Solutions to Chapter 14 Problems

S.14.1

Suppose that the mass of the aircraft is m and its vertical deceleration is a. Then referring to Fig. S.14.1(a) and resolving forces in a vertical direction

$$ma + 135 - 2 \times 200 = 0$$

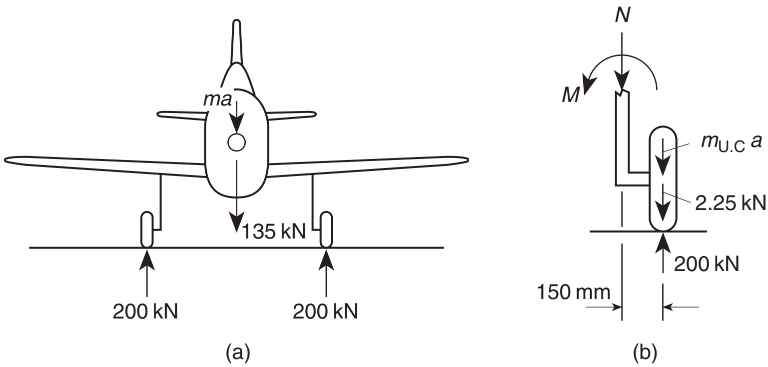

Fig. S.14.1(a) and (b)

which gives

$$ma = 265\,\text{kN}$$

Therefore

$$a = \frac{265}{m} = \frac{265}{135/g}$$

i.e.

$$a = 1.96\,g$$

Now consider the undercarriage shown in Fig. S.14.1(b) and suppose that its mass is $m_{\text{U.C.}}$ Then resolving forces vertically

$$N + m_{\text{U.C}}a + 2.25 - 200 = 0 \tag{i}$$

in which

$$m_{\text{U.C}}a = \frac{2.25}{g} \times 1.96\,g = 4.41\,\text{kN}$$

Substituting in Eq. (i) gives

$$N = 193.3\,\text{kN}$$

Now taking moments about the point of contact of the wheel and the ground

$$M + N \times 0.15 = 0$$

which gives

$$M = -29.0\,\text{kN m} \quad \text{(i.e. clockwise)}$$

The vertical distance, s, through which the aircraft moves before its vertical velocity is zero, i.e. the shortening of the oleo strut, is obtained using elementary dynamics; the compression of the tyre is neglected here but in practice could be significant. Thus, assuming that the deceleration a remains constant

$$v^2 = v_0^2 + 2as$$

in which $v_0 = 3.5$ m/s and $v = 0$. Then

$$s = -\frac{3.5^2}{2(-1.96\,g)} = \frac{3.5^2}{2 \times 1.96 \times 9.81}$$

i.e.

$$s = 0.32\,\text{m}$$

Let the mass of the wing outboard of the section AA be m_{W}. Then, referring to Fig. S.14.1(c) and resolving forces vertically the shear force, S, at the section AA is given by

$$S - m_{\text{W}}a - 6.6 = 0$$

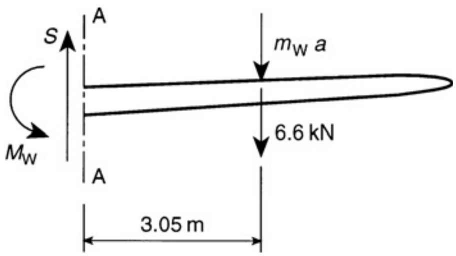

Fig. S.14.1(c)

i.e.

$$S - \frac{6.6}{g} \times 1.96\,g - 6.6 = 0$$

which gives

$$S = 19.5\,\text{kN}$$

Now taking moments about the section AA

$$M_{\text{W}} - m_{\text{W}}a \times 3.05 - 6.6 \times 3.05 = 0$$

or

$$M_w = \frac{6.6}{g} \times 1.96\,g \times 3.05 + 6.6 \times 3.05$$

i.e.

$$M_w = 59.6\,\text{kNm}$$

S.14.2

From Example 14.2 the time taken for the vertical velocity of the aircraft to become zero is 0.099 s. During this time the aircraft moves through a vertical distance, s, which, from elementary dynamics, is given by

$$s = v_0 t + \tfrac{1}{2} a t^2$$

where $v_0 = 3.7$ m/s and $a = -3.8\,g$ (see Example 14.2). Then

$$s = 3.7 \times 0.099 - \tfrac{1}{2} \times 3.8 \times 9.81 \times 0.099^2$$

i.e.

