

ENGINEERING MECHANICS

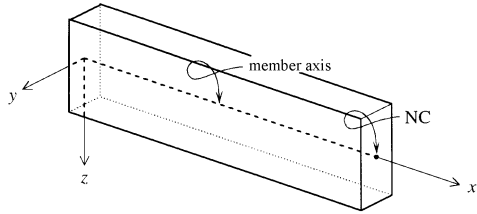
Volume 1 Equilibrium

C. Hartsuijker and J.W. Welleman



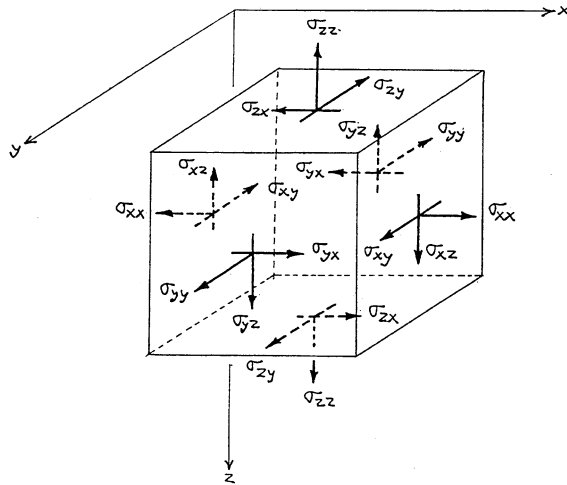
Coordinate system in a member (bar, beam, column, etc.)

The x axis is selected along the *member axis*, through the *normal (force) centre* NC of the consecutive cross-sections. The y and z axis are chosen parallel to the plane of a cross-section.



Normal and shear forces

σ_{ij} is the stress on a plane with the normal in the i direction ($i = x, y, z$), and acting in the j direction. σ_{ij} is a normal stress when $i = j$ and a shear stress when $i \neq j$. The positive directions are shown in the figure.



Relationship between section forces and stresses in the cross-section

Normal force

$$N = \int_A \sigma_{xx} dA$$

Shear forces

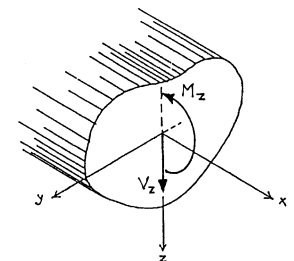
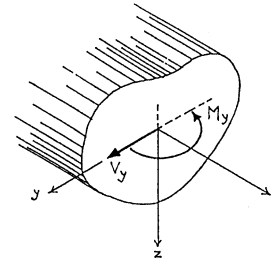
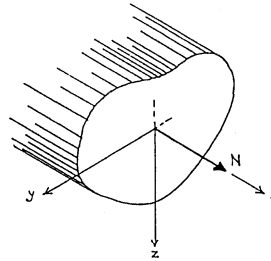
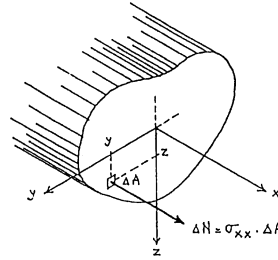
$$V_y = \int_A \sigma_{xy} dA$$

$$V_z = \int_A \sigma_{xz} dA$$

Bending moments

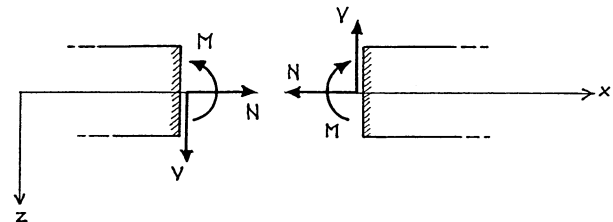
$$M_y = \int_A y \sigma_{xx} dA$$

$$M_z = \int_A z \sigma_{xx} dA$$



Positive directions of N , V_z and M_z

The figure below shows the positive directions of the normal force N , shear force $V_z = V$ and bending moment in the xz plane $M_z = M$.



