Using signs: always a difficult thing

When you're calculating all kinds of formulas, there's always the matter whether to substract values from eachother, or to add them up. There is a method which saves you that trouble, which works in normal coordinate systems (positive x-axis pointing right, positive y-axis pointing up). If you have to define a coordinate system yourself, it is often handy to define such a coordinate system, since it is most often used. (In later chapters we will also assume such a coordinate system is used.)

1 Which direction is positive, and which is negative?

First, I just state the following things: To the right (along the x-axis) is positive. Upward (along the y-axis) is positive. To the left (along the x-axis) is negative. Downward (along the y-axis) is negative. A counter-clockwise torque is positive. A clockwise torque is negative. All these rules are important, otherwise I wouldn't write them.

2 What's the use of it?

So what's the use of this rule? As long as you always use these values for any forces along the axes (when you dissect a force along axes, you always have to check the sign again, but you don't have to do it during calculations, which is the huge advantage of this method), you don't have to worry about signs or directions of forces or torques, simply because you stated the rules above.

Adding forces up is now a lot easier. You just have to put a plus between all the forces. When the force is negative, it will automatically get subtracted, because of its sign, so there's no need to worry about that. For example, just write: $F_5 = F_1 + F_2 + F_3$. When, for example, F_2 points downward, you know its value is negative, and you don't have to write: $F_4 = F_1 - F_2 + F_3$.

3 But what about torques?

Even when calculating torques, this method works. As you know, you calculate a torque T with the formula $T = d\,F$, where F is the force and d is the distance between the force and the center of rotation. But for this method to work, you also have to make sure the distance d is negative if it's pointed downward or to the left. This is quite important, and works really well if you're too lazy to ask yourself whether a force is directed clockwise or counterclockwise.

But I still haven't told you another technique to calculate torques. You simply have to use the cross product, which you might know from vector mathematics. For every force F, the torque T of the force is equal to:

$$M = d \times F = \begin{bmatrix} d_x \\ d_y \end{bmatrix} \times \begin{bmatrix} F_x \\ F_y \end{bmatrix} = d_x F_y - d_y F_x \tag{1}$$

The minus here is one of the few minus's you see in this method, which makes it ever more important. You also should keep in mind the order in which to multiply d and F. This is because $d \times F = -F \times d$. Of course if you already dissected all the forces along the axes, you don't have to use the cross product.

But you still might be wondering, why is the minus there? Suppose d_y is positive, and F_x is negative. Then the torque is directed counter-clockwise, so it must be positive. But $d_y\,F_x$ is negative, so there must be a minus sign. In any other case you will also conclude that the minus sign ought to be present.

4 Two-force members

Two-force members are an important thing in statics when trusses are present. But how to handle signs in that case? There is also a simple rule for that: assume tension. Tension is positive and compression negative, so if you assume there is tension, you always wind up with the right sign.

But what if you have a two-force member F_A, and want to dissect the force along the axes to find $F_{A;x}$ and $F_{A;y}$? There is a quite simple rule for that. Assume that the two-force member you're dissecting is in tension. If the tensile force points upward (or left-upward or right-upward), then the vertical component $F_{A;y}$ has the same sign as the two-force member F_A. If it points downward, $F_{A;y}$ has a different sign than F_A. The situation is analog for the horizontal component $F_{A;x}$.

But do remember that tensile forces, when present in two-force members, point inward. Most people find this strange, because an outward force is necessary to put a two-force member in tension. This is true, but the force we're talking about is not the force an object is acting on the two-force member, but the force that the two-force member is acting on an object. And according to Newton's third law, this force is oppositely directed, and thus points inward.

5 The conclusion

So by applying this rule in a very consistent way (forget it once, and you'll screw up your entire solution, pardon me for saying it), you don't have to worry about signs at all. At the end of a difficult series of calculations, you, for example, only have to conclude: "*oh, the sign is negative, so the torque is clockwise!*". Isn't it easy?

By the way, this is just a method of getting rid of those awful problems with signs and such. If you already have an own method which serves you really well, please use it! This is just a useful method for those people who just keep having trouble with the signs, and have no idea how to solve it.