Solutions to Chapter 17 Problems

S.17.1

In Fig. S.17.1 the x axis is an axis of symmetry (i.e. $I_{xy}=0$) and the shear centre, S, lies on this axis. Suppose S is a distance ξ_S from the web 24. To find ξ_S an arbitrary shear load S_y is applied through S and the internal shear flow distribution determined. Since $I_{xy}=0$ and $S_x=0$, Eq. (17.14) reduces to

$$q_s = -\frac{S_y}{I_{xx}}\int_0^s ty\,\mathrm{d}s \tag{i}$$

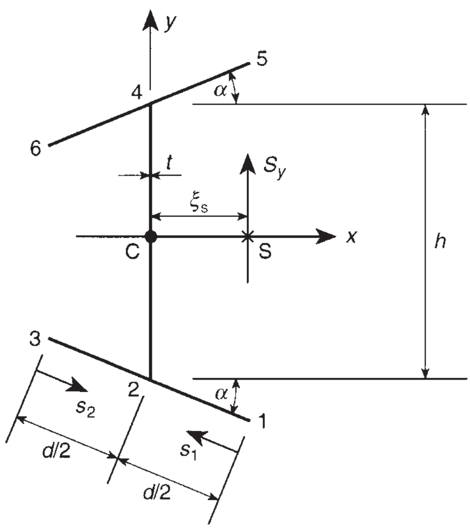

Fig. S.17.1

in which

$$I_{xx} = \frac{th^3}{12} + 2\left[\frac{td^3\sin^2\alpha}{12} + td\left(\frac{h}{2}\right)^2\right]$$

i.e.

$$I_{xx} = \frac{th^3}{12}(1 + 6\rho + 2\rho^3\sin^2\alpha) \tag{ii}$$

Then

$$q_{12} = -\frac{S_y}{I_{xx}}\int_0^{s_1} ty\,\mathrm{d}s_1$$

i.e.

$$q_{12} = \frac{S_y}{I_{xx}}\int_0^{s_1} t\left[\frac{h}{2} + \left(\frac{d}{2} - s_1\right)\sin\alpha\right]\mathrm{d}s_1$$

so that

$$q_{12} = \frac{S_y t}{2I_{xx}}(hs_1 + ds_1 \sin\alpha - s_1^2 \sin\alpha) \qquad \text{(iii)}$$

Also

$$q_{32} = -\frac{S_y}{I_{xx}}\int_0^{s_2} ty\,\mathrm{d}s_2 = \frac{S_y t}{I_{xx}}\int_0^{s_2}\left[\frac{h}{2} - \left(\frac{d}{2} - s_2\right)\sin\alpha\right]\mathrm{d}s_2$$

whence

$$q_{32} = \frac{S_y t}{2I_{xx}}(hs_2 - ds_2 \sin\alpha + s_2^2 \sin\alpha) \qquad \text{(iv)}$$

Taking moments about C in Fig. S.17.1

$$S_y \xi_S = -2\int_0^{d/2} q_{12}\frac{h}{2}\cos\alpha\,\mathrm{d}s_1 + 2\int_0^{d/2} q_{32}\frac{h}{2}\cos\alpha\,\mathrm{d}s_2 \qquad \text{(v)}$$

Substituting in Eq. (v) for q_{12} and q_{32} from Eqs (iii) and (iv)

$$S_y \xi_S = \frac{S_y th\cos\alpha}{I_{xx}}\left[\int_0^{d/2} -(hs_1 + ds_1\sin\alpha - s_1^2\sin\alpha)\mathrm{d}s_1 + \int_0^{d/2}(hs_2 - ds_2\sin\alpha + s_2^2\sin\alpha)\mathrm{d}s_2\right]$$

from which

$$\xi_S = -\frac{thd^3 \sin\alpha\cos\alpha}{12I_{xx}} \qquad \text{(vi)}$$

Now substituting for I_{xx} from Eq. (ii) in (vi)

$$\xi_S = -d\frac{\rho^2 \sin\alpha\cos\alpha}{1 + 6\rho + 2\rho^3\sin^2\alpha}$$

S.17.2

The x axis is an axis of symmetry so that $I_{xy} = 0$ and the shear centre, S, lies on this axis (see Fig. S.17.2). Therefore, an arbitrary shear force, S_y, is applied through S and the internal shear flow distribution determined.

Since $S_x = 0$ and $I_{xy} = 0$, Eq. (17.14) reduces to

$$q_s = -\frac{S_y}{I_{xx}}\int_0^s ty\,\mathrm{d}s \qquad \text{(i)}$$

in which, from Fig. S.17.2.,

$$I_{xx} = 2\left[\frac{a^3 t\sin^2\alpha}{12} + at\left(a\sin\alpha + \frac{a}{2}\sin\alpha\right)^2 + \frac{a^3 t\sin^2\alpha}{12} + at\left(\frac{a}{2}\sin\alpha\right)^2\right]$$

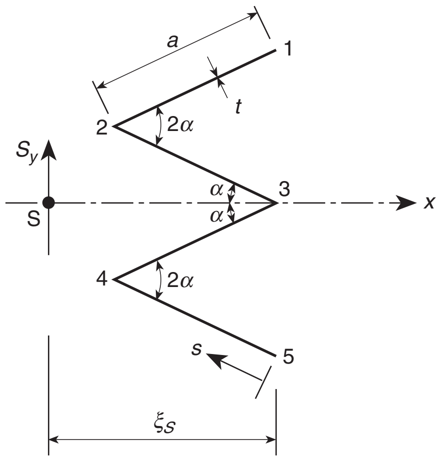

Fig. S.17.2

which gives

$$I_{xx} = \frac{16a^3 t \sin^2 \alpha}{3} \tag{ii}$$

For the flange 54, from Eq. (i)

$$q_{54} = -\frac{S_y}{I_{xx}} \int_0^s t(s - 2a) \sin \alpha \, \mathrm{d}s$$

from which

$$q_{54} = -\frac{S_y t \sin \alpha}{I_{xx}} \left(\frac{s^2}{2} - 2as \right) \tag{iii}$$

Taking moments about the point 3

$$S_y \xi_S = 2 \int_0^a q_{54} a \sin 2\alpha \, \mathrm{d}s \tag{iv}$$

Substituting in Eq. (iv) for q_{54} from Eq. (iii)

$$S_y \xi_S = -\frac{2a \sin 2\alpha S_y t \sin \alpha}{I_{xx}} \int_0^a \left(\frac{s^2}{2} - 2as \right) \mathrm{d}s$$

which gives

$$\xi_S = \frac{2at \sin 2\alpha \sin \alpha}{I_{xx}} \left(\frac{5a^3}{6} \right) \tag{v}$$

