9 Trusses

A *truss* is *by definition* a structure assembled with *straight bars* (*members*), which are connected by *hinged joints*, and loaded by forces which have their point of application at these joints.

In comparison to heavily-built structures, trusses need little material, and therefore have a relatively small dead weight. If we consider the use of little material, and the reduced costs for foundation because of the small dead weight, they can be cost saving. On the other hand, constructing trusses is often labour-intensive due to the complexity of the joints, and labour costs can be higher. Nevertheless, the total costs may be lower, and trusses can be an interesting type of structure from an economic perspective.

Trusses are often used in roof structures, bridges, cranes, and so forth. Scaffoldings are also often trusses.

Section 9.1 addresses the difference between a space truss and a plane truss. The rest of this chapter only looks at *plane trusses*. For this type of truss, all the members are located in the same plane, and the load acts in the plane of the truss. Section 9.1 also looks at the modelling of a structure as a truss, the nomenclature for the members in a truss, and the conventions used to label the joints and members.

Section 9.2 explains the relationship between the number of members and joints in a *simple* or *self-contained truss* and a *compound truss* respectively.

Next, the kinematic/static (in)determinacy of a truss is investigated and the relationship between the number of support reactions, members and joints is considered.

Calculating the member forces in a truss is addressed in Section 9.3. There are several methods for this, two of which are discussed:
- the method of sections;
- the method of joints.

In the *method of sections*, we make a suitable section in the truss, and calculate the member forces from the equilibrium of one of the bisected (isolated) parts. In the *method of joints*, we calculate the member forces from the equilibrium of the joints.

The methods mentioned are *manual calculation methods* and are applicable only to statically determinate trusses; they demand the necessary insight if they are to be used effectively. Sometimes it is useful to use both methods in combination.

Nowadays, we generally use computer programs to calculate trusses. Many of these programs use the so-called *displacement method*, which can be used for both statically determinate and statically indeterminate trusses.[1]

Even though increasing numbers of calculations are performed using computer programs, the manual calculation methods remain valuable, even if only because they can be used as a relatively simple check. This is true particularly for the method of sections, which offers a superb way of checking computer-based results. It allows us to check for errors that may be, for example, the result of incorrect data entry by the user.

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[1] The displacement method not only uses the equilibrium relationships, but also the behaviour of the material (the constitutive relationships) and the compatibility of the structure (the kinematic relationships). The constitutive and kinematic relationships are covered in Volume 2: *Stresses, Deformations, Displacements*.

9.1 Plane trusses

This section addresses the difference between a space truss and a plane truss. From here on, we will look only at plane trusses, with all members in the same plane, and the load acting in the plane of the truss. We will also look at the way in which a structure is modelled as a truss, the nomenclature for the members in a truss, the various types of trusses, and the conventions for labelling the joints and members.

9.1.1 Plane and space trusses

A *truss* is defined as a structure constructed with *straight bars* (*members*), which are connected by *hinges* at so-called *joints*, and loaded by forces which have their point of application at these joints.

There are *plane* trusses and *space trusses*. In plane trusses, all the members are in the same plane, and forces only act in the plane of the structure. In space trusses, the members are not all in the same plane (see Figure 9.1).

Many space trusses in fact consist of plane trusses, such as the structure in Figure 9.1b. The load shown is transferred to the supports via the plane trusses ABCD and ABEG.

From here on, we will look only at plane trusses. The open circles, which indicate the hinged joints, will be omitted since in a truss all joints are *hinged by definition*.

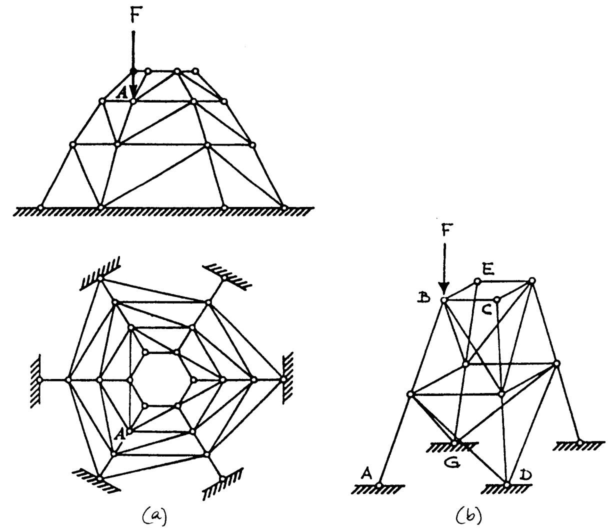

Figure 9.1 Space trusses. (a) Side view and top view of a truncated truss dome. (b) A space truss constructed from plane trusses.

9.1.2 Modelling a structure as a truss

Calculating a plane truss, hereafter referred to as truss, is based on the following assumptions:

- all members are straight;
- all members are connected at hinged joints;
- the load consists of forces that act in the plane of the structure and apply at the joints.

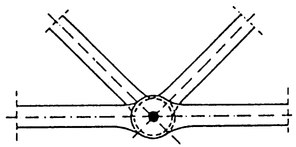

Figure 9.2 A hinged joint.

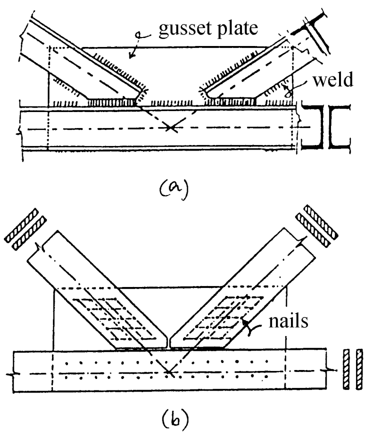

Figure 9.3 In trusses, the member axes intersect at one point. The members are usually rigidly connected to one another by a gusset plate. The joints (a) in a steel truss and (b) in a wooden truss are examples of this.

This implies that all the members in the truss behave as two-force members and can only transfer tensile and compressive forces between the joints (see also Sections 3.2.2 and Figure 3.35).

In the past, one tried to realise the connections in the joints as real hinges (see Figure 9.2). These days, all the members are rigidly connected, either directly or via a so-called *gusset plate*. Figure 9.3 shows two examples of a joint with a gusset plate: one made of steel (a) and the other made of wood (b). It is clear that these joints are not hinged. One can show, however, that whether or not the joints are hinged, this in fact has little impact on the force flow. A condition is, however, that the member axes intersect at the joints – clearly the case in Figure 9.3 – and that the load is applied at the joints.[1] This must be taken into account seriously when designing a truss.

Figures 9.4a to 9.4d show four structures with rigid joints, for which the load consists of forces that act at the joints. These structures behave as trusses, and can be calculated as such only if the structure remains kinematically determinate when all the rigid joints are replaced by hinged joints.

Figures 9.4e to 9.4h show the same structures as in (a) to (d), but now with hinged joints. After applying hinges, structures (a) and (b) are kinematically determinate and can therefore be considered trusses. With structures (c) and (d), a mechanism is formed after introducing hinged joints. They are now kinematically indeterminate and cannot be calculated as trusses. The force flow in these structures occurs mainly by bending.

The simple truss bridge in Figure 9.5 shows how to ensure that the load on the bridge ends up at the joints of the truss. The bridge consists of two *main beams* constructed as plane trusses. *Cross beams* have been introduced

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[1] The proof for this cannot be given at this stage, but is based on the characteristic that the members in a truss are relatively weak with respect to bending, and relatively stiff with respect to extension (changing length).

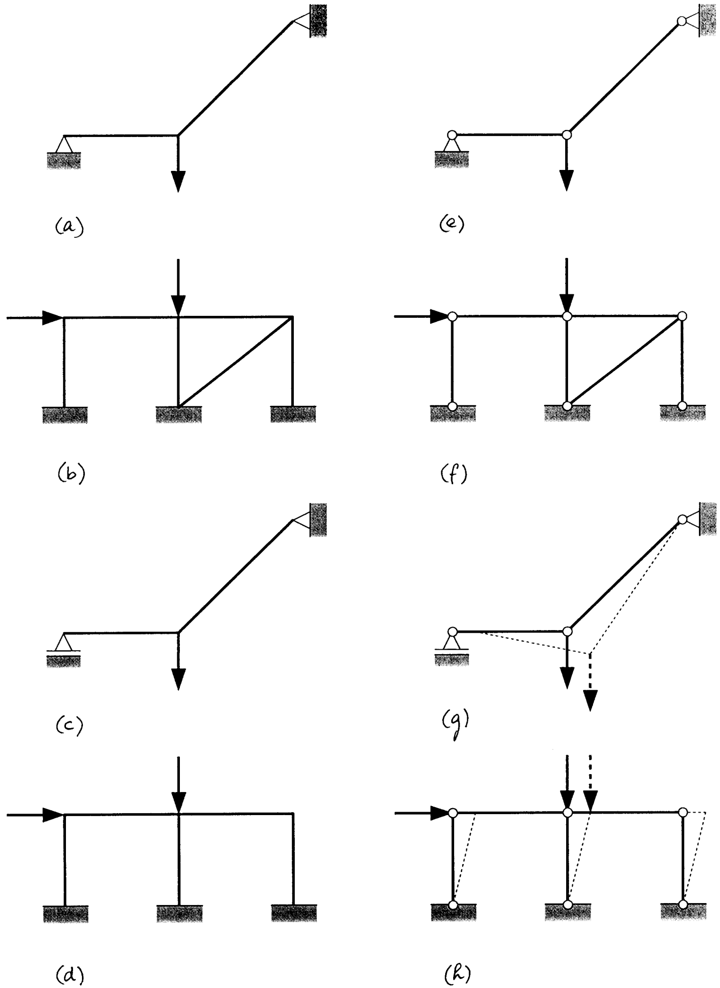


Figure 9.4

Figure 9.4 (a) to (d) Four structures with rigid joints and loaded by forces at the joints. (e) to (h) The same structures, but now all the rigid joints are replaced by hinged joints. With hinged joints (a) ad (b) are kinematically determinate and can be considered to be trusses. For (c) and (d), the use of hinged joints generates a mechanism; they cannot be considered trusses. The force flow in (c) and (d) occurs mainly by bending.

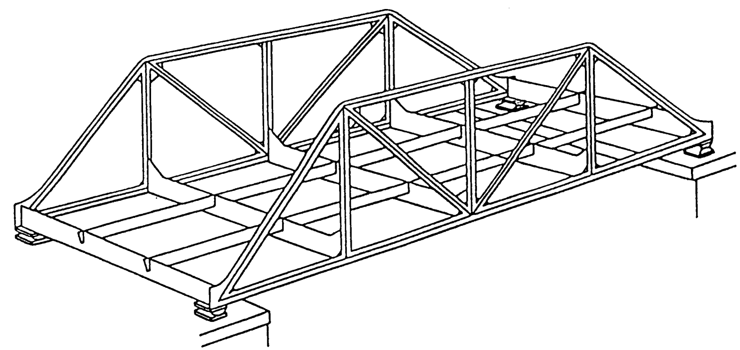


Figure 9.5 The structure of a simple truss bridge.

between the main beams, which are supported at the joints of the truss. Between the cross beams, *stringers* carry the *deck* (not shown). In this way, the traffic loading is directed via deck, stringers and cross beams as joint loads onto the trusses.

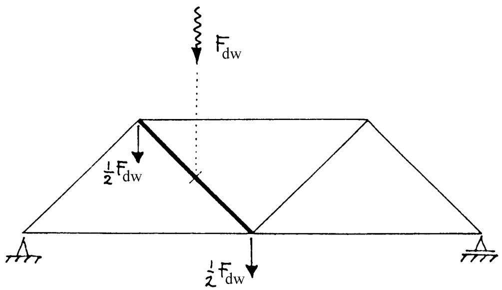


Figure 9.6 We assume that the dead weight of a truss applies in the joints. The total dead weight F_{dw} of a truss member is equally distributed over both adjacent joints.

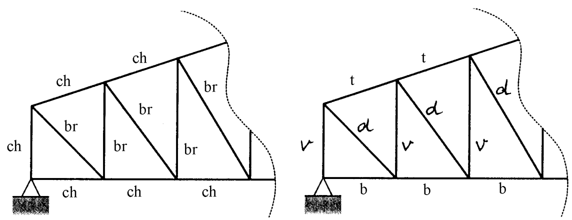


Figure 9.7 The members along the chord or circumference of the truss are chord members (ch), the others are known as bracing members (br). Chord members can be divided into top chord members (t) and bottom chord members (b), while for bracing members we distinguish between verticals (v) and diagonals (d). A vertical chord member is also referred to as a vertical.

One also often assumes that the *dead weight* of a truss applies at the joints. The total dead weight F_{dw} of a truss member is split up into two equal forces in both adjacent joints (see Figure 9.6). This is a rough model of reality, but since the dead weight is generally small with respect to the other loads that the truss has to bear, the deviations that occur are relatively small.

9.1.3 Nomenclature members and truss types

Figure 9.7 shows part of a truss. The letters show the names of the members in the truss. The members along the chord or perimeter of the truss are called *chord members* (ch), the others are referred to as *bracing members* (br). Chord members can be divided into *top chord members* (t) and bottom chord members (b). For bracing members, we distinguish between *verticals* (v) and *diagonals* (d), depending on whether the members are positioned vertically or obliquely. Vertical chord members are also referred to as verticals. In certain cases, one distinguishes between *rising diagonals* (rd) and *falling diagonals* (fd), depending on their position, seen from the perspective of the nearest support, towards the centre (see Figure 9.8).

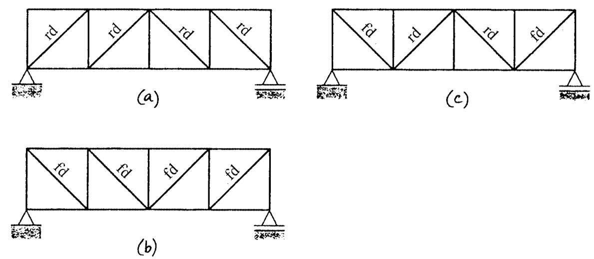


Figure 9.8 Trusses with (a) rising diagonals (rd), (b) falling diagonals (fd) and (c) alternating falling and rising diagonals.

In the following you will find a number of types of trusses. Several trusses have been named after their designer or after the region where they were developed. We will not discuss this nomenclature further, which differs per language area. We will also not address the benefits and disadvantage of the various trusses. We will briefly discuss only the motive for choosing rising or falling diagonals.

Figure 9.9 shows a number of trusses that are commonly used in roofs.

In a *Belgian truss* (a) the bracing consists of members at right angles to the top chord, and diagonals. In an *English truss* (*Howe truss*) (b) the bracing consists of verticals and diagonals. Trusses (c) and (d) have gently sloping top chords and alternating rising and falling diagonals. Truss (c) is suitable for a transom window. In a *Polonceau truss* (*Fink truss*) (e) one can recognise a three-hinged truss with a tie rod. Truss (f) is used in *saw tooth roofs*; glass is placed in the sheer sloping roof planes.

The truss in Figure 9.10 can be used in canopies and is therefore also referred to as a *canopy truss*.

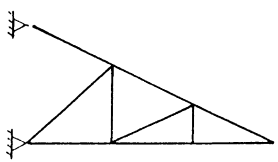


Figure 9.10 A canopy truss.

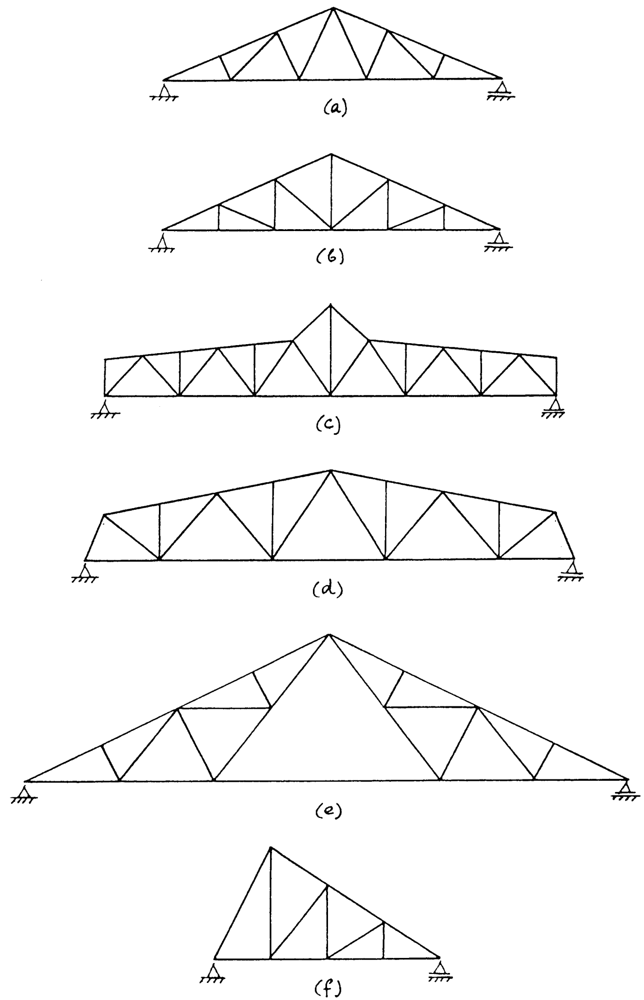


Figure 9.9 Trusses applied in roofs.

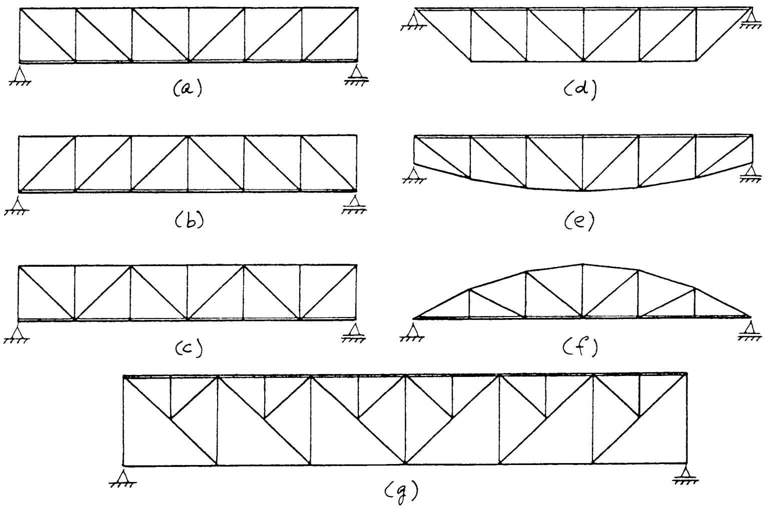


Figure 9.11 Trusses applied in bridges. The bridge deck is shown by means of a double line.

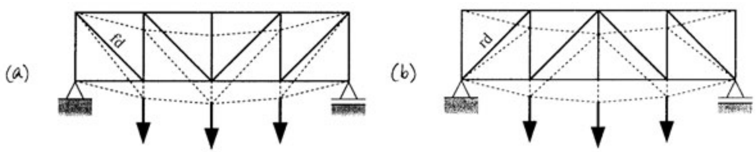


Figure 9.12 From the expected deformation due to a full load, we can deduce that (a) falling diagonals will extend and be subject to tension, and that (b) rising diagonals will shorten and be subject to compression.

You will find the trusses in Figure 9.11 in bridges. The deck is shown by means of a double line. Bridges (a), (b), (c) and (f) have a *lower deck*. The other bridges have an *upper deck*.

Since these trusses have the same function as a beam, they are often called *truss beams*. Trusses (a), (b), (c) and (g) are known as *parallel truss beams*, as a result of their parallel top and bottom chords. If the end verticals are omitted from a parallel beam, as in truss (d), the truss is referred to as a *trapezoidal truss beam*.

Truss beam (e) has a *curved bottom chord*. Truss beam (f) has a *curved top chord*. In a curved chord, the joints of the chord are located on a curve. The chord members are straight. The curve is often a parabola. This is known as a *parabolic truss beam* if the points of support are also part of the parabola, as in truss (f). If this is not the case, as in truss (e), it is called a *half-parabolic truss beam*.

Truss (g) is found in large spans. By creating an auxiliary truss within the main truss, additional points of support are created for the bridge deck, allowing the structure to be lighter.

Trusses (a) and (d) to (g) have falling diagonals, truss (b) has rising diagonals, and in truss (c) the diagonals alternate between falling and rising.

Due to the dead load, falling diagonals are loaded by tensile forces, and rising diagonals are loaded by compressive forces. This is shown in Figure 9.12 in a general sketch of the expected deformation of the truss beam subject to full loading. The falling diagonals in case (a) extend and are loaded by tensile forces. The rising diagonals in case (b) shorten and are loaded by compressive forces.

In steel trusses, falling diagonals (tension diagonals) are used most frequently, as (usually slender) steel members subject to compression run the risk of buckling. Preferably apply them as tension members.

In contrast, rising diagonals (compression diagonals) are most often used in wooden trusses, as in general a wooden joint is more suitable to transfer compressive forces rather than tensile forces.

An example close to home is the simple *garden gate* in Figure 9.13, with (a) a steel version (falling diagonal) and (b) a wooden version (rising diagonal). The wooden *lock-gate* in Figure 9.14 is another example. The wooden *diagonal strut* is a rising diagonal and acts as a compression member under influence of the dead weight of the gate. The wooden planking is facing the same way as the diagonal strut. The steel falling diagonal is a *tension bar*.

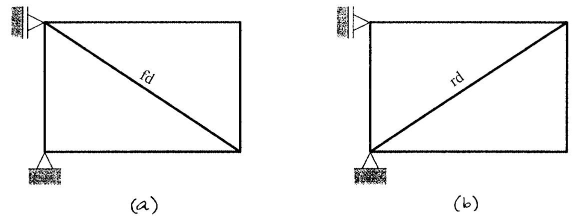


Figure 9.13 Model of (a) a steel garden gate with falling diagonal (tension diagonal) and (b) a wooden garden gate with rising diagonal (compression diagonal).

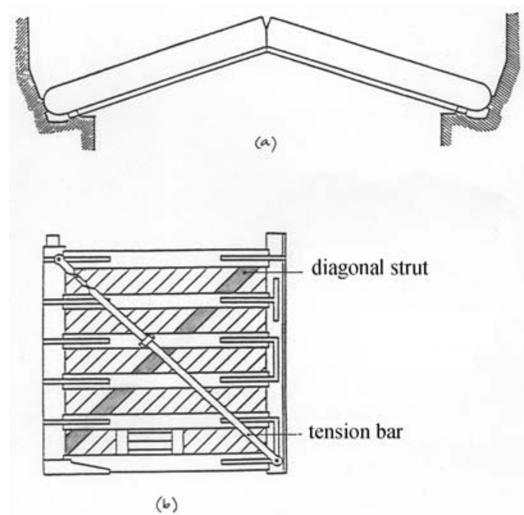


Figure 9.14 (a) The mitre gates of a simple navigation lock in closed position. (b) Interior view of the left-hand lock door, as found in older wooden mitre gates, with a diagonal strutt (compression diagonal) and a steel tension bar (tension diagonal).

9.1.4 Labelling joints and members

The joints in a truss are numbered or lettered (see Figure 9.15). The numbers or letters used to indicate the joints can be used as an index. For example, x_4; y_4 gives the x and y coordinates of joint 4, and $F_{x;\mathrm{C}}$ is the x component of force F on joint C. It is customary to use the joint label as *sub-index*.