$$s = 0.184\,\text{m}$$

The angle of rotation, θ_1, during this time is given by

$$\theta_1 = \omega_0 t + \tfrac{1}{2} \alpha t^2$$

in which $\omega_0 = 0$ and $\alpha = 3.9\,\text{rad/s}^2$ (from Example 14.2). Then

$$\theta_1 = \tfrac{1}{2} \times 3.9 \times 0.099^2 = 0.019\,\text{rad}$$

The vertical distance, s_1, moved by the nose wheel during this rotation is, from Fig. 14.5

$$s_1 = 0.019 \times 5.0 = 0.095\,\text{m}$$

Therefore the distance, s_2, of the nose wheel from the ground after the vertical velocity at the CG of the aircraft has become zero is given by

$$s_2 = 1.0 - 0.184 - 0.095$$

i.e.

$$s_2 = 0.721\,\text{m}$$

It follows that the aircraft must rotate through a further angle θ_2 for the nose wheel to hit the ground where

$$\theta_2 = \frac{0.721}{5.0} = 0.144\,\text{rad}$$

During the time taken for the vertical velocity of the aircraft to become zero the vertical ground reactions at the main undercarriage will decrease from 1200 to 250 kN and, assuming the same ratio, the horizontal ground reaction will decrease from 400 kN to $(250/1200) \times 400 = 83.3$ kN. Therefore, from Eqs (ii) and (iii) of Example 14.2, the angular acceleration of the aircraft when the vertical velocity of its CG becomes zero is

$$\alpha_1 = \frac{250}{1200} \times 3.9 = 0.81\ \text{rad/s}^2$$

Thus the angular velocity, ω_1, of the aircraft at the instant the nose wheel hits the ground is given by

$$\omega_1^2 = \omega_0^2 + 2\alpha_1\theta_2$$

where $\omega_0 = 0.39$ rad/s (see Example 14.2). Then

$$\omega_1^2 = 0.39^2 + 2 \times 0.81 \times 0.144$$

which gives

$$\omega_1 = 0.62\ \text{rad/s}$$

The vertical velocity, v_{NW}, of the nose wheel is then

$$v_{\text{NW}} = 0.62 \times 5.0$$

i.e.

$$v_{\text{NW}} = 3.1\ \text{m/s}$$

S.14.3

With the usual notation the loads acting on the aircraft at the bottom of a symmetric manoeuvre are shown in Fig. S.14.3.

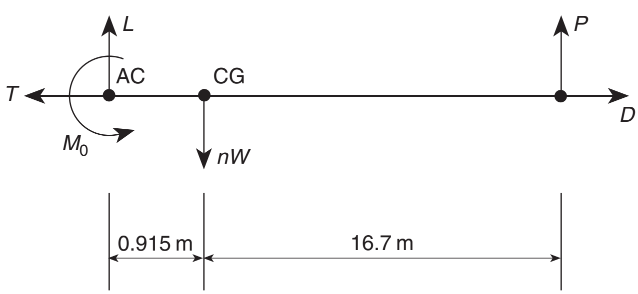

Fig. S.14.3

Taking moments about the CG

$$0.915L - M_0 = 16.7P \tag{i}$$

and for vertical equilibrium

$$L + P = nW \tag{ii}$$

Further, the bending moment in the fuselage at the CG is given by

$$M_{\mathrm{CG}} = nM_{\mathrm{LEV.FLT}} - 16.7P \tag{iii}$$

Also

$$M_0 = \frac{1}{2}pV^2S\bar{c}C_{M_0} = \frac{1}{2} \times 1.223 \times 27.5 \times 3.05^2 \times 0.0638V^2$$

i.e.

$$M_0 = 9.98V^2 \tag{iv}$$

From Eqs (i) and (iii)

$$0.915(nW - P) - M_0 = 16.7P$$

Substituting for M_0 from Eq. (iv) and rearranging

$$P = 0.052nW - 0.567V^2 \tag{v}$$

In cruise conditions where, from Fig. P.14.3, $n=1$ and $V=152.5\,\mathrm{m/s}$, P, from Eq. (v) is given by

$$P = -2994.3\,\mathrm{N}$$

Then, from Eq. (iii) when $n=1$

$$600\,000 = M_{\mathrm{LEV.FLT}} + 16.7 \times 2994.3$$

which gives

$$M_{\mathrm{LEV.FLT}} = 549\,995\,\mathrm{Nm}$$

Now, from Eqs (iii) and (v)

