Therefore

$$\tau_{max} = 170\,\text{N/mm}^2$$

The maximum shear stress in the open part is, from Eqs (18.12) and (18.13)

$$\tau_{max} = \pm 25\,000 \times 2 \times 18.5 \times 10^{-6} = \pm 0.9\,\text{N/mm}^2$$

Solutions to Chapter 20 Problems

S.20.1

From either Eq. (20.1) or (20.2)

$$B_1 = 60 \times 10 + 40 \times 10 + \frac{500 \times 10}{6}(2+1) + \frac{300 \times 10}{6}(2-1)$$

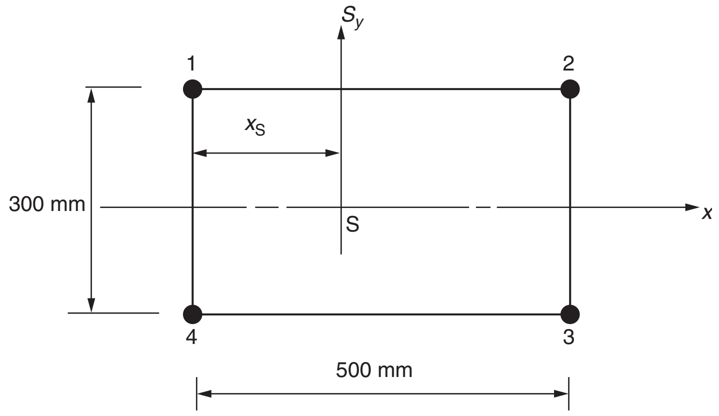

Fig. S.20.1(a)

i.e.

$$B_1 = 4000\,\text{mm}^2 = B_4$$

$$B_2 = 50 \times 8 + 30 \times 8 + \frac{500 \times 10}{6}(2+1) + \frac{300 \times 8}{6}(2-1)$$

i.e.

$$B_2 = 3540\,\text{mm}^2 = B_3$$

Since the section is now idealized, the shear flow distribution due to an arbitrary shear load S_y applied through the shear centre is, from Eq. (20.11), given by

$$q_s = -\frac{S_y}{I_{xx}} \sum_{r=1}^{n} B_r y_r + q_{s,0} \qquad \text{(i)}$$

in which

$$I_{xx} = 2 \times 4000 \times 150^2 + 2 \times 3540 \times 150^2 = 339 \times 10^6\,\text{mm}^4.$$

'Cut' the section in the wall 12. Then

$$q_{b,12} = q_{b,43} = 0$$

$$q_{b,41} = -\frac{S_y}{I_{xx}} \times 4000 \times (-150) = 1.77 \times 10^{-3} S_y$$

$$q_{b,32} = -\frac{S_y}{I_{xx}} \times 3540 \times (-150) = 1.57 \times 10^{-3} S_y$$

Since the shear load is applied through the shear centre the rate of twist is zero and $q_{s,0}$ is given by Eq. (17.28) in which

$$\oint \frac{\mathrm{d}s}{t} = 2 \times \frac{500}{10} + \frac{300}{10} + \frac{300}{8} = 167.5$$

Then

$$q_{s,0} = -\frac{1}{167.5} S_y \left(1.57 \times 10^{-3} \times \frac{300}{8} - 1.77 \times 10^{-3} \times \frac{300}{10}\right)$$

which gives

$$q_{s,0} = -0.034 \times 10^{-3} S_y$$

The complete shear flow distribution is then as shown in Fig. S.20.1(b).

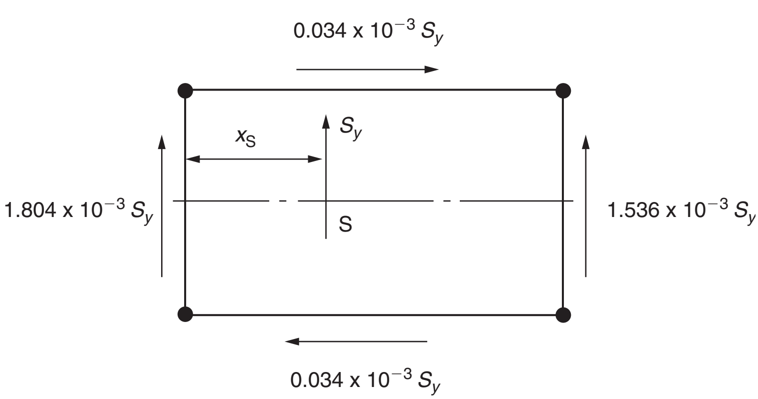

Fig. S.20.1(b)

