Hence

$$w = \frac{1}{2a}\frac{d\theta}{dz}(y^3 - 3x^2y).$$

S.3.5

The torsion constant, J, for the complete cross-section is found by summing the torsion constants of the narrow rectangular strips which form the section. Then, from Eq. (3.29)

$$J = 2\frac{at^3}{3} + \frac{bt^3}{3} = \frac{(2a+b)t^3}{3}$$

Therefore, from the general torsion equation (3.12)

$$\frac{d\theta}{dz} = \frac{3T}{G(2a+b)t^3} \tag{i}$$

The maximum shear stress follows from Eqs (3.28) and (i), hence

$$\tau_{\max} = \pm Gt\frac{d\theta}{dz} = \pm\frac{3T}{(2a+b)t^2}.$$

Solutions to Chapter 4 Problems

S.4.1

Give the beam at D a virtual displacement δ_D as shown in Fig. S.4.1. The virtual displacements of C and B are then, respectively, $3\delta_D/4$ and $\delta_D/2$.

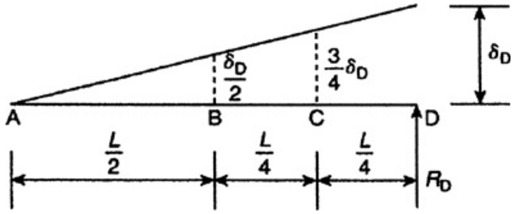

Fig. S.4.1

The equation of virtual work is then

$$R_D\delta_D - \frac{2W\delta_D}{2} - \frac{W3\delta_D}{4} = 0$$

from which

$$R_D = 1.75W$$

It follows that

$$R_A = 1.25W.$$

S.4.2

The beam is given a virtual displacement δ_C at C as shown in Fig. S.4.2.

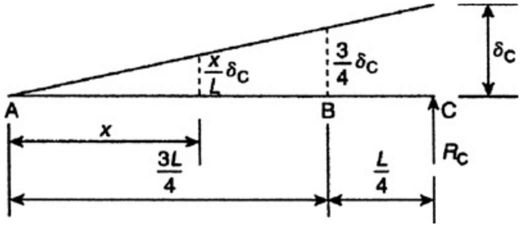

Fig. S.4.2

The virtual work equation is then

$$R_C\delta_C - \frac{W3\delta_C}{4} - \int_0^L w\left(\frac{x}{L}\right)\delta_C\,\mathrm{d}x = 0$$

from which

$$R_C = \frac{3W + 2wL}{4}$$

so that

$$R_A = \frac{W + 2wL}{4}.$$

S.4.3

The beam is given a virtual rotation θ_A at A as shown in Fig. S.4.3.

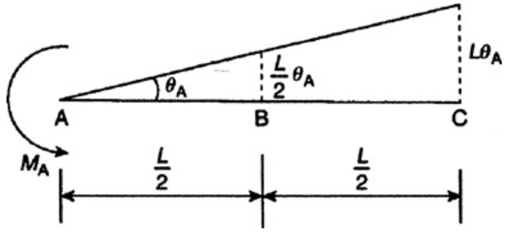

Fig. S.4.3

The virtual work equation is then

$$M_A\theta_A - \frac{WL\theta_A}{2} - 2WL\theta_A = 0$$

from which

$$M_A = 2.5WL$$

and

$$R_A = 3W.$$

S.4.4

Give the beam virtual rotations α and β at A and B, respectively as shown in Fig. S.4.4. Then, at C, $(3L/4)\alpha = (L/4)\beta$ so that $\beta = 3\alpha$.

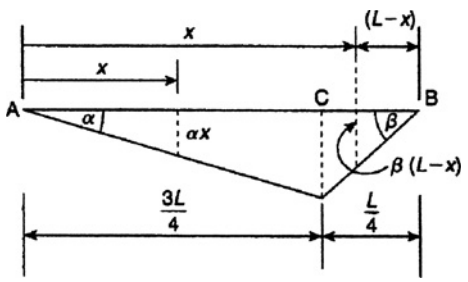

Fig. S.4.4

The relative rotation of AB and BC at C is $(\alpha+\beta)$ so that the equation of virtual work is $M_C(\alpha+\beta) = \int_0^{3L/4} w\alpha x\,dx + \int_{3L/4}^{L} w3\alpha(L-x)dx$
i.e.

$$4M_C\alpha = w\alpha\left[\int_0^{3L/4} x\,dx + 3\int_{3L/4}^{L}(L-x)dx\right]$$

from which

$$M_C = \frac{3wL^2}{32}.$$

S.4.5

Suppose initially that the portion GCD of the truss is given a small virtual rotation about C so that G moves a horizontal distance δ_G and D a vertical distance δ_D as shown in Fig. S.4.5(a).