$$M_{\mathrm{CG}} = 549\,995n - 16.7(0.052nW - 0.567V^2)$$

or

$$M_{\mathrm{CG}} = 379\,789n + 9.47V^2 \tag{vi}$$

From Eq. (vi) and Fig. P.14.3 it can be seen that the most critical cases are $n=3.5$, $V=152.5\,\mathrm{m/s}$ and $n=2.5$, $V=183\,\mathrm{m/s}$. For the former Eq. (vi) gives

$$M_{\mathrm{CG}} = 1\,549\,500\,\mathrm{Nm}$$

and for the latter

$$M_{\mathrm{CG}} = 1\,266\,600\,\mathrm{Nm}$$

Therefore the maximum bending moment is $1\,549\,500\,\mathrm{Nm}$ at $n=3.5$ and $V=152.5\,\mathrm{m/s}$.

S.14.4

With the usual notation the loads acting on the aeroplane are shown in Fig. S.14.4; ΔP is the additional tail load required to check the angular velocity in pitch. Then

$$\Delta P \times 12.2 = 204\,000 \times 0.25$$

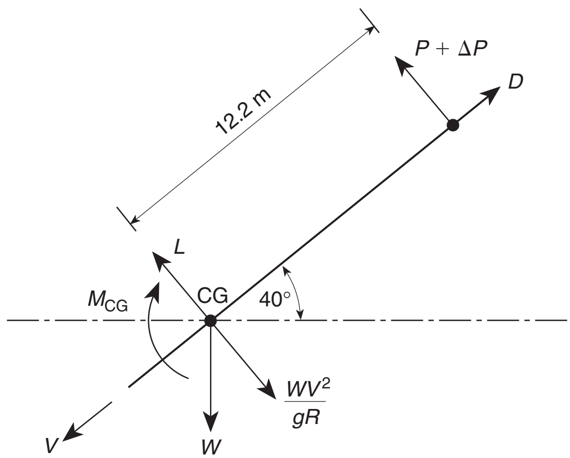

Fig. S.14.4

i.e.

$$\Delta P = 4180\,\text{N}$$

Now resolving perpendicularly to the flight path

$$L + (P + \Delta P) = \frac{WV^2}{gR} + W\cos 40^\circ \tag{i}$$

Then resolving parallel to the flight path

$$fW + W\sin 40^\circ = D \tag{ii}$$

where f is the forward inertia coefficient, and taking moments about the CG

$$(P + \Delta P) \times 12.2 = M_{\text{CG}} \tag{iii}$$

Assume initially that

$$L = W\cos 40^\circ + \frac{WV^2}{gR}$$

i.e.

$$L = 230\,000\cos 40^\circ + 238\,000 \times 215^2/(9.81 \times 1525)$$

which gives

$$L = 917\,704\,\text{N}$$

Then

$$C_L = \frac{L}{\frac{1}{2}\rho V^2 S} = \frac{917\,704}{\frac{1}{2} \times 1.223 \times (215)^2 \times 88.5} = 0.367$$

and

$$M_{CG} = \tfrac{1}{2}\rho V^2 S(0.427C_L - 0.061)$$

i.e.

$$M_{CG} = \tfrac{1}{2} \times 1.223 \times 215^2 \times 88.5(0.427 \times 0.367 - 0.061)$$

from which

$$M_{CG} = 239\,425\,\text{Nm}$$

Then, from Eq. (iii)

$$P + \Delta P = \frac{239\,425}{12.2}$$

i.e.

$$P + \Delta P = 19\,625\,\text{N}$$

Thus, a more accurate value for L is

$$L = 917\,704 - 19\,625 = 898\,079\,\text{N}$$

which then gives

$$C_L = \frac{898\,079}{\frac{1}{2} \times 1.223 \times 215^2 \times 88.5} = 0.359$$

Hence

$$M_{CG} = \tfrac{1}{2} \times 1.223 \times 215^2 \times 88.5(0.427 \times 0.359 - 0.061)$$

i.e.

$$M_{CG} = 230\,880\,\text{Nm}$$

and, from Eq. (iii)

$$P + \Delta P = 18\,925\,\text{N}$$

Then

$$L = 917\,704 - 18\,925 = 898\,779\,\text{N}$$

so that

$$n = \frac{898\,779}{238\,000} = 3.78$$

At the tail

$$\Delta n = \frac{\ddot{\theta} l}{g} = \frac{230\,880}{204\,000} \times \frac{12.2}{9.81} = 1.41$$

Thus the total n at the tail $= 3.78 + 1.41 = 5.19$.

