6 Loads

All influences acting on a structure can be considered as loads. In mechanics, we generally restrict ourselves to loads that occur as a result of forces and prescribed deformations or displacements. In doing so, we make a distinction between *static* and *dynamic loads*. This is covered in Section 6.1.

In vibration-sensitive structures, dynamic loads can generate far greater forces and deformations than one would find from a static calculation. Dynamic calculations are more complex than static calculations and are beyond the scope of this book.

For traditional structures, *regulations* or *codes* are prescribed with respect to loads and loading combinations. These are based on experience, measurements and common sense. Section 6.2 briefly describes the loads mentioned in the regulations. For special structures, the regulations are often not sufficient, and loading analyses may demand extensive study.

Whereas up until now a load has consisted of one or more concentrated forces, this chapter addresses *distributed loads*; we distinguish between *volume loads, surface loads*, and *line loads*.

A system of forces on a structure (which is considered a rigid body) can, for equilibrium purposes, be replaced by its resultant. The same applies for a distributed load. Section 6.3 addresses how to calculate the resultant of a

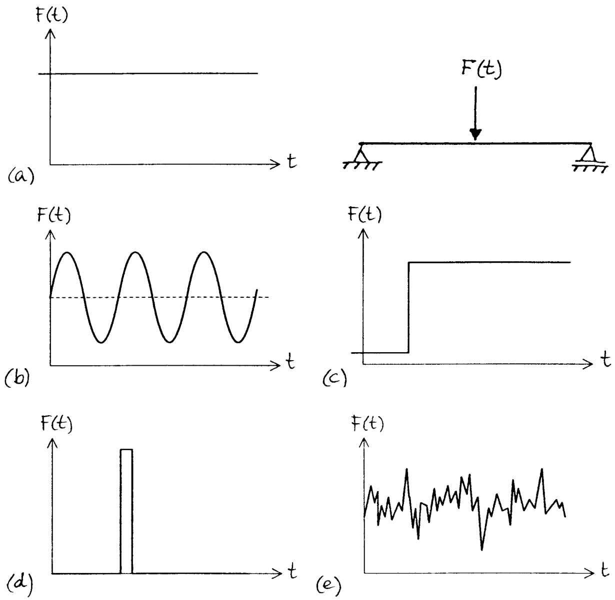

Figure 6.1 (a) A load that does not change with time is called a static load. Dynamic loads are loads that change in time, such as (b) periodic loads, (c) suddenly applied loads, (d) loads of short duration and collision phenomena (impact loads), and (e) stochastic loads.

distributed load. Line loads on a member will be treated more extensively.

How the load is determined depends to a great extent on the manner in which a structure or structural element is modelled. For example, the dead weight of a bar depicted as a line element is not treated as a volume load but rather as a line load. In the same way, the dead weight of a slab (plane element) is considered as surface load. This issue is illustrated in Section 6.4 using a simple building.

Section 6.5 addresses the concept of stress. The transfer of forces in and between materials is the result of extremely small interactions between adjacent particles. Spreading all the forces evenly over a section leads to the concept of *stress*.

6.1 Loads in mechanics

6.1.1 Influences on structures

All influences that can act on a structure can be considered as loads. In general, we distinguish between the following:

- *Loads due to forces*
  This could for example be the weight of traffic on a bridge. In addition to the traffic, the bridge must also be able to carry its dead weight.
- *Loads due to prescribed deformations or displacements*
  The settlement of a support is an example of a prescribed displacement. Other examples are the influences of temperature changes, shrinkage and creep.
- *Loads due to other influences*
  If the structure is located in an aggressive environment in which the material is affected, this effect on the material can be seen as a load. Fire is also seen as a load.

Structures have to be designed and constructed in such a way that they offer sufficient resistance to all these influences so that the function of the structure is not endangered in any way.

In mechanics, we generally restrict ourselves to loading by forces, and prescribed deformations and displacements. A further distinction here is that between *static* and *dynamic* loads.

6.1.2 Static and dynamic loads

If a load due to forces or prescribed displacements does not change (or changes very little) in time, as in Figure 6.1a, it is called a *static load*. In contrast, *dynamic loads* do change with time, as in Figures 6.1b to 1e.

The wave action on a structure at sea and the forces exerted by a machine on its foundations are examples of dynamic loading by *forces*. Another example of a dynamic load is an earthquake. In an earthquake, one refers to a *prescribed displacement*: the earth starts to move and the structure is forced to follow the movement of the earth via its foundations.

In general, one can distinguish between four different types of dynamic loading:

- *Periodic loads* (Figure 6.1b)
  This type of load is caused, for example, by rotating machines, ringing bells, eddies in a stream, or people jumping on a floor.
- *Suddenly-applied loads* (Figure 6.1c)
  This could include a load resulting from a snapping wire (see Figure 6.2a). Another example is the cableway in Figure 6.2b, which was used at the Haringvliet[1] dam to unload concrete blocks.
- *Loads of short duration and collision phenomena (impact loads)* (Figure 6.1d)

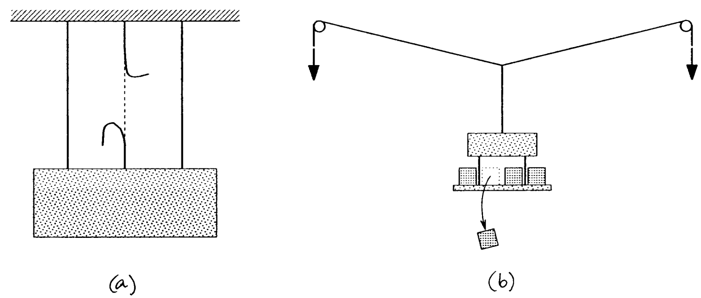

Figure 6.2 (a) When a block is suspended by three wires, and one of the wires suddenly breaks, there is a sudden change in the loading on the remaining wires. (b) Model of the cableway used to close the Haringvliet. Discarding a concrete block causes a sudden change in the loading on the cable gondola.

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[1] A see arm. The enclosure of the Haringvliet is one of the Delta Works in the Netherlands.

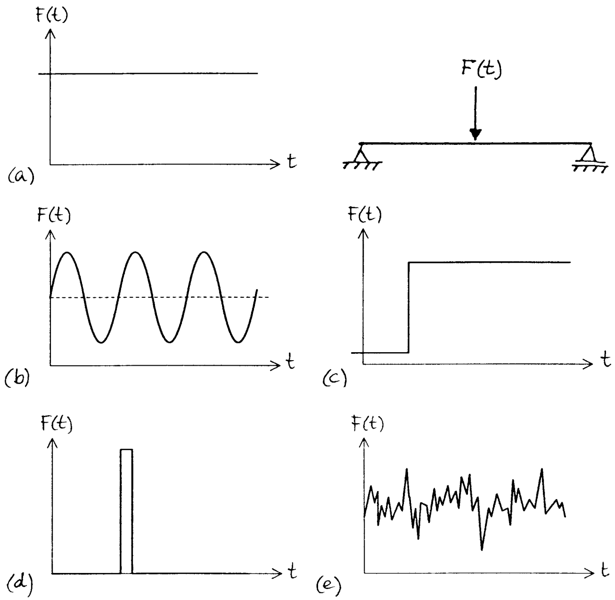

Figure 6.1 (a) A load that does not change with time is called a static load. Dynamic loads are loads that change in time, such as (b) periodic loads, (c) suddenly applied loads, (d) loads of short duration and collision phenomena (impact loads), and (e) stochastic loads.

Examples include explosions, wave impact, gusts of wind, or a falling pile hammer on a pile.

- *Stochastic loads* (Figure 6.1e)
  This includes loads of a variable and unpredictable character, such as those resulting from wind, waves, traffic or earthquakes.

In vibration-sensitive structures, dynamic loads can generate much larger forces and deformations than one would find from a static calculation. This will be illustrated using the simply supported beam in Figure 6.3a, which in the middle of the span has to carry a block with mass m and weight $G = mg$. To simplify matters, the mass of the beam will be disregarded.

The block is suspended from the cable, and touches the beam without resting on it. If the block is carefully placed on the beam by letting out the cable very slowly, both the vertical support reactions will slowly increase to $\frac{1}{2}G$, after which they do not change in time (see Figure 6.3b). The load is static.

It would also be possible to have the weight of the block act on the beam suddenly, not by slowly letting out the cable, but by cutting it. The beam with the block will then start to vibrate around the *static equilibrium position* (see Figure 6.3c). The vertical support reactions are now twice as large (albeit of short duration) as in the case with the static loading.[1] As a result of the ever-present *damping*, the amplitude of the vibration will decrease in time, and the block will finally come to rest in the static equilibrium position, as indicated in Figure 6.3d.

Due to a suddenly applied load, the forces in the structure are twice as large as would be determined by means of a static calculation. If the block is dropped from a certain height, the acting forces are even larger.

In the case of a *periodic load* (soldiers walking in step across a bridge, people jumping up and down on a floor, bells ringing in towers, foundations

1 The evidence cannot be given at this stage and is beyond the scope of this book.

for machines, turbines, engines, and so forth) the structure must be designed in such a way that the *natural frequencies*[1] of the structure clearly differ from the *frequency of the loading*. If this is not the case, there is the danger of *resonance*, in which the forces and deformations in the structure can become extremely large.

Dynamic calculations are more complex than static calculations. In regulations and codes, the dynamic influences have often been taken into account by increasing the load so that a static calculation is enough. For example, the static load of people on floors is approximately 3 kN/m^2. Due to the movement of the people, the load changes by frequencies of 1 to 2.5 Hz.[2] The resulting forces are approximately twice the static value. Regulations therefore prescribe that a static equivalent of about 6 kN/m^2 has to be taken into account.[3]

6.1.3 Volume loads, surface loads, line loads, and point loads

So far, we have always imagined that loads are concentrated forces that have their points of application on the structure. In reality, a force never acts on a single point, but acts across a particular area. The following distinctions are made, depending on the dimension of the area of application:

- *Volume loads* (forces per volume; N/m^3)
  For example: a material's dead weight.
- *Surface loads* p (forces per surface area; N/m^2)

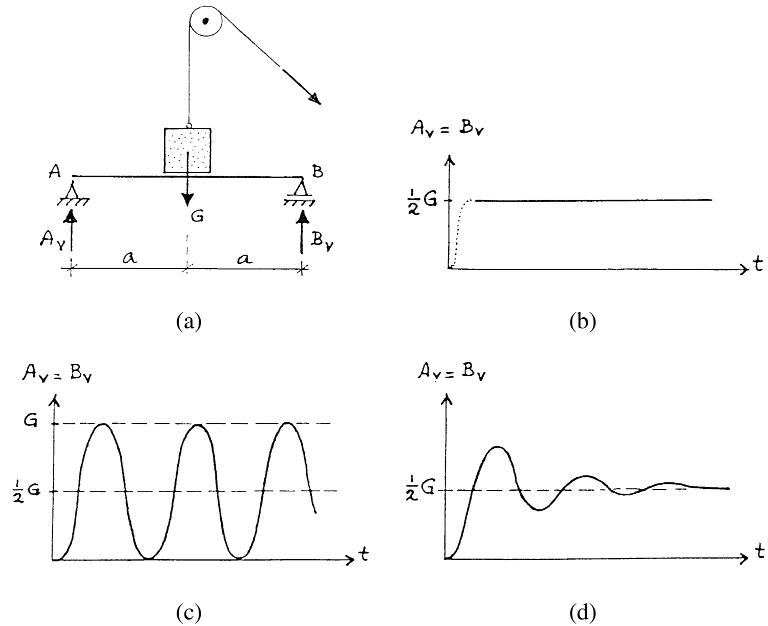

Figure 6.3 (a) A simply supported beam has to carry a block with weight G in the middle of the span. Initially the block is hanging from a cable and touches the beam without resting on it. (b) The vertical support reactions under static conditions after gently letting out the cable. (c) If the cable is not let out slowly, but is cut, the beam with the block starts to vibrate and the vertical support reactions (albeit of short duration) are twice as large as they are under static conditions. (d) As a result of the ever-present damping, the amplitude of the vibration decreases in time, and the block finally ends up at rest in the static equilibrium position.

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[1] A natural frequency is a frequency with which (part of) a structure can vibrate freely.

[2] The unit of frequency (Hz = hertz = s^{-1}) is named after Heinrich Rudolf Hertz (1857–1894), German physicist.

[3] In the regulations, the *value for the load* is found from the *characteristic load* by multiplying this by a *load factor* (see Section 6.2.5).

Example: wind and snow loading, gas, liquid, and earth pressures.

- *Line loads q* (forces per length; N/m)
  Example: the weight of a dividing wall on a floor.
- *Point loads*, or concentrated forces F (N)

Volume loads, surface loads, and line loads are referred to as *distributed loads*. In equilibrium analysis, distributed loads may be replaced by their resultant. Section 6.3 addresses the calculation of the magnitude and line of action of this resultant.

6.2 Loads in regulations

For traditional structures, *regulations* or *codes* are prescribed with respect to loads and loading combinations. These have been created on the basis of experience, measurements and common sense. For special structures, the regulations are generally insufficient and the load analysis may demand extensive study.

In the regulations, two important main groups are generally distinguished:

- *dead loads*
- *live loads*

The live load due to the (vertical) traffic load on bridges is known as a *moving load*.

6.2.1 Dead loads

Dead loads are loads that are always present for the entire lifecycle of the structure. The dead load can often be determined quite easily and accurately.

Examples of dead loads include:

- *Dead weight*
  This is the weight of the (bearing) structural element under consideration.

