7

Gas Pressure and Hydrostatic Pressure

Sometimes, an important part of the loading on a structure consists of gas pressure (such as with an air-supported hall or pneumatic structure) or hydrostatic pressure (such as with lock-gates, barrages or reservoirs). We look at this type of loading more closely in this chapter.

Because of the loose structure of material particles in stationary gases and fluids, there are no shear stresses. *As a result, the stresses in stationary gases and fluids always act normal to any bounding plane*.

In Section 7.1, we will show that, if there are no shear stresses, the stress at a particular point is independent of the orientation of the plane on which the stress is acting. This property is known as *Pascal's Law*. Such a stress situation is known as an *isotropic or spherical state of stress*.

Sections 7.2 and 7.3 provide examples of structures on which the loading is caused by gas and hydrostatic pressures respectively. The difference between the two is that the pressure in a gas is constant within the closed space in question.[1] In a fluid, the pressure increases linearly with depth due to its dead weight. The latter is referred to as a *hydrostatic pressure distribution*.

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[1] Other conditions apply when looking at the air pressure within the earth's atmosphere, for example; it depends on the distance to the surface of the earth, and is influenced by currents (wind).

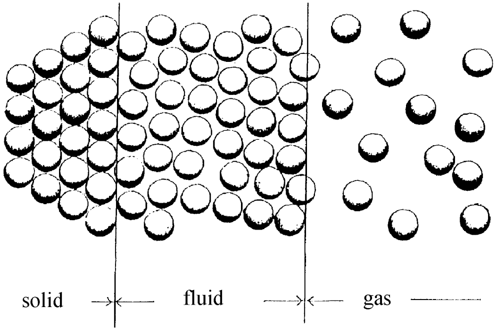

Figure 7.1 Because of the loose structure of the material particles in stationary gases and fluids, there are no shear stresses.

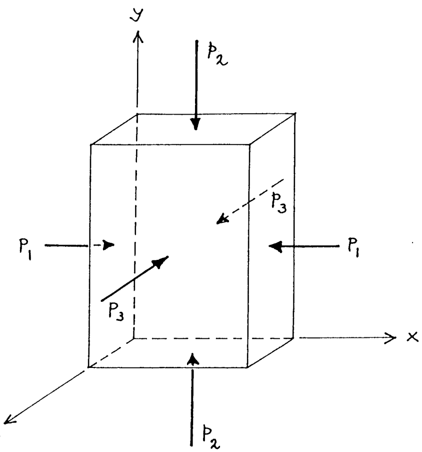

Figure 7.2 A small, rectangular volume element isolated from a gas or fluid, with compressive forces p_1; p_2; p_3 on its boundaries. Since there are no shear stresses, the stresses are normal to the sides in question. Here, the arrows should not be interpreted as forces.

7.1 Pascal's law – All-round pressure

Gases and fluids are distinct from solids in that they lack a solid shape. They can flow and adapt their shape to the environment. As such, gases do not have their own volume: all gas quantities distribute themselves throughout the available space. One of the reasons for this is their loose particle structure (see Figure 7.1).

Because of the weak bonding, gas and fluid particles can easily move with respect to one another. As a result, we could (rather boldly) state that no shear stresses can be transmitted in gases and fluids. This is, however, not the case with *flowing* gases and fluids; because of the differences in speed between adjacent layers, shear stresses can occur, although they are far weaker than in solids.

Below, it is assumed that no shear stresses occur in gases and fluids at rest. *This means that the stresses in stationary gases and fluids always act normal to any bounding plane.*

In Figure 7.2, a rectangular volume element has been isolated from a gas or fluid. Compressive stresses p_1; p_2; p_3 act on the boundary of the element. The volume element is so small that, for all the stresses on the boundary, it can be assumed that they are uniformly distributed. In that case, one does not have to draw the entire stress distribution, but a single arrow[1] is sufficient.

The condition that no shear stresses can act in the material implies that the stresses on the boundary of the volume element have to be of the same magnitude:

$$p_1 = p_2 = p_3.$$

[1] Note: the arrows here cannot be interpreted as forces.

To demonstrate this, a small triangular part has been isolated from the material parallel to the xy plane in Figure 7.3. The oblique side has an area ΔA. The area of the vertical side is therefore $\Delta A \cos\alpha$, while that of the horizontal side is $\Delta A \sin\alpha$. The triangular part is so small that, for all the stresses on the boundary, it can be assumed that they are uniformly distributed. Assume that a compressive stress p is acting on the oblique side. This stress acts normal to the side as there is no shear stress.

In Figure 7.4, the forces (force = stress × area) on the edges of the triangular part are shown. The lines of action of the forces pass through a single point. This means that there is moment equilibrium in the xy plane. Here it is assumed that the element is so small that its dead weight can be neglected.

The equations for the force equilibrium in respectively the x and y direction are

$$\sum F_x = p_1 \Delta A \cos\alpha - p\Delta A \cos\alpha = 0,$$

$$\sum F_y = p_2 \Delta A \sin\alpha - p\Delta A \sin\alpha = 0,$$

so that

$$p_1 = p_2 = p_3.$$

The result is independent of angle α.

In the same way, using the equilibrium of a triangular section parallel to the xz plane, we derive

$$p_1 = p_3 = p.$$

This means that the stress at a particular point is independent of the orientation of the plane that the stress acts on. This characteristic is known as

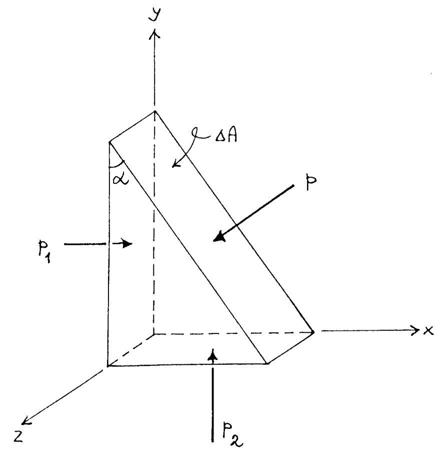

Figure 7.3 Stresses normal to the sides of a triangular volume element.

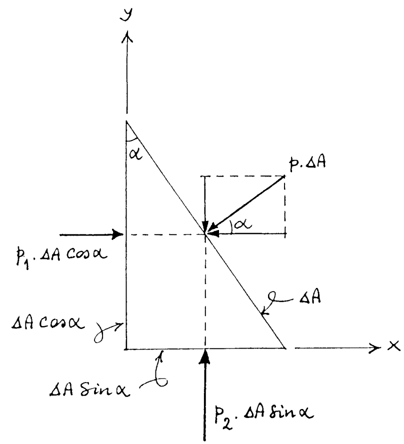

Figure 7.4 Forces on the sides of the triangular volume element.

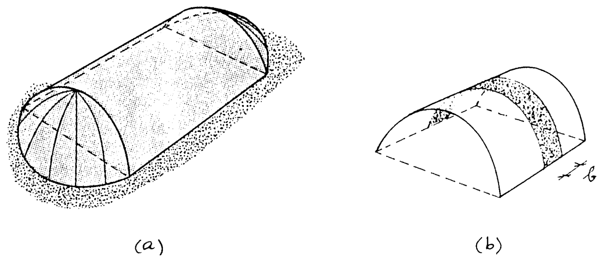

Figure 7.5 (a) A pneumatic structure consists of a membrane that maintains its shape through internal overpressure. (b) A strip with width *b* from the circular cylindrical midsection in more detail.

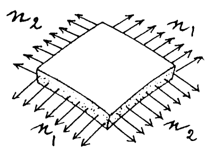

Figure 7.6 A membrane can transfer forces only in the direction of its curved plane.

Pascal's Law.[1] This state of stress is known as *isotropic* or *spherical*. With gases and fluids, in which only compressive stresses occur, we also speak of *all-round pressure*.

7.2 Working with gas pressures

The type of structure in Figure 7.5a which is sometimes used as a tennis hall, is an *air-supported hall* or *pneu*. Pneu is an abbreviation of *pneumatic structure*. This type of structure consists of a thin, flaccid skin (*membrane*), which can transfer tensile forces only in its curved plane (see Figure 7.6). The structure maintains its shape through internal *overpressure*.[2] The same holds, for example, for an inflated balloon, or the inner tube of a bike. We will look at three examples for this type of structure. In the first two examples the load is a gas pressure (the overpressure in the pneu). The third example concerns a body subjected to an all-round pressure.

Example 1

The pneu in Figure 7.5a consists of a circular cylindrical midsection that is closed by means of spherical ends. The diameter of the circular cylinder is r, the aperture angle is α, and the internal overpressure is p.

Question:

Determine the distributed support reactions (forces per length) for the circular cylindrical midsection of the pneu.

Solution:

In Figure 7.5b, a strip of width b has been isolated. This strip is modelled in Figure 7.7a as a curved line element with a distributed load pb. In addition

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1 Blaise Pascal (1623–1662), French mathematician, physicist and writer. With Fermat, he was one of the founders of the theory of probability. As a writer he is known for his Pensées, a collection of loose notes published posthumously.

2 This is the difference between the pressure inside and outside the structure.

to the support reactions at A and B, no forces other than those shown in the figure act in the plane of the drawing.

Since a membrane can transfer forces only in its (curved) plane, the support reactions at A and B act along the tangents of the circular cross-section (see Figure 7.7b). Because of mirror symmetry, the support reactions at A and B are of equal magnitude. Assume these are tensile forces N. With an aperture angle α, the horizontal and vertical components of N are:

$$N_{\mathrm{h}} = N\cos\alpha,$$

$$N_{\mathrm{v}} = N\sin\alpha.$$

The vertical component N_{v} can be derived from the vertical force equilibrium. In doing so, a tricky point is that the distributed load pb changes direction. The calculation can, however, be considerably simplified by isolating the structure from its surroundings, not "*by itself*", but "*with content*" (see Figure 7.7c). The overpressure on plane AB (with width b) is equal to pb. If the dead weight of the gas and the membrane can be ignored, the equation for the vertical force equilibrium is

$$2N_{\mathrm{v}} - pb \cdot 2r\sin\alpha = 0$$

so that

$$N_{\mathrm{v}} = pbr\sin\alpha.$$

The horizontal component of N is then

$$N_{\mathrm{h}} = pbr\cos\alpha$$

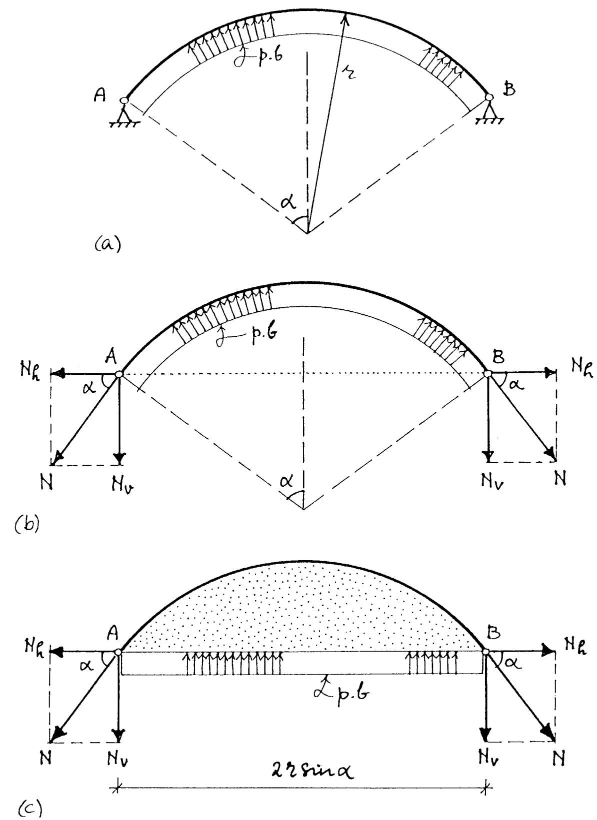

Figure 7.7 (a) A strip with width b from the cylindrical midsection of the pneu, modelled as a line element. (b) The support reactions at A and B. (c) The vertical component of the support reactions at A and B are found from the vertical equilibrium of the "strip with content".