Now

$$C_D = 0.0075 + 0.045 \times \left(\frac{898\,779}{\frac{1}{2}\rho V^2 S} \right)^2 + 0.0128$$

i.e.

$$C_D = 0.026$$

so that

$$D = \tfrac{1}{2}\rho V^2 S \times 0.026 = 65\,041\,\text{N}$$

Thus, from Eq. (ii)

$$f = -0.370$$

S.14.5

From Eq. (14.21) ϕ, in Fig. 14.10, is given by

$$\tan\phi = \frac{V^2}{gR} = \frac{168^2}{9.81 \times 610} = 4.72$$

so that

$$\phi = 78.03^\circ$$

From Eq. (14.20)

$$n = \sec\phi = 4.82$$

Thus, the lift generated in the turn is given by

$$L = nW = 4.82 \times 133\,500 = 643\,470\,\text{N}$$

Then

$$C_L = \frac{L}{\frac{1}{2}\rho V^2 S} = \frac{643\,470}{\frac{1}{2} \times 1.223 \times 168^2 \times 46.5} = 0.80$$

Hence

$$C_D = 0.01 + 0.05 \times 0.80^2 = 0.042$$

and the drag

$$D = \tfrac{1}{2} \times 1.223 \times 168^2 \times 46.5 \times 0.042 = 33\,707\,\text{N}$$

The pitching moment M_0 is given by

$$M_0 = \tfrac{1}{2}\rho V^2 S\bar{c}C_{M,0} = -\tfrac{1}{2} \times 1.223 \times 168^2 \times 46.5 \times 3.0 \times 0.03$$

i.e.

$$M_0 = -72\,229\,\text{Nm} \qquad \text{(i.e nose down)}$$

The wing incidence is given by

$$\alpha = \frac{C_L}{dC_L/d\alpha} = \frac{0.80}{4.5} \times \frac{180}{\pi} = 10.2°$$

The loads acting on the aircraft are now as shown in Fig. S.14.5.

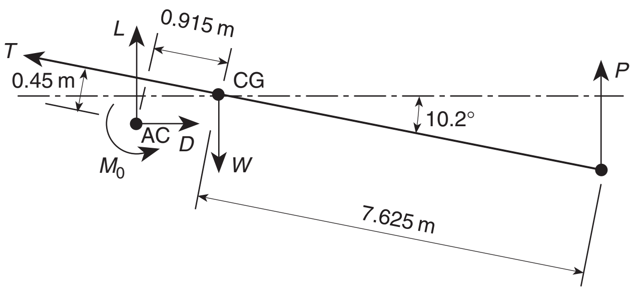

Fig. S.14.5

Taking moments about the CG

$$L(0.915\cos 10.2° + 0.45\sin 10.2°) - D(0.45\cos 10.2° - 0.915\sin 10.2°) - M_0$$
$$= P \times 7.625\cos 10.2° \qquad \text{(i)}$$

Substituting the values of L, D and M_0 in Eq. (i) gives

$$P = 73\,160\,\text{N}$$

S.14.6

(a) The forces acting on the aircraft in the pull-out are shown in Fig. S.14.6. Resolving forces perpendicularly to the flight path

$$L = \frac{WV^2}{gR} + W\cos\theta \qquad \text{(i)}$$

The maximum allowable lift is $4.0W$ so that Eq. (i) becomes

$$\frac{V^2}{gR} = 4 - \cos\theta$$

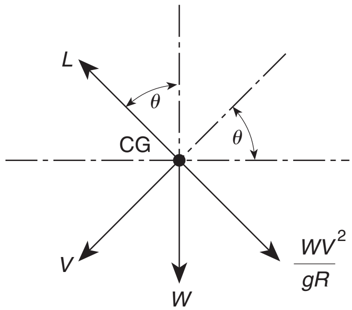

Fig. S.14.6

or

$$\frac{V\omega}{g} = 4 - \cos\theta \tag{ii}$$

where ω $(= V/R)$ is the angular velocity in pitch. In Eq. (ii) ω will be a maximum when $\cos\theta$ is a minimum, i.e. when θ reaches its maximum allowable value (60°). Then, from Eq. (ii)