- *Permanent loads*
  This is the weight of non-bearing elements that rest permanently on the structural element under consideration, either directly or indirectly. Examples include the weight of the insulation plates and waterproof roofing material for a roof or the weight of the topping of a bridge deck.
- *Loads due to prestressing*

The effect of the dead load can sometimes be most unfavourable *during construction*, when the structure has not yet been completed and the dead load is not yet present everywhere. A similar situation can occur when the structure is being *converted* or *demolished*.

6.2.2 Live loads (buildings)

Live loads are loads that do not act permanently on the structural element in question. At times, they are present, while at other times they are absent. It is often not as easy to determine the magnitude of live loads as it is to determine dead loads. The values prescribed in the regulations are the result of many years of experience and research.

Live loads include snow on a roof, people on a dance floor, goods in a warehouse or traffic on a bridge. Traffic loads are referred to as *moving loads* (see Section 6.2.3).

In calculations, one has to assume the most unfavourable situation.

In order to simplify calculations, regulations often prescribe the live load on floors, balconies, stairs, roofs, porches, and so forth, in three different guises:
- A uniformly distributed surface load p;
- A uniformly distributed line load q;
- A concentrated load F (a force acting on a small area).

For the live load prescribed for floors, the weight of a standard inventory

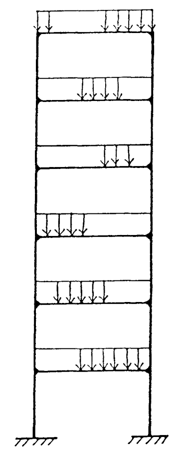

Figure 6.4 Since it is unlikely that the floors in a building are all maximally loaded at the same time, the live load may in certain cases be reduced.

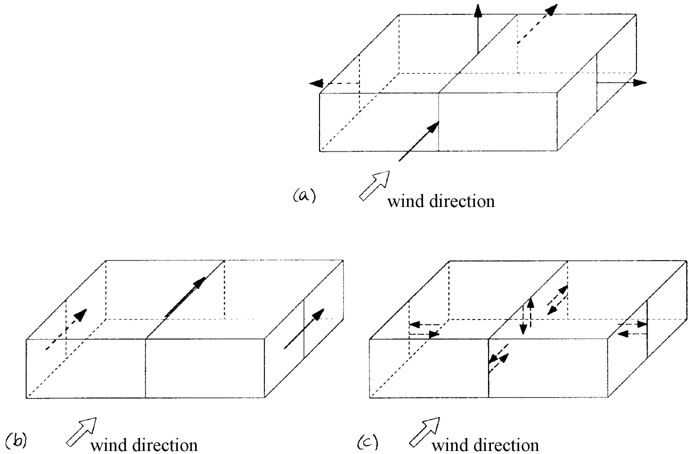

Figure 6.5 Wind loads are distributed loads. The direction of the load is shown by means of an open arrow: (a) wind pressure and wind suction, (b) wind friction, and (c) overpressure and underpressure.

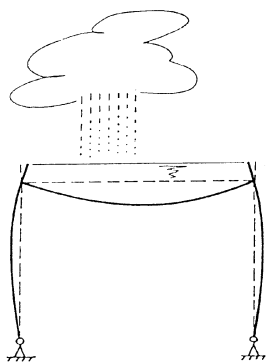

Figure 6.6 For flat roofs, one has to take the risk of water accumulation into account: the deflection of the roof due to the rain water allows the storage of an increasing amount of water. If the roof is not sufficiently rigid, it may eventually collapse.

is included in addition to the weight of people. Furthermore, the dynamic effects of walking, jumping, dancing, stamping, and so forth, are taken into account as well. The line loads and concentrated loads are introduced as they may occur during removals.

The live load has to be calculated separately for machines, archives, and so forth.

Since it is unlikely that in a building all the floors are maximally loaded at the same time, as in Figure 6.4, the live load can be reduced in certain cases.

For roofs where local *snow accumulation* is possible, the associated load concentration has to be taken into account. If the wind loading is predominant, the snow as well as people or tools on the roof can be ignored.

Wind loading is also a live load, but is generally defined separately. A distinction is made between:

- Wind pressure and wind suction (Figure 6.5a);
- Wind friction (Figure 6.5b);
- Overpressure and underpressure (Figure 6.5c).

For *rainwater*, the load of gutters and rainwater pipes filled with water as a result of blockages have to be taken into account.

For flat roofs, the possibility that the water cannot drain away has to be considered. This incurs the risk of *water accumulation*: the deflection of the roof due to the water allows for the storage of an increasing amount of water (see Figure 6.6). If the roof is not sufficiently rigid, this can result in its collapse at times of continuing rainfall.

6.2.3 Live loads (bridges)

Vertical live loads on bridges due to traffic are referred to as *moving loads*. In regulations, this load is a uniformly distributed surface load together with a limited number of concentrated loads (see Figure 6.7).

The uniformly distributed load is a representation of the actual load that can occur over large lengths. This load becomes more important for longer spans.

The system of point loads, with the underlying part of the uniformly distributed load, represents the load caused by a few very heavy trucks or locomotives. This load is important for bridge elements of limited length.

It may occur that certain structural elements are loaded more unfavourably if the load is omitted over a certain length. For this reason, the fact that the traffic load on bridges may be missing along that length has to be considered.

In the hinged beam in Figure 6.8, for example, the vertical support reaction at A as a compression force has its maximum when field AB is loaded and field BC is unloaded. In this case, the support reaction is qa. In the event of full loading, the support reaction in A is half the magnitude. The maximum tensile force that support A has to transfer is $\frac{1}{2}qa$, and occurs when only field BC is loaded.

For *railway bridges*, the train is always a continuous load, even though it can consist partially of empty carriages. A lower load is prescribed for the empty carriages.

The influence of impacts and vibrations are taken into account by multiplying the moving loads by an *impact factor S* ($S > 1$).

In longer *traffic bridges*, it is increasingly less likely that the maximum moving load occurs, unless there is a traffic jam. In that case, the impact factor will lead to a too heavy load, and a reduction is justified. This reduction is achieved by multiplying the traffic loading by a *load factor B* ($B < 1$).

For *railway bridges*, there is no load factor.

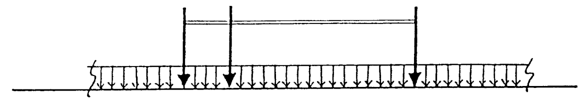

Figure 6.7 In regulations, the mobile load is seen as a uniformly distributed surface load together with a limited number of concentrated loads.

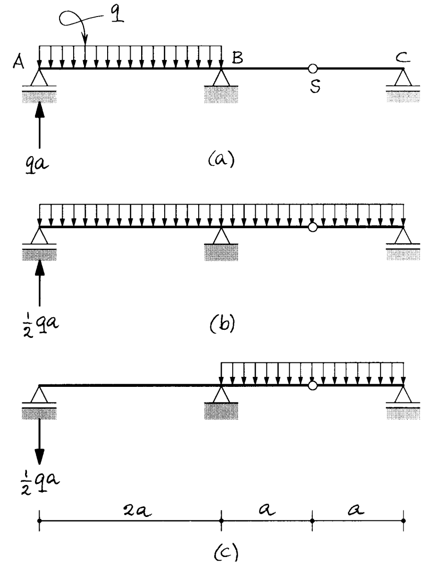

Figure 6.8 For bridges, one has to take into account that certain structural elements are loaded less favourably if the load is omitted over a certain length. (a) The vertical support reaction at A as a compressive force is a maximum when field AB is loaded and field BC is unloaded. (b) Due to a full load, the support reaction at A is half as large. (c) The largest tensile force that support A has to transfer occurs when only field BC is loaded.

In addition to the vertical traffic loads, horizontal loads such as brake forces and wind loads have to be taken into account.

6.2.4 Limit states

For each structure, it has to be shown that it is *reliable* (safe), and will not collapse prematurely, and that the structure meets the requirements related to *serviceability*.

In order to be able to check a structure on these various aspects, the concept of *limit state* was introduced. A limit state is a state in which the structure just meets (or just does not meet) certain demands regarding the structure. A distinction is made between two groups of limit states, which are directly related to the concepts reliability and serviceability:

- *ultimate limit states*
- *serviceability limit states*

Ultimate limit states
If a load is gradually increased, a moment arises at which the structure will collapse, for example because its strength limit is reached (exceeded), or because its equilibrium is no longer reliable (instability). Limit states used to test a structure for its reliability (or more generally speaking, structural safety) are referred to as *ultimate limit states*, or also as *failure states*.

Serviceability limit states
If a structure is insufficiently rigid, this can negatively influence its serviceability. Examples include doors that start to jam if the deformations become too large, and windows that may shatter. Another example is a floor that sags too much. This sort of floor elicits feelings of insecurity and is unusable, even if there is no risk of failure. Annoying cracking can also lead to a situation in which a structure is no longer serviceable (leakage through the cracks or corrosion of the reinforcement in reinforced concrete). Limit states used to check a structure for its serviceability are referred to as *serviceability limit states*.

When checking ultimate limit states, the structure is subjected to an *overload*. When checking *serviceability limit state*, the load at *serviceability level* is used.

The following section provides a brief summary of how, with design codes, the loads (and strength) that have to be used in the calculations are determined.

6.2.5 Characteristic values and design values

With loads and (material) strengths, it is not possible to indicate their precise values beforehand. In practice, they are subject to dispersion. One can only indicate with what *probability* certain values will occur. Loads and strengths are therefore *stochastic quantities*.[1]

Stochastic quantities can be defined by means of a *probability density function*, of which the *normal distribution* is the best-known. Most stochastic quantities that play a part in the assessment of the behaviour of a structure follow the normal distribution. Figure 6.9 shows the curve for the normal distribution of a quantity x. This curve, also known as the *Gauss curve*,[2] is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}.$$

The normal distribution is characterised by two parameters: the *mean value* μ and the *standard deviation* σ. The curve is in the shape of a bell, with a vertical symmetry axis and two points of inflection, and approaches zero for $x \to -\infty$ and $x \to +\infty$. The mean value μ coincides with the symmetry

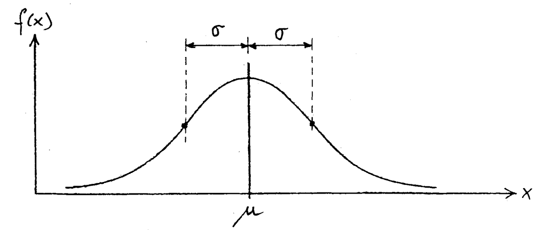

Figure 6.9 The normal distribution or Gauss curve is characterised by the mean value μ and standard deviation σ.

1 From the Greek στοχαξομαι (to guess, to suspect).

2 Carl Friedrich Gauss (1777–1855), German mathematician and astronomer.

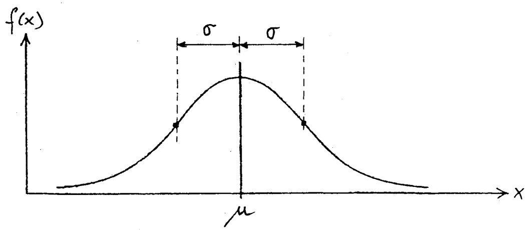

Figure 6.9 The normal distribution or Gauss curve is characterised by the mean value μ and standard deviation σ.

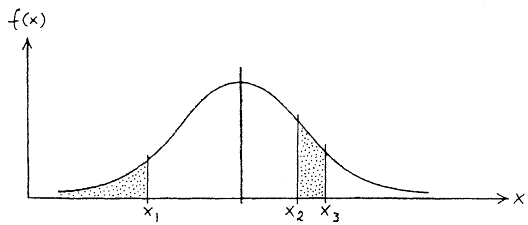

Figure 6.10 The probability that the value of x is smaller than x_1 is equal to the area under the curve for $x < x_1$; the probability of a value of x between x_2 and x_3 is equal to the area under the curve between x_2 and x_3.

axis. The standard deviation σ is the distance from the symmetry axis to the points of inflection.

The probability $P(x < x_1)$ that the value of x is smaller than x_1 is equal to the surface area under the Gauss curve for $x < x_1$ (see Figure 6.10):

$$P(x < x_1) = \int_{-\infty}^{x_1} f(x)\,\mathrm{d}x.$$

There are tables available for this integral.

The probability of a value of x between x_2 and x_3 is equal to the area under the Gauss curve between x_2 and x_3.

The total area under the curve is equal to 1: there is a probability of 100% that the value of x lies between $-\infty$ and $+\infty$.

The probability $P(x < x_1)$ can be shown in various ways. The area under the curve gives a value (smaller or equal to 1), such as

$$P(x < x_1) = 0.0025 = 2.5 \times 10^{-3}.$$

This value can also be written as a ratio:

$$P(x < x_1) = 1 : 400.$$

The probability is also often shown as the percentage of the total area under the probability density curve (which is equal to 1). In this example, the probability is

$$0.0025 \times 100\% = 0.25\%.$$

The ultimate limit state is a check for strength. This means that, on the one hand, the strength R of the structure has to be determined, and that on the other, we have to determine the load S. In the building regulations, the procedure used is based on the so-called *characteristic values* for strength and load.

The *characteristic strength* R_k is defined as the strength that is *exceeded* with a probability of 95%; in other words, the strength is therefore less than the characteristic strength in 5% of all cases (see Figure 6.11a).

The *characteristic load* S_k is defined as the load that with a probability of 95% is *not exceeded* throughout the lifetime of the structure; only 5% of all occurring loads are larger than the characteristic load (see Figure 6.11b).

The symbols R for *strength* and S for *load* are used internationally.[1] They have a broad meaning. *Strength R* (generally) relates to the largest forces and stresses that can be transferred by a structure, such as the *admissible tensile force* in a tie-rod, or the *compressive strength* of the material. *Load S* (generally) relates to the force or stress exerted on the structure (or part of a structure), or in other words, the *acting tensile force* in the tie-rod or the *acting compressive stress* in the material.