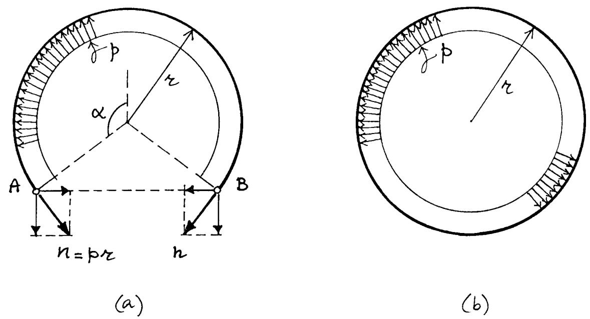

Figure 7.8 (a) The membrane force $n = pr$ in the circumferential direction is independent of the aperture angle α. (b) The formula $n = pr$ is also referred to as the boiler formula as calculating the force in the walls of a steam boiler was an important field of application.

Figure 7.9 A spherical pneu designed by Frei Otto to cover a settlement in Antarctica.

and the resulting support reaction is

$$N = pbr.$$

The calculation relates to a strip with width b. The requested support reactions per length are:[1]

$$n = \frac{N}{b} = pr.$$

Note that the magnitude of the force $n = pr$ is independent of the aperture angle α (see Figure 7.8a). Obviously, in the circular cylinder pneu, the (distributed) circumferential tensile forces have the same magnitude everywhere. The formula is also applicable for a closed ring (see Figure 7.8b, where $\alpha = 180°$) and is known as the *boiler formula*, as calculating forces in the walls of a steam boiler was an important field of application.

Example 2
The second example relates to a pneu designed by architect Frei Otto[2] to cover a settlement in Antarctica (see Figure 7.9). This design is discussed in his book "*Zugbeanspruchte Konstruktionen*".

The pneu is shaped like a segment of a sphere and rests on a concrete ring beam. The diameter of the sphere is $r = 2200$ m. The diameter of the ring beam is $r_{\text{beam}} = 1000$ m. The segment of the sphere is 240 m high (see Figure 7.10). The weight of the roof is 82 N/m^2. The pneu maintains its shape through an internal overpressure of 350 N/m^2.

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1 It is the convention to use a lower case letter for distributed forces.

2 Frei Otto (1925), German architect. Renowned designer of pneumatic structures and cable networks. One of his most famous designs was the roof of the Olympic Stadium in Munich (1972). Also see Chapter 14, Section 14.3, Example 4.

Questions:

a. Determine the weight of the ring beam so that the foundation is not subjected to tension.
b. Determine the compressive force in the ring beam.

Solution:

a. To calculate the forces in the pneu, it is assumed that the dead weight of the roof acts in the direction of the centre of the sphere instead of the centre of the earth. This assumption introduces only a minor discrepancy. In this case, the resulting overpressure in the pneu is

$$p = 350 - 82 = 268\ \text{N/m}^2.$$

This overpressure generates tensile forces n (forces per length) in the membrane. The vertical component n_v can be deduced from the vertical force equilibrium of the segment of the sphere "*with content*" (see Figure 7.11). Here, n_v acts on the circumference of the ring beam and the overpressure p acts on the area within the ring beam. The equilibrium equation is

$$n_\text{v} \cdot 2\pi r_\text{beam} - p \cdot \pi r_\text{beam}^2 = 0$$

so that

$$n_\text{v} = \frac{p \cdot \pi r_\text{beam}^2}{2\pi r_\text{beam}} = \tfrac{1}{2} p r_\text{beam}.$$

With $n_\text{v} = n \sin\alpha$ and $r_\text{beam} = r \sin\alpha$, the (distributed) tensile forces are

$$n = \tfrac{1}{2} pr.$$

Here too, the (distributed) tensile forces are independent of the aperture angle α. They are, however, half as large as the circumferential tensile forces in the circular cylindrical pneu from the previous example.

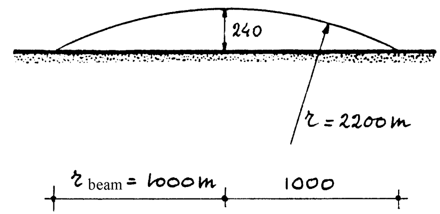

Figure 7.10 Dimensions of the spherical pneu.

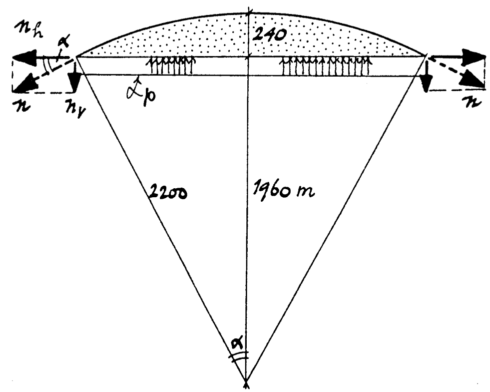

Figure 7.11 The vertical component of the membrane force is found from the vertical equilibrium of the sphere segment with content. The membrane force acts on the circumference of the ring beam. The overpressure p acts on the area within the ring beam.

In this example

$$n = \tfrac{1}{2}pr = \tfrac{1}{2} \times (268\ \mathrm{N/m^2})(2200\ \mathrm{m}) = 295\ \mathrm{kN/m}.$$

The aperture angle α is (see Figure 7.11)

$$\alpha = \arccos\left(\frac{1960\ \mathrm{m}}{2200\ \mathrm{m}}\right) = 27°.$$

At the ring beam, the horizontal and vertical components of n are

$$n_\mathrm{h} = n\cos\alpha = (295\ \mathrm{kN/m}) \times \cos 27° = 263\ \mathrm{kN/m},$$

$$n_\mathrm{v} = n\sin\alpha = (295\ \mathrm{kN/m}) \times \sin 27° = 134\ \mathrm{kN/m}.$$

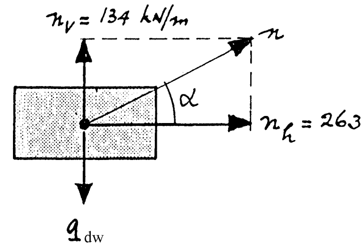

Figure 7.12 The forces acting on the ring beam.

Figure 7.12 shows all the forces acting on the ring beam. These are the distributed force n, which the pneu exerts on the ring beam, and the dead weight q_dw of the ring beam (also a force per length).

The vertical component n_v tries to lift the ring beam. In order to prevent this, the dead weight q_dw has to be larger than $n_\mathrm{v} = 134$ kN/m. If the ring beam is made of concrete, with a specific weight of 24 kN/m^3, then the cross-section A of the beam has to obey

$$q_\mathrm{dw} = A \times (24\ \mathrm{kN/m^3}) \geq n_\mathrm{v} = 134\ \mathrm{kN/m} \Rightarrow A \geq \frac{134\ \mathrm{kN/m}}{24\ \mathrm{kN/m^3}} = 5.6\ \mathrm{m^2}.$$

The cross-section of the ring beam has to be at least 5.6 m^2.

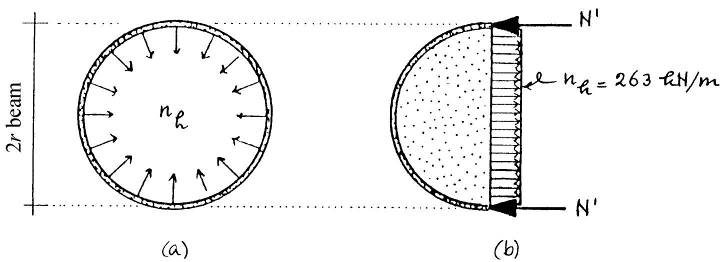

Figure 7.13 (a) Due to the horizontal component of the membrane force, the ring beam is pulled inwards on all sides. Here we can recognise the analogy of a closed ring with underpressure. (b) Compressive forces are generated in the ring. They can be determined using the boiler formula, or directly from the equilibrium of half a ring beam.

b. Due to the horizontal forces n_h, the ring beam is pulled inwards from all sides (see Figure 7.13a). Here, you will recognise the loading case of the closed ring from Figure 7.8b, but now with an *underpressure* instead of an *overpressure*.

A compressive force N'[1] is formed in the ring. This can be calculated using the *boiler formula* from the previous example, or directly from the equilibrium of the half ring beam in Figure 7.13b:

$$N' = \frac{2r_{\text{beam}}n_{\text{h}}}{2} = r_{\text{beam}}n_{\text{h}} = (1000\text{ m})(263\text{ kN/m}) = 263\text{ MN}.$$

Comment: This force is relatively large for a concrete cross-section of 5.6 m^2. The compressive force in the ring may therefore call for a larger cross-section.

Example 3

A uniformly distributed load q is acting on the plane body in Figure 7.14. The load acts in the plane of the body along the entire outline and normal to the body.

Questions:

a. Show that the resultant of the distributed load on the body is zero, regardless of the shape of the body.
b. Determine the resultant of the load above section AB.

Solution:

a. In Figure 7.15a, a minor force ΔF is acting perpendicular to the given boundary element with small length Δs:

$$\Delta F = q\Delta s.$$

The horizontal and vertical components of ΔF are respectively

$$\Delta F_x = q\Delta s\cos\varphi,$$

[1] The convention is that N as tensile force is positive. The prime for a switch in sign indicates that compressive forces are now positive (see Section 6.5).

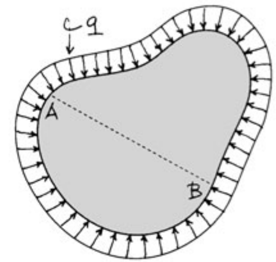

Figure 7.14 A plane body loaded over its entire outline by a uniformly distributed load normal to the body.

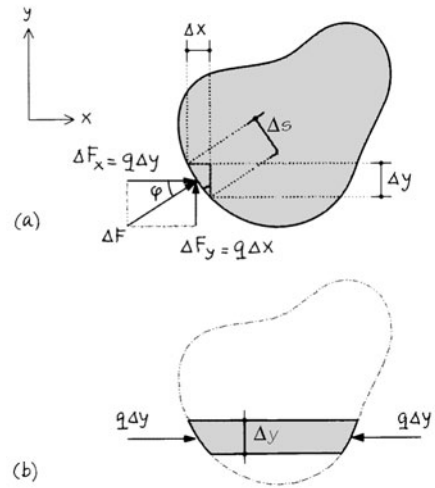

Figure 7.15 (a) As a result of the distributed load q, a small force $\Delta F = q\Delta s$ is acting on a small boundary element with length Δs. (b) The horizontal components of the load on the boundary elements of a horizontal strip are equal and opposite. Together they form an equilibrium system with zero resultant.

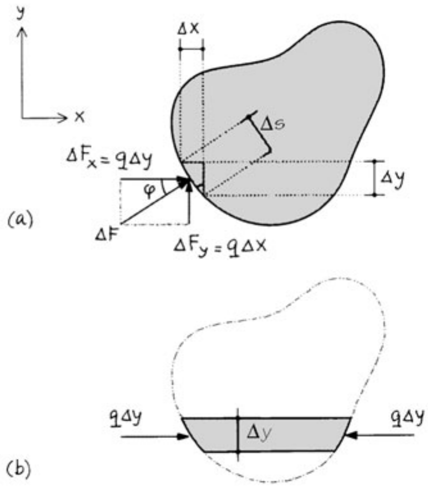

Figure 7.15 (a) As a result of the distributed load q, a small force $\Delta F = q\Delta s$ is acting on a small boundary element with length Δs. (b) The horizontal components of the load on the boundary elements of a horizontal strip are equal and opposite. Together they form an equilibrium system with zero resultant.

$$\Delta F_y = q\,\Delta s \sin\varphi.$$

Since $\Delta s \cos\varphi = \Delta y$ and $\Delta s \sin\varphi = \Delta x$, we can also write

$$\Delta F_x = q\,\Delta y,$$

$$\Delta F_y = q\,\Delta x.$$

The components ΔF_x and ΔF_y of force ΔF on boundary element Δs are equal to the product of the distributed load q and the projection of Δs on the y axis and the x axis respectively.

