Virtual Work

Virtual work, if mastered well, can make it very easy to solve certain problems. While normal solving methods can involve numerous calculations, virtual work sometimes enables you to solve a problem with just one equation, and that's the benefit. The downside is that it's easy to make errors.

1 Increasing the degree of freedom

Most structures that occur in problems are statically determinant. They therefore can not move. To use virtual work, you must increase the degree of freedom in such a way, that the structure can move. But how to do that is sometimes complicated.

Every part in the structure can pass on forces. But which forces depends on the type of structure. In table 1 is an overview of all the structure types, and which forces they can transfer. As you can see, there are a lot of different connection types, some of which are familiar, some of which are not, but for which you just have to use your imagination to find out what they look like. The important part is that they can move in certain directions, and can not move in other directions.

Structure type	**Normal forces**	**Shear forces**	**Bending moments**
Normal bar	X	X	X
Horizontal sliding bar		X	X
Vertical sliding bar	X		X
Hinge	X	X	
Hinge on wheel support (tangential to the bar)	X		
Hinge on wheel support (perpendicular to the bar)		X	
Sliding connection (unable to rotate)			X
No connection			

Table 1: Overview of structure types, and their degree of freedom.

To use virtual work, you need to replace a certain connection by a connection that has just one more degree of freedom (one less X). The additional degree of freedom given should be replaced by an external force or torque. Suppose you want to know the normal force caused by a normal hinge on wheel support, you have to replace it by just a force (no connection), acting in the same direction as the reaction force caused by the support was acting. Suppose you want to know the bending moment in a bar, you have to replace part of the bar by a hinge and two moments (one on each side of the hinge, oppositely directed).

2 Making the structure move

After the degree of freedom has been created, the structure should move. Just imagine that the structure moves (rotates/slides) at the point at which you've replaced a connection, and draw it in the picture. Give the distance/angle that this point has moved a name (for example δu for distances or $\delta\theta$ for angles). Now draw the rest of the "new" structure, with the corresponding movements. But do keep in mind that bars remain straight, fixed hinges can not move, and hinges on wheel supports can only move in 1 direction.

Now look at every significant point in the structure that has moved/rotated in any way. If it has moved, express the movement in δu (if δu hasn't been defined yet, just define it as a certain movement). If it has rotated, express the rotation in $\delta\theta$ (define it if necessary). Now you should have a drawing on which is clearly visible what point moved what distance in what direction.

3 Setting up the equation

The most important thing now, is to set up the virtual work equation. As you should know, work is force times distance traveled ($\delta A = F\,\delta u$). Also, work is torque times angle rotated ($\delta A = M\,\delta\theta$) ($\delta\theta$ in radians!). It is very important to remember the following rule: If a force points in the same direction as the movement (thus the force partially causes the movement), the work done is positive. If a force points in the opposite direction as the movement (thus the force partially counteracts the movement), the work done is negative. And if the force is perpendicular to the movement, then the work done is 0. In formula, this can be written as $\delta A = F\,\delta u\,\cos\alpha$, where α is the angle between the force and the movement.

4 Working out the equation

When the work done by all the forces and the torques has been written down (with the right sign), it should be equal to 0, since the structure is in equilibrium. Now the equation should be solved. However, next to the unknown force/torque, there are often 2 unknown variables: δu and $\delta\theta$. To solve the equation, you first have to express one of those variables in the other one. When doing this, the small angle approximation should be used: $\tan\theta = \theta$. This makes solving the equation a lot simpler.

When one of the two unknown variables has been filled in, the equation should be solved. While solving it, the other unknown variable will also disappear (if you've done everything right), so that the only unknown left is the unknown force/torque, expressed in other known forces and torques. Of course you need to evaluate your answer (check whether it has the right unit, etc.), but if you have followed the steps, the answer should be right.