The strength R must not be smaller than the load S. With respect to the characteristic values, this means:

$$R_k \geq S_k.$$

In this situation, however, the probability of failure is considered too great. In order to reduce this probability, the calculation is not carried out with the characteristic values, but with a lesser strength R_d and a larger load S_d, known as the design values.[2]

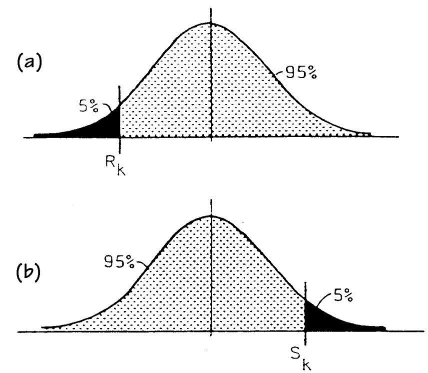

Figure 6.11 (a) The characteristic strength R_k is the strength that is *exceeded* with a 95% probability; in only 5% of all cases, the strength is therefore less than the characteristic strength. (b) The characteristic load S_k is the load that is *not exceeded* throughout the lifecycle of the structure with 95% probability; only 5% of all occurring loads are larger than the characteristic load.

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1 From French: R of *Résistance* (resistance, stamina) and S from *Sollicitation* (load).

2 The index $\underline{d}$ is derived from *design*.

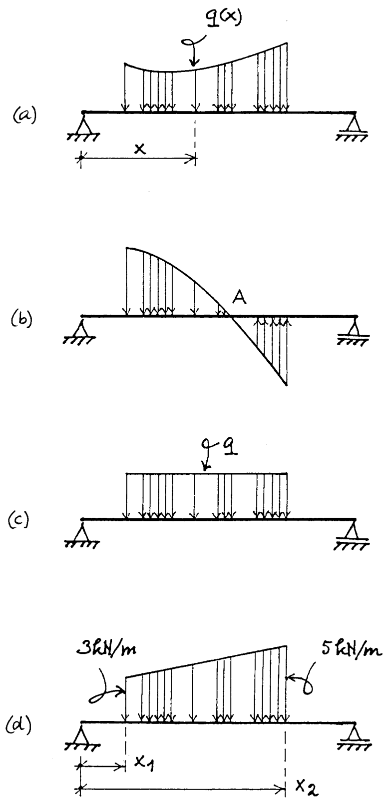


Figure 6.12 Distributed loads normal to the member axis: (a) a distributed load $q(x)$ as a function of x; (b) a distributed load that changes direction at A; (c) a uniformly distributed load; (d) a linearly distributed load.

The *design value for strength* is derived from the characteristic strength by dividing it by a *material factor* γ_R:

$$R_d = \frac{R_k}{\gamma_R}.$$

The material factor accounts for insecurities in construction. As such, steel has a lower material factor than, for example, cast in situ concrete.

The *design value of the load* is derived from the characteristic load by multiplying it by a *load factor* γ_S:

$$S_d = \gamma_S S_k.$$

Amongst other things, the magnitude of the load factor depends on the type of load (dead or live, and whether its effect is favourable or unfavourable), the safety class (office building or shed), and the limit state in question. To check an *ultimate limit state*, the structure is subjected to an overload, and the design value of the load is larger than the characteristic value. To check a *serviceability limit state*, the *load at serviceability level* is used: in this case, the design value of the load is equal to the characteristic value.

Structures are considered sufficiently strong if the design value of the strength is not smaller than the design value of the load:

$$R_d \geq S_d$$

or

$$\frac{R_k}{\gamma_R} \geq \gamma_S S_k.$$

Each limit state has its own load factor. For information concerning load factors and material factors, please refer to the regulations, building codes and relevant books.

In this book, all the examples use only design values.

6.3 Working with distributed loads

When working with a distributed load, it can sometimes be useful to replace it (temporarily) by its resultant. This section addresses the calculation of the resultant of a distributed load. Most attention is devoted to line loads on a member.

6.3.1 Resultant of a line load on a member

Figure 6.12a provides a schematic representation of a *line load* q on (a part of) a member. The direction of the distributed load is shown by means of arrows. The load in Figure 6.12a is acting normal to the member axis and is a function of x. Other examples of distributed loads acting normal to the member axis are shown in Figures 6.12b to 6.12d.

In the special case that the distributed load is constant, we refer to a *uniformly distributed load* (see Figure 6.12c).

The distributed load in Figure 6.12d is known as a *linearly distributed load*; it varies linearly from $q(x_1) = 3$ kN/m to $q(x_2) = 5$ kN/m.

A distributed load can also act in the direction of the member axis. Figure 6.13, for example, shows the uniformly distributed load q on a column as a result of its dead weight.

A distributed load q, acting at an angle to a member, can be resolved into directions parallel to and normal to the member axis (see Figure 6.14). In the xz coordinate system shown, $q_x = q\cos\alpha$ and $q_z = q\sin\alpha$ are called the components of q.

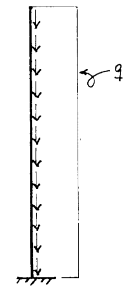


Figure 6.13 A uniformly distributed load in the direction of the member axis.

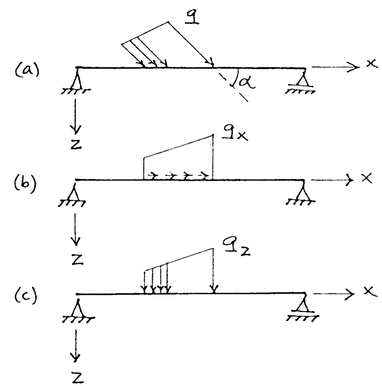


Figure 6.14 (a) A distributed load, at an angle to a member, can be resolved into components (b) parallel, and (c) normal to the member axis.

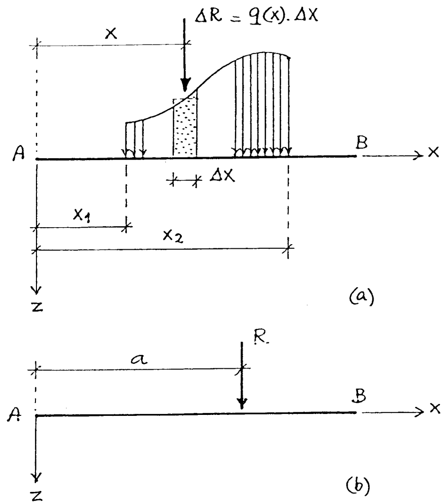


Figure 6.15 (a) An arbitrarily distributed load $q(x)$ normal to the member axis. A small force $\Delta R = q(x)\Delta x$ acts on a small element at length Δx. The magnitude of the resultant R of the distributed load is equal to the sum of all parallel forces ΔR. (b) The magnitude of the resultant R is equal to the area of the load diagram; its line of action passes through the centroid of the load diagram.

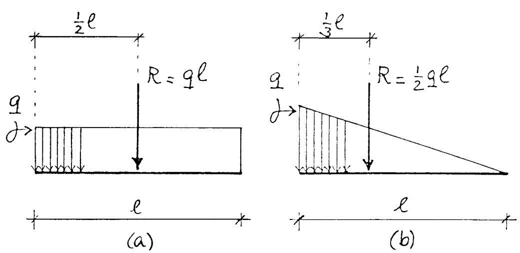


Figure 6.16 Magnitude and line of action of resultant R with (a) a rectangular, and (b) a triangular load diagram.

Note: the distributed load has the dimension of force per length. So far, the length was always measured *along the member axis*. With inclined members, the distributed load is also sometimes expressed per length projected on the (horizontal) ground surface, for example in the case of a snow load. See Example 3 in this section.

When considering the equilibrium of a system of forces on a structure, considered as a rigid body, we can replace the system of forces by its resultant.

In Figure 6.15, an arbitrarily distributed force $q(x)$ is acting normal to the axis of member AB between $x = x_1$ and $x = x_2$. A small force ΔR is acting on a small element at length Δx:

$$\Delta R = q(x)\Delta x.$$

The magnitude of the resultant R of the distributed force is equal to the sum of all small parallel forces ΔR:

$$R = \sum \Delta R = \sum q(x)\Delta x = \int_{x_1}^{x_2} q(x)\,\mathrm{d}x.$$

Conclusion: *The magnitude of R is equal to the area enclosed by the load diagram.*

The line of action of R is found using Varignon's Moment Theorem: the resultant R and the distributed load $q(x)$ have to produce the same moment about an arbitrary point. The moment about point A, for example, gives

$$aR = \sum (x\Delta R) = \int_{x_1}^{x_2} x q(x)\,\mathrm{d}x$$

so that

$$a = \frac{\int_{x_1}^{x_2} x q(x)\,\mathrm{d}x}{R} = \frac{\int_{x_1}^{x_2} x q(x)\,\mathrm{d}x}{\int_{x_1}^{x_2} q(x)\,\mathrm{d}x}.$$

By definition, a is the x coordinate of the *centroid* of the load diagram.[1]

Conclusion: *The line of action of R passes through the centroid of the load diagram.*

Figures 6.16a and 6.16b give the magnitude and location of the resultant for a rectangular and triangular load diagram respectively.

Example 1
Determine the vertical support reactions at A and B of the simply supported beam AB in Figure 6.17a, with a distributed load that increases linearly from 4 kN/m at A to 12 kN/m at B.

Solution (units kN and m):
For the distributed load, with $\ell = 6$ m, applies

$$q(x) = 4 + 8\frac{x}{\ell}\ \mathrm{kN/m}.$$

The resultant R of the distributed load is

$$R = \int_0^{\ell} q(x)\,\mathrm{d}x = \int_0^6 \left(4 + 8\frac{x}{6}\right)\mathrm{d}x = \left(4x + 4\frac{x^2}{6}\right)\Bigg|_{x=0}^{x=6} = 48\ \mathrm{kN}.$$

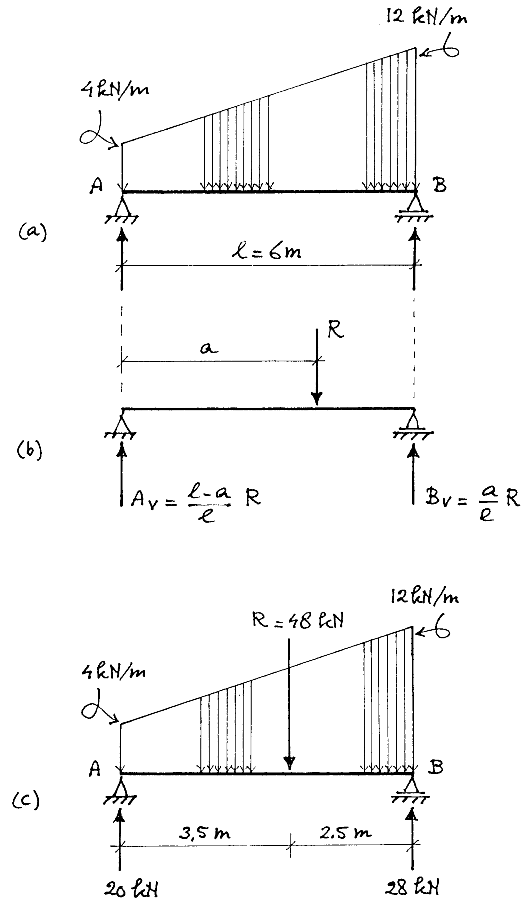


Figure 6.17 (a) A simply supported beam with a linearly distributed load. (b) The support reactions due to resultant R. (c) The magnitude and line of action of resultant R of the distributed load and the associated support reactions.

[1] Volume 2, *Stresses, Deformations, Displacements*, addresses the calculation of centroids. Here it is assumed that the reader is aware of the location of centroids in simple figures.

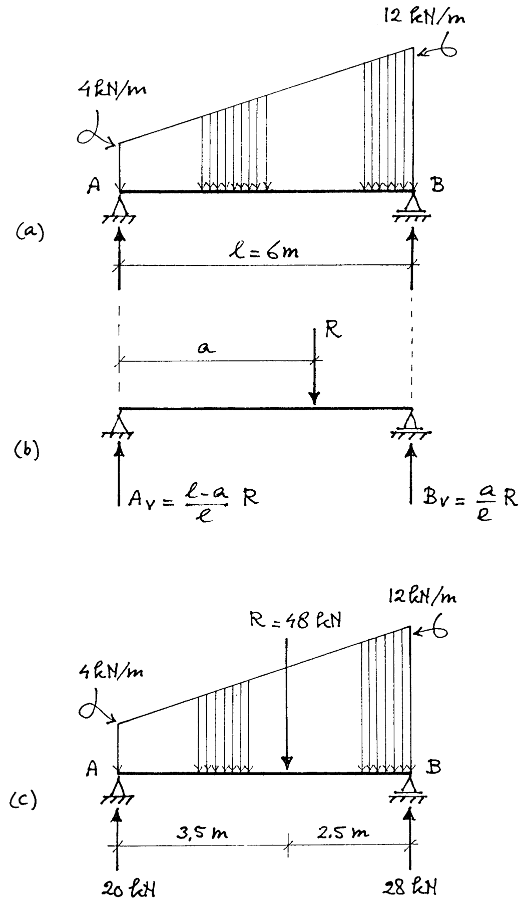


Figure 6.17 (a) A simply supported beam with a linearly distributed load. (b) The support reactions due to resultant R. (c) The magnitude and line of action of resultant R of the distributed load and the associated support reactions.

