Distributed Loads

Distributed loads don't have to be difficult, as long as you know how to deal with them. It's all about replacing them by normal forces, so you can use the normal statics rules on them. There are multiple kinds of distributed loads. We'll handle the simple ones first.

1 Constant distributed loads

Constant lineair loads are quite easy to handle. You can simply replace them by a normal force. As you probably should know, the resultant force caused by a distributed load is equal to the area under the distributed load. If the size of the load is q (N/m) and the length is a (m), then the size of the load is $a\,q$ (N). Interesting here, is to note the units of the values. The position of the resultant force of the load is simple too: right in the middle of the distributed load.

However, when there are hinges in the field of a distributed load, you may not always replace the entire distributed load by a resultant force, but have to split the distributed load up in two parts, and find a resultant force for every separate part. This is only necessary when you make a cut through the hinge though, but there may be a very few small exceptions to this rule.

2 Lineair distributed loads

Sometimes distributed loads are lineair, and usually they have the shape of a triangle. It is a bit more difficult to replace them by a resultant force, but it's nevertheless possible. The size of a distributed load, which ranges from 0 to q (N/m) and has length a, is simply $\frac{1}{2}a\,q$, which is the area of the triangle. The resultant force isn't in the middle of the load, but it's on $\frac{2}{3}$ of it, or of course at $\frac{1}{3}$ if you turn the triangle around, and have a value of q on the left and 0 on the right of the distributed load.

3 Combined lineair and constant loads

Sometimes lineair distributed loads don't have the shape of a triangle, but go from q_1 to q_2. If this is the case, you can simply cut the distributed load up in 2 parts, of which one has the shape of a rectangle (and thus is a constant distributed load with size q_1 (assuming $q_1 < q_2$, otherwise it's q_2)) and the other one has the shape of a triangle, which we also have previously discussed. You only have to find the resultant force of each of the 2 distributed loads. After that you may simplify it as mush as you wish.

4 Other distributed loads

There are also other distributed loads. These are quite difficult to handle. If there is no data about the shape/size of the distributed load, you can't calculate with it, of course. But sometimes there is a formula which gives the size of the distributed load on a point x, or sometimes you have to find that formula yourself. Let's do an example. Suppose $D(x) = -x^2 + 6x$, where $D(x)$ is the size of the distributed load, and x ranges from 0 to 6.

Since the resultant force of a distributed load is equal to the area under the distributed load, the resultant force can be calculated using an integral. So:

$$F_R = \int_0^6 D(x)dx = \left[-\frac{1}{3}x^3 + 3x^2\right]_0^6 = 3 \cdot 36 - \frac{1}{3} \cdot 216 = 36 \tag{1}$$

Now we have found the resultant force, so we need to know where it applies. For that, we take the torque about point 0. Since $T = F\,d$ where F is the force and d is the arm, we know that the torque caused by the distributed load at point x is $x\, D(x)$. So we can calculate T of the entire distributed load, around point 0:

$$T_0 = \int_0^6 D(x)\, x\, dx = [-\frac{1}{4}x^4 + 2x^3]_0^6 = 432 - 324 = 108 \tag{2}$$

So the torque around point 0 is 108, while the resultant force itself is 36. The resultant force must therefore apply at a distance $\frac{T}{F} = \frac{108}{36} = 3$ from point 0. In this case it is in the middle of the distributed load, which is quite logical because the shape of it was a symmetric parabola. Of course this isn't always the case. But this example does show how to calculate the resultant force of a strange distributed load.