Members are always numbered. The member numbers are often placed between brackets. For quantities that relate to a particular member, the member number is used as an *upper index*. The length ℓ of member (2) is recorded as $\ell^{(2)}$, and $N^{(1)}$ is the normal force in member (1).

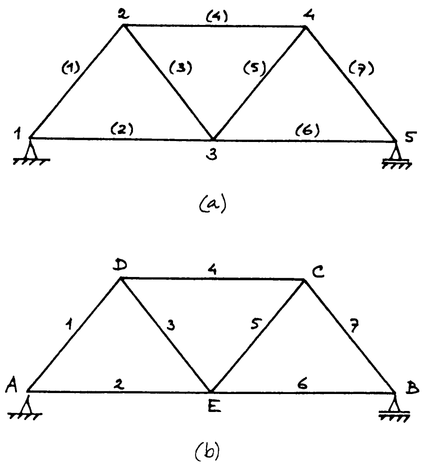


Figure 9.15 Joint and member numbering in (a) computer calculations and (b) manual calculations.

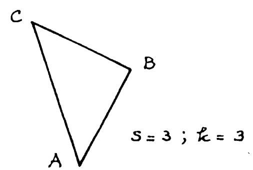


Figure 9.16 A triangle is the basic form of a simple or self-contained truss, defined as a truss that can retain its shape.

In computer calculations, it is customary to use the labelling in Figure 9.15a; computer programs can deal better with numbers than with letters. In manual calculations, the labelling in Figure 9.15b is used most. Occasionally, the brackets about the member numbers are omitted. Their context must then show whether ℓ^2 means "*the square of* ℓ", or "*the length of member* 2". If there is a chance of confusion, the member number has to remain between brackets.

9.2 Kinematically/statically (in)determinate trusses

In this section, we discuss the relationship between the number of members and joints in a simple or self-contained truss and a compound truss respectively. Subsequently a systematic procedure will be introduced to calculate the degree of kinematic/static (in)determinacy of a truss and the relationship between the number of support reactions, members, and joints.

9.2.1 Simple and compound trusses

A *simple* or *self-contained*[1] *truss* is defined as a truss that retains its shape. The basic element of a simple truss is the triangle with $s = 3$ members and $k = 3$ joints, like triangle ABC in Figure 9.16.

Unlike a triangle, a (hinged) quadrangle cannot retain its shape.[2] Figure 9.17 shows the displacements with respect to AG for quadrangle ABEG. One can imagine that BE is connected with AG via the two-force members AB and EG. The displacement of BE with respect to AG consists of a rotation about $\mathrm{RC}^{(\mathrm{BE})}$, the *centre of rotation* of BE, that coincides with

1 The concept *self-contained* was covered earlier in Section 4.5.1.

2 The open circles for hinged joints are consistently omitted (see Section 9.1.1).

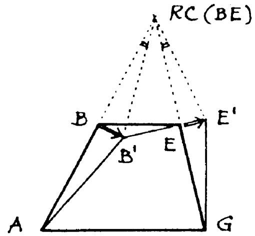


Figure 9.17 A hinged quadrangle cannot retain its shape.

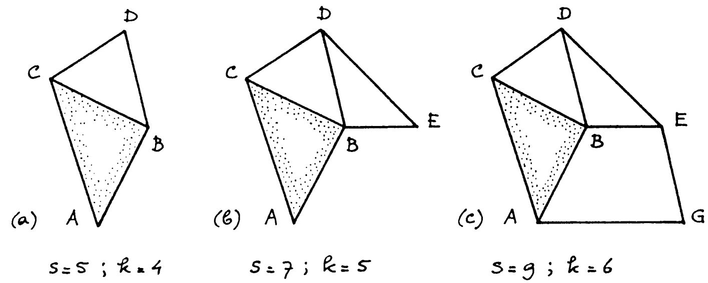


Figure 9.18 Based on a simple triangle, we can repeatedly create a new joint by adding two members.

the intersection of two-force members AB and EG. See also Section 4.5.1 and Figure 4.38c. When looking at the deformed quadrangle once more, it is important to note that the displacements are depicted large in the figure as compared to the length of the members.

The simplest way of constructing self-contained trusses is to start with a triangle, and, as in Figure 9.18, repeatedly create a new joint with two members. To retain its shape the truss does not have to consist only of triangles. For example, the quadrangle ABEG from Figure 9.17 is found again in the self-contained truss in Figure 9.18c.

Figure 9.19 shows a number of trusses that were constructed using this method. Trusses (a) and (b) consist entirely of triangles and are clearly self-contained. This is harder to determine for trusses (c) and (d) as they do not consist entirely of triangles.[1] They retain their shape however as they can be constructed from the dark triangle in the middle by repeatedly creating a new joint by adding two members to two existing joints.

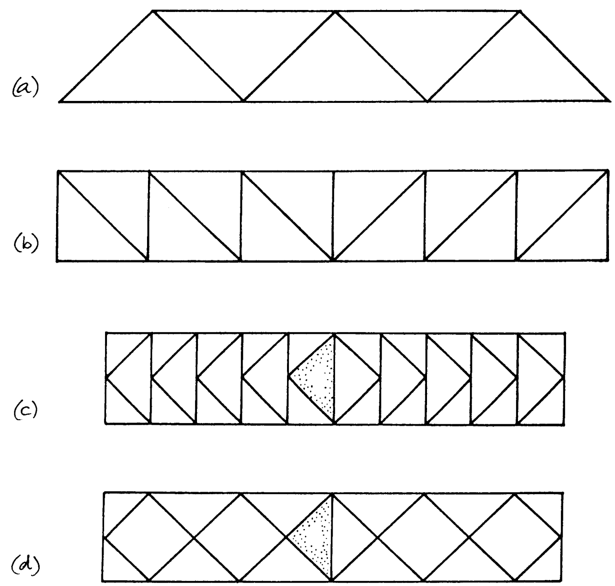


Figure 9.19 Simple trusses constructed in the way shown in Figure 9.18. In (c) and (d) we can start with the dark triangle in the middle. For all these trusses it holds that $s = 2k - 3$.

[1] The eight triangles in truss (c) are not “real” triangles but quadrangles.

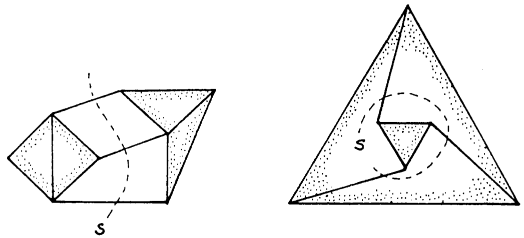


Figure 9.20 Simple trusses with a more complicated structure. The two dark self-contained parts are connected by three members. The formula $s = 2k - 3$ is also applicable to these trusses.

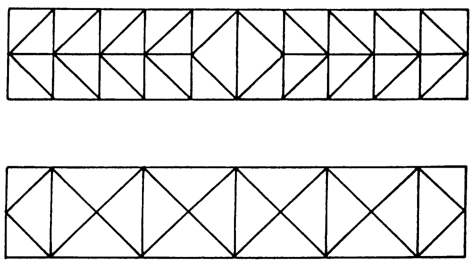


Figure 9.21 Simple trusses that contain more members than needed for being self-contained. For these trusses, it holds that $s > 2k - 3$.

The following relationship holds between the number of members s and the number of joints k for a truss created in the way described above:

$$s = 2k - 3.$$

This can be derived as follows. Three members are needed for the first three joints in the truss, which forms the first triangle. For the remaining $(k - 3)$ joints $2(k - 3)$ members are needed. The total number of members s is therefore:

$$s = 3 + 2(k - 3) = 2k - 3.$$

Figure 9.20 shows two examples of simple trusses that cannot be constructed as shown in Figure 9.18. They clearly have a more complicated structure. If we look more closely, we notice that the structures consist of two dark coloured simple trusses of the type described earlier, which are connected to one another by three members. The structures retain their shape only when the three members do not intersect at one point, and neither are parallel. In the figure, a section s has been introduced across the three members. The same formula $s = 2k - 3$ also applies to these more complicated trusses.

The formula $s = 2k - 3$ is a minimum condition for a truss that will retain its shape. By adding additional members to a simple truss, without creating new joints, the structure remains self-contained. In this way, the trusses in Figure 9.21 were created by adding additional members to trusses (c) and (d) in Figure 9.19. The trusses are still self-contained, but now the number of members is

$$s > 2k - 3$$

One would imagine that a truss is always self-contained if the number of members s is at least equal to $2k - 3$. This is a misconception, however, as is shown for the truss in Figure 9.22 with $s = 16$ and $k = 9$. This truss consists of two self-contained parts, which could both lose a member without losing their shape. Both parts are connected by means of a hinge, and can move with respect to one another. The structure is therefore *not self-contained*, although the number of members $s = 16$ is greater than $2k - 3 = 15$.

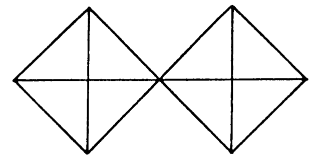


Figure 9.22 A truss that cannot retain its shape is called a compound truss.

Hereafter, a *truss* that cannot retain its shape is referred to as a *compound truss*.[1]

The formula $s = 2k - 3$ is clearly not a good criterion for a truss that will retain its shape. One can say that for each self-contained truss, the following relationship must hold:

$$s \geq 2k - 3.$$

The reverse is not true, however. Not every truss with $s \geq 2k - 3$ is self-contained. This is demonstrated by the counterexample in Figure 9.22. The formula does not indicate the *functionalism* for which the various members were introduced. The formula $s \geq 2k - 3$ is a *necessary* although *insufficient* condition for a truss that will retain its shape.

To summarise:

$s < 2k - 3$ The truss is a compound truss (the truss cannot retain its form).

$s \geq 2k - 3$ Necessary condition for a self-contained truss, but not a sufficient condition. As a result of the application of inefficient members, the truss may still not be capable of retaining its shape. One can be sure only when the truss has been investigated from joint to joint.

[1] The literature often defines compound trusses as those of the type in Figure 9.20, but sometimes also those in Figure 9.22. Here, as in Section 4.5.3, a compound truss is defined as one that, when isolated from its supports, is not capable of retaining its shape.

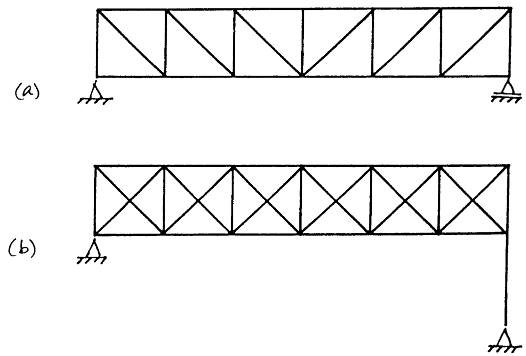


Figure 9.23 (At least) three support reactions are needed for an immovable support of a simple truss. Here they are provided by (a) a hinged and roller support and (b) a hinged and bar support.

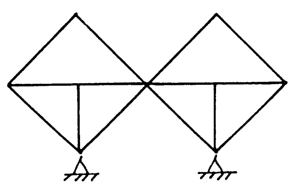


Figure 9.24 More than three support reactions are needed for an immovable support of a compound truss, as the internal degrees of freedom also have to be eliminated. In this case, four support reactions are required, provided by two hinged supports.

If the truss retains its shape, the following cases can be distinguished:

$s = 2k - 3$ The truss needs all the members to retain its shape.

$s > 2k - 3$ The truss can miss $s - (2k - 3)$ members without losing its capability to retain its shape. These members cannot be selected arbitrarily; they are determined by the way the truss is assembled.

9.2.2 Determining kinematic/static (in)determinacy

If a truss is supported so that is has no possibility of moving, the truss is defined as *immovable* or as *kinematically determinate*. This type of truss can resist all types of load. If a truss is to be kinematically determinate, it needs at least as many support reactions as degrees of freedom; one degree of freedom is removed for each support reaction (interaction force between truss and the immovable environment).

A simple truss may be considered as a rigid body. Since (in a plane) it has three degrees of freedom (one rotation and the two components of a translation), at least three independent support reactions are needed for immovability. For example by means of a hinged support together with a roller support or bar support, as shown in Figure 9.23.

Compound trusses can be seen as systems of rigid bodies that have a certain degree of freedom with respect to one another. The possible movements with respect to one another are known as the *internal degrees of freedom*. The immovability of a compound truss always needs more than three support reactions, as the internal degrees of freedom also have to be eliminated. In this way, the truss in Figure 9.24 is not shape-retaining in itself, as the two constituent parts can rotate with respect to one another. The two hinged supports ensure the kinematic determinacy of the truss.

In the examples, bar supports, roller supports, and hinged supports have been used. It should be clear that fixed supports are not used in trusses.

Instead of looking at the degrees of freedom for simple or compound trusses, we can also determine how many support reactions are needed to keep the truss in equilibrium under every imaginable load. This procedure was explained in Section 4.5.3 for an arbitrary structure. The answers are somewhat easier to determine for trusses as here the members can transfer only tensile and compressive forces between the joints. These forces are called normal forces. Normal forces are represented by means of a capital letter N. According to the *sign convention* N is positive for a tensile force, and negative for a compressive force.

The truss in Figure 9.25a is supported at A on a hinge and at B on a roller, and is loaded by the forces F_1; F_2; F_3. In Figure 9.25b, all the joints in the truss have been isolated. It has been assumed that all the member forces N are positive (all the members transfer tensile forces, and therefore pull at the joints).

An arbitrary truss has k joints, s members and r support reactions. The unknown force quantities in the truss are then the r support reactions and the s member forces. In total, there are therefore $(r + s)$ unknowns in the truss.

The equilibrium can be investigated for each joint. The conditions for moment equilibrium are automatically met as all the forces intersect at the joint. All that remains is the force equilibrium. Two equations can be created per joint. These contain both known forces (the loads) and unknown forces (member forces and support reactions). With k joints, there are therefore $2k$ equilibrium equations.

Let n be the difference between the number of unknown forces and the number of available equilibrium equations:

$$n = r + s - 2k.$$

If $n < 0$, there are more equations than unknowns. It is always possible to choose the (arbitrary) load in such a way that a number of the (redundant)

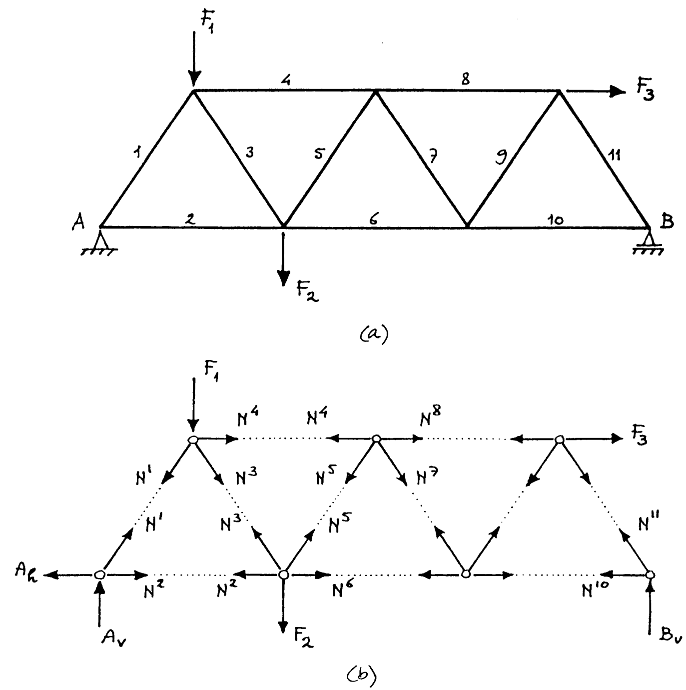


Figure 9.25 (a) A truss for which in (b) all the joints have been isolated. It has been assumed that the normal force N in each member is a tensile force. If so, all members pull at the joints.

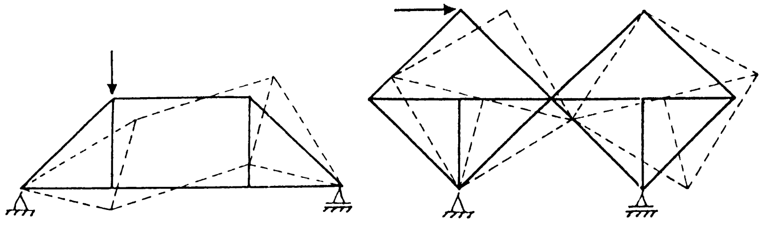


Figure 9.26 Kinematically indeterminate trusses or mechanisms. (a) $n = -1$; a diagonal member is missing in the middle. (b) $n = -1$; the hinged and roller support are insufficient to eliminate all possible movement of the compound truss.

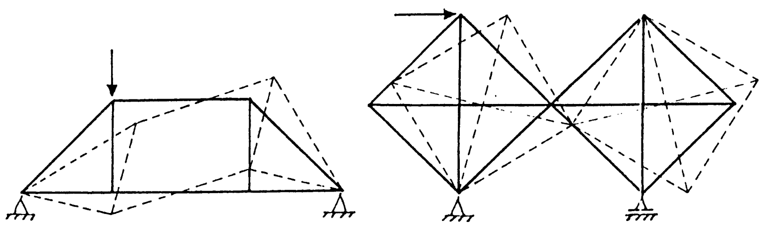


Figure 9.27 Kinematically indeterminate trusses or mechanisms with $n \geq 0$: (a) $n = 0$ and (b) $n = 1$.

equations become inconsistent. This means that, with that load, the equilibrium conditions cannot be met at all the joints. The truss is a mechanism and is kinematically indeterminate (not immovable). Examples are shown in Figure 9.26.

In Figure 9.26a (with $r = 3$, $s = 8$ and $k = 6$, and therefore $n = -1$) the kinematic indeterminacy results from the missing diagonal member in the centre field. In Figure 9.26b (with $r = 3$, $s = 14$ and $k = 9$, and therefore $n = -1$), the method of support is inadequate to remove all the degrees of freedom of the compound truss.

From the above, we can conclude that $n \geq 0$ is a *necessary condition* for kinematic determinacy. Since the value of n is the result of a calculation in which the *functionalism* of the members and supports present is not taken into account, this necessary condition is an *insufficient condition*. Even when $n \geq 0$, there is always the possibility that the structure is kinematically indeterminate. Examples of this are shown in Figure 9.27.

The structure in Figure 9.27a (with $r = 4$, $s = 8$, $k = 6$, and therefore $n = 0$) is the same as the structure in Figure 9.26a, except that the roller support is replaced by a hinged support. Since, for motion as a mechanism, the roller in Figure 9.26a remains in place, this change makes no difference whatsoever – the truss remains kinematically indeterminate.

In Figure 9.27b (with $r = 3$, $s = 16$, $k = 9$, and therefore $n = 1$) the support of the compound truss is equally inadequate as in Figure 9.26b. The only difference is that the two constituent parts now contain more members than required for retaining their shape.

The kinematic determinacy of a truss cannot be assessed based on a calculation alone; one always has to take the construction of the truss into account.

With $n = r + s - 2k$ the following is true for a truss:

$n < 0$ *The truss is kinematically indeterminate*. This is also known as a mechanism.

$n \geq 0$ Necessary but insufficient condition for a *kinematically determinate truss*. As a result of non-effective members and/or supports, the truss can still be kinematically indeterminate and a mechanism.

In kinematically determinate trusses, $n \geq 0$, and as in Section 4.5.3 we can distinguish the following cases:

$n = 0$ *The truss is statically determinate*.
The number of unknowns is equal to the number of available equilibrium equations. All unknowns (member forces and support reactions) can be derived directly from the equilibrium.

$n > 0$ *The truss is statically indeterminate*.
There are more unknowns than equilibrium equations. One or more of the member forces and/or support reactions cannot be determined directly from the equilibrium. In principle, there is an infinite number of solutions that satisfy the equilibrium conditions (the solution is undetermined). The correct solution can be found by taking into account the deformation behaviour of the structure. The surplus of unknowns, n, is known as the *degree of static indeterminacy*.

Figure 9.28 provides examples of statically determinate trusses.

The shape-retaining truss ABCD in Figure 9.28a is immovable supported by a hinge at A and a bar at B. The hinged support provides two support reactions, and the bar support provides one, so that $r = 3$. With $s = 25$ and $k = 14$, $n = 0$. The truss is therefore statically determinate. If the bar support is considered as one of the truss members, B′ has to be seen as a hinged support. In that case, $r = 4$, $s = 26$ and $k = 15$, and again $n = 0$.

In the simple truss in Figure 9.28b, the diagonal members cross one another. The truss is immovable, supported on a roller and by a hinge. Here $r = 3$, $s = 13$ and $k = 8$, so that $n = 0$. The truss is therefore statically determinate.

The compound truss in Figure 9.28c is also immovable supported. With $r = 4$, $s = 14$ and $k = 9$, $n = 0$. The truss is statically determinate.

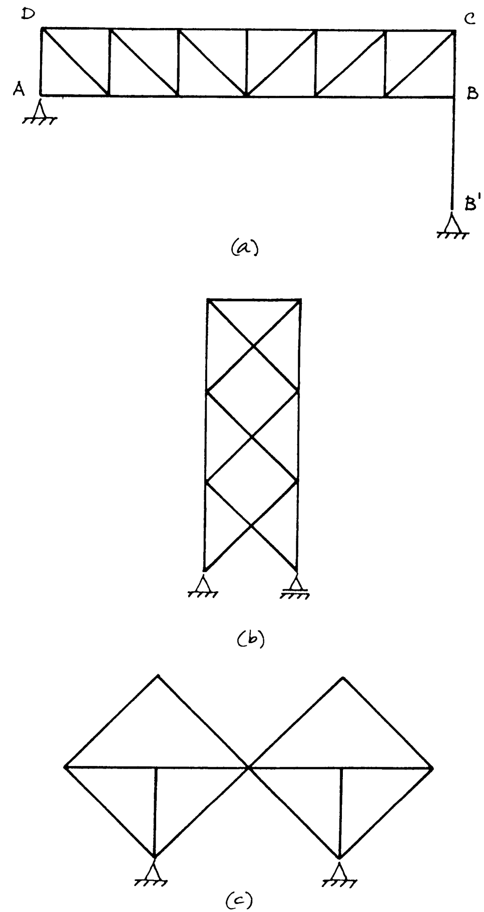


Figure 9.28 Statically determinate trusses.

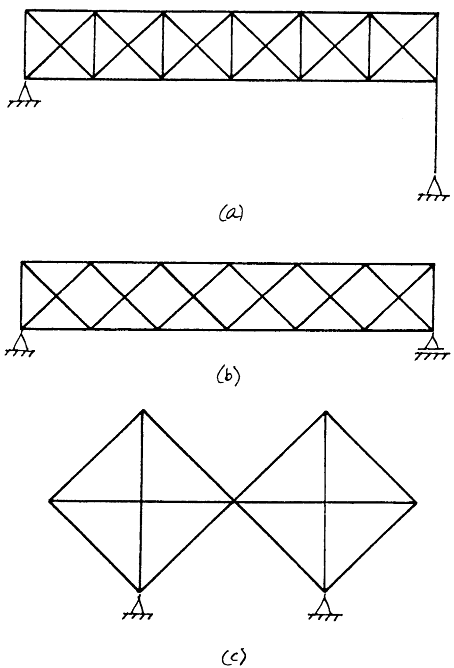


Figure 9.29abc Statically indeterminate trusses. Trusses (a) to (c) are supported with static determinacy. These trusses are also said to be externally statically determinate and internally statically indeterminate.