Taking moments about the intersection of the horizontal axis of symmetry and the left-hand web

$$S_y x_S = 1.536 \times 10^{-3} S_y \times 300 \times 500 - 2 \times 0.034 \times 10^{-3} S_y \times 500 \times 150$$

from which

$$x_S = 225\,\text{mm}$$

S.20.2

From Eq. (20.6)

$$q_s = -\frac{S_y}{I_{xx}} \sum_{r=1}^{n} B_r y_r$$

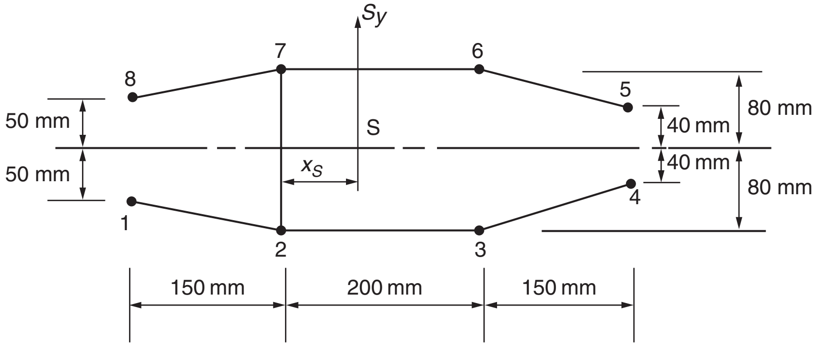

Fig. S.20.2(a)

where

$$I_{xx} = 4 \times 2.0 \times 80^2 + 2 \times 200 \times 50^2 + 2 \times 200 \times 40^2$$

i.e.

$$I_{xx} = 8.04 \times 10^6 \text{ mm}^4$$

Then

$$q_s = -1.86 \times 10^{-4} \sum_{r=1}^{n} B_r y_r$$

from which

$$q_{12} = -1.86 \times 10^{-4} \times 200 \times (-50) = 1.86 \text{ N/mm}$$

$$q_{43} = -1.86 \times 10^{-4} \times 200 \times (-40) = 1.49 \text{ N/mm}$$

$$q_{32} = 1.49 - 1.86 \times 10^{-4} \times 250 \times (-80) = 5.21 \text{ N/mm}$$

$$q_{27} = 1.86 + 5.21 - 1.86 \times 10^{-4} \times 250(-80) = 10.79 \text{ N/mm.}$$

The remaining shear flow distribution follows from symmetry; the complete distribution is shown in Fig. S.20.2(b).

Taking moments about the mid-point of web 27

$$S_y x_S = 2(q_{12} \times 150 \times 80 - q_{32} \times 200 \times 80 - q_{43} \times 150 \times 80 - q_{43} \times 40 \times 200)$$

which gives

$$x_S = -122 \text{ mm} \quad \text{(i.e. to the left of web 27)}$$

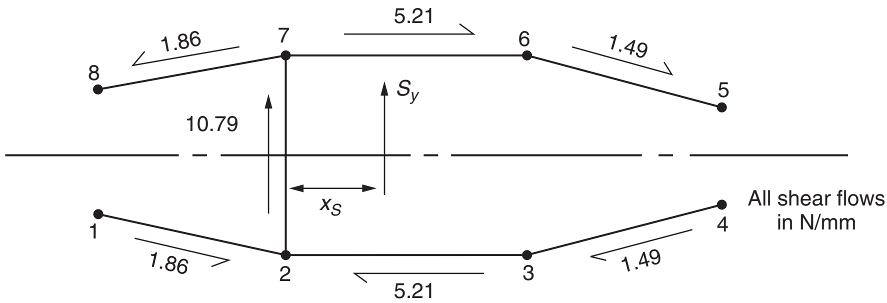

Fig. S.20.2(b)

S.20.3

The shear centre, S, lies on the horizontal axis of symmetry, the x axis. Therefore apply an arbitrary shear load, S_y, through S (Fig. S.20.3(a)). The internal shear flow distribution is given by Eq. (20.11) which, since $I_{xy} = 0$, $S_x = 0$ and $t_D = 0$, simplifies to

$$q_s = -\frac{S_y}{I_{xx}} \sum_{r=1}^{n} B_r y_r + q_{s,0} \tag{i}$$

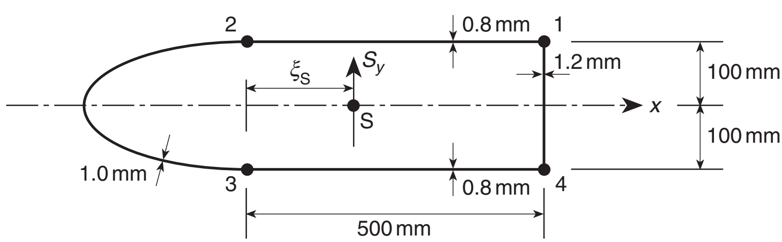

Fig. S.20.3(a)

in which

$$I_{xx} = 2 \times 450 \times 100^2 + 2 \times 550 \times 100^2 = 20 \times 10^6 \text{ mm}^4$$

Equation (i) then becomes

$$q_s = -0.5 \times 10^{-7} S_y \sum_{r=1}^{n} B_r y_r + q_{s,0} \tag{ii}$$

The first term on the right-hand side of Eq. (ii) is the q_b distribution (see Eq. (17.16)). To determine q_b 'cut' the section in the wall 23. Then

$$q_{b,23} = 0$$

$$q_{b,34} = -0.5 \times 10^{-7} S_y \times 550 \times (-100) = 2.75 \times 10^{-3} S_y = q_{b,12}$$

$$q_{b,41} = 2.75 \times 10^{-3} S_y - 0.5 \times 10^{-7} S_y \times 450 \times (-100) = 5.0 \times 10^{-3} S_y$$

