Solutions to Chapter 18 Problems

S.18.1

Referring to Fig. P.18.1 the maximum torque occurs at the built-in end of the beam and is given by

$$T_{\max} = 20 \times 2.5 \times 10^3 = 50\,000\,\text{N m}$$

From Eq. (18.1)

$$\tau_{\max} = \frac{q_{\max}}{t_{\min}} = \frac{T_{\max}}{2At_{\min}}$$

i.e.

$$\tau_{\max} = \frac{50\,000 \times 10^3}{2 \times 250 \times 1000 \times 1.2}$$

so that

$$\tau_{\max} = 83.3\,\text{N/mm}^2$$

From Eq. (18.4)

$$\frac{\mathrm{d}\theta}{\mathrm{d}z} = \frac{T}{4A^2} \oint \frac{\mathrm{d}s}{Gt}$$

i.e.

$$\frac{\mathrm{d}\theta}{\mathrm{d}z} = \frac{20(2500 - z) \times 10^3 \times 2}{4 \times (250 \times 1000)^2} \left(\frac{1000}{18\,000 \times 1.2} + \frac{250}{26\,000 \times 2.1} \right)$$

which gives

$$\frac{\mathrm{d}\theta}{\mathrm{d}z} = 8.14 \times 10^{-9}(2500 - z)$$

Then

$$\theta = 8.14 \times 10^{-9} \left(2500z - \frac{z^2}{2} \right) + C_1$$

When $z = 0$, $\theta = 0$ so that $C_1 = 0$, hence

$$\theta = 8.14 \times 10^{-9} \left(2500z - \frac{z^2}{2} \right)$$

Thus θ varies parabolically along the length of the beam and when $z = 2500\,\text{mm}$

$$\theta = 0.0254\,\text{rad} \quad \text{or} \quad 1.46^\circ$$

S.18.2

The shear modulus of the walls of the beam is constant so that Eq. (18.5) may be written

$$w_s - w_0 = \frac{T\delta}{2AG}\left(\frac{\delta_{Os}}{\delta} - \frac{A_{Os}}{A}\right) \tag{i}$$

in which

$$\delta = \oint \frac{ds}{t} \quad \text{and} \quad \delta_{Os} = \int_0^s \frac{ds}{t}$$

Also, the warping displacement will be zero on the axis of symmetry, i.e. at the mid-points of the walls 61 and 34. Therefore take the origin for s at the mid-point of the wall 61, then Eq. (i) becomes

$$w_s = \frac{T\delta}{2AG}\left(\frac{\delta_{Os}}{\delta} - \frac{A_{Os}}{A}\right) \tag{ii}$$

in which

$$l_{23} = \sqrt{500^2 + 100^2} = 509.9\,\text{mm} \quad \text{and} \quad l_{12} = \sqrt{890^2 + 150^2} = 902.6\,\text{mm}$$

Then

$$\delta = \frac{200}{2.0} + \frac{300}{2.5} + \frac{2 \times 509.9}{1.25} + \frac{2 \times 902.6}{1.25} = 2479.9$$

and

$$A = \tfrac{1}{2}(500 + 200) \times 890 + \tfrac{1}{2}(500 + 300) \times 500 = 511\,500\,\text{mm}^2$$

Equation (ii) then becomes

$$w_s = \frac{90\,500 \times 10^3 \times 2479.9}{2 \times 511\,500 \times 27\,500}\left(\frac{\delta_{Os}}{2479.9} - \frac{A_{Os}}{511\,500}\right)$$

i.e.

$$w_s = 7.98 \times 10^4(4.03\delta_{Os} - 0.0196A_{Os}) \tag{iii}$$

The walls of the section are straight so that δ_{Os} and A_{Os} vary linearly within each wall. It follows from Eq. (iii) that w_s varies linearly within each wall so that it is only necessary to calculate the warping displacement at the corners of the section. Thus, referring to Fig. P.18.2

$$w = 7.98 \times 10^{-4}\left(4.03 \times \frac{100}{2.0} - 0.0196 \times \frac{1}{2} \times 890 \times 100\right)$$

i.e.

