Solutions to Chapter 19 Problems

S.19.1

From Example 19.1

$$I_{xx} = 14.5 \times 10^6 \text{ mm}^4$$

From Eq. (16.18) in which $M_y = 0$ and $I_{xy} = 0$

$$\sigma_z = \frac{M_x}{I_{xx}} y$$

Therefore

$$\sigma_z = \frac{20 \times 10^6}{14.5 \times 10^6} y = 1.38y \qquad \text{(i)}$$

The C_x axis is 75 mm (see Example 19.1) from the upper wall 2367 so that, from Eq. (i), the maximum direct stress due to bending will occur in the wall 45 where $y = -125$ mm. Then

$$\sigma_z(\text{max}) = 1.38 \times (-125) = -172.5 \text{ N/mm}^2 \quad \text{(compression)}$$

S.19.2

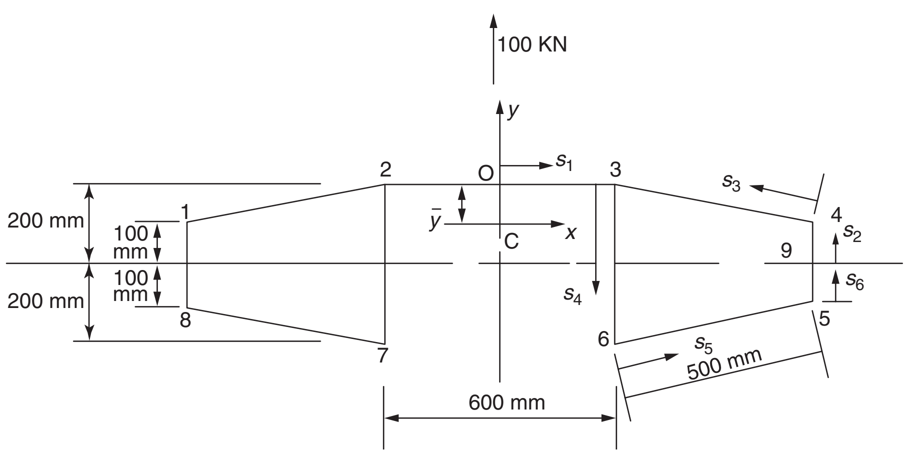

Fig. S.19.2

Take moments of areas about 23

$$2(4 \times 500 + 2 \times 200 + 2 \times 400 + 600)\bar{y} = 2(2 \times 500 \times 50 + 2 \times 500 \times 350 + 2 \times 400 \times 200 + 2 \times 200 \times 200)$$

from which

$$\bar{y} = 168.4 \text{ mm}$$

Then (see Section 16.4.5)

$$I_{xx} = \frac{4 \times 500^3 \times 2}{12}\left(\frac{100}{500}\right)^2 + 2 \times 500 \times 2 \times 118.4^2 + 2 \times 500 \times 2 \times 181.6^2$$
$$+ 2 \times \frac{2 \times 400^3}{12} + 2 \times 2 \times 400 \times 31.6^2 + 2 \times \frac{2 \times 200^3}{12}$$
$$+ 2 \times 2 \times 200 \times 31.6^2 + 2 \times 600 \times 168.4^2$$

i.e.

$$I_{xx} = 157.8 \times 10^6\,\text{mm}^4$$

Since O lies on an axis of symmetry q at O is zero. Then, from Eq. (17.14), the 'basic' or 'open section' shear flows are

$$q_{\text{O}3} = -\frac{S_y}{I_{xx}}t(168.4)s_1$$

and

$$q_3 \text{ in O3} = -\frac{S_y t}{I_{xx}} \times 168.4 \times 300 = -50\,520\frac{S_y t}{I_{xx}}$$

'Cut' the section at mid-point of 54. Then

$$q_{94} = -\frac{S_y t}{I_{xx}}\int_0^{s_2}(s_2 - 31.6)\text{d}s_2 = -\frac{S_y t}{I_{xx}}\left(\frac{s_2^2}{2} - 31.6s_2\right)$$

Then

$$q_4 = -1840\frac{S_y t}{I_{xx}}$$
$$q_{43} = -\frac{S_y t}{I_{xx}}\left[\int_0^{s_3}\left(31.6 + \frac{100}{500}s_3\right)\text{d}s_3 + 1840\right]$$

which gives

$$q_{43} = -\frac{S_y t}{I_{xx}}\left(31.6s_3 + \frac{s_3^2}{10} + 1840\right)$$

and

$$q_3 \text{ (in 43)} = -42\,640\frac{S_y t}{I_{xx}}$$
$$q_{36} = -\frac{S_y t}{I_{xx}}\left[\int_0^{s_4}(168.4 - s_4)\text{d}s_4 + 50\,520 + 42\,640\right]$$

i.e.

$$q_{36} = -\frac{S_y t}{I_{xx}}\left(168.4 s_4 - \frac{s_4^2}{2} + 93\,160\right)$$

and

$$q_6 = -80\,520\frac{S_y t}{I_{xx}}$$

Similarly

$$q_{65} = -\frac{S_y t}{I_{xx}}\left(231.6 s_5 + \frac{s_5^2}{10} + 80\,520\right)$$

and

$$q_5 = 10\,280\frac{S_y t}{I_{xx}}$$

Also

$$q_{59} = -\frac{S_y t}{I_{xx}}\left(131.6 s_6 - \frac{s_6^2}{2} - 10\,280\right)$$

