# 5

# Calculating Support Reactions and Interaction Forces

In this chapter, we will see how to calculate support reactions and interaction forces in statically determinate bar-type structures from the equilibrium equations using a number of examples.

For compound structures, if you write down all the available equilibrium equations and then try to solve the system, you soon end up with a large number of calculations. To prevent this, you have to select the equilibrium equations in a sensible order, preferably in such a way that an unknown force can be calculated directly with each new equilibrium equation.

The strategy for determining all the forces as efficiently as possible depends to a large degree on the type of structure. For this reason, in addition to self-contained structures, we will also look at compound and related structures, such as hinged beams, three-hinged frames (with or without tie-rods), shored structures and trussed beams.

The loading remains limited to a few point loads. In one case, the structure is loaded by a concentrated couple.

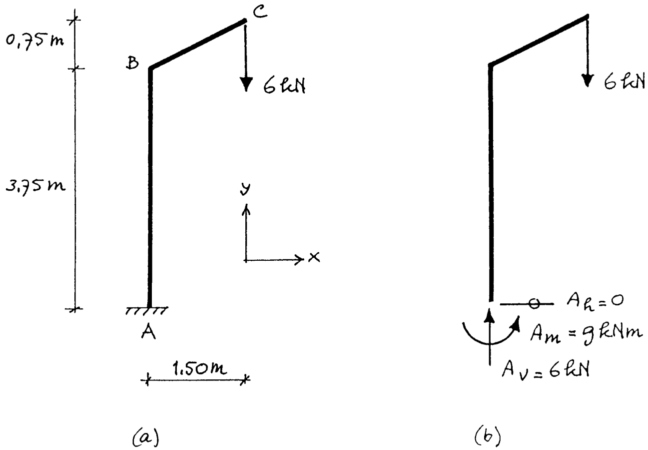


***Figure 5.1*** (a) A light mast fixed at A with (b) the support reactions.

## 5.1 Self-contained structures

In this section, we will use five examples to show how, for statically determinate self-contained structures, it is possible to determine the support reactions and interaction forces directly from the equilibrium.

**Example 1**
The light mast ABC in Figure 5.1a is fixed at A and is loaded at C by a vertical force of 6 kN.

*Question*:
Draw the support reactions at A as they are expected to act and determine them.

*Solution*:
No horizontal loading is being exerted on the mast. The horizontal support reaction at A is therefore zero. The vertical support reaction at A must generate an equilibrium with the vertical force of 6 kN, and will therefore be pointed upwards. In order to determine the fixed-end moment, the isolated structure is considered to be pinned at A. The load causes a clockwise rotation about A. The fixed-end moment has to prevent this rotation and will therefore act counter-clockwise. The support reactions are shown in Figure 5.1b. The equilibrium equations are

$$\sum F_x = A_\mathrm{h} = 0,$$

$$\sum F_y = -(6\ \mathrm{kN}) + A_\mathrm{v} = 0,$$

$$\sum T_z|\mathrm{A} = -(6\ \mathrm{kN}) \times (1.5\ \mathrm{m}) + A_\mathrm{m} = 0.$$

The solution is

$$A_\mathrm{h} = 0,\ \ A_\mathrm{v} = 6\ \mathrm{kN}\ \ \text{and}\ \ A_\mathrm{m} = 9\ \mathrm{kNm}.$$

The fact that the solutions found are positive confirms the correctness of the directions assumed for these support reactions.

Note that the support reaction $A_\text{v}$ and the force of 6 kN at C together form a couple that is in equilibrium with the fixed-end moment $A_\text{m}$.

**Example 2**
In Figure 5.2a, a block with a weight of 60 kN is supported on three bars.

*Questions*:

a. Determine the support reactions at A and B.
b. Determine the forces $N^{(a)}$, $N^{(b)}$ and $N^{(c)}$ in the bars[1] with the correct sign for tension and compression, based on the convention that a force $N$ as tensile force is positive and as compressive force is negative.

*Solution* (units in kN and m):
a. Figure 5.2b shows the support reactions. The directions of $A_\text{h}$ and $A_\text{v}$ are such that the line of action of their resultant coincides with two-force member (a). For the others, the directions of the support reactions have been assumed arbitrarily.

On the basis of the slope of bar (a), it follows that $A_\text{h}/A_\text{v} = 4/3$, or $A_\text{h} = (4/3)A_\text{v}$. $A_\text{v}$ can be determined using the moment equilibrium about B:

$$\sum T_z|\text{B} = -A_\text{v} \times 4 - 60 \times 4 = 0 \Rightarrow A_\text{v} = -60\ \text{kN}$$

so that

$$A_\text{h} = \tfrac{4}{3}A_\text{v} = -80\ \text{kN}.$$

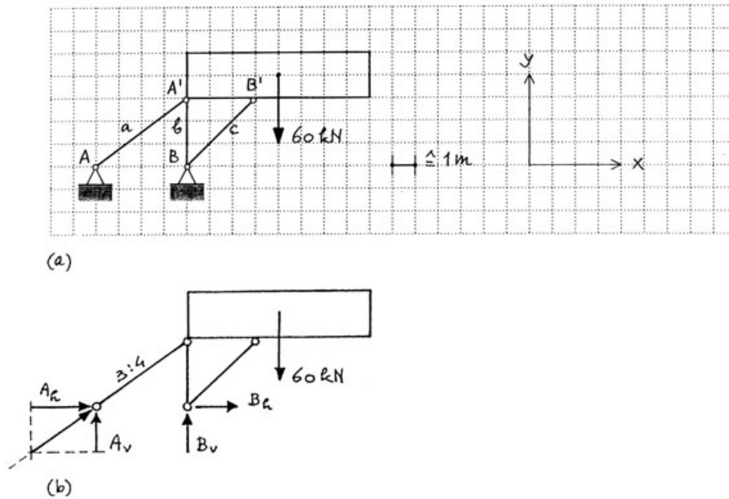


***Figure 5.2*** (a) A block supported on three bars; (b) the assumed directions of the support reactions at A and B.

[1] The upper index indicates the relevant bar. The brackets can be omitted as they do not create any confusion.

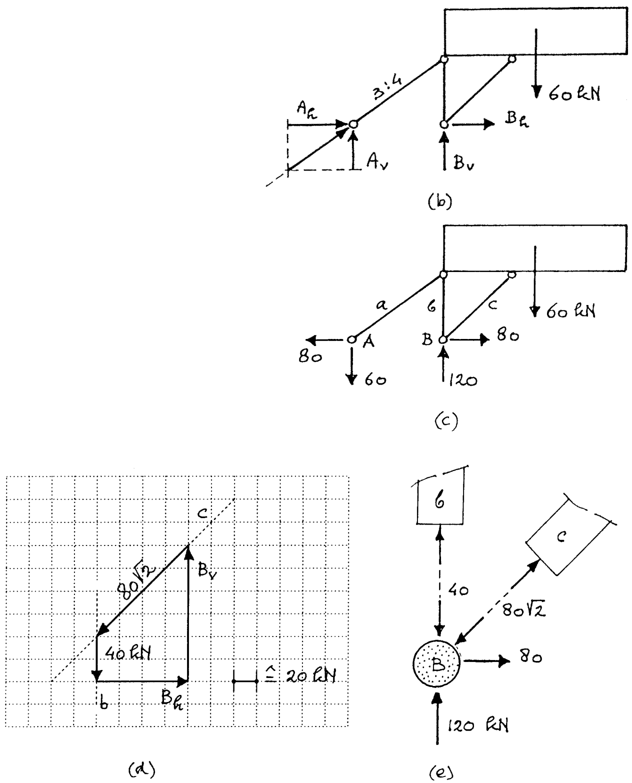


***Figure 5.2*** (b) The assumed directions of the support reactions in A and B; (c) the support reactions as they are actually acting; (d) the closed force polygon for the force equilibrium of joint B; (e) isolated joint B with all the forces acting on it.

$B_h$ is found from the horizontal force equilibrium:

$$\sum F_x = A_h + B_h = -80 + B_h = 0 \Rightarrow B_h = 80 \text{ kN}.$$

$B_v$ follows from the vertical force equilibrium:

$$\sum F_y = A_v + B_v - 60 = -60 + B_v - 60 = 0 \Rightarrow B_v = 120 \text{ kN}.$$

$B_v$ can also be determined from the moment equilibrium about A.

In Figure 5.2c, the support reactions are shown as they act in reality. Only the direction of the support reactions at A was falsely assumed.

b. Figure 5.2c shows directly that a tensile force is acting in bar (a):

$$N^{(a)} = \sqrt{A_h^2 + A_v^2} = \sqrt{80^2 + 60^2} = 100 \text{ kN}.$$

The forces in the bars (b) and (c) can be determined from the force equilibrium of joint B. The force polygon in Figure 5.2d shows that bar (b) exerts a force of 40 kN on joint B. This force "pushes" against the joint. Figure 5.2e shows the interaction forces between bar and joint. In bar (b), there is a compressive force $N^{(b)} = -40$ kN. Bar (c) is exerting a force of $80\sqrt{2}$ kN on joint B, also a compressive force, so that $N^{(c)} = -80\sqrt{2}$ kN.

*Alternative solution* (units in kN and m):
The questions a and b are now answered in reverse order.

b. In Figure 5.3a, the block has been isolated at A′ and B′. $N^{(a)}$, $N^{(b)}$ and $N^{(c)}$ are the (tensile) forces that the bars are exerting on the block. In Figure 5.3b, they have been resolved into their components.

$N^{(c)}$ follows from the moment equilibrium about A′:

$$\sum T_z|\mathrm{A}' = -60 \times 4 - \tfrac{1}{2}\sqrt{2}\,N^{(c)} \times 3 = 0 \Rightarrow N^{(c)} = -80\sqrt{2}\ \mathrm{kN}.$$

$N^{(a)}$ now follows from the horizontal force equilibrium:

$$\begin{aligned}\sum F_x &= -\tfrac{4}{5}N^{(a)} - \tfrac{1}{2}\sqrt{2}N^{(c)}\\ &= -\tfrac{4}{5}N^{(a)} - (-80) = 0 \Rightarrow N^{(a)} = 100\ \mathrm{kN}.\end{aligned}$$

Finally, $N^{(b)}$ can be derived from the vertical force equilibrium:

$$\begin{aligned}\sum F_y &= -60 - \left(N^{(b)} + \tfrac{3}{5}N^{(a)}\right) - \tfrac{1}{2}\sqrt{2}N^{(c)}\\ &= -60 - (N^{(b)} + 60) - (-80) = 0 \Rightarrow N^{(b)} = -40\ \mathrm{kN}.\end{aligned}$$

$N^{(a)}$ is a tensile force; $N^{(b)}$ and $N^{(c)}$ are compressive forces (see Figure 5.3c).

a. The support reactions at A and B now follow from the force equilibrium of the joints A and B. Figure 5.3d shows the forces that the bars are exerting on the joints. It is not difficult to see that the support reactions are acting in the directions shown in the figure. This gives

$$A_\mathrm{h} = 80\ \mathrm{kN},\ A_\mathrm{v} = 60\ \mathrm{kN},\ B_\mathrm{h} = 80\ \mathrm{kN}\ \text{ and }\ B_\mathrm{v} = 40 + 80 = 120\ \mathrm{kN}.$$

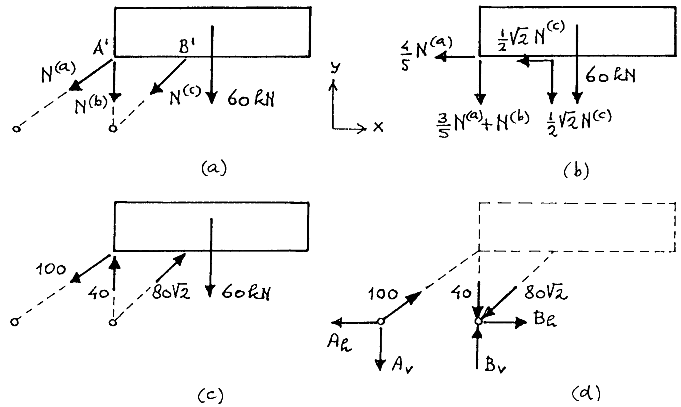


***Figure 5.3*** (a) The block isolated at A′ and B′, assuming that all the bars are tension members; (b) the forces acting on the block resolved into horizontal and vertical components; (c) the forces actually exerted by the bars on the block; (d) the support reactions at A and B are found from the force equilibrium of the joints A and B.

**Example 3**

Figure 5.4 represents a schematisation of a *retaining wall on piles* shown as a two-dimensional problem. Assume the piles are exclusively transferring forces in their longitudinal direction. In that case they can be considered two-force members. The resultant of the total loading carried by the piles

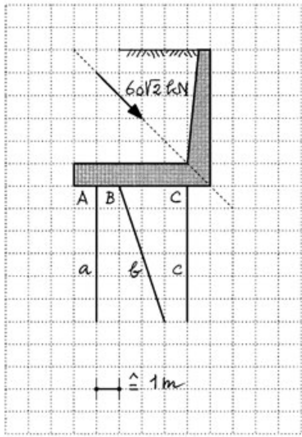


***Figure 5.4*** A retaining wall on piles.

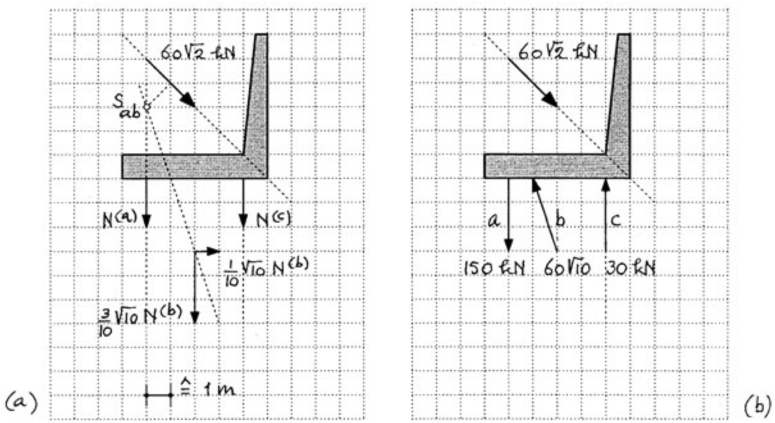


***Figure 5.5*** (a) The isolated retaining wall, in which it has been assumed that all the piles exert tensile forces on the bottom plate; (b) the pile forces as they really act on the bottom plate.

is a force of $60\sqrt{2}$ kN, of which the direction and line of action are shown in the figure.

