which gives

$$\int_{\text{sect}} \frac{q_0 q_1}{Gt} \mathrm{d}s = \frac{1359 p_0}{1000 Gt}(L - z)$$

Hence

$$\int_0^L \left(\int_{\text{sect}} \frac{q_0 q_1}{Gt} \mathrm{d}s \right) \mathrm{d}z = \frac{1359 p_0}{1000 Gt} \int_0^L (L - z)\, \mathrm{d}z = \frac{1359 p_0 L^2}{2000 Gt} \qquad \text{(iv)}$$

Substituting in Eq. (i) from Eqs (ii)–(iv) gives

$$\Delta = \frac{p_0 L^2}{8Gt} + \frac{9 p_0 L^4}{80 E b^2 t} + \frac{1359 p_0 L^2}{2000 Gt}$$

Thus

$$\Delta = \frac{p_0 L^2}{t} \left(\frac{9L^2}{80 E b^2} + \frac{1609}{2000 G} \right)$$

Solutions to Chapter 21 Problems

S.21.1

Referring to Fig. P.21.1 the bending moment at section 1 is given by

$$M_1 = \frac{15 \times 1^2}{2} = 7.5\,\text{kN m}$$

Thus

$$P_{z,\text{U}} = -P_{z,\text{L}} = \frac{7.5}{300 \times 10^{-3}} = 25\,\text{kN}$$

Also

$$P_{y,\text{U}} = 0 \quad \text{and} \quad P_{y,\text{L}} = -25 \times \frac{100}{1 \times 10^3} = -2.5\,\text{kN} \quad \text{(see Eqs (21.1))}$$

Then

$$P_\text{U} = \sqrt{P_{z,\text{U}}^2 + P_{y,\text{U}}^2} = 25\,\text{kN} \quad \text{(tension)}$$

$$P_\text{L} = -\sqrt{25^2 + 2.5^2} = -25.1\,\text{kN} \quad \text{(compression)}$$

The shear force at section 1 is $15 \times 1 = 15$ kN. This is resisted by $P_{y,\text{L}}$, the shear force in the web. Thus

$$\text{shear in web} = 15 - 2.5 = 12.5\,\text{kN}$$

Hence

$$q = \frac{12.5 \times 10^3}{300} = 41.7\,\text{kN/mm}$$

At section 2 the bending moment is

$$M_2 = \frac{15 \times 2^2}{2} = 30\,\text{kN m}$$

Hence

$$P_{z,\text{U}} = -P_{z,\text{L}} = \frac{30}{400 \times 10^{-3}} = 75\,\text{kN}$$

Also

$$P_{y,\text{U}} = 0 \quad \text{and} \quad P_{y,\text{L}} = -75 \times \frac{200}{2 \times 10^3} = -7.5\,\text{kN}$$

Then

$$P_\text{U} = 75\,\text{kN} \quad \text{(tension)}$$

and

$$P_\text{L} = -\sqrt{75^2 + 7.5^2} = -75.4\,\text{kN} \quad \text{(compression)}$$

The shear force at section 2 is $15 \times 2 = 30$ kN. Hence the shear force in the web $= 30 - 7.5 = 22.5$ kN which gives

$$q = \frac{22.5 \times 10^3}{400} = 56.3\,\text{N/mm}$$

S.21.2

The bending moment at section 1 is given by

$$M = \frac{15 \times 1^2}{2} = 7.5\,\text{kN m}$$

The second moment of area of the beam cross-section at section 1 is

$$I_{xx} = 2 \times 500 \times 150^2 + \frac{2 \times 300^3}{12} = 2.7 \times 10^7\,\text{mm}^4$$

The direct stresses in the flanges in the z direction are, from Eq. (16.18)

$$\sigma_{z,\text{U}} = -\sigma_{z,\text{L}} = \frac{7.5 \times 10^6 \times 150}{2.7 \times 10^7} = 41.7\,\text{N/mm}^2$$

Then

$$P_{z,\text{U}} = 41.7 \times 500 = 20\,850\,\text{N} = P_\text{U} \quad \text{(tension)}$$

Also

$$P_{z,\mathrm{L}} = -20\,850\,\mathrm{N} \quad \text{(compression)}$$

Hence

$$P_{y,\mathrm{L}} = -20\,850 \times \frac{100}{1 \times 10^3} = -2085\,\mathrm{N} \quad \text{(compression)}$$

Therefore, the shear force in the web at section 1 is given by

$$S_y = -15 \times 1 \times 10^3 + 2085 = -12\,915\,\mathrm{N}$$

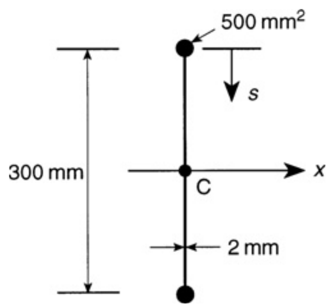

Fig. S.21.2

The shear flow distribution is obtained using Eq. (21.6). Thus, referring to Fig. S.21.2

$$q = \frac{12\,915}{2.7 \times 10^7}\left[\int_0^s 2(150 - s)\mathrm{d}s + 500 \times 150\right]$$

Hence

$$q = 4.8 \times 10^{-4}(300s - s^2 + 75\,000)$$

The maximum value of q occurs when $s = 150$ mm, i.e.