Substituting for I_{xx} from Eq. (ii) in (v) gives

$$\xi_S = \frac{5a \cos \alpha}{8}$$

S.17.3

The shear centre, S, lies on the axis of symmetry a distance ξ_S from the point 2 as shown in Fig. S.17.3. Thus, an arbitrary shear load, S_y, is applied through S and since $I_{xy}=0$, $S_x=0$, Eq. (17.14) simplifies to

$$q_s = -\frac{S_y}{I_{xx}}\int_0^s ty\,\mathrm{d}s \qquad \text{(i)}$$

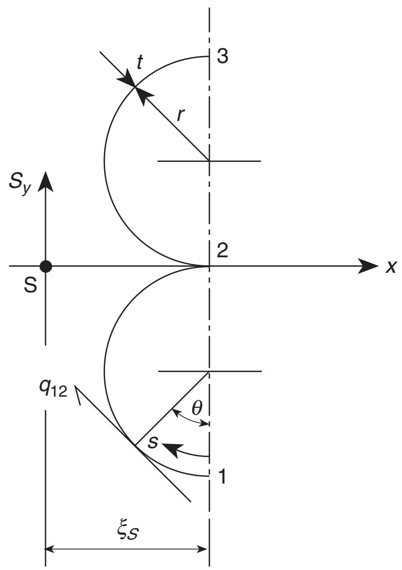

Fig. S.17.3

in which I_{xx} has the same value as the section in S.16.8, i.e. $3\pi r^3 t$. Then Eq. (i) becomes

$$q_{12} = \frac{S_y}{I_{xx}}\int_0^\theta t(r + r\cos\theta)r\,\mathrm{d}\theta$$

or

$$q_{12} = \frac{S_y}{3\pi r}[\theta + \sin\theta]_0^\theta$$

i.e.

$$q_{12} = \frac{S_y}{3\pi r}(\theta + \sin\theta) \qquad \text{(ii)}$$

Taking moments about the point 2

$$S_y\xi_S = 2\int_0^\pi q_{12}(r + r\cos\theta)r\,\mathrm{d}\theta \qquad \text{(iii)}$$

Substituting in Eq. (iii) for q_{12} from Eq. (ii)

$$S_y\xi_S = \frac{2S_y r}{3\pi}\int_0^{\pi}(\theta + \sin\theta)(1 + \cos\theta)\mathrm{d}\theta$$

Thus

$$\xi_S = \frac{2r}{3\pi}\int_0^{\pi}(\theta + \theta\cos\theta + \sin\theta + \sin\theta\cos\theta)\mathrm{d}\theta$$

i.e.

$$\xi_S = \frac{2r}{3\pi}\left[\frac{\theta^2}{2} + \theta\sin\theta - \frac{\cos 2\theta}{4}\right]_0^{\pi}$$

from which

$$\xi_S = \frac{\pi r}{3}$$

S.17.4

The x axis is an axis of symmetry so that $I_{xy} = 0$ and the shear centre, S, lies on this axis (see Fig. S.17.4). Further $S_x = 0$ so that Eq. (17.14) reduces to

$$q_s = -\frac{S_y}{I_{xx}}\int_0^s ty\,\mathrm{d}s \tag{i}$$

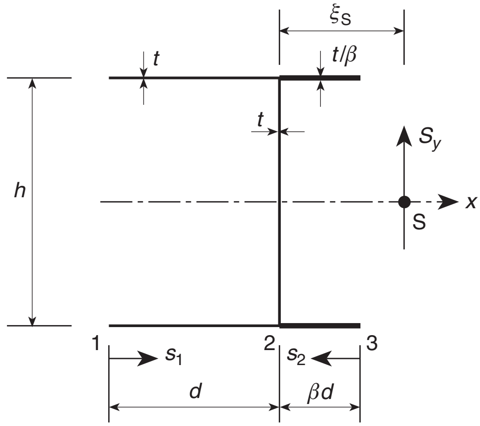

Fig. S.17.4

Referring to Fig. S.17.4

$$I_{xx} = \frac{th^3}{12} + 2\left[td\left(\frac{h}{2}\right)^2 + \frac{t}{\beta}\beta d\left(\frac{h}{2}\right)^2\right] = th^2\left(\frac{h}{12} + d\right)$$

From Eq. (i)

$$q_{12} = -\frac{S_y}{I_{xx}} \int_0^{s_1} t\left(-\frac{h}{2}\right) \mathrm{d}s_1$$

i.e.

$$q_{12} = \frac{S_y th}{2I_{xx}} s_1 \tag{ii}$$

Also

$$q_{32} = -\frac{S_y}{I_{xx}} \int_0^{s_2} \frac{t}{\beta}\left(-\frac{h}{2}\right) \mathrm{d}s_2$$

so that

$$q_{32} = \frac{S_y th}{2\beta I_{xx}} s_2 \tag{iii}$$

Taking moments about the mid-point of the web

$$S_y \xi_S = 2\int_0^d q_{12}\frac{h}{2}\mathrm{d}s_1 - 2\int_0^{\beta d} q_{32}\frac{h}{2}\mathrm{d}s_2 \tag{iv}$$

Substituting from Eqs (ii) and (iii) in Eq. (iv) for q_{12} and q_{32}

$$S_y \xi_S = \frac{S_y th^2}{2I_{xx}} \int_0^d s_1\,\mathrm{d}s_1 - \frac{S_y th^2}{2\beta I_{xx}} \int_0^{\beta d} s_2\,\mathrm{d}s_2$$

i.e.

$$\xi_S = \frac{th^2}{2I_{xx}}\left(\frac{d^2}{2} - \beta\frac{d^2}{2}\right)$$

i.e.

$$\xi_S = \frac{th^2 d^2(1-\beta)}{4th^3(1+12d/h)/12}$$

so that

$$\frac{\xi_S}{d} = \frac{3\rho(1-\beta)}{(1+12\rho)}$$

S.17.5

Referring to Fig. S.17.5 the shear centre, S, lies on the axis of symmetry, the x axis, so that $I_{xy} = 0$. Therefore, apply an arbitrary shear load, S_y, through the shear centre and determine the internal shear flow distribution. Thus, since $S_x = 0$, Eq. (17.14) becomes

$$q_s = -\frac{S_y}{I_{xx}} \int_0^s ty\,\mathrm{d}s \tag{i}$$

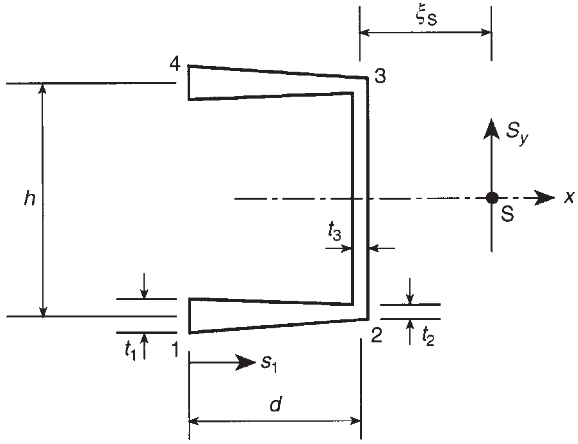

Fig. S.17.5

in which

$$I_{xx} = \frac{t_3 h^3}{12} + 2\frac{(t_1 + t_2)}{2} d \left(\frac{h}{2}\right)^2$$

i.e.

$$I_{xx} = \frac{h^2}{12}[t_3 h + 3(t_1 + t_2)d] \quad \text{(ii)}$$

The thickness t in the flange 12 at any point s_1 is given by

$$t = t_1 - \frac{(t_1 - t_2)}{d} s_1 \quad \text{(iii)}$$

Substituting for t from Eq. (iii) in (i)

$$q_{12} = -\frac{S_y}{I_{xx}} \int_0^{s_1} \left[t_1 - \frac{(t_1 - t_2)}{d} s_1 \right] \left(-\frac{h}{2}\right) \mathrm{d}s_1$$