*Question*:
Determine the pile forces, with the correct sign for tension and compression:

a. analytically;
b. graphically.

*Solution*:
a. Analytical method (units in kN and m):
In Figure 5.5a, the retaining wall has been isolated and the (tensile) forces $N^{(a)}$, $N^{(b)}$ and $N^{(c)}$ that the piles exert on the bottom plate are shown. To keep the picture simple, $N^{(b)}$ has been shifted somewhat along its line of action.

$N^{(b)}$ follows from the horizontal force equilibrium:

$$\sum F_x = \tfrac{1}{10}\sqrt{10}\,N^{(b)} + 60 = 0 \Rightarrow N^{(b)} = -60\sqrt{10}\text{ kN}.$$

$N^{(c)}$ can be found from the moment equilibrium about A. One could also take the moment equilibrium about intersection $\mathrm{S_{ab}}$ of the piles (a) and (b), which works faster in this case:

$$\sum T_z|\mathrm{S_{ab}} = -60\sqrt{2} \times \sqrt{2} - N^{(c)} \times 4 = 0 \Rightarrow N^{(c)} = -30\text{ kN}.$$

Finally, $N^{(a)}$ can be derived from the vertical force equilibrium:

$$\sum F_y = -N^{(a)} - \tfrac{3}{10}\sqrt{10}\,N^{(b)} - N^{(c)} - 60 = 0 \Rightarrow N^{(a)} = +150\text{ kN}.$$

Figure 5.5b shows the forces as they act on the structure. In pile (a) there is a tensile force while there is a compressive force in piles (b) and (c).

b. Graphical method (see Section 3.1.8):
The pile forces can also be found graphically. Imagine $\vec{F}^{(a)}$, $\vec{F}^{(b)}$ and $\vec{F}^{(c)}$ are the forces that the piles exert on the structure. These forces have to be in equilibrium with the load $\vec{F}$, so that:

$$\vec{F}^{(a)} + \vec{F}^{(b)} + \vec{F}^{(c)} = \vec{F}$$

or

$$\vec{F}^{(a)} + \vec{F}^{(b)} = \vec{F} - \vec{F}^{(c)}.$$

In addition to the force equilibrium there also has to be moment equilibrium. Therefore $(\vec{F}^{(a)} + \vec{F}^{(b)})$ and $(\vec{F} - \vec{F}^{(c)})$ have a common line of action. The line of action of $(\vec{F}^{(a)} + \vec{F}^{(b)})$ passes through $S_{ab}$ and that of $(\vec{F} - \vec{F}^{(c)})$ passes through P, see the line of action figure in Figure 5.6a. The common line of action is therefore $PS_{ab}$.

Since $(\vec{F}^{(a)} + \vec{F}^{(b)})$ and $\vec{F}^{(c)}$ are in equilibrium with $\vec{F}$ in P, $(\vec{F}^{(a)} + \vec{F}^{(b)})$ and $\vec{F}^{(c)}$ can be obtained from a force polygon (see Figure 5.6b). $(\vec{F}^{(a)} + \vec{F}^{(b)})$ can then be resolved in $S_{ab}$ into $\vec{F}^{(a)}$ and $\vec{F}^{(b)}$. The force polygon in Figure 5.6b now shows:

$$\vec{F}^{(a)} = 150\text{ kN}\downarrow;\ \vec{F}^{(b)} = 60\sqrt{10}\text{ kN}\uparrow \quad \text{and} \quad \vec{F}^{(c)} = 30\text{ kN}\uparrow.$$

These are the forces that the piles exert on the retaining wall. Translated into the pile forces with the correct sign for tension and compression one now finds:

$$N^{(a)} = +150\text{ kN};\ N^{(b)} = -60\sqrt{10}\text{ kN} \ \text{and}\ N^{(c)} = -30\text{ kN}.$$

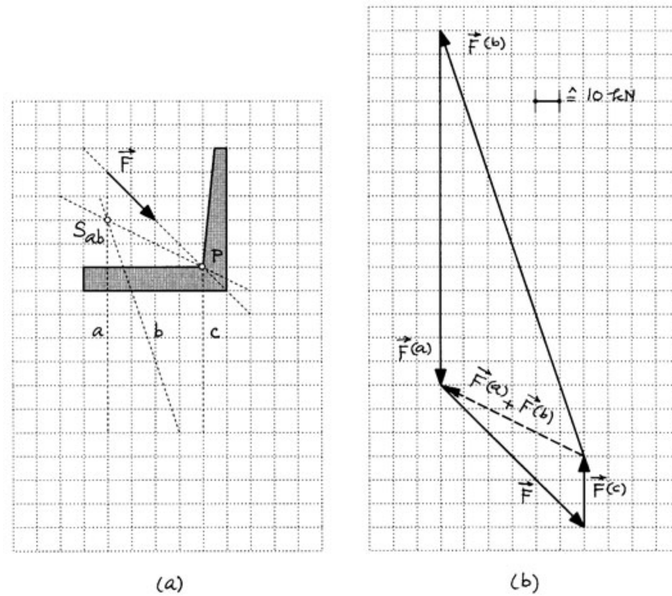


***Figure 5.6*** Graphical method for finding the pile forces: (a) line of action figure and (b) force polygon.

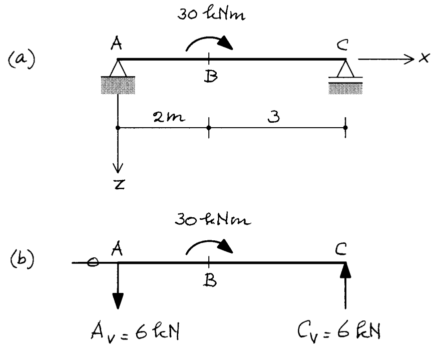


***Figure 5.7*** (a) A beam loaded by a couple at joint B; (b) the isolated beam with its support reactions.

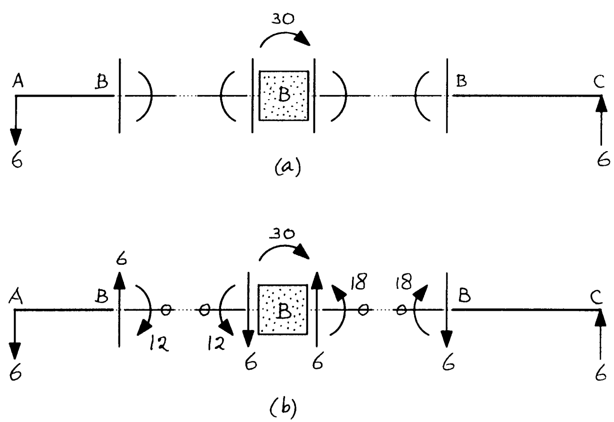


***Figure 5.8*** (a) The interaction forces between joint B and members AB and BC can be found from the equilibrium of these members; (b) the interaction forces as they are actually acting; joint B must meet the conditions of force and moment equilibrium.

**Example 4**

The simply supported beam ABC in Figure 5.7a consists of the members AB and BC that are connected rigidly in joint B. The beam is loaded at joint B by a couple of 30 kNm.

*Question*:
Isolate joint B and draw all the forces[1] acting on it.

*Solution*:
From the horizontal force equilibrium, it follows that the horizontal support reaction A is zero. Only vertical support reactions are therefore acting at A and C. In order to find the direction of the vertical support reaction at C, one considers the isolated beam to be pinned by a hinge at A. Due to the couple of 30 kNm, beam ABC will try to rotate clockwise about A. The vertical support reaction in C must prevent this rotation and therefore acts upwards (see Figure 5.7b).

The vertical equilibrium requires that the vertical support reactions at A and C must be of equal magnitude and opposite direction. The vertical support reaction $A_\mathrm{v}$ at A therefore acts downwards (see Figure 5.7b).

$C_\mathrm{v}$ and $A_\mathrm{v}$ are found with the following equilibrium equations:

$$\sum T_y|\mathrm{A} = -(30\ \mathrm{kNm}) + C_\mathrm{v} \times (5\ \mathrm{m}) = 0 \ \Rightarrow C_\mathrm{v} = 6\ \mathrm{kN},$$
$$\sum F_z = -A_\mathrm{v} + C_\mathrm{v} = 0 \qquad \Rightarrow A_\mathrm{v} = 6\ \mathrm{kN}.$$

In Figure 5.8a, the members AB and BC have been isolated at joint B. In this figure, the calculated support reactions are shown, as are (without indicating their direction) the currently unknown interaction forces[2] be-

[1] The forces are intended here in a *generalised* sense (see Section 4.2.2).

[2] Remember that three interaction forces act at a rigid connection (see Section 4.2.2).

tween the members and the joint. The forces that are exerted at B on the member ends are found from the force and moment equilibrium of respectively member AB and BC. The law of action and reaction requires that the member ends exert equal and opposite forces on joint B. Figure 5.8b shows the interaction forces according to their direction and magnitude (forces in kN and moments in kNm).

*Check*: At joint B the force and moment equilibrium is satisfied.

**Example 5**

Of the bar-type structure in Figure 5.9, parts AC, BC and DC are rigidly connected at joint C.

*Questions*:

a. Determine and draw the support reactions.
b. Graphically check the force and moment equilibrium.
c. Isolate AC, BC, and DC at joint C and draw all the support reactions and interaction forces.

*Solution*:

a. The support reactions are found from the three equilibrium equations for the structure as a whole. The result is shown in Figure 5.10a.

b. The lines of action of the three (resulting) forces at A, B and D intersect at one point. This means that there is moment equilibrium. In Figure 5.10b these forces form a closed force polygon; there is therefore also force equilibrium.

c. In Figure 5.11a, all the parts connected at joint C have been isolated. The forces acting at C on AC can be determined using the known support reactions at A. Equal and opposite forces are acting on joint C. The forces between joint C and the parts BC and CD can be calculated in the same way. The result is shown in Figure 5.11b.

*Check*: At joint C, the force and moment equilibrium is satisfied.

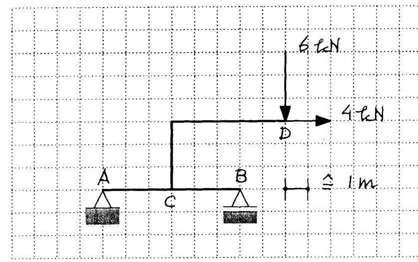


***Figure 5.9*** A structure of which the parts AC, BC and DC are connected rigidly at joint C.

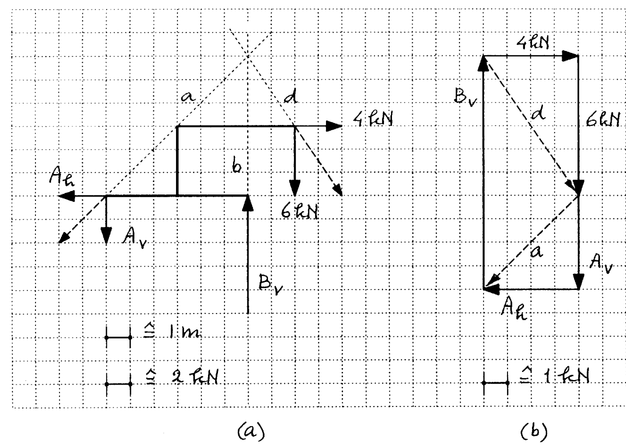


***Figure 5.10*** (a) Graphical check of the moment equilibrium: the lines of action of the three resulting forces at A, B and D pass through a single point; (b) graphical check of the force equilibrium: all the forces form a closed force polygon.

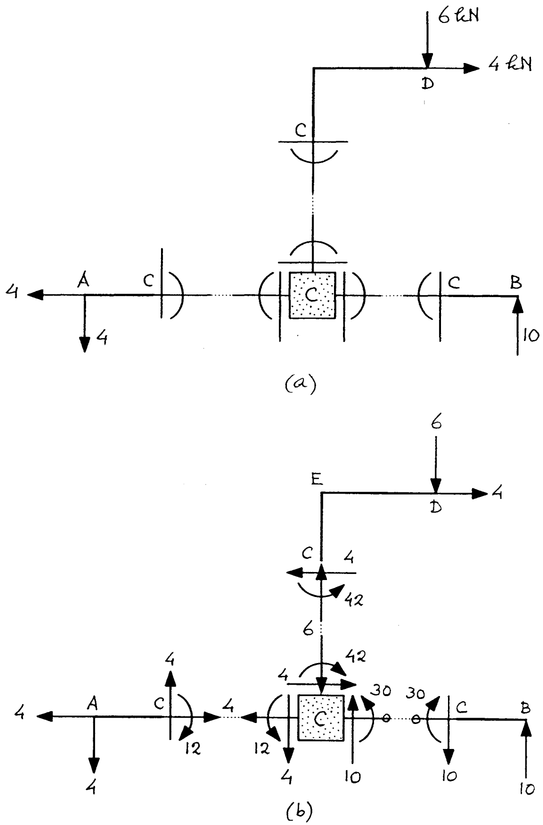


*Figure 5.11* (a) The interaction forces between joint C and parts AC, BC and DC can be found from the equilibrium of these parts; (b) the interaction forces as they are really acting; joint C must meet the conditions of the force and moment equilibrium.