One can also determine R directly from the area of the trapezoidal load diagram:

$$R = \tfrac{1}{2} \times 6 \times (4 + 12) = 48 \text{ kN}.$$

In Figure 6.17b, the distributed load has been replaced by the resultant R. The line of action of R is determined by:

$$aR = \int_{x_1}^{x_2} x q(x)\,\mathrm{d}x = \int_0^6 \left(4x + 8\frac{x^2}{6}\right) \mathrm{d}x$$

$$= \left(2x^2 + 8\frac{x^3}{18}\right)\Bigg|_{x=0}^{x=6} = 168 \text{ kNm}$$

so that

$$a = \frac{168}{R} = \tfrac{168}{48} = 3.5 \text{ m}.$$

The magnitude and location of the resultant R of the distributed load are shown in Figure 6.17c.

The vertical support reactions at A and B are now found from the moment equilibrium about B and A respectively:

$$\sum T|\mathrm{B} = 0 \Rightarrow A_\mathrm{v} = \tfrac{2.5}{6} \times 48 = 20 \text{ kN},$$

$$\sum T|\mathrm{A} = 0 \Rightarrow B_\mathrm{v} = \tfrac{3.5}{6} \times 48 = 28 \text{ kN}.$$

A distributed load q may also be split up into loads q_1 and q_2, as in Figure 6.18, where the individual influences may be added. It always holds that

$$R = \int_{x_1}^{x_2} q(x)\,\mathrm{d}x = \int_{x_1}^{x_2} q_1(x)\,\mathrm{d}x + \int_{x_1}^{x_2} q_2(x)\,\mathrm{d}x = R_1 + R_2$$

and

$$aR = \int_{x_1}^{x_2} xq(x)\,\mathrm{d}x = \int_{x_1}^{x_2} xq_1(x)\,\mathrm{d}x + \int_{x_1}^{x_2} xq_2(x)\,\mathrm{d}x = a_1R_1 + a_2R_2.$$

The fact that the influences of the individual loads can be added is referred to as the *principle of superposition*. This principle is based on the fact that the relationships between the various quantities are linear.

If a load diagram can be split into a number of simpler diagrams, such as a number of rectangles and triangles, the abovementioned approach often leads to a result more quickly. This is illustrated in the next example.

Example 2
For the simply supported beam in Figure 6.19, the trapezoidal load diagram has been split into triangles and/or rectangles in four different ways.

Question:
Show that the same support reactions at A and B are found in all four cases.

Solution (units in kN and m):
In Figure 6.19a, the trapezoidal load has been split up into two triangular loads. The determination of the support reactions is shown below:

$$R_1 = \tfrac{1}{2} \times 6 \times 4 = 12\ \text{kN},$$

$$R_2 = \tfrac{1}{2} \times 6 \times 12 = 36\ \text{kN},$$

$$\sum T|\text{B} = 0 \Rightarrow A_\text{v} = \tfrac{4}{6}R_1 + \tfrac{2}{6}R_2 = 8 + 12 = 20\ \text{kN},$$

$$\sum T|\text{A} = 0 \Rightarrow B_\text{v} = \tfrac{2}{6}R_1 + \tfrac{4}{6}R_2 = 4 + 24 = 28\ \text{kN}.$$

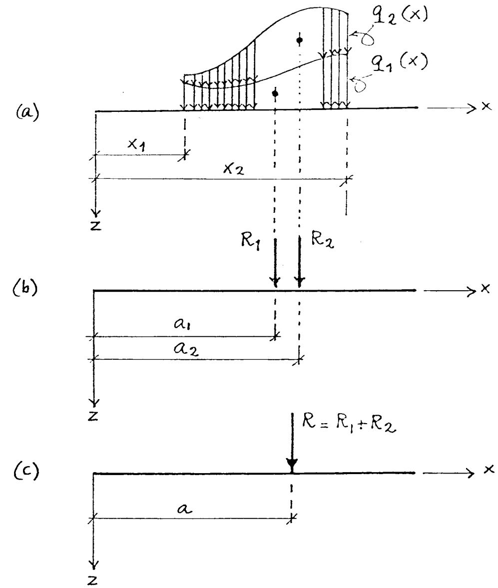


Figure 6.18 Principle of superposition: One can (a) split a distributed load q into loads q_1 and q_2, with (b) resultants R_1 and R_2, and (c) add their individual influences.

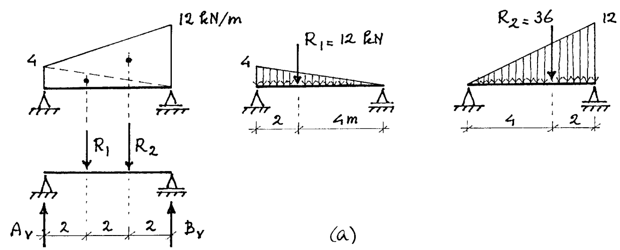


Figure 6.19 A simply supported beam with the trapezoidal load diagram split up into two triangles.

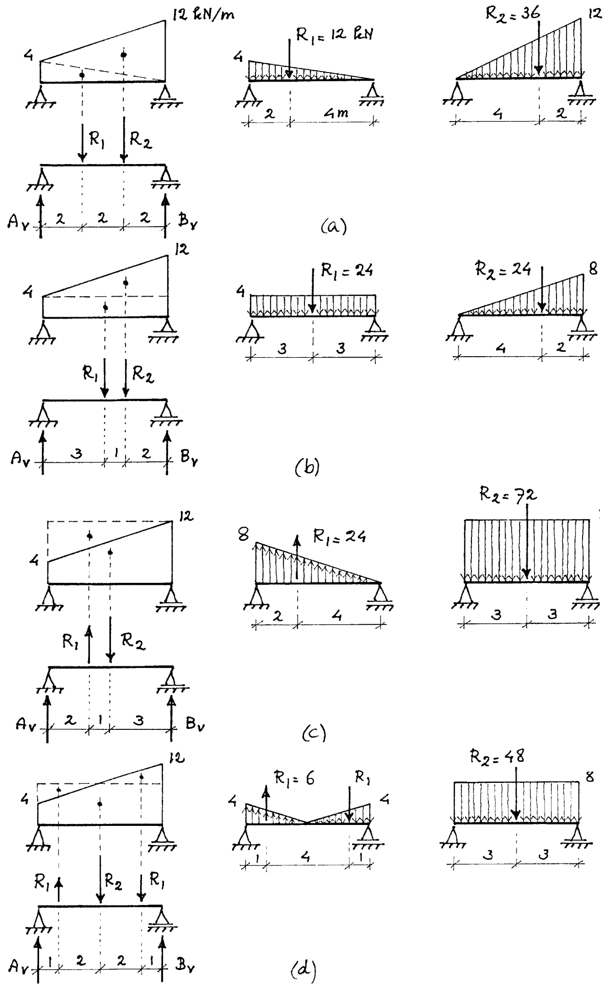


Figure 6.19 A simply supported beam with the trapezoidal load split up in four different ways into triangles and/or rectangles to determine the support reactions.

In Figure 6.19b, the trapezoidal load is divided into a rectangular and a triangular load diagram:

$$R_1 = 6 \times 4 = 24 \text{ kN},$$

$$R_2 = \tfrac{1}{2} \times 6 \times 8 = 24 \text{ kN},$$

$$\sum T|\text{B} = 0 \Rightarrow A_\text{v} = \tfrac{3}{6}R_1 + \tfrac{2}{6}R_2 = 12 + 8 = 20 \text{ kN},$$

$$\sum T|\text{A} = 0 \Rightarrow B_\text{v} = \tfrac{3}{6}R_1 + \tfrac{4}{6}R_2 = 12 + 16 = 28 \text{ kN}.$$

The trapezoidal load can be split into a uniformly distributed load and a triangular load in many other ways, as for example in Figure 6.19c:

$$R_1 = \tfrac{1}{2} \times 6 \times 8 = 24 \text{ kN},$$

$$R_2 = 6 \times 12 = 72 \text{ kN},$$

$$\sum T|\text{B} = 0 \Rightarrow A_\text{v} = -\tfrac{4}{6}R_1 + \tfrac{3}{6}R_2 = -16 + 36 = 20 \text{ kN},$$

$$\sum T|\text{A} = 0 \Rightarrow B_\text{v} = -\tfrac{2}{6}R_1 + \tfrac{3}{6}R_2 = -8 + 36 = 28 \text{ kN}.$$

If the trapezoidal load is split as shown in Figure 6.19d, it follows that

$$R_1 = \tfrac{1}{3} \times 3 \times 4 = 6 \text{ kN},$$

$$R_2 = 6 \times 8 = 48 \text{ kN},$$

$$\sum T|\text{B} = 0 \Rightarrow A_\text{v} = -\tfrac{5}{6}R_1 + \tfrac{3}{6}R_2 + \tfrac{1}{6}R_1 = -5 + 24 + 1 = 20 \text{ kN},$$

$$\sum T|\text{A} = 0 \Rightarrow B_\text{v} = -\tfrac{1}{6}R_1 + \tfrac{3}{6}R_2 + \tfrac{5}{6}R_1 = -1 + 24 + 5 = 28 \text{ kN}.$$

Irrespective of how the load diagram is split, the support reactions are always the same.

Example 3

Part of a roof modelled as the line element in Figure 6.20a is supported by a hinge at A, and a roller with vertical roller track at B. The member is loaded by three uniformly distributed (line) loads, of which the load diagrams are shown in Figures 6.20b to 6.20d:

- the *dead weight* q_{dw} (vertical force per length measured along the member axis);
- a *snow load* q_{sn} (vertical force per horizontally measured length);
- a *wind load* q_{w} normal to the member axis (force per length measured along the member axis).

Unlike dead weight and wind load, the snow load is given as a load per length projected on the horizontal ground plane. The load diagram for snow in Figure 6.20c is drawn differently therefore.

Question:
Determine the support reactions at A and B for all three loads.

Solution:
When calculating the support reactions, we can replace the distributed loads by their resultants. The dead weight and the wind load act over a length of $15a$, while the snow load acts over a length of $12a$, so that

$$R_{\mathrm{dw}} = 15aq_{\mathrm{dw}},$$

$$R_{\mathrm{sn}} = 12aq_{\mathrm{sn}},$$

$$R_{\mathrm{w}} = 15aq_{\mathrm{w}}.$$

The resultants and their lines of action are shown in Figures 6.20e to 6.20g. The same figures also show the associated support reactions.

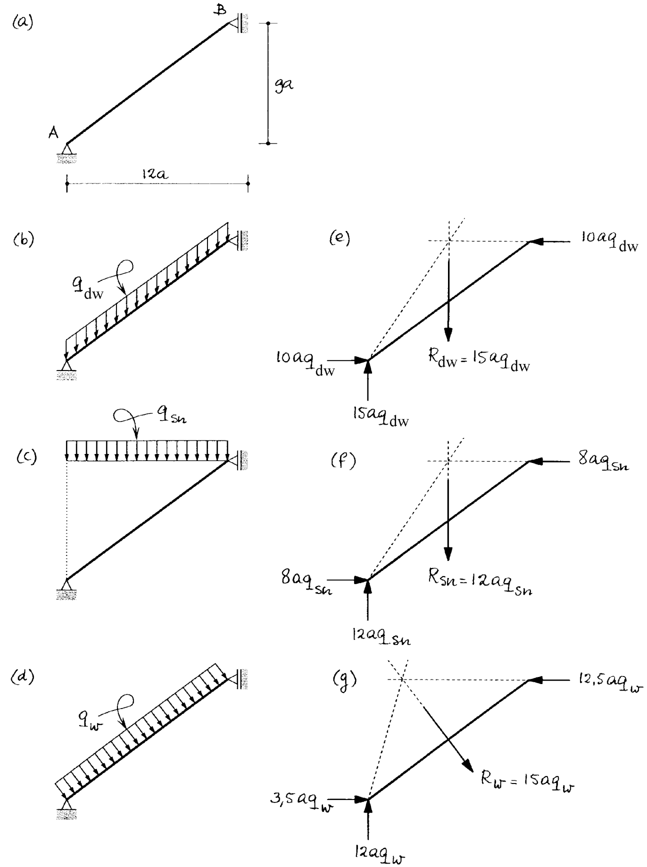


Figure 6.20 (a) Part of a roof, modelled as a line element, is loaded by (b) its dead weight, (c) snow, and (d) wind. The resultant and support reactions due to (e) the dead weight, (f) snow load, and (g) wind load.

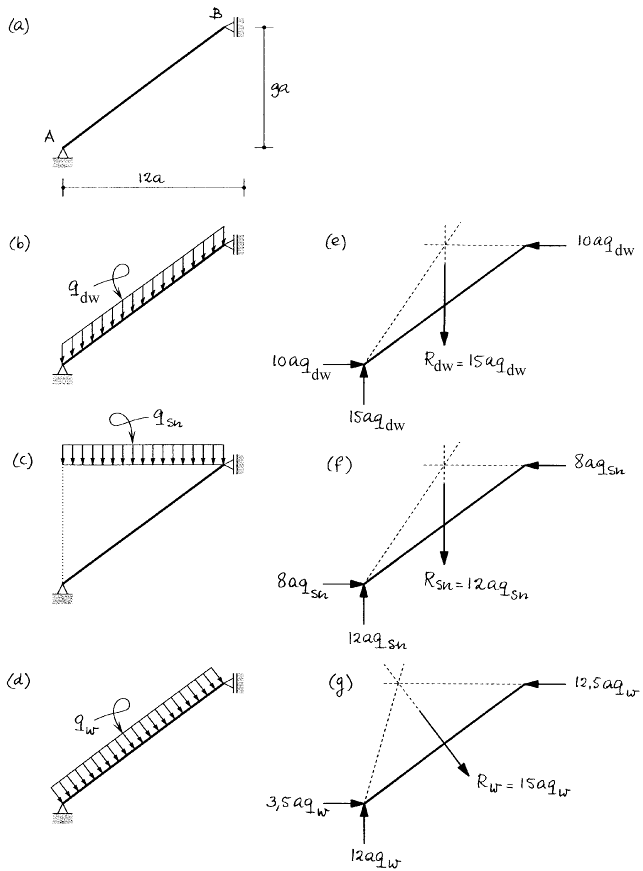


Figure 6.20 (a) Part of a roof, modelled as a line element, is loaded by (b) its dead weight, (c) snow, and (d) wind. The resultant and support reactions due to (e) the dead weight, (f) snow load, and (g) wind load.