Hence

$$q_{12} = \frac{S_y h}{2I_{xx}} \left[t_1 s_1 - \frac{(t_1 - t_2)}{d} \frac{s_1^2}{2} \right] \quad \text{(iv)}$$

Taking moments about the mid-point of the web

$$S_y \xi_S = 2 \int_0^d q_{12} \left(\frac{h}{2}\right) \mathrm{d}s_1$$

i.e.

$$S_y \xi_S = \frac{S_y h^2}{2I_{xx}} \left[t_1 \frac{s_1^2}{2} - \frac{(t_1 - t_2)}{d} \frac{s_1^3}{6} \right]_0^d$$

from which

$$\xi_S = \frac{h^2 d^2}{12 I_{xx}}(2t_1 + t_2)$$

Substituting for I_{xx} from Eq. (ii)

$$\xi_S = \frac{d^2(2t_1 + t_2)}{3d(t_1 + t_2) + ht_3}$$

S.17.6

The beam section is shown in Fig. S.17.6(a). Clearly the x axis is an axis of symmetry so that $I_{xy} = 0$ and the shear centre, S, lies on this axis. Thus, apply an arbitrary shear load, S_y, through S and determine the internal shear flow distribution. Since $S_x = 0$, Eq. (17.14) simplifies to

$$q_s = -\frac{S_y}{I_{xx}} \int_0^s ty\, \mathrm{d}s \tag{i}$$

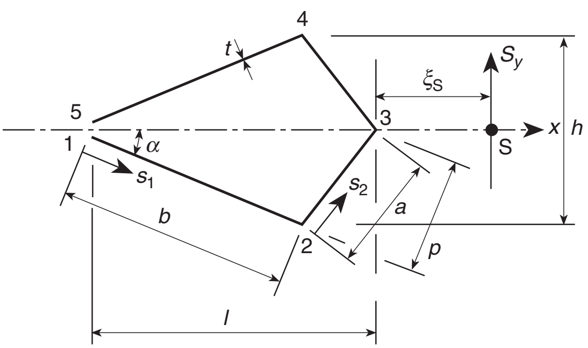

Fig. S.17.6(a)

in which, from Fig. S.17.6(a)

$$I_{xx} = 2\left[\int_0^b t\left(\frac{h}{2b}s\right)^2 \mathrm{d}s + \int_0^a t\left(\frac{h}{2a}s\right)^2 \mathrm{d}s\right] \tag{ii}$$

where the origin of s in the first integral is the point 1 and the origin of s in the second integral is the point 3. Equation (ii) then gives

$$I_{xx} = \frac{th^2(b + a)}{6} \tag{iii}$$

From Eq. (i)

$$q_{12} = -\frac{S_y}{I_{xx}} \int_0^{s_1} t\left(-\frac{h}{2b}s_1\right) \mathrm{d}s_1$$

from which

$$q_{12} = \frac{S_y th}{2bI_{xx}} \frac{s_1^2}{2}$$

or, substituting for I_{xx} from Eq. (iii)

$$q_{12} = \frac{3S_y}{2bh(b+a)} s_1^2 \tag{iv}$$

and

$$q_2 = \frac{3S_y b}{2h(b+a)} \tag{v}$$

Also

$$q_{23} = -\frac{S_y}{I_{xx}} \int_0^{s_2} t \left[-\frac{h}{2a}(a - s_2) \right] \mathrm{d}s_2 + q_2$$

Substituting for I_{xx} from Eq. (iii) and q_2 from Eq. (v)

$$q_{23} = \frac{3S_y}{h(b+a)} \left(s_2 - \frac{s_2^2}{2a} + \frac{b}{2} \right) \tag{vi}$$

and

$$q_3 = \frac{3S_y}{2h} \tag{vii}$$

Equation (iv) shows that q_{12} varies parabolically but does not change sign between 1 and 2; also $\mathrm{d}q_{12}/\mathrm{d}s_1 = 0$ when $s_1 = 0$. From Eq. (vi) $q_{23} = 0$ when $s_2 - s_2^2/2a + b/2 = 0$, i.e. when

$$s_2^2 - 2as_2 - ba = 0 \tag{viii}$$

Solving Eq. (viii)

$$s_2 = a \pm \sqrt{a^2 + ba}$$

Thus, q_{23} does not change sign between 2 and 3. Further

$$\frac{\mathrm{d}q_{23}}{\mathrm{d}s_2} = \frac{3S_y}{h(b+a)} \left(1 - \frac{s_2}{a} \right) = 0 \quad \text{when } s_2 = a$$

Therefore q_{23} has a turning value at 3. The shear flow distributions in the walls 34 and 45 follow from antisymmetry; the complete distribution is shown in Fig. S.17.6(b).

Referring to Fig. S.17.6(a) and taking moments about the point 3

$$S_y \xi_S = 2 \int_0^b q_{12} p \, \mathrm{d}s_1 \tag{ix}$$

where p is given by

$$\frac{p}{l} = \sin\alpha = \frac{h}{2b} \quad \text{i.e. } p = \frac{hl}{2b}$$

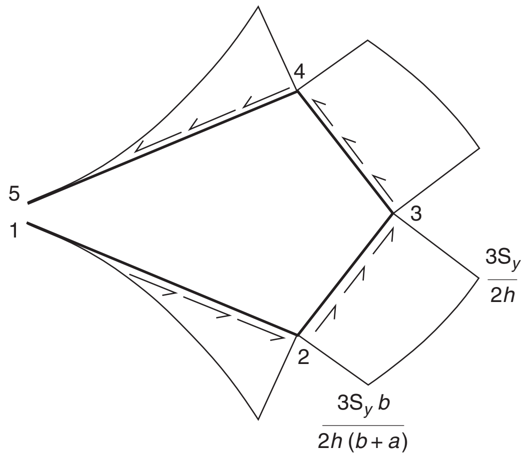

Fig. S.17.6(b)

Substituting for p and q_{12} from Eq. (iv) in (ix) gives

$$S_y \xi_S = \frac{3S_y}{bh(b+a)} \int_0^b \frac{hl}{2b} s_1^2 \, ds_1$$

from which

$$\xi_S = \frac{l}{2(1+a/b)}$$

S.17.7

Initially the position of the centroid, C, must be found. From Fig. S.17.7, by inspection $\bar{y} = a$. Also taking moments about the web 23

$$(2at + 2a2t + a2t)\bar{x} = a2t\frac{a}{2} + 2ata$$

from which $\bar{x} = 3a/8$.