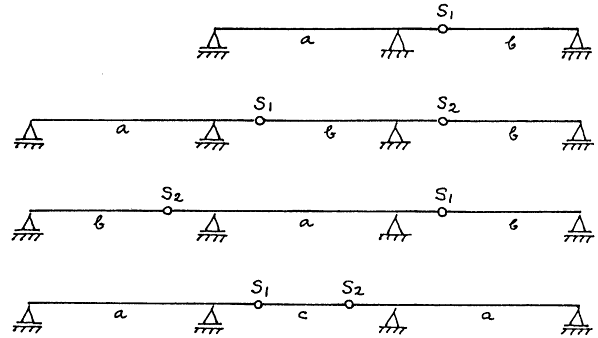


*Figure 5.12* Hinged beams.

## 5.2 Hinged beams

A *hinged beam* is a structure in which several beams are linked through consecutive hinges. Figure 5.12 shows examples of hinged beams. Hinged beams are found in roof girders and bridges.

In Figure 5.12, the beams with an overhang are depicted with (a). These beams are referred to as being supported at *fixed* points. The beams (b) and (c) are sometimes referred to as being supported at *floating* points, as they rest on the non-fixed supporting points $S_1$ and/or $S_2$. Beam (c) is called a suspended beam; it can be placed at a later stage during construction.

Statically determinate hinged beams are also known as *Gerber beams* after the German *Gerber*,[1] who first used this type of structure in the second half of the 19th century.

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1 Heinrich Gerber (1832–1912), German engineer.

By choosing an adequate place for the hinges, it is possible to influence the force distribution in the structure positively. However, you have to make sure that the structure does not become kinematically indeterminate, as for example in Figure 5.13.

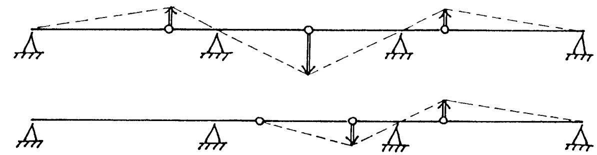

*Figure 5.13* With too many hinges, or inadequate placement, the structure becomes kinematically indeterminate and changes into a mechanism.

A possibility for hinge $S_1$ in a bridge structure is shown in Figure 5.14. The right-hand part (the suspended beam) is supported at a hinge on the left-hand part. In this example, the hinge works only if the right-hand part exerts a downward force onto the left-hand part. This requirement is usually fulfilled as a result of the relatively large dead weight of the suspended beam.

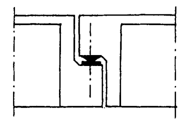

*Figure 5.14* Example of a hinge in a bridge structure.

From now on we assume that all hinges in a hinged beam can transfer both tensile and compressive forces.

**Example 1**
The hinged beam in Figure 5.15a consists of parts AS and CS, which are connected at a hinge in S.

*Questions*:
a. Determine the support reactions.
b. Determine the forces exerted on hinge S.

*Solution* (units in kN and m):
a. There are three equilibrium equations available for the structure. With the directions assumed for the support reactions in Figure 5.15b, the following applies for the given $xy$ coordinate system:

$$\sum F_x = A_\mathrm{h} = 0, \tag{a}$$

$$\sum F_y = -40 - 60 + A_\mathrm{v} + B_\mathrm{v} + C_\mathrm{v} = 0, \tag{b}$$

$$\sum T_z|A = -40 \times 4 - 60 \times 12 + B_\mathrm{v} \times 8 + C_\mathrm{v} \times 16 = 0. \tag{c}$$

The moment equilibrium for the entire structure can also be applied for a point other than A.

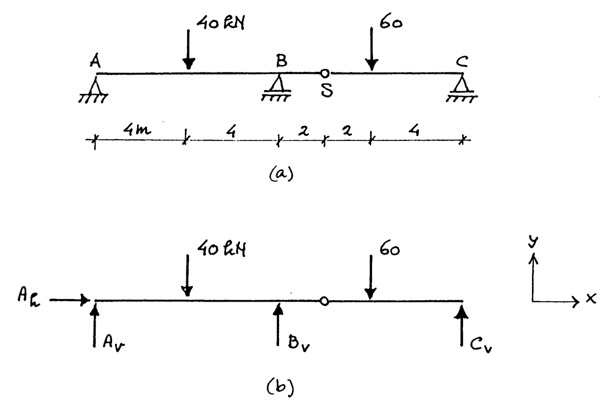


*Figure 5.15* (a) A hinged beam on three supports; (b) the assumed directions for the support reactions.

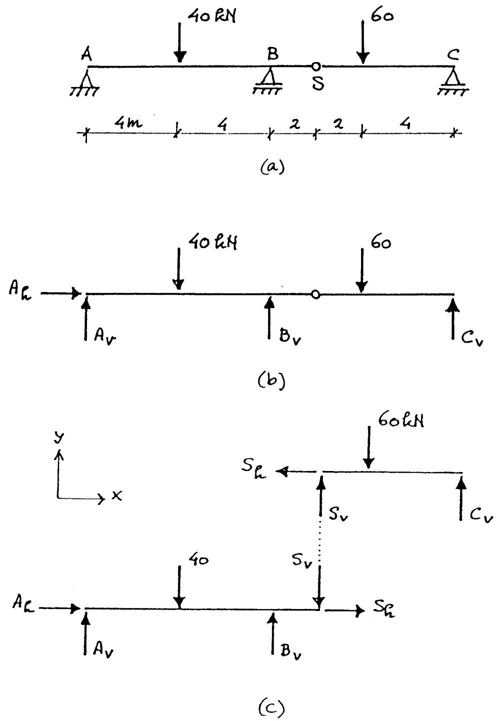


***Figure 5.15*** (a) A hinged beam on three supports; (b) the assumed directions for the support reactions. (c) The hinge forces in S.

The three equations (a) to (c) are insufficient for finding all the support reactions. A fourth equation is required. This equation relates to the property that no couple can be transferred at hinge S. If parts AS and CS are isolated at S, we are left with the interaction forces $S_h$ and $S_v$ (see Figure 5.15c). The missing equation is now found from the moment equilibrium about S of one of the individual parts.

For the left-hand part AS one finds[1]

$$\sum T_z^{(AS)}|S = 40 \times 6 - A_v \times 10 - B_v \times 2 = 0 \qquad \text{(d)}$$

and for the right-hand part CS

$$\sum T_z^{(CS)}|S = -60 \times 2 + C_v \times 6 = 0. \qquad \text{(e)}$$

Both equations (d) and (e) are of equal value, but it should be clear that equation (e) is preferable as it is simpler.

The support reactions are therefore most easily found as follows:

$$\text{(e)} \quad \sum T_z^{(CS)}|S = 0 \quad \Rightarrow C_v = 20 \text{ kN},$$

$$\text{(c)} \quad \sum T_z^{(AC)}|A = 0 \quad \Rightarrow Bv = 70 \text{ kN},$$

$$\text{(b)} \quad \sum F_y^{(AC)} = 0 \quad \Rightarrow A_v = 10 \text{ kN},$$

$$\text{(a)} \quad \sum F_x^{(AC)} = 0 \quad \Rightarrow A_h = 0.$$

It seems that the correct direction was assumed for all the support reactions. The support reactions are shown in Figure 5.16.

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[1] In $\sum T_z^{(AS)}|S = 0$, the upper index indicates the part AS to which the equilibrium equation relates. This notation is particularly useful if the equilibrium has to be written down for the various parts of the same structure.

b. The hinge forces follow from the equilibrium of the separate parts. Taking the right-hand part CS in Figure 5.16 we find

$$\sum F_x^{(\mathrm{CS})} = -S_\mathrm{h} = 0 \quad \Rightarrow S_\mathrm{h} = 0,$$

$$\sum F_z^{(\mathrm{CS})} = S_\mathrm{v} - 60 + 20 = 0 \quad \Rightarrow S_\mathrm{v} = 40 \text{ kN}.$$

The same values are found from the force equilibrium for the left-hand part AS.

$S_\mathrm{h}$ and $S_\mathrm{v}$ are the forces that are acting at S on AS and CS. The forces acting on the hinged joint S are the same magnitude, but of opposite direction (see Figure 5.17).

*Alternative solution*:
The *floating* supported part CS can be seen as a beam, supported on a roller and a hinge (see Figure 5.18). The support reactions at S and C follow from the equilibrium of CS:

$$\sum F_x^{(\mathrm{CS})} = 0 \quad \Rightarrow S_\mathrm{h} = 0,$$

$$\sum T_z^{(\mathrm{CS})}|\mathrm{C} = 0 \quad \Rightarrow S_\mathrm{v} = 40 \text{ kN},$$

$$\sum T_z^{(\mathrm{CS})}|\mathrm{S} = 0 \quad \Rightarrow C_\mathrm{v} = 20 \text{ kN}.$$

With $S_\mathrm{h}$ and $S_\mathrm{v}$ we now know the load on the overhang of ABS and we can determine the support reactions at A and B:

$$\sum T_z^{(\mathrm{AS})}|\mathrm{A} = 0 \quad \Rightarrow B_\mathrm{v} = 70 \text{ kN},$$

$$\sum T_z^{(\mathrm{AS})}|\mathrm{B} = 0 \quad \Rightarrow A_\mathrm{v} = 10 \text{ kN},$$

$$\sum F_x^{(\mathrm{AS})} = 0 \quad \Rightarrow A_\mathrm{h} = 0.$$

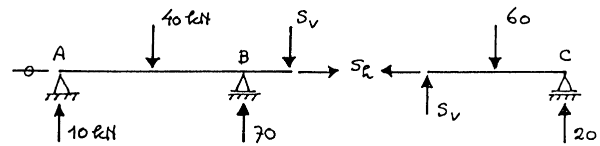



***Figure 5.16*** The support reactions as they really act; the hinge forces follow from the equilibrium of AS or CS.

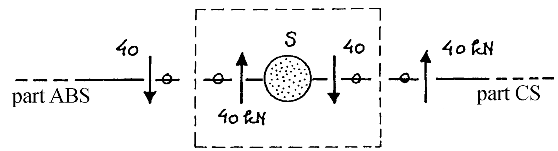



***Figure 5.17*** The forces acting on the isolated hinged joint S.

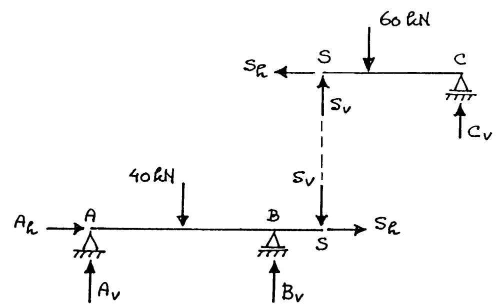



***Figure 5.18*** The support reactions and interaction forces can also be found by first working out the equilibrium of SC and then the equilibrium of AS.

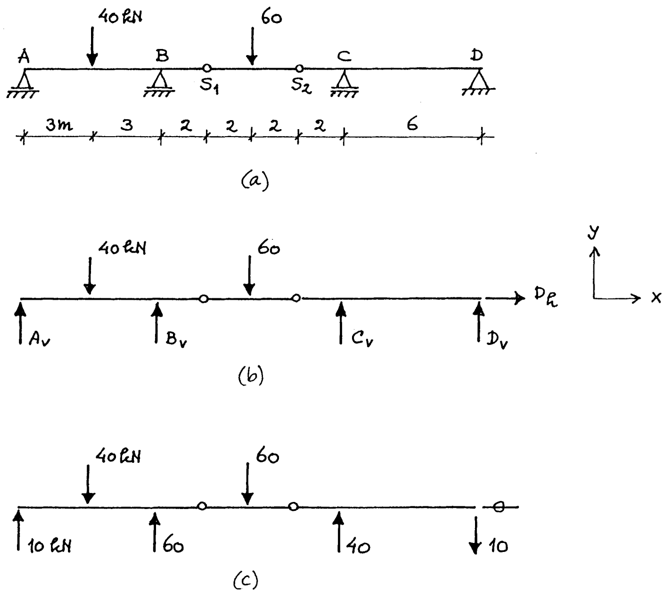



***Figure 5.19*** (a) A hinged beam with four supports; (b) the assumed directions of the support reactions; (c) the support reactions as they really act.

**Example 2**
The hinged beam in Figure 5.19a is given.

*Question*:
Determine the support reactions.

*Solution* (units in kN and m):
In Figure 5.19b, the following applies for the assumed directions of the support reactions in the given coordinate system, and for the system as a whole:

$$\sum F_x = D_\mathrm{h} = 0, \tag{a}$$

$$\sum F_y = -40 - 60 + A_\mathrm{v} + B_\mathrm{v} + C_\mathrm{v} + D_\mathrm{v} = 0, \tag{b}$$

$$\begin{aligned}\sum T_z|\mathrm{A} = {} & -40 \times 3 - 60 \times 10 \\ & + B_\mathrm{v} \times 6 + C_\mathrm{v} \times 14 + D_\mathrm{v} \times 20 = 0.\end{aligned} \tag{c}$$

We have three equations with five unknowns. The two missing equations are found from the condition that the hinges $S_1$ and $S_2$ cannot transfer couples. Therefore the following applies for the isolated part $S_2D$:

$$\sum T_z^{(\mathrm{S_2D})}|\mathrm{S}_2 = C_\mathrm{v} \times 2 + D_\mathrm{v} \times 8 = 0. \tag{d}$$

and for the isolated part $S_1D$:

$$\sum T_z^{(\mathrm{S_1D})}|\mathrm{S}_1 = -60 \times 2 + C_\mathrm{v} \times 6 + D_\mathrm{v} \times 12 = 0. \tag{e}$$

Here the moment equilibrium has been associated with the parts to the right of the hinges. One could just as well look at the moment equilibrium of the parts to the left of both hinges, although doing so would involve more calculations.