Relationship between section forces and load

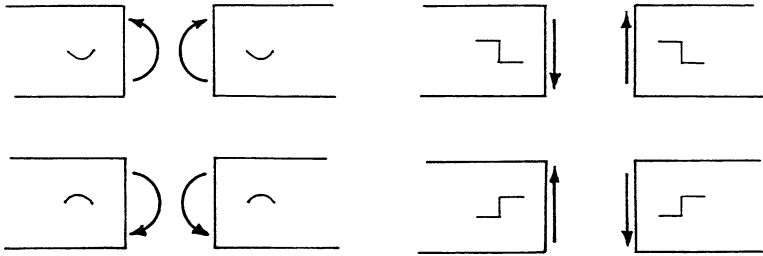
$$\frac{dN}{dx} + q_x = 0, \quad N = - \int q_x dx,$$

$$\frac{dV_z}{dx} + q_z = 0, \quad V_z = - \int q_z dx,$$

$$\frac{dM_z}{dx} - V_z = 0, \quad M_z = \int V_z dx = - \iint q_z dx dx.$$

Deformation symbols

The deformation symbols for bending are given in the left-hand figure, those for the shear forces are shown in the right-hand figure.

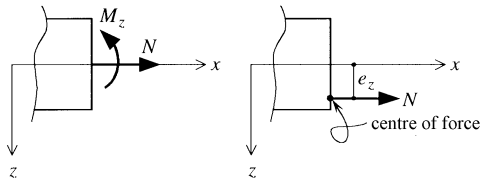


Cable equation

$$H \frac{d^2z}{dx^2} = -q_z.$$

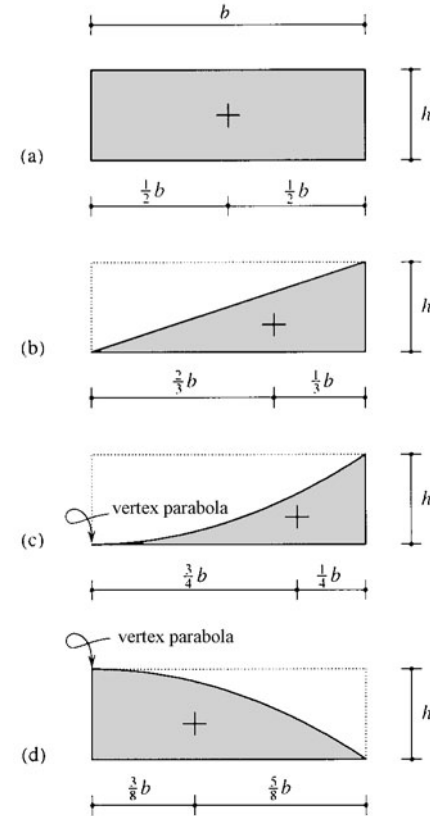
Centre of force

$$e_z = \frac{M_z}{N}, \text{ see figure below.}$$



Area A and centroid (+) of a number of simple shapes

Shape	Area A	Figure
Rectangle	bh	a
Right-angles triangle	$\frac{1}{2}bh$	b
Parabola (concave)	$\frac{1}{3}bh$	c
Parabola (convex)	$\frac{2}{3}bh$	d



ENGINEERING MECHANICS

Engineering Mechanics

Volume 1: Equilibrium

by

C. HARTSUIJKER

Delft University of Technology, Delft, The Netherlands

and

J.W. WELLEMAN

Delft University of Technology, Delft, The Netherlands

A C.I.P. Catalogue record for this book is available from the Library of Congress.

ISBN-10 1-4020-4120-9 (HB)

ISBN-13 978-1-4020-4120-4 (HB)

ISBN-10 1-4020-5483-1 (e-book)

ISBN-13 978-1-4020-5483-9 (e-book)

Published by Springer,

P.O. Box 17, 3300 AA Dordrecht, The Netherlands.

www.springer.com

This is a translation of the original Dutch work “Toegepaste Mechanica, Deel 1: Evenwicht”, 1999,
Academic Service, The Hague, The Netherlands.

Printed on acid-free paper

All Rights Reserved

© 2006 Springer

No part of this work may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, electronic, mechanical, photocopying, microfilming, recording or otherwise, without written permission from the Publisher, with the exception of any material supplied specifically for the purpose of being entered and executed on a computer system, for exclusive use by the purchaser of the work.