Figure 7.15b shows a horizontal strip from the body with a small width Δy. The horizontal components of the load on the boundary elements are equal and opposite. They form an equilibrium system with resultant zero. Since this applies to all the horizontal strips of which the body is composed, the resulting horizontal load on the body is zero.

By dividing the body into vertical strips, and looking at the vertical component of the load on the boundary elements, we can similarly deduce that the resulting vertical load on the body is zero.

Conclusion: *If a uniformly distributed load acts on a plane body in the plane of the body along its entire outline, and everywhere normal to the body, the load forms an equilibrium system with resultant zero.*

One can show that this is also true in three-dimensional cases: *If a uniformly distributed load acts on a body in space on its entire surface, and everywhere normal to the body, the load forms an equilibrium system with resultant zero.*

b. In Figure 7.16a, the part of the body above section AB has been isolated. Assume the resultant of the distributed load on the outside between A and B is R.

If a uniformly distributed load q is also applied to section AB, as in Figure 7.16b, the total load on the isolated part of the body forms an equilibrium system: the resultant R of the load on the outside of the body is equal and opposite to the resultant R_{section} of the load on the section. Therefore $R = R_{\text{section}} = qa$, in which a is the length of the section. The line of action of R coincides with the perpendicular bisector of AB.

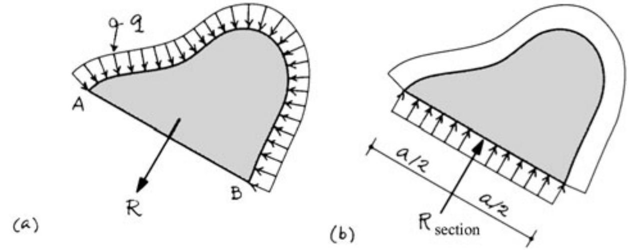

Figure 7.16 (a) The resultant R of the uniformly distributed load on the outside AB is equal and opposite to (b) the resultant R_{section} of an equally large uniformly distributed load on section AB.

7.3 Working with hydrostatic pressures

In a fluid at rest, the (all-round or isotropic) pressure increases linearly with depth. This can be derived from the vertical force equilibrium of the fluid column in Figure 7.17. Using density ρ of the fluid and the gravitational field intensity $g = 10$ N/kg the specific weight γ is

$$\gamma = \rho g.$$

The weight ΔG of the fluid column, with height z and cross-section ΔA is

$$\Delta G = \gamma z \Delta A.$$

At the base of the fluid column, there is a compressive force $p\Delta A$, the resultant of the compressive stresses p on the area ΔA. The vertical force equilibrium of the column (there are no shear stresses) now gives

$$p\Delta A = \Delta G$$

or

$$p = \frac{\Delta G}{\Delta A} = \frac{\gamma z \Delta A}{\Delta A} = \gamma z.$$

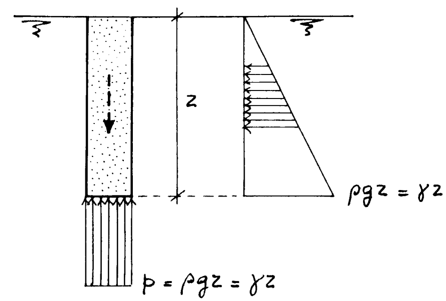

Figure 7.17 In a fluid at rest, the (all-round) pressure increases linearly with depth as a result of its dead weight. This is derived from the vertical force equilibrium of the fluid column.

The isotropic compressive stress p increases linearly with depth z. This is referred to as a *hydrostatic pressure distribution*.

Below you will find a number of examples covering loads due to a hydrostatic pressure. We assume that in all cases the *fluid is at rest* and that the pressure distribution is *hydrostatic*. At any point the *hydrostatic pressure is equally large in all directions* (isotropic state of stress) and *always acts normal to the plane in question* (as there are no shear stresses).

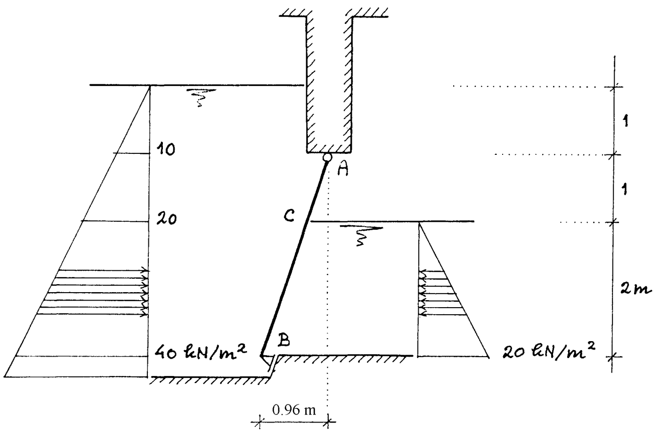

Figure 7.18 Longitudinal section of a channel with the 4-metre wide flap AB. The distribution of the water pressure is shown on both sides of the flap.

Example 1

Figure 7.18 shows the longitudinal section of a channel with a 4-metre wide flap AB. The flap is supported at A by a hinge and is resting at B on a sill. The support in B can be seen as a roller support. The water level on both sides of the flap is shown in the figure. The density of water is 1000 kg/m^3. The gravitational field intensity is 10 N/kg.

Question:
Determine the support reactions at A and B due to the total water pressure. The dead weight of the flap should be ignored.

Solution:
The linear distribution of the water pressure on both sides of the flap is shown in Figure 7.18.

To the left of the flap, the water pressure at A is

$$(1000\ \text{kg/m}^3)(10\ \text{N/kg})(1\ \text{m}) = 10\ \text{kN/m}^2$$

and at B it is

$$(1000\ \text{kg/m}^3)(10\ \text{N/kg})(4\ \text{m}) = 40\ \text{kN/m}^2.$$

To the right of the flap, the water pressure at B is

$$(1000\ \text{kg/m}^3)(10\ \text{N/kg})(2\ \text{m}) = 20\ \text{kN/m}^2.$$

In Figure 7.19a, the 4-metre wide flap is modelled as a line element, with line loads due to the water pressures normal to it.

To the left of the flap, the distributed load varies linearly from

$$(4\ \text{m})(10\ \text{kN/m}^2) = 40\ \text{kN/m at A},$$

to

$$(4\ \text{m})(40\ \text{kN/m}^2) = 160\ \text{kN/m at B}.$$

To the right of the flap, the load increases linearly from 0 at C to

$$(4\ \text{m})(20\ \text{kN/m}^2) = 80\ \text{kN/m at B}.$$

Figure 7.19b represents the load diagram for the resulting water pressure.

The length of flap AB is (see Figure 7.18)

$$\sqrt{(3\ \text{m})^2 + (0.96\ \text{m})^2} = 3.15\ \text{m}.$$

The distances BC and CA are respectively 2.10 m and 1.05 m.

To work quickly, the load diagram in Figure 7.20 has been placed horizontally and is split up into a number of areas for which the resultants can be easily calculated:

$$R_1 = (2.10\ \text{m}(80\ \text{kN/m}) = 168\ \text{kN},$$

$$R_2 = \tfrac{1}{2} \times (1.05\ \text{m})(80\ \text{kN/m}) = 42\ \text{kN},$$

$$R_3 = \tfrac{1}{2} \times (1.05\ \text{m})(40\ \text{kN/m}) = 21\ \text{kN}.$$

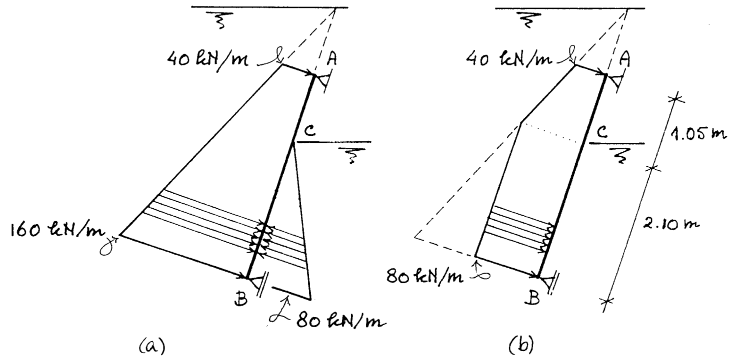

Figure 7.19 The 4-metre wide flap modelled as a line element with (a) the water pressure normal to the flap and (b) the resulting load diagram.

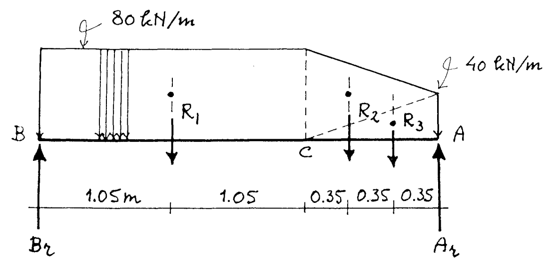

Figure 7.20 The load diagram split up into three areas for which the resultants are easy to find with respect to their magnitudes and lines of action.

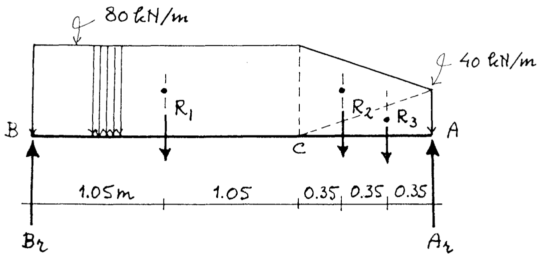

Figure 7.20 The load diagram split up into three areas for which the resultants are easy to find with respect to their magnitudes and lines of action.

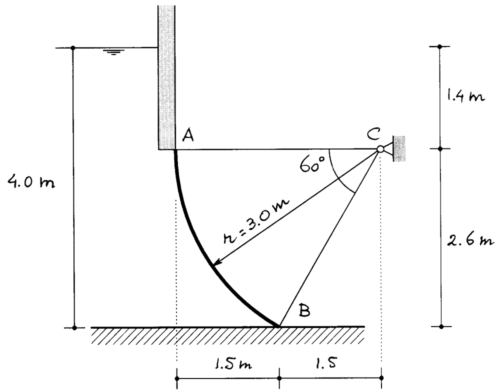

Figure 7.21 A moveable dam consisting of a circular cylindrical slide AB hinged at C and joined to an rigid vertical partition wall at A. There is no water to the right of the dam.

The support reaction A_r at A is found from the moment equilibrium of the flap about B:

$$A_r = \frac{(1.05 \text{ m}) \times R_1 + (2.45 \text{ m}) \times R_2 + (2.80 \text{ m}) \times R_3}{3.15 \text{ m}} = 107.3 \text{ kN}.$$

The support reaction B_r in B is found from the force equilibrium:

$$B_r = R_1 + R_2 + R_3 - A_r = 123.7 \text{ kN}.$$

Example 2

The moveable dam in Figure 7.21 consists of a circular cylindrical slide AB hinged at C and joining a rigid vertical partition wall at A. There is no water to the right of the dam. The specific weight of water is $\gamma_w = 10 \text{ kN/m}^3$. All other information required can be found in the figure.

Question:

Find the magnitude and direction of the resultant water pressure on a 1-metre strip from the circular cylindrical slide.

Solution:

In Figure 7.22, the 1-metre wide strip from the slide is modelled as a line element. The figure also shows the water pressure, increasing from 14 kN/m at top A of the slide to 40 kN/m near base B.