To find the horizontal position of the shear centre, S, apply an arbitrary shear load, S_y, through S. Since $S_x = 0$ Eq. (17.14) simplifies to

$$q_s = \frac{S_y I_{xy}}{I_{xx}I_{yy} - I_{xy}^2} \int_0^s tx \, ds - \frac{S_y I_{yy}}{I_{xx}I_{yy} - I_{xy}^2} \int_0^s ty \, ds$$

i.e.

$$q_s = \frac{S_y}{I_{xx}I_{yy} - I_{xy}^2} \left(I_{xy} \int_0^s tx \, ds - I_{yy} \int_0^s ty \, ds \right) \tag{i}$$

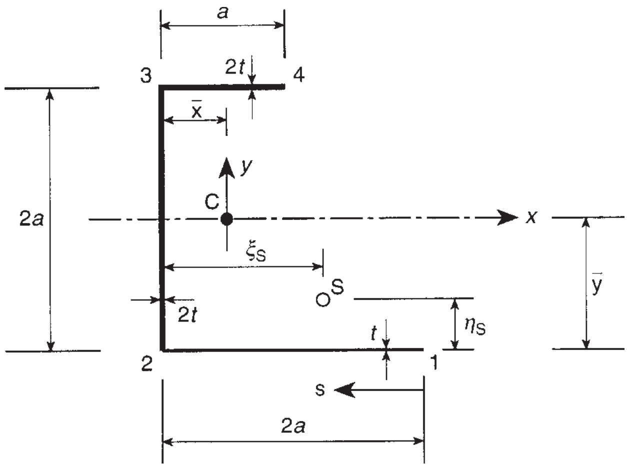

Fig. S.17.7

in which, referring to Fig. S.17.7

$$I_{xx} = a2t(a)^2 + 2at(a)^2 + t(2a)^3/12 = 16a^3t/3$$

$$I_{yy} = 2ta^3/12 + 2ta(a/8)^2 + t(2a)^3/12 + 2at(5a/8)^2 + 4at(3a/8)^2 = 53a^3t/24$$

$$I_{xy} = a2t(a/8)(a) + 2at(5a/8)(-a) = -a^3t$$

Substituting for I_{xx}, I_{yy} and I_{xy} in Eq. (i) gives

$$q_s = \frac{9S_y}{97a^3t}\left(-\int_0^s tx\,\mathrm{d}s - \frac{53}{24}\int_0^s ty\,\mathrm{d}s\right) \qquad \text{(ii)}$$

from which

$$q_{12} = \frac{9S_y}{97a^3}\left[-\int_0^s\left(\frac{13a}{8} - s\right)\mathrm{d}s - \frac{53}{24}\int_0^s(-a)\mathrm{d}s\right] \qquad \text{(iii)}$$

i.e.

$$q_{12} = \frac{9S_y}{97a^3}\left(\frac{7as}{12} + \frac{s^2}{2}\right) \qquad \text{(iv)}$$

Taking moments about the corner 3 of the section

$$S_y\xi_S = -\int_0^{2a} q_{12}(2a)\mathrm{d}s \qquad \text{(v)}$$

Substituting for q_{12} from Eq. (iv) in (v)

$$S_y\xi_S = -\frac{18S_y}{97a^2}\int_0^{2a}\left(\frac{7as}{12} + \frac{s^2}{2}\right)\mathrm{d}s$$

from which

$$\xi_S = -\frac{45a}{97}$$

Now apply an arbitrary shear load S_x through the shear centre, S. Since $S_y = 0$ Eq. (17.14) simplifies to

$$q_s = -\frac{S_x}{I_{xx}I_{yy} - I_{xy}^2}\left(I_{xx}\int_0^s tx\,\mathrm{d}s - I_{xy}\int_0^s ty\,\mathrm{d}s\right)$$

from which, by comparison with Eq. (iii)

$$q_{12} = -\frac{9S_x}{97a^3 t}\left[\frac{16}{3}\int_0^s t\left(\frac{13a}{8} - s\right)\mathrm{d}s + \int_0^s t(-a)\mathrm{d}s\right]$$

i.e.

$$q_{12} = -\frac{3S_x}{97a^3}(23as - 8s^2) \tag{vi}$$

Taking moments about the corner 3

$$S_x(2a - \eta_S) = -\int_0^{2a} q_{12}(2a)\mathrm{d}s$$

Substituting for q_{12} from Eq. (vi)

$$S_x(2a - \eta_S) = \frac{6S_x}{97a^2}\int_0^{2a}(23as - 8s^2)\mathrm{d}s$$

which gives

$$\eta_S = \frac{46a}{97}$$

S.17.8

The shear centre is the point in a beam cross-section through which shear loads must be applied for there to be no twisting of the section.

The x axis is an axis of symmetry so that the shear centre lies on this axis. Its position is found by applying a shear load S_y through the shear centre, determining the shear flow distribution and then taking moments about some convenient point. Equation (17.14) reduces to

$$q_s = -\frac{S_y}{I_{xx}}\int_0^s ty\,\mathrm{d}s \tag{i}$$

in which, referring to Fig. S.17.8

$$I_{xx} = 2\left(\frac{tr^3}{3} + 2rtr^2 + \int_0^{\pi/2} tr^2\cos^2\theta r\,\mathrm{d}\theta\right)$$

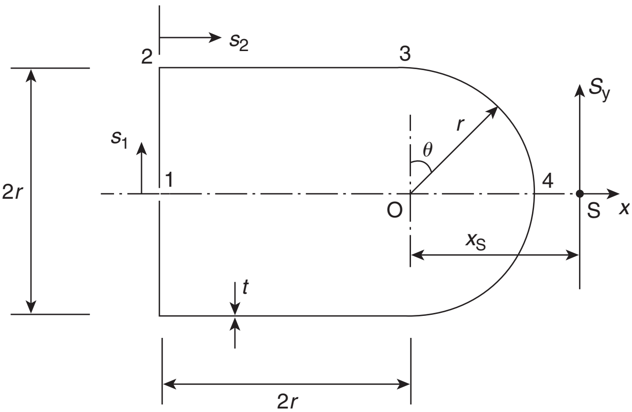

Fig. S.17.8

i.e.

$$I_{xx} = 6.22tr^3$$

In the wall 12, $y = s_1$. Therefore substituting in Eq. (i)

$$q_{12} = -\frac{S_y}{I_{xx}} \int_0^s ts_1 \, \mathrm{d}s = -\frac{S_y}{I_{xx}} \frac{ts_1^2}{2}$$

Then

$$q_2 = -\frac{S_y}{I_{xx}} \frac{tr^2}{2}$$

In the wall 23, $y = r$, then

$$q_{23} = -\left(\frac{S_y}{I_{xx}}\right)\left(\int_0^s tr \, \mathrm{d}s + \frac{tr^2}{2}\right)$$

i.e.

$$q_{23} = -\left(\frac{S_y}{I_{xx}}\right)\left(trs_2 + \frac{tr^2}{2}\right)$$

and

$$q_3 = -5\frac{S_y}{I_{xx}} \frac{tr^2}{2}$$

In the wall 34, $y = r\cos\theta$, then

$$q_{34} = -\frac{S_y}{I_{xx}}\left(\int_0^\theta tr^2 \cos\theta \, \mathrm{d}\theta + \frac{5tr^2}{2}\right)$$

i.e.

$$q_{34} = -\frac{S_y}{I_{xx}} tr^2 \left(\sin\theta + \frac{5}{2}\right)$$

Taking moments about O

$$S_y x_S = -2\left[\int_0^r q_{12} 2r\,\mathrm{d}s + \int_0^{2r} q_{23} r\,\mathrm{d}s + \int_0^{\pi/2} q_{34} r^2\,\mathrm{d}\theta\right]$$

The negative sign arises from the fact that the moment of the applied shear load is in the opposite sense to the moments produced by the internal shear flows. Substituting for q_{12}, q_{23} and q_{34} from the above

$$S_y x_S = \frac{S_y}{I_{xx}} t\left[\int_0^r \left(\frac{s_1^2}{2}\right) 2r\,\mathrm{d}s + \int_0^{2r}\left(rs_2 + \frac{r^2}{2}\right) r\,\mathrm{d}s + \int_0^{\pi/2} r^4\left(\sin\theta + \frac{5}{2}\right)\mathrm{d}\theta\right]$$

which gives

$$x_S = 2.66r$$

S.17.9

In this problem the axis of symmetry is the vertical y axis and the shear centre will lie on this axis so that only its vertical position is required. Therefore, we apply a horizontal shear load S_x through the shear centre, S, as shown in Fig. S.17.9.