The value of shear flow at the 'cut' is obtained using Eq. (17.28) which, since $G =$ constant becomes

$$q_{s,0} = -\frac{\oint (q_b/t)\mathrm{d}s}{\oint \mathrm{d}s/t} \tag{iii}$$

In Eq. (iii)

$$\oint \frac{\mathrm{d}s}{t} = \frac{580}{1.0} + 2 \times \frac{500}{0.8} + \frac{200}{1.2} = 1996.7$$

Then, from Eq. (iii) and the above q_b distribution

$$q_{s,0} = -\frac{S_y}{1996.7}\left(2 \times \frac{2.75 \times 10^{-3} \times 500}{0.8} + \frac{5.0 \times 10^{-3} \times 200}{1.2}\right)$$

i.e.

$$q_{s,0} = -2.14 \times 10^{-3} S_y$$

The complete shear flow distribution is shown in Fig. S.20.3(b).

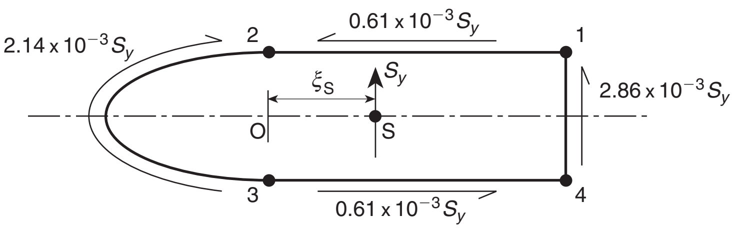

Fig. S.20.3(b)

Now taking moments about O in Fig. S.20.3(b) and using the result of Eq. (20.10)

$$\begin{aligned} S_y\xi_S =& 2 \times 0.61 \times 10^{-3} S_y \times 500 \times 100 + 2.86 \times 10^{-3} S_y \times 200 \times 500 \\ &- 2.14 \times 10^{-3} S_y \times 2(135\,000 - 500 \times 200) \end{aligned}$$

which gives

$$\xi_S = 197.2\,\text{mm}$$

S.20.4

The x axis is an axis of symmetry so that $I_{xy} = 0$, also the shear centre, S, lies on this axis. Apply an arbitrary shear load, S_y, through S. The internal shear flow distribution is then given by Eq. (20.11) in which $S_x = 0$ and $I_{xy} = 0$. Thus

$$q_s = -\frac{S_y}{I_{xx}}\left(\int_0^s t_D y\,\mathrm{d}s + \sum_{r=1}^{n} B_r y_r\right) + q_{s,0} \tag{i}$$

in which from Fig. S.20.4

$$I_{xx} = 4 \times 100 \times 40^2 + 2 \times 0.64 \times 240 \times 40^2 + \frac{0.36 \times 80^3}{12} + \frac{0.64 \times 80^3}{12}$$

i.e.

$$I_{xx} = 1.17 \times 10^6 \text{ mm}^4$$

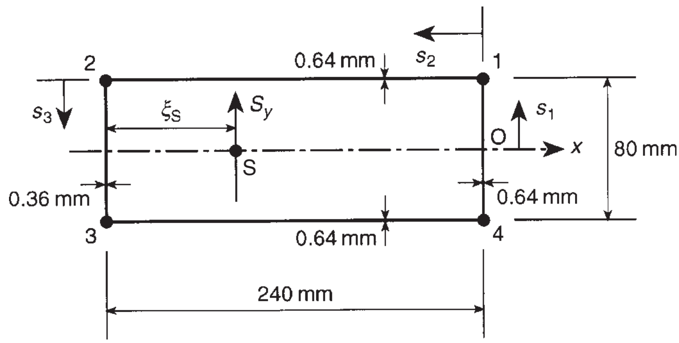

Fig. S.20.4

'Cut' the section at O. Then, from the first two terms on the right-hand side of Eq. (i)

$$q_{b,O1} = -\frac{S_y}{I_{xx}} \int_0^{s_1} 0.64 s_1 \, ds_1$$

i.e.

$$q_{b,O1} = -0.27 \times 10^{-6} S_y s_1^2 \qquad \text{(ii)}$$

and

$$q_{b,1} = -4.32 \times 10^{-4} S_y$$

Also

$$q_{b,12} = -\frac{S_y}{I_{xx}} \left(\int_0^{s_2} 0.64 \times 40 \, ds_2 + 100 \times 40 \right) - 4.32 \times 10^{-4} S_y$$

i.e.

$$q_{b,12} = -10^{-4} S_y (0.22 s_2 + 38.52) \qquad \text{(iii)}$$

whence

$$q_{b,2} = -91.32 \times 10^{-4} S_y$$

Finally

$$q_{b,23} = -\frac{S_y}{I_{xx}} \left[\int_0^{s_3} 0.36(40 - s_3) ds_3 + 100 \times 40 \right] - 91.32 \times 10^{-4} S_y$$

i.e.