Table 6.1

	R	$A_v(\uparrow)$	$A_h(\rightarrow)$	$B_h(\leftarrow)$
dead weight	$R_{dw} = 15aq_{dw}$	$15aq_{dw}$	$10aq_{dw}$	$10aq_{dw}$
snow	$R_{sn} = 12aq_{sn}$	$12aq_{sn}$	$8aq_{sn}$	$8aq_{sn}$
wind	$R_w = 15aq_w$	$12aq_w$	$3.5aq_w$	$12.5aq_w$

All the values are shown Table 6.1.

The reader is asked to verify the correctness of the support reactions.

Example 4

The hinged beam in Figure 6.21a carries a uniformly distributed load of 6 kN/m over part CD.

Question:
Determine the support reactions.

Solution:
In an equilibrium system, a distributed load may be replaced by its resultant. Therefore, when looking at the equilibrium of the hinged beam as a whole, we can use the resultant of the entire distributed load (see Figure 6.21b). For the directions assumed for the support reactions, this gives

$$\sum F_x^{(\mathrm{ASD})} = -A_h = 0 \Rightarrow A_h = 0,$$

$$\sum F_z^{(\mathrm{ASD})} = -A_v - B_v - C_v + (36\ \mathrm{kN}) = 0, \tag{a}$$

$$\sum T_y^{((\mathrm{ASD})}|\mathrm{A} = +B_v \times (4\ \mathrm{m}) + C_v \times (8\ \mathrm{m}) - (36\ \mathrm{kN})(5\ \mathrm{m}) = 0. \tag{b}$$

The two equations (a) and (b) are not sufficient to determine all vertical support reactions. The additional equation required is found from the moment equilibrium of parts SA or SD about hinge S. In this case it is not possible to work with the resultant in Figure 6.21b; this resultant has to be replaced by the resultants of the distributed loads on the individual parts (see Figure 6.21c).

With equations (a) and (b), an efficient way of obtaining results is to consider the moment equilibrium of SD:

$$\sum T_y^{(\mathrm{SD})}|\mathrm{S} = +C_\mathrm{v} \times (2\ \mathrm{m}) - (12\ \mathrm{kN})(1\ \mathrm{m}) = 0. \qquad \text{(c)}$$

From (c) we find

$$C_\mathrm{v} = +6\ \mathrm{kN}$$

which then gives the following from (b) and (a)

$$B_\mathrm{v} = +33\ \mathrm{kN},$$

$$A_\mathrm{v} = -3\ \mathrm{kN}.$$

Figure 6.21d shows the support reactions in the directions in which they are really acting. Only the direction of the vertical support reaction at A was assumed falsely.

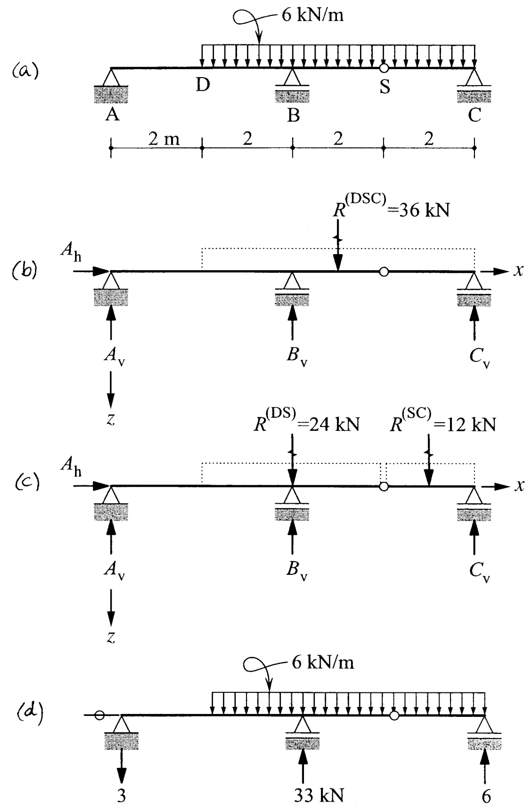


Figure 6.21 (a) Hinged beam with uniformly distributed load. (b) For the equilibrium of the structure as a whole, the total distributed load can be replaced by its resultant. (c) For the equilibrium of the separate parts, each part has its own resultant, and one may no longer use the resultant of the total distributed load. (d) Support reactions.

6.3.2 Resultant of a surface load on a plate

In Figure 6.22a, a plate in the xy plane is loaded normal to its plane by an arbitrarily distributed load $p(x, y)$.

The resultant of the distributed load on a small area ΔA is a small force ΔR:

$$\Delta R = p(x, y)\Delta A.$$

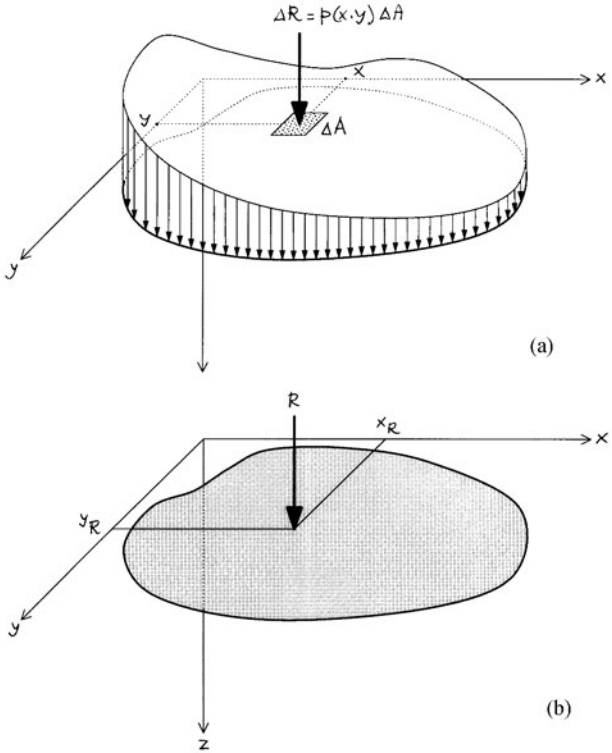


Figure 6.22 (a) A plate in the xy plane is loaded normal to its plane by an arbitrarily distributed load $p(x, y)$. A small force $\Delta R = p(x, y)\Delta A$ is acting on a small area ΔA. (b) The resultant R of the distributed load and the location (x_R, y_R) where the line of action of R intersects the xy plane. The magnitude of R is equal to the volume of the load diagram. The line of action of R passes through the centroid of the load diagram.

The magnitude of the resultant R of the distributed load is equal to the sum of all parallel forces ΔR:

$$R = \sum \Delta R = \sum p(x, y)\Delta A = \int_A p(x, y)\,\mathrm{d}A.$$

Conclusion: *The magnitude of R is equal to the volume of the load diagram.*

The location (x_R, y_R) of the line of action of R is found using Varignon's theorem[1] (see Figure 6.22b):

$$\sum T_y = -x_R R = -\sum (x\Delta R) = -\int_A xp(x, y)\,\mathrm{d}A,$$

$$\sum T_x = +y_R R = +\sum (y\Delta R) = +\int_A yp(x, y)\,\mathrm{d}A$$

so that

$$x_R = \frac{\int_A xp(x, y)\,\mathrm{d}A}{R} = \frac{\int_A xp(x, y)\,\mathrm{d}A}{\int_A p(x, y)\,\mathrm{d}A},$$

$$y_R = \frac{\int_A yp(x, y)\,\mathrm{d}A}{R} = \frac{\int_A yp(x, y)\,\mathrm{d}A}{\int_A p(x, y)\,\mathrm{d}A}.$$

By definition, x_R and y_R are the x and y coordinates of the centroid of the load diagram.

---

1 See also Examples 1 and 2 in Section 3.3.4.

Conclusion: *The line of action of R passes through the centroid of the load diagram.*

Example

In Figure 6.23a, a rectangular plate in the xy plane, with an area $A = ab$, is loaded by a distributed load normal to its plane:

$$p(x, y) = \hat{p}\frac{x(b-y)}{ab}.$$

Question:

Determine the magnitude of the resultant R and the coordinates $(x_R; y_R)$ where the line of action of R intersects the xy plane.

Solution:

The magnitude of the resultant R is equal to the volume of the load diagram:

$$\begin{aligned} R &= \int_A p(x, y)\,\mathrm{d}A = \int_0^a \int_0^b \hat{p}\frac{x(b-y)}{ab}\,\mathrm{d}x\,\mathrm{d}y \\ &= \frac{\hat{p}}{ab}\int_0^a x\,\mathrm{d}x \int_0^b (b-y)\,\mathrm{d}y \\ &= \frac{\hat{p}}{ab}\frac{x^2}{2}\bigg|_0^a \left(by - \frac{y^2}{2}\right)\bigg|_0^b \\ &= \frac{\hat{p}ab}{4}. \end{aligned}$$

The line of action of R passes through the centroid of the load diagram (see Figure 6.23b). For the coordinates (x_R, y_R) of the centroid, the formulas derived earlier can be used.

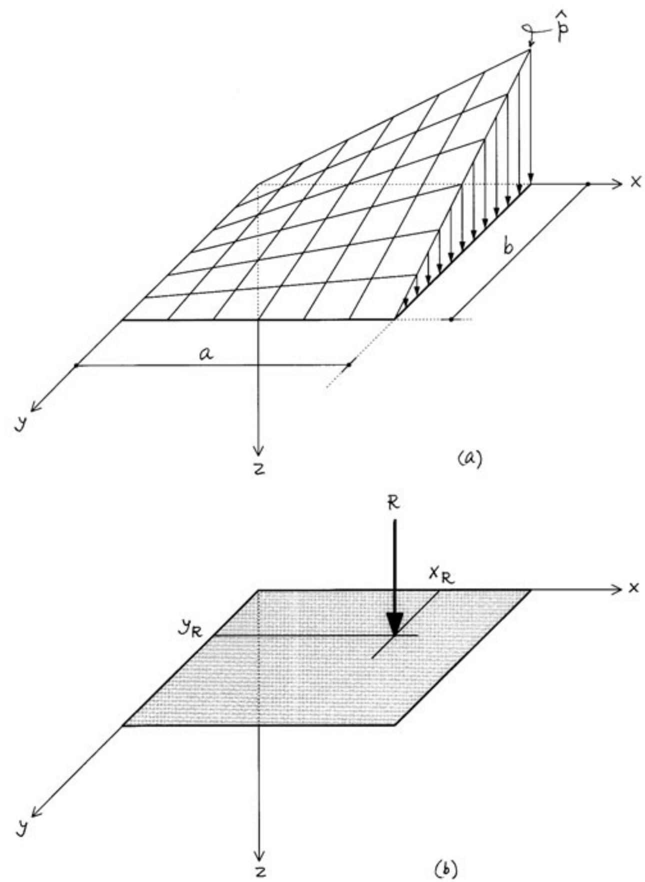
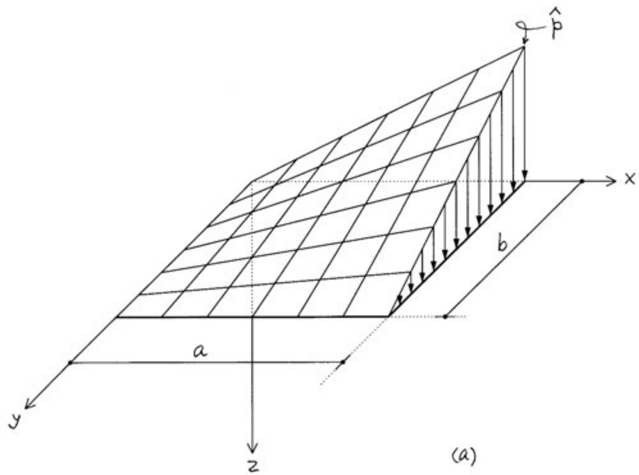


Figure 6.23 (a) A rectangular plate in the xy plane is loaded normal to its plane by a distributed load. (b) The resultant R of the distributed load and the location (x_R, y_R) where the line of action of R intersects the xy plane.



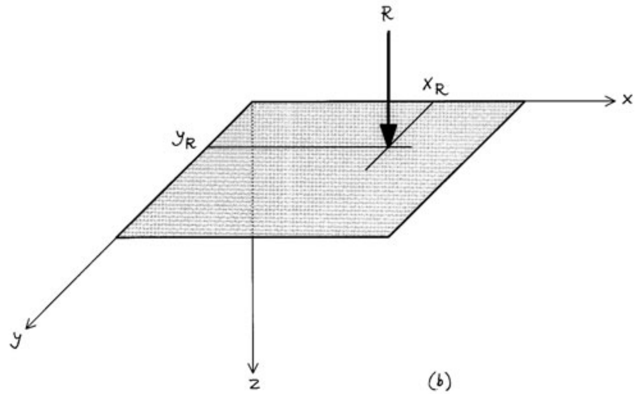


Figure 6.23 (a) A rectangular plate in the xy plane is loaded normal to its plane by a distributed load. (b) The resultant R of the distributed load and the location (x_R, y_R) where the line of action of R intersects the xy plane.