Figure 9.29 provides examples of statically indeterminate trusses. All the trusses are kinematically determinate. The degree of static indeterminacy can be determined with

$$n = r + s - 2k.$$

The truss in Figure 9.29a, in which the diagonal members cross one another, has a hinged support and a bar support. With $r = 3$, $s = 31$, $k = 14$, one finds $n = 6$. The truss is six-fold statically indeterminate. If we compare the truss with the statically determinate structure in Figure 9.28a, we see that the truss has 6 redundant diagonal members.

For the truss in Figure 9.29b, with crossing diagonals, $r = 3$, $s = 26$, $k = 14$ and so $n = 1$. The truss is therefore statically indeterminate to the first degree.

For the compound truss in Figure 9.29c, $r = 4$, $s = 16$ and $k = 9$, so that $n = 2$. A member could be omitted in each of two self-contained parts (see also Figure 9.28c).

The truss in Figure 9.29d is statically indeterminate to the first degree, with $r = 4$, $s = 19$, $k = 11$ and so $n = 1$. The structure can be made statically determinate by, for example, removing one of the roller supports. You could also remove an arbitrary top or bottom chord member.

For the truss in Figure 9.29e, $r = 6$, $s = 13$ and $k = 8$, so that $n = 3$. The truss is statically indeterminate to the third degree. The simple truss has three redundant support reactions and/or members.

In statically determinate trusses, all the force members and support reactions can be determined directly from the equilibrium. This is not possible for statically indeterminate trusses. Sometimes, for statically indeterminate trusses, it is possible to find all the support reactions from the equilibrium equations, but not all the member forces. Examples of this type of truss are shown in (a) to (c) in Figure 9.29. The support of these trusses is statically determinate. Their static indeterminacy is caused by redundant members in their self-contained parts. These types of trusses are also known as *externally statically determinate* and *internally statically indeterminate*.

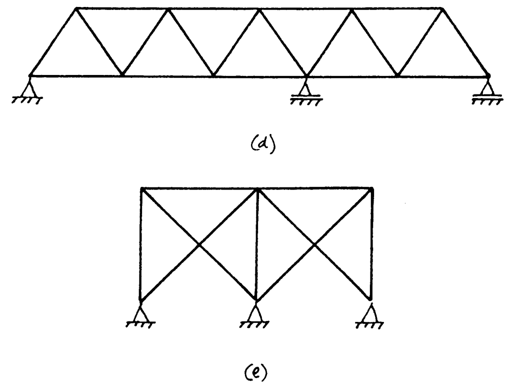


Figure 9.29de Statically indeterminate trusses.

9.3 Determining member forces

There are various methods for calculating member forces in statically determinate trusses. We will look at two:

- the method of sections;
- the method of joints.

In the *method of sections*, one introduces a suitable section across the truss and calculates the member forces from the equilibrium of one of the isolated parts. In the *method of joints*, we consistently determine the member forces from the equilibrium of the joints.

9.3.1 Method of sections

In the *method of sections*, the member forces in a (statically determinate) truss are determined by introducing a *section* and investigating at the equilibrium of one of the isolated parts. Since there are only three equilibrium equations available, you have to select a section such that there are no more than three unknowns. In general, the support reactions have to be determined previously. The method is demonstrated using a number of examples.

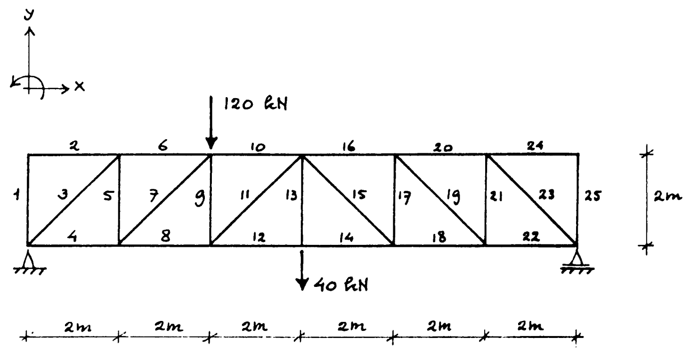


Figure 9.30 Truss, with parallel top and bottom chord, for which the forces in members 6 to 9 and in member 13 have to be calculated using the method of sections.

Example 1

The first example relates to the truss beam in Figure 9.30, with parallel top chord and bottom chord. The load consists of the two vertical forces shown in the figure of respectively 120 kN and 40 kN.

Question:

Determine the forces in the members 6 to 9 and in member 13, with the correct signs for tension and compression. In the calculation, use the

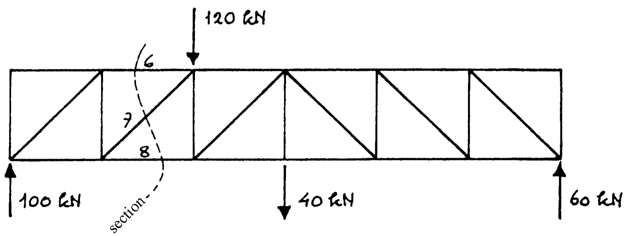


Figure 9.31 The isolated truss with support reactions. To calculate the forces in members 6, 7 and 8, a section is introduced across these members in the truss.

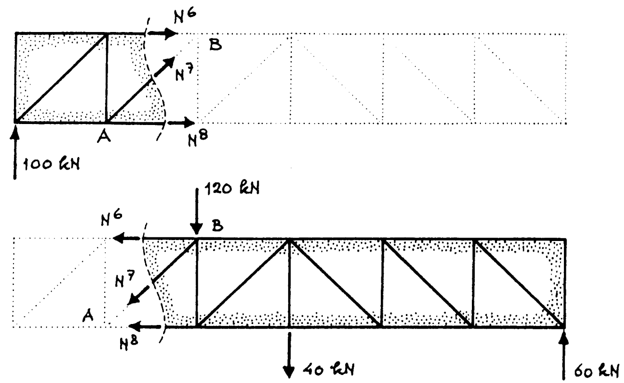


Figure 9.32 The isolated parts to the left and right of the section across members 6, 7 and 8. The interaction forces, the normal forces N, are shown as tensile forces because tensile forces are by definition positive.

coordinate system shown.

Solution:
In Figure 9.31, the truss has been isolated and the support reactions are shown. For calculating the forces in members 6, 7 and 8, we introduce a section across these members.

In Figure 9.32, the parts to the left and to the right of the section have been isolated. The as yet unknown member forces N^6, N^7 and N^8 are introduced as tensile forces. Here we use the sign convention that the normal force in a member is positive when it is a tensile force. If the member has to transfer a compressive force, this will become clear later through a negative value for the normal force N.

The normal force N^6 in member 6 is most easily determined by looking at the moment equilibrium of the left-hand part about intersection A of members 7 and 8:

$$\sum T_z|\text{A} = -(100\text{ kN})(2\text{ m}) - N^6 \times (2\text{ m}) = 0 \Rightarrow N^6 = -100\text{ kN}.$$

The minus sign shows that member 6 is a *compression member*. The 100 kN force is therefore acting opposite to the direction shown in Figure 9.32.

Instead of the left-hand part, we can also look at the right-hand part. From the moment equilibrium about A of the right-hand part, it follows that

$$\sum T_z|\text{A} = -(120\text{ kN})(2\text{ m}) - (40\text{ kN})(4\text{ m}) + (60\text{ kN})(10\text{ m}) + \\ + N^6 \times (2\text{ m}) = 0.$$

Of course, $N^6 = -100$ kN also here, except that it took a little more work to find the answer as more forces are acting on the right-hand part than on the left-hand part.

When calculating the member forces, it does not make a difference whether you look at the equilibrium on the left-hand side or the right-hand side of the section. It is sensible to choose the part that offers the simplest calculation.

The force in member 7 is most easily determined from the vertical force equilibrium for the part to the left of the section:

$$\sum F_y = (100\text{ kN}) + \tfrac{1}{2}N^7\sqrt{2} = 0 \Rightarrow N^7 = -100\sqrt{2}\text{ kN}.$$

Diagonal member 7 is also a *compression member*. Calculating this member force is easy as the parallel top and bottom chords members do not have a vertical component.

Member force N^8 is most easily determined from the moment equilibrium of the left-hand part about the intersection of members 6 and 7:

$$\sum T_z|\text{B} = -(100\text{ kN})(4\text{ m}) + N^8 \times (2\text{ m}) = 0 \Rightarrow N^8 = +200\text{ kN}.$$

Bottom chord member 8 is a *tension member*.

To check the above, we calculate whether there is horizontal force equilibrium in the left-hand part:

$$\begin{aligned}\sum F_x &= N^6 + \tfrac{1}{2}\sqrt{2}\times N^7 + N^8\\ &= (-100\text{ kN}) + \tfrac{1}{2}\sqrt{2}\times(-100\sqrt{2}\text{ kN}) + (200\text{ kN}) = 0.\end{aligned}$$

With the values found for N^6, N^7 and N^8, the conditions for horizontal force equilibrium are indeed satisfied.

Please note the parallel with calculating the support reaction for a structure on three bar supports, as in Examples 2 and 3 in Section 5.1.

The forces in the other members of the truss can be determined in the same way. For example, we find the force in member 9 by introducing a section across members 8, 9 and 10, as shown in Figure 9.33. From the vertical

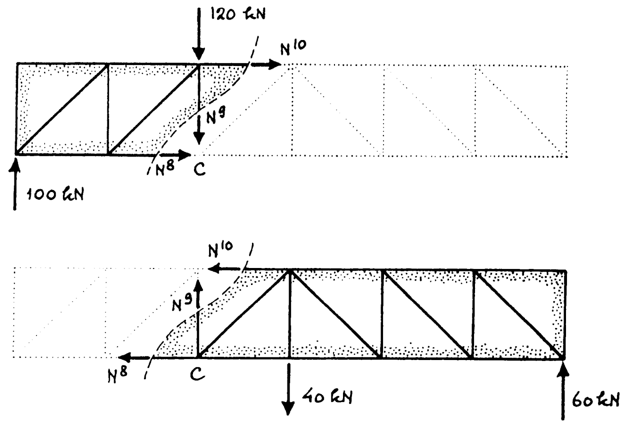


Figure 9.33 The section for calculating the forces in members 8, 9 and 10.

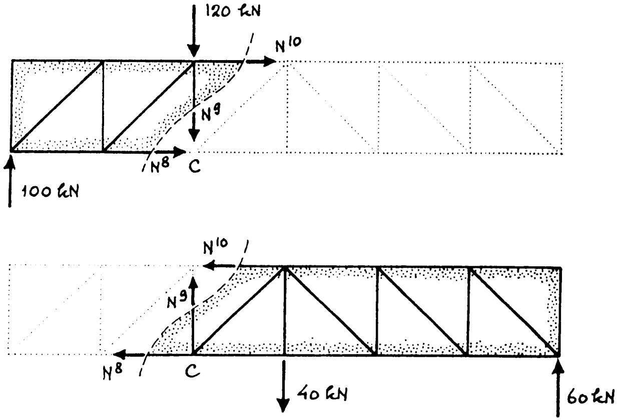


Figure 9.33 The section for calculating the forces in members 8, 9 and 10.

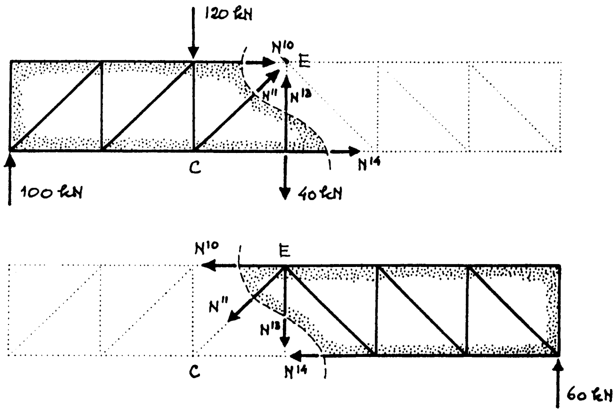


Figure 9.34 The section across member 13 cuts four members, one too many to be able to determine all the member forces from the equilibrium. With this section, we can only find N^{14} from the moment equilibrium about E.

force equilibrium for the left-hand part it follows that

$$\sum F_y = (100\text{ kN}) - (120\text{ kN}) - N^9 = 0 \Rightarrow N^9 = -20\text{ kN}.$$

Member 9 is a compression member.

When determining the force in member 13, the problem arises that a section across member 13 cuts more than three members. The section in Figure 9.34. for example, cuts through members 10, 11, 13 and 14, which is one too many to be able to determine all the forces from the equilibrium. If one of the member forces N^{10} of N^{11} is known, then it is possible to determine the other three. We therefore look at a second section to first determine one of the forces N^{10} or N^{11}.

From the moment equilibrium about C of one of the isolated parts in Figure 9.33 we find

$$N^{10} = -200\text{ kN}.$$

Using this information, we find from the moment equilibrium about C of one of the isolated parts in Figure 9.34 (here we select the right-hand part) the force in member 13 is

$$\sum T_z|\text{C} = -N^{10} \times (2\text{ m}) + (60\text{ kN})(8\text{ m}) - N^{13} \times (2\text{ m}) = 0$$
$$\Rightarrow N^{13} = +40\text{ kN}.$$

By chance, we can also find the force in member 13 using an easier method, namely by using the section in Figure 9.35 across the members 12, 13 and 14. N^{13} is found from the vertical force equilibrium of one of the parts. Here, we actually determine the force N^{13} from the force equilibrium of joint D, where three members come together, of which two in a direct line.

Note that it is not possible to determine the section forces N^{12} and N^{14} from

Table 9.1 Member forces Example 1.

Mem. no. i	N^i (kN)
1	0
2	0
3	$-100\sqrt{2}$
4	$+100$
5	$+100$
6	-100
7	$-100\sqrt{2}$
8	$+200$
9	-20
10	-200
11	$+20\sqrt{2}$
12	$+180$
13	$+40$
14	$+180$
15	$-60\sqrt{2}$
16	-120
17	$+60$
18	$+120$
19	$-60\sqrt{2}$
20	-60
21	$+60$
22	$+60$
23	$-60\sqrt{2}$
24	0
25	0

the equilibrium of one of the parts isolated in Figure 9.35. It is possible to determine only that $N^{12} = N^{14}$ from the horizontal force equilibrium, but we cannot determine their magnitude.

Table 9.1 provides a summary of all the member forces in the truss.

Example 2

The second example relates to the truss in Figure 9.36, with non-parallel top and bottom chords. The load consists of a single vertical force of 120 kN.

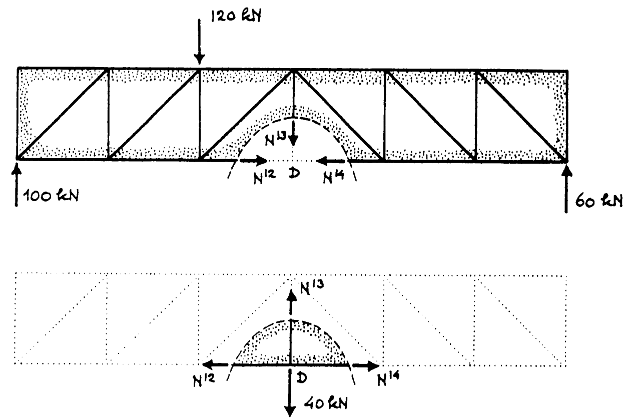


Figure 9.35 With this section across three members, we actually isolate joint D. N^3 follows directly from the vertical force equilibrium of the joint. We do not need to know N^{12} and N^{14} to do so.

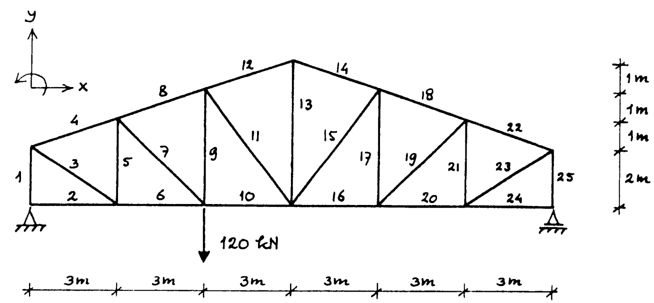


Figure 9.36 Truss with non-parallel top and bottom chord.

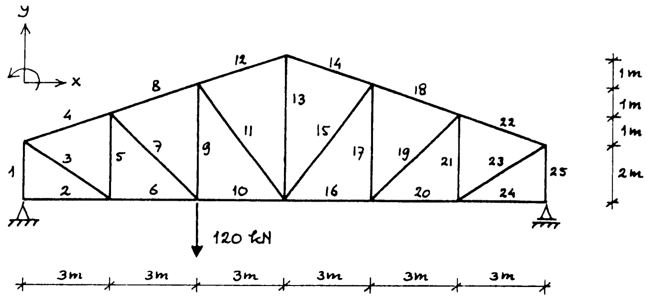


Figure 9.36 Truss with non-parallel top and bottom chord.

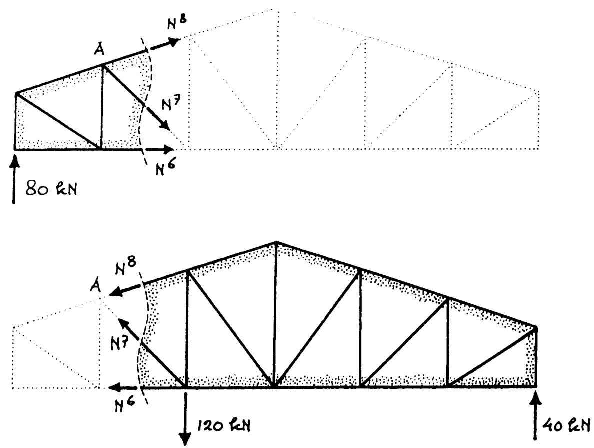


Figure 9.37 Section for calculating the forces in members 6, 7 and 8.

Question:
Determine the forces in members 6 to 9 and in member 13, with the correct sign for tension and compression. Use the coordinate system given.

Solution:
We first determine the support reactions. For the left-hand and right-hand support reactions, we find 80 and 40 kN respectively, both vertically and directed upwards.

For determining the three unknown member forces N^6, N^7 and N^8, a section has been introduced across members 6, 7, and 8 in Figure 9.37, and the parts on both sides of the section have been isolated. The member forces N^6, N^7 and N^8 follow from the equilibrium of one of the parts to the left or right of the section. The force N^6 is most easily determined. This follows directly from the moment equilibrium about intersection A of members 7 and 8. For the left-hand part, we find

$$\sum T_z|\text{A} = -(80\ \text{kN})(3\ \text{m}) + N^6 \times (3\ \text{m}) = 0 \Rightarrow N^6 = +80\ \text{kN}.$$

Member 6 is a tension member.

If we use the right-hand section, the equation for the moment equilibrium about A demands a little more effort:

$$\sum T_z|\text{A} = (40\ \text{kN})(15\ \text{m}) - (120\ \text{kN})(3\ \text{m}) - N^6 \times (3\ \text{m}) = 0.$$

Of course, this way round we also find a tensile force of 80 kN in member 6.

The force in member 7 is found from the moment equilibrium about intersection B of the members 6 and 8 (see Figure 9.38a), where only the left-hand part is shown:

$$\sum T_z|B = (80\text{ kN})(6\text{ m}) - N^7 \times (6\sqrt{2}\text{ m}) = 0 \Rightarrow N^7 = +40\sqrt{2}\text{ kN}.$$

Member 7 is a tension member.

If the distance of point B to the line of action of N^7 is difficult to find (not the case here), force N^7 can be shifted along its line of action to a more suitable position.[1] In Figure 9.38b, N^7 has been shifted to point C, where it has been resolved into components. The equation for the moment equilibrium about B now only contains the vertical component of N^7:

$$\sum T_z|B = (80\text{ kN})(6\text{ m}) - \left(\tfrac{1}{2}N^7\sqrt{2}\right)(12\text{ m}) = 0.$$

As found earlier, this gives $N^7 = +40\sqrt{2}$ kN. If N^6 is known, N^7 can also be determined from the moment equilibrium about a point other than B on the line of action of N^8, such as about point D.

For the left-hand part we find (see Figure 9.38b)

$$\sum T_z|D = -(80\text{ kN})(6\text{ m}) + N^6 \times (4\text{ m}) + \left(\tfrac{1}{2}N^7\sqrt{2}\right)(4\text{ m}) = 0.$$

With $N^6 = +80$ kN, we find $N^7 = +40\sqrt{2}$ kN, as expected. This sort of approach can offer benefits if the intersection B of the members 6 and 8 is far away or is difficult to find.

The force in member 8 is found from the moment equilibrium about intersection C of the members 6 and 7. Here, it is useful that force N^8 can be shifted along its line of action to point D (see Figure 9.39). For the left-hand part we find

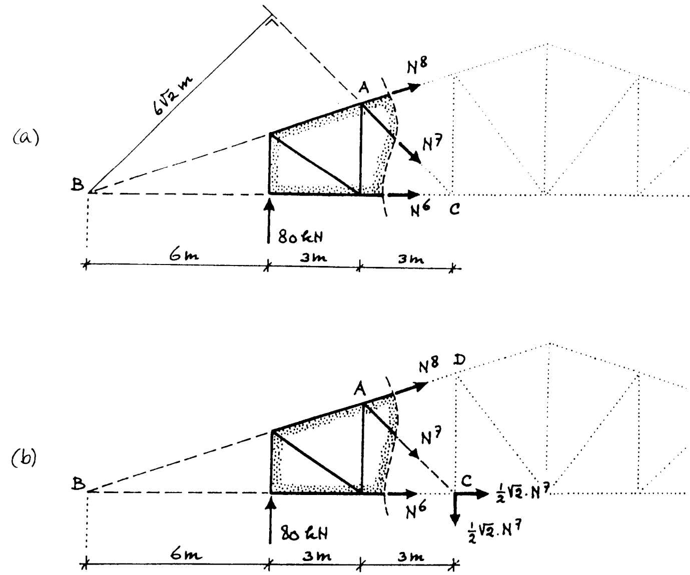


Figure 9.38 N^7 is found from the moment equilibrium about B. (a) Here we have to determine the distance from B to the line of action of N^7. (b) We can also shift N^7 to C and there resolve it into a horizontal and vertical component.

[1] See also Section 3.1.5 with Figure 3.17.

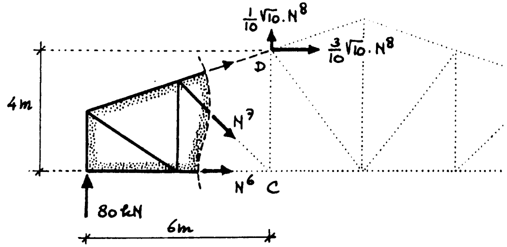


Figure 9.39 N^8 is found from the moment equilibrium about C.