$$q_{b,23} = -10^{-4}S_y(0.12s_3 - 0.15 \times 10^{-2}s_3^2 + 125.52) \tag{iv}$$

The remaining q_b distribution follows from symmetry. From Eq. (17.27)

$$q_{s,0} = -\frac{\oint (q_b/t)\mathrm{d}s}{\oint \mathrm{d}s/t} \tag{v}$$

in which

$$\oint \frac{\mathrm{d}s}{t} = \frac{80}{0.64} + \frac{2 \times 240}{0.64} + \frac{80}{0.36} = 1097.2$$

Now substituting in Eq. (v) for $q_{b,O1}$, $q_{b,12}$ and $q_{b,23}$ from Eqs (ii)–(iv), respectively

$$q_{s,0} = \frac{2 \times 10^{-4}S_y}{1097.2}\left[\int_0^{40} \frac{0.27 \times 10^{-2}}{0.64}s_1^2\,\mathrm{d}s_1 + \int_0^{240} \frac{1}{0.64}(0.22s_2 + 38.52)\mathrm{d}s_2 \right.$$
$$\left. + \int_0^{40} \frac{1}{0.64}(0.12s_3 - 0.15 \times 10^{-2}s_3^2 + 125.52)\mathrm{d}s_3\right]$$

from which

$$q_{s,0} = 70.3 \times 10^{-4}S_y$$

The complete shear flow distribution is then

$$q_{O1} = -10^{-4}S_y(0.27 \times 10^{-2}s_1^2 - 70.3) \tag{vi}$$

$$q_{12} = q_{34} = -10^{-4}S_y(0.22s_2 - 31.78) \tag{vii}$$

$$q_{23} = -10^{-4}S_y(0.12s_3 - 0.15 \times 10^{-2}s_3^2 - 55.22) \tag{viii}$$

Taking moments about the mid-point of the wall 23

$$S_y\xi_S = 2\left[\int_0^{40} q_{O1} \times 240\,\mathrm{d}s_1 + \int_0^{240} q_{12} \times 40\,\mathrm{d}s_2\right] \tag{ix}$$

Substituting for q_{O1} and q_{12} from Eqs (vi) and (vii) in Eq. (ix)

$$S_y\xi_S = -2 \times 10^{-4}S_y\left[\int_0^{40} (0.27 \times 10^{-2}s_1^2 - 70.3) \times 240\,\mathrm{d}s_1 \right.$$
$$\left. + \int_0^{240} (0.22s_2 - 31.78) \times 40\,\mathrm{d}s_2\right]$$

from which

$$\xi_S = 142.5\,\mathrm{mm}$$

S.20.5

Referring to Fig. S.20.5(a) the x axis of the beam cross-section is an axis of symmetry so that $I_{xy} = 0$. Further, S_y at the end A is equal to -4450 N and $S_x = 0$. The total deflection, Δ, at one end of the beam is then, from Eqs (20.17) and (20.19)

$$\Delta = \int_L \frac{M_{x,1}M_{x,0}}{EI_{xx}}\mathrm{d}z + \int_L \left(\int_{\text{sect}} \frac{q_0 q_1}{Gt}\mathrm{d}s\right)\mathrm{d}z \tag{i}$$

in which q_0, from Eqs (20.20) and (20.11) is given by

$$q_0 = -\frac{S_{y,0}}{I_{xx}}\sum_{r=1}^{n} B_r y_r + q_{s,0} \tag{ii}$$

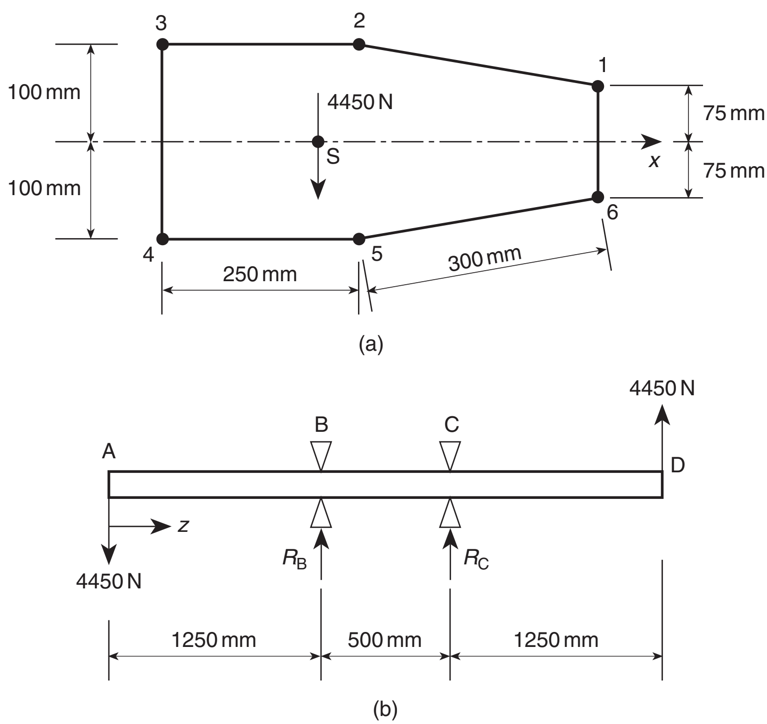

Fig. S.20.5

and

$$q_1 = \frac{q_0}{4450}$$

Since the booms carrying all the direct stresses, I_{xx} in Eq. (i) is, from Fig. S.20.5(a)

$$I_{xx} = 2 \times 650 \times 100^2 + 2 \times 650 \times 75^2 + 2 \times 1300 \times 100^2 = 46.3 \times 10^6\ \text{mm}^4$$