$$w_1 = -0.53\,\text{mm} = -w_6 \text{ from antisymmetry}$$

Also

$$w_2 = 7.98 \times 10^{-4}\left(4.03 \times \frac{902.6}{1.25} - 0.0196 \times \frac{1}{2} \times 250 \times 890\right) - 0.53$$

i.e.

$$w_2 = 0.05\,\text{mm} = -w_5$$

Finally

$$w_3 = 7.98 \times 10^{-4}\left(4.03 \times \frac{509.9}{1.25} - 0.0196 \times \frac{1}{2} \times 250 \times 500\right) + 0.05$$

i.e.

$$w_3 = 0.38\,\text{mm} = -w_4$$

S.18.3

Referring to Fig. P.18.3 and considering the rotational equilibrium of the beam

$$2R = 2 \times 450 \times 1.0 \times 2000$$

so that

$$R = 1450\,\text{Nm}$$

In the central portion of the beam

$$T = 450 + 1.0(1000 - z) - 1450 = -z\,\text{Nm} \quad (\text{z in mm}) \tag{i}$$

and in the outer portions

$$T = 450 + 1.0(1000 - z) = 1450 - z\,\text{Nm} \quad (\text{z in mm}) \tag{ii}$$

From Eq. (i) it can be seen that T varies linearly from zero at the mid-span of the beam to -500 Nm at the supports. Further, from Eq. (ii) the torque in the outer portions of the beam varies linearly from 950 Nm at the support to 450 Nm at the end. Therefore $T_{\text{max}} = 950$ Nm and from Eq. (18.1)

$$\tau_{\text{max}} = \frac{q_{\text{max}}}{t_{\text{min}}} = \frac{T_{\text{max}}}{2At_{\text{min}}}$$

i.e.

$$\tau_{\text{max}} = \frac{950 \times 10^3}{2 \times \pi \times 50^2 \times 2.5} = 24.2\,\text{N/mm}^2$$

For convenience the datum for the angle of twist may be taken at the mid-span section and angles of twist measured relative to this point. Thus, from Eqs (18.4) and (i), in the central portion of the beam

$$\frac{d\theta}{dz} = \frac{z \times 10^3 \times \pi \times 100}{4(\pi \times 50^2)^2 \times 30\,000 \times 2.5}$$

i.e.

$$\frac{d\theta}{dz} = -1.70 \times 10^{-8} z$$

Then

$$\theta = -1.70 \times 10^{-8} \frac{z^2}{2} + B$$

When $z = 0$, $\theta = 0$ (datum point) so that $B = 0$. Then

$$\theta = -0.85 \times 10^{-8} z^2 \qquad \text{(iii)}$$

In the outer portions of the beam, from Eqs (18.4) and (ii)

$$\frac{d\theta}{dz} = \frac{(1450 - z) \times 10^3 \times \pi \times 100}{4(\pi \times 50^2)^2 \times 30\,000 \times 2.5}$$

i.e.

$$\frac{d\theta}{dz} = 1.70 \times 10^{-8}(1450 - z)$$

Hence

$$\theta = 1.70 \times 10^{-8}\left(1450z - \frac{z^2}{2}\right) + C \qquad \text{(iv)}$$

When $z = 500$ mm, $\theta = -2.13 \times 10^{-3}$ rad from Eq. (iii). Thus, substituting this value in Eq. (iv) gives $C = -12.33 \times 10^{-3}$ and Eq. (iv) becomes

$$\theta = 1.70 \times 10^{-8}\left(1450z - \frac{z^2}{2}\right) - 12.33 \times 10^{-3} \text{ rad} \qquad \text{(v)}$$

The distribution of twist along the beam is then obtained from Eqs (iii) and (v) and is shown in Fig. S.18.3. Note that the distribution would be displaced upwards by 2.13×10^{-3} rad if it were assumed that the angle of twist was zero at the supports.