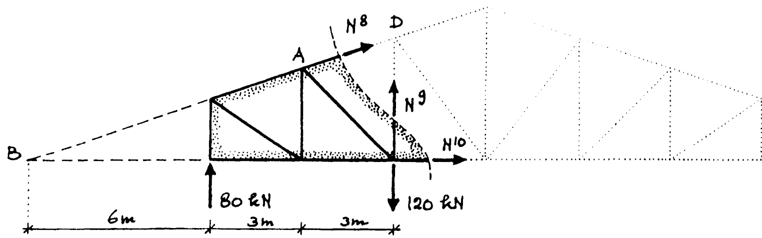


Figure 9.40 Section for calculating the force in member 9. N^9 follows from the moment equilibrium about B. As an interim step, we can also first determine N^{10} from the moment equilibrium about D, and then determine N^9 from the moment equilibrium about A.

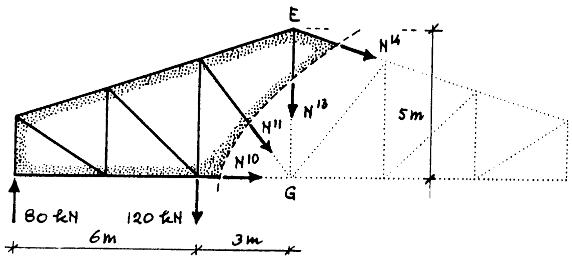


Figure 9.41 This section across member 13 intersects four members. We can only determine N^{14} from the moment equilibrium about G.

$$\sum T_z|\text{C} = -(80\text{ kN})(6\text{ m}) - \left(\tfrac{3}{10}N^8\sqrt{10}\right)(4\text{ m}) = 0$$
$$\Rightarrow N^8 = -40\sqrt{10}\text{ kN}.$$

N^8 is a compressive force.

To verify the three values we have determined for N^6, N^7 and N^8, we can check whether the conditions for force equilibrium are satisfied for the left-hand part. This leads to the following two equations:

$$\sum F_x = \tfrac{3}{10}N^8\sqrt{10} + \tfrac{1}{2}N^7\sqrt{2} + N^6 = 0,$$
$$\sum F_y = \tfrac{1}{10}N^8\sqrt{10} - \tfrac{1}{2}N^7\sqrt{2} + (80\text{ kN}) = 0.$$

The values we found indeed meet these equilibrium conditions.

To determine the force in member 9, a section has been introduced in Figure 9.40 across the members 8, 9 and 10. The force N^9 follows directly from the moment equilibrium about the intersection B of the members 8 and 10. Written out in full, the left-hand part gives

$$\sum T_z|\text{B} = (80\text{ kN})(6\text{ m}) - (120\text{ kN})(12\text{ m}) + N^9 \times (12\text{ m}) = 0$$
$$\Rightarrow N^9 = +80\text{ kN}.$$

Member 9 is a tension member.

If determining the location of point B is complicated (not the case here) you could also first determine N^{10} from the moment equilibrium about D and then derive N^9 from the moment equilibrium about A, for example:

$$\sum T_z|\text{D} = -(80\text{ kN})(6\text{ m}) + N^{10} \times (4\text{ m}) = 0 \Rightarrow N^{10} = +120\text{ kN},$$

$$\sum T_z|A = -(80 \text{ kN})(3 \text{ m}) - (120 \text{ kN})(3 \text{ m}) + N^{10} \times (3 \text{ m}) + N^9 \times (3 \text{ m}) = 0 \Rightarrow N^9 = +80 \text{ kN}.$$

When determining the force in member 13, we again encounter the problem that a section across member 13 cuts four members (see Figure 9.41). In this case, the problem cannot be easily solved from the equilibrium of joint E isolated in Figure 9.42. In order to find member force N^{13} from the force equilibrium of joint E, we first have to know one of the member forces N^{12} or N^{14}.

Here there is a special case, in which we can determine the force in member 14 by means of the section in Figure 9.41, even though it passes over four members. Since, in this section, three of the four unknown member forces intersect at point G, the fourth force, in this case N^{14}, can be derived directly from the moment equilibrium about G. This gives (for the part shown to the left of the section, with force N^{14} moved to point E)

$$\sum T_z|G = -(80 \text{ kN})(9 \text{ m}) + (120 \text{ kN})(3 \text{ m}) - \tfrac{3}{10} N^{14}\sqrt{10} \times (5 \text{ m}) = 0$$

so that $N^{14} = -24\sqrt{10}$ kN. The horizontal force equilibrium of joint E (Figure 9.42) now gives

$$N^{12} = N^{14} = -24\sqrt{10} \text{ kN}.$$

The vertical force equilibrium gives

$$N^{13} = -\tfrac{1}{10}\sqrt{10} \times (N^{12} + N^{14}) = +48 \text{ kN}.$$

Member 13 is therefore a tension member.

Table 9.2 provides a summary of all the member forces in the truss.

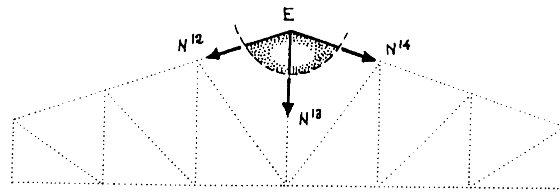


Figure 9.42 Once N^{14} is known we can find N^{13} from the equilibrium of joint E.

Table 9.2 Member forces Example 2.

Mem. no. i	N^i (kN)	Mem. no. i	N^i (kN)
1	−80	14	075.89
2	0	15	+20
3	+96.15	16	+60
4	−84.33	17	−20
5	−53.33	18	−63.25
6	+80	19	+28.28
7	+56.57	20	+40
8	−126.49	21	−26.67
9	+80	22	−42.16
10	+120	23	+48.07
11	−80	24	0
12	−75.89	25	40
13	+48		

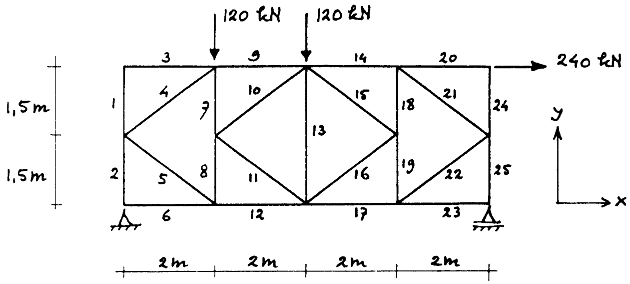


Figure 9.43 A K-truss.

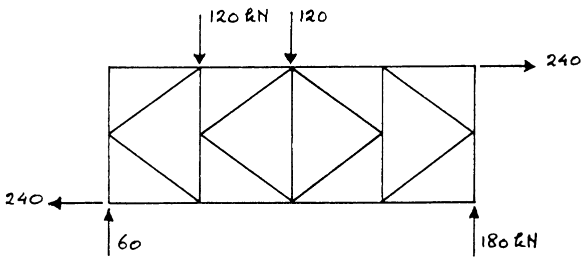


Figure 9.44 The isolated K-truss with support reactions.

Example 3

The third example relates to the somewhat more complicated truss in Figure 9.43, a so-called *K-truss*. This type of truss is sometimes used as *wind bracing* in bridges. Here, the K-truss has four fields and is loaded by two vertical forces of 120 kN and a horizontal force of 240 kN.

Question:
Determine the forces in members 7 to 13, with the correct sign for tension and compression. In the calculations, use the coordinate system given.

Solution:
In Figure 9.44, the truss has been isolated and the support reactions have been shown. Using the method of sections, we now encounter the difficulty that, for most of the members, no section can be found that intersects only three members. Sometimes it is possible to determine a member force if the section passes through more than three members, but in most cases, additional information is required that has to be obtained by selecting a section in a clever way, or by considering a combination of sections. Since the top chord and bottom chord members are easiest to determine, we will start with them.

To determine the normal force N^9 in top chord member 9, we introduce a section across members 5, 6, 7 and 9. Figure 9.45 shows only the part to the left of the section. Four unknown member forces are acting in the section. Since the lines of action of the forces N^5, N^6 and N^7 intersect one another at point A, only one force is unknown in the equation for the moment equilibrium about A, which can be determined directly. This gives

$$\sum T_z|\mathrm{A} = -(60\text{ kN})(2\text{ m}) - N^9 \times (3\text{ m}) = 0 \rightarrow N^9 = -40\text{ kN}.$$

Member 9 is a compression member.

The same equation is found from the moment equilibrium about A of the part to the left of the section over members 7, 8, 9 and 12 (see Figure 9.46). This section offers the advantage that the forces in the members of both the top chord and the bottom chord can be found. In this way, force N^{12} in member 12 follows directly from the moment equilibrium about B:

$$\sum T_z|B = -(240\text{ kN})(3\text{ m}) - (60\text{ kN})(2\text{ m}) + N^{12} \times (3\text{ m}) = 0$$

$$\Rightarrow N^{12} = +280\text{ kN}.$$

A tensile force is acting in member 12.

The section in Figure 9.46 has the additional benefit that the values found for N^9 and N^{12} can be checked using the horizontal force equilibrium of the isolated part, without having to know the forces in the diagonal members or verticals:

$$\sum F_x = -(240\text{ kN}) + N^9 + N^{12}$$

$$= -(240\text{ kN}) + (-40\text{ kN}) + (280\text{ kN}) = 0.$$

The isolated section in Figure 9.46 therefore meets the conditions for horizontal force equilibrium.

With the section in Figure 9.46, we can quickly determine the forces in the top chord member 9 and bottom chord member 12, but not the forces in the verticals 7 and 8. The forces in these verticals are found from the equilibrium of joints A and B, but we do not have enough information to do so yet.

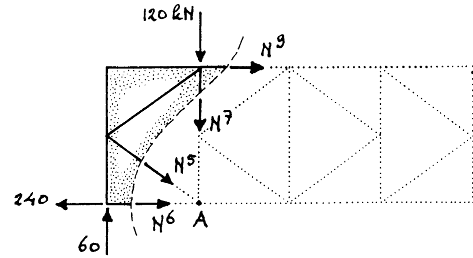


Figure 9.45 Section for determining the force in member 9. The force is found from the moment equilibrium about A.

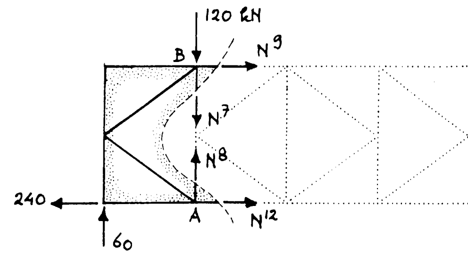


Figure 9.46 Section for determining the forces in members 9 and 12. They are found from the moment equilibrium about respectively A and B.

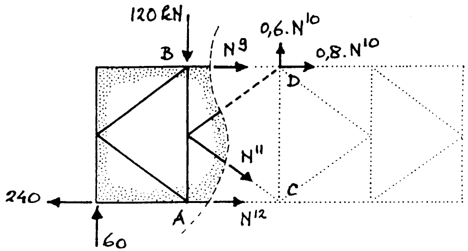


Figure 9.47 Section for determining the force in member 10. This force follows from the moment equilibrium about C. However, we do have to know N^9 first.

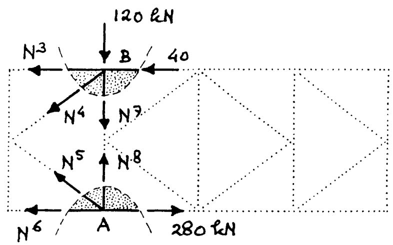


Figure 9.48 N^7 and N^8, the forces in the verticals, are found from the equilibrium of joints B and A, although we must first know the forces in members 3 or 4, respectively 5 or 6.

The forces in the diagonal members 10 and 11 are found using the section in Figure 9.47 over members 9, 10, 11 and 12. Since we already know N^9, we can find N^{10} from the moment equilibrium about C. This gives the following (with force N^{10} shifted along its line of action to point D, only the horizontal component of N^{10} is left in the equation for the moment equilibrium)

$$\sum T_z|\mathrm{C} = -(60 \text{ kN})(4 \text{ m}) + (120 \text{ kN})(2 \text{ m}) - N^9 \times (3 \text{ m}) + \\ - 0.8N^{10} \times (3 \text{ m}) = 0.$$

With $N^9 = -40$ kN this gives

$$N^{10} = +50 \text{ kN}.$$

In the same way, from the moment equilibrium about D we find

$$N^{11} = -50 \text{ kN}.$$

Since we know both N^9 and N^{12}, we can find N^{10} and N^{11} from the two equations for the force equilibrium of the isolated part in Figure 9.47:

$$\sum F_x = N^9 + 0.8N^{10} + 0.8N^{11} + N^{12} - (240 \text{ kN}) = 0,$$
$$\sum F_y = 0.6N^{10} - 0.6N^{11} + (60 \text{ kN}) - (120 \text{ kN}) = 0.$$

With $N^9 = -40$ kN and $N^{12} = +280$ kN these equations are now

$$0.8N^{10} + 0.8N^{11} = 0,$$
$$0.6N^{10} - 0.6N^{11} = +60 \text{ kN}.$$

The solution is

$$N^{10} = +50 \text{ kN},$$

$$N^{11} = -50 \text{ kN}.$$

This is in agreement with earlier results.

The forces in the verticals 7 and 8 follow from the (force) equilibrium of joints B and A respectively, although we do first have to know the forces in one of members 3 and 4 and one of members 5 or 6 (see Figure 9.48).

The forces N^3 and N^6 can be found in the section in Figure 9.49 from the moment equilibrium about G and E respectively. In fact, with this section on the end of the truss, we isolate joints E and G. N^3 and N^6 can therefore also be found directly from the horizontal force equilibrium of joints E and G:

$$N^3 = 0,$$

$$N^6 = +240 \text{ kN}.$$

In Figure 9.50, joints A and B have been isolated, and all the known forces N^3, N^6, N^9 and N^{12} are shown as they act in reality on the joints.

At joint B, two forces are still unknown: N^4 and N^7. From the equilibrium for this joint we find

$$\sum F_x = -(40 \text{ kN}) - 0.8N^4 = 0,$$

$$\sum F_y = -(120 \text{ kN}) - 0.6N^4 - N^7 = 0$$

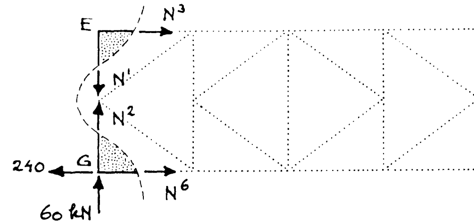


Figure 9.49 N^3 and N^6 follow from the force equilibrium of respectively joint E and G.

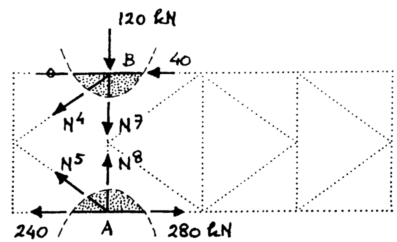


Figure 9.50 If $N^3 = 0$ and $N^6 = 240$ kN, then N^7 and N^8 are found from the force equilibrium of respectively joint B and A.

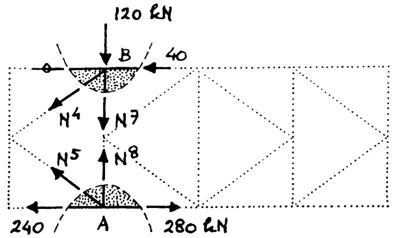


Figure 9.50 If $N^3 = 0$ and $N^6 = 240$ kN, then N^7 and N^8 are found from the force equilibrium of respectively joint B and A.

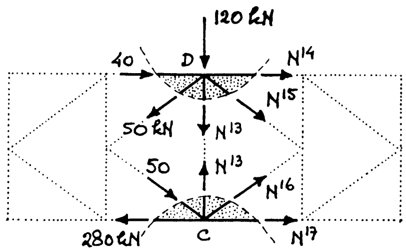


Figure 9.51 The force in member 13 is found from the force equilibrium of joint D or C, although we must first determine one of the member forces N^{14} and N^{15}, or one of N^{16} and N^{17}.

with the solution

$$N^4 = -50 \text{ kN},$$
$$N^7 = -90 \text{ kN}.$$

In the same way, we can determine N^5 and N^8 from the equilibrium of joint A:

$$\sum F_x = -(240 \text{ kN}) + (280 \text{ kN}) - 0.8N^5 = 0,$$
$$\sum F_y = 0.6N^5 + N^8 = 0$$

such that

$$N^5 = +50 \text{ kN},$$
$$N^8 = -30 \text{ kN}.$$

The force in member 13 is the most complicated one to determine. This force is found from the equilibrium of joint D or C. However, we first have to determine one of the member forces N^{14} and N^{15}, or one of N^{16} and N^{17} (see Figure 9.51).

With the section in Figure 9.52, N^{14} is found from the moment equilibrium about H:

$$\sum T_z|\text{H} = (180 \text{ kN})(2 \text{ m}) - (240 \text{ kN})(3 \text{ m}) + N^{14} \times (3 \text{ m}) = 0$$
$$\Rightarrow N^{14} = +120 \text{ kN}.$$

At joint D, N^{13} and N^{14} are now the only unknowns (see Figure 9.51). The two equations for the force equilibrium of the joint are:

$$\sum F_x = (40\text{ kN}) + N^{14} - 0.8 \times (50\text{ kN}) + 0.8 \times N^{15} = 0,$$

$$\sum F_y = -(120\text{ kN}) - N^{13} - 0.6 \times (50\text{ kN}) - 0.6 \times N^{15} = 0.$$

Here substitute $N^{14} = +120$ kN to find the solution:

$$N^{15} = -150\text{ kN},$$

$$N^{13} = -60\text{ kN}.$$

Table 9.3 provides a summary of all the member forces.

In the *method of sections*, member forces are determined from the equilibrium of a sectioned part of the truss. In the examples, the *sectioned part* sometimes degenerates into a *joint*. The following section looks at the *method of joints*. With this method, all the member forces are consistently derived from the equilibrium of the joints.

9.3.2 The method of joints

In the *method of joints*, all the joints are isolated, and we investigate the *force equilibrium* of the individual joints.

For the truss in Figure 9.53a, all the joints have been isolated in Figure 9.53b. On the isolated joints are acting

- loads (joints C and D);
- support reactions (joints A and B);
- member forces.

Here, the support reactions and member forces are the unknown forces.

Since only two equations for the force equilibrium are available per joint, we have to start the calculation at a joint where no more than two forces

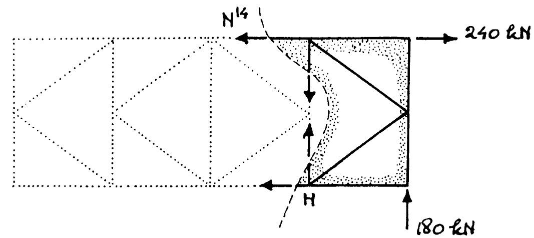


Figure 9.52 In this section, N^{14} is found from the moment equilibrium about H.

Table 9.3 Member forces Example 3.

Mem. no. i	N^i (kN)	Mem. no. i	N^i (kN)
1	0	14	+120
2	−60	15	−150
3	0	16	+150
4	−50	17	+120
5	+50	18	+90
6	+240	19	−90
7	−90	20	+240
8	−30	21	−150
9	−40	22	+150
10	+50	23	0
11	−50	24	0
12	+280	25	−180
13	−60		

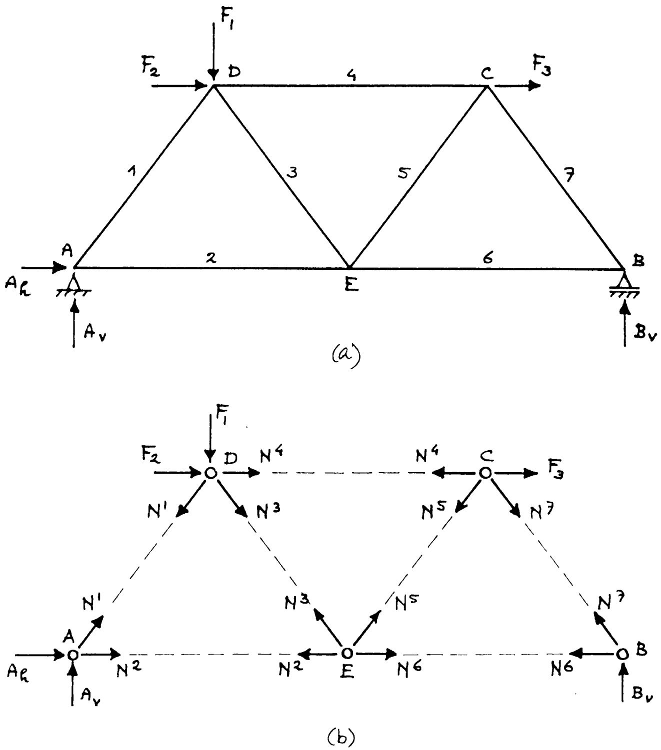


Figure 9.53 (a) Truss with support reactions and (b) all isolated joints of the truss with all the forces acting on them.

are unknown. These forces are determined from the joint equilibrium, after which we move to the next joint where, again, no more than two forces are unknown. In this way, we pass along each of the joints in the truss.

If there are k joints, it is not the intention to first generate all $2k$ equations for the force equilibrium, and then to solve them together as a system of equations. We will often encounter the problem in which we cannot start with a joint with only two unknowns, as in Figure 9.53. This can be avoided by previously determining the support reactions from the truss as a whole. In Figure 9.53b, we can now start the procedure at one of the joints A or B.

The method of joints is mostly used if one wants to find all the member forces in a truss. If you want only to calculate the member force somewhere in the middle of the truss, you will often have to work out the equilibrium for several joints. In that case, the method of sections is faster.

Calculating the two unknown forces per joint can be done either *analytically* or *graphically*. The graphical approach is preferable; it is not only faster but also gives a better insight in the force flow. The method of joints is illustrated using a number of examples.

Example 1
The truss crane in Figure 9.54 is loaded at A by means of a vertical force $4F$.

Question:
Determine all the member forces, with the correct sign for tension and compression.

Solution:
In this case, we do not have to determine the support reactions as we can start directly at joint A. Here, two forces are unknown: N^1 and N^2. These forces can be determined both *analytically* and *graphically*.

Analytical solution for the equilibrium of joint A:
In Figure 9.55a, all the forces acting on joint A are shown. In this figure, the member forces are again shown as tensile forces. For a tensile force, N is by convention positive.

For the angles α^1 and α^2 shown in the figure, the equilibrium equations are

$$\sum F_x = -N^1 \cos\alpha^1 - N^2 \cos\alpha^2 = 0,$$
$$\sum F_y = -N^1 \sin\alpha^1 - N^2 \sin\alpha^2 - 4F = 0.$$

From the slopes of the members 1 and 2 we find

$$\sin\alpha^1 = \cos\alpha^1 = \tfrac{1}{2}\sqrt{2},$$
$$\sin\alpha^2 = \tfrac{1}{5}\sqrt{5} \text{ and } \cos\alpha^2 = \tfrac{2}{5}\sqrt{5}.$$

Both equations in N^1 and N^2 now become

$$-N^1 \times \tfrac{1}{2}\sqrt{2} - N^2 \times \tfrac{2}{5}\sqrt{5} = 0,$$
$$-N^1 \times \tfrac{1}{2}\sqrt{2} - N^2 \times \tfrac{1}{5}\sqrt{5} = 4F$$

with solution:

$$N^1 = -8F\sqrt{2},$$
$$N^2 = +4F\sqrt{5}.$$

Member 1 is a *compression member* and exerts a compressive force on joint A. Member 2 is a *tension member*. Figure 9.55b shows the forces as they really act on both the joint and on the two members.