Table of Contents

Preface	ix	4 Structures	111
Foreword	xi	4.1 Structural elements	113
1 Introduction	1	4.2 Joints between structural elements	116
1.1 Mechanics	1	4.3 Supports	120
1.2 Quantities, units, dimensions	8	4.4 Planar structures	126
1.3 Vectors	11	4.5 Kinematic/static (in)determinate structures	130
1.4 Newton's Laws	18	4.6 Problems	145
2 Statics of a Particle	23	5 Calculating Support Reactions and Interaction Forces	153
2.1 Coplanar forces	23	5.1 Self-contained structures	154
2.2 Forces in space	30	5.2 Hinged beams	162
2.3 Equilibrium of a particle	35	5.3 Three-hinged frames	168
2.4 Problems	39	5.4 Three-hinged frames with tie-rod	173
3 Statics of a Rigid Body	51	5.5 Shored structures	176
3.1 Coplanar forces and moments	52	5.6 Trussed beams	184
3.2 Equilibrium of a rigid body in a plane	71	5.7 Strengthened beams	186
3.3 Forces and moments in space	80	5.8 Problems	190
3.4 Equilibrium of a rigid body in space	93	6 Loads	205
3.5 Problems	99	6.1 Loads in mechanics	206
		6.2 Loads in regulations	210
		6.3 Working with distributed loads	219

6.4	Modelling load flow	231	11	Mathematical Description of the Relationship between Section Forces and Loading	431
6.5	Stress concept; normal stress and shear stress	233	11.1	Differential equations for the equilibrium	432
6.6	Problems	236	11.2	Mathematical elaboration of the relationship between N and q_x (extension)	436
7	Gas Pressure and Hydrostatic Pressure	245	11.3	Mathematical elaboration of the relationship between M , V and q_z (bending)	441
7.1	Pascal's law – All-round pressure	246	11.4	Problems	456
7.2	Working with gas pressures	248	12	Bending Moment, Shear Force and Normal Force Diagrams	461
7.3	Working with hydrostatic pressures	255	12.1	Rules for drawing V and M diagrams more quickly	462
7.4	Summary	269	12.2	Rules for drawing the N diagram more quickly	493
7.5	Problems	271	12.3	Bent and compound bar type structures	495
8	Earth Pressures	285	12.4	Principle of superposition	505
8.1	Stresses in soil	286	12.5	Schematisations and reality	508
8.2	Vertical earth pressures	288	12.6	Problems	517
8.3	Horizontal earth pressures	292	13	Calculating M, V and N Diagrams	545
	Appendix 8.1	307	13.1	Self-contained structures	545
	Appendix 8.2	308	13.2	Compound and associated structures	586
8.4	Problems	309	13.3	Statically indeterminate structures	596
9	Trusses	319	13.4	Problems	608
9.1	Plane trusses	321	14	Cables, Lines of Force and Structural Shapes	631
9.2	Kinematically/statically (in)determinate trusses	328	14.1	Cables	632
9.3	Determining member forces	337	14.2	Centre of force and line of force	674
9.4	Problems	370	14.3	Relationship between cable, line of force and structural shape	679
10	Section Forces	387	14.4	Problems	694
10.1	Force flow in a member	388	15	Virtual Work	709
10.2	Diagrams for the normal force, shear force and bending moment	401	15.1	Work and strain energy	710
10.3	Deformation symbols for shear forces and bending moments	416			
10.4	Summary sign conventions for the N , V and M diagrams	420			
10.5	Problems	421			

15.2	Virtual work equation for a particle	713	16	Influence Lines	743
15.3	Virtual work equation for a rigid body	719	16.1	Influence lines using equilibrium equations	744
15.4	Virtual work equation for mechanisms	725	16.2	Influence lines using virtual work	748
15.5	Calculating forces using virtual work	729	16.3	Working with influence lines	755
15.6	Problems	739	16.4	Problems	763

Preface

This Volume is the first of a series of two:

- Volume 1 : Equilibrium
- Volume 2 : Stresses, deformations and displacements

Volume 1 introduces the fundamentals of structural and continuum mechanics in a comprehensive and consistent way. All theoretical developments are presented in the text and by means of an extensive set of figures. Numerous examples support the theory and provide a link to engineering practice. Combined with an extensive set of problems in each chapter, students are given ample opportunities to exercise.