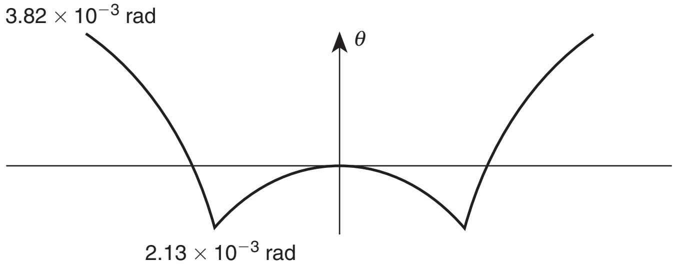

Fig. S.18.3

S.18.4

The total torque applied to the beam is $20 \times 4 \times 10^3$ Nm. From symmetry the reactive torques at A and D will be equal and are 40×10^3 Nm. Therefore,

$$T_{AB} = 40\,000\,\text{N m}$$

$$T_{BC} = 40\,000 - 20(z - 1000) = 60\,000 - 20z\,\text{N m} \quad (z \text{ in mm})$$

Note that the torque distribution is antisymmetrical about the centre of the beam. The maximum torque in the beam is therefore 40 000 Nm so that, from Eq. (18.1)

$$\tau_{max} = \frac{40\,000 \times 10^3}{2 \times 200 \times 350 \times 4} = 71.4\,\text{N/mm}^2$$

The rate of twist along the length of the beam is given by Eq. (18.4) in which

$$\oint = \frac{2 \times 200}{4} + \frac{2 \times 350}{6} = 216.7$$

Then

$$\frac{\mathrm{d}\theta}{\mathrm{d}z} = \left[\frac{216.7}{4 \times (200 \times 350)^2 \times 70\,000}\right] T = 15.79 \times 10^{-14} T$$

In AB, $T_{AB} = 40\,000$ Nm so that

$$\theta_{AB} = 6.32 \times 10^{-6} z + B$$

When $z = 0$, $\theta_{AB} = 0$ so that B = 0 and when $z = 1000$ mm, $\theta_{AB} = 0.0063$ rad (0.361°)

In BC, $T_{BC} = 60\,000 - 20z$ N m. Then, from Eq. (18.4)

$$\theta_{BC} = 15.79 \times 10^{-14}(60\,000z - 10z^2) \times 10^3 + C$$

When $z = 1000$ mm, $\theta_{BC} = 0.0063$ so that $C = -0.0016$. Then

$$\theta_{BC} = 1.579 \times 10^{-10}(60\,000z - 10z^2) - 0.0016$$

At mid-span where $z = 3000$ mm, θ_{BC}=0.0126 rad (0.722°).

S.18.5

The torque is constant along the length of the beam and is 1 kN m. Also the thickness is constant round the beam section so that the shear stress will be a maximum where the area enclosed by the mid-line of the section wall is a minimum, i.e. at the free end. Then

$$\tau_{max} = \frac{1000 \times 10^3}{2 \times 50 \times 150 \times 2} = 33.3\,\text{N/mm}^2$$

The rate of twist is given by Eq. (18.4) in which $\oint \mathrm{d}s/t$ varies along the length of the beam as does the area enclosed by the mid-line of the section wall. Then

$$\oint \frac{\mathrm{d}s}{t} = \left[\frac{2\times 50 + \left(150 + \dfrac{50z}{2500}\right)}{2}\right] = 125 + 0.01z$$

Also

$$A = 50\left(150 + \frac{50z}{2500}\right) = 7500 + z$$

Then

$$\frac{\mathrm{d}\theta}{\mathrm{d}z} = \left(\frac{1\times 10^6}{4\times 25\,000}\right)\frac{(125 + 0.01z)}{(7500 + z)^2}$$

or

$$\frac{\mathrm{d}\theta}{\mathrm{d}z} = 10\left[\frac{12\,500 + z}{100(7500 + z)^2}\right]$$

i.e.