It is also possible to start at once with Varignon's theorem:

$$\sum T_y = -x_R R = -\int_A x p(x, y)\,\mathrm{d}A,$$

$$\sum T_x = +y_R R = +\int_A y p(x, y)\,\mathrm{d}A.$$

This gives

$$x_R = \frac{\displaystyle\int_A x p(x, y)\,\mathrm{d}A}{R} = \frac{\dfrac{\hat{p}}{ab}\displaystyle\int_0^a x^2\,\mathrm{d}x \int_0^b (b - y)\,\mathrm{d}y}{\dfrac{\hat{p}ab}{4}}$$

$$= \frac{\dfrac{\hat{p}}{ab} \times \dfrac{a^3}{3} \times \dfrac{b^2}{2}}{\dfrac{\hat{p}ab}{4}} = \tfrac{2}{3}a,$$

$$y_R = \frac{\displaystyle\int_A y p(x, y)\,\mathrm{d}A}{R} = \frac{\dfrac{\hat{p}}{ab}\displaystyle\int_0^a x\,\mathrm{d}x \int_0^b (by - y^2)\,\mathrm{d}y}{\dfrac{\hat{p}ab}{4}}$$

$$= \frac{\dfrac{\hat{p}}{ab} \times \dfrac{a^2}{2} \times \dfrac{b^3}{6}}{\dfrac{\hat{p}ab}{4}} = \tfrac{1}{3}b.$$

6.4 Modelling load flow

How the load is taken into account depends greatly on the way in which a structure or structural element is modelled. For example, the dead weight of a member modelled as a line element is not considered as volume load, but rather as a line load. In the same way, the dead weight of a plate (surface element) will be taken into account as a surface load.

This will be demonstrated in an example using the simple concrete building in Figure 6.24a. For this example we will investigate how the vertical loads on the building are transferred to the foundation.

The building consists of two frames that, 4 metres apart, carry the roof slabs. Figure 6.24b shows one of the frames. Each frame consists of an 8-metre beam that is simply supported at both ends by a column. The 5-metre columns are rigidly joined to a square footing, located at a certain depth below ground level. The dead weight of the roof slabs is 2 kN/m^2. The weight of the waterproof roof covering and insulation is set at 0.3 kN/m^2. In addition, a live load of 0.5 kN/m^2 is taken into account. The total load on the roof slabs is therefore

$$p = (2 \text{ kN/m}^2) + 0.3 \text{ kN/m}^2) + (0.5 \text{ kN/m}^2) = 2.8 \text{ kN/m}^2.$$

Figure 6.25a shows a load of 2.8 kN acting on a square metre. If one takes an arbitrary strip of the roof of 1-metre width, the total load on the strip would be

$$(4 \text{ m})(1 \text{ m})(2.8 \text{ kN/m}^2) = 11.2 \text{ kN}.$$

Each beam carries half of this, or in other words, 5.6 kN over a 1-metre length, (see Figure 6.25b). Over the full length, the beam is therefore loaded by a uniformly distributed load of 5.6 kN/m. We also have to include the dead weight of the beam. If we assume a dead weight of 6 kN/m, the total

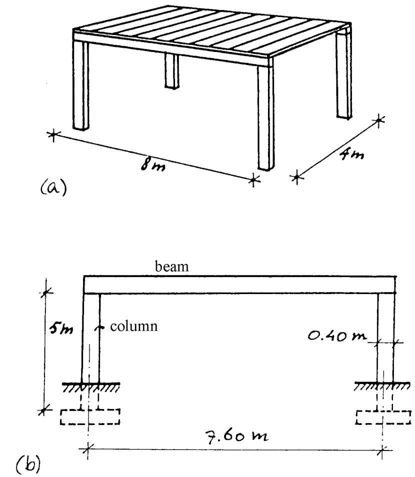


Figure 6.24 (a) A simple concrete building consisting of two frames, covered by roof slabs. (b) Each frame consists of an 8-metre beam that at each end is simply supported on a column. The 5-metre columns are rigidly joined to a square footing, which is located at a certain depth below ground level.

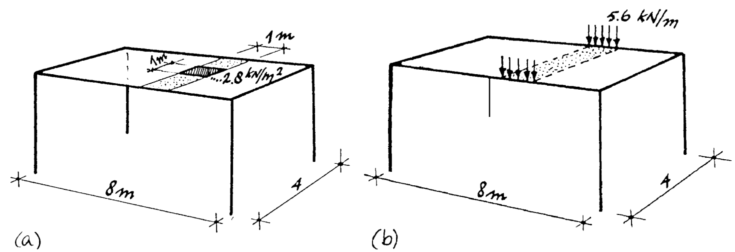


Figure 6.25 (a) The total roof load is 2.8 kN/m^2; (b) this generates a uniformly distributed load on the beam equal to 5.6 kN/m.

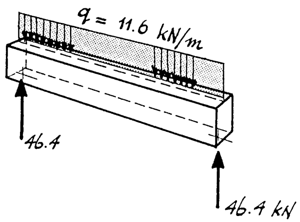


Figure 6.26 All the forces acting on the isolated beam. The distributed load is composed of the roof load and a dead weight of 6 kN/m. The support reactions of 46.4 kN have to be provided by the columns.

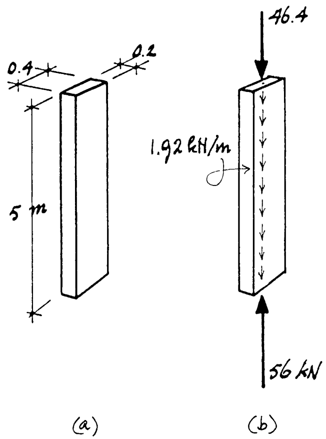


Figure 6.27 (a) The column dimensions and (b) all the forces acting on the isolated column. The dead weight of the column is a uniformly distributed load parallel to the column axis.

load on the beam is (see Figure 6.26)

$$q = (5.6\ \text{kN/m}) + (6\ \text{kN/m}) = 11.6\ \text{kN/m}.$$

The beam is simply supported. The support reactions, which have to be provided by the columns, amount to

$$\tfrac{1}{2} \times (11.6\ \text{kN/m})(8\ \text{m}) = 46.4\ \text{kN}.$$

Equal and opposite forces are acting on the columns.

Figure 6.27a shows the cross-sectional dimensions of the columns. With mass density $\rho = 2400$ kg/m, the specific weight of concrete is

$$\rho g = (2400\ \text{kg/m}^3) \times (10\ \text{N/kg}) = 24000\ \text{N/m}^3) = 24\ \text{kN/m}^3.$$

For the cross-sectional dimensions of the column in Figure 6.27a, the dead weight per length is

$$(0.2\ \text{m})(0.4\ \text{m})(24\ \text{kN/m}^3) = 1.92\ \text{kN/m}.$$

This is a uniformly distributed load acting in the direction of the column axis (see Figure 6.27b).

The total dead weight of the column is

$$(5\ \text{m})(1.92\ \text{kN/m}) = 9.6\ \text{kN}.$$

At its base, the column has to be kept in equilibrium by a force of

$$(46.4\ \text{kN}) + (5\ \text{m})(1.92\ \text{kN/m}) = 56\ \text{kN}.$$

An equally large, opposite force is acting on the footing of the column. If the footing is square, and has the dimensions given in Figure 6.28a, the dead weight of the footing is

$$(0.8 \text{ m})(0.8 \text{ m})(0.3 \text{ m})(24 \text{ kN/m}^3) = 4.6 \text{ kN}.$$

The earth pressure on the bottom of the footing has to be in equilibrium with the force of 56 kN from the column, and the footing's dead weight of 4.6 kN (see Figure 6.28b)

$$(56 \text{ kN}) + (4.6 \text{ kN}) = 60.6 \text{ kN}.$$

If the earth pressure is uniformly distributed, it equals

$$p = \frac{60.6 \times 10^3 \text{ N}}{(800 \text{ mm})(800 \text{ mm})} = 0.095 \text{ N/mm}^2.$$

In general, the earth pressure is not uniformly distributed. The value given for p is then referred to as the *average earth pressure*.

How the load exerted by the footing is transferred further into the ground, is a problem addressed by the special field of *soil mechanics*.

6.5 Stress concept; normal stress and shear stress

In reality, the earth pressure on the footing in Figure 6.29 consists of a very large number of very small forces provided by the grains of soil. Spreading all these forces evenly into a distributed surface load implies an idealisation of reality: the soil as a discontinuous material is replaced by a continuous material.

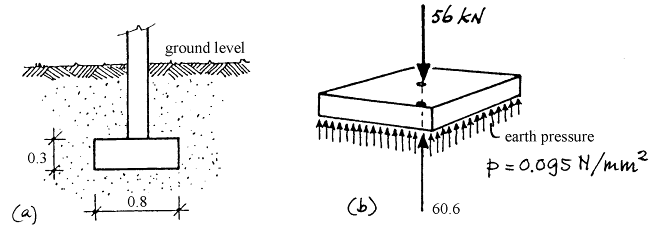


Figure 6.28 (a) The dimensions of the square footing. (b) On the bottom of the footing, the earth pressure has to provide an equilibrium with the force of 56 kN from the column and the footing's dead weight of 4.6 kN. The resultant of the earth pressure is therefore 60.6 kN.

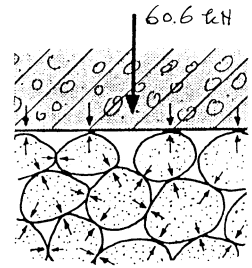


Figure 6.29 In reality, the earth pressure on the footing consists of a very large number of small forces provided by the grains of soil. Spreading all these forces evenly into a distributed surface load implies an idealisation of reality.

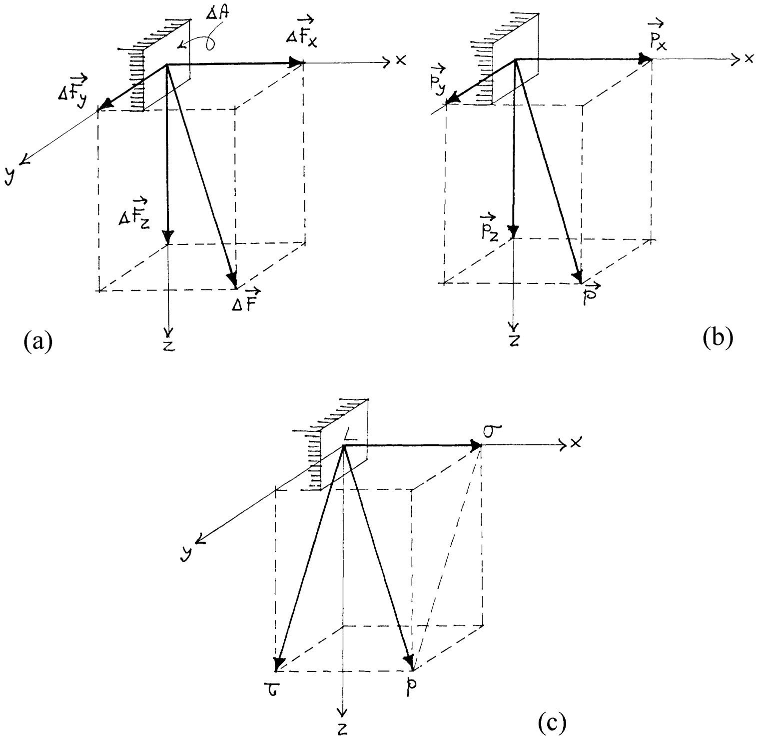


Figure 6.30 (a) Force $\Delta\vec{F}$ is the resultant of all the forces acting on a small yet finite area ΔA. (b) Stress vector $\vec{p}$ with its components. (c) Stress p (in visual notation) resolved into the normal stress σ perpendicular to the section plane and the shear stress τ in the section plane.

In fact, as a result of their atomic structure, all materials are discontinuous. The force flow in and between materials is the result of a very large number of small interactions between adjacent elementary particles.

Mathematically, the transfer of forces in and between materials is described using the concept *stress*. This concept is explained using Figure 6.30, in which a part of a body has been isolated from its environment.

Imagine that force $\Delta\vec{F}$ in Figure 6.30a is the resultant of all the small forces acting on a small, but finite area ΔA. As ΔA is chosen to be smaller, $\Delta\vec{F}$ is also smaller. The limit of the relationship between $\Delta\vec{F}$ and ΔA as ΔA approaches zero is defined as the *stress vector* $\vec{p}$:

$$\vec{p} = \lim_{\Delta A \to 0} \frac{\Delta\vec{F}}{\Delta A}.$$

When introducing the stress concept, one uses an idealised model of reality (a continuous material). The justification of this model is given *post hoc* by the agreement between model and reality. This agreement only exists if the stresses vary gradually. In areas with a major change in stresses (in the surroundings of *stress peaks*), one has to take into account the differences between model and reality.

The stress vector $\vec{p}$ (in space) has three components: p_x, p_y and p_z (see Figure 6.30b).

The stress vector can also be resolved into a component σ normal to the section plane and a component τ in the section plane. See Figure 6.30c, in which the visual notation is used. σ is known as *normal stress* and τ is referred to as *shear stress*.[1]

In mechanics, it is common practice to define the normal stress σ in solids as positive if it is a tensile stress. Sometimes, if dealing mainly with compressive stresses, it can be useful to define compressive stresses as positive. We often use a *prime* to indicate a change in sign. In such a case, $\sigma' = 300\ \text{N/m}^2$ means the same as $\sigma = -300\ \text{N/m}^2$. However, be aware that this notation is not always used, for instance in the cases of gas, liquid and earth pressures.