The water pressure is acting normal to the slide everywhere. In other words, all the forces on the slide pass through C, the centre of arc AB. Therefore, the resultant R of the total water pressure on the slide also passes through C.

To determine the resultant water pressure, please refer to Figure 7.23, which shows all symbols used. The water pressure as a function of φ is

$$q(\varphi) = \frac{1 + r \sin \varphi}{d} \cdot \hat{q}.$$

The resultant of the water pressure on a small part of the slide with length $r \,\mathrm{d}\varphi$ is a small force $\mathrm{d}F$:

$$\mathrm{d}F = q(\varphi) \cdot r \,\mathrm{d}\varphi$$

with components

$$\mathrm{d}F_x = \mathrm{d}F \cos \varphi = q(\varphi) r \cos \varphi \,\mathrm{d}\varphi,$$

$$\mathrm{d}F_y = \mathrm{d}F \sin \varphi = q(\varphi) r \sin \varphi \,\mathrm{d}\varphi.$$

The components R_x and R_y of the resulting water pressure are found by summing up all the contributions $\mathrm{d}F_x$, respectively $\mathrm{d}F_y$, over the length of slide AB. This summation is done by integrating between the limits $\varphi = 0$ and $\varphi = 60° = \pi/3$ rad:

$$R_x = \int_0^{\pi/3} q(\varphi) r \cos \varphi \,\mathrm{d}\varphi,$$

$$R_y = \int_0^{\pi/3} q(\varphi) r \sin \varphi \,\mathrm{d}\varphi.$$

Using the previously deduced expression for $q(\varphi)$ and the formulas in Table 7.1, the integrals are elaborated:

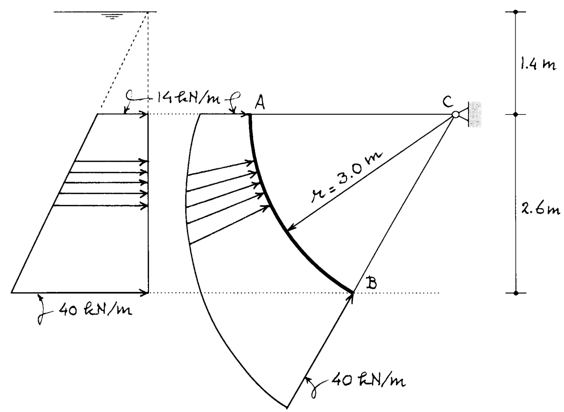

Figure 7.22 The distribution of the water pressure on a 1-metre wide strip from the slide.

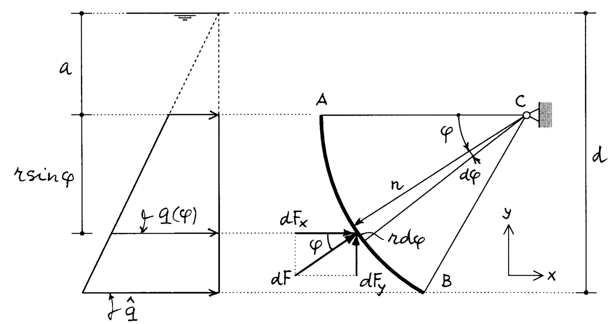

Figure 7.23 To calculate the resulting water pressure, all the data has been shown as symbols.

$$R_x = \frac{\hat{q}r}{d}\int_0^{\pi/3}(a + r\sin\varphi)\cos\varphi\,\mathrm{d}\varphi = \frac{\hat{q}r}{d}\left[a\sin\varphi + \tfrac{1}{2}r\sin^2\varphi\right]_{\varphi=0}^{\varphi=\pi/3}$$

$$= \frac{\hat{q}r}{d}(a \times 0.86 + r \times 0.375),$$

$$R_y = \frac{\hat{q}r}{d}\int_0^{\pi/3}(a + r\sin\varphi)\sin\varphi\,\mathrm{d}\varphi$$

$$= \frac{\hat{q}r}{d}\left[-a\cos\varphi + r\left(-\tfrac{1}{4}\sin 2\varphi + \tfrac{1}{2}\varphi\right)\right]_{\varphi=0}^{\varphi=\pi/3}$$

$$= \frac{\hat{q}r}{d}(a \times 0.5 + r \times 0.307).$$

Table 7.1

$\int \sin\varphi\cos\varphi\,\mathrm{d}\varphi = \frac{1}{2}\sin^2\varphi$
$\int \sin^2\varphi\,\mathrm{d}\varphi = -\frac{1}{4}\sin 2\varphi + \frac{1}{2}\varphi$

By substituting $\hat{q} = 40$ kN/m, $r = 3.0$ m, $d = 4.0$ m and $a = 1.4$ m, we find

$$R_x = 70.1\text{ kN},$$

$$R_y = 48.6\text{ kN}.$$

The vertical component of the water pressure generates an *upward* force on the slide.

The resulting water pressure R on the 1-metre strip from the slide is shown in Figure 7.24:

$$R = \sqrt{(70.1\text{ kN})^2 + (48.6\text{ kN})^2} = 85.3\text{ kN}.$$

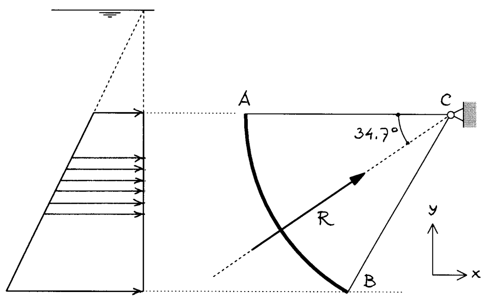

Figure 7.24 The resultant of the water pressure on the circular cylindrical slide passes through C, the centre of arc AB.

The line of action, as shown earlier, passes through C and is at an angle of α to the horizontal:

$$\alpha = \arctan\left(\frac{48.6\text{ kN}}{70.1\text{ kN}}\right) = 34.7°.$$

Alternative solution:
Since the shape of the slide is actually rather simple, the question can also be answered without integrals. To do so, Figure 7.25 shows the isolated slide including water mass ADB. Assume that $R_{h;w}$ is the resultant of the horizontal water pressure on AD and $R_{v;w}$ is the resultant of the vertical water pressure on BD:

$$R_{h;w} = \tfrac{1}{2} \times (2.6\ \text{m})\{(14\ \text{kN/m}) + (40\ \text{kN/m})\} = 70.2\ \text{kN},$$

$$R_{v;w} = (1.5\ \text{m})(40\ \text{kN/m}) = 60\ \text{kN}.$$

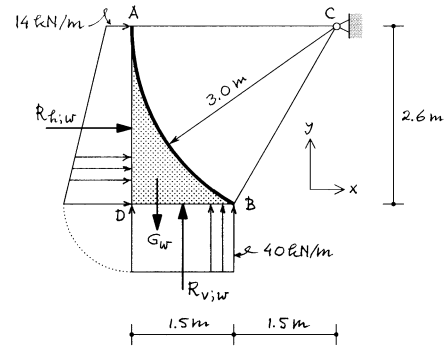

Figure 7.25 The resultant of the water pressure on the slide can also be found by looking at the forces acting on the isolated slide together with water mass ADB.

Assume that G_w is the weight of the volume of water enclosed by ADB. We are looking at a 1-metre wide strip from the slide:

$$G_w = \gamma_w A^{(ADB)}(1\ \text{m}).$$

Here $A^{(ADB)}$ is the area of ADB. This is equal to the area of trapezium ADBC, reduced by the area of circle sector ABC. The area of trapezium ADBC is

$$A^{(ADBC)} = \tfrac{1}{2} \times (2.6\ \text{m})\{(3.0\ \text{m}) + (1.5\ \text{m})\} = 5.85\ \text{m}^2.$$

The area of circle sector ABC, with an aperture angle of 60°, is equal to one sixth of the area of the entire circle:

$$A^{(ABC)} = \frac{60^\circ}{360^\circ} \times \pi(3.0\ \text{m})^2 = 4.71\ \text{m}^2.$$

With

$$A^{(ADB)} = A^{(ADBC)} - A^{(ABC)} = (5.85\ \text{m}^2) - (4.71\ \text{m}^2) = 1.14\ \text{m}^2$$

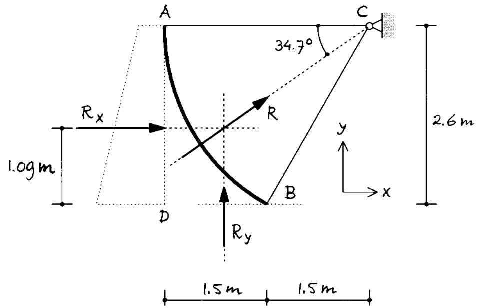

Figure 7.26 The lines of action of the horizontal and vertical component of the resulting water pressure on the slide. The horizontal component R_x is independent of the shape of the slide and can be directly found from the trapezoidal load diagram for the water pressure on the vertical AD.

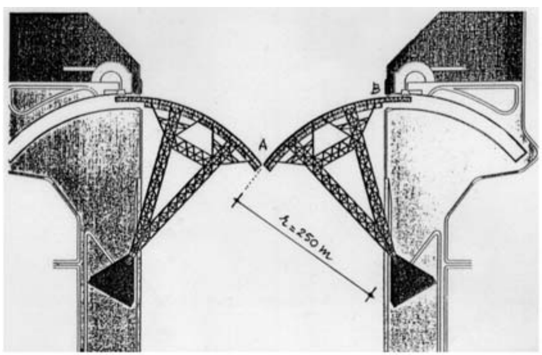

Figure 7.27 A storm barrier consisting of two sector doors with a radius $r = 250$ m.

one finds

$$G_w = \gamma_w A^{(ADB)}(1\text{ m}) = (10\text{ kN/m}^3)(1.14\text{ m}^2)(1\text{ m}) = 11.4\text{ kN}.$$

The resulting water pressure on the slide is

$$R_x = R_{h;w} = 70.2\text{ kN},$$

$$R_y = R_{v;w} - G_w = (60\text{ kN}) - (11.4\text{ kN}) = 48.6\text{ kN}.$$

The results agree with those of the first calculation, with the exception of a minor difference in the magnitude of R_x. This is because in the alternative solution, the height of the slide ($= r\sin 60°$) was rounded off to 2.6 m.

From the alternative approach, one can conclude the following: *The resultant of the horizontal water pressure on the slide is independent of the shape of the slide and is exclusively determined by the height of the slide and the depth at which it is located under the water surface.*

Figure 7.26 shows the lines of action of R_x and R_y. The line of action of R_x can be found directly from the trapezoidal load diagram on AD.[1]

Example 3

At Hoek van Holland, near Rotterdam in the Netherlands, the Maeslantkering became operational in 1997. This storm barrier in the Nieuwe Waterweg consists of two sector doors with a radius $r = 250$ m (see Figure 7.27). The arc length of AB is 209.5 m. The door is 22.5 m in height. Figure 7.28 is a sketch of the longitudinal section of the door, with the water levels on both sides. The specific weight of water is $\gamma_w = 10.25\text{ kN/m}^3$. To simplify the question, the part of the door within the parking dock is ignored.

1 The calculation is left to the reader. See Section 6.3.1, Example 1.

In addition, it is assumed that the water levels in front and behind the dam are present over the entire length of arc AB and that the pressure distribution on both sides is hydrostatic.

Question:
Determine the resulting horizontal water pressure on part AB of the right-hand sector door.