$$\frac{\mathrm{d}\theta}{\mathrm{d}z} = 0.1\left[\frac{5000}{(7500 + z)^2} + \frac{1}{7500 + z}\right]$$

Then

$$\theta = 0.1\left[\frac{-5000}{(7500 + z)} + \log_e(7500 + z) + B\right]$$

When $z = 2500$ mm, $\theta = 0$ so that $B = -6.41$ and

$$\theta = 0.1\left[\frac{-5000}{(7500 + z)} + \log_e(7500 + z) - 6.41\right]\text{rad}$$

When $z = 0$, $\theta = 10.6°$, etc.

S.18.6

In Eq. (18.4), i.e.

$$\frac{\mathrm{d}\theta}{\mathrm{d}z} = \frac{T}{4A^2}\oint \frac{\mathrm{d}s}{Gt}$$

$Gt = \text{constant} = 44\,000$ N/mm. Thus, referring to Fig. S.18.6

$$\frac{\mathrm{d}\theta}{\mathrm{d}z} = \frac{4500\times 10^3}{4(100\times 200 + \pi\times 50^2/2)^2}\left(\frac{2\times 200 + 100 + \pi\times 50}{44\,000}\right)$$

i.e.

$$\frac{\mathrm{d}\theta}{\mathrm{d}z} = 29.3 \times 10^{-6}\,\mathrm{rad/mm}$$

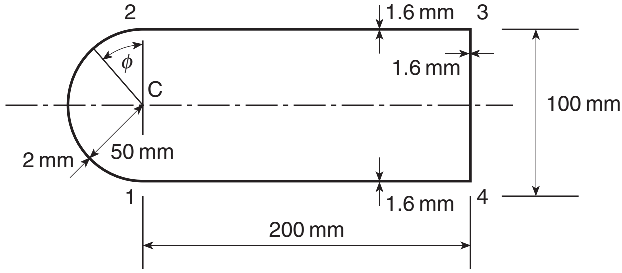

Fig. S.18.6

The warping displacement is zero on the axis of symmetry so that Eq. (18.5) becomes

$$w_s = \frac{T\delta}{2A}\left(\frac{\delta_{\mathrm{O}s}}{\delta} - \frac{A_{\mathrm{O}s}}{A}\right) \tag{i}$$

where

$$\delta = \oint \frac{\mathrm{d}s}{Gt} \quad \text{and} \quad \delta_{\mathrm{O}s} = \int_0^s \frac{\mathrm{d}s}{Gt}$$

Since $Gt = \text{constant}$, Eq. (i) may be written

$$w_s = \frac{T}{2AGt}\oint \mathrm{d}s\left(\frac{\int_0^s \mathrm{d}s}{\oint \mathrm{d}s} - \frac{A_{\mathrm{O}s}}{A}\right) \tag{ii}$$

in which

$$\oint \mathrm{d}s = 2 \times 200 + 100 + \pi \times 50 = 657.1\,\mathrm{mm}$$

and

$$A = 100 \times 200 + \pi \times 50^2/2 = 23\,927.0\,\mathrm{mm}^2$$

Equation (ii) then becomes

$$w_s = \frac{4500 \times 10^3 \times 657.1}{2 \times 23\,927.0 \times 44\,000}\left(\frac{\int_0^s \mathrm{d}s}{657.1} - \frac{A_{\mathrm{O}s}}{23\,927.0}\right)$$

i.e.

$$w_s = 1.40 \times 10^{-3}\left(1.52\int_0^s \mathrm{d}s - 4.18 \times 10^2 A_{\mathrm{O}s}\right) \tag{iii}$$