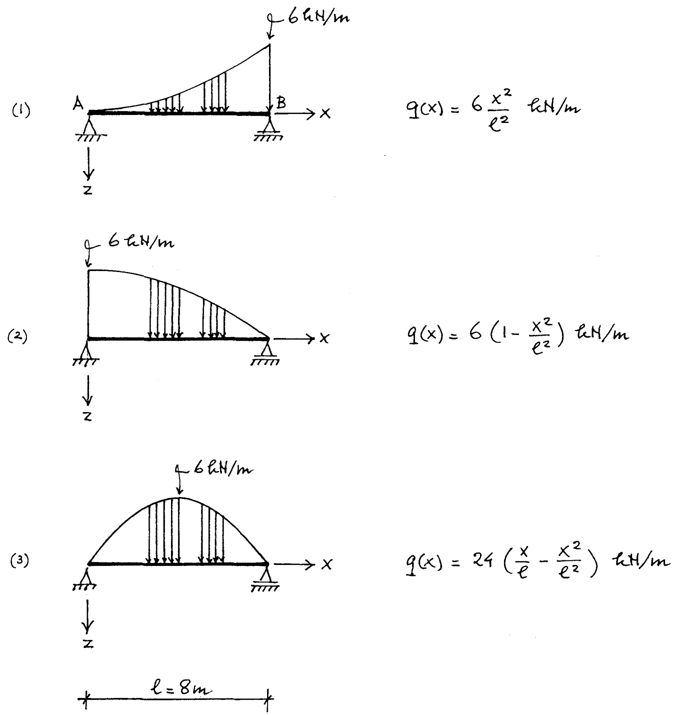
---

1 The normal stress and shear stress are shown (for the present) as the components of a stress vector. Using the normal stress and shear stress to describe the *interaction* in the section plane, this presentation is not complete. In that case the normal vector on the section plane has to be considered. The complete definition is addressed in Chapter 10, where we look at the section forces in a member. Here it becomes clear that also for the shear stress it is possible to have an unequivocal sign convention.

6.6 Problems

Resultant of a line load on a member (Section 6.3.1)

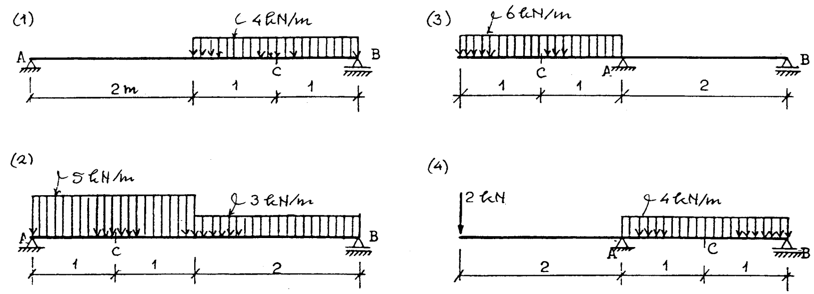
6.1: 1–3 The same simply supported beam AB is carrying three parabolically distributed loads.



Questions:

a. Determine the line of action and magnitude of the resultant of the distributed load.
b. Determine the support reactions at A and B.

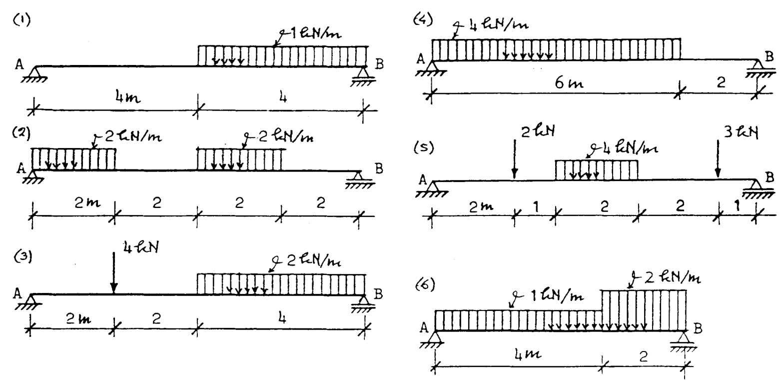
6.2: 1–4



Questions:

a. Determine the support reaction at A.
b. Determine the support reaction at B.
c. Determine the interaction forces at C.

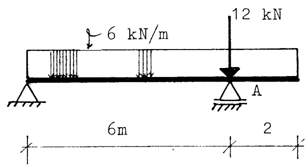
6.3: 1–6



Questions:

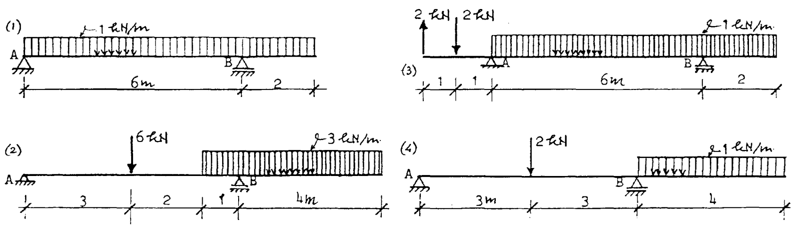
a. Determine the support reaction at A.
b. Determine the support reaction at B.

6.4



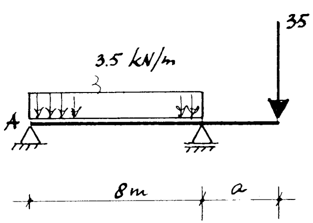
Question: Determine the support reaction at A.

6.5: 1–4



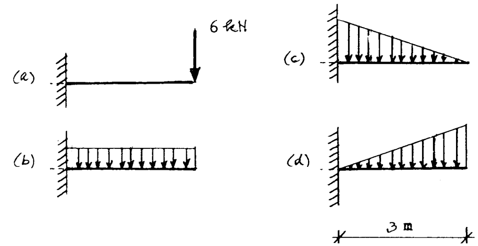
Questions:
a. Determine the support reaction at A.
b. Determine the support reaction at B.

6.6



Question:
For which length *a* of the cantilever is the support reaction at A zero for the given load?

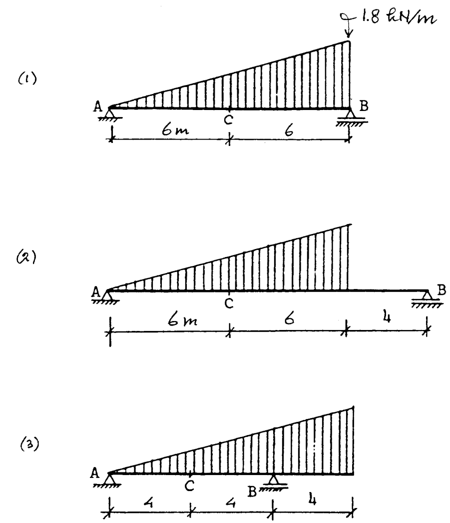
6.7 The same fixed beam is loaded in four different ways.



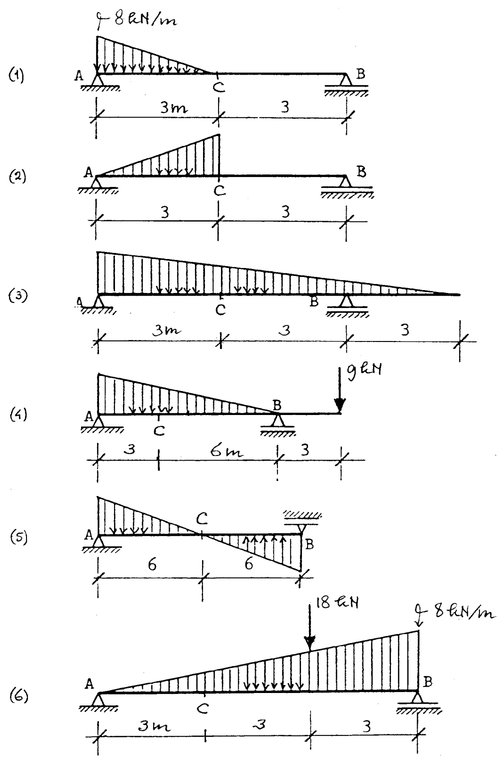
Question:
Determine the (peak value of the) distributed loads so that the fixed-end moment in all four cases is the same.

6.8: 1–3 Three beams are given with a linearly distributed load. The peak value of the distributed load is 1.8 kN/m for all cases.

Questions:
a. Determine the support reaction at A.
b. Determine the support reaction at B.
c. Determine the interaction forces at C.



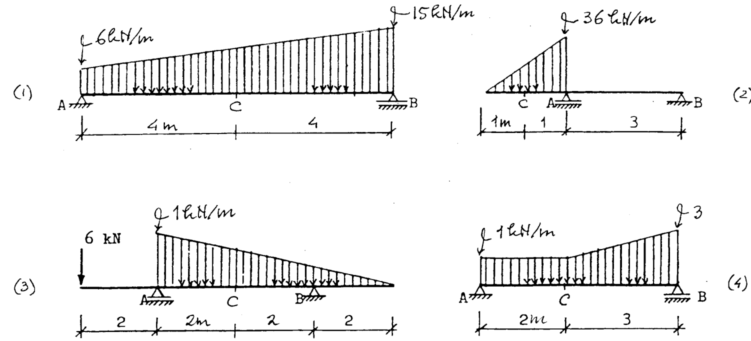
6.9: 1–6 A number of beams are given with a linearly distributed load and also a point load in two cases. The figures are not all shown to the same scale. The top value of the linearly distributed load is 8 kN/m in all cases. The magnitude of the point loads is given in the figure.



Questions:

a. Determine the support reaction at A.
b. Determine the support reaction at B.
c. Determine the interaction forces at C.

6.10: 1–4

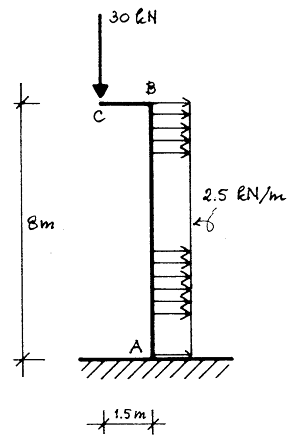


Questions:

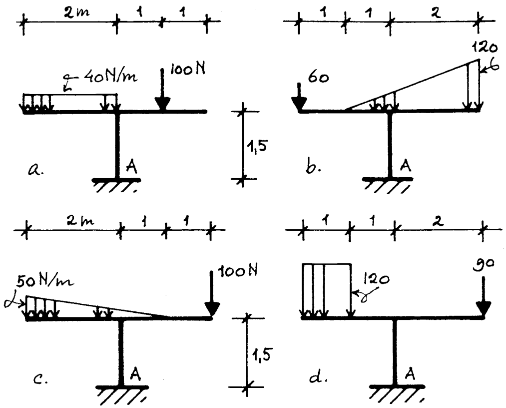
a. Determine the support reaction at A.
b. Determine the support reaction at B.
c. Determine the interaction forces at C.

6.11 The fixed member AB is loaded by an eccentric compressive force, and a uniformly distributed horizontal load.

Question:
Determine the support reactions at A.

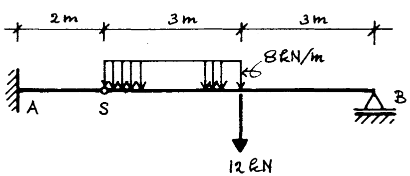


6.12 The same shelter is loaded in four different ways.



Question:
In which case is the fixed-end moment at most?

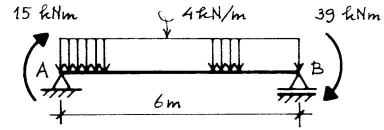
6.13 Hinged beam ASB is fixed at A.



Question:
Determine the fixed-end moment at A.

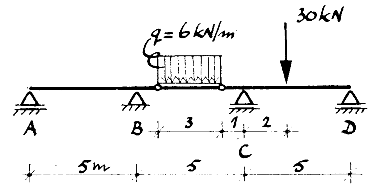
6.14 The simply supported beam AB is carrying a uniformly distributed load over the entire length and is also loaded by couples at the supports.

Questions:
a. Determine the support reaction at A.
b. Determine the support reaction at B.



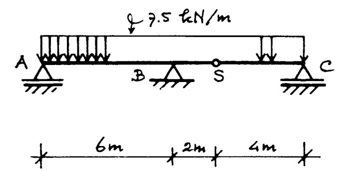
6.15 You are given a hinged beam.

Questions:
a. Determine the support reaction at A.
b. Determine the support reaction at D.
c. Determine the other support reactions.

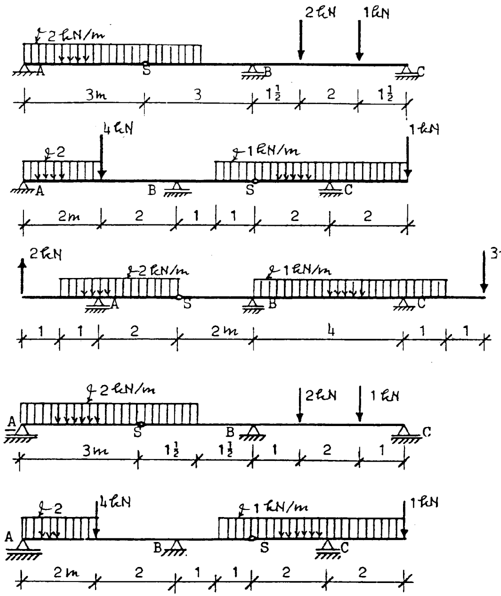


6.16 You are given a hinged beam.

Questions:
a. Determine the support reaction a A.
b. Determine the other support reactions.



