Cables and pressure vessels

Next to rigid bars, structures can also consist of cables and pressure valves. This is often a difficult subject, since cables are rarely straight. Of course the normal equilibrium equations still apply, but these are often not enough to solve the problem. But there are a few tricks that can be used to solve problems involving these subjects.

1 Cables without distributed loads

Cables bend, and bars don't. That's why cables are sometimes a bit more difficult to calculate with. However, cables are often a bit like structures containing normal bars. Suppose there is a cable, on which one or more normal forces (no distributed loads) apply. If you replace the cable by bars, and place hinges at the points where the forces apply, the structures are more or less equivalent, and the normal solving methods for trusses can be applied. However, when distributed loads are present, it is not possible to put an infinite amount of hinges in the bar, so this trick won't work.

2 Cables without horizontal forces

In almost all problems involving cables, all the forces that are acting on the cable are downwards (except at the two ends of the cable). Thus the horizontal component of the cable tension does not change in the entire cable, and is therefore constant. So if you at every point know in which direction the cable points towards, and if you know the horizontal component of the cable tension, you can easily calculate the cable tension at any given point in the cable.

3 The rigid bar trick for cables

If all the forces acting on the cable are directed downward (or sometimes upward, just as long as there's no horizontal component), there is another trick you can use. Suppose there is a cable that spans from point A to point B, which is subjected to a number of vertical forces. Assume that there is a rigid and straight bar AB between points A and B (which don't have to be at the same height), and that the vertical forces acting on the rope act on the bar. Draw the bending moment diagram of the bar, and turn it upside down (positive downward). Now divide the entire diagram by the horizontal component of the cable tension H_{cable}. The diagram that appears shows the distance between the bar and the cable at any given point. In formula, this is:

$$y_{bar} - y_{cable} = -\frac{M_{bar}}{H_{cable}} \tag{1}$$

The variable y indicates the height, and M_{cable} the bending moment in the cable. But the bending moment is the integral of the shear force diagram, and the shear force diagram is the integral of the "force diagram". So also the following function applies:

$$(y_{bar} - y_{cable})'' = -\frac{q_x}{H_{cable}} \tag{2}$$

Where q_x is the force (often the magnitude of the distributed load) acting on the cable at any point x. Note that the second derivative of the height is taken. If you choose a nice and simple root for your coordinate system, it is often easy to find a function for the height of the cable at any given point.

4 Pressure vessels

Pressure vessels are just like cables. However, instead of a distributed load directed downward, there is a distributed load directed perpendicular to the vessel hull. This might seem difficult to calculate with (if you want, you can use integrals to find resultant forces and such, but I wouldn't recommend this), but there is a trick which makes it simple. It is called Pascal's law.

Suppose you have a pressure vessel with the shape of a circle (radius r). Now cut it in half by a vertical line right through the middle. The pressure being exerted on the cut is of course $2r \cdot p$, where p is the pressure inside the vessel. The pressure exerted on the other side is $\pi r \cdot p$. However, this pressure consists of forces directed in multiple directions. Pascal's law states that the resultant force of the latter pressure is of equal size and opposite direction with respect to the resultant force of the pressure exerted on the cut. Also the lines of action of these two resultant forces are equal.

So by using this law, it can be proven that the tension in the pressure vessel is $r \cdot p$. The 2 disappeared, because we cut twice through the pressure vessel (once on top of the circle, once on the bottom). However, this law holds not only for circular shapes but for any shape. And if you know this law, you just have to cut the pressure vessel in the right way to find what you need to know.