Also, from Fig. S.20.5(b) and taking moments about C

$$R_B \times 500 - 4450 \times 1750 - 4450 \times 1250 = 0$$

from which

$$R_B = 26\,700\,\text{N}$$

Therefore in AB

$$M_{x,0} = 4450z \quad M_{x,1} = z$$

and in BC

$$M_{x,0} = 33.4 \times 10^6 - 22\,250z \quad M_{x,1} = 7500 - 5z$$

Thus the deflection, Δ_M, due to bending at the end A of the beam is, from the first term on the right-hand side of Eq. (i)

$$\Delta_M = \frac{1}{EI_{xx}}\left\{\int_0^{1250} 4450z^2\,\mathrm{d}z + \int_{1250}^{1500} 4450(7500 - 5z)^2\mathrm{d}z\right\}$$

i.e.

$$\Delta_M = \frac{4450}{69\,000 \times 46.3 \times 10^6}\left\{\left[\frac{z^3}{3}\right]_0^{1250} - \frac{1}{15}[(7500 - 5z)^3]_{1250}^{1500}\right\}$$

from which

$$\Delta_M = 1.09\,\text{mm}$$

Now 'cut' the beam section in the wall 12. From Eq. (20.11), i.e.

$$q_s = -\frac{S_y}{I_{xx}}\sum_{r=1}^{n} B_r y_r + q_{s,0} \tag{iii}$$

$$q_{b,12} = 0$$

$$q_{b,23} = -\frac{S_y}{I_{xx}} \times 1300 \times 100 = -130\,000\frac{S_y}{I_{xx}}$$

$$q_{b,34} = -130\,000\frac{S_y}{I_{xx}} - \frac{S_y}{I_{xx}} \times 650 \times 100 = -195\,000\frac{S_y}{I_{xx}}$$

$$q_{b,16} = -\frac{S_y}{I_{xx}} \times 650 \times 75 = -48\,750\frac{S_y}{I_{xx}}$$

The remaining distribution follows from symmetry. The shear load is applied through the shear centre of the cross-section so that $\mathrm{d}\theta/\mathrm{d}z = 0$ and $q_{s,0}$ is given by Eq. (17.28), i.e.

$$q_{s,0} = -\frac{\oint q_b\,\mathrm{d}s}{\oint \mathrm{d}s} \quad (t = \text{constant})$$

in which

$$\oint \mathrm{d}s = 2 \times 300 + 2 \times 250 + 2 \times 100 + 2 \times 75 = 1450\,\text{mm}$$

i.e.

$$q_{s,0} = -\frac{2S_y}{1450I_{xx}}(-130\,000 \times 250 - 195\,000 \times 100 + 48\,750 \times 75)$$

from which

$$q_{s,0} = 66\,681 S_y/I_{xx}$$

Then

$$\begin{aligned}
q_{12} &= 66\,681 S_y/I_{xx} \\
q_{23} &= -63\,319 S_y/I_{xx} \\
q_{34} &= -128\,319 S_y/I_{xx} \\
q_{16} &= -115\,431 S_y/I_{xx}
\end{aligned}$$

Therefore the deflection, Δ_S, due to shear is, from the second term in Eq. (i)

$$\Delta_S = \int_L \left(\int_{\text{sect}} \frac{q_0 q_1}{Gt} \mathrm{d}s \right) \mathrm{d}z$$

i.e.

$$\Delta_S = \int_L \left\{ 2\frac{S_{y,0}S_{y,1}}{GtI_{xx}^2}(115\,431^2 \times 75 + 66\,681^2 \times 300 + 63\,319^2 \times 250 \right.$$
$$\left. + 128\,319^2 \times 100) \right\} \mathrm{d}z$$

Thus

$$\Delta_S = \int_L 2\frac{S_{y,0}S_{y,1} \times 4.98 \times 10^{12}}{26\,700 \times 2.5 \times (46.3 \times 10^6)^2} \mathrm{d}z = \int_L 6.96 \times 10^{-8} S_{y,0}S_{y,1}\, \mathrm{d}z$$