In the straight walls $\int_0^s \mathrm{d}s$ and $A_{\mathrm{O}s}$ are linear so that it is only necessary to calculate the warping displacement at the corners. Thus

$$w_3 = -w_4 = 1.40 \times 10^{-3}(1.52 \times 50 - 4.18 \times 10^{-2} \times \tfrac{1}{2} \times 200 \times 50) = -0.19\,\text{mm}$$

$$w_2 = -w_1 = 1.40 \times 10^{-3}(1.52 \times 200 - 4.18 \times 10^{-2} \times \tfrac{1}{2} \times 200 \times 50) - 0.19$$

i.e.

$$w_2 = -w_1 = -0.056\,\text{mm}$$

In the wall 21

$$\int_0^s \mathrm{d}s = 50\phi \quad \text{and} \quad A_{\mathrm{O}s} = \tfrac{1}{2} \times 50^2\phi$$

Then Eq. (iii) becomes

$$w_{21} = 1.40 \times 10^{-3}(1.52 \times 50\phi - 4.18 \times 10^{-2} \times \tfrac{1}{2} \times 50^2\phi) - 0.056$$

i.e.

$$w_{21} = 0.033\phi - 0.056 \qquad \text{(iv)}$$

Thus w_{21} varies linearly with ϕ and when $\phi = \pi/2$ the warping displacement should be zero. From Eq. (iv), when $\phi = \pi/2$, $w_{21} = -0.004$ mm; the discrepancy is due to rounding off errors.

S.18.7

Suppose the mass density of the covers is ρ_a and of the webs ρ_b. Then

$$\rho_a = k_1 G_a \quad \rho_b = k_1 G_b$$

Let W be the weight/unit span. Then

$$W = 2at_a\rho_a g + 2bt_b\rho_b g$$

so that, substituting for ρ_a and ρ_b

$$W = 2k_1 g(at_a G_a + bt_b G_b) \qquad \text{(i)}$$

The torsional stiffness may be defined as $T/(\mathrm{d}\theta/\mathrm{d}z)$ and from Eq. (8.4)

$$\frac{\mathrm{d}\theta}{\mathrm{d}z} = \frac{T}{4a^2b^2}\left(\frac{2a}{G_a t_a} + \frac{2b}{G_b t_b}\right) \qquad \text{(ii)}$$

Thus, for a given torsional stiffness, $\mathrm{d}\theta/\mathrm{d}z = $ constant, i.e.

$$\frac{a}{G_a t_a} + \frac{b}{G_b t_b} = \text{constant} = k_2 \qquad \text{(iii)}$$

Let $t_b/t_a = \lambda$. Equation (iii) then becomes

$$t_a = \frac{1}{k_2}\left(\frac{a}{G_a} + \frac{b}{\lambda G_b}\right)$$

and substituting for t_a in Eq. (i)

$$W = 2k_1 g t_a(aG_a + \lambda b G_b) = 2\frac{k_1}{k_2}g\left(a^2 + b^2 + \frac{abG_a}{\lambda G_b} + \frac{\lambda abG_b}{G_a}\right)$$

For a maximum

$$\frac{\mathrm{d}W}{\mathrm{d}\lambda} = 0$$

i.e.

$$\lambda^2 = \left(\frac{G_a}{G_b}\right)^2$$

from which

$$\lambda = \frac{G_a}{G_b} = \frac{t_b}{t_a}$$

For the condition $G_a t_a = G_b t_b$ knowing that a and b can vary. Eq. (i) becomes

$$W = 2k_1 G_a t_a g(a + b) \tag{iv}$$

From Eq. (ii), for constant torsional stiffness

$$\frac{a + b}{a^2 b^2} = \text{constant} = k_3 \tag{v}$$

Let $b/a = x$. Equation (iv) may then be written

$$W = 2k_1 G_a t_a g a(1 + x) \tag{vi}$$

and Eq. (v) becomes

$$k_3 = \frac{1 + x}{a^3 x^2}$$

which gives

$$a^3 = \frac{1 + x}{k_3 x^2}$$

Substituting for a in Eq. (vi)

$$W = \frac{2k_1 G_a t_a g}{k_3^{1/3}}\left(\frac{1 + x}{x^2}\right)^{1/3}(1 + x)$$

i.e.