The book consists of distinct modules, each divided into sections which are conveniently sized to be used as lectures. Both formal and intuitive (engineering) arguments are used in parallel to derive the important principles, for instance in bending moment diagrams and shear force diagrams. An important feature of the book is the straightforward and consistent sign convention, based on the stress definitions of continuum mechanics which will be used in Volume 2.

The modular content of the book shows a clear order of topics, starting with the introduction of forces and equilibrium of a particle followed by the extension to moments and the equilibrium of rigid bodies. An important

aspect that is used throughout the series is the interaction between rigid bodies and the forces that act upon rigid bodies. These forces play an important role in Chapter 4, where structural elements and support conditions are introduced, followed by Chapter 5, which deals with the interaction forces and support reactions. A comprehensive chapter on loads gives an overview not only of the origin of loads, but also provides an introduction how to treat loads in engineering codes and in structural calculations. Examples of specific loads from gases, from liquids and from soils can be found in Chapters 7 and 8. These chapters can be regarded as an introduction in soil and fluid mechanics, and can be omitted when treating only structural mechanics.

After the basic theory of equilibrium of rigid bodies, boundary conditions and the method of calculating the reactions, the focus shifts to the section forces (internal forces) in trusses (Chapter 9), and beam and frame structures (Chapters 10 to 13). The formal treatment of the beam theory of Chapter 11 uses as little mathematics as possible and shows the fundamental relations between bending moments, shear forces and distributed loads. This fundamental approach is supported with an extensive intuitive approach based on the visual use of bending moment diagrams and shear force diagrams. Chapters 12 and 13 are therefore the most important chapters, and use all previously introduced definitions and sign conventions.

The last part of Volume 1 consists of some special topics like cables (Chapter 14), virtual work and influence lines (Chapters 15 and 16). Virtual work is introduced as an alternative to the ordinary equilibrium conditions as used in the first part of this book. Using the principle of virtual work proves to be a fast method to calculate sectional forces and reactions in statically determinate structures. The theory of virtual work is also needed to obtain influence lines. Chapter 16 can therefore only be used in combination with Chapter 15.

Although the books introduce the fundamentals of engineering mechanics, not much mathematical knowledge is required. Examples in which use is made of integral calculus or differential equations can be omitted, although they contribute to the mathematical explanation of the relations between bending moments, shear forces and distributed loads. The educational value is therefore not only fundamental knowledge. It is also a demonstration how to translate physical problems into abstract models, which can be solved with mathematical tools.

Finding the right balance between the abstract fundamentals and practical application should be the challenge for the lecturer.

Coenraad Hartsuijker
Hans Welleman
Delft, The Netherlands
July 2006



Foreword

Structural or Engineering Mechanics is one of the core courses for new students in engineering studies. At Delft University of Technology a joint educational program for Statics and Strength of Materials has been developed by the Koiter Institute, and has subsequently been incorporated in the curricula of faculties like Civil Engineering, Aeronautical Engineering, Architectural Engineering, Mechanical Engineering, Maritime Engineering and Industrial Design.

In order for foreign students also to be able to benefit from this program an English version of the Dutch textbook series written by Coenraad Hartsuijker, which were already used in most faculties, appeared to be necessary. It is fortunate that in good cooperation between the writers, Springer and the Koiter Institute Delft, an English version of two text books could be realized, and it is believed that this series of books will greatly help the student to find his or her way into Engineering or Structural Mechanics.

Indeed, the volumes of this series offer some advantages not found elsewhere, at least not to this extent. Both formal and intuitive approaches are used, which is more important than ever. The books are modular and can also be used for self-study. Therefore, they can be used in a flexible manner and will fit almost any educational system. And finally, the SI system

is used consistently. For these reasons it is believed that the books form a very valuable addition to the literature.

René de Borst
Scientific Director, Koiter Institute Delft