Then

$$\Delta_S = \int_0^{1250} 6.96 \times 10^{-8} \times 4450 \times 1\, \mathrm{d}z + \int_{1250}^{1500} 6.96 \times 10^{-8} \times 22\,250 \times 5\, \mathrm{d}z$$

from which

$$\Delta_S = 2.32\,\text{mm}$$

The total deflection, Δ, is then

$$\Delta = \Delta_M + \Delta_S = 1.09 + 2.32 = 3.41\,\text{mm}$$

S.20.6

At any section of the beam the applied loading is equivalent to bending moments in vertical and horizontal planes, to vertical and horizontal shear forces through the shear centre (the centre of symmetry C) plus a torque. However, only the vertical deflection of A is required so that the bending moments and shear forces in the horizontal plane do not contribute directly to this deflection. The total deflection is, from Eqs (20.14), (20.17) and (20.19)

$$\Delta = \int_L \frac{T_0 T_1}{GJ}\mathrm{d}z + \int_L \frac{M_{x,1}M_{x,0}}{EI_{xx}}\mathrm{d}z + \int_L \left(\int_{\text{sect}} \frac{q_0 q_1}{Gt}\mathrm{d}s\right)\mathrm{d}z \qquad \text{(i)}$$

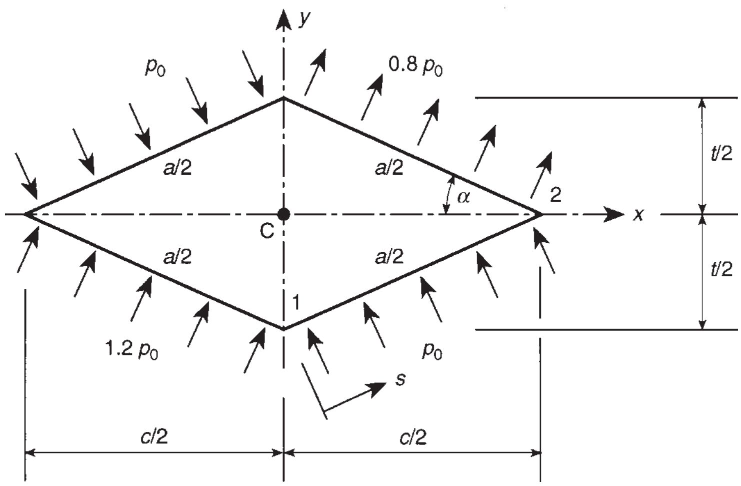

Fig. S.20.6

Referring to Fig. S.20.6 the vertical force/unit length on the beam is

$$1.2p_0\frac{c}{2} + p_0\frac{c}{2} + 0.8p_0\frac{c}{2} - p_0\frac{c}{2} = p_0 c \quad \text{(upwards)}$$

acting at a distance of $0.2c$ to the right of the vertical axis of symmetry. Also the horizontal force/unit length on the beam is

$$1.2p_0\frac{t}{2} + p_0\frac{t}{2} + 0.8p_0\frac{t}{2} - p_0\frac{t}{2} = p_0 t$$

acting to the right and at a distance $0.2t$ above the horizontal axis of symmetry. Thus, the torque/unit length on the beam is

$$p_0 c \times 0.2c - p_0 t \times 0.2t = 0.2p_0(c^2 - t^2)$$

acting in an anticlockwise sense. Then, at any section, a distance z from the built-in end of the beam

$$T_0 = 0.2p_0(c^2 - t^2)(L - z) \quad T_1 = -1\frac{c}{2} \quad \text{(unit load acting upwards at } A\text{)}$$

Comparing Eqs (3.12) and (18.4)

$$J = \frac{4A^2}{\oint \frac{\mathrm{d}s}{t}}$$

i.e.

$$J = 4\left(2\frac{t}{2}\frac{c}{2}\right)^2 \bigg/ \frac{2a}{t_0} = \frac{t^2c^2t_0}{2a}$$

Then

$$\int_0^L \frac{T_0T_1}{GJ}\mathrm{d}z = -\int_0^L \frac{0.1p_0(c^2-t^2)c}{Gt^2c^2t_0/2a}(L-z)\mathrm{d}z = \frac{0.1p_0aL^2(t^2-c^2)}{Gt^2t_0c} \qquad \text{(ii)}$$

The bending moment due to the applied loading at any section a distance z from the built-in end is given by

$$M_{x,0} = -\frac{p_0c}{2}(L-z)^2 \quad \text{also } M_{x,1} = -1(L-z)$$

Thus

$$\int_0^L \frac{M_{x,1}M_{x,0}}{EI_{xx}}\mathrm{d}z = \frac{p_0c}{2EI_{xx}}\int_0^L (L-z)^3\mathrm{d}z$$

in which

$$I_{xx} = 2\frac{(a)^3t_0\sin^2\alpha}{12} = \frac{a^3t_0}{6}\left(\frac{t/2}{a/2}\right)^2 = \frac{at^2t_0}{6}$$

Then

$$\int_0^L \frac{M_{x,1}M_{x,0}}{EI_{xx}}\mathrm{d}z = \frac{3p_0c}{Eat^2t_0}\left[-\frac{1}{4}(L-z)^4\right]_0^L = \frac{3p_0cL^4}{4Eat^2t_0} \qquad \text{(iii)}$$