Solution:
With a specific weight of $\gamma_w = 10.25\ \text{kN/m}^3$, the water pressure increases for each metre of depth by $10.25\ \text{kN/m}^2$. At the base of the door, the water pressure on the sea-side is

$$22 \times (10.25\ \text{kN/m}^2) = 225.5\ \text{kN/m}^2,$$

while on the river-side it is

$$14 \times (10.25\ \text{kN/m}^2) = 143.5\ \text{kN/m}^2.$$

Figure 7.29 shows the distribution of the horizontal water pressure on a 1-metre wide vertical strip of the door. The horizontal water pressure on the door is independent of the shape of the door.[1] The resultant of the horizontal water pressure on the 1-metre wide vertical strip is

$$\tfrac{1}{2} \times (22\ \text{m})(225.5\ \text{kN/m}) - \tfrac{1}{2} \times (14\ \text{m})(143.5\ \text{kN/m}) = 1476\ \text{kN}.$$

We have shown, therefore, that per metre in the circumferential direction, the door is subject to a force of 1476 kN. In other words, the horizontal water pressure on the door consists of a uniformly distributed load

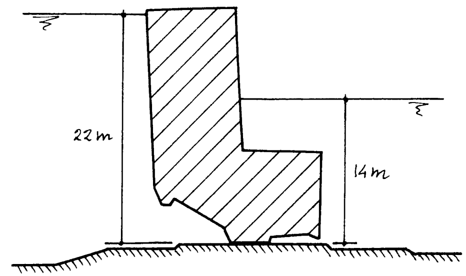

Figure 7.28 A sketch of the cross-section of the door, with the water levels on both sides.

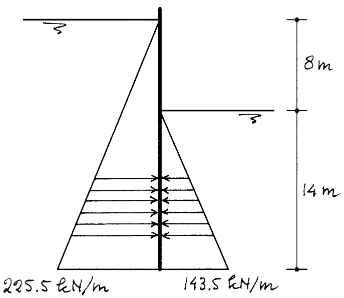

Figure 7.29 The distribution of the horizontal water pressure on a 1-metre wide vertical strip from the door. The horizontal water pressure is independent of the shape of the door.

[1] See the previous example.

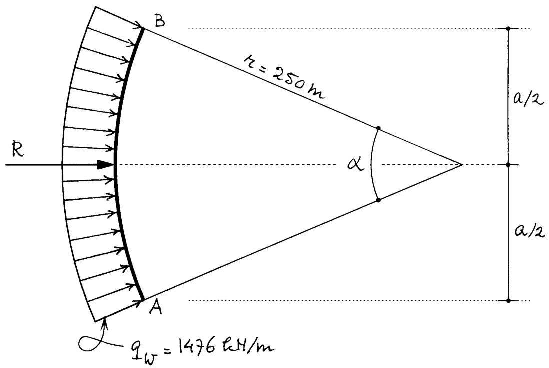

Figure 7.30 A horizontal force of 1476 kN is acting on the door per metre in the circumferential direction. In other words, the horizontal water pressure on the door consists of a uniformly distributed load $q_\mathrm{w} = 1476$ kN/m in radial direction.

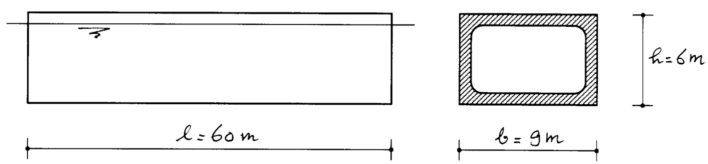

Figure 7.31 A floating element of a two-track metro tunnel, ready to be transported to the sinking site.

$q_w = 1476$ kN/m in radial direction (see Figure 7.30).

With an arc length of 209.5 m for AB and a radius of $r = 250$ m, the aperture angle α is

$$\alpha = \frac{\text{arc length AB}}{2\pi r} \cdot 360^\circ = \frac{209.5\ \text{m}}{2\pi \times (250\ \text{m})} \times 360^\circ = 48^\circ.$$

The resultant R of the horizontal water pressure on arc AB is equal to the resultant of the horizontal water pressure on chord AB (see Section 7.2, Example 2). This gives:

$$R = q_\mathrm{w} a = q_\mathrm{w} 2r \sin(\alpha/2)$$

$$= (1476\ \text{kN/m}) \times 2 \times (250\ \text{m}) \times \sin 24^\circ = 300\ \text{MN}.$$

Example 4

Figure 7.31 represents an element of a floating two-track metro tunnel, ready to be transported to its sinking site. The tunnel element is considered a rigid body.

Dimensions: length $\ell = 60$ m, width $b = 9$ m and height $h = 6$ m.
Dead weight of the tunnel element: $q_\mathrm{dw} = 524$ kN/m.
Weight of each of the temporary bulkheads: $F_\mathrm{head} = 235$ kN.
Specific weight of water: $\gamma_\mathrm{w} = 10$ kN/m^3.

Questions:

a. Determine the water pressure at the base of the tunnel element.
b. Determine the resultant of the horizontal water pressure on a bulkhead.

Solution:
a. The total dead weight R_{dw} of the tunnel element is

$$R_{\mathrm{dw}} = q_{\mathrm{dw}}\ell + 2F_{\mathrm{head}}$$
$$= (524\ \mathrm{kN/m})(60\ \mathrm{m}) + 2 \times (235\ \mathrm{kN}) = 31910\ \mathrm{kN}.$$

Figure 7.32 shows the distribution of the water pressures on the tunnel element. With a specific weight γ_{w}, the water pressure p_{w} at a depth d is

$$p_{\mathrm{w}} = \gamma_{\mathrm{w}} d.$$

The vertical water pressure on the base of the tunnel element gives an upward force $R_{\mathrm{v;w}}$:

$$R_{\mathrm{v;w}} = p_{\mathrm{w}} b\ell = \gamma_{\mathrm{w}} d b\ell.$$

The upward force is equal to the weight of the displaced water.

The tunnel element will sink in the water until the upward force is in equilibrium with the total dead weight R_{dw}:

$$R_{\mathrm{dw}} = R_{\mathrm{v;w}} = \gamma_{\mathrm{w}} d b\ell$$

so that

$$d = \frac{R_{\mathrm{dw}}}{\gamma_{\mathrm{w}} b\ell} = \frac{31910\ \mathrm{kN}}{(10\ \mathrm{kN/m^3})(9\ \mathrm{m})(60\ \mathrm{m})} = 5.91\ \mathrm{m}.$$

The water pressure at the base of the tunnel element is therefore

$$p_{\mathrm{w}} = \gamma_{\mathrm{w}} d = (10\ \mathrm{kN/m^3})(5.91\ \mathrm{m}) = 59.1\ \mathrm{kN/m^2}.$$

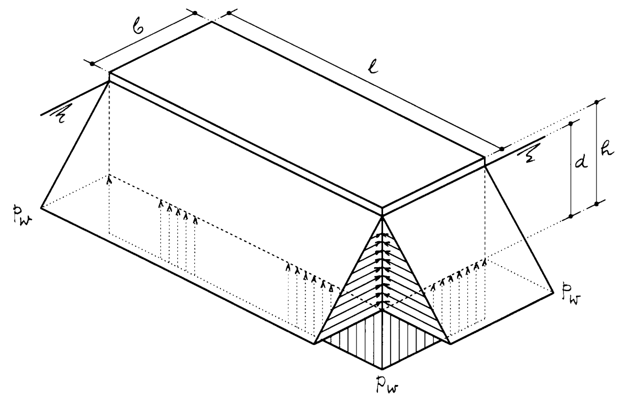

Figure 7.32 The distribution of the water pressures on the tunnel element.

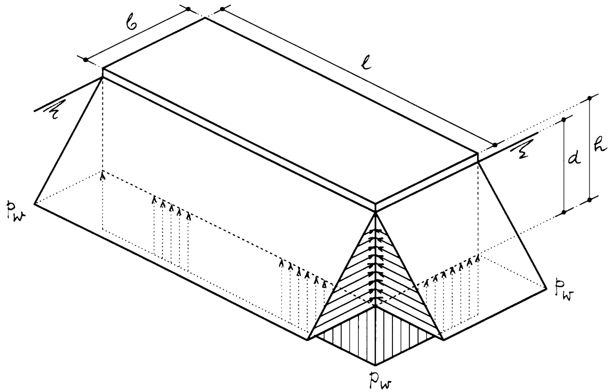

Figure 7.32 The distribution of the water pressures on the tunnel element.

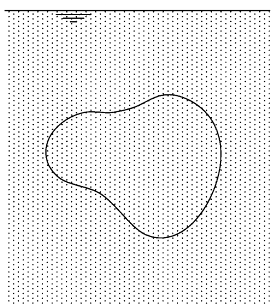

Figure 7.33 If the fluid is in equilibrium, the vertical component of the hydrostatic pressures on the outside of a contained space has to provide an upward force that equals the weight of the fluid within the contained space.

b. The resultant $R_{h;w}$ of the horizontal water pressure on a bulkhead is equal to the volume of the load diagram (see Figure 7.32):

$$R_{h;w} = \frac{1}{2} p_w b d = \frac{1}{2} \times (59.1\ \text{kN/m}^2)(9\ \text{m})(5.91\ \text{m}) = 1572\ \text{kN}.$$

In the calculation, it was noted that the vertical water pressure on the tunnel element exerts an upward force that is equal in magnitude to the weight of the displaced water. This is not a coincidence, but applies in general, regardless of the shape of the body, and is known as *Archimedes' Law*.[1] The general proof can be found below.

Take a contained space of arbitrary shape within a fluid (see Figure 7.33). If there is an equilibrium, the vertical component of the hydrostatic pressures has to provide an upward force on the outside of the contained space that equals the weight of the fluid within the contained space. The upward force does not change if the contained space is taken up by a body.

Conclusion: *A body in a fluid is exposed to an upward force that is equal to the weight of the displaced volume of fluid.*

Example 5

In the water-retaining wall in Figure 7.34 there is a circular partition of radius r. The centroid C of the partition is at a depth z_C.

Question:
Determine the resultant R of the water pressure on the partition.

Solution:
The water pressure on the partition varies linearly. At a depth z the water pressure is $\rho g z$, whereby ρ is the density of water, and g is the gravitational

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1 Archimedes (287–212 BC), Greek scientist from Syracuse. He addressed issues relating to integral calculus and was one of the founders of statics (equilibrium of solids) and hydrostatics (equilibrium of fluids).

field intensity (see Figure 7.35a).

For the partition, take a very narrow horizontal strip dz at depth z (see Figure 7.35b). The width of the strip is $b(z)$. The water pressure on this narrow strip is constant and equal to $\rho g z$. This contribution dR of the strip to the resulting water pressure R on the strip is

$$\mathrm{d}R = \rho g z \cdot b(z) \cdot \mathrm{d}z, \qquad (1)$$

whereby

$$b(z) = 2r\cos\varphi, \qquad (2)$$

$$z = z_C - r\cos\varphi, \qquad (3)$$

$$\mathrm{d}z = \frac{\mathrm{d}z}{\mathrm{d}\varphi}\,\mathrm{d}\varphi = r\sin\varphi\,\mathrm{d}\varphi. \qquad (4)$$

Substitute (2) to (4) into (1) and we find

$$\mathrm{d}R = 2\rho g r^2 (z_C - r\cos\varphi)(\sin\varphi)^2\,\mathrm{d}\varphi.$$

To find the resulting water pressure R, one has to sum up the contributions of all the strips. This is done by integrating between the limits $\varphi = 0$ and $\varphi = \pi$:

$$R = 2\rho g r^2 \int_0^{\pi} (z_C - r\cos\varphi)(\sin\varphi)^2\,\mathrm{d}\varphi.$$

Using the formulas in Table 7.2 we find

$$\int_0^{\pi} z_C(\sin\varphi)^2\,\mathrm{d}\varphi = z_C\left[-\tfrac{1}{4}\sin 2\varphi + \tfrac{1}{2}\varphi\right]_{\varphi=0}^{\varphi=\pi} = \frac{\pi}{2} z_C$$

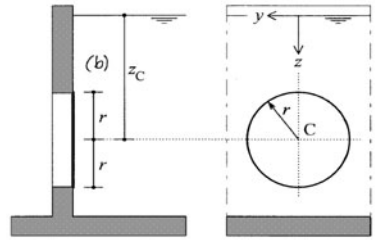


Figure 7.34 A circular partition in a water-retaining wall.