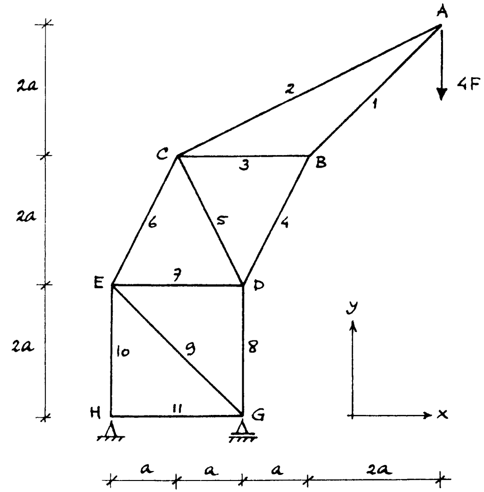


Figure 9.54 A truss crane. We can start the method of joints in A without having to first determine the support reactions.

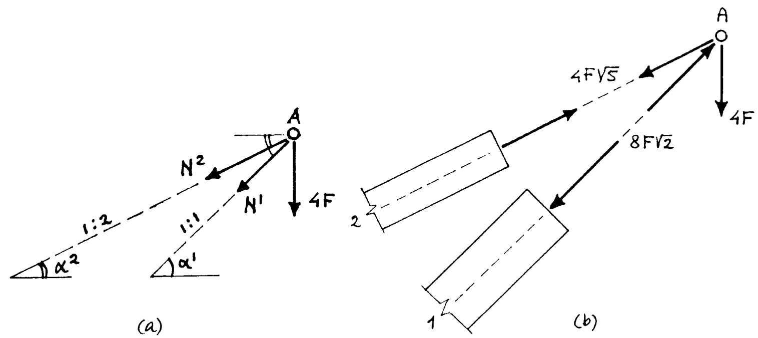


Figure 9.55 (a) The isolated joint A. The unknown forces N^1 and N^2 exerted by the members 1 and 2 on joint A are shown as tensile forces. (b) The interaction forces between joint A and members 1 and 2 as they really act. Member 1 is a compression member and member 2 is a tension member.

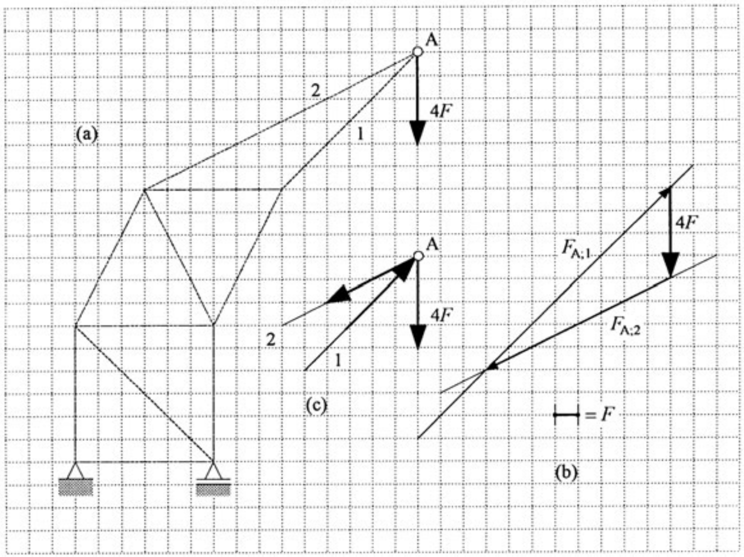


Figure 9.56 (a) The forces in members 1 and 2 follow from the equilibrium of joint A. (b) The closed force polygon for the equilibrium of joint A. $F_{A;1}$ and $F_{A;2}$ are the forces that members 1 and 2 exert on joint A. (c) Joint A with all the forces acting on it. From this figure we can see that N^1 is a compressive force and N^2 is a tensile force.

Graphical solution for the equilibrium of joint A:
$F_{A;1}$ and $F_{A;2}$ are the forces that members 1 and 2 exert on joint A. The forces $F_{A;1}$ and $F_{A;2}$ have their line of action along the members 1 and 2, but we do not know their magnitudes, nor their directions (see Figure 9.56a). Joint A is in equilibrium if all forces acting on joint A form a *closed force polygon*. Figure 9.56b shows the closed force polygon for the equilibrium of joint A. From here, we can read off the magnitude of $F_{A;1}$ and $F_{A;2}$ (or calculate it):

$$F_{A;1} = 8F\sqrt{2},$$

$$F_{A;2} = 4F\sqrt{5}.$$

From the force polygon, we can also find the directions of $F_{A;1}$ and $F_{A;2}$, but we cannot see whether they are tensile or compressive forces. To do so, we first have to draw the forces found as they act on joint A, see Figure 9.56c. Only then we can see that $F_{A;1}$ is a compressive force, and $F_{A;2}$ is a tensile force, so that

$$N^{(1)} = -F_{A;1} = -8F\sqrt{2},$$

$$N^{(2)} = +F_{A;2} = +4F\sqrt{5}.$$

Note that the forces in the force polygon have not been denoted as N. The force polygon provides information only on the magnitude of the member forces, and not on the sign for tension or compression.

The order in which one writes down the forces in a force polygon does not influence the result (vector addition is associative and commutative). Figure 9.57 shows two equivalent force polygons. The first force polygon is created by ranking the various forces acting on joint A in an order that is associated with an anti-clockwise rotation about joint A: $4F \Rightarrow F_{A;2} \Rightarrow F_{A;1}$.

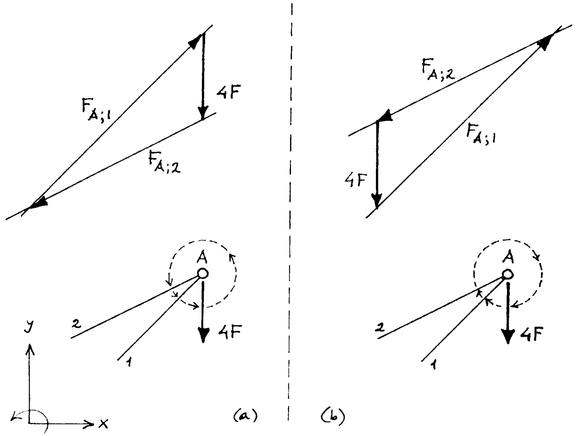


Figure 9.57 The order in which the forces in a force polygon are plotted does not influence the result (vector addition is associative and commutative).

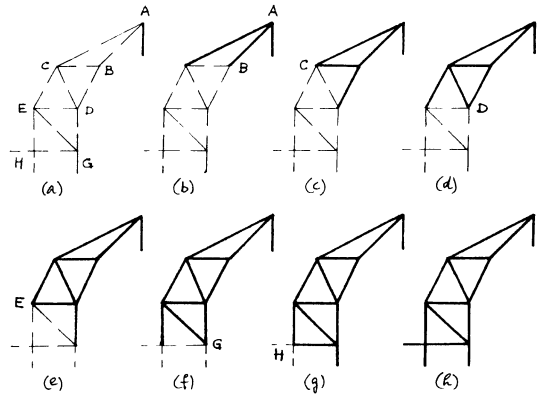


Figure 9.58 The order (a) to (h) shows how we can repeatedly determine two member forces per joint (and finally the support reactions at G and H). The members for which the normal force is known are shown in bold.

The second force polygon arises from ranking the forces in a clockwise order, so that $4F \Rightarrow F_{\mathrm{A};1} \Rightarrow F_{\mathrm{A};2}$.

In Figure 9.58, the order (a) to (h) shows how, per joint, we can consecutively calculate two member forces (and then the support reactions in G and H). The members for which the forces are known are shown in bold.

Figure 9.58a shows the initial situation. A is the only joint with two unknown member forces. Once we have calculated these, we get the situation shown in Figure 9.58b. Now B is the only joint with only two unknown member forces. Once these have been determined, we get the situation in Figure 9.58c, and so forth. The order in which the joint equilibrium is determined, with no more than two unknowns per joint, is

$$\mathrm{A} \Rightarrow \mathrm{B} \Rightarrow \mathrm{C} \Rightarrow \mathrm{D} \Rightarrow \mathrm{E} \Rightarrow \mathrm{G} \Rightarrow \mathrm{H}.$$

For calculating the still unknown member forces, we now use the graphical method. After A, the next joint is B, where we can calculate the member forces. Joint B is subject to the forces $F_{B;1}$, $F_{B;3}$ and $F_{B;4}$, of which $F_{B;1}$ is known.

Earlier, we found that the force in member 1 is a compressive force: $N^1 = -8F\sqrt{2}$. Member 1 therefore exerts a compressive force on joint B of $8F\sqrt{2}$, so that $F_{B;1} = 8F\sqrt{2}$ (see Figure 9.59a).

The two unknowns $F_{B;3}$ and $F_{B;4}$ can be determined from the closed force polygon for the equilibrium of joint B (see Figure 9.59b):

$$F_{B;3} = 4F,$$
$$F_{B;4} = 4F\sqrt{5}.$$

In Figure 9.59c, the forces from the force polygon are shown as they act on joint B in reality. Here we see that $F_{B;3}$ and $F_{B;4}$ are both compressive forces. Converted into the normal forces in the members 3 and 4, with the correct sign for tension and compression, we therefore get

$$N^3 = -F_{B;3} = -4F,$$
$$N^4 = -F_{B;4} = -4F\sqrt{5}.$$

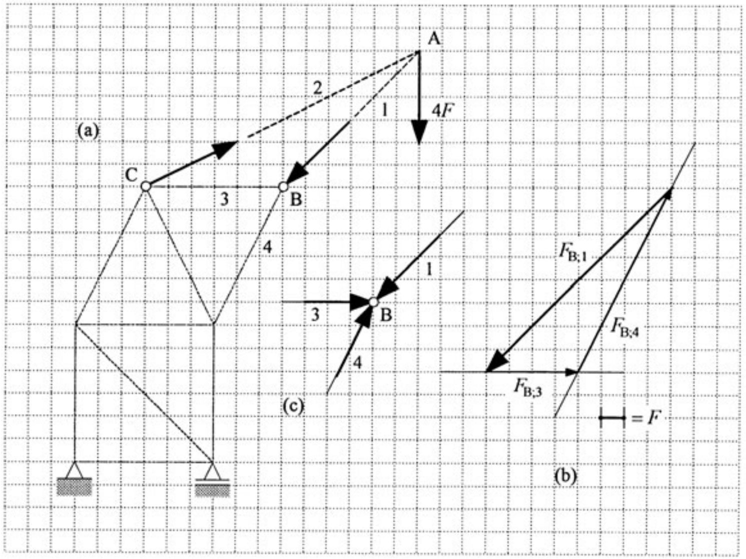


Figure 9.59 (a) The forces in members 3 and 4 follow from the equilibrium of joint B. (b) The closed force polygon for the equilibrium of joint B. $F_{B;1}$ is known. (c) Joint B with all the forces acting on it. From this figure we can see that N^3 and N^4 are compressive forces.

The following joint with only two unknowns is C. The forces that the members 2 and 3 exert on the joint are known (see Figure 9.60a):

$$F_{C;2} = 4F\sqrt{5},$$
$$F_{C;3} = 4F.$$

The unknown forces $F_{C;5}$ and $F_{C;6}$ follow from the force polygon in Figure 9.60b:

$$F_{C;5} = F\sqrt{5},$$
$$F_{C;6} = 3F\sqrt{5}.$$

In Figure 9.60c, all the forces are shown as they act on joint C in reality. Member 5 presses against the joint and is a compression member, member 6 pulls on the joint and is a tension member:

$$N^5 = -F\sqrt{5},$$
$$N^6 = +3F\sqrt{5}.$$

In Figures 9.61 to 9.64, the other member forces are calculated using the same method.

Table 9.4 provides a summary of all the member forces.

Table 9.4 Member forces Example 1.

Mem. no. i	**N^i (kN)**
1	$-8F\sqrt{2}$
2	$+4F\sqrt{5}$
3	$-4F$
4	$-4F\sqrt{5}$
5	$-F\sqrt{5}$
6	$+3F\sqrt{5}$
7	$-3F$
8	$-10F$
9	0
10	$+6F$
11	0

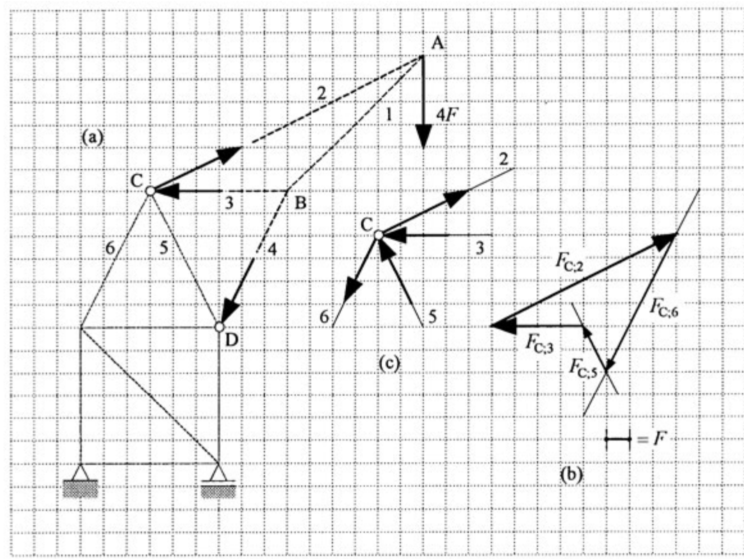


Figure 9.60 (a) The forces in members 5 and 6 follow from the equilibrium of joint C. (b) The closed force polygon for the equilibrium of joint C. $F_{C;2}$ and $F_{C;3}$ are known forces. (c) Joint C with all the forces acting on it. From this figure we can see that N^5 is compressive and N^6 is tensile.

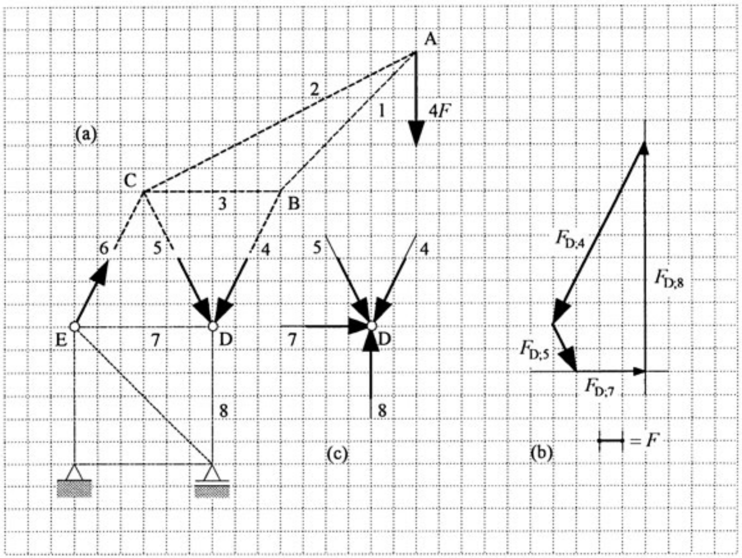


Figure 9.61 (a) The forces in members 7 and 8 follow from the equilibrium of joint D. (b) The closed force polygon for the equilibrium of joint D. $F_{D;4}$ and $F_{D;5}$ are known forces. (c) Joint D with all the forces acting on it. From this figure we can see that N^7 and N^8 are compressive.

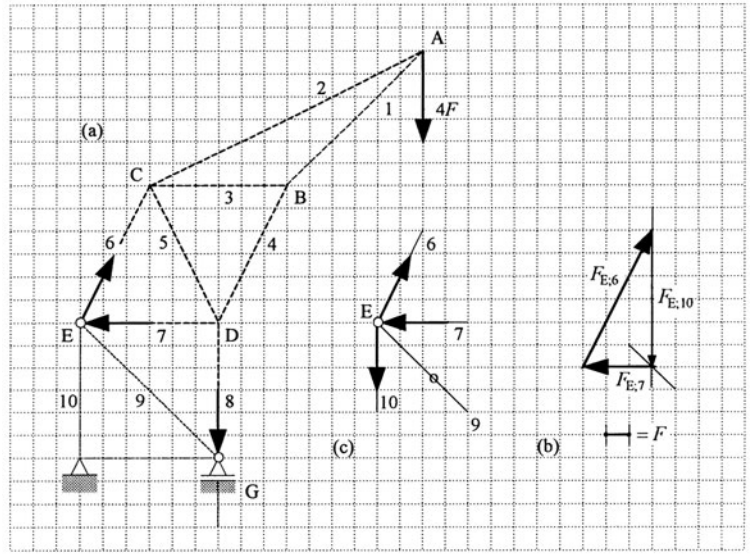


Figure 9.62 (a) The forces in members 9 and 10 follow from the equilibrium of joint E. (b) The closed force polygon for the equilibrium of joint E. $F_{E;6}$ and $F_{E;7}$ are known. (c) Joint E with all the forces acting on it. From this figure we can see that N^{10} is tensile. Member 9 is a zero-force member.

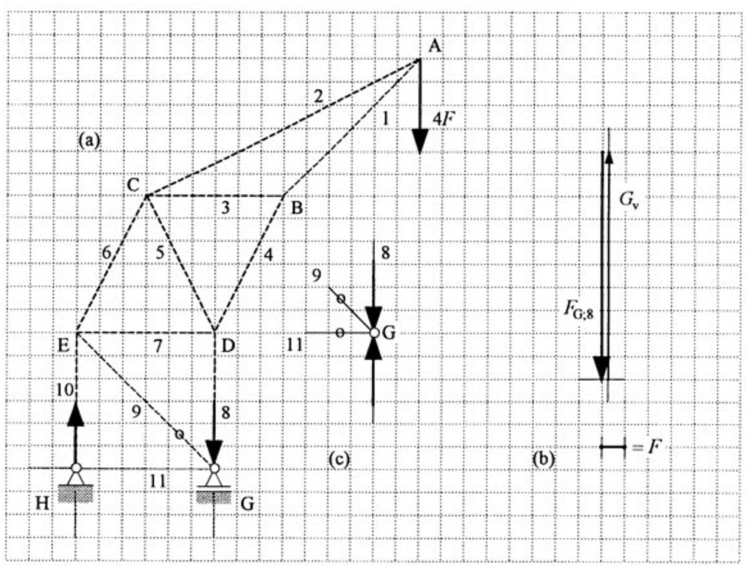


Figure 9.63 (a) The force in member 11 and the vertical support reaction at G is found from the equilibrium of joint G. (b) The closed force polygon for the equilibrium of joint G. Member 9 is a zero-force member and does not participate. $F_{G;8}$ is known. (c) Joint G with all the forces acting on it. Member 11 is a zero-force member. The vertical support reaction at G is a compressive force.

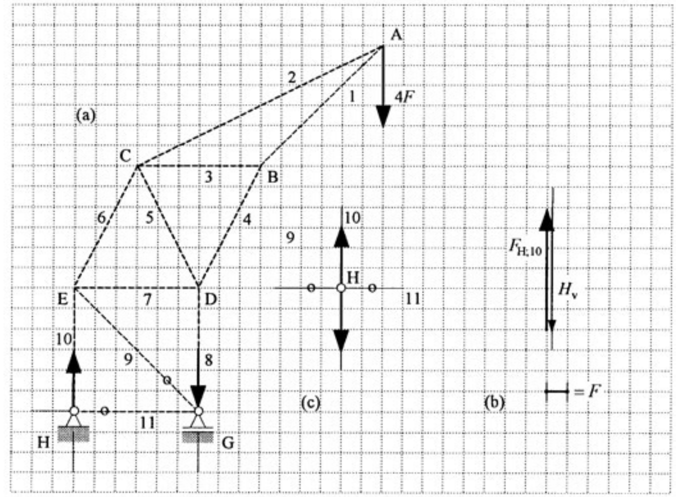


Figure 9.64 (a) The horizontal and vertical support reaction at H is found from the equilibrium of joint H. (b) The closed force polygon for the equilibrium of joint H. Member 11 is a zero-force member and does not participate. $F_{H;10}$ is known. (c) Joint H with all the forces acting on it. The horizontal support reaction at H is zero. The vertical support reaction in H is a tensile force.

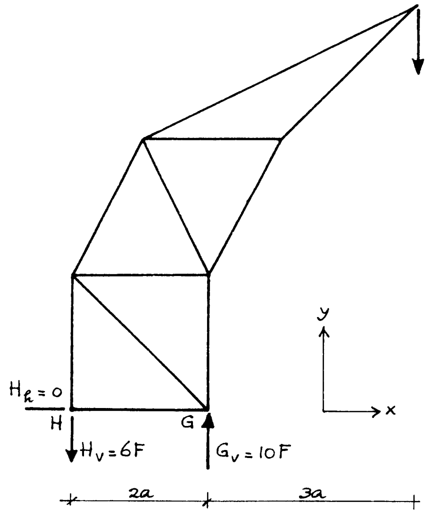


Figure 9.65 The truss crane with the support reactions as they are acting in reality.

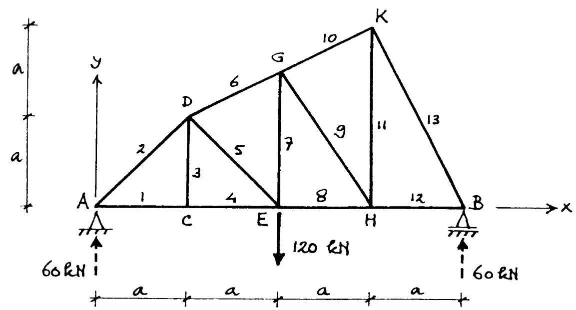


Figure 9.66 In this truss, we can apply the method of joints only when we know the support reactions.

In Figures 9.63 and 9.64 the support reactions in G and H have also been calculated:

$$G_v = 10F,$$

$$H_h = 0,$$

$$H_v = 6F.$$

In Figure 9.65, the support reactions are shown with the directions in which they are acting.

To check the calculation, we can look at the equilibrium of the truss as a whole:

$$\sum F_x = H_h = 0,$$

$$\sum F_y = G_v - H_v - 4F = 10F - 6F - 4F = 0,$$

$$\sum T_z|\mathrm{H} = G_v \times 2a - 4F \times 5a = 10F \times 2a - 4F \times 5a = 0.$$

The truss as a whole meets the equilibrium conditions.

Example 2
The truss in Figure 9.66 is loaded at joint E by a vertical force of 120 kN.

Question:
Calculate the member forces, with the correct sign for tension and compression.

Solution:
In this truss, we cannot find a joint with only two unknown forces. Before we can start the procedure for the joint equilibrium, we first have to determine the support reactions from the truss as a whole. Then we can start the calculation at joint A or B.