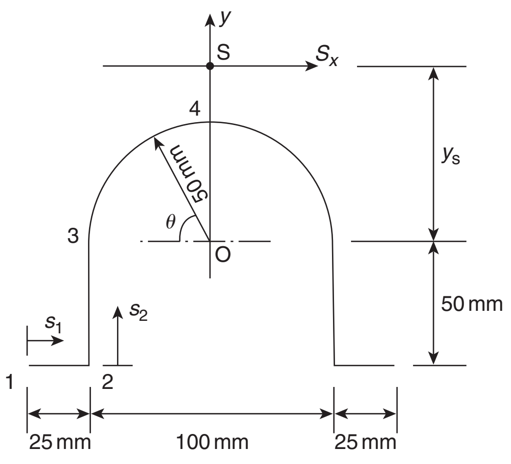

Fig. S.17.9

The thickness of the section is constant and will not appear in the answer for the shear centre position, therefore assume the section has unit thickness.

Equation (17.14), since $I_{xy} = 0$, $t = 1$ and only S_x is applied, reduces to

$$q_s = -\frac{S_x}{I_{yy}}\int_0^s x\,\mathrm{d}s \qquad \text{(i)}$$

where

$$I_{yy} = \frac{25^3}{12} + 25 \times 62.5^2 + 50 \times 50^2 + \int_0^{\pi/2} (50\cos\theta)^2 50\,\mathrm{d}\theta$$

i.e.

$$I_{yy} = 6.44 \times 10^5\ \mathrm{mm}^4$$

In the flange 12, $x = -75 + s_1$ and

$$q_{12} = -\frac{S_x}{I_{yy}} \int_0^s (-75 + s_1)\mathrm{d}s = -\frac{S_x}{I_{yy}} \left(-75s_1 + \frac{s_1^2}{2} \right)$$

and when $s_1 = 25$ mm, $q_2 = 1562.5S_x/I_{yy}$

In the wall 23, $x = -50$ mm, then

$$q_{23} = -\frac{S_x}{I_{yy}} \left(\int_0^s -50\,\mathrm{d}s - 1562.5 \right) = \frac{S_x}{I_{yy}}(50s_2 + 1562.5)$$

when $s_2 = 50$ mm, $q_3 = 4062.5S_x/I_{yy}$.

In the wall 34, $x = -50\cos\theta$, therefore

$$q_{34} = -\frac{S_x}{I_{yy}} \left(\int_0^\theta -50\cos\theta\, 50\,\mathrm{d}\theta - 4062.5 \right) = \frac{S_x}{I_{yy}}(2500\sin\theta + 4062.5)$$

Now taking moments about O

$$S_x y_S = 2\left(-\int_0^{25} q_{12} 50\,\mathrm{d}s_1 + \int_0^{50} q_{23} 50\,\mathrm{d}s_2 + \int_0^{\frac{\pi}{2}} q_{34} 50^2\,\mathrm{d}\theta \right)$$

Note that the moments due to the shear flows in the walls 23 and 34 are opposite in sign to the moment produced by the shear flow in the wall 12. Substituting for q_{12}, etc. gives

$$y_S = 87.5\ \mathrm{mm}$$

S.17.10

Apply an arbitrary shear load S_y through the shear centre S. Then, since the x axis is an axis of symmetry, $I_{xy} = 0$ and Eq. (17.14) reduces to

$$q_s = -\frac{S_y}{I_{xx}} \int_0^s ty\,\mathrm{d}s$$

$$I_{xx} = 2\left[\int_0^r ty^2\,\mathrm{d}s + \int_0^{\pi/4} t(r\sin\theta)^2 r\,\mathrm{d}\theta \right]$$

$$= 2\left[\int_0^r t(2r\sin 45^\circ - s_1 \sin 45^\circ)^2 \mathrm{d}s_1 + \int_0^{\pi/4} tr^3 \sin^2\theta\,\mathrm{d}\theta \right]$$

$$= 2\left[t\sin^2 45^\circ \int_0^r (4r^2 - 4rs_1 + s_1^2)\mathrm{d}s_1 + \frac{tr^3}{2} \int_0^\pi (1 - \cos 2\theta)\mathrm{d}\theta \right]$$

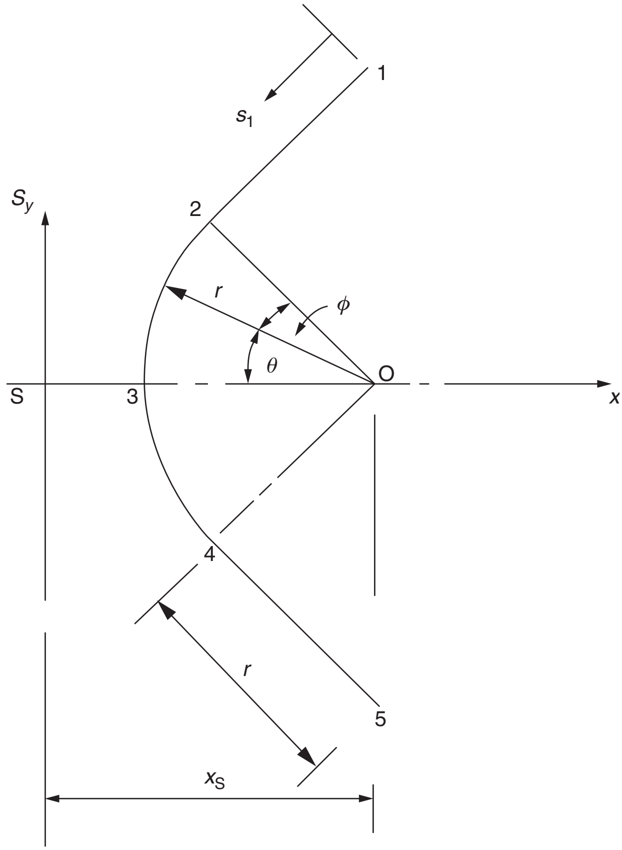

Fig. S.17.10

which gives

$$I_{xx} = 2.62\,tr^3$$

Then

$$q_{12} = -\frac{S_y}{I_{xx}} \int_0^{s_1} t(2r \sin 45° - s_1 \sin 45°)\mathrm{d}s_1$$

i.e.

