C and B

$$P_{CB} = 21.6z \quad \text{(iii)}$$

so that P_{CB} varies linearly from zero at C to 6480 N (tension) at B. Also at any section z between B and A

$$P_{\mathrm{BA}} = 21.6 \times 300 + 10(z - 300)$$

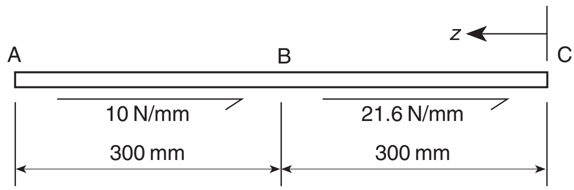

Fig. S.24.2(a)

i.e.

$$P_{\mathrm{BA}} = 3480 + 10z \tag{iv}$$

Thus P_{BA} varies linearly from 6480 N (tension) at B to 9480 N (tension) at A.

Referring to Fig. S.24.2(b) for the bottom flange HGF, the flange load P_{FG} at any section z is given by

$$P_{\mathrm{FG}} = 1.6z \tag{v}$$

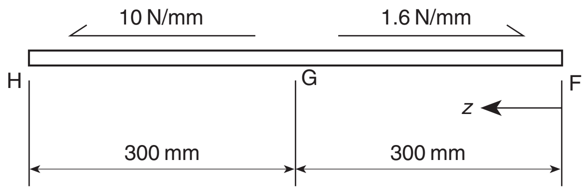

Fig. S.24.2(b)

Thus P_{FG} varies linearly from zero at F to 480 N (tension) at G. Also at any section z between G and H

$$P_{\mathrm{GH}} + 10(z - 300) - 1.6 \times 300 = 0$$

i.e.

$$P_{\mathrm{GH}} = 3480 - 10z \tag{vi}$$

Hence P_{GH} varies linearly from 480 N (tension) at G to −2520 N (compression) at H.

The forces acting on the stiffener DE are shown in Fig. S.24.2(c). At any section a distance z from D

$$P_{\mathrm{DE}} + 21.6z + 1.6z - 8000 \cos 30^\circ = 0$$

i.e.

$$P_{\mathrm{DE}} = -23.2z + 6928.2 \tag{vii}$$

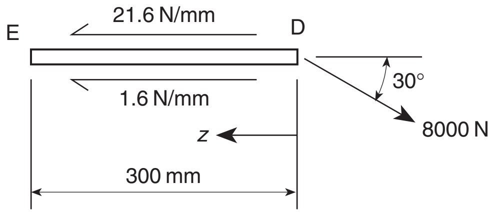

Fig. S.24.2(c)

Therefore P_{DE} varies linearly from 6928 N (tension) at D to zero at E. (The small value of P_{DE} at E given by Eq. (vii) is due to rounding off errors in the values of the shear flows.)

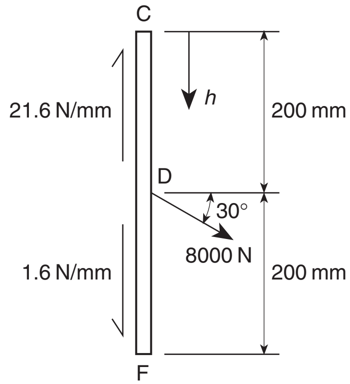

Fig. S.24.2(d)

The forces in the stiffener CDF are shown in Fig. S.24.2(d). At any section in CD a distance h from C the stiffener load, P_{CD}, is given by

$$P_{CD} = 21.6h \tag{viii}$$

so that P_{CD} varies linearly from zero at C to 4320 N (tension) at D. In DF

$$P_{DF} + 8000 \sin 30^\circ + 1.6(h - 200) - 21.6 \times 200 = 0$$

from which

$$P_{DF} = 640 - 1.6h \tag{ix}$$

Hence P_{DF} varies linearly from 320 N (tension) at D to zero at F.

The stiffener BEG is shown in Fig. S.24.2(e). In BE at any section a distance h from B

$$P_{BE} + 21.6h - 10h = 0$$

i.e.

$$P_{BE} = -11.6h \tag{x}$$

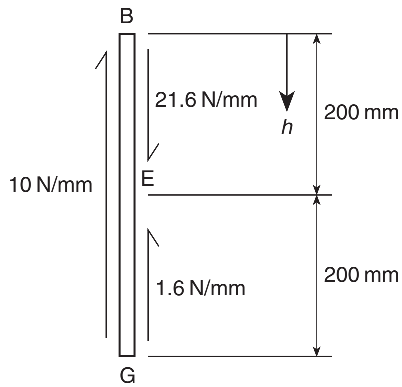

Fig. S.24.2(e)

P_{BE} therefore varies linearly from zero at B to −2320 N (compression) at E. In EG

$$P_{EG} - 1.6(h - 200) + 21.6 \times 200 - 10h = 0$$

i.e.

$$P_{EG} = 11.6h - 4640 \tag{xi}$$

Thus P_{EG} varies linearly from −2320 N (compression) at E to zero at G.