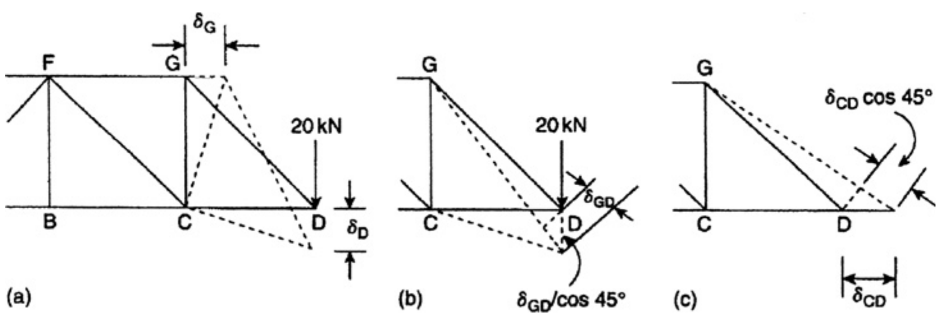

Fig. S.4.5

Then, since CG = CD, $\delta_G = \delta_D$ and the equation of virtual work is

$$\mathrm{FG}\delta_G = 20\delta_D$$

so that

$$\mathrm{FG} = +20\,\mathrm{kN}$$

The virtual displacement given to G corresponds to an extension of FG which, since the calculated value of FG is positive, indicates that FG is tensile.

Now suppose that GD is given a small virtual increase in length δ_{GD} as shown in Fig. S.4.5(b). The vertical displacement of D is then $\delta_{GD}/\cos 45°$ and the equation of virtual work is

$$\mathrm{GD}\delta_{GD} = 20\delta_{GD}/\cos 45°$$

from which

$$\mathrm{GD} = +28.3\,\mathrm{kN} \quad \text{(tension)}$$

Finally suppose that CD is given a small virtual extension δ_{CD} as shown in Fig. S.4.5(c). The corresponding extension of GD is $\delta_{CD}\cos 45°$. Then the equation of virtual work is, since the 20 kN load does no work

$$\mathrm{CD}\delta_{CD} + \mathrm{GD}\delta_{CD}\cos 45° = 0$$

Substituting for GD from the above gives

$$\mathrm{CD} = -20\,\mathrm{kN} \quad \text{(compression).}$$

S.4.6

First determine the deflection at the quarter-span point B. Then, referring to Fig. S.4.6 the bending moment due to the actual loading at any section is given by

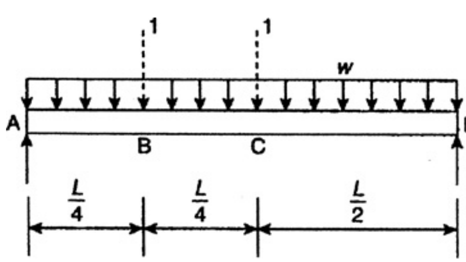

Fig. S.4.6

$$M_A = \frac{wLx}{2} - \frac{wx^2}{2} = \frac{w(Lx - x^2)}{2}$$

and due to the unit load placed at B is

$$M_1 = \frac{3x}{4} \text{ in AB} \quad \text{and} \quad M_1 = \frac{(L - x)}{4} \text{ in BD}$$

Then substituting in Eq. (4.20)

$$v_B = \frac{w}{8EI}\left[\int_0^{L/4} 3(Lx^2 - x^3)\mathrm{d}x + \int_{L/4}^{L} (Lx - x^2)(L - x)\mathrm{d}x\right]$$

which gives

$$v_B = \frac{57wL^4}{6144EI}$$

For the deflection at the mid-span point the bending moment at any section due to the actual loading is identical to the expression above. With the unit load applied at C

$$M_1 = \frac{x}{2} \text{ in AC} \quad \text{and} \quad M_1 = \frac{(L - x)}{2} \text{ in CD}$$

Substituting in Eq. (4.20)

$$v_C = \frac{w}{4EI}\left[\int_0^{L/2} (Lx^2 - x^3)\,\mathrm{d}x + \int_{L/2}^{L} (Lx - x^2)(L - x)\mathrm{d}x\right]$$

from which

$$v_C = \frac{5wL^4}{384EI}.$$

Solutions to Chapter 5 Problems

S.5.1

This problem is most readily solved by the application of the unit load method. Therefore, from Eq. (5.20), the vertical deflection of C is given by

$$\Delta_{V,C} = \sum \frac{F_0 F_{1,V} L}{AE} \tag{i}$$

and the horizontal deflection by

$$\Delta_{H,C} = \sum \frac{F_0 F_{1,H} L}{AE} \tag{ii}$$

in which $F_{1,V}$ and $F_{1,H}$ are the forces in a member due to a unit load positioned at C and acting vertically downwards and horizontally to the right, in turn, respectively. Further, the value of L/AE ($= 1/20$ mm/N) for each member is given and may be omitted from the initial calculation. All member forces (see Table S.5.1) are found using the method of joints which is described in textbooks on structural analysis, for example, *Structural and Stress Analysis* by T. H. G. Megson (Elsevier, 2005).