$$\omega = \frac{g}{V}(4 - 0.5) = \frac{3.5\,g}{V} \tag{iii}$$

From Eq. (iii) ω will be a maximum when V is a minimum which occurs when $C_{\mathrm{L}} = C_{\mathrm{L.MAX}}$. Thus

$$\tfrac{1}{2}\rho V^2 S C_{\mathrm{L.MAX}} = 4 \times \tfrac{1}{2}\rho V_{\mathrm{s}}^2 S C_{\mathrm{L.MAX}}$$

whence

$$V = 2V_{\mathrm{s}} = 2 \times 46.5 = 93.0\,\mathrm{m/s}$$

Therefore, from Eq. (iii)

$$\omega_{\max} = \frac{3.5 \times 9.81}{93.0} = 0.37\,\mathrm{rad/s}$$

(b) Referring to Fig. 14.10, Eq. (14.17) gives

$$nW\sin\phi = \frac{WV^2}{gR}$$

i.e.

$$4\sin\phi = \frac{V\omega}{g} \tag{iv}$$

Also, from Eq. (14.20) $\sec\phi = 4$ whence $\sin\phi = 0.9375$. Then Eq. (iv) becomes

$$\omega = 3.87\frac{g}{V} \tag{v}$$

Thus, ω is a maximum when V is a minimum, i.e. when $V = 2V_s$ as in (a). Therefore

$$\omega_{max} = \frac{3.87 \times 9.81}{2 \times 46.5} = 0.41 \text{ rad/s}$$

The maximum rate of yaw is $\omega_{max} \cos\phi$, i.e.

$$\text{maximum rate of yaw} = 0.103 \text{ rad/s}$$

S.14.7

The forces acting on the airliner are shown in Fig. S.14.7 where α_w is the wing incidence. As a first approximation let $L = W$. Then

$$\frac{1}{2}\rho V^2 S\alpha_w \frac{\partial C_L}{\partial \alpha} = 1\,600\,000$$

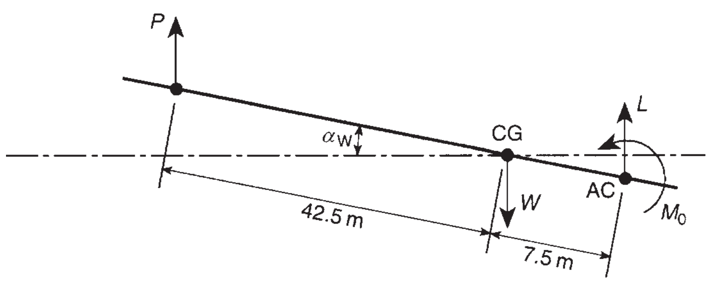

Fig. S.14.7

i.e.

$$\alpha_w = \frac{1\,600\,000 \times 180}{\frac{1}{2} \times 0.116 \times 610^2 \times 280 \times 1.5 \times \pi}$$

so that

$$\alpha_w = 10.1^\circ$$

From vertical equilibrium

$$L + P = W \tag{i}$$

and taking moments about the CG.

$$P \times 42.5 \cos 10.1^\circ = L \times 7.5 \cos 10.1^\circ + M_0 \tag{ii}$$

Substituting for L from Eq. (i) in Eq. (ii)

$$P \times 42.5 \cos 10.1^\circ = (1\,600\,000 - P)7.5 \cos 10.1^\circ$$
$$+ \tfrac{1}{2} \times 0.116 \times 610^2 \times 280 \times 22.8 \times 0.01$$

from which

$$P = 267\,963\,\text{N}$$

Thus, from Eq. (i)

$$L = 1\,332\,037\,\text{N}$$

giving

$$\alpha_{\text{w}} = 8.4^{\circ}$$

Then, taking moments about the CG

$$P \times 42.5 \cos 8.4^{\circ} = (1\,600\,000 - P)7.5 \cos 8.4^{\circ} + \tfrac{1}{2} \times 0.116 \times 610^2 \times 280 \times 22.8 \times 0.01$$

which gives

$$P = 267\,852\,\text{N}$$

This is sufficiently close to the previous value of tail load to make a second approximation unnecessary.