S.24.3

A three flange wing section is statically determinate (see Section 23.1) so that the shear flows applied to the wing rib may be found by considering the equilibrium of the wing rib. From Fig. S.24.3(a) and resolving forces horizontally

$$600q_{12} - 600q_{34} - 1200 = 0$$

whence

$$q_{12} - q_{34} = 20 \tag{i}$$

Now resolving vertically and noting that $q_{51} = q_{45}$

$$400q_{45} - 400q_{23} + 8000 = 0$$

i.e.

$$q_{45} - q_{23} = -20 \tag{ii}$$

Taking moments about 4

$$q_{12} \times 600 \times 400 + 2\left(\frac{\pi \times 200^2}{2} + \frac{1}{2} \times 400 \times 600\right) q_{23} - 12\,000 \times 200 - 8000 \times 600 = 0$$

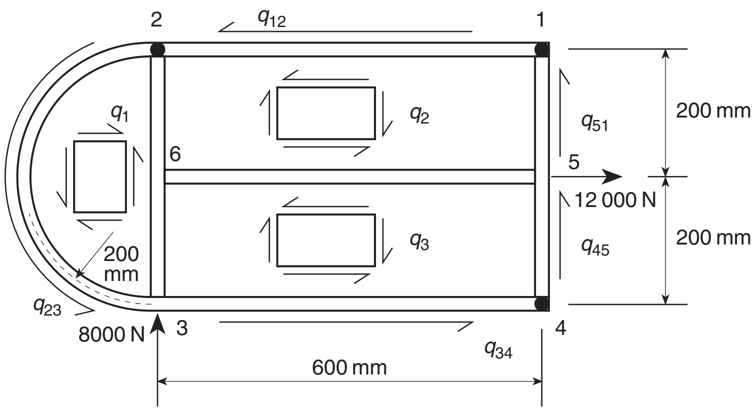

Fig. S.24.3(a)

so that

$$q_{12} + 1.52q_{23} = 30 \tag{iii}$$

Subtracting Eq. (iii) from (i) and noting that $q_{34} = q_{23}$

$$-2.52q_{23} = -10$$

or

$$q_{23} = 4.0\,\text{N/mm} = q_{34}$$

Then from Eq. (i)

$$q_{12} = 24.0\,\text{N/mm}$$

and from Eq. (ii)

$$q_{45} = -16.0\,\text{N/mm} = q_{51}$$

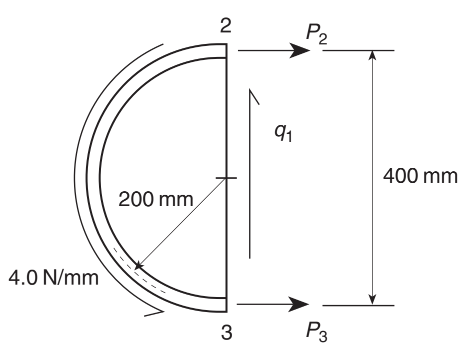

Fig. S.24.3(b)

Consider the nose portion of the wing rib in Fig. S.24.3(b). Taking moments about 3

$$P_2 \times 400 - 2 \times \frac{\pi \times 200^2}{2} \times 4.0 = 0$$

from which

$$P_2 = 1256.6\,\text{N} \quad \text{(tension)}$$

From horizontal equilibrium

$$P_3 + P_2 = 0$$

whence

$$P_3 = -1256.6\,\text{N} \quad \text{(compression)}$$

and from vertical equilibrium

$$q_1 = 4.0\,\text{N/mm}$$

From the vertical equilibrium of the stiffener 154 in Fig. S.24.3(c)

$$q_2 \times 200 + q_3 \times 200 - 16 \times 400 = 0$$

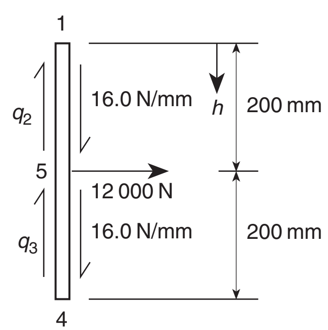

Fig. S.24.3(c)

i.e.

$$q_2 + q_3 = 32 \tag{iv}$$

Also, in 15 at any distance h from 1

$$P_{15} + 16h - q_2 h = 0$$

i.e.

$$P_{15} = (q_2 - 16)h \tag{v}$$

and in 54

$$P_{54} + 16h - q_2 \times 200 - q_3(h - 200) = 0$$

whence

$$P_{54} = 200(q_2 - q_3) + (q_3 - 16)h \qquad \text{(vi)}$$

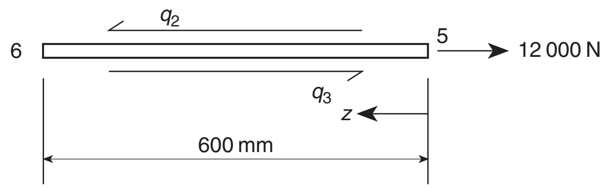

Fig. S.24.3(d)