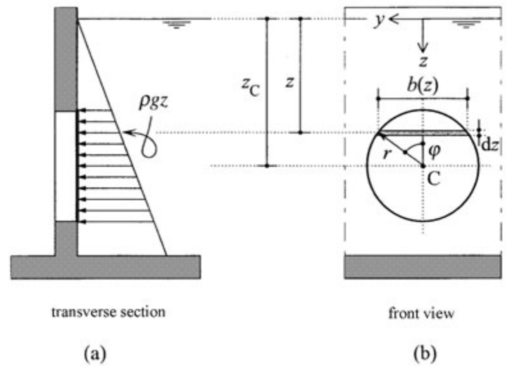


Figure 7.35 (a) The water pressure on the partition increases linearly with the depth. (b) The water pressure is constant on a small horizontal strip and is equal to $\rho g z$.

Table 7.2

$$\int (\sin\varphi)^2\,\mathrm{d}\varphi = -\tfrac{1}{4}\sin 2\varphi + \tfrac{1}{2}\varphi$$

$$\int \cos\varphi(\sin\varphi)^2\,\mathrm{d}\varphi = -\tfrac{1}{3}(\sin\varphi)^3$$

Table 7.2

$$\int (\sin\varphi)^2 \, \mathrm{d}\varphi = -\tfrac{1}{4}\sin 2\varphi + \tfrac{1}{2}\varphi$$

$$\int \cos\varphi(\sin\varphi)^2 \, \mathrm{d}\varphi = -\tfrac{1}{3}(\sin\varphi)^3$$

and

$$\int_0^\pi r\cos\varphi(\sin\varphi)^2 \, \mathrm{d}\varphi = r\left[-\tfrac{1}{3}(\sin\varphi)^3\right]_{\varphi=0}^{\varphi=\pi} = 0$$

so that

$$R = \rho g z_\mathrm{C} \cdot \pi r^2.$$

Conclusion: *The resulting water pressure on the partition is equal to the water pressure at the centroid, multiplied by the area of the partition.*

Although derived for a circular partition, this characteristic is generally applicable. The proof can be provided easily if one knows that the z coordinate of the centroid C of a plane figure with area A is defined as:[1]

$$z_\mathrm{C} = \frac{\int_A z \, \mathrm{d}A}{A}. \tag{5}$$

The resultant of the water pressure on a small area $\mathrm{d}A$ at depth z is

$$\mathrm{d}R = \rho g z \cdot \mathrm{d}A.$$

The resulting water pressure is found by summing up all the contributions $\mathrm{d}R$ for the entire area A. This is performed by integrating with respect to the area A:

$$R = \int_A \rho g z \, \mathrm{d}A = \rho g \int_A z \, \mathrm{d}A. \tag{6}$$

[1] Volume 2, *Stresses, Deformations, Displacements*, addresses the definition and calculation of centroids in detail. Here, it is assumed that readers know the location of the centroid for simple plane figures.

Definition (5) gives

$$\int_A z \, \mathrm{d}A = z_\mathrm{C} A. \tag{7}$$

Substitute (7) in (6) and we find

$$R = \rho g z_\mathrm{C} \cdot A.$$

Conclusion: *The resulting water pressure R on a plane figure is equal to the water pressure $\rho g z_\mathrm{C}$ at the point of centroid* C *of the figure, multiplied by the area A of the figure.*

Note: This does not give the line of action of the resultant R which passes through the *centroid of the load diagram* (see Section 6.3.2).

7.4 Summary

The various characteristics of gas pressures and hydrostatic pressures in this chapter are summarised below.

1. Since there are no shear stresses, the compressive forces in a gas and fluid always act normal to any bounding plane (see Section 7.1).

2. In a gas and fluid, the pressure at a particular point is independent of the orientation of the plane on which the pressure acts. It is also said that, in that point, the stress is of equal magnitude in all directions (isotropic or spherical state of stress) (see Section 7.1).

3. Gas pressure is constant in a contained volume.

4. If a uniformly distributed force acts on the entire area of a body, and

normal to that body, this load forms an equilibrium system, and the resultant is zero (see Section 7.2, Example 3).

5. In a fluid, pressure increases linearly with depth (hydrostatic pressure distribution) (see Section 7.3).

6. The resultant of the hydrostatic pressure on a flat plate is equal to the pressure at the centroid of the plate, multiplied by the area of the plate (see Section 7.3, Example 5).

7. The horizontal component of the resulting hydrostatic pressure on a body is equal to the resultant of the hydrostatic pressure on the horizontal projection of the body on a vertical plane (see Section 7.3, Example 2).

8. The vertical component of the resulting hydrostatic pressure on a body is an upward force that is equal to the weight of the volume of water displaced by the body (Archimedes' Law) (see Section 7.3, Example 4).

7.5 Problems

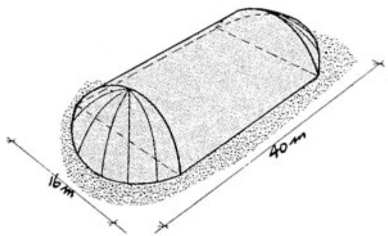
Remark: If necessary, assume that the gravitational field intensity is $g = 10$ N/kg.

Working with gas pressures (Section 7.2)

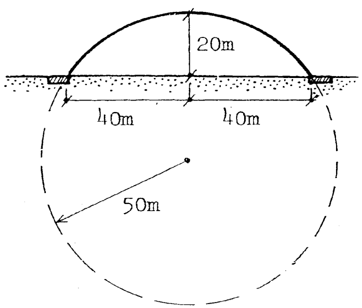
7.1 A cylindrical pneu with a length of 40 m and a width of 16 m has a semi-circular cross-section. The internal overpressure is 375 N/m^2.

Questions:

a. Determine the support reactions for the cylindrical part of the pneu.
b. Determine the membrane force in the circumferential direction at the cylindrical pneu.
c. Determine the force in the longitudinal direction of the cylindrical pneu.



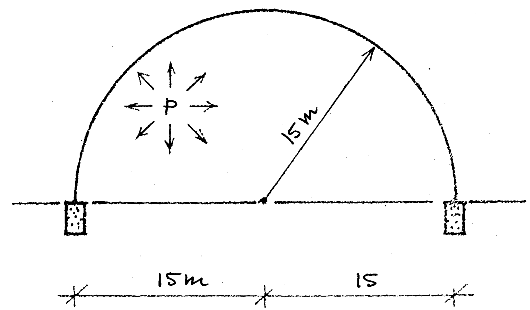
7.2 A spherical pneumatic hall is supported on a concrete ring beam. The internal overpressure is 350 N/m^2.



Questions:

a. Determine the membrane force in the pneu.
b. Determine the vertical forces that the pneu exerts on the ring beam.
c. Determine the horizontal forces that the pneu exerts on the ring beam.
d. Determine the (normal) force in the ring beam. Is it a tensile force or a compressive force?

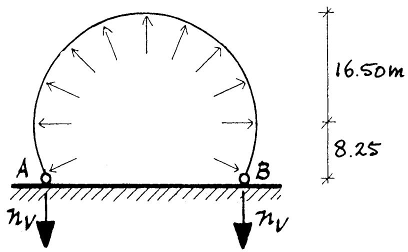
7.3 A pneumatic structure has the shape of a hemisphere with a radius of 15 m and an internal overpressure of 400 N/m^2. The forces in the pneu are transferred to a concrete ring beam. The weight of the ring beam ensures that the pneu is not lifted. The specific weight of concrete is 24 kN/m^3.



Questions:

a. Determine the membrane force in the pneu.
b. Determine the normal pressure in the ring beam.
c. Determine the required diameter of the ring beam to prevent the pneu from lifting.

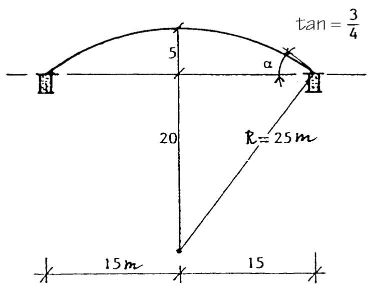
7.4 In a spherical pneu with radius 16.5 m there is an overpressure of 350 N/m^2. The horizontal support reactions are transferred by a ring belt applied around the pneu.



Questions:

a. Determine the membrane force in the pneu.
b. Determine the vertical support reactions n_V.
c. Determine the force in the ring belt.

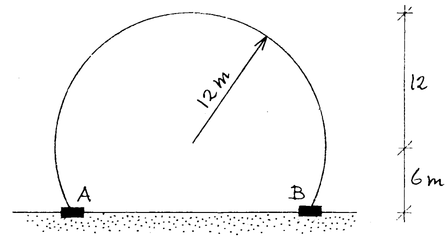
7.5 A pneumatic structure, with a spherical shape, is connected to a circular ring beam that rests freely on the ground. The overpressure in the pneu is 400 N/m^2. The dead weight of the pneu can be neglected.



Questions:

a. Determine the membrane force in the pneu.
b. How large must the weight per metre of the ring beam be to prevent lifting?
c. Determine the normal force in the ring beam. Is this a tensile force or a compressive force?

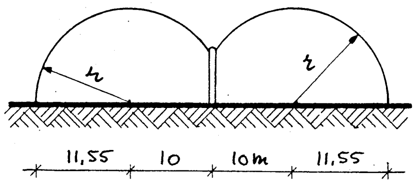
7.6 A (long) cylindrical pneu with an internal pressure of 400 N/m^2 has a circular cross-section as indicated in the figure with a radius of 12 m. The weight of the concrete beams at A and B has to prevent the pneu from lifting. Tie-rods have been applied between the beams A and B every 2.5 m. The specific weight of concrete is 24 kN/m^3.



Questions:

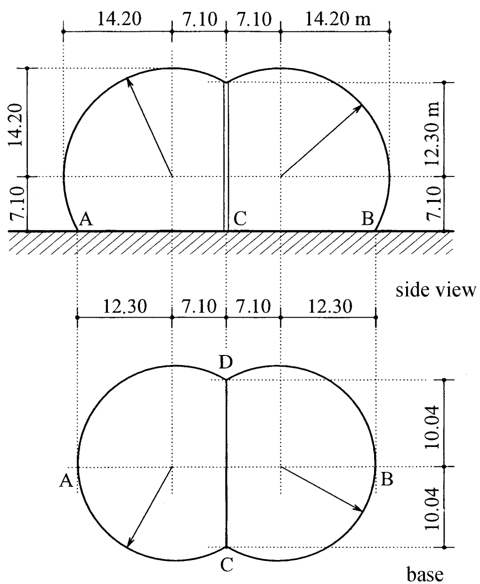
a. Determine the membrane force (in the circumferential direction) in the pneu.
b. Determine the required cross-section of the beams to prevent lifting.
c. Determine the force in a tie-rod between A and B.

7.7 Two spherical pneus with radius $r = 11.55$ m have been placed adjacent to one another and joined. A cable has been placed over the pneus at the connection. The overpressure in the pneu is 400 N/m^2.



Questions:
a. Determine the membrane forces in the pneu.
b. Determine the forces that the pneus exert on the cable.
c. Determine the tensile force in the cable.

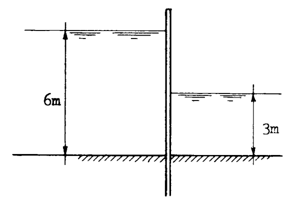
7.8 Two spherical pneus with a radius of 14.20 m have been placed adjacent to one another and joined. A cable has been laid over the pneus at the line of joining. The overpressure in the pneu is 410 N/m^2. The pneus are attached to concrete foundation beams.



Questions:
a. Determine the membrane forces in the pneu.
b. Determine the vertical support reactions for the pneu.
c. Determine the (normal) force in the ring beams. Are they tensile forces or compressive forces?
d. Determine the forces that the pneus exert on the cable.
e. Determine the tensile force in the cable.
f. Determine the (normal) force in beam CD. Is this a tensile force or a compressive force?