To summarise, a good strategy for solving this is as follows:

$$\text{(a)} \quad \sum F_x^{(\mathrm{AD})} = 0 \qquad \Rightarrow D_\mathrm{h} = 0\,\mathrm{kN},$$

$$\left.\begin{array}{l} \text{(e)} \quad \sum T_z^{(\mathrm{S_1D})}|\mathrm{S_1} = 0 \\ \\ \text{(d)} \quad \sum T_z^{(\mathrm{S_2D})}|\mathrm{S_2} = 0 \end{array}\right\} \Rightarrow C_v = 40\,\mathrm{kN} \;\text{ and }\; D_\mathrm{v} = -10\,\mathrm{kN},$$

$$\text{(c)} \quad \sum T_z^{(\mathrm{AD})}|\mathrm{A} = 0 \qquad \Rightarrow B_\mathrm{v} = 60\,\mathrm{kN},$$

$$\text{(b)} \quad \sum F_y^{(\mathrm{AD})} = 0 \qquad \Rightarrow A_\mathrm{v} = 10\,\mathrm{kN}.$$

Figure 5.19c shows the support reactions as they act in reality. Apparently, only the direction of $D_\mathrm{v}$ was initially assumed falsely.

*Alternative solution*:
The most efficient approach however is to first look at the moment equilibrium of the suspended beam $\mathrm{S_1S_2}$ (see Figure 5.20):

$$\sum T_z^{(\mathrm{S_1S_2})}|\mathrm{S_1} = 0 \qquad \Rightarrow \quad S_{2;\mathrm{v}} = 30\,\mathrm{kN},$$

$$\sum T_z^{(\mathrm{S_1S_2})}|\mathrm{S_2} = 0 \qquad \Rightarrow \quad S_{1;\mathrm{v}} = 30\,\mathrm{kN},$$

With $S_{1;\mathrm{v}}$ and $S_{2;\mathrm{v}}$, we know the vertical forces that the suspended beam exerts on the overhangs of beams $\mathrm{AS_1}$ and $\mathrm{S_2D}$. For these beams, the vertical support reactions can be determined from the moment equilibrium:

$$\sum T_z^{(\mathrm{AS_1})}|\mathrm{B} = 0 \qquad \Rightarrow \quad A_\mathrm{v} = 10\,\mathrm{kN},$$

$$\sum T_z^{(\mathrm{AS_1})}|\mathrm{A} = 0 \qquad \Rightarrow \quad B_\mathrm{v} = 60\,\mathrm{kN},$$

$$\sum T_z^{(\mathrm{S_2D})}|\mathrm{D} = 0 \qquad \Rightarrow \quad C_\mathrm{v} = 40\,\mathrm{kN},$$

$$\sum T_z^{(\mathrm{S_2D})}|\mathrm{C} = 0 \qquad \Rightarrow \quad D_\mathrm{v} = -10\,\mathrm{kN},$$

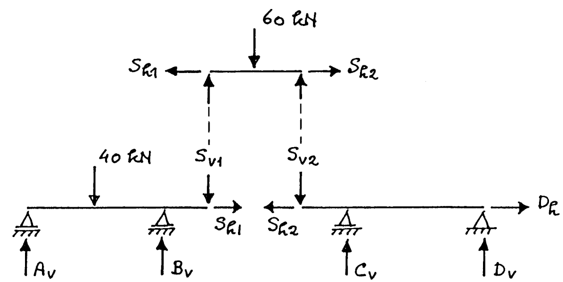



***Figure 5.20*** The support reactions can also be found by first working out the moment equilibrium of suspended beam $\mathrm{S_1S_2}$ and then the equilibrium of AS and DS.

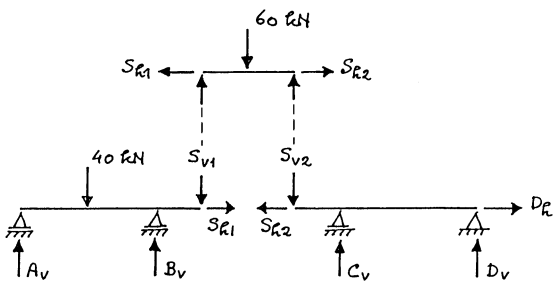



***Figure 5.20*** The support reactions can also be found by first working out the moment equilibrium of suspended beam $S_1S_2$ and then the equilibrium of AS and DS.

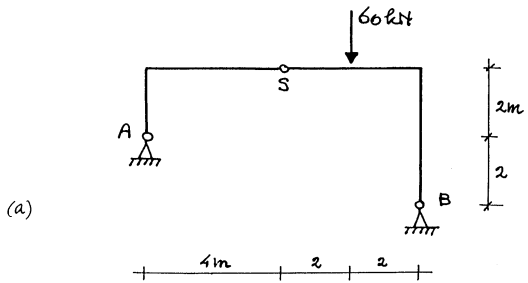



***Figure 5.21*** (a) A three-hinged frame with the hinge bearings at different levels.

Finally, the horizontal force equilibrium for each of the structural members gives

$$S_{1;h} = S_{2;h} = D_h = 0 \text{ kN}.$$

## 5.3 Three-hinged frames

Figure 5.21a is an example of a *three-hinged frame*. The frame consists of two self-contained parts AS and BS that are connected at S by means of a hinge, and are supported at A and B by a hinge. The whole is statically determinate. Three-hinged frames are often used as covering structures. They were previously mentioned in Sections 3.2.2 and 4.4.4.

A three-hinged frame has four unknown support reactions. In order to be able to calculate these, we need four equilibrium equations. Three of these are found from the equilibrium of the structure as a whole. The fourth equation follows from the condition that the hinged joint at S cannot transfer a couple.

**Example 1**
In the three-hinged frame in Figure 5.21a, the hinge bearings at A and B are at different levels. The frame is loaded by a vertical force of 60 kN that acts on the right-hand part BS.

*Questions*:

a. Determine the support reactions.
b. Determine the forces that parts AS and BS in S exert on one another.
c. Perform a graphical check of the equilibrium.

*Solution* (units in kN and m):

a. For the given coordinate system and the directions assumed for the support reactions in Figure 5.21b the following applies for the structure as a whole:

$$\sum F_x^{(\mathrm{ASB})} = A_\mathrm{h} - B_\mathrm{h} = 0, \tag{a}$$

$$\sum F_y^{(\mathrm{ASB})} = -60 + A_\mathrm{v} + B_\mathrm{v} = 0, \tag{b}$$

$$\sum T_z^{(\mathrm{ASB})}|A = -60 \times 6 - B_\mathrm{h} \times 2 + B_\mathrm{v} \times 8 = 0. \tag{c}$$

The missing fourth equation is found from the moment equilibrium about S of one of the separate parts AS or BS. For the left-hand part AS one finds

$$\sum T_z^{(\mathrm{AS})}|\mathrm{S} = A_\mathrm{h} \times 2 - A_\mathrm{v} \times 4 = 0. \tag{d}$$

For the right-hand part BS, one finds

$$\sum T_z^{(\mathrm{BS})}|\mathrm{S} = -60 \times 2 - B_\mathrm{h} \times 4 + B_\mathrm{v} \times 4 = 0. \tag{e}$$

The equations (d) and (e) are equivalent. Either of them is sufficient for calculating the support reactions in combination with the equations (a) to (c). The other equation can then be used to check the values found.

Equation (e) is preferable in finding the solution as, in combination with equation (c), it leads directly to the support reactions at B:

$$B_\mathrm{h} = 20\ \mathrm{kN};\ \ B_\mathrm{v} = 50\ \mathrm{kN}.$$

From (a) and (b) we find

$$A_\mathrm{h} = 20\ \mathrm{kN};\ \ A_\mathrm{v} = 10\ \mathrm{kN}.$$

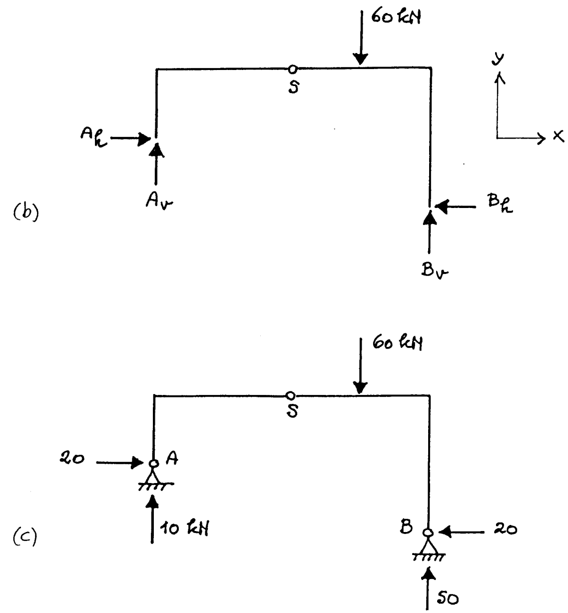



***Figure 5.21*** (b) The assumed directions for the support reactions; (c) the support reactions as they really act.

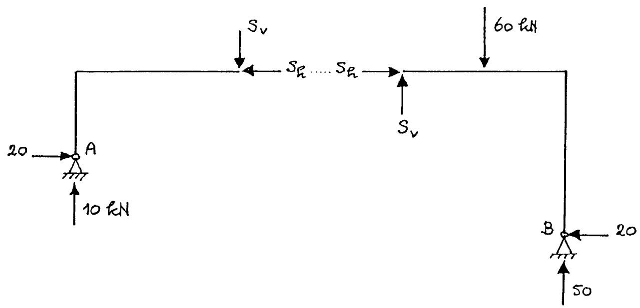



***Figure 5.22*** The interaction forces at S follow from the force equilibrium of AS or BS.

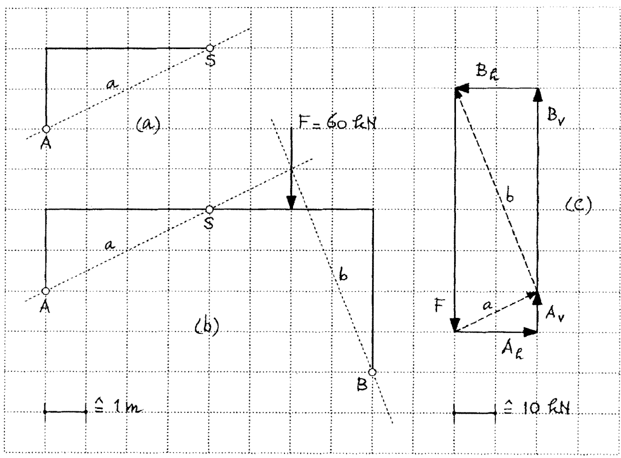



***Figure 5.23*** (a) The left frame half AS is in equilibrium if the two forces at A and S are equal and opposite and have a common line of action; (b) the three-hinged frame is in moment equilibrium if the lines of action of force $F$ and the support reactions at A and B intersect in a single point; (c) the three-hinged frame is in force equilibrium if force $F$ and the support reactions at A and B form a closed force polygon.

The support reactions are shown in Figure 5.21c. Since there is only vertical and no horizontal loading, the horizontal support reactions are equal and opposite.

*Check*: The solution is true in equation (d).

b. The forces that parts AS and BS in S exert on one another (the interaction forces at S) follow from the force equilibrium of one of the separate parts AS or BS (see Figure 5.22). The force equilibrium for the left-hand part AS gives

$$S_h = 20 \text{ kN} \quad \text{and} \quad S_v = 10 \text{ kN}.$$

The same values follow from the force equilibrium for the right-hand part BS. This therefore offers an opportunity for checking.

c. Since the load only acts on one half of the frame, one can also easily check the solution graphically (see Section 3.2.2).

Only two forces are acting on the left-hand part AS: the support reaction at A and the hinge force at S. The left-hand part AS can be in equilibrium only if the two forces that act on AS at A and S are equal and opposite. Both forces must also have the same line of action (see Figure 5.23a). The line of action of the support reaction at A will therefore pass through S and is thus determined.

Three forces are acting on the entire frame (the two support reactions at A and B and the load) that together have to form an equilibrium system. This is possible only if the lines of action of the three forces intersect in a single point (if not, there is no moment equilibrium). The line of action of the support reaction at B must therefore pass through the intersection of the line of action of the point load and the known line of action of the support reaction at A (see Figure 5.23b).

With the known lines of action for both support reactions, the magnitude and direction can be found by means of the force polygon in Figure 5.23c.

The figure shows that the support reactions at A and B correspond in magnitude and direction with those calculated previously.

**Example 2**

The left-hand column of the three-hinged frame from the previous example is extended in such a way that the hinge bearings at A and B are at equal level (see Figure 5.24a). The load remains unchanged.

*Questions*:

a. Determine the support reactions at A and B.
b. Determine the forces that AS and BS at S exert on one another.
c. Determine the forces acting on joint D.

*Solution* (units in kN and m):

a. For a three-hinged frame with the hinge bearings at equal level, the vertical support reactions can be determined directly from the moment equilibrium of the structure as a whole.

With the directions assumed for the support reactions in Figure 5.24b the following applies for the given coordinate system for the frame as a whole:

$$\sum T_z^{(\mathrm{ASB})}|\mathrm{A} = -60 \times 6 + B_\mathrm{v} \times 8 = 0 \Rightarrow B_\mathrm{v} = 45 \text{ kN}, \quad \text{(a)}$$

$$\sum T_z^{(\mathrm{ASB})}|\mathrm{B} = 60 \times 2 - A_\mathrm{v} \times 8 = 0 \quad \Rightarrow A_\mathrm{v} = 15 \text{ kN}. \quad \text{(b)}$$

One of these equations for the moment equilibrium can be replaced by the equation for the vertical force equilibrium.

The horizontal force equilibrium of the structure as a whole gives

$$\sum F_x^{(\mathrm{ASB})} = A_\mathrm{h} - B_\mathrm{h} = 0. \quad \text{(c)}$$

Since there is no horizontal loading, the horizontal support reactions are equal and opposite. The magnitude of the horizontal support reactions follow from the moment equilibrium about S of one of the parts AS or BS.