$$q_{12} = \frac{-0.27S_y}{r^3}\left(2rs_1 - \frac{s_1^2}{2}\right) \tag{i}$$

and

$$q_2 = \frac{-0.4S_y}{r}$$

Also

$$q_{23} = -\frac{S_y}{I_{xx}} \int_0^{\phi} t\left[r \sin(45° - \phi)r\,\mathrm{d}\phi - \frac{0.4S_y}{r}\right]$$

from which

$$q_{23} = -\frac{S_y}{2.62r}\cos(45^\circ - \phi) - \frac{0.13S_y}{r} \tag{ii}$$

Taking moments about O

$$S_y x_S = -2\left[\int_0^r q_{12} r\,\mathrm{d}s_1 + \int_0^{\pi/4} q_{23} r^2\,\mathrm{d}\phi\right] \tag{iii}$$

Substituting for q_{12} and q_{23} from Eqs (i) and (ii) in Eq. (iii) gives

$$x_S = 1.2r$$

S.17.11

Since the x axis is an axis of symmetry and only S_y is applied Eq. (17.14) reduces to

$$q_s = -\frac{S_y}{I_{xx}}\int_0^s ty\,\mathrm{d}s$$

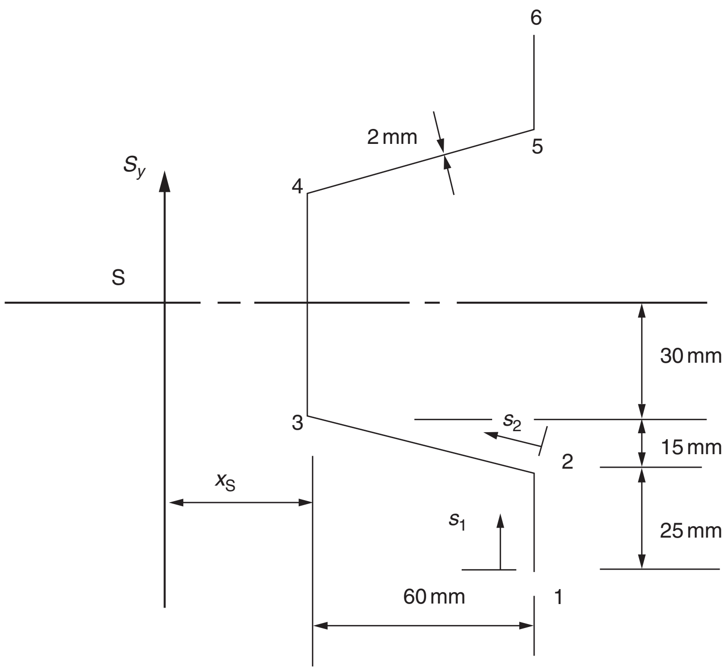

Fig. S.17.11

Also

$$s_{32} = (15^2 + 60^2)^{1/2} = 61.8\,\text{mm}$$

and

$$I_{xx} = 2\left[\frac{2\times 25^3}{12} + 2\times 25\times 57.5^2 + \frac{2\times 61.8^3}{12}\times\left(\frac{15}{61.8}\right)^2 + 2\times 61.8\times 37.5^2 + \frac{2\times 60^3}{12}\right]$$

which gives

$$I_{xx} = 724\,094\ \text{mm}^4$$

Then

$$q_{12} = -\frac{S_y}{I_{xx}}\int_0^{s_1} 2(-70+s_1)\,\mathrm{d}s_1$$

i.e.

$$q_{12} = \frac{S_y}{I_{xx}}(140s_1 - s_1^2) \qquad \text{(i)}$$

and

$$q_2 = \frac{2875S_y}{I_{xx}}$$

Also

$$q_{23} = -\frac{S_y}{I_{xx}}\int_0^{s_2} 2\left(-45+\frac{15}{61.8}s_2\right)\mathrm{d}s_2 + \frac{2875S_y}{I_{xx}}$$

Then

$$q_{23} = \frac{S_y}{I_{xx}}\left(90s_2 - \frac{15}{61.8}s_2^2 + 2875\right) \qquad \text{(ii)}$$

Taking moments about the mid-point of the web 34 (it therefore becomes unnecessary to determine q_{34})

$$S_y x_S = 2\left[-\int_0^{25} 60q_{12}\,\mathrm{d}s_1 + \int_0^{61.8} 30\times\frac{60}{61.8}q_{23}\,\mathrm{d}s_2\right]$$

Substituting for q_{12} and q_{23} from Eqs (i) and (ii)

$$x_S = 20.2\ \text{mm}$$

S.17.12

Referring to Fig. S.17.12 the x axis is an axis of symmetry so that $I_{xy} = 0$ and since $S_x = 0$. Eq. (17.15) reduces to

$$q_s = -\frac{S_y}{I_{xx}}\int_0^s ty\,\mathrm{d}s + q_{s,0} \qquad \text{(i)}$$

in which

$$I_{xx} = \frac{(2r)^3 t \sin^2 45°}{12} + 2\int_0^{\pi/2} t(r\sin\theta)^2 r\,\mathrm{d}\theta$$

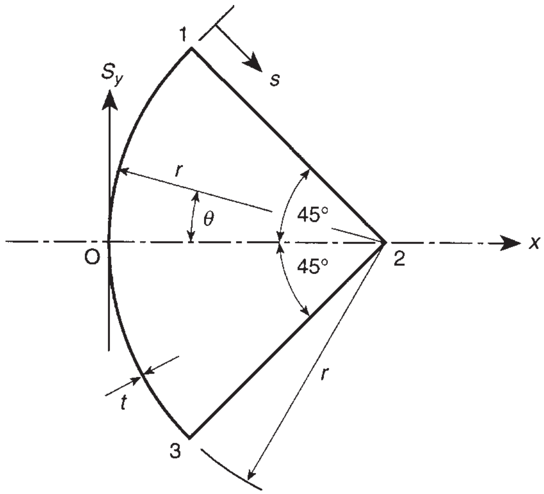

Fig. S.17.12

i.e.

$$I_{xx} = 0.62tr^3$$

'Cut' the section at O. Then, from the first term on the right-hand side of Eq. (i)

$$q_{\mathrm{b,O1}} = -\frac{S_y}{0.62tr^3}\int_0^\theta tr\sin\theta r\,\mathrm{d}\theta$$

i.e.

$$q_{\mathrm{b,O1}} = -\frac{S_y}{0.62r}[-\cos\theta]_0^\theta$$

so that

$$q_{\mathrm{b,O1}} = -\frac{S_y}{0.62r}(\cos\theta - 1) = 1.61\frac{S_y}{r}(\cos\theta - 1) \qquad \text{(ii)}$$

and

$$q_{\mathrm{b,1}} = -\frac{0.47S_y}{r}$$

Also

$$q_{b,12} = -\frac{S_y}{0.62tr^3}\int_0^s t(r-s)\sin 45°\,\mathrm{d}s - \frac{0.47S_y}{r}$$

which gives

$$q_{b,12} = \frac{S_y}{r^3}(-1.14rs + 0.57s^2 - 0.47r^2) \qquad \text{(iii)}$$

Now take moments about the point 2

$$S_y r = 2\int_0^{\pi/4} q_{b,O1} r r \, d\theta + 2 \times \frac{\pi r^2}{4} q_{s,0}$$

Substituting in Eq. (iv) for $q_{b,O1}$ from Eq. (ii)

$$S_y r = 2\int_0^{\pi/4} 1.61 \frac{S_y}{r}(\cos\theta - 1) r^2 \, d\theta + \frac{\pi r^2}{2} q_{s,0}$$

i.e.

$$S_y r = 3.22 S_y r[\sin\theta - \theta]_0^{\pi/4} + \frac{\pi r^2}{2} q_{s,0}$$

so that

$$q_{s,0} = \frac{0.80 S_y}{r}$$

Then, from Eq. (ii)

$$q_{O1} = \frac{S_y}{r}(1.61\cos\theta - 0.80)$$

and from Eq. (iii)

$$q_{12} = \frac{S_y}{r^3}(0.57s^2 - 1.14rs + 0.33r^2)$$

The remaining distribution follows from symmetry.