In Figure 9.67, the order (a) to (g) shows how, starting at A, we can consecutively determine two member forces per joint. The members for which we know the normal force are shown in bold. We will look at the joints in the following order:

$$A \Rightarrow C \Rightarrow D \Rightarrow E \Rightarrow G \Rightarrow H \Rightarrow K \text{ or } B.$$

The last two joints K and B both offer an opportunity to check the results: both force polygons have to be closed and give the same force in member 13.

Table 9.5 provides a summary of all the member forces.

Table 9.5 Member forces Example 2.

Mem. no. i	**N^i (kN)**
1	$+60$
2	$-60\sqrt{2}$
3	0
4	$+60$
5	$+20\sqrt{2}$
6	$-40\sqrt{5}$
7	$+100$
8	$+80$
9	$-25\sqrt{13}$
10	$-15\sqrt{5}$
11	$+75$
12	$+30$
13	$-30\sqrt{5}$

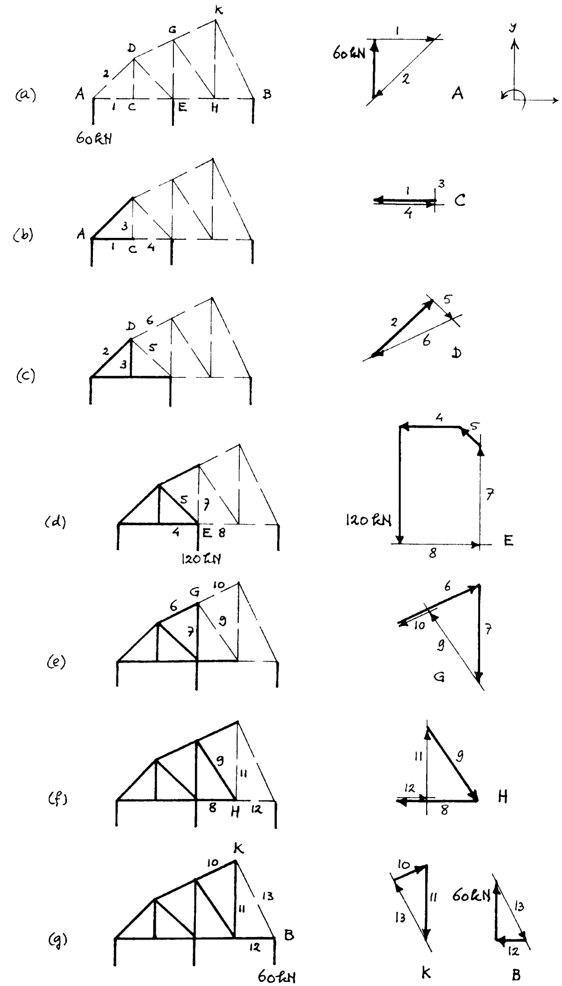


Figure 9.67 The order (a) to (g) shows how, starting at A, we can consecutively determine two member forces per joint. The members for which we know the normal force are shown in bold.

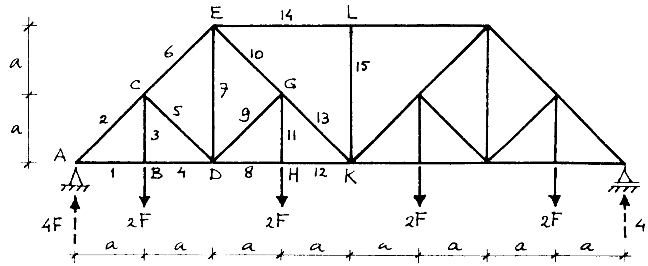


Figure 9.68 Truss with support reactions.

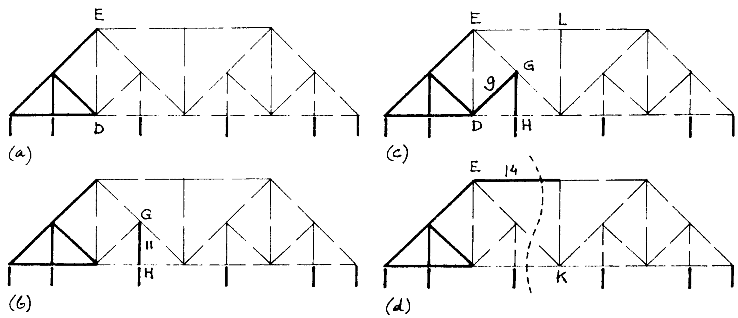


Figure 9.69 The members for which we know the normal force are shown in bold. (a) The method of joints gets stuck at joints E and D, as more than two member forces are unknown. (b) The force in member 11 follows from the vertical equilibrium of joint H, (c) after which we can find the force in member 9 from the force equilibrium of joint G. The method of joints can now continue at D. (d) We could also switch to the method of sections to calculate the force in member 14. The method of joints can then be resumed at E.

Example 3
You are given the (Baltimore) truss beam in Figure 9.68.

Question:
Determine the member forces N^1 to N^{15} using the method of joints. In which order should we handle the joint equilibrium?

Solution:
After first determining the support reactions from the equilibrium of the truss as a whole, we can determine the forces in members 1 to 6 from the equilibrium of joints A, B and C respectively. In the situation shown in Figure 9.69a we get stuck, as more than two member forces are unknown in both D and E.

Since members 8 and 12, and 10 and 13 are in a direct line with one another, we can determine the forces in the members 11 and 9 from the equilibrium of joints H and G.

The vertical equilibrium of joint H in Figure 9.70 gives

$$N^{11} = 2F.$$

We now have the situation as shown in Figure 9.69b. From the equilibrium in G in the direction normal to members 10 and 13 we find next (see Figure 9.70):

$$N^9 + \tfrac{1}{2}N^{11}\sqrt{2} = N^9 + \tfrac{1}{2} \times 2F \times \sqrt{2} = 0 \Rightarrow N^9 = -F\sqrt{2}.$$

Now that N^9 is known (see Figure 9.69c), we can find the remaining member forces by consecutively elaborating the equilibrium of joints D, E, G, H and L. The order in which we handle the joints is therefore

$$\mathrm{A} \Rightarrow \mathrm{B} \Rightarrow \mathrm{C} \Rightarrow \mathrm{H} \Rightarrow \mathrm{G} \Rightarrow \mathrm{D} \Rightarrow \mathrm{E} \Rightarrow \mathrm{G} \Rightarrow \mathrm{H} \Rightarrow \mathrm{L}.$$

Instead of determining N^9 and N^{11} from the equilibrium of joints H and G, it is far easier to revert to the method of sections. With the section shown in Figure 9.69d across members 12, 13 and 14, we can determine the force in member 14 from $\sum T_z|\text{K} = 0$. The other member forces are then found from the equilibrium for the successive joints E, D, G, H and L.

In certain cases, it can be useful to switch from one method to the other at the right moment.

Table 9.6 provides a summary of member forces N^1 to N^{15}.

Table 9.6 Member forces Example 3.

Mem. no. i	N^i (kN)	Mem. no. i	N^i (kN)
1	$+4F$	9	$-F\sqrt{2}$
2	$-4F\sqrt{2}$	10	$+F\sqrt{2}$
3	$+2F$	11	$+2F$
4	$+4F$	12	$+4F$
5	$-F\sqrt{2}$	13	0
6	$-3F\sqrt{2}$	14	$-4F$
7	$+2F$	15	0
8	$+4F$		

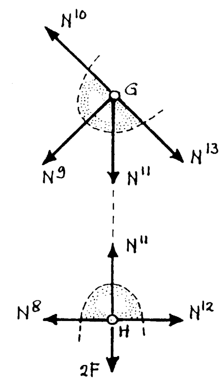


Figure 9.70 The isolated joints G and H.

9.3.3 Zero-force members and continuous members; simplifying the calculation

We can often shorten the calculation that needs to be done by first looking for *zero-force members* in a truss. Zero-force members are members in which no forces are acting ($N = 0$) due to the present loading.

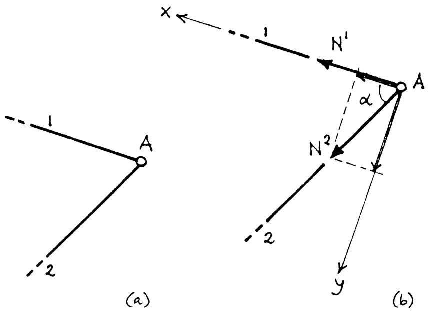


Figure 9.71 (a) If two members meet in an unloaded joint, both members are zero-force members. (b) The forces acting on isolated joint A.

There are three situations of frequent occurrence in which zero-force members can be easily recognised:

1. *If only two members meet in an unloaded joint, both are zero-force members (see Figure 9.71).*
2. *If three members meet in an unloaded joint of which two are in a direct line with one another, then the third is a zero-force member (see Figure 9.72).*
3. *If two members meet in an unloaded joint and the line of action of the load coincides with one of the members, the other member is a zero-force member (see Figure 9.73).*

These three rules are the direct consequence of the joint equilibrium, as shown below for each of the cases.

Rule 1. Two members meet in unloaded joint A in Figure 9.71. The force in one of the members has a component normal to the direction of the other member. If we write down the equilibrium of joint A in the given (local) xy coordinate system, we find

$$\sum F_x = N^1 + N^2 \cos\alpha = 0,$$
$$\sum F_y = N^2 \sin\alpha = 0$$

with the solution (because $\sin\alpha \neq 0$):[1]

$$N^1 = N^2 = 0.$$

Equilibrium is possible only if both member forces are zero.

[1] In a kinematically determinate truss, members 1 and 2 cannot be an extension of one another, so that $\alpha \neq 0$ and $\alpha \neq 180°$.

Rule 2. In Figure 9.72, three members meet in joint B, of which members 1 and 3 are in a direct line with one another. The force in member 2 has a component normal to members 1 and 3. There can be equilibrium only if this component is zero, or in other words, if $N^2 = 0$. If we write down the equilibrium of joint B in the given (local) xy coordinate system, we find

$$\sum F_x = N^1 + N^2 \cos\alpha - N^3 = 0,$$
$$\sum F_y = N^2 \sin\alpha = 0$$

so that

$$N^2 = 0 \quad \text{and} \quad N^1 = N^3.$$

In addition to the fact that member 2 is a zero-force member, the normal forces in the continuous members 1 and 3, which are in a direct line with one another, are equal.

Rule 3. The situation in Figure 9.73 is clearly similar to that in Figure 9.71. The equations for the equilibrium of joint C are

$$\sum F_x = N^1 + N^2 \cos\alpha - F = 0,$$
$$\sum F_y = N^2 \sin\alpha = 0$$

so that

$$N^2 = 0 \quad \text{and} \quad N^2 = F.$$

By using rules 1 to 3 to determine the zero-force members first, you can often shorten the required calculation. A fourth rule with which we can shorten the calculation relates to an unloaded joint, in which four members meet and in pairs are in a direct line with one another. This situation is

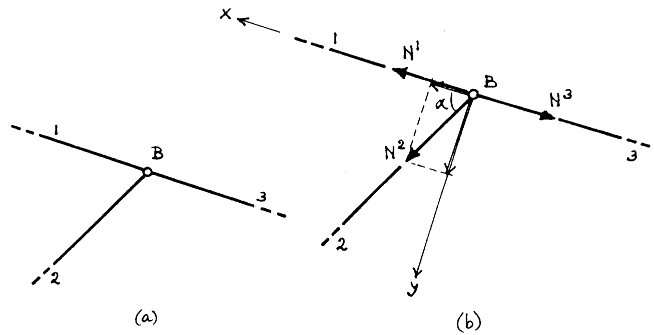


Figure 9.72 (a) If three members meet in an unloaded joint of which two are in a direct line with one another, the third member is a zero-force member. The normal forces in continuous members 1 and 3 are equal. (b) The forces acting on isolated joint B.

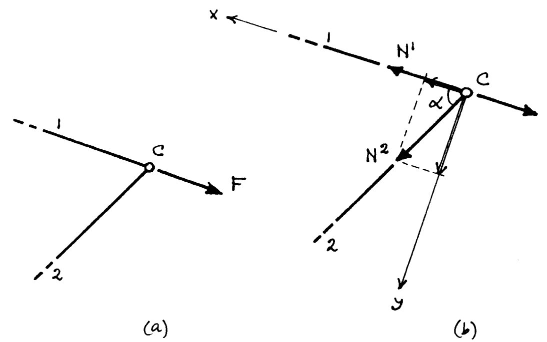


Figure 9.73 (a) If two members meet in an unloaded joint and the line of action of the load is in a direct line with one of the members, the other member is a zero-force member. (b) The forces acting on isolated joint C.

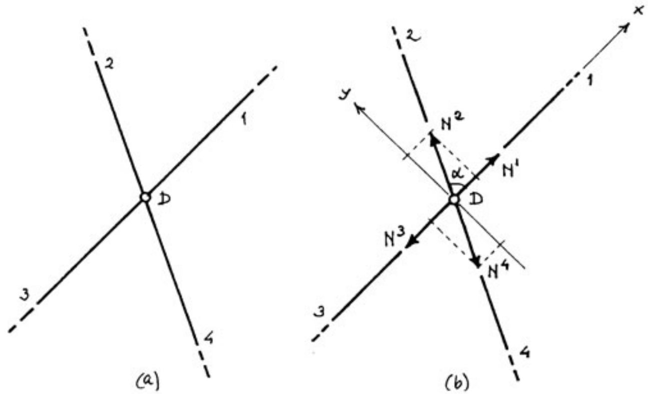


Figure 9.74 If four members meet in an unloaded joint that in pairs are in a direct line with one another, these members can be considered crossing members as far as the transfer of forces is concerned. (b) The forces acting on isolated joint D.

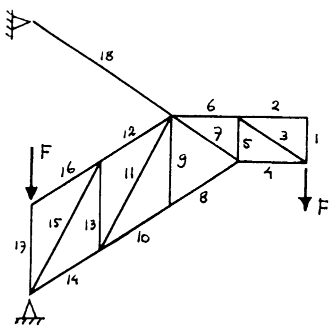


Figure 9.75 A truss.

shown in Figure 9.74. For the given xy coordinate system, the equilibrium of joint D gives

$$\sum F_x = N^1 + N^2 \cos\alpha - N^3 - N^4 \cos\alpha = 0,$$
$$\sum F_y = N^2 \sin\alpha - N^4 \sin\alpha = 0$$

so that

$$N^1 = N^3 \quad \text{and} \quad N^2 = N^4.$$

Conclusion:

Rule 4. If four members meet in an unloaded joint that in pairs are in a direct line with one another, these members can be considered crossing members *as far as the transfer of forces is concerned.*

The three examples below show how it is possible to simplify the calculation with these four rules.

Example 1
You are given the truss in Figure 9.75.

Question:
Which members are zero-force members for the given load?

Solution:
A is an unloaded joint in which two members meet (see Figure 9.76). Both members are zero-force members (rule 1), so that

$$N^1 = 0 \quad \text{and} \quad N^2 = 0.$$

B is an unloaded joint in which three members meet, and of which two are in a direct line with one another. The third member is therefore a zero-force member (rule 2), so that

$$N^9 = 0.$$

C is a loaded joint where two members meet, and where the line of action of the load coincides with member 17. Thus (rule 3)

$$N^{16} = 0.$$

The zero-force members are shown in Figure 9.76 with a "0" through the member axis.

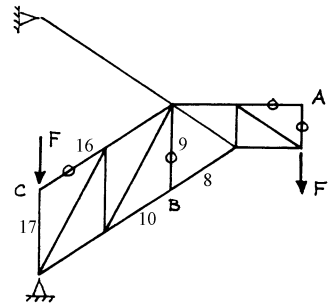


Figure 9.76 The zero-force members in the truss.

Zero-force members do not participate in the force flow for the present load. When calculating the forces in the other members, you can leave out the zero-force members from the truss. If you leave out zero-force member 9 from the truss, you immediately notice that

$$N^8 = N^{10}.$$

If you leave out zero-force member 16, you see that

$$N^{17} = -F.$$

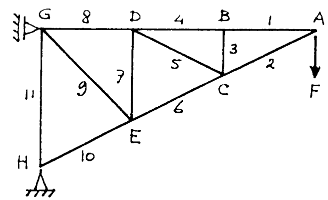


Figure 9.77 A truss.

That this (imaginary) omission of zero-force members can significantly reduce the effort in calculating is further emphasised in the following two examples.

Example 2
You are given the truss in Figure 9.77.

Question:
Determine all the zero-force members for the given load.

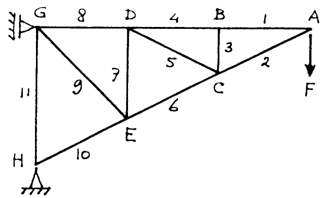


Figure 9.77 A truss.

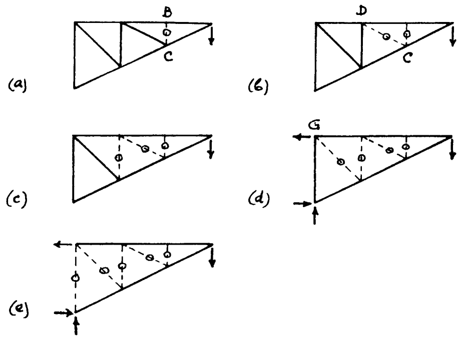


Figure 9.78 A zero-force member does not participate in the transfer of forces and can be omitted from the calculation. This is shown here by depicting the member with a dashed line. (a) to (e) shows the order in which one can find the zero-force members. (e) In the end, all bracing members turn out to be zero-force members, and the top and bottom chord members only transfer the load to the supports.

Solution:
From the equilibrium in joint B, it follows that member 3 is a zero-force member (rule 2). With the given load, this member does not participate in the transfer of forces, and could therefore be omitted. In Figure 9.78a, the member is now shown by means of a dashed line. The equilibrium of joint C means that member 5 is also a zero-force member (rule 2 again) (see Figure 9.78b). If we continue, we notice that members 7 and 9 are also zero-force members (see Figures 9.78c and 9.78d).

Since the support reaction in G is horizontal, member 11 is also a zero-force member (rule 3) (see Figure 9.78e).

All the verticals and diagonals are zero-force members. The load is therefore fully transferred by the bottom and top chord members. For the (continuous) top chord members we find

$$N^1 = N^4 = N^8$$

For the (continuous) bottom chord members we find

$$N^2 = N^6 = N^{10}.$$

When we talk about *omitting* zero-force members, this is done only to simplify the calculation. If the zero-force members are removed from the truss in reality, the truss becomes kinematically indeterminate.

Zero-force members therefore have a genuine function in the truss. On the one hand they ensure the truss retains its shape, while on the other they can prevent *buckling* (in the plane of the structure) of (long) compressed members, such as the bottom chord in Figure 9.77, or the top chord in Figure 9.79.

Example 3

You are given the truss in Figure 9.80. The diagonals are crossing members.

Question:
Determine all the zero-force members for the given load.

Solution:
In this truss, it is not possible to find a section across three members (that do not intersect in one point), nor is there a joint with less than two unknowns (member forces or support reactions). We therefore cannot determine the member forces with the method of sections, or with the method of joints, unless we first determine the support reactions.

For determining the zero-force members in the truss, it is enough to know that the support reaction at the point of the roller is vertical, so that $N^2 = 0$. This means that $N^7 = 0$, and so forth (see Figures 9.81a–9.81d). We subsequently discover that members 10, 12 and 13 are zero-force members.

Determining the other member forces is now a relatively simple task. Note that the force flow does not change when the crossing diagonals are joined at the point where they cross (rule 4).

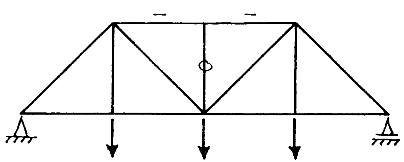


Figure 9.79 Zero-force members have a definite function in a truss. On the one hand, they ensure the truss retains its shape. On the other hand they can prevent buckling (in the plane of the structure) of (long) compressed members, such as the top chord in this truss.

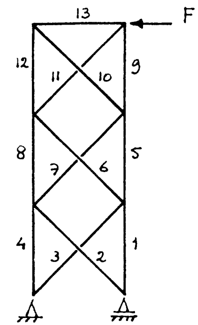


Figure 9.80 A truss in which the diagonals cross one another.

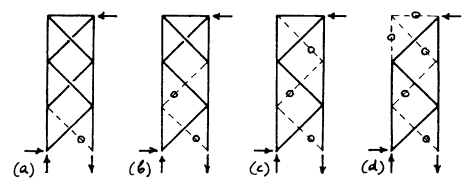
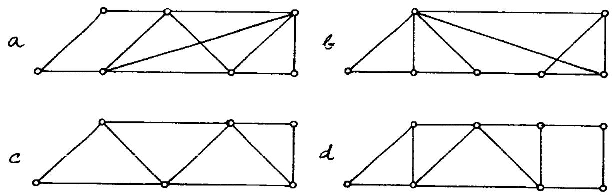


Figure 9.81 The zero-force members in the truss. (a) to (d) represent the order in which the zero-force members can be found.

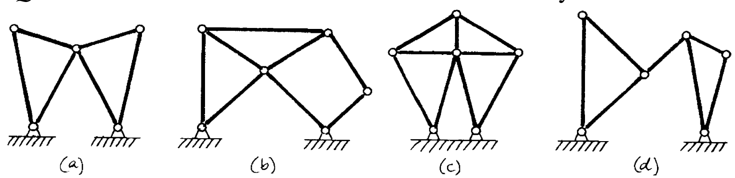
9.4 Problems

Kinematically/statically (in)determinate trusses (Section 9.2)

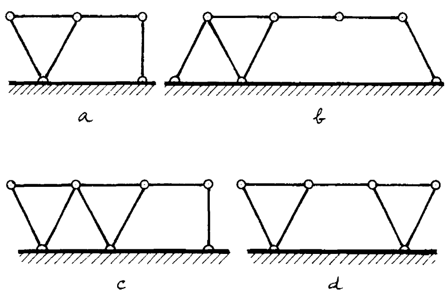
9.1 *Question*: Which of these structures retains its shape?



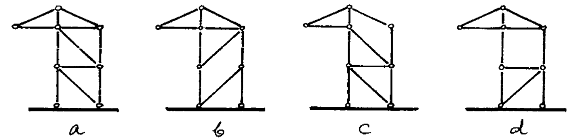
9.2 *Question*: Which of these structures is kinematically indeterminate?



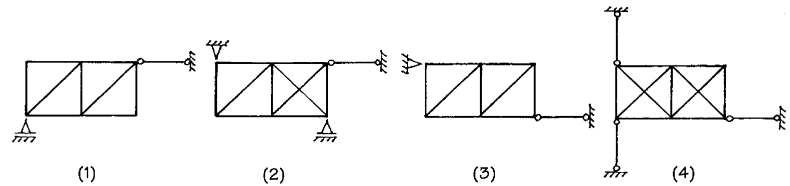
9.3 *Question*: Which structure is kinematically determinate?



9.4 *Question*: Which structure is kinematically determinate?