$$W = \frac{2k_1 G_a t_a g}{k_3^{1/3}} \frac{(1+x)^{4/3}}{x^{2/3}}$$

Hence for $(\mathrm{d}W/\mathrm{d}x) = 0$

$$0 = \frac{4}{3}\frac{(1+x)^{1/3}}{x^{2/3}} - \frac{2}{3}x^{-5/3}(1+x)^{4/3}$$

i.e.

$$4x - 2(1+x) = 0$$

so that

$$x = 1 = b/a$$

S.18.8

The maximum shear stress in the section is given by Eq. (18.13) in which, from Eqs. (18.11)

$$J = 2 \times 2^3 \left(\frac{20 + 15 + 25 + 25}{3}\right) = 453.3\,\mathrm{mm}^4$$

Then

$$\tau_{\max} = \frac{50 \times 10^3 \times 2}{453.3} = 220.6\,\mathrm{N/mm}^2$$

From Eq. (18.12)

$$\frac{\mathrm{d}\theta}{\mathrm{d}z} = \frac{T}{GJ}$$

i.e.

$$\frac{\mathrm{d}\theta}{\mathrm{d}z} = \frac{50 \times 10^3}{25\,000 \times 453.3} = 0.0044\,\mathrm{rad/mm}$$

S.18.9

The rate of twist/unit torque is given by Eq. (18.12).
i.e.

$$\frac{\mathrm{d}\theta}{\mathrm{d}z} = \frac{1}{GJ}$$

where

$$J = \sum \frac{st^3}{3} = \frac{8}{3}(2 \times 25 + 2 \times 61.8 + 60) = 623\,\mathrm{mm}^4$$

Then

$$\frac{d\theta}{dz} = \frac{1}{25\,000 \times 623} = 6.42 \times 10^{-8}\text{rad/mm}$$

S.18.10

From the second of Eqs (18.13) the maximum shear stress is given by

$$\tau_{\text{max}} = \pm\frac{tT}{J} \tag{i}$$

in which J, from Eqs (18.11), is given by (see Fig. P.18.10)

$$J = \frac{100 \times 2.54^3}{3} + 2 \times \frac{38 \times 1.27^3}{3} + \frac{2}{3}\int_0^{50}\left(1.27 + 1.27\frac{s}{50}\right)^3 ds$$

where the origin for s is at the corner 2 (or 5). Thus

$$J = 854.2\,\text{mm}^4$$

Substituting in Eq. (i)

$$\tau_{\text{max}} = \pm\frac{2.54 \times 100 \times 10^3}{854.2} = \pm 297.4\,\text{N/mm}^2$$

The warping distribution is given by Eq. (18.20) and is a function of the swept area, A_R (see Fig. 18.11). Since the walls of the section are straight A_R varies linearly around the cross-section. Also, the warping is zero at the mid-point of the web so that it is only necessary to calculate the warping at the extremity of each wall. Thus

$$\begin{aligned} w_1 &= -2A_R\frac{T}{GJ} = -2 \times \frac{1}{2} \times 25 \times 50 \times \frac{100 \times 10^3}{26\,700 \times 854.2} \\ &= -5.48\,\text{mm} = -w_6 \text{ from antisymmetry} \end{aligned}$$

Note that p_R, and therefore A_R, is positive in the wall 61.

$$w_2 = -5.48 + 2 \times \frac{1}{2} \times 50 \times 50 \times \frac{100 \times 10^3}{26\,700 \times 854.2} = 5.48\,\text{mm} = -w_5$$

(p_R is negative in the wall 12)

$$w_3 = 5.48 + 2 \times \frac{1}{2} \times 38 \times 75 \times \frac{100 \times 10^3}{26\,700 \times 854.2} = 17.98\,\text{mm} = -w_4$$