From Eq. (17.28)

$$\oint q_{\mathrm{b}}\,\mathrm{d}s = -\int_0^{300} q_{\mathrm{O3}}\,\mathrm{d}s_1 - \int_0^{100} q_{94}\,\mathrm{d}s_2 - \int_0^{500} q_{43}\,\mathrm{d}s_3 - \int_0^{400} q_{36}\,\mathrm{d}s_4 - \int_0^{500} q_{65}\,\mathrm{d}s_5 - \int_0^{100} q_{59}\,\mathrm{d}s_6$$

Then, substituting for q_{O3}, etc.

$$\oint q_{\mathrm{b}}\,\mathrm{d}s = -65\,885\,801\frac{S_y t}{I_{xx}}$$

Also

$$\oint \mathrm{d}s = 4 \times 500 + 2 \times 400 + 2 \times 200 + 600 = 3800$$

Then

$$q_{\mathrm{s},0} = 34\,677\frac{S_y t}{I_{xx}}$$

Hence, the total shear flows are

$$q_{\mathrm{O3}} = -168.4\frac{S_y t}{I_{xx}} s_1$$

$$q_{36} = -\frac{S_y t}{I_{xx}}\left(168.4 s_4 - \frac{s_4^2}{2} + 93\,160 + 34\,677\right)$$

and so on and at the mid-point of 36.

$$q = -179.4\,\text{N/mm} \quad \text{(in direction 63)}$$

and the shear stress is

$$\frac{179.4}{2} = 89.7\,\text{N/mm}^2$$

S.19.3

For the closed part of the section, from Eq. (18.4)

$$GJ\ \text{(closed)} = \frac{4A^2G}{\oint \frac{\mathrm{d}s}{t}} = \frac{4A^2 \times 25\,000}{\frac{2(400 + 200 + 2 \times 500)}{2}} \tag{i}$$

But

$$A = \frac{1}{2}(400 + 200)(500^2 - 100^2)^{1/2} \times 2 = 293\,938.8\,\text{mm}^2$$

Substituting in Eq. (i)

$$GJ\ \text{(closed)} = 5.4 \times 10^{12}\,\text{N mm}^2$$

From Eq. (18.11)

$$GJ\ \text{(open)} = G\sum \frac{st^3}{3} = \frac{25\,000 \times 600 \times 2^3}{3} = 40 \times 10^6\,\text{N mm}^2$$

(negligible compared to GJ (closed))

Therefore

$$\text{Total } GJ = 5.4 \times 10^{12}\,\text{N mm}^2$$

From Eq. (18.4)

$$\frac{\mathrm{d}\theta}{\mathrm{d}z} = \frac{100 \times 10^6}{5.4 \times 10^{12}} = 18.5 \times 10^{-6}\,\text{rad/mm}$$

Then

$$q = \frac{T}{2A} = \frac{GJ\ \text{(closed)}}{2A}\frac{\mathrm{d}\theta}{\mathrm{d}z} = \frac{5.4 \times 10^{12} \times 18.5 \times 10^{-6}}{\frac{2 \times 293\,938.8}{2}}$$

i.e.

$$q = 340\,\text{N/mm} \quad \text{(in closed part)}$$

Therefore

$$\tau_{max} = 170\,\text{N/mm}^2$$

The maximum shear stress in the open part is, from Eqs (18.12) and (18.13)

$$\tau_{max} = \pm 25\,000 \times 2 \times 18.5 \times 10^{-6} = \pm 0.9\,\text{N/mm}^2$$

Solutions to Chapter 20 Problems

S.20.1

From either Eq. (20.1) or (20.2)

$$B_1 = 60 \times 10 + 40 \times 10 + \frac{500 \times 10}{6}(2 + 1) + \frac{300 \times 10}{6}(2 - 1)$$

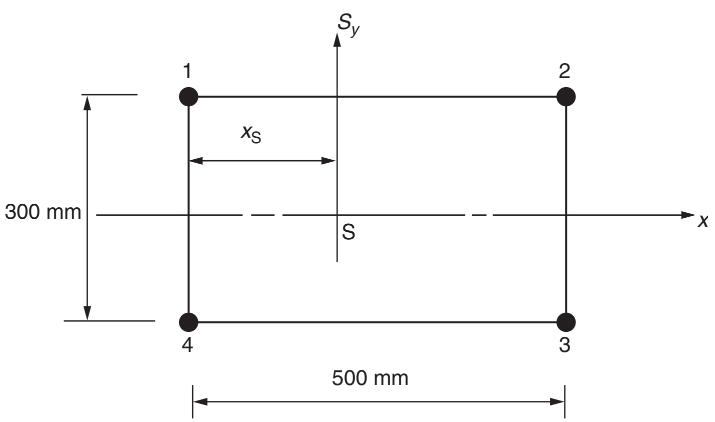

Fig. S.20.1(a)

i.e.

$$B_1 = 4000\,\text{mm}^2 = B_4$$

$$B_2 = 50 \times 8 + 30 \times 8 + \frac{500 \times 10}{6}(2 + 1) + \frac{300 \times 8}{6}(2 - 1)$$

i.e.

$$B_2 = 3540\,\text{mm}^2 = B_3$$

Since the section is now idealized, the shear flow distribution due to an arbitrary shear load S_y applied through the shear centre is, from Eq. (20.11), given by

$$q_s = -\frac{S_y}{I_{xx}} \sum_{r=1}^{n} B_r y_r + q_{s,0} \qquad \text{(i)}$$