The change $\Delta\alpha$ in wing incidence due to the gust is given by

$$\Delta\alpha = \frac{18}{610} = 0.03\,\text{rad}$$

Thus the change ΔP in the tail load is

$$\Delta P = \frac{1}{2}\rho V^2 S_{\text{T}} \frac{\partial C_{\text{L.T}}}{\partial \alpha} \Delta\alpha$$

i.e.

$$\Delta P = \tfrac{1}{2} \times 0.116 \times 610^2 \times 28 \times 2.0 \times 0.03 = 36\,257\,\text{N}$$

Also, neglecting downwash effects, the change ΔL in wing lift is

$$\Delta L = \frac{1}{2}\rho V^2 S \frac{\partial C_{\text{L}}}{\partial \alpha} \Delta\alpha$$

i.e.

$$\Delta L = \tfrac{1}{2} \times 0.116 \times 610^2 \times 280 \times 1.5 \times 0.03 = 271\,931\,\text{N}$$

The resultant load factor, n, is then given by

$$n = 1 + \frac{36\,257 + 271\,931}{1\,600\,000}$$

i.e.

$$n = 1.19$$

S.14.8

As a first approximation let $L = W$. Then

$$\frac{1}{2}\rho V^2 S \frac{\mathrm{d}C_{\mathrm{L}}}{\mathrm{d}\alpha}\alpha_{\mathrm{w}} = 145\,000$$

Thus

$$\alpha_{\mathrm{w}} = \frac{145\,000}{\frac{1}{2} \times 1.223 \times 250^2 \times 50 \times 4.8} = 0.0158\,\mathrm{rad} = 0.91^\circ$$

Also

$$C_{\mathrm{D}} = 0.021 + 0.041 \times 0.08^2$$

i.e.

$$C_{\mathrm{D}} = 0.0213$$

Referring to Fig. P.14.8 and taking moments about the CG and noting that $\cos 0.91^\circ \simeq 1$

$$L \times 0.5 - D \times 0.4 + M_0 = P \times 8.5$$

i.e.

$$0.5(145\,000 - P) - 0.4 \times \frac{1}{2}\rho V^2 S C_{\mathrm{D}} + \frac{1}{2}\rho V^2 S\bar{c} C_{\mathrm{M},0} = 8.5P$$

Thus

$$0.5(145\,000 - P) - 0.4 \times \frac{1}{2} \times 1.223 \times 250^2 \times 50 \times 0.0213 - \frac{1}{2}$$
$$\times 1.223 \times 250^2 \times 50 \times 2.5 \times 0.032 = 8$$

which gives

$$P = -10\,740\,\mathrm{N}$$

Hence

$$L = W - P = 145\,000 + 10\,740 = 155\,740\,\mathrm{N}$$

The change ΔP in the tail load due to the gust is given by

$$\Delta P = \frac{1}{2}\rho V^2 S_{\mathrm{T}} \frac{\partial C_{\mathrm{L.T}}}{\partial \alpha}\Delta\alpha$$

in which

$$\Delta\alpha = -\frac{6}{250} = -0.024\,\mathrm{rad}$$

Thus

$$\Delta P = -\frac{1}{2} \times 1.223 \times 250^2 \times 9.0 \times 2.2 \times 0.024 = -18\,162\,\mathrm{N}$$

Therefore the total tail load $= -10\,740 - 18\,162 = -28\,902\,\mathrm{N}$.

The increase in wing lift ΔL due to the gust is given by

$$\Delta L = -\frac{1}{2}\rho V^2 S \frac{\partial C_L}{\partial \alpha}\Delta\alpha = -\frac{1}{2} \times 1.223 \times 250^2 \times 50 \times 4.8 \times 0.024$$

i.e.

$$\Delta L = -220\,140\,\text{N}$$

Hence

$$n = 1 - \frac{(220\,140 + 18\,162)}{145\,000} = -0.64$$

Finally the forward inertia force fW is given by

$$fW = D = \tfrac{1}{2}\rho V^2 S C_D = \tfrac{1}{2} \times 1.223 \times 250^2 \times 50 \times 0.0213$$

i.e.

$$fW = 40\,703\,\text{N}$$

Solutions to Chapter 15 Problems

S.15.1

Substituting the given values in Eq. (15.3)

$$S_a = 2 \times 230\left(1 - \frac{S_a}{2 \times 870}\right)$$

from which

$$S_a = 363\,\text{N/mm}^2$$

S.15.2

From Eq. (15.4)

$$S_a = 2 \times 230\left[1 - \left(\frac{S_a}{2 \times 870}\right)^2\right]$$

i.e.

$$S_a = 460 - 1.519 \times 10^{-4} S_a^2$$

or

$$S_a^2 + 6581.7 S_a - 3\,027\,600 = 0$$

Solving,

$$S_a = 432\,\text{N/mm}^2.$$