The shear load at any section a distance z from the built-in end produced by the actual loading system is given by

$$S_{y,0} = p_0c(L-z) \quad \text{also } S_{y,1} = 1$$

From Eq. (17.15), in which $I_{xy} = 0$ and $S_x = 0$

$$q_s = -\frac{S_y}{I_{xx}}\int_0^s ty\,\mathrm{d}s + q_{s,0} \qquad \text{(iv)}$$

If the origin of s is taken at the point 1, $q_{s,0} = 0$ since the shear load is applied on the vertical axis of symmetry, Eq. (iv) then becomes

$$q_s = -\frac{S_y}{I_{xx}}\int_0^s ty\,\mathrm{d}s$$

and

$$q_{12} = -\frac{6S_y}{at^2 t_0} \int_0^s t_0 \left(-\frac{t}{2} + s \sin \alpha \right) \mathrm{d}s$$

i.e.

$$q_{12} = \frac{6S_y}{at^2} \left(\frac{t}{2} s - \frac{t}{a} \frac{s^2}{2} \right)$$

Thus

$$q_{12} = \frac{3S_y}{at} \left(s - \frac{s^2}{a} \right)$$

The remaining distribution follows from symmetry. Then

$$\int_{\text{sect}} \frac{q_0 q_1}{Gt} \mathrm{d}s = 4 \times \frac{9p_0 c(L-z)}{Ga^2 t^2 t_0} \int_0^{a/2} \left(s - \frac{s^2}{a} \right)^2 \mathrm{d}s$$

i.e.

$$\int_{\text{sect}} \frac{q_0 q_1}{Gt} \mathrm{d}s = \frac{3p_0 ca(L-z)}{5Gt^2 t_0}$$

Then

$$\int_0^L \left(\int_{\text{sect}} \frac{q_0 q_1}{Gt} \mathrm{d}s \right) \mathrm{d}z = \frac{3p_0 ca}{5Gt^2 t_0} \int_0^L (L-z) \mathrm{d}z = \frac{3p_0 caL^2}{10Gt^2 t_0} \qquad \text{(v)}$$

Now substituting in Eq. (i) from Eqs (ii), (iii) and (v)

$$\Delta = \frac{0.1 p_0 a L^2 (t^2 - c^2)}{Gt^2 t_0 c} + \frac{3p_0 cL^4}{4Eat^2 t_0} + \frac{3p_0 caL^2}{10Gt^2 t_0}$$

i.e.

$$\Delta = \frac{p_0 L^2}{t^2 t_0} \left[\frac{a(t^2 - c^2)}{10Gc} + \frac{3cL^2}{4Ea} + \frac{3ca}{10G} \right]$$

Substituting the given values and taking $a \simeq c$

$$\Delta = \frac{p_0 (2c)^2}{(0.05c)^2 t_0} \left[\frac{c[(0.05c)^2 - c^2]}{4E} + \frac{3c(2c)^2}{4Ec} + \frac{3c^2}{4E} \right]$$

Neglecting the term $(0.05c)^2$ in $[(0.05c)^2 - c^2]$ gives

$$\Delta = \frac{5600 p_0 c^2}{E t_0}$$

S.20.7

The pressure loading is equivalent to a shear force/unit length of $3bp_0/2$ acting in the vertical plane of symmetry together with a torque $= 3bp_0(3b/2 - b)/2 = 3b^2p_0/4$ as shown in Fig. S.20.7. The deflection of the beam is then, from Eqs (20.14), (20.17) and (20.19)

$$\Delta = \int_L \frac{T_0 T_1}{GJ} \mathrm{d}z + \int_L \frac{M_{x,1} M_{x,0}}{EI_{xx}} \mathrm{d}z + \int_L \left(\int_{\text{sect}} \frac{q_0 q_1}{Gt} \mathrm{d}s \right) \mathrm{d}z \tag{i}$$

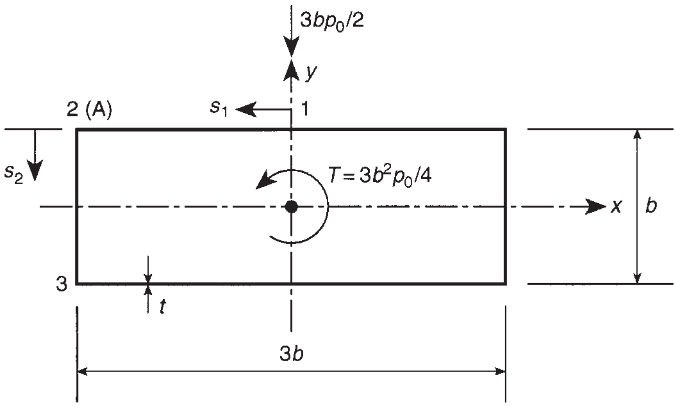

Fig. S.20.7

Now

$$T_0 = 3b^2 p_0 (L - z)/4 \quad T_1 = 3b/2$$

Also, from Eqs (3.12) and (18.4)

$$J = \frac{4A^2}{\oint \mathrm{d}s/t} = \frac{4(3b^2)^2}{8b/t} = \frac{9b^3 t}{2}$$