9.5: 1–4 You are given four simple or self-contained trusses that are supported in four different ways:



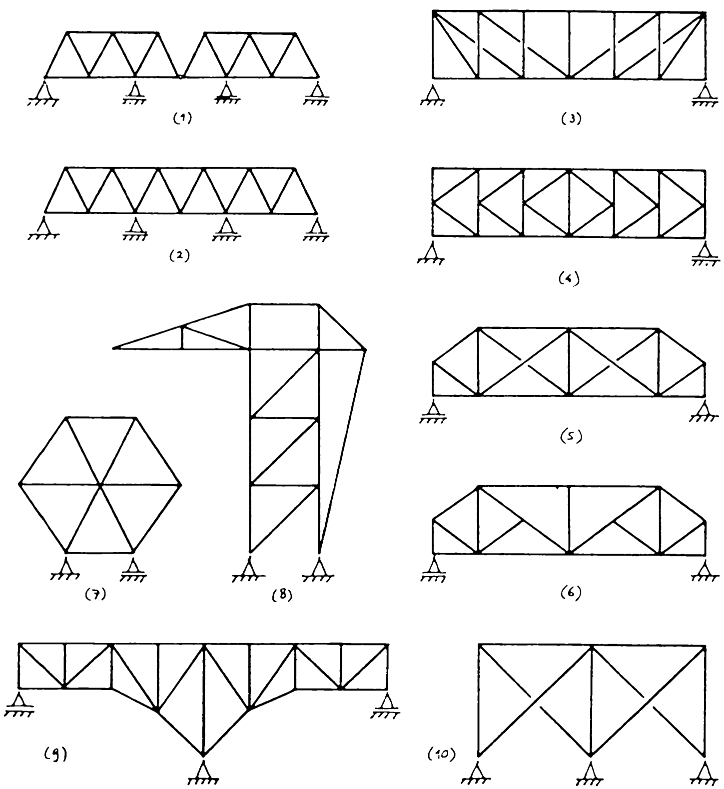
Questions:

a. What is the essential difference between a kinematically determinate and a kinematically indeterminate structure?
b. What is the essential difference between a statically determinate and a statically indeterminate structure?
c. Indicate whether the structure is
   – kinematically determinate (kd) or kinematically indeterminate (ki), and (if kinematically determinate) whether the structure is
   – statically determinate (sd) or statically indeterminate (si).

9.6: 1–10 The given trusses are kinematically determinate.

Questions (for each truss):

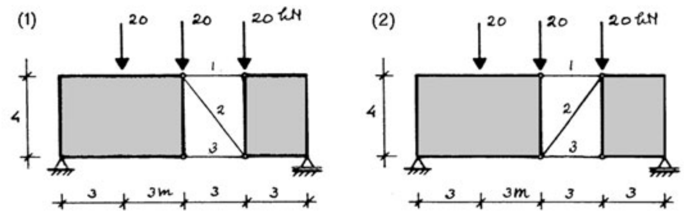
a. Is the truss statically determinate or statically indeterminate?
b. What is the degree of static indeterminacy if the truss is statically indeterminate?



Method of sections (Section 9.3.1)

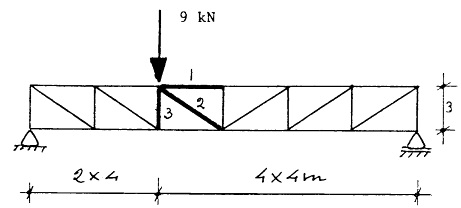
Note: Unless indicated otherwise, all structures in the problems are trusses.

9.7: 1–2 Two weightless blocks are connected by means of three bars. Structure (1) is different to structure (2) owing to the different placement of diagonal member 2.



Question: Determine the normal force in bars 1 to 3.

9.8 to 9.53 *Question*: Determine the normal force in the member(s) shown in bold.



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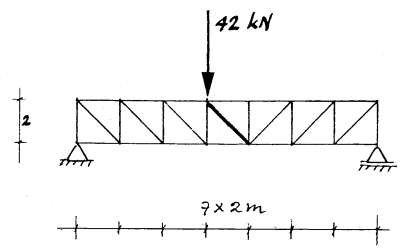

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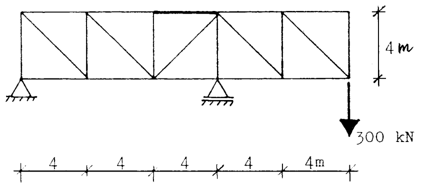

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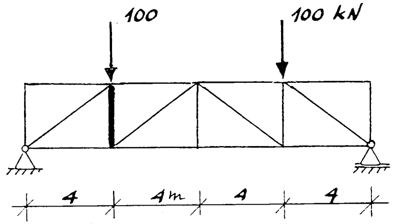

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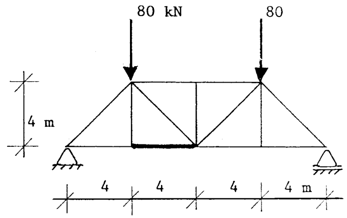

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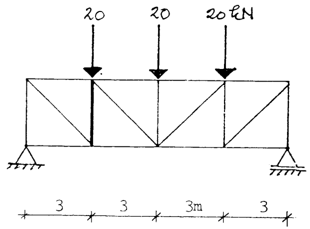

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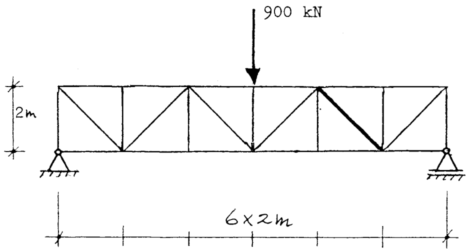

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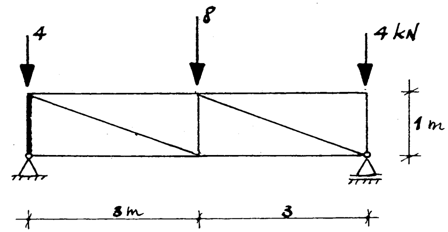

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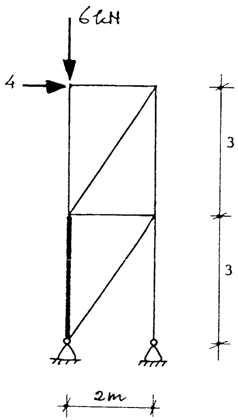

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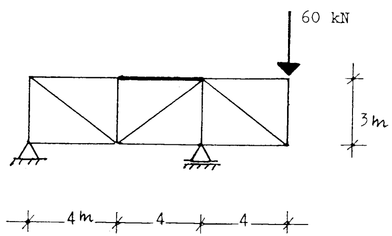

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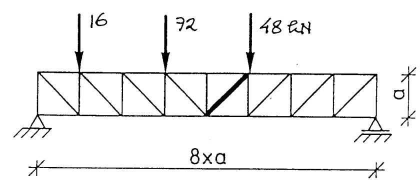

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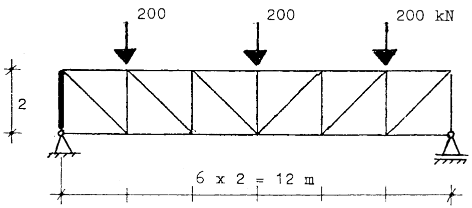

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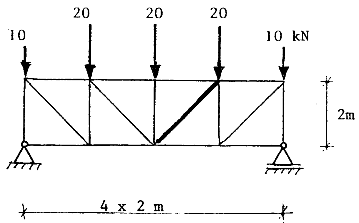

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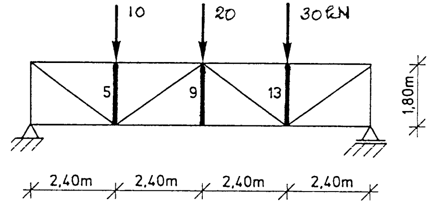

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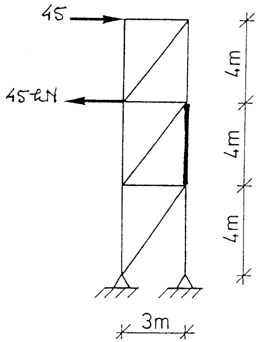

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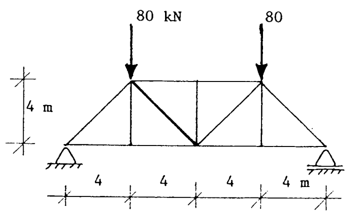

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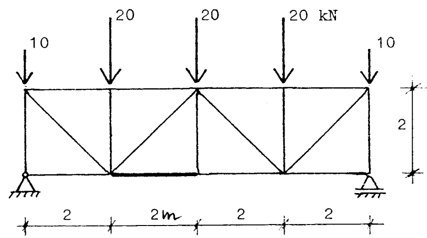

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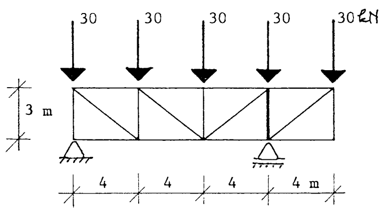

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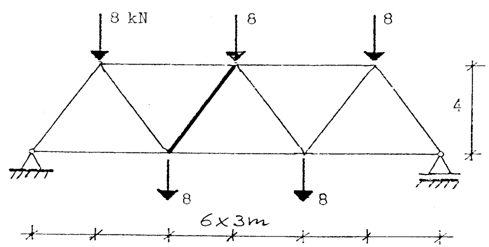

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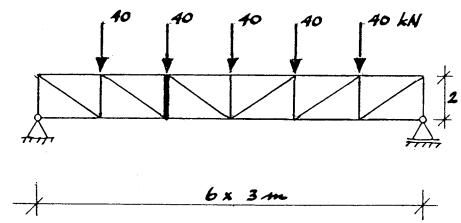

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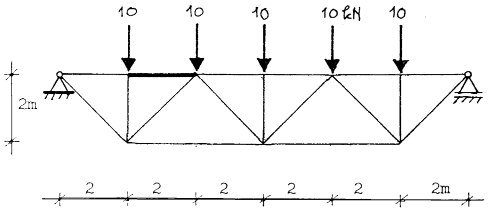

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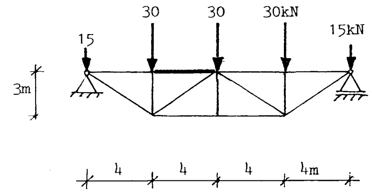

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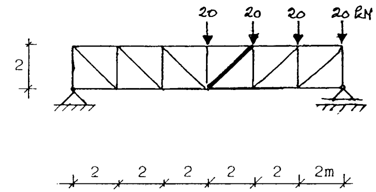

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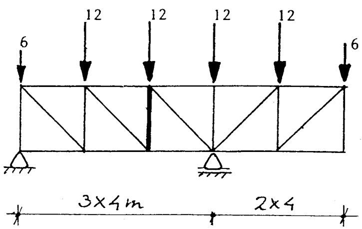

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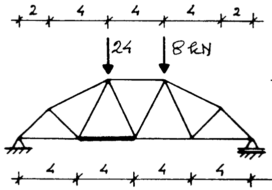

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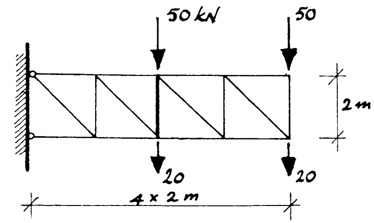

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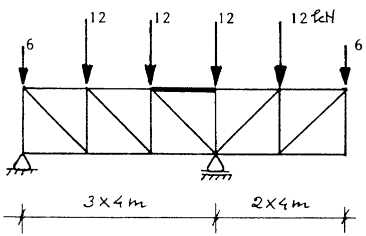

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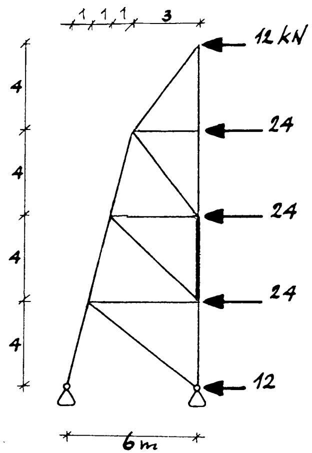

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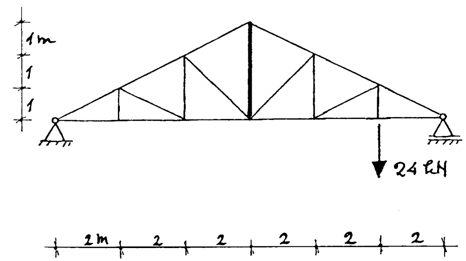

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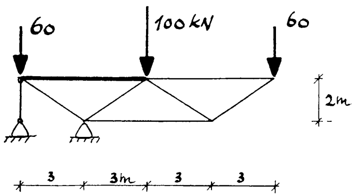

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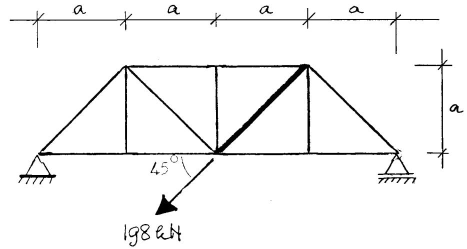

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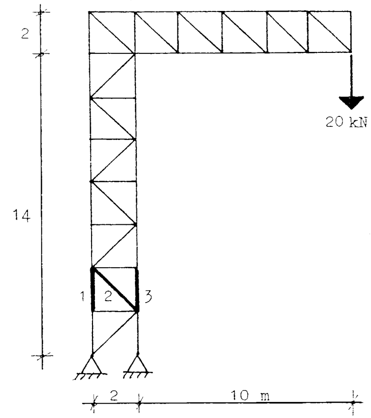

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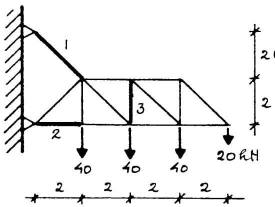

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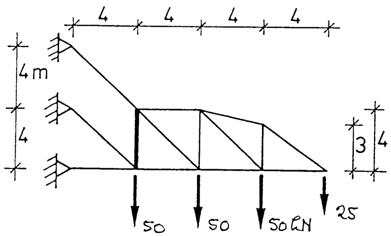

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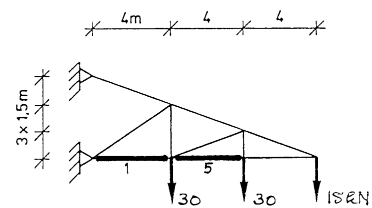

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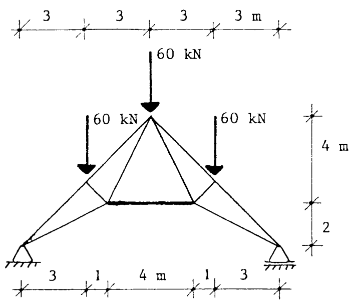

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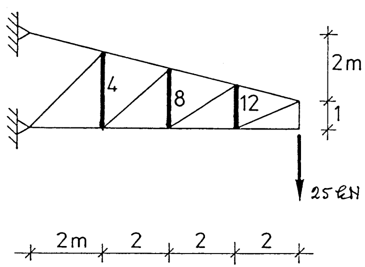

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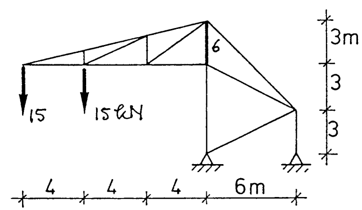

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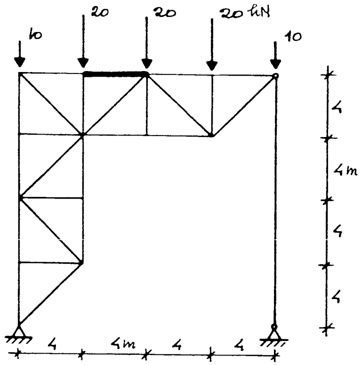

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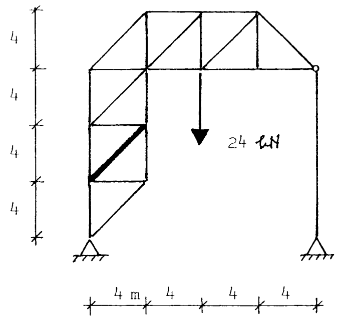

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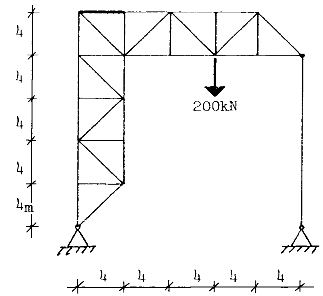

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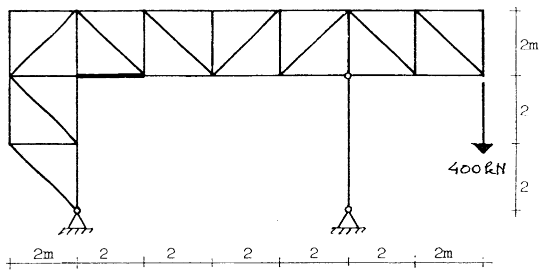

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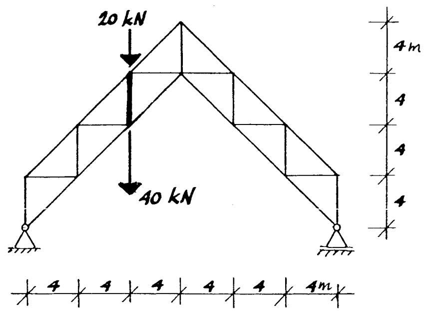

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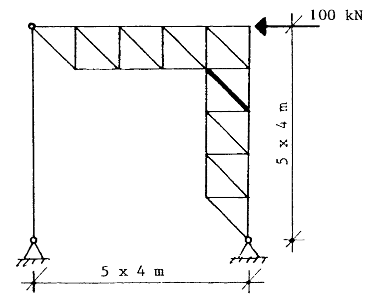

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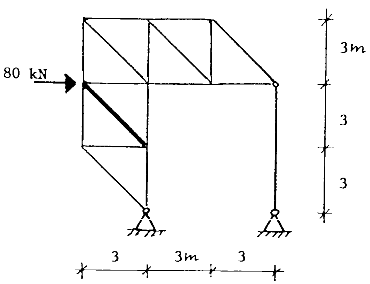
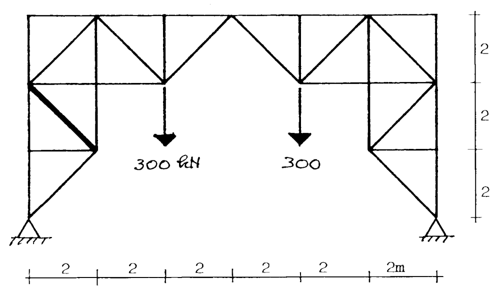


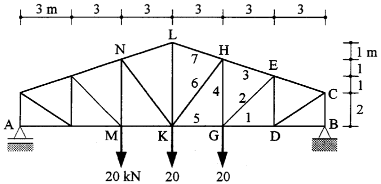
(9.52)



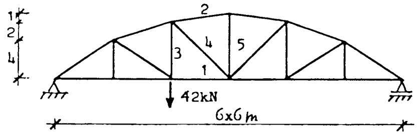
(9.53)

9.54 *Questions*:

a. Using the method of sections, determine the forces in members 1 to 7.
b. Draw the force polygon for the equilibrium of joint G. Plot the forces in the order 1, 2, 4, etc.

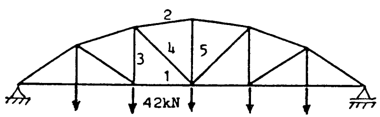


9.55 You are given a parabolic truss beam whose bottom chord is loaded by a single force of 42 kN.



Question: Using the method of sections, determine the normal force in the members 1 to 5.

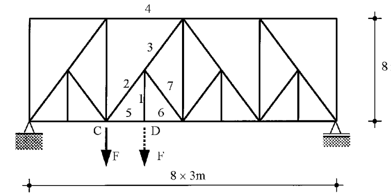
9.56 The truss from the previous problem is now loaded at the joints on the bottom chord by five equally large forces of 42 kN.



Question: Using the method of sections, determine the normal force in members 1 to 5.

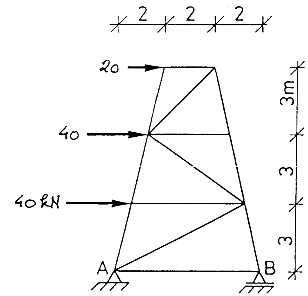
9.57 Use the method of sections to determine the normal force in members 1 to 7.

a. for $F = 160$ kN at joint C;
b. for $F = 160$ kN at joint D.

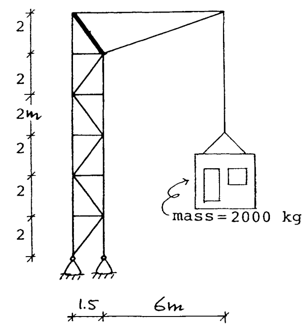


Method of joints (Section 9.3.2)

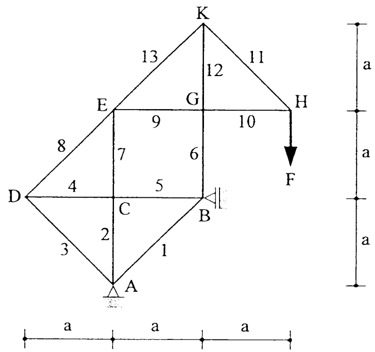
9.58 *Question*: Determine the normal force in member AB.



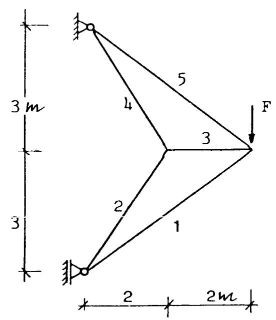
9.59 *Question*: Determine the normal force in the member shown in bold.



9.60 *Question*:
Using the method of joints, determine all the member forces.

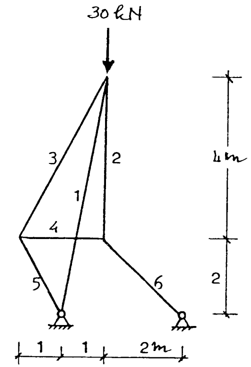


9.61 *Question*:
Using the method of joints, determine all the member forces due to $F = 6$ kN.



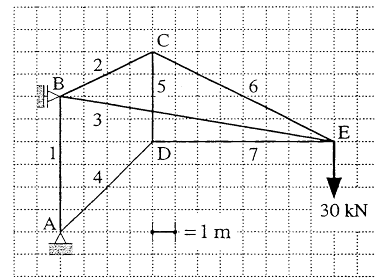
9.62 You are given a truss in which members 1 and 4 cross one another.

Question:
Using the method of joints, determine the normal forces in members 1 to 6 due to the vertical force of 30 kN in the top of the truss.

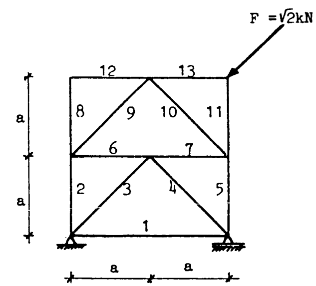


9.63 You are given a truss in which members 3 and 5 cross one another.

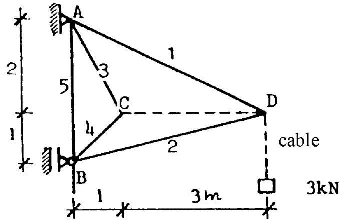
Question:
Using the method of joints, determine all the member forces. To do so, draw the force polygon for all the joints.