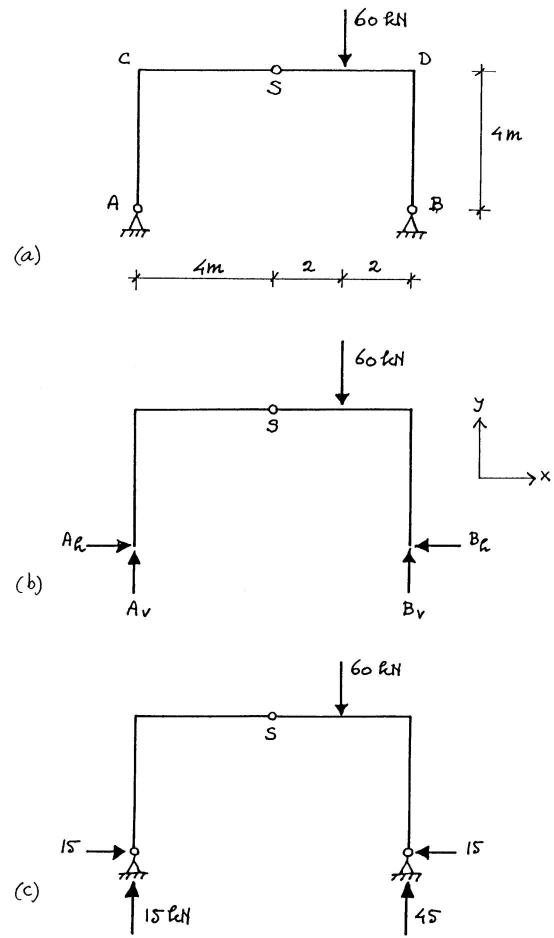



***Figure 5.24*** (a) A three-hinged frame with the hinged supports at the same level; (b) the assumed directions for the support reactions; (c) the support reactions as they really act.

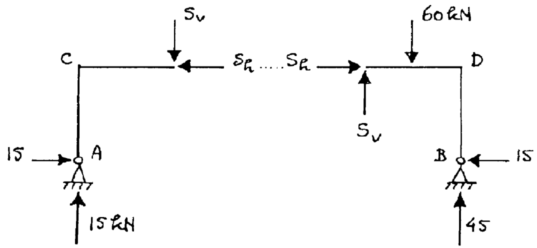



*Figure 5.25* The hinge forces at S follow from the force equilibrium of AS or BS.

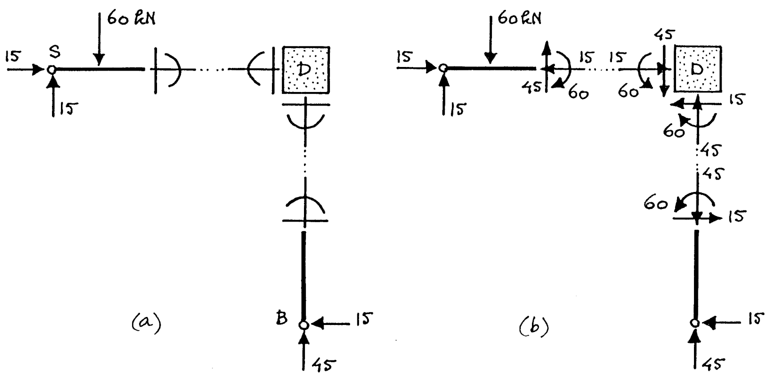



*Figure 5.26* (a) The interaction forces between joint D and members SD and BD are found from the equilibrium of the separate members; (b) the interaction forces as they really act.

If one selects the left-hand part AS, this gives

$$\sum T_z^{(\mathrm{AS})}|\mathrm{S} = A_\mathrm{h} \times 4 - A_\mathrm{v} \times 4 = 0 \qquad \text{(d)}$$

or, if one assumes the right-hand part BS

$$\sum T_z^{(\mathrm{BS})}|\mathrm{S} = -60 \times 2 - B_\mathrm{h} \times 4 + B_\mathrm{v} \times 4 = 0. \qquad \text{(e)}$$

Both equations are equivalent. The solution is

$$A_\mathrm{h} = B_\mathrm{h} = 15 \text{ kN}.$$

All the support reactions are shown in Figure 5.24c.

b. The interaction forces in hinge S follow from the force equilibrium of AS or BS (see Figure 5.25). The equilibrium of the left-hand part AS gives

$$S_\mathrm{h} = S_\mathrm{v} = 15 \text{ kN}.$$

*Check*: For these forces, the left-hand part BS is also in equilibrium.

c. To find the forces acting on joint D, the joint is isolated (see Figure 5.26a). There are three interaction forces acting between joint D and member SD. The magnitude of these forces is found from the equilibrium of member SD. In the same way, one can use the equilibrium of BD to find the magnitude of the three interaction forces between joint D and member BD. The result is shown in Figure 5.26b.

*Check*: Joint D has to meet the conditions of force and moment equilibrium.

## 5.4 Three-hinged frames with tie-rod

The previous section shows that a vertical load on a three-hinged frame generates not only vertical, but also horizontal support reactions (see Figures 5.27a and 5.27b). Horizontal forces on foundations in soft soil often cause problems. To reduce the horizontal forces on the foundation, one can decide to link the bearings A and B of the three-hinged frame by means of a so-called tie-rod. In this way a self-contained structure is created that can be supported by a roller and a hinge (see Figure 5.27c). This is referred to as a *three-hinged frame with tie-rod*. Tie-rod AB ensures that the roller support B stays in place and carries the horizontal support reactions. Vertical loading now generates exclusively vertical support reactions.

Whether rod AB is subject to tension or compression depends on the loading. The name *tie-rod* indicates that such a structure is used only if tension can be expected in the rod.

**Example**
In Figure 5.28, a vertical and a horizontal load is acting on a three-hinged frame with tie-rod.

*Questions*:
a. Determine the support reactions.
b. Determine the force in rod AB.
c. Determine the interaction forces at S.
d. Determine the forces acting on joint A.

*Solution* (units in kN and m):
a. In Figure 5.29a, the structure has been isolated from its supports. The support reactions follow from the equilibrium of the structure as a whole. For the directions assumed for $A_h$, $A_v$ and $B_v$ we find

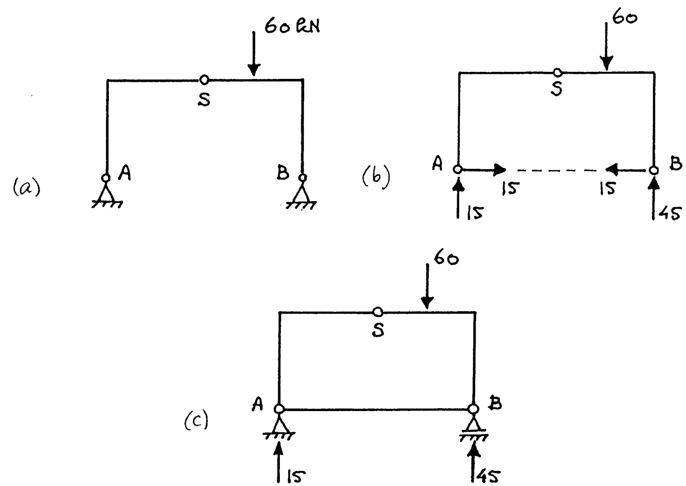



***Figure 5.27*** (a) A vertical load on a three-hinged frame gives (b) not only vertical but also horizontal support reactions; (c) by linking the bearings A and B of the three-hinged frame by a tie rod, the horizontal support reactions can be eliminated.

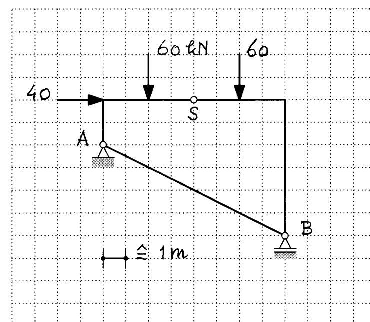



***Figure 5.28*** A three-hinged frame with tie rod, with a horizontal and vertical load.

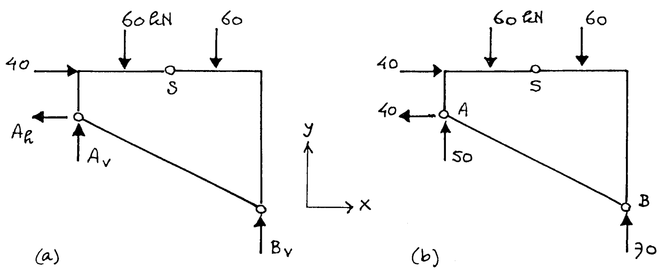



***Figure 5.29*** (a) The assumed directions of the support reactions; (b) the support reactions as they really act.

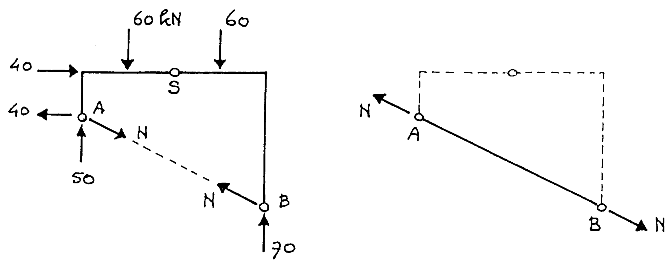



***Figure 5.30*** Three-hinged frame ASB and rod AB isolated from one another, assuming that a tensile force $N$ acts in rod AB.

$$\sum F_x^{(\mathrm{ASB})} = 40 - A_\mathrm{h} = 0 \qquad \Rightarrow A_\mathrm{h} = 40\ \mathrm{kN},$$

$$\sum F_y^{(\mathrm{ASB})} = -60 - 60 + A_\mathrm{v} + B_\mathrm{v}$$

$$= -60 - 60 + A_\mathrm{v} + 70 = 0 \ \Rightarrow A_\mathrm{v} = 50\ \mathrm{kN},$$

$$\sum T_z^{(\mathrm{ASB})}|\mathrm{A} = -40 \times 2 - 60 \times 2$$

$$-60 \times 6 + B_\mathrm{v} \times 8 = 0 \ \Rightarrow B_\mathrm{v} = 70\ \mathrm{kN}.$$

The support reactions are shown in Figure 5.29b.

b. To calculate the force in rod AB, it is isolated from ASB in Figure 5.30. We can immediately recognise a two-force member in rod AB: the rod is loaded only by forces at its ends A and B and can therefore be in equilibrium only if these forces are equal and opposite with AB as common line of action. It is assumed that a tensile force $N$ acts in rod AB.

The magnitude of $N$ follows from the moment equilibrium about S of one of the parts AS or BS. In Figure 5.31a both parts have been isolated at S. In order to simplify the calculation, $N$ has been resolved into a horizontal component $N_\mathrm{h}$ and a vertical component $N_\mathrm{v}$:

$$N_\mathrm{h} = \tfrac{2}{5}\sqrt{5}\,N,$$

$$N_\mathrm{v} = \tfrac{1}{5}\sqrt{5}\,N.$$

Taking the right-hand part BS we find

$$\sum T_z^{(\mathrm{BS})}|\mathrm{S} = -60 \times 2 + 70 \times 4 - N_\mathrm{h} \times 6 + N_\mathrm{v} \times 4$$

$$= 160 - \tfrac{8}{5}\sqrt{5}\,N = 0$$

from which it follows that

$$N = 20\sqrt{5}\,\text{kN}$$

and

$$N_\text{h} = 40\ \text{kN},$$

$$N_\text{v} = 20\ \text{kN}.$$

Since $N$ is positive, the force in rod AB is indeed a tensile force.

The equation for the moment equilibrium about S can be simplified by shifting $N$ along its line of action to a convenient position, for example to the point vertically under S. In that case it follows that

$$\sum T_z^{(\text{SB})}|\text{S} = -60 \times 2 + 70 \times 4 - N_\text{h} \times 4 = 0 \Rightarrow N_\text{h} = 40\ \text{kN}.$$

*Check*: For the value determined for $N$, the left-hand part AS must also satisfy the conditions for moment equilibrium:

$$\sum T_z^{(\text{AS})}|\text{S} = 0.$$

c. The hinge forces at S follow from the force equilibrium of the left-hand or right-hand part of the frame. With the directions of $S_\text{h}$ and $S_\text{v}$ assumed in Figure 5.31a we find for the right-hand part BS

$$\sum F_x^{(\text{BS})} = -N_\text{h} - S_\text{h} = -40 - S_\text{h} = 0 \ \Rightarrow S_\text{h} = -40\ \text{kN},$$

$$\sum F_z^{(\text{BS})} = -60 + 70 + N_\text{v} + S_\text{v}$$

$$= -60 + 70 + 20 + S_\text{v} = 0 \qquad \Rightarrow S_\text{v} = -30\ \text{kN}.$$

Clearly the wrong direction was assumed in Figure 5.31a for both hinge forces. Figure 5.31b shows all the forces as they act in reality.

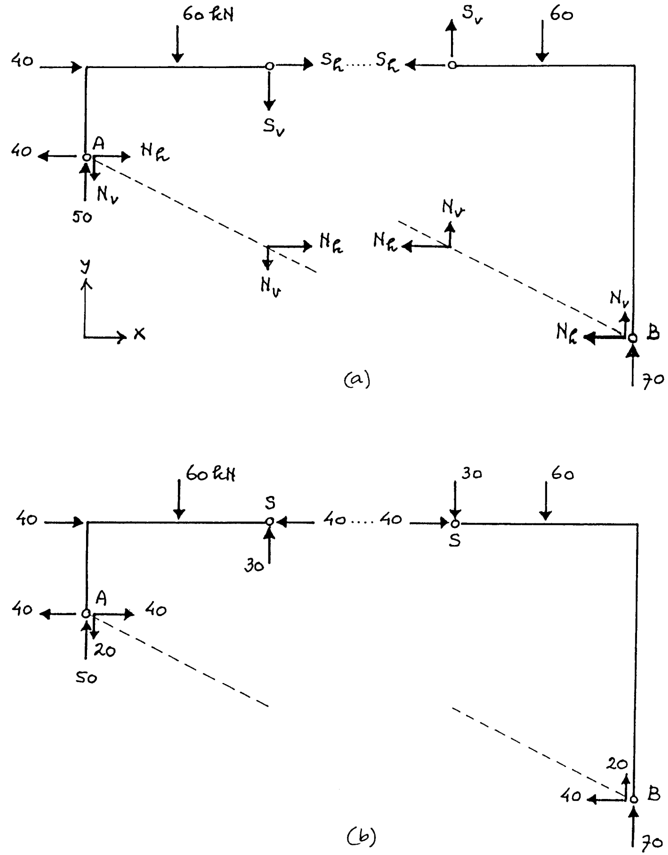



***Figure 5.31*** (a) The magnitude of $N$ follows from the moment equilibrium of one of the frame halves about S, after which the interaction forces at S follow from the force equilibrium of the frame halves; (b) all the forces as they really act on the frame halves.