Thus

$$\int_L \frac{T_0 T_1}{GJ} \mathrm{d}z = \int_0^L \frac{p_0}{4Gt}(L - z)\mathrm{d}z = \frac{p_0 L^2}{8Gt} \tag{ii}$$

Also

$$M_{x,0} = 3bp_0(L - z)^2/4 \quad M_{x,1} = 1(L - z)$$

Then

$$\int_L \frac{M_{x,1} M_{x,0}}{EI_{xx}} \mathrm{d}z = \int_0^L \frac{3bp_0}{4EI_{xx}}(L - z)^3 \mathrm{d}z$$

in which

$$I_{xx} = 2 \times 3bt \times (b/2)^2 + 2tb^3/12 = 5b^3 t/3$$

Thus

$$\int_L \frac{M_{x,1}M_{x,0}}{EI_{xx}}\mathrm{d}z = \frac{9p_0}{20Eb^2t}\int_0^L (L-z)^3\mathrm{d}z = \frac{9p_0L^4}{80Eb^2t} \tag{iii}$$

Further

$$S_{y,0} = -\frac{3bp_0}{2}(L-z) \quad S_{y,1} = -1$$

Taking the origin for s at 1 in the plane of symmetry where $q_{s,0} = 0$ and since $I_{xy} = 0$ and $S_x = 0$, Eq. (17.15) simplifies to

$$q_s = -\frac{S_y}{I_{xx}}\int_0^s ty\,\mathrm{d}s$$

Then

$$q_{12} = -\frac{3S_y}{5b^3t}\int_0^{s_1} t\left(\frac{b}{2}\right)\mathrm{d}s_1$$

i.e.

$$q_{12} = -\frac{3S_y}{10b^2}s_1$$

from which

$$q_2 = -\frac{9S_y}{20b}$$

Also

$$q_{23} = -\frac{S_y}{I_{xx}}\int_0^{s_2} t\left(\frac{b}{2} - s_2\right)\mathrm{d}s_2 - \frac{9S_y}{20b}$$

i.e.

$$q_{23} = -\frac{3S_y}{5b^3}\left(\frac{b}{2}s_2 - \frac{s_2^2}{2}\right) - \frac{9S_y}{20b}$$

Hence

$$q_{23} = -\frac{3S_y}{20b}\left(2\frac{s_2}{b} - 2\frac{s_2^2}{b^2} + 3\right)$$

Then

$$\int_{\text{sect}} \frac{q_0q_1}{Gt}\mathrm{d}s = 4\int_0^{3b/2} \frac{3bp_0(L-z)}{2Gt}\left(\frac{3}{10b^2}\right)^2 s_1^2\mathrm{d}s_1$$
$$+ 2\int_{3b/2}^{b} \frac{3bp_0(L-z)}{2Gt}\left(\frac{3}{20}\right)^2\left(2\frac{s_2}{b} - 2\frac{s_2^2}{b^2} + 3\right)^2 \mathrm{d}s_2$$

which gives

$$\int_{\text{sect}} \frac{q_0 q_1}{Gt} \mathrm{d}s = \frac{1359 p_0}{1000Gt}(L - z)$$

Hence

$$\int_0^L \left(\int_{\text{sect}} \frac{q_0 q_1}{Gt} \mathrm{d}s \right) \mathrm{d}z = \frac{1359 p_0}{1000Gt} \int_0^L (L - z)\, \mathrm{d}z = \frac{1359 p_0 L^2}{2000Gt} \qquad \text{(iv)}$$

Substituting in Eq. (i) from Eqs (ii)–(iv) gives

$$\Delta = \frac{p_0 L^2}{8Gt} + \frac{9 p_0 L^4}{80Eb^2 t} + \frac{1359 p_0 L^2}{2000Gt}$$

Thus

$$\Delta = \frac{p_0 L^2}{t} \left(\frac{9L^2}{80Eb^2} + \frac{1609}{2000G} \right)$$

Solutions to Chapter 21 Problems

S.21.1

Referring to Fig. P.21.1 the bending moment at section 1 is given by

$$M_1 = \frac{15 \times 1^2}{2} = 7.5\,\text{kN m}$$

Thus

$$P_{z,\text{U}} = -P_{z,\text{L}} = \frac{7.5}{300 \times 10^{-3}} = 25\,\text{kN}$$

Also

$$P_{y,\text{U}} = 0 \quad \text{and} \quad P_{y,\text{L}} = -25 \times \frac{100}{1 \times 10^3} = -2.5\,\text{kN} \quad \text{(see Eqs (21.1))}$$

Then

$$P_\text{U} = \sqrt{P_{z,\text{U}}^2 + P_{y,\text{U}}^2} = 25\,\text{kN} \quad \text{(tension)}$$

$$P_\text{L} = -\sqrt{25^2 + 2.5^2} = -25.1\,\text{kN} \quad \text{(compression)}$$

The shear force at section 1 is $15 \times 1 = 15$ kN. This is resisted by $P_{y,\text{L}}$, the shear force in the web. Thus

$$\text{shear in web} = 15 - 2.5 = 12.5\,\text{kN}$$