(p_R is negative in the wall 23)

S.18.11

The maximum shear stress in the section is given by the second of Eqs (18.13), i.e.

$$\tau_{\max} = \pm\frac{t_{\max}T_{\max}}{J} \tag{i}$$

in which $t_{\max} = t_0$ and the torsion constant J is obtained using the second of Eqs (18.11). Thus

$$J = 2\left[\frac{1}{3}\int_0^a \left(\frac{s}{a}t_0\right)^3 \mathrm{d}s + \frac{1}{3}\int_0^{3a}\left(\frac{s}{3a}t_0\right)^3 \mathrm{d}s + \frac{at_0^3}{3}\right]$$

In the first integral s is measured from the point 7 while in the second s is measured from the point 1. Then

$$J = \frac{4at_0^3}{3}$$

Substituting in Eq. (i)

$$\tau_{\max} = \pm\frac{t_0 T}{4at_0^3/3} = \pm\frac{3T}{4at_0^2}$$

The warping distribution is given by Eq. (18.19). Thus, for unit rate of twist

$$w_s = -2A_{\mathrm{R}} \tag{ii}$$

Since the walls are straight A_{R} varies linearly in each wall so that it is only necessary to calculate the warping displacement at the extremities of the walls. Further, the section is constrained to twist about O so that $w_0 = w_3 = w_4 = 0$. Then

$$w_7 = -2\times\tfrac{1}{2}aa = -a^2 = -w_8 \quad (p_{\mathrm{R}} \text{ is positive in 37})$$

$$w_2 = -2\times\tfrac{1}{2}a2a\cos 45^\circ = \sqrt{2}a^2 = -w_5 \quad (p_{\mathrm{R}} \text{ is negative in 32})$$

$$w_1 = \sqrt{2}a^2 + 2\times\tfrac{1}{2}a(2a\sin 45^\circ + a) = a^2(1+2\sqrt{2}) = -w_6 \quad (p_{\mathrm{R}} \text{ is negative in 21})$$

S.18.12

The torsion constant J is given by the first of Eqs (18.11)
i.e.

$$J = \frac{1}{3}(\pi rt^3 + 4rt^3) = 2.38rt^3$$

The maximum shear stress/unit torque is, from Eqs (18.13)

$$\tau_{\max} = \pm\frac{t}{2.38rt^3} = \pm 0.42/rt^2$$

The warping distribution is obtained from Eq. (18.19)

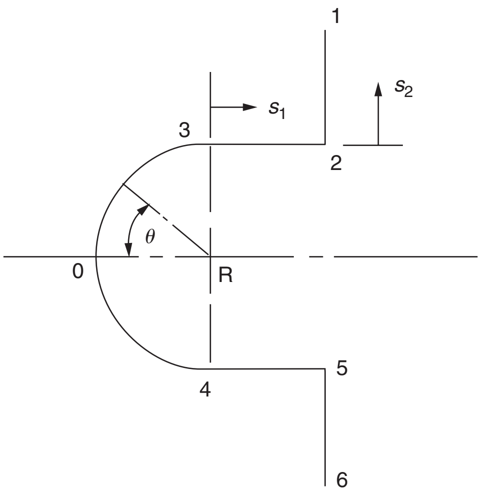

Fig. S.18.12

i.e.