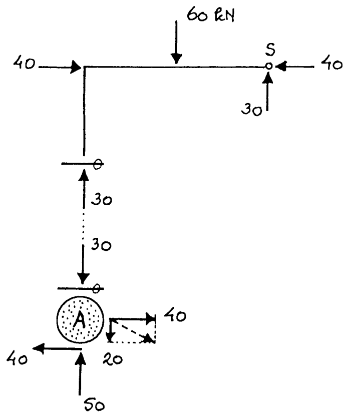



*Figure 5.32* The forces acting on joint A and frame half AS.

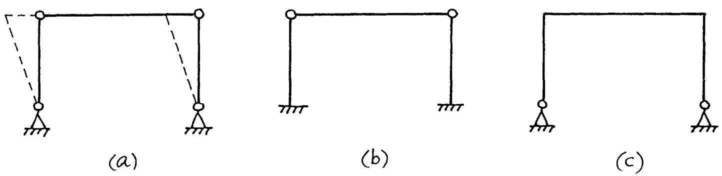



*Figure 5.33* (a) This portal-like structure with only hinged joints is kinematically indeterminate and can tilt. To prevent tilting one can (b) fix the columns or (c) replace the hinged joints between the columns and the beams by rigid joints.

*Check*: With the hinge forces calculated, the left-hand frame part AS must also be in force equilibrium.

d. The following forces are acting on joint A:

- the support reactions $A_h = 40$ kN and $A_v = 50$ kN;
- the force $N$ exerted by the tie-rod AB, with components $N_h = 40$ kN and $N_v = 20$ kN;
- the forces exerted by the left-hand frame part AS.

The last-mentioned forces can be found from the force equilibrium of joint A. All the forces on the joint are shown in Figure 5.32.

*Check*: The part AS isolated from joint A has to be in force equilibrium.

Note that here the horizontal load of 40 kN is transferred via a long detour to the support at A.

## 5.5 Shored structures

The portal-like structure in Figure 5.33a, with only hinged joints, is kinematically indeterminate. The structure can *tilt*. To prevent this, one can fix one or more of the columns (Figure 5.33b). Or one can replace one or more of the hinges between column and beam by rigid connections (Figure 5.33c). It is also possible to prevent the construction from tilting by applying so-called *shoring bars*, indicated in Figure 5.34 with the letter s.

If the shoring bar s in Figure 5.34b can transfer only compressive forces, a single shoring bar is not enough. The shoring bar applied does prevent tilting to the left, as in Figure 5.33a (the shoring bar has to shorten and therefore comes under pressure), but not tilting to the right (the shoring bar would be subject to tensile pressure, and may fall or come loose). In that case, two shoring bars would be required, as shown in Figure 5.34c.

The solution with shoring bars, also known simply as *shoring*, stems from the time when stiff corner joints were hard to achieve. You will often find them in (older) timber structures.

An example of this is the wooden roof structure in Figure 5.35a. This type of structure, still often used at the turn of the century, is called a *mansart roof truss*.[1] Figure 5.35b gives the structural model.

*Strut* B ensures that the horizontal forces are transferred to the beam layer that operates as a *tie-rod*. Strut B, in combination with the *hammer beam* C, can be seen as a shore that ensures a certain restraint of *rafter* A, in the same way as the shoring bar in Figure 5.34a, but in this case placed on the inside. *Brace* G fixes the corner between rafter A and *collar beam* D. They operate like the shoring bar in Figure 5.34b.

Figure 5.35a clearly shows that brace G is connected to rafter A and collar beam D by means of *toothed joints*. Since toothed joints work only under pressure, the upper struts can transfer only compressive forces. For the shoring bars, one still often refers to bars that are loaded by compressive forces.

Shoring bars are used not only to make a structure kinematically determinate, but also to influence the force flow positively, as the *shores* F in Figure 5.35. These shores provide additional support to rafter A, which can therefore be made lighter.

Shoring is found not only in old structures. Shores are still used to influence force flow positively, so that less material is required to meet the demands of strength and rigidity.

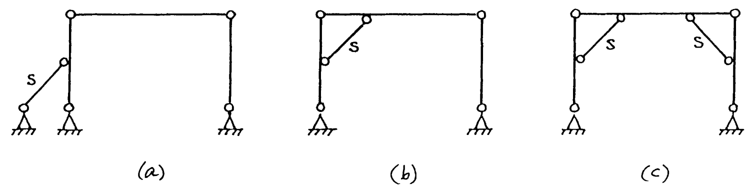



***Figure 5.34*** (a) A fixed end and (b) a rigid corner connection, both created by using a shoring bar. (c) If the shoring bar can transfer only compressive forces, two shoring bars are required to prevent the tilting to the left and to the right.

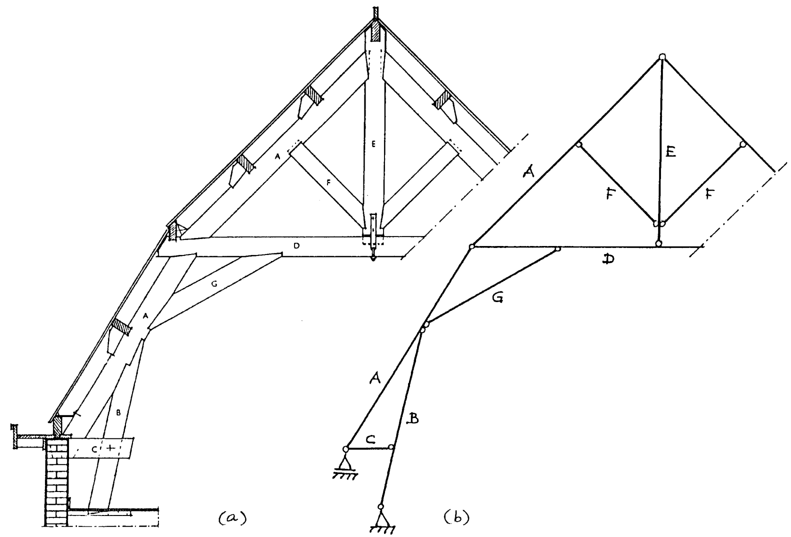



***Figure 5.35*** (a) A Mansart truss with (b) the structural model.

[1] Named after Jules Hardouin Mansart (1646–1708), French architect. He built the Dôme des Invalides in Paris and major sections of the palace in Versailles.

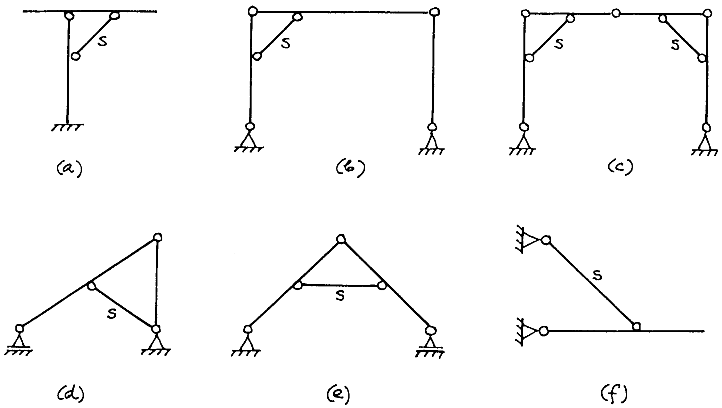



*Figure 5.36* Examples of shored structures. In examples (e) to (f) one refers to a tie rod rather than a shoring bar.

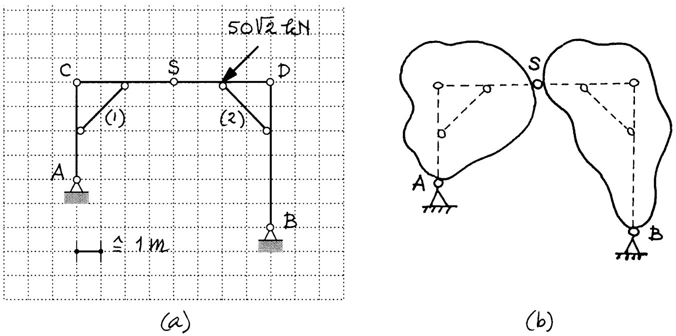



*Figure 5.37* (a) A shored three-hinged frame. (b) A three-hinged frame in principle consists of two self-contained parts that are connected by a hinge at S and supported by hinges at A and B.

The following examples will be limited to statically determinate structures. It is assumed that shoring bars can transfer both tensile and compressive forces.

Figure 5.36 shows a number of statically determinate shored structures. In cases (e) and (f) one refers to a *tie-rod*[1] rather than to a *shoring bar*, even though the tie-rod is actually fulfilling the role of a shore.

**Example 1**

The shored structure in Figure 5.37a is loaded by the force $F = 50\sqrt{2}$ kN.

*Questions*:

a. Determine the support reactions.
b. Determine the forces in the shoring bars (with the correct sign for tension and compression).
c. Determine all the forces acting on bar SD.

*Solution*:

a. You will recognise a three-hinged frame in the structure. There are two self-contained parts that are connected in a hinge at S and are supported by hinges at A and B (see Figure 5.37b). The structure in Figure 5.37a is therefore also referred to as a *shored three-hinged frame*. The support reactions can be derived in the standard way for a three-hinged frame (see Section 5.3). The calculation, which will be left to the reader, leads to the support reactions shown in Figure 5.38.

b. The shoring bars are loaded only by forces at the end of the bars and therefore act as two-force members. Suppose that a tensile force $N^{(1)}$ acts in the left shoring bar (1) and a tensile force $N^{(2)}$ in the right shoring bar (2). In Figure 5.39, AC and BD have been isolated. The unknown interaction forces at C and D are not shown here.

---

[1] Since the vertical weight causes tension in these bars.

It is now possible to deduce $N^{(1)}$ from the moment equilibrium of AC about C:

$$\sum T_z^{(AC)}|C = +(40\sqrt{2}\,\text{kN})(2\sqrt{2}\,\text{m}) + N^{(1)} \times (\sqrt{2}\,\text{m}) = 0$$

so that

$$N^{(1)} = -80\sqrt{2}\,\text{kN}.$$

There is a compressive force in shoring bar (1).

In the same way, one can find $N^{(2)}$ from the moment equilibrium of DB about D:

$$\sum T_z^{(BD)}|D = +(10\sqrt{2}\,\text{kN})(3\sqrt{2}\,\text{m}) - N^{(2)} \times (\sqrt{2}\,\text{m}) = 0$$

so that

$$N^{(2)} = 30\sqrt{2}\,\text{kN}.$$

Shoring bar 2 is a tension bar.

To demonstrate clearly how the shoring bars act on frame ASB, the frame and the shoring bars have been isolated from one another in Figure 5.40.

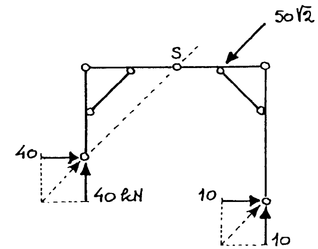



***Figure 5.38*** The support reactions of the frame as they really act.

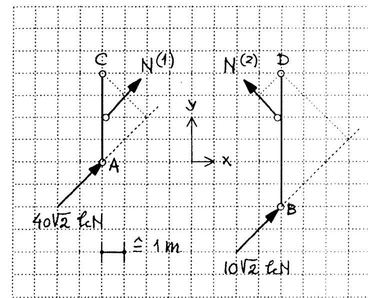



***Figure 5.39*** The isolated columns AC and BD. The unknown interaction forces at C and D are not shown here.

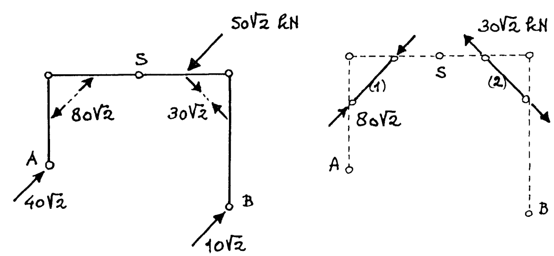



***Figure 5.40*** To see which forces the shoring bars and frame are exerting on one another, they have been isolated.

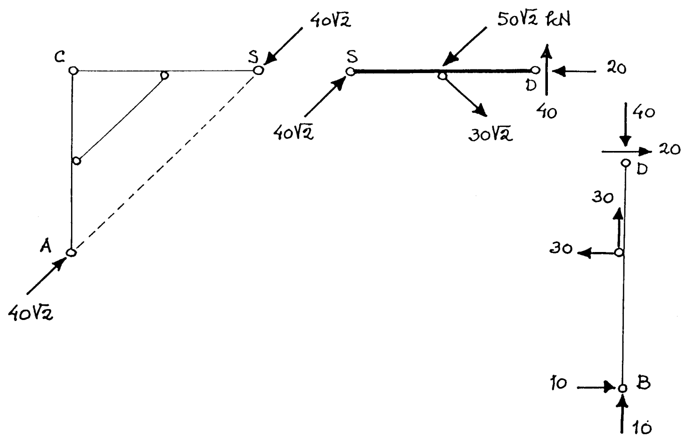



***Figure 5.41*** The forces acting on SD are found from the equilibrium of the isolated parts.

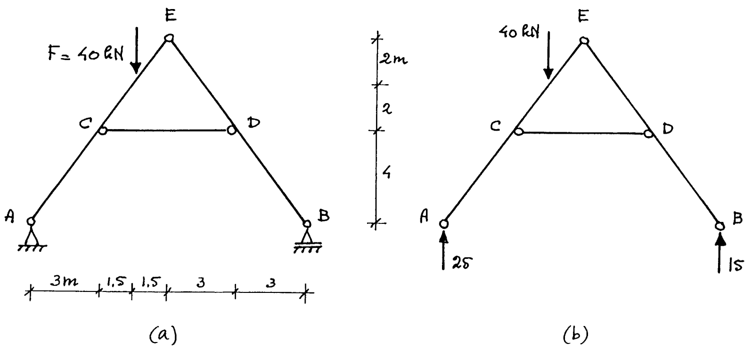



***Figure 5.42*** (a) A structure loaded by a vertical force of 40 kN on the left-hand rafter, with (b) its support reactions.

c. The force acting at S on SD is equal to the support reaction at A. The force that shoring bar (2) exerts on SD is also known. Still unknown are the components of the force exerted on SD at D. These are found via the force equilibrium of column BD (see Figure 5.41).