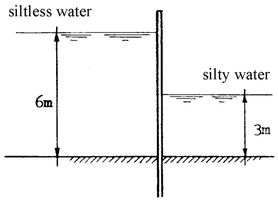
Working with hydrostatic pressures (Section 7.3)

7.9 A steel sheet-pile wall is fixed in a concrete floor. There is 6 m of water against one side of the wall, and 3 m on the other. Mass density of water: 1000 kg/m^3.



Questions:
a. Draw the distribution of the water pressure on the wall.
b. Determine the horizontal support reaction per metre wall.
c. Determine the fixed-end moment per metre wall.

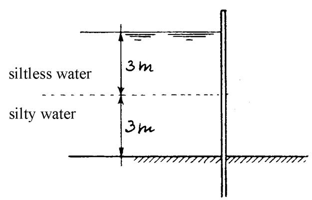
7.10 A steel sheet-pile wall is fixed in a concrete floor. There is 6 m of water against one side of the wall with a mass density of 1000 kg/m^3. On the other side, there is 3 m of water with, due to a high silt content, a mass density of 1400 kg/m^3.



Questions:
a. Draw the distribution of the water pressure on the wall.
b. Determine the horizontal support reaction per metre wall.
c. Determine the fixed-end moment per metre wall.

7.11 Like the previous question, but now with the silty water to the left, and the siltless water to the right of the wall. The mass density of the silty water is 1200 kg/m^3, and that without silt is 1000 kg/m^3.

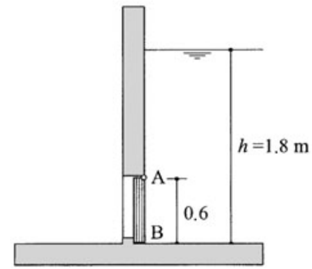
7.12 A steel sheet-piling is fixed in a concrete floor, and is retaining 6 m of water. The mass density of the upper 3 metres is 1000 kg/m^3. The lower three metres have a mass density of 1400 kg/m^3 as a result of the silt present.



Questions:
a. Draw the distribution of the water pressure on the sheet-piling.
b. Determine the horizontal support reaction per metre of sheet-piling.
c. Determine the fixed-end moment per metre sheet-piling.

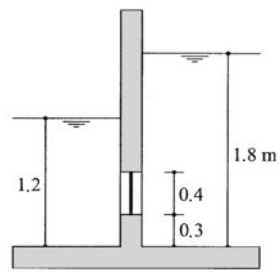
7.13 A water-retaining wall contains a square flap that is hinged at A, and supported by a sill at B.

Questions:
a. Draw the distribution of the water pressure against the wall.
b. Determine the resultant of the water pressure on the flap.
c. Determine the line of action of this resultant.
d. Determine the support reactions at A and B.



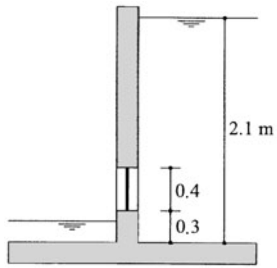
7.14 What is the water depth h if the total water pressure on the square flap from the previous question is 3.6 kN?

7.15 A connection between two reservoirs is sealed by means of a circular valve.



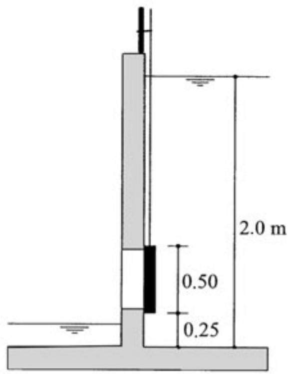
Questions:
a. Draw the distribution of the water pressure on both sides of the wall.
b. Draw the resulting water pressure on the wall.
c. Determine the resulting water pressure on the sealing valve.

7.16 A connection between two reservoirs is closed by means of a circular valve.



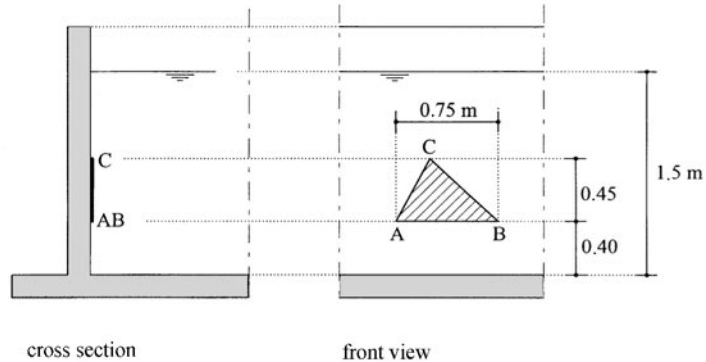
Question:
Determine the resulting water pressure on the valve.

7.17 An opening in a water-retaining wall is closed by means of a slide. The slide is 0.5 m high and 0.4 m wide.



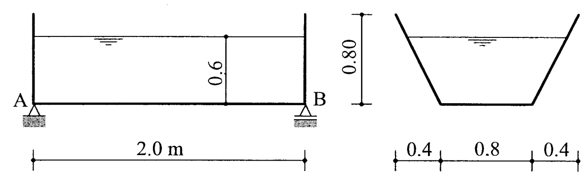
Question:
Determine the resulting water pressures on the slide.

7.18 You are given a water-retaining wall with the triangular area ABC as shown.



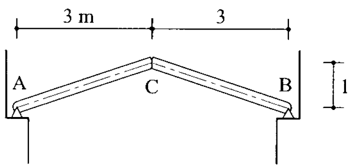
Questions:
a. Determine the resulting water pressure on triangle ABC
b. Determine the resulting water pressure on triangle ABC if base AB is above top C.

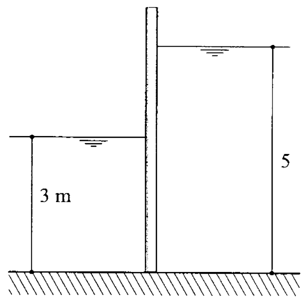
7.19 You are given the longitudinal section and the cross-section of a water trough.



Questions:
a. Determine the support reactions at A and B.
b. Determine the resultant of the water pressure on an end-partition.

7.20 You are given a wooden mitre gate in a small lock. The depth of the water outside the lock is 5 m and 3 m inside.





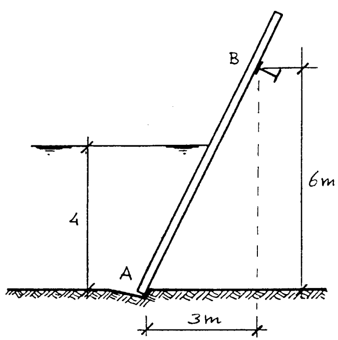
Questions:

a. Determine the resulting water pressures on door AC.
b. Determine the forces that the doors at A and B exert on the lock walls.
c. Determine the force that the doors at C exert on one another.

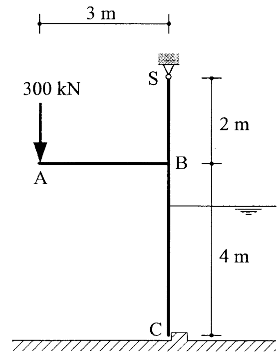
7.21 A barrage is made up of partitions that at base A are resting against a groove and at top B against an I-section. The I-section beam is supported in the walls of the barrage.

Questions:

a. Draw the distribution of the water pressure on the walls.
b. Determine the support reactions at A and B for a partition with a width of 1.5 m.



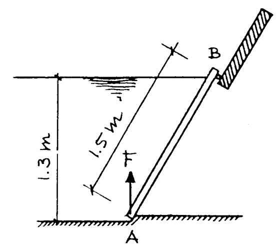
7.22 A lock door is supported by a hinge at S and is pressed against sill C by a force of 300 kN at A. There is only water to the right of the door. The lock door has a width of 4 m. The weight of the door can be ignored.



Question:

At which water level will the door open?

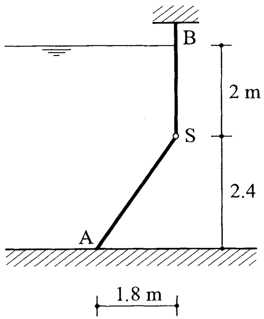
7.23 A barrage contains a flap AB with a width of 1 m. The flap is resting in a groove at A and is supported by a hinge at B.



Question:

Determine the vertical force F required to open flap AB.

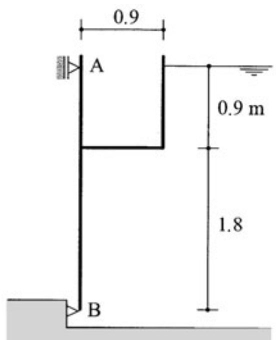
7.24 In a barrage, flap AS is resting at A on an entirely flat base and is connected in a hinge at S with SB. The flap is 2.5 m wide. There is only water to the left of the barrage.



Questions:

a. Determine the (total) support reaction at A.
b. Determine the horizontal and vertical component of the hinge force at S. Also clearly indicate the directions.

7.25 A barrage with the shape shown is located in a 1-m wide channel.

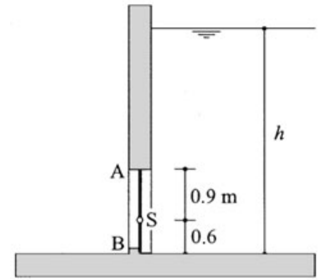


Questions:

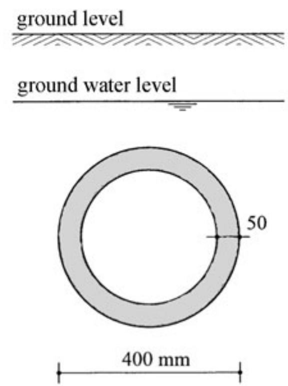
a. Draw the distribution of the water pressure on the barrage.
b. Determine the support reactions at A and B.

7.26 A barrage contains a flap AB that can rotate about a hinge at S.

Question:
Determine the water level h at which the flap will open.

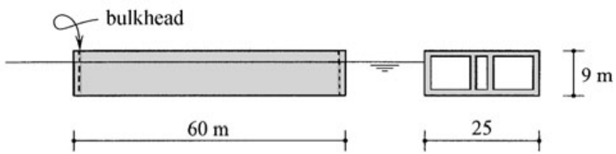


7.27 A concrete sewer pipe with Ø400 mm and a wall thickness of 50 mm is located in an area with sandy soil and a high water level. The pipe is located below ground water level. The specific weight of concrete is 24 kN/m^3.



Question:
Determine whether there is a danger of lifting if the weight of the soil above the pipe is not taken into account as a load.

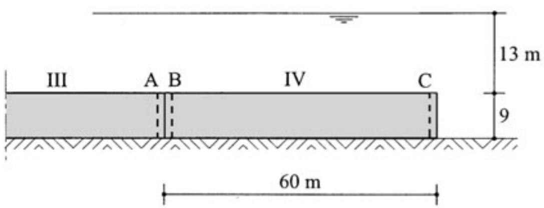
7.28 A concrete tunnel element is afloat, waiting to be transported to the sinking site. The element is 60 m long and has a cross-section of 25×9 m^2. The outer walls of the tunnel are 1.20 m thick, the inner walls are 0.75 m thick. The two temporary bulkheads each have a weight of 1320 kN. The specific weight of concrete is 25 kN/m^3, while that of water is 10 kN/m^3.



Questions:

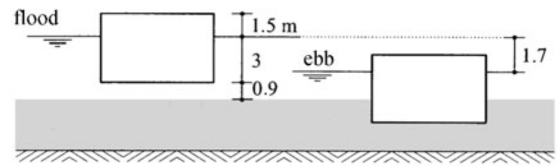
a. How much is the tunnel above water level?
b. How many litres of water have to be used to fill the ballast tanks to sink the tunnel element?