S.17.13

The x axis is an axis of symmetry so that $I_{xy} = 0$ and, since $S_x = 0$, Eq. (17.15) simplifies to

$$q_s = -\frac{S_y}{I_{xx}}\int_0^s ty \, ds + q_{s,0} \qquad \text{(i)}$$

in which, from Fig. S.17.13(a)

$$I_{xx} = \frac{th^3}{12} + \frac{(2d)^3 t \sin^2\alpha}{12} = \frac{th^2}{12}(h + 2d) \qquad \text{(ii)}$$

'Cut' the section at 1. Then, from the first term on the right-hand side of Eq. (i)

$$q_{b,12} = -\frac{S_y}{I_{xx}}\int_0^{s_1} t(-s_1 \sin\alpha) ds_1 = \frac{S_y t \sin\alpha}{2I_{xx}} s_1^2$$

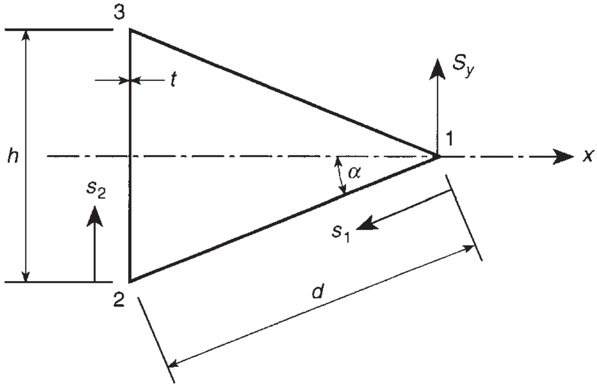

Fig. S.17.13(a)

Substituting for I_{xx} and $\sin\alpha$

$$q_{b,12} = \frac{3S_y}{hd(h + 2d)} s_1^2 \tag{iii}$$

and

$$q_{b,2} = \frac{3S_y d}{h(h + 2d)}$$

Also

$$q_{b,23} = -\frac{S_y}{I_{xx}} \int_0^{s_2} t\left(-\frac{h}{2} + s_2\right) \mathrm{d}s_2 + q_{b,2}$$

so that

$$q_{b,23} = \frac{6S_y}{h(h + 2d)} \left(s_2 - \frac{s_2^2}{h} + \frac{d}{2}\right) \tag{iv}$$

Now taking moments about the point 1 (see Eq. (17.18))

$$0 = \int_0^h q_{b,23} d \cos\alpha \, \mathrm{d}s_2 + 2\frac{h}{2} d \cos\alpha q_{s,0}$$

i.e.

$$0 = \int_0^h q_{b,23} \, \mathrm{d}s_2 + hq_{s,0} \tag{v}$$

Substituting in Eq. (v) for $q_{b,23}$ from Eq. (iv)

$$0 = \frac{6S_y}{h(h + 2d)} \int_0^h \left(s_2 - \frac{s_2^2}{h} + \frac{d}{2}\right) \mathrm{d}s_2 + hq_{s,0}$$

which gives

$$q_{s,0} = -\frac{S_y(h+3d)}{h(h+2d)} \tag{vi}$$

Then, from Eqs (iii) and (i)

$$q_{12} = \frac{3S_y}{hd(h+2d)}s_1^2 - \frac{S_y(h+3d)}{h(h+2d)}$$

i.e.

$$q_{12} = \frac{S_y}{h(h+2d)}\left(\frac{3s_1^2}{d} - h - 3d\right) \tag{vii}$$

and from Eqs (iv) and (vi)

$$q_{23} = \frac{S_y}{h(h+2d)}\left(6s_2 - \frac{6s_2^2}{h} - h\right) \tag{viii}$$

The remaining distribution follows from symmetry.

From Eq. (vii), q_{12} is zero when $s_1^2 = (hd/3) + d^2$, i.e. when $s_1 > d$. Thus there is no change of sign of q_{12} between 1 and 2. Further

$$\frac{dq_{12}}{ds_1} = \frac{6s_1}{d} = 0 \quad \text{when } s_1 = 0$$

and

$$q_1 = -\frac{S_y(h+3d)}{h(h+2d)}$$

Also, when $s_1 = d$

$$q_2 = -\frac{S_y}{(h+2d)}$$

From Eq. (viii) q_{23} is zero when $6s_2 - (6s_2^2/h) - h = 0$, i.e. when $s_2^2 - s_2h + (h^2/6) = 0$. Then

$$s_2 = \frac{h}{2} \pm \frac{h}{\sqrt{12}}$$

Thus q_{23} is zero at points a distance $h/\sqrt{12}$ either side of the x axis. Further, from Eq. (viii), q_{23} will be a maximum when $s_2 = h/2$ and $q_{23}(\max) = S_y/2(h+2d)$. The complete distribution is shown in Fig. S.17.13(b).

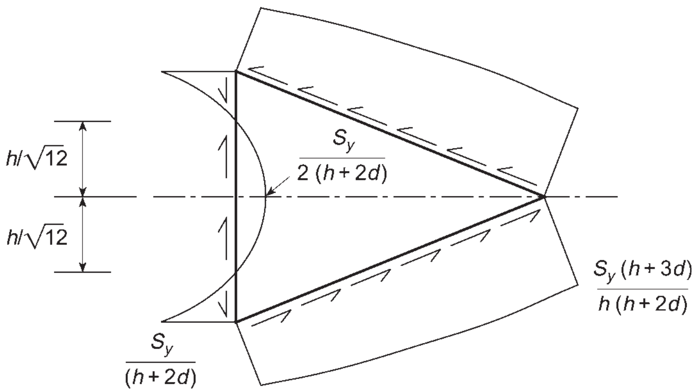

Fig. S.17.13(b)

S.17.14

Since the section is doubly symmetrical the centroid of area, C, and the shear centre, S, coincide. The applied shear load, S, may be replaced by a shear load, S, acting through the shear centre together with a torque, T, as shown in Fig. S.17.14. Then

$$T = Sa\cos 30^\circ = 0.866Sa \tag{i}$$

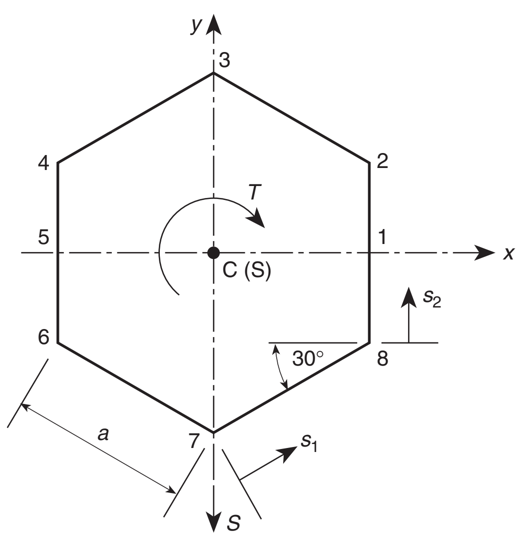

Fig. S.17.14

The shear flow distribution produced by this torque is given by Eq. (18.1), i.e.