$$q_{\max} = 46.8\,\mathrm{N/mm}$$

S.21.3

The beam section at a distance of 1.5 m from the built-in end is shown in Fig. S.21.3. The bending moment, M, at this section is given by

$$M = -40 \times 1.5 = -60\,\mathrm{kN\,m}$$

Since the x axis is an axis of symmetry $I_{xy} = 0$; also $M_y = 0$. The direct stress distribution is then, from Eq. (16.18)

$$\sigma_z = \frac{M_x}{I_{xx}}y \qquad \text{(i)}$$

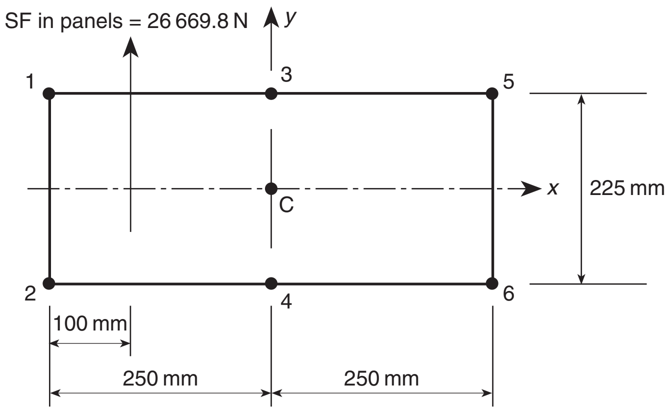

Fig. S.21.3

in which $I_{xx} = 2 \times 1000 \times 112.5^2 + 4 \times 500 \times 112.5^2 = 50.63 \times 10^6\ \text{mm}^4$. Then, from Eq. (i), the direct stresses in the flanges and stringers are

$$\sigma_z = \pm \frac{60 \times 10^6 \times 112.5}{50.63 \times 10^6} = \pm 133.3\ \text{N/mm}^2$$

Therefore

$$P_{z,1} = -P_{z,2} = -133.3 \times 1000 = -133\,300\ \text{N}$$

and

$$P_{z,3} = P_{z,5} = -P_{z,4} = -P_{z,6} = -133.3 \times 500 = -66\,650\ \text{N}$$

From Eq. (21.9)

$$P_{y,1} = P_{y,2} = 133\,300 \times \frac{75}{3 \times 10^3} = 3332.5\ \text{N}$$

and

$$P_{y,3} = P_{y,4} = P_{y,5} = P_{y,6} = 66\,650 \times \frac{75}{3 \times 10^3} = 1666.3\ \text{N}$$

Thus the total vertical load in the flanges and stringers is

$$2 \times 3332.5 + 4 \times 1666.3 = 13\,330.2\ \text{N}$$

Hence the total shear force carried by the panels is

$$40 \times 10^3 - 13\,330.2 = 26\,669.8\ \text{N}$$

The shear flow distribution is given by Eq. (20.11) which, since $I_{xy}=0$, $S_x=0$ and $t_D=0$ reduces to

$$q_s = -\frac{S_y}{I_{xx}} \sum_{r=1}^{n} B_r y_r + q_{s,0}$$

i.e.

$$q_s = -\frac{26\,669.8}{50.63 \times 10^6} \sum_{r=1}^{n} B_r y_r + q_{s,0}$$

or

$$q_s = -5.27 \times 10^{-4} \sum_{r=1}^{n} B_r y_r + q_{s,0} \qquad \text{(ii)}$$

From Eq. (ii)

$$q_{b,13} = 0$$
$$q_{b,35} = -5.27 \times 10^{-4} \times 500 \times 112.5 = -29.6\,\text{N/mm}$$
$$q_{b,56} = -29.6 - 5.27 \times 10^{-4} \times 500 \times 112.5 = -59.2\,\text{N/mm}$$
$$q_{b,12} = -5.27 \times 10^{-4} \times 1000 \times 112.5 = -59.3\,\text{N/mm}$$

The remaining distribution follows from symmetry. Now taking moments about the point 2 (see Eq. (17.17))

$$26\,669.8 \times 100 = 59.2 \times 225 \times 500 + 29.6 \times 250 \times 225 + 2 \times 500 \times 225_{q_{s,0}}$$

from which

$$q_{s,0} = -36.9\,\text{N/mm} \quad \text{(i.e. clockwise)}$$

Then

$$q_{13} = 36.9\,\text{N/mm} = q_{42}$$
$$q_{35} = 36.9 - 29.6 = 7.3\,\text{N/mm} = q_{64}$$
$$q_{65} = 59.2 - 36.9 = 22.3\,\text{N/mm}$$
$$q_{21} = 36.9 + 59.3 = 96.2\,\text{N/mm}$$

Finally

$$P_1 = -\sqrt{P_{z,1}^2 + P_{y,1}^2} = -\left(\sqrt{133\,300^2 + 3332.5^2}\right) \times 10^{-3} = -133.3\,\text{kN} = -P_2$$
$$P_3 = -\sqrt{P_{z,3}^2 + P_{y,3}^2} = -\left(\sqrt{66\,650^2 + 1666.3^2}\right) \times 10^{-3}$$
$$= -66.7\,\text{kN} = P_5 = -P_4 = -P_6$$