*Check*: SD must be in equilibrium.

**Example 2**

The structure in Figure 5.42a is loaded on rafter ACE by a vertical force $F = 40$ kN.

*Questions*:

a. Determine the support reactions.
b. Determine the force in bar CD (with the correct sign for tension and compression).
c. Determine the hinge force at E.

*Solution*:

a. The support reactions follow directly from the equilibrium of the structure as a whole. There are only vertical support reactions. They are shown in Figure 5.42b.

b. Suppose the tensile force in CD is $N^{(\mathrm{CD})}$. In Figure 5.43, CD has been isolated from AEB. The magnitude of $N^{(\mathrm{CD})}$ follows from the moment equilibrium about E of one of the rafters AE or BE. The unloaded rafter BE is simpler with respect to the amount of arithmetic:

$$\sum T_z^{(\mathrm{BE})}|E = -N^{(\mathrm{CD})} \times (4\text{ m}) + (15\text{ kN}(6\text{ m}) = 0 \Rightarrow N^{(\mathrm{CD})} = 22.5\text{ kN}$$

CD is a tension member.

c. The hinge force at E is subsequently found from the force equilibrium of one of the rafters AE or BE. Again, the unloaded right-hand rafter BE is preferable. In Figure 5.44a, BE has been isolated, and the result of the calculation is shown.

The forces acting on BDE at D and E can also be determined graphically. The lines of action b and d are known (see Figure 5.44a). Line of action e of the hinge force at E must pass through the intersection of b and d (moment equilibrium of a body subjected to three forces). In a force polygon, one can now determine the forces at D and E that ensure equilibrium with the support reaction at B (see Figure 5.44b).

*Check*: The left-hand rafter ACE must also be in equilibrium. You can see immediately that there is force equilibrium in Figure 5.45. To check the moment equilibrium, write down the moment equation for all the forces about an arbitrary point.

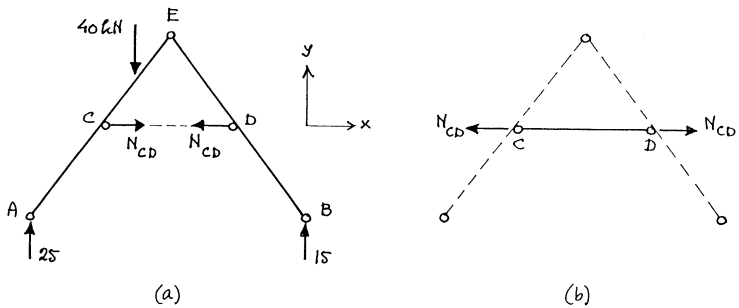



***Figure 5.43*** To see how rafter AEB and bar CD exert forces on one another, they have been isolated.

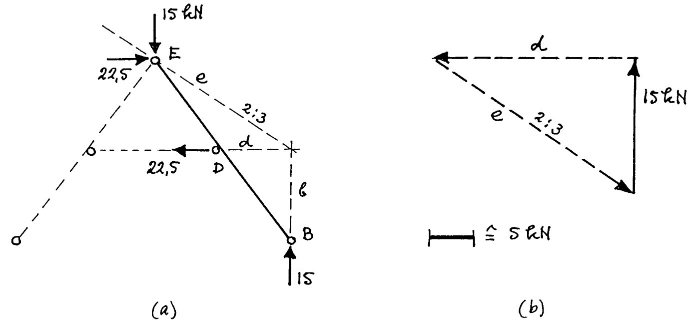



***Figure 5.44*** Graphical determination of the forces acting at D and E on the right-hand rafter BDE: (a) line of action figure and (b) force polygon.

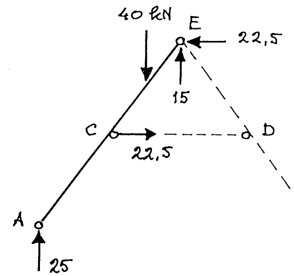



***Figure 5.45*** The forces acting on the left-hand rafter ACE.

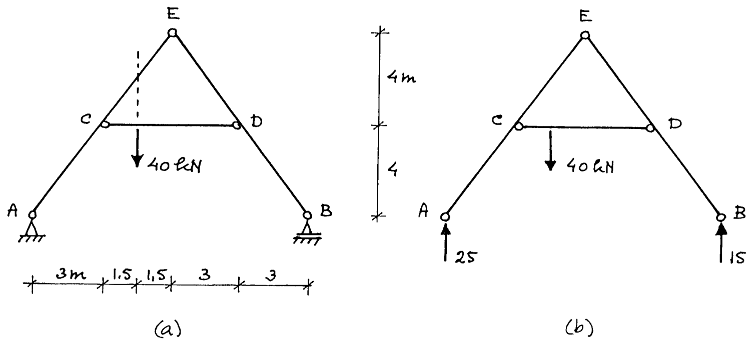



*Figure 5.46* (a) A structure loaded by a vertical force of 40 kN on the tie rod, with (b) its support reactions. For self-contained structures the support reactions do not change if one shifts a loading force along its line of action; on the other hand the interaction forces do change.

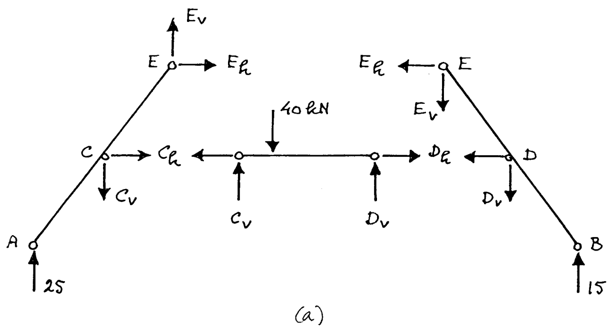



*Figure 5.47* (a) The interaction forces between the isolated parts AE, BE and CD. The interaction forces $C_v$ and $D_v$ are found from the moment equilibrium of CD.

**Example 3**

Figure 5.46a uses the same structure as in Example 2, except that this time, the vertical force $F = 40$ kN has been shifted along its line of action to a point of application on member CD.

*Questions*:

a. Determine the support reactions.
b. Determine the forces acting on the isolated parts ACE, BDE, and DE.
c. Perform a graphical check of the moment equilibrium for each of the parts.

*Solution*:

a. The support reactions are the same as those in example 2. They are shown in Figure 5.46b. Note that for a self-contained structure, the support reactions do not change if one shifts a force along its line of action. The forces *within* the structure do change, however, as is shown below.

b. In Figure 5.47a, the various structural parts have been isolated, and all the interaction forces are shown.

First look at the equilibrium of CD. From the moment equilibrium about C follows

$$D_v = 10 \text{ kN}.$$

From the moment equilibrium about D follows

$$C_v = 30 \text{ kN}.$$

The horizontal force equilibrium gives

$$C_h = D_h.$$

Next look at the right-hand rafter AE (see Figure 5.47b). The moment equilibrium about E gives

$$C_h = 15 \text{ kN}$$

so that

$$D_h = 15 \text{ kN}.$$

The force equilibrium gives

$$E_h = -15 \text{ kN},$$

$$E_v = 5 \text{ kN}.$$

The direction of $E_h$ was obviously assumed falsely.

In Figure 5.48, all the interaction forces are shown as they act in reality.

*Check*: BE must also meet the conditions of the force and moment equilibrium. Figure 5.48 shows that the force equilibrium conditions are satisfied. Only the moment equilibrium has to be checked.

c. If three forces act on a body, there is moment equilibrium only if the lines of action of the forces intersect at one point. In Figure 5.48, this check for moment equilibrium has been performed for each of the structural parts.

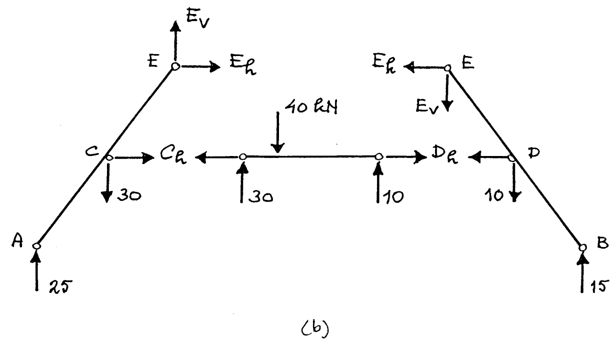



***Figure 5.47*** (b) The equilibrium of AE and BE is then used to find the other interaction forces.

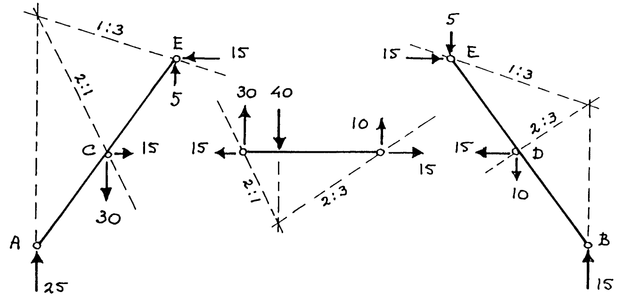


***Figure 5.48*** Graphical check of the moment equilibrium of AE, CD and BE: in all the cases, the lines of action of the three (resulting) forces pass through a single point.

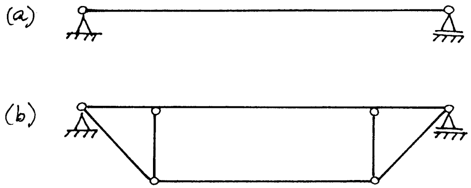



***Figure 5.49*** (a) A beam and (b) a trussed beam.

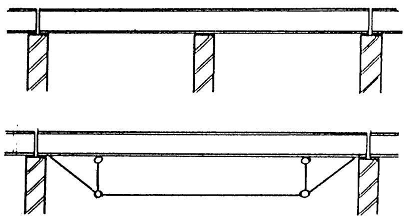


***Figure 5.50*** After a dividing wall has been demolished, the bearing capacity of a beam can be restored by introducing intermediate supports.

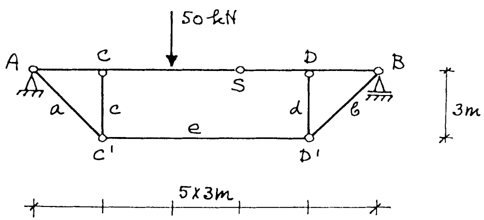



***Figure 5.51*** A statically determinate trussed beam.

## 5.6 Trussed beams

The bearing capacity of the beam in Figure 5.49a can be increased by introducing intermediate supports. These structures are referred to as *trussed beams* when these intermediate supports are realised by a bar system applied directly to the beam (see Figure 5.49b).

Trussed beams are used in simple bearing structures and for auxiliary structures in the construction industry (formwork bearers). You may also see them in restoration activities when, for example, after a dividing wall has been demolished, the bearing capacity of the floor beams is no longer adequate, as a result of the enlarged span (see Figure 5.50).

In the examples given, the trussed beams are (internally) statically indeterminate. In the following will address only statically determinate structures.

**Example**
The trussed beam ASB in Figure 5.51 consists of the two beam segments AS and SB joined by a hinge at S. The structure is loaded by a vertical force of 50 kN.

*Questions*:

a. Determine the support reactions.
b. Determine the forces in the bars (a) to (e) (with the correct sign for tension and compression).
c. Draw the forces acting on beam segments AS and SB.
d. Draw the forces acting on joint D.

*Solution*:
a. The support reactions follow directly from the equilibrium of the structure as a whole. They are shown in Figure 5.52.

b. The bars (a) to (e) are loaded only at their ends. They are therefore two-force members. Note: ACS and SDB are not two-force members!

In Figure 5.52, the isolated structure has been dissected across bar (e) and the hinged joint at S. Suppose there is a tensile force in bar (e) of $N^{(e)}$. The magnitude of $N^{(e)}$ follows from the moment equilibrium about S of the left-hand or right-hand part. The simpler equation is obtained with the unloaded right-hand part:

$$\sum T_z^{(SB)}|S = (20\text{ kN})(6\text{ m}) - N^{(e)} \times (3\text{ m}) = 0 \Rightarrow N^{(e)} = 40\text{ kN}.$$

Bar (e) is therefore a tension member.

The moment equilibrium of the left-hand part about S can be used to check the solution.

The forces in the bars (a) and (c) follow from the force equilibrium of joint C′. In Figure 5.53 these forces have been determined using a force polygon. In bar (a) there is a tensile force, while there is a compressive force in bar (c):

$$N^{(a)} = 40\sqrt{2}\text{ kN} \quad \text{and} \quad N^{(c)} = -40\text{ kN}.$$

In the same way, the force equilibrium of joint D′ gives

$$N^{(b)} = 40\sqrt{2}\text{ kN} \quad \text{and} \quad N^{(d)} = -40\text{ kN}.$$

c. Figure 5.54 shows all the forces acting on the beam segments AS and SB. The components of the hinge force S follow from the force equilibrium of the part to the left or to the right of S.