$$q_T = \frac{T}{2A} = \frac{0.866Sa}{2A} \quad \text{(from Eq. (i))}$$

where

$$A = a2a\cos 30^\circ + 2 \times a\cos 30^\circ \times a\sin 30^\circ = 2.6a^2$$

Then

$$q_T = \frac{0.17S}{a} \quad \text{(clockwise)} \tag{ii}$$

The rate of twist is obtained from Eq. (18.4) and is

$$\frac{\mathrm{d}\theta}{\mathrm{d}z} = \frac{0.866Sa}{4(2.6a^2)^2G}\left(\frac{6a}{t}\right)$$

i.e.

$$\frac{\mathrm{d}\theta}{\mathrm{d}z} = \frac{0.192S}{Gta^2} \tag{iii}$$

The shear load, S, through the shear centre produces a shear flow distribution given by Eq. (17.15) in which $S_y = -S$, $S_x = 0$ and $I_{xy} = 0$. Hence

$$q_s = \frac{S}{I_{xx}}\int_0^s ty\,\mathrm{d}s + q_{s,0} \tag{iv}$$

in which

$$I_{xx} = 2\frac{ta^3}{12} + 4\int_0^a t(-a + s_1\sin 30^\circ)^2\mathrm{d}s_1 = \frac{5a^3t}{2}$$

Also on the vertical axis of symmetry the shear flow is zero, i.e. at points 7 and 3. Therefore, choose 7 as the origin of s in which case $q_{s,0}$ in Eq. (iv) is zero and

$$q_s = \frac{S}{I_{xx}}\int_0^s ty\,\mathrm{d}s \tag{v}$$

From Eq. (v) and referring to Fig. S.17.14

$$q_{78} = \frac{S}{I_{xx}}\int_0^{s_1} t(-a + s_1\sin 30^\circ)\mathrm{d}s_1$$

i.e.

$$q_{78} = \frac{2S}{5a^3}\int_0^s\left(-a + \frac{s_1}{2}\right)\mathrm{d}s$$

so that

$$q_{78} = \frac{S}{5a^3}\left(2as_1 - \frac{s_1^2}{2}\right) \tag{vi}$$

and

$$q_8 = -\frac{3S}{10a} \tag{vii}$$

Also

$$q_{81} = \frac{S}{I_{xx}} \int_0^{s_2} t\left(-\frac{a}{2} + s_2\right) \mathrm{d}s_2 + q_8$$

i.e.

$$q_{81} = \frac{2S}{5a^3} \int_0^{s_2} \left(-\frac{a}{2} + s_2\right) \mathrm{d}s_2 - \frac{3S}{10a}$$

from which

$$q_{81} = \frac{S}{10a^3}(-2as_2 + 2s_2^2 - 3a^2) \qquad \text{(viii)}$$

Thus

$$q_1 = -\frac{7S}{20a}$$

The remaining distribution follows from symmetry.

The complete shear flow distribution is now found by superimposing the shear flow produced by the torque, T, (Eq. (ii)) and the shear flows produced by the shear load acting through the shear centre. Thus, taking anticlockwise shear flows as negative

$$q_1 = -\frac{0.17S}{a} - \frac{0.35S}{a} = -\frac{0.52S}{a}$$

$$q_2 = q_8 = -\frac{0.17S}{a} - \frac{0.3S}{a} = -\frac{0.47S}{a} \quad \text{(from Eq. (vii))}$$

$$q_3 = q_7 = -\frac{0.17S}{a}$$

$$q_4 = q_6 = -\frac{0.17S}{a} + \frac{0.3S}{a} = \frac{0.13S}{a}$$

$$q_5 = -\frac{0.17S}{a} + \frac{0.35S}{a} = \frac{0.18S}{a}$$

The distribution in all walls is parabolic.

S.17.15

Referring to Fig. P.17.15, the wall DB is 3 m long so that its cross-sectional area, $3 \times 10^3 \times 8 = 24 \times 10^3\ \text{mm}^2$, is equal to that of the wall EA, $2 \times 10^3 \times 12 = 24 \times 10^3\ \text{mm}^2$. If follows that the centroid of area of the section lies mid-way between DB and EA on the vertical axis of symmetry. Also since $S_y = 500$ kN, $S_x = 0$ and $I_{xy} = 0$, Eq. (17.15) reduces to

$$q_s = -\frac{500 \times 10^3}{I_{xx}} \int_0^s ty\, \mathrm{d}s + q_{s,0} \qquad \text{(i)}$$

If the origin for s is taken on the axis of symmetry, say at O, then $q_{s,0}$ is zero. Also

$$I_{xx} = 3 \times 10^3 \times 8 \times (0.43 \times 10^3)^2 + 2 \times 10^3 \times 12 \times (0.43 \times 10^3)^2 + 2 \times (1 \times 10^3)^3 \times 10 \times \sin^2 60°/12$$

i.e.

$$I_{xx} = 101.25 \times 10^8 \text{ mm}^4$$

Equation (i) then becomes

$$q_s = -4.94 \times 10^{-5} \int_0^s ty \, ds$$

In the wall OA, $y = -0.43 \times 10^3$ mm. Then

$$q_{OA} = 4.94 \times 10^{-5} \int_0^s 12 \times 0.43 \times 10^3 \, ds = 0.25 s_A$$

and when $s_A = 1 \times 10^3$ mm, $q_{OA} = 250$ N/mm.

In the wall AB, $y = -0.43 \times 10^3 + s_B \cos 30°$. Then

$$q_{AB} = -4.94 \times 10^{-5} \int_0^s 10(-0.43 \times 10^3 + 0.866 s_B) ds + 250$$

i.e.

$$q_{AB} = 0.21 s_B - 2.14 \times 10^{-4} s_B^2 + 250$$

When $s_B = 1 \times 10^3$ mm, $q_{AB} = 246$ N/mm.

In the wall BC, $y = 0.43 \times 10^3$ mm. Then

$$q_{BC} = -4.94 \times 10^{-5} \int_0^s 8 \times 0.43 \times 10^3 \, ds + 246$$

i.e.

$$q_{BC} = -0.17 s_C + 246$$

Note that at C where $s_C = 1.5 \times 10^3$ mm, q_{BC} should equal zero; the discrepancy, -9 N/mm, is due to rounding off errors.

The maximum shear stress will occur in the wall AB (and ED) mid-way along its length (this coincides with the neutral axis of the section) where $s_B = 500$ mm. This gives, from Eq. (ii), $q_{AB}(\text{max}) = 301.5$ N/mm so that the maximum shear stress is equal to $301.5/10 = 30.2$ N/mm^2.