$$w = -2A_R/\text{unit rate of twist}$$

In 03

$$A_R = -\frac{1}{2}r^2\theta$$

so that

$$w_{03} = r^2\theta$$

and

$$w_3 = \frac{r^2\pi}{2} = 1.571\, r^2 = -w_4$$

In 32

$$A_R = -\frac{\pi r^2}{4} - \frac{1}{2}s_1 r$$

and

$$w_{32} = \frac{r}{2}(\pi r + 2s_1)$$

Then

$$w_2 = \frac{r}{2}(\pi r + 2r) = 2.571\, r^2 = -w_5$$

In 21

$$A_R = -\frac{r}{4}(\pi r + 2r) + \frac{1}{2}s_2 r$$

which gives

$$w_{21} = -\frac{r}{2}(2s_2 - 5.142r)$$

and

$$w_1 = +1.571r^2 = -w_6$$

With the centre of twist at 0

$$A_{R,1} = -\left(\frac{\pi r^2}{4} - \frac{r^2}{2}\right) - \frac{1}{2}r^2 + \frac{1}{2}r2r = +0.215\,r^2$$

and

$$w_1 = -0.43r^2$$

Maximum shear stress is unchanged but torsional stiffness increases since the warping is reduced.

S.18.13

The loading is equivalent to a pure torque of $1 \times 25 = 25$ kN/mm acting as shown in Fig. S.18.13 together with a shear load of 1 kN acting at 2 (the shear centre).

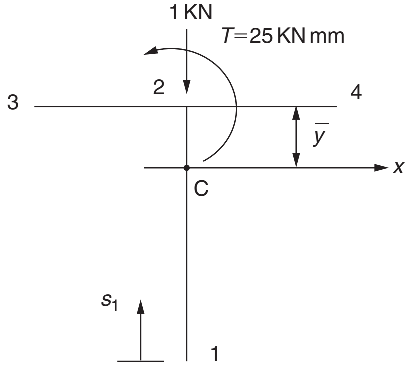

Fig. S.18.13

The maximum shear stress due to the torque is given by Eq. (18.13) in which

$$J = \frac{100 \times 3^3}{3} + \frac{80 \times 2^3}{3} = 1113.3\,\text{mm}^4$$

Then

$$\tau_{\max}(324) = \frac{25 \times 10^3 \times 3}{1113.3} = 67.4\,\text{N/mm}^2$$

$$\tau_{\max}(12) = \frac{25 \times 10^3 \times 2}{1113.3} = 44.9\,\text{N/mm}^2$$

From Eq. (18.12)

$$\frac{d\theta}{dz} = \frac{25 \times 10^3}{25\,000 \times 1113.3} = 9.0 \times 10^{-4}\,\text{rad/mm}$$

The shear flow distribution due to shear is given by Eq. (17.14) in which $S_x = 0$ and $I_{xy} = 0$, i.e.

$$q_s = -\frac{S_y}{I_{xx}} \int_0^s ty\,ds$$

Taking moments of area about the top flange

$$(100 \times 3 + 80 \times 2)\bar{y} = 80 \times 2 \times 40$$

i.e.

$$\bar{y} = 13.9\,\text{mm}$$

Then

$$I_{xx} = 100 \times 3 \times 13.9^2 + \frac{2 \times 80^3}{12} + 80 \times 2 \times 26.1^2 = 252\,290\,\text{mm}^4$$

Therefore

$$q_{12} = -\frac{S_y}{I_{xx}} \int_0^{s_1} 2(-66.1 + s_1)ds_1$$

i.e.

$$q_{12} = -7.93 \times 10^{-3}\left(66.1s_1 - \frac{s_1^2}{2}\right) \quad \text{(i)}$$

From Eq. (i), q_{12} is a maximum when $s_1 = 66.1$ mm. Then

$$q_{12}(\text{max}) = -17.4\,\text{N/mm}$$

and

$$\tau_{12}(\text{max}) = -8.7\,\text{N/mm}^2$$

Also, from Eq. (i) the shear flow at 2 in 12 = −16.6 N/mm so that the maximum shear flow in the flange occurs at 2 and is −16.6/2 = −8.3 N/mm. Then the maximum shear stress in the flange is −8.3/3 = −2.8 N/mm^2 in the directions 32 and 42.

The maximum shear stress due to shear and torsion is then 67.4 + 2.8 = 70.2 N/mm^2 on the underside of 24 at 2 or on the upper surface of 32 at 2.