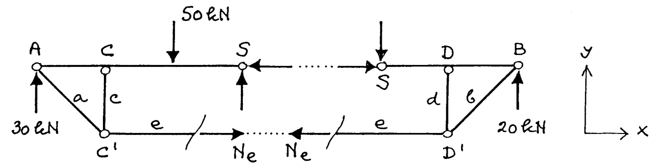



***Figure 5.52*** The trussed beam, isolated from its supports, has been "cut" across hinged joint S and bar (e). It has been assumed that bar (e) is a tension member.

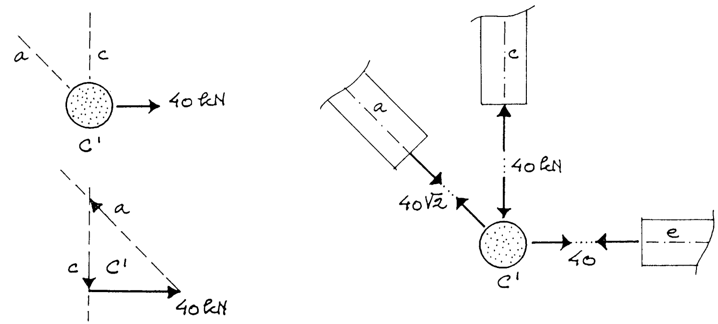



***Figure 5.53*** The forces in the bars (a) and (c) follow from the force equilibrium of joint C′: in bar (a) there is a tensile force and in bar (c) there is a compressive force.

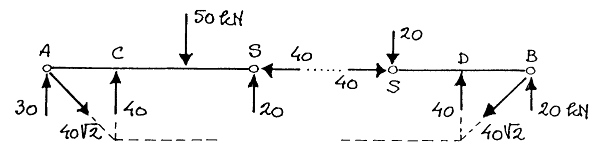
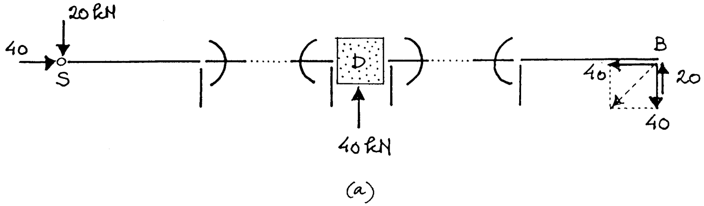



***Figure 5.54*** The isolated beam segments AS and SB, with all the forces as they really act.



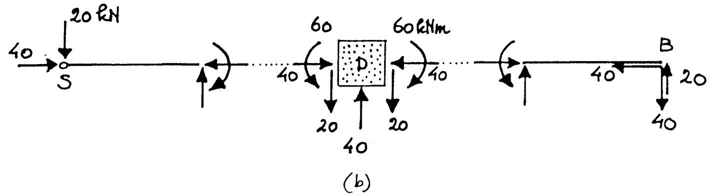




***Figure 5.55*** (a) Joint D isolated from the beam segments SD and DB. Three interaction forces are acting in the rigid connections. These can be found from the equilibrium of the separate beam segments. (b) The interaction forces as they really act. Joint D satisfies the conditions for force and moment equilibrium.

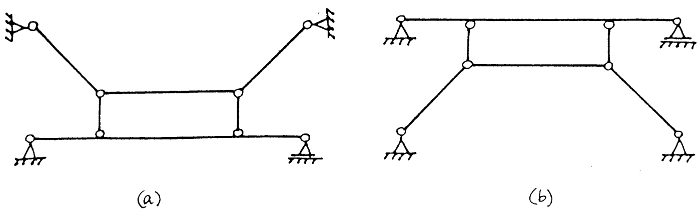



***Figure 5.56*** A beam (a) suspended from and (b) leaning upon a strengthening bar system.

d. In Figure 5.55a, joint D has been isolated from SD and DB. Three interaction forces are acting in the rigid connections. The forces exerted by joint D on SD and DB can be found from the equilibrium of these parts. Equal and opposite forces act on joint D (see Figure 5.55b).

*Check*: Joint D is in force equilibrium and in moment equilibrium.

## 5.7 Strengthened beams

The strengthened beams in Figure 5.56 are in many ways comparable to trussed beams. An important difference is that in here the strengthening bar system is supported outside the beam. In Figure 5.56a the beam is suspended from the strengthening bar system, in Figure 5.56b the beam is leaning upon it.

These structures are used in bridges. They are used also as auxiliary structures during building activities.

The structures in Figure 5.56 are statically indeterminate to the first degree. In the following we will address only statically determinate examples.

**Example**
The structure in Figure 5.57 is loaded by a vertical force of 40 kN.

*Questions*:
a. Determine the support reactions at A and B.
b. Determine the forces in bars (1) to (3) and (a) to (d).
c. Draw the forces acting on the hinged joint S.

*Solution* (units in kN and m):
a. This compound structure has five support reactions:

- two at hinged support A,
- one at roller support B,
- one at hinge A′, and
- one at hinge B′.

The three equilibrium equations for the structure as a whole are not sufficient for finding the five unknown support reactions. The solutions have to be found by means of the strengthening bar system.

Bars (1) to (3) and (a) to (d) are all two-force members. If one of the bar forces is known, all the others follow from the force equilibrium of the joints S′, C′ and D′. This is shown graphically in Figure 5.58 on the assumption that there is a tensile force $N$ in bar (2):

$$N^{(2)} = N.$$

The force equilibrium of joint S′ then gives

$$N^{(b)} = N^{(c)} = \tfrac{1}{2}\sqrt{17}\,N.$$

The force equilibrium of joint C′ gives

$$N^{(1)} = \tfrac{3}{2}N \quad \text{and} \quad N^{(a)} = 2\sqrt{2}\,N,$$

while the equilibrium of joint D′ gives

$$N^{(3)} = \tfrac{3}{2}N \quad \text{and} \quad N^{(d)} = 2\sqrt{2}\,N.$$

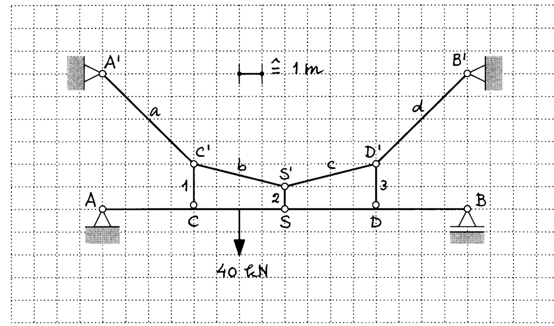



***Figure 5.57*** A statically determinate strengthened beam.

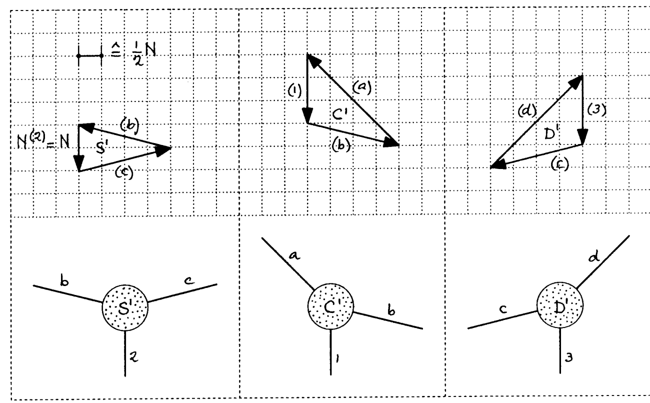



***Figure 5.58*** Assuming that a tensile force $N$ acts in bar (2), the force equilibrium of the hinged joints S′, C′ and D′ can be used to express the forces in the bars (a) to (d) and (1) and (3) in terms of $N$.

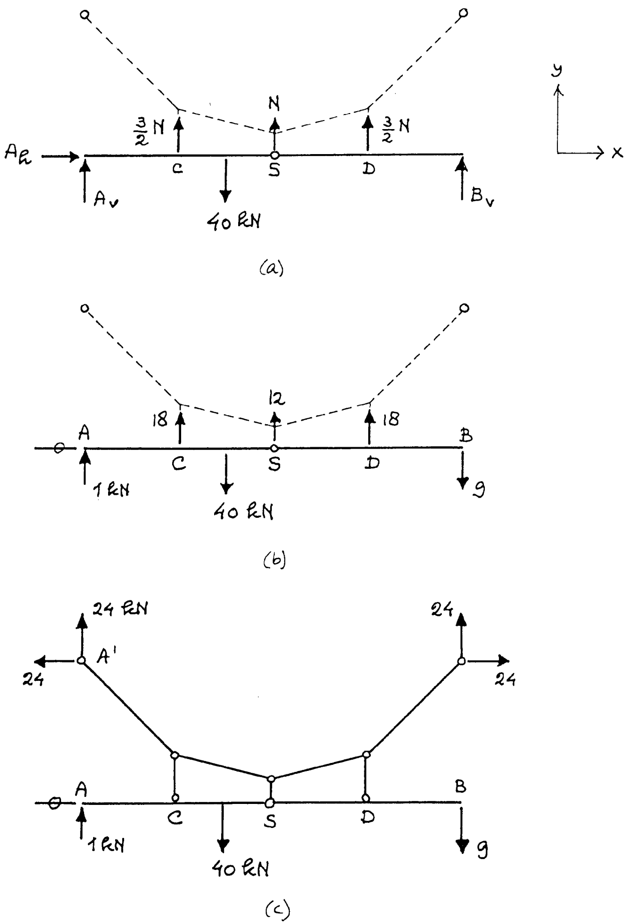



*Figure 5.59* (a) The isolated beam ASB. (b) All the forces acting on the beam ASB as they really act. (c) The entire structure with all the support reactions.

In Figure 5.59a, hinged beam ASB has been isolated and all the forces acting on it are shown. The horizontal force equilibrium of the hinged beam as a whole then gives

$$A_\mathrm{h} = 0.$$

The vertical support reactions $A_\mathrm{v}$ and $B_\mathrm{v}$, and the unknown force $N$ are calculated in the same way as for a hinged beam (see Section 5.2).

For the beam as a whole applies

$$\begin{aligned}\sum T_z|\mathrm{A} &= \tfrac{3}{2}N \times 4 + N \times 8 + \tfrac{3}{2}N \times 12 + B_\mathrm{v} \times 16 - 40 \times 6 \\ &= 32N + 16B_\mathrm{v} - 240 = 0.\end{aligned} \tag{a}$$

For the right-hand section SB

$$\sum T_z|\mathrm{S} = 4 \times \tfrac{3}{2}N + 8 \times B_\mathrm{v} = 0. \tag{b}$$

These two equations with $N$ and $B_\mathrm{v}$ as unknowns give

$$N = 12\ \mathrm{kN} \;\text{ and }\; B_\mathrm{v} = -9\ \mathrm{kN}.$$

The vertical support reaction at B therefore acts opposite to the direction assumed in Figure 5.59a.

The vertical support reaction at A follows from the vertical force equilibrium of beam ASB as a whole:

$$\sum F_y = A_\mathrm{v} + \tfrac{3}{2}N + N + \tfrac{3}{2}N + B_\mathrm{v} - 40 = 0$$

so that

$$A_\mathrm{v} = -4N - B_\mathrm{v} + 40 = -4 \times 12 - (-9) + 40 = 1\ \mathrm{kN}.$$

Figure 5.59b shows all the forces on beam ASB as they really act.

b. The forces in the bars (1) to (3) and (a) to (d) were previously expressed in terms of $N$ (see Figure 5.58). With $N = 12$ kN the result is

$$N^{(1)} = N^{(3)} = \tfrac{3}{2}N = 18\ \mathrm{kN},$$

$$N^{(2)} = N = 12\ \mathrm{kN},$$

$$N^{(a)} = N^{(d)} = 2\sqrt{2}\,N = 24\sqrt{2}\ \mathrm{kN},$$

$$N^{(b)} = N^{(c)} = \tfrac{1}{2}\sqrt{17}\,N = 6\sqrt{17}\ \mathrm{kN}.$$

All bar forces are tensile forces.

Figure 5.59c gives the entire structure with all the support reactions.

*Check*: The structure as a whole satisfies the conditions of the force and moment equilibrium.

c. In Figure 5.60a, the beam segments AS and BS and the hinged joint S have been isolated. The values of all the known forces are shown. The forces acting on joint S are found via the equilibrium of the segments AS and SB. They are shown in Figure 5.60b.

*Check*: Joint S is in equilibrium.

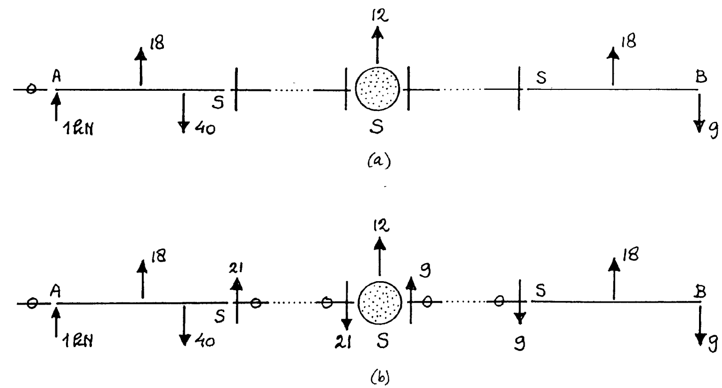


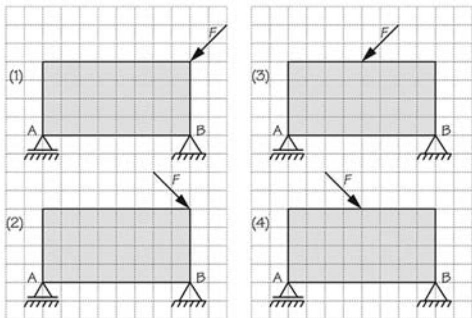

***Figure 5.60*** (a) The hinged joint S isolated from the beam segments AS and SB. The interaction forces can be found from the equilibrium of the segments AS and SB. (b) All the forces as they really act. Joint S is in equilibrium.

## 5.8 Problems