9.64 *Question*: Using the method of joints, determine all the member forces.

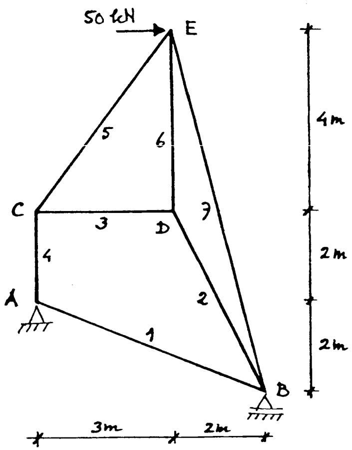


9.65 In the truss shown, the dashed line is a cable that is connected to the truss at C and runs over a pulley (without friction) at D. A load of 3 kN hangs from the cable.

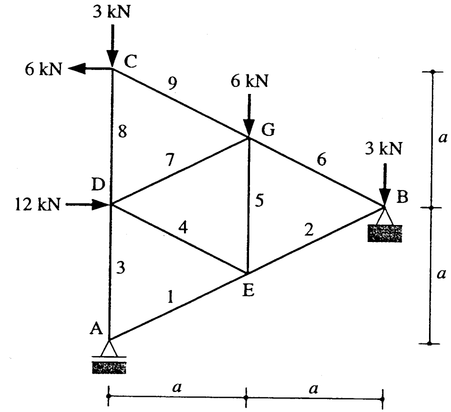


Question:
Using the method of joints, determine all the member forces.

9.66 *Questions*:
a. Using the method of joints, determine the forces in members 1 to 7.
b. Draw the force polygons for joints B and E.

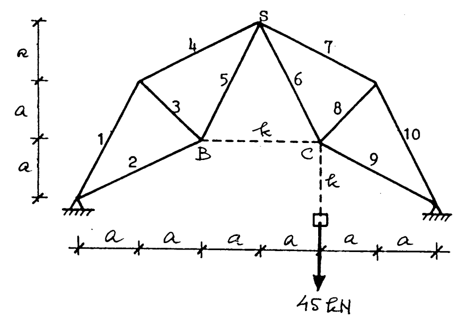


9.67 *Questions*:
a. Determine and draw the support reactions at A and B.
b. Determine all the member forces. To do so, draw the force polygon for the equilibrium for all the joints. Choose a scale of 5 mm ≡ 1 kN for the forces.



9.68 In the truss shown, the dashed line k is a cable that is joined to the truss at B and runs over a trolley (without friction) at C. The cable is loaded with a weight of 45 kN.

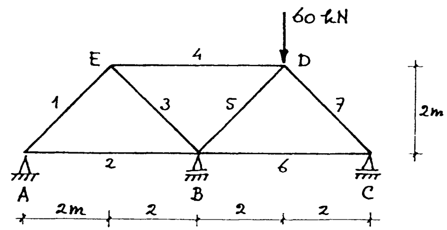
Questions:
a. Draw the forces that the cable exerts at B and C on the truss.
b. Using the method of joints, determine the forces in members 3 and 8.
c. Draw the force polygon for the equilibrium of joint C.



9.69 In the truss shown, there is a tensile force in member 4 of 20 kN: $N^4 = +20$ kN.

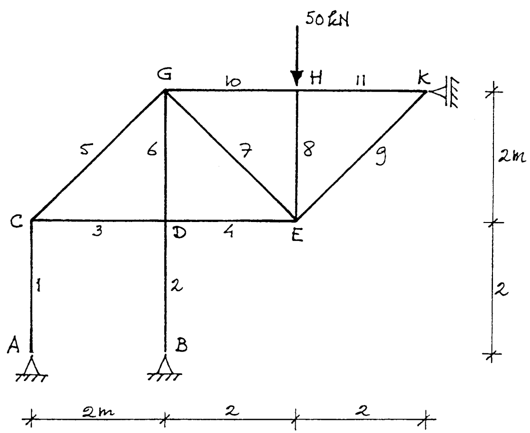
Questions:

a. Show that the truss is statically indeterminate to the first degree.
b. Using the method of joints, determine all the member forces.
c. Draw the support reactions in the direction in which they act, and give their values.
d. Draw the force polygon for joint B. Plot the forces in the order 2, 3, 5 and 6. Use 10 mm ≡ 10 kN as force scale.



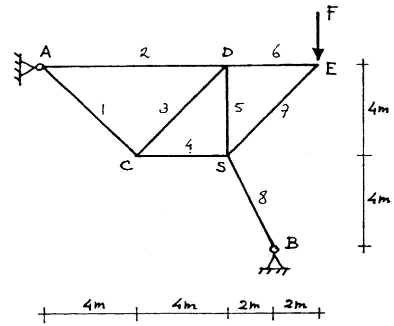
9.70 *Questions*:

a. Draw the support reactions as they act in reality on the structure and give their values.
b. Using the method of joints, determine the forces in members 1 to 11.



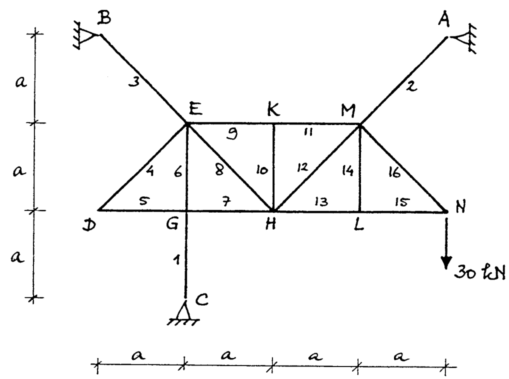
9.71 *Questions*:

a. Using the method of joints, determine all the member forces.
b. Draw the force polygon for the equilibrium in joint S.

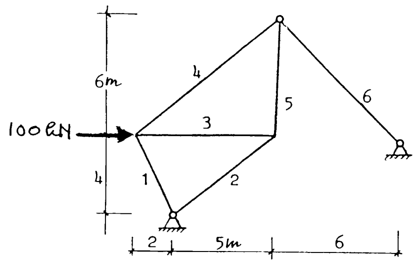


9.72 *Questions*:

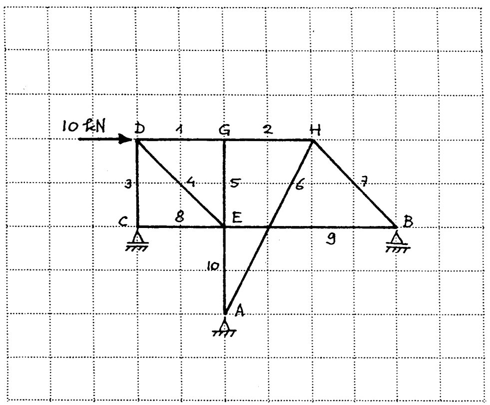
a. Determine the support reactions at A, B and C.
b. Using the method of joints, determine the forces in members 1 to 16.
c. Check the force equilibrium of joint E graphically.



9.73 *Question*: Using the method of joints, determine all the member forces.



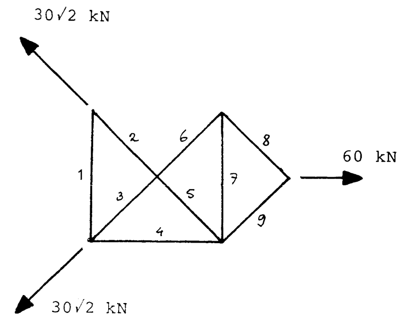
9.74 You are given a truss that is supported on a hinge at A and on rollers at B and C. The truss is loaded by means of a horizontal force of 10 kN at D. The members 6 and 9 cross one another.



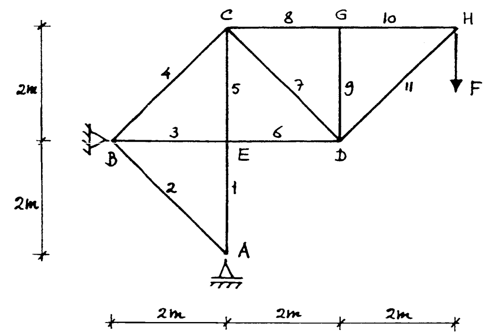
Questions:

a. First determine (as far as possible) the support reactions.
b. Determine the force in member 6.
c. Also determine all the other member forces.
d. Draw all the support reactions as they act in reality on the structure and give their values.

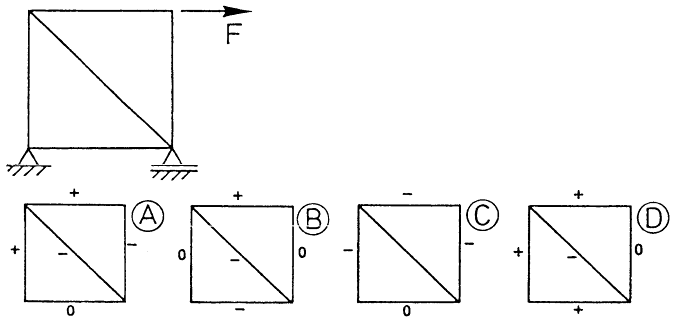
9.75 *Question*: How many tension members does this truss have due to the given load?



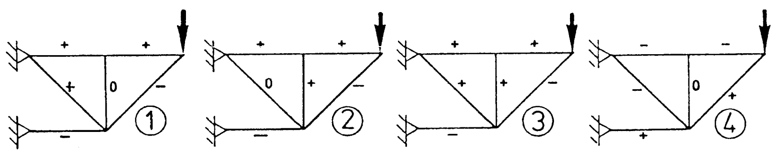
9.76 *Question*: For each of the members in the truss, indicate whether it is a zero-force member, a tension member, or a compression member. You do not have to calculate the member forces.



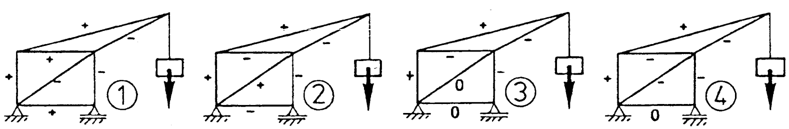
9.77 *Question*: In which figure are the correct signs for the member forces given?



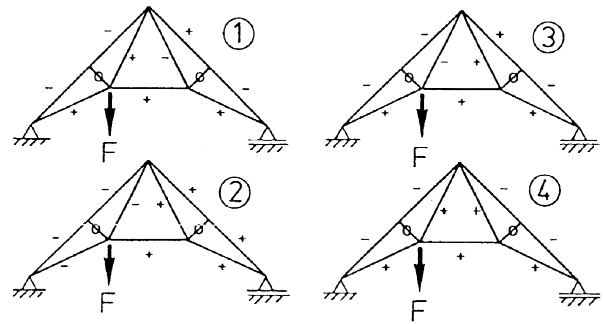
9.78 *Question*: In which figure are the correct signs for the member forces given?



9.79 *Question*: In which figure are the correct signs for the member forces given?

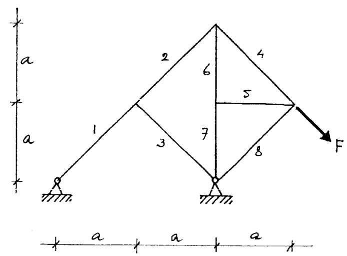


9.80 *Question*: In which figure are the correct signs for the member forces given?

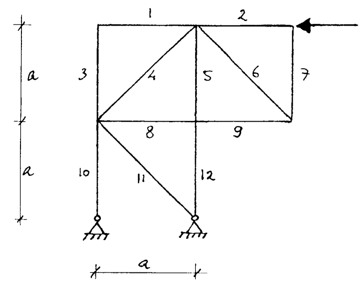


Zero-force members (Section 9.3.3)

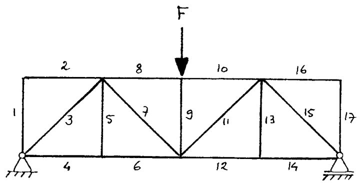
9.81 to 9.92 *Question*: Which of the members are zero-force members?



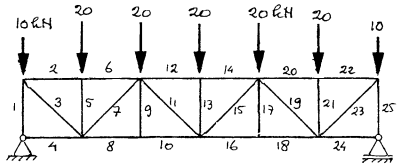
(9.81)



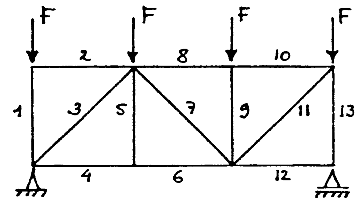
(9.82)



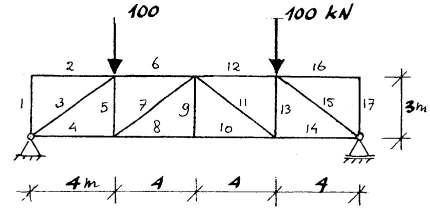
(9.83)



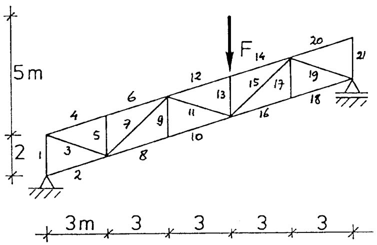
(9.84)



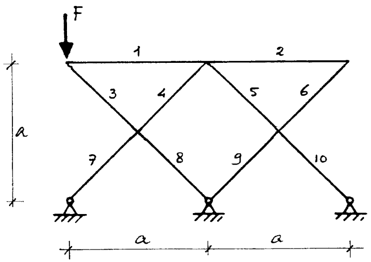
(9.89)



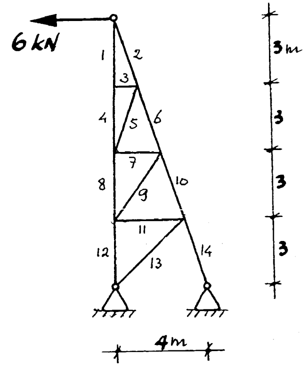
(9.90)



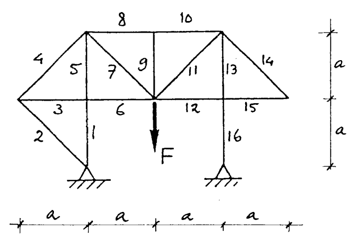
(9.85)



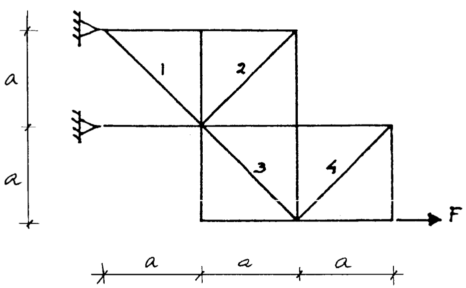
(9.86)



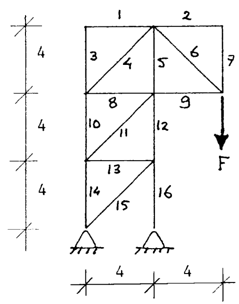
(9.91)



(9.92)



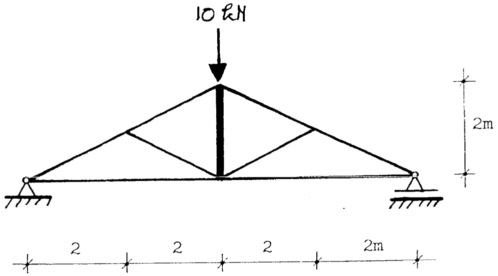
(9.87)



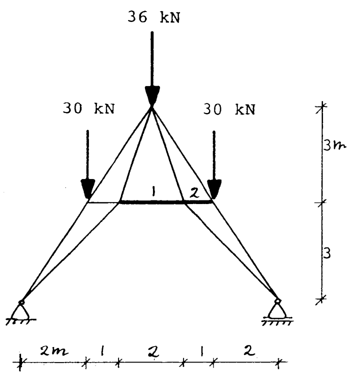
(9.88)

Mixed problems (Section 9.3)

9.93 *Question*: Determine the normal force in the vertical shown in bold.

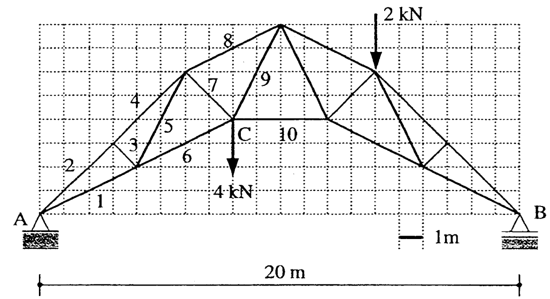


9.94 *Question*: Determine the normal force in members 1 and 2 shown in bold.



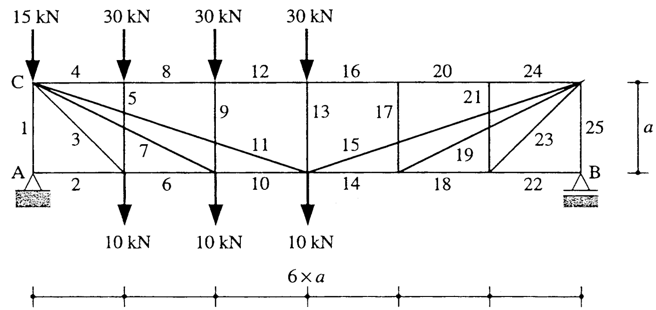
9.95 *Questions*:

a. Determine the member forces 1 to 10, with the correct sign for tension and compression.
b. Draw the force polygon for joint C in the order 6, 7, 9 and 10.



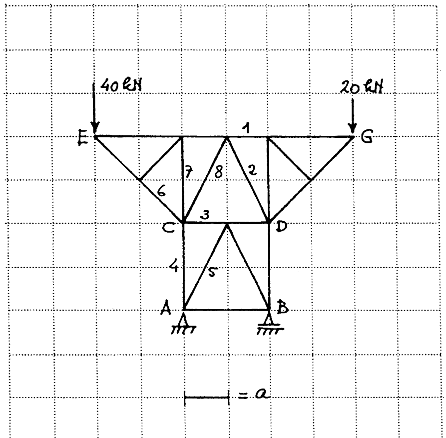
9.96 *Questions*:

a. Determine all the member forces.
b. Draw the force polygon for joint C.



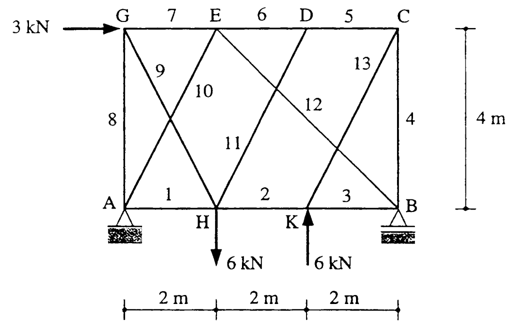
9.97 *Questions*:

a. Using the method of sections, determine the forces in members 1 to 4.
b. Using a method of your own choice, determine the forces in members 5 to 8.
c. Draw the force polygon for joint C. Plot the forces in the order 3, 4, 6, 7 and 8. Use a force scale of 10 mm ≡ 10 kN.
d. How many zero-force members are there in the truss? Indicate them (clearly) in the truss.



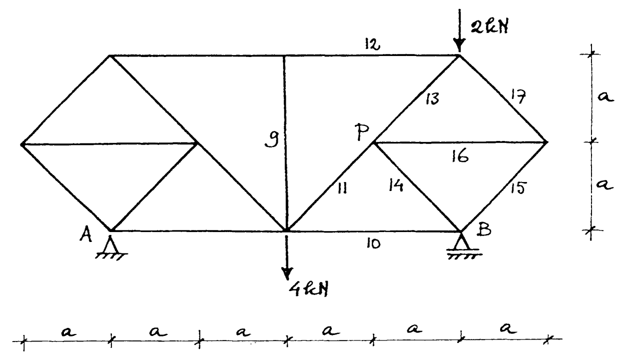
9.98 *Questions*:

a. The truss is kinematically determinate. What does that mean?
b. Show that the truss is statically determinate.
c. Determine all the member forces.
d. Draw the force polygon for the equilibrium of joint E, in the order 6, 7, 10 and 12. Use a scale of 5 mm ≡ 1 kN for the forces.



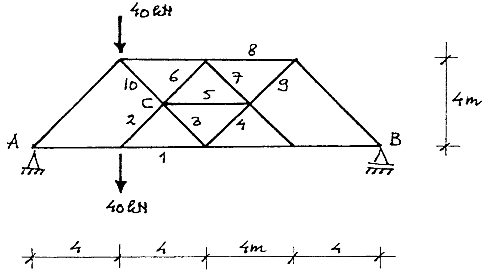
9.99 *Questions*:

a. Determine the forces in the members 9 to 17.
b. Draw the force polygon for the equilibrium at joint P.

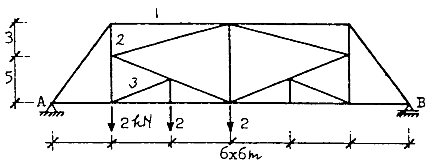


9.100 *Questions*:

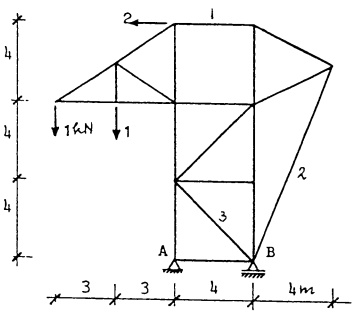
a. Determine the forces in the members 1 to 9.
b. Draw the force polygon for the equilibrium at joint C. Plot the member forces in the order 2, 3, 5, 6 and 10.



9.101: 1–2 *Question*: Determine the forces in the members 1 to 3.



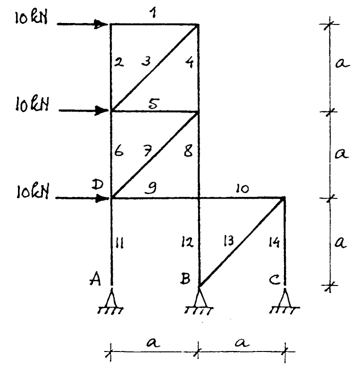
(1)



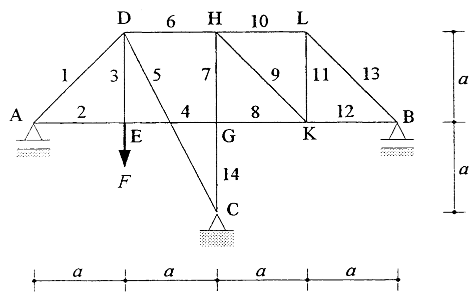
(2)

9.102 *Questions*:

a. Determine the support reactions at A, B and C. Draw them as they act in reality on the structure and write down their values alongside.
b. Determine the forces in the members 1 to 14.
c. Draw the force polygon for the equilibrium of joint D. Plot the forces in the order 11, 9, 7 and 6.



9.103 You are given a truss in which the members 4 and 5 cross one another.



Questions:

a. Determine the member forces.
b. Draw the support reactions as they are acting in reality.
c. Draw the force polygon for joint H. Plot the member forces in the order 6, 7, 9 and 10.