Fig. S.24.3(d) shows the stiffener 56. From horizontal equilibrium

$$600q_2 - 600q_3 - 12\,000 = 0$$

or

$$q_2 - q_3 = 20 \qquad \text{(vii)}$$

Adding Eqs (iv) and (vii)

$$2q_2 = 52$$

i.e.

$$q_2 = 26\,\text{N/mm}$$

and from Eq. (iv)

$$q_3 = 6\,\text{N/mm}$$

Then, from Eq. (v)

$$P_{15} = 10h \qquad \text{(viii)}$$

and P_{15} varies linearly from zero at 1 to 2000 N (tension) at 5. From Eq. (vi)

$$P_{54} = 200(26 - 6) + (6 - 16)h$$

i.e.

$$P_{54} = 4000 - 10h \qquad \text{(ix)}$$

so that P_{54} varies linearly from 2000 N (tension) at 5 to zero at 4. Now from Fig. S.24.3(d) at any section z

$$P_{56} + q_2 z - q_3 z - 12\,000 = 0$$

i.e.

$$P_{56} = -20z + 12\,000 \qquad \text{(x)}$$

Thus P_{56} varies linearly from 12 000 N (tension) at 5 to zero at 6.

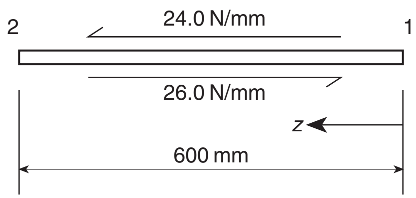

Fig. S.24.3(e)

Consider the flange 12 in Fig. S.24.3(e). At any section a distance z from 1

$$P_{12} + 24z - 26z = 0$$

i.e.

$$P_{12} = 2z \tag{xi}$$

Hence P_{12} varies linearly from zero at 1 to 1200 N (tension) at 2.

Now consider the bottom flange in Fig. S.24.3(f). At any section a distance z from 4

$$P_{43} + 6z - 4z = 0$$

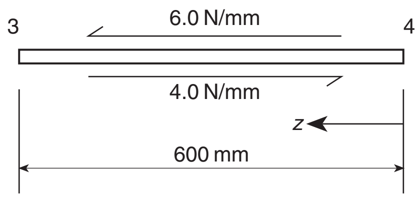

Fig. S.24.3(f)

i.e.

$$P_{43} = -2z \tag{xii}$$

Thus P_{43} varies linearly from zero at 4 to -1200 N (compression) at 3. (The discrepancy between P_2 in 12 and P_2 in 23 and between P_3 in 43 and P_3 in 23 is due to the rounding off error in the shear flow q_1.)

In Fig. S.24.3(g) the load in the stiffener at any section a distance h from 2 is given by

$$P_{26} + 26h + 4h = 0$$

i.e.

$$P_{26} = -30h \tag{xiii}$$

Therefore P_{26} varies linearly from zero at 2 to -6000 N (compression) at 6. In 63

$$P_{63} + 26 \times 200 + 4h + 6(h - 200) = 0$$

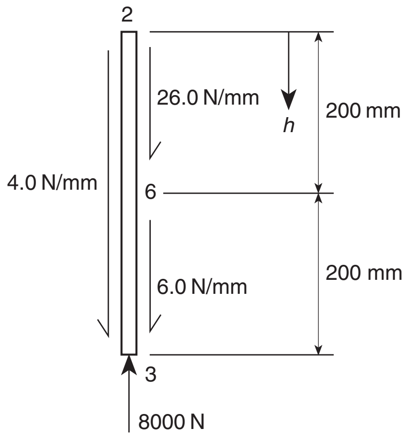

Fig. S.24.3(g)

i.e.

$$P_{63} = -4000 - 10h \tag{xiv}$$

Thus P_{63} varies linearly from -6000 N (compression) at 6 to -8000 N (compression) at 3.

Solutions to Chapter 25 Problems

S.25.1

From Eq. (25.5) the modulus of the bar is given by

$$E_1 = 140\,000 \times \frac{100 \times 10}{100 \times 55} + 3000 \times \frac{100 \times 45}{100 \times 55}$$

i.e.

$$E_1 = 27\,909.1\,\text{N/mm}^2$$

The overall direct stress in the longitudinal direction is given by

$$\sigma_1 = \frac{500 \times 10^3}{100 \times 55} = 90.9\,\text{N/mm}^2$$

Therefore, from Eq. (25.2), the longitudinal strain in the bar is

$$\varepsilon_1 = \frac{90.9}{27\,909.1} = 3.26 \times 10^{-3}$$

The shortening, Δ_1, of the bar is then

$$\Delta_1 = 3.26 \times 10^{-3} \times 1 \times 10^3 = 3.26\,\text{mm}$$