6.17: 1–5 You are given five different hinged beams.

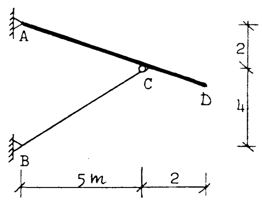


Question:
Determine the support reaction at A, B and C.

6.18 You are given a canopy roof ACD modelled as a line element.

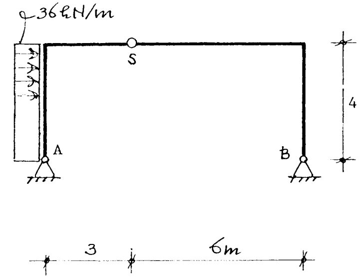
Questions: Determine the horizontal and vertical support reactions at A and the force in member BC (with the correct sign) due to the following uniformly distributed loads on ACD:

a. Dead weight of 2 kN/m.
b. Wind load of 3 kN/m.
c. Snow load of 4 kN/m.



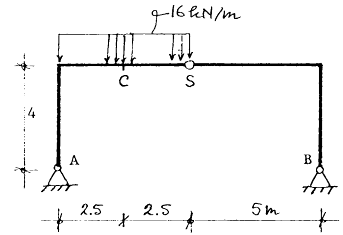
6.19 A uniformly distributed horizontal load is acting on the left-hand post of the three-hinged frame ASB.

Questions:
a. Determine the support reactions at A.
b. Determine the support reactions at B.



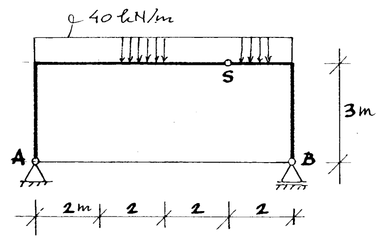
6.20 A uniformly distributed load is acting on the left-hand side of the three-hinged frame ASB.

Questions:
a. Determine the support reactions at A.
b. Determine the support reactions at B.
c. Determine the interaction forces at C.

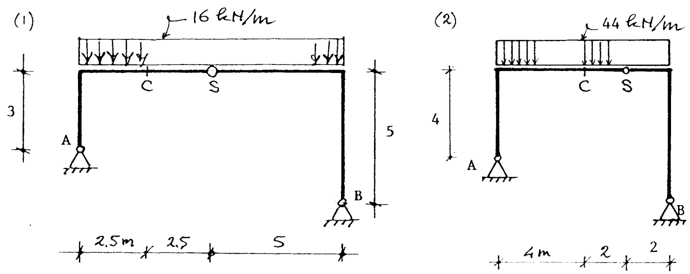


6.21 A three-hinged frame with tie-rod is carrying a uniformly distributed load of 40 kN/m.

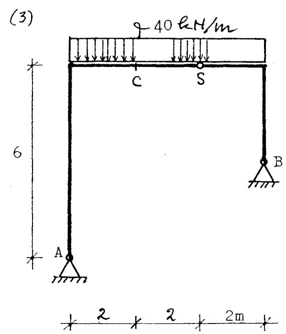
Questions:
a. Determine the support reaction at A.
b. Determine the force in tie-rod AB.



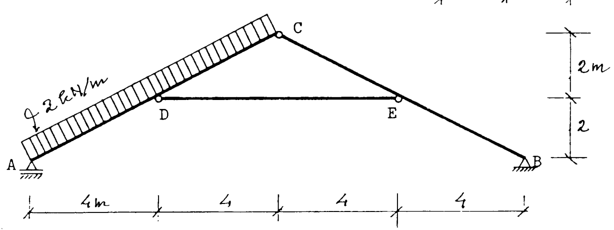
6.22: 1–3 You are given three different three-hinged frames with unequal post lengths.



Questions:
a. Determine the support reactions at A.
b. Determine the support reactions at B.
c. Determine the interaction forces at C.



6.23

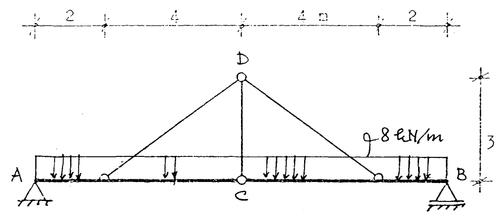


Questions:
a. Determine the support reactions at A and B.
b. Determine the force in bar DE, with the correct sign.

6.24 For the structure in problem 6.23, the roller and hinged support are exchanged.

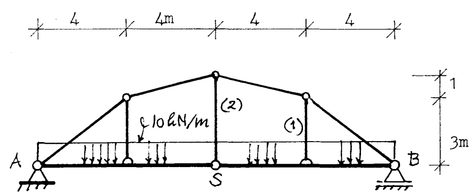
Questions:
a. Determine the support reactions at A and B.
b. Determine the force in bar DE, with the correct sign.

6.25 Trussed beam ACB is carrying over its entire length a uniformly distributed load of 8 kN/m.



Question:
Determine the force in bar CD. Is it a tensile force or a compressive force?

6.26 Trussed beam ASB is carrying over its entire length a uniformly distributed load of 10 kN/m.

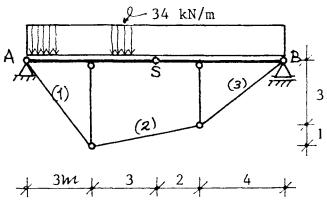


Questions:
a. Determine the force in bar 1.
b. Determine the force in bar 2.
c. Draw the closed force polygon for the equilibrium of joint B.

6.27 Trussed beam ASB is carrying over its entire length a uniformly distributed load of 34 kN/m.

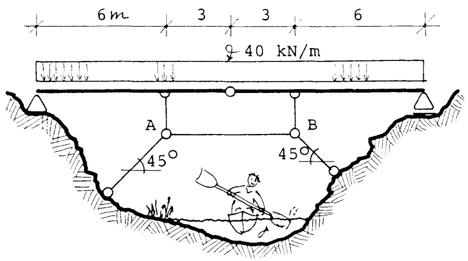
Questions:

a. Determine the force in bar 1.
b. Determine the force in bar 2.
c. Determine the force in bar 3.
d. Draw the closed force polygon for the equilibrium of joint A.
e. Draw the closed force polygon for the equilibrium of joint B.



6.28 You are given a queen post truss with a uniformly distributed load of 40 kN/m.

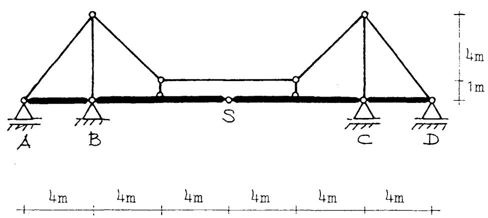
Question:
Determine the force in member AB. Is this a tensile force or a compressive force?



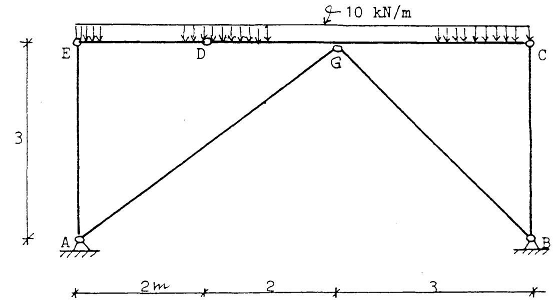
6.29 The dead weight of beam ABSCD is 125 kN/m.

Questions:

a. Determine the support reaction at A due to this dead weight.
b. Determine the other support reactions.



6.30 In the compound structure shown, ED and DC are connected by a hinge at D.

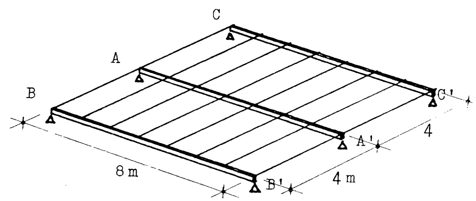


Questions:

a. Determine the vertical support reactions at A and B.
b. Determine the forces in the members AE and BC, with the correct signs.
c. Determine the forces in the members AG and BG, with the correct signs.
d. Determine the horizontal support reactions at A and B.
e. Draw the closed force polygon for the equilibrium of joint A.
f. Draw the closed force polygon for the equilibrium of joint B.

Modelling load flow (Section 6.4)

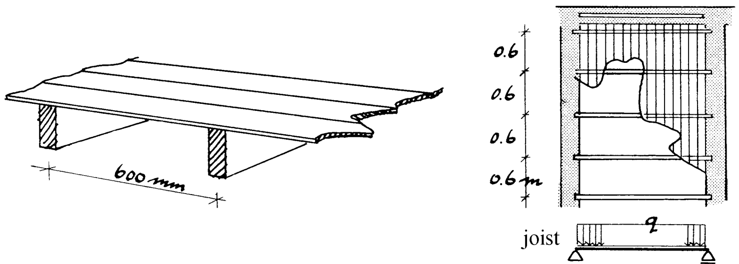
6.31 Steel beams AA′, BB′ and CC′ are carrying roof slabs. The dead weight of the roof slabs, together with the live load, equals 4 kN/m^2. The dead weight of the beams is estimated as 1 kN/m.



Questions:

a. Determine the (uniformly) distributed line load which has to be taken into account for beam AA′.
b. Determine the (uniformly) distributed line load which has to be taken into account for beam BB′.

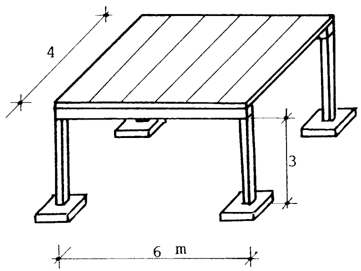
6.32 You are given a wooden joisting whereby the joists have a lateral distance of 0.6 m. The joists have a mass of 10 kg/m, and the floor has a mass of 10 kg/m^2. The live load is 1.5 kN/m^2. The load on an arbitrary joist (no edge joist) is modelled by as a uniformly distributed line load q. Let $g = 10$ N/kg.



Question:
How large is q?

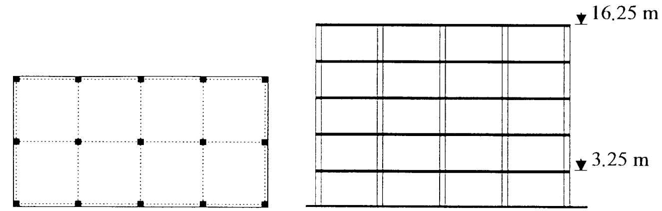
6.33 For the building on spread foundation the following is given:

- *Roof*
  - Live load 500 N/m^2
  - Dead load 300 N/m^2
  - Dead weight 1500 N/m^2
- *Beams*
  - Dead weight 3000 N/m
- *Columns*
  - Dead weight 1500 N/m



Question:
Determine the load on one of the footings.

6.34 You are given a concrete skeleton with the columns on a grid of 5.5×5.5 m^2 and a height between floors of 3.25 m. The floors and the roof are 0.25 m thick. All the columns have cross-sectional dimensions of 0.5×0.5 m^2. The specific weight of concrete is 24 kN/m^3. The dead load is 1.5 kN/m^2.

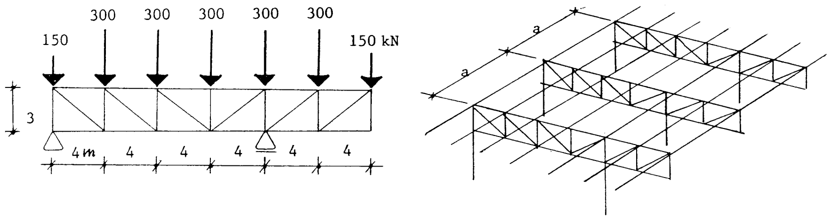


ground plan side view

Question:
Determine the load on the lower columns of the skeleton (or make a good estimation) due to the dead weight and dead load. Distinguish between:

a. a centre column,
b. an outer column, and
c. a corner column.

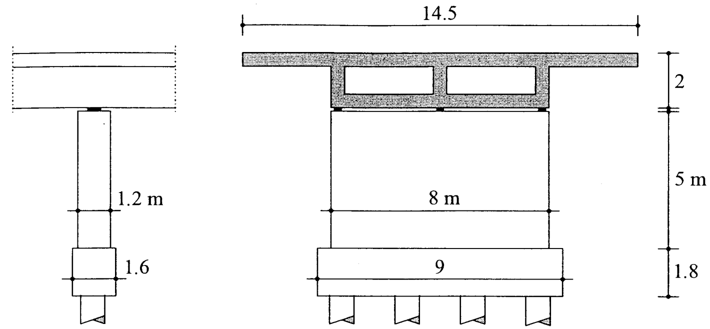
6.35 A roof truss is loaded by the forces shown. The truss spacing is a. The roof load (including the dead weight of the roof) is a uniformly distributed surface load p.



Question:
Which combination of truss spacing a and load p occur according to the given forces on the roof truss?

	Truss spacing a **(m)**	**Load** p **(kN/m^2)**
a.	4	22.5
b.	4	25
c.	3.75	20
d.	3	22.5
e.	3	25

6.36 For a concrete box girder bridge with a large number of spans, all the spans have the same length of 42 m. The piers have a pile foundation. The cross-sectional dimensions of the bridge and piers are shown in the figure. The box girder bridge has the same wall thickness of 0.4 m everywhere. The specific weight of concrete is 24 kN/m^3.



side view at pier   cross-section tubular bridge

Questions:
Due to the dead weight, determine:
a. The load on the bridge modelled as a line element.
b. The load on a centre pier.
c. The load on a pile under the centre pier (assuming all piles are loaded equally).