7.29 Once tunnel element IV has been sunk and placed at the correct level, the space between the bulkheads A and B is pumped empty. The tunnel has a rectangular cross-section of 25×9 m^2. The specific weight of the water is 10 kN/m^3.



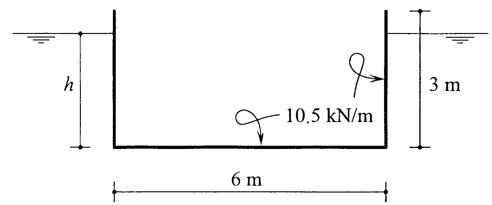
Question:
Determine the force that tunnel element IV exerts on element III?

7.30 At high tide, a barge with rectangular cross-section is 1.5 m above the water and 3.0 m below the water. At low tide, the water level is 1.7 m less, and the base of the barge ends up in a muddy layer of sediment. The muddy layer has a mass density of 1400 kg/m^3 and behaves like a liquid. The water above the muddy layer has a mass density of 1050 kg/m^3.



Question:
How much does the barge stick out of the water at low tide?

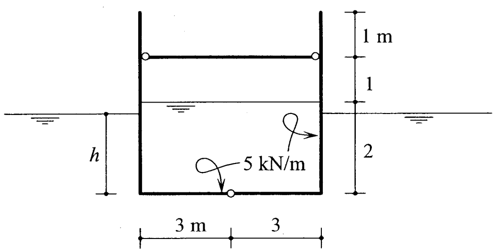
7.31 A 1-metre strip has been isolated from the length of a long barge and is modelled as a line element. The dead weight of the isolated line element (walls and base) is 10.5 kN/m.



Questions:

a. Determine depth h of the barge.
b. Draw the distribution of the water pressure on the walls and the base.
c. Isolate the base of the barge and draw all the forces acting on it.

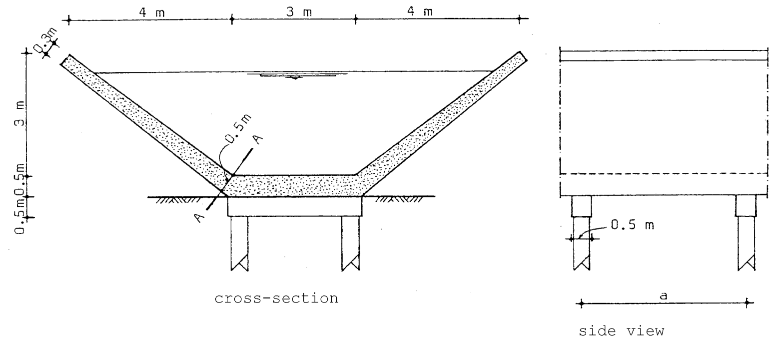
7.32 In the middle of the base of a long barge there is a continuous hinge. There are bars between the walls of the barge three metres above its base. The centre to centre distance of these bars is 2.5 m. The barge is filled with petroleum up to 2 m. The dead weight of the petroleum is 7.5 kN/m^3. A 1-metre strip has been isolated in the length of the barge and modelled as a line element. The dead weight of the isolated line element (walls and base) is 5 kN/m.



Questions:

a. Determine the depth h of the barge.
b. Draw the distribution of the hydrostatic pressures on walls and base.
c. Isolate a wall and draw all the forces acting on it.
d. Determine the force in a bar. Is it a tensile force or a compressive force?

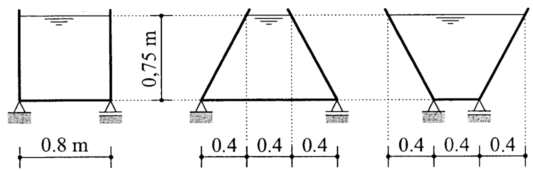
7.33 A sketch is shown with a number of estimated thicknesses of a long concrete channel for the transport of water. The water can rise to the upper edge of the channel The specific weight of concrete is 24 kN/m^3.



Questions:

a. Determine the centre to centre distance a of the cross-beams to an accuracy of 0.1 m, such that the piles are not loaded by more than 600 kN.
b. Determine the section forces (interaction forces) per metre in cross-section A–A of the channel.

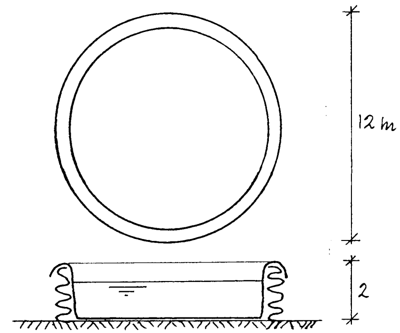
7.34: 1–3 A 1-metre strip has been isolated from a channel filled with water and is modelled as a line element. There are three different shapes of channel.



Questions:

a. Determine the support reactions.
b. Draw the distribution of the water pressure on the walls and the base.
c. Isolate the base and draw all the forces acting on it.

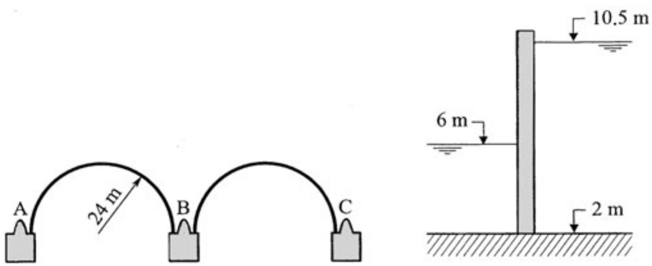
7.35 At a nursery, an open tank is being built to store water. The round tank has a diameter of 12 m and a height of 2 m. The wall of the tank consists of corrugated steel plates that are connected by means of bolts. The water retention is achieved by means of a membrane.



Questions:

a. Determine the (normal) force in the circumferential direction if the tank is three-quarters full. Is this a tensile force or a compressive force?
b. Determine the (normal) force in the circumferential direction if the tank is filled to the top.

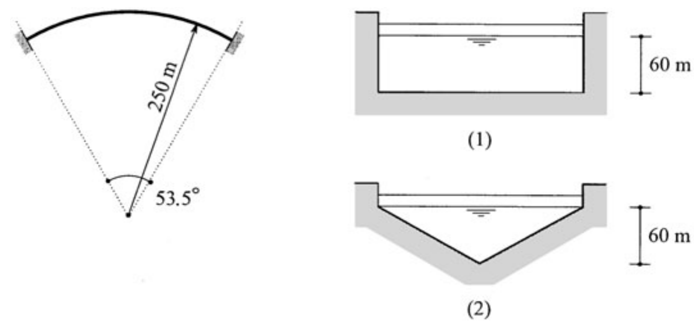
7.36 You are given two curved weirs. The weirs are a semi-circle and have a radius of 24 m. The bulging sides of the weirs are pointing downstream. The water levels are shown in the figure.



Questions:

a. Determine the (normal) force in the circumferential direction of the weir. Is this a compressive force or a tensile force?
b. Determine the total force that the weirs AB and BC exert on pier B.

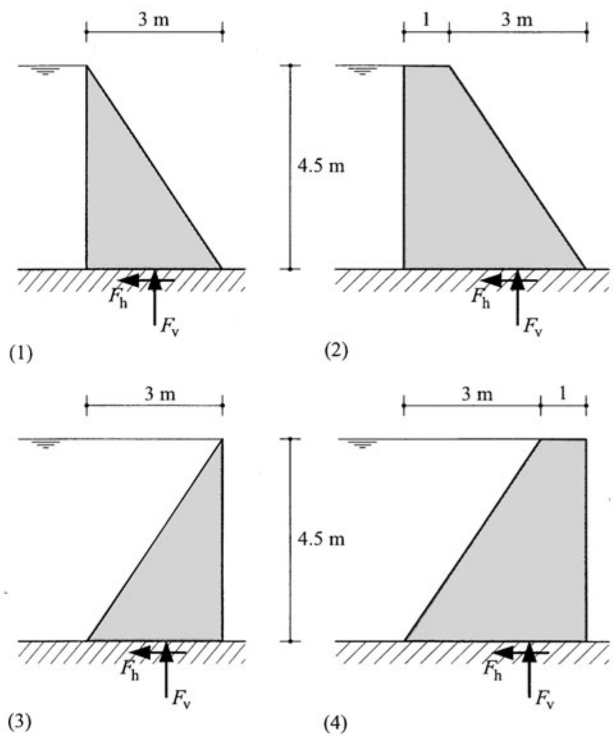
7.37: 1–2 Sixty metres of water is standing against a circular storage dam with a radius of 250 m and an aperture angle of 53.5°. In case (1), the transverse section of the closed valley is rectangular, while in case (2) it is an isosceles triangle.



Question:

Determine the horizontal resultant of the water pressure on the dam.

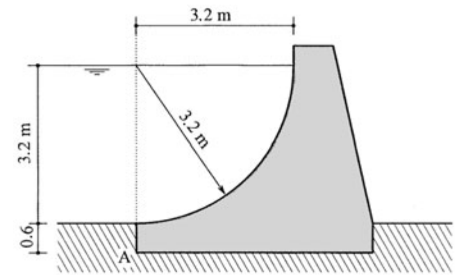
7.38: 1–4 A concrete wall is retaining 4.5 m of water. The support reactions F_h and F_v exerted by the foundation provide equilibrium. The specific weight of concrete is 24 kN/m^3 and that of water is 10 kN/m^3.



Questions (for 1 m retaining wall):

a. Determine support reaction F_h.
b. Determine support reaction F_v.
c. Determine the line of action of F_v.

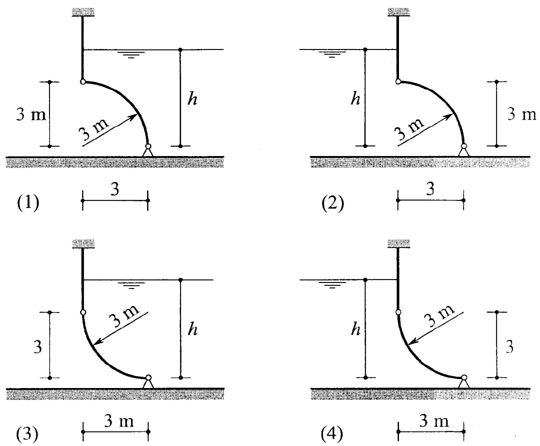
7.39 The cross-section of a retaining wall is circular on the water-retaining side.



Questions (for 1 m retaining wall):

a. Determine the vertical component of the water pressure.
b. Determine the horizontal component of the water pressure.
c. At which distance from A does the line of action of the resulting water pressure intersect the base of the retaining wall?

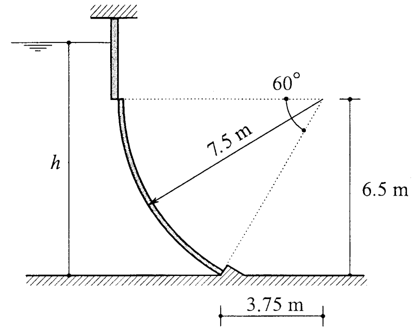
7.40: 1–4 A 5-metre wide dam is retaining a water level h. The dam is composed of a flat vertical wall and a circular cylindrical wall. The cross-section of the cylindrical wall is a quarter-circle with a radius of 3 m.



Question:
Determine the resultant of the water pressure on the 5-metre long circular cylindrical wall, with its line of action, when:

a. $h = 4.80$ m.
b. $h = 3.00$ m.
c. $h = 1.50$ m.

7.41 A circular cylindrical slide with a length of 20 m, a radius of 7.5 m and an aperture angle of 60°, is retaining a water level h.



Question:
Determine the resultant of the water pressure on the slide, with its line of action, if:

a. $h = 8.0$ m.
b. $h = 6.5$ m.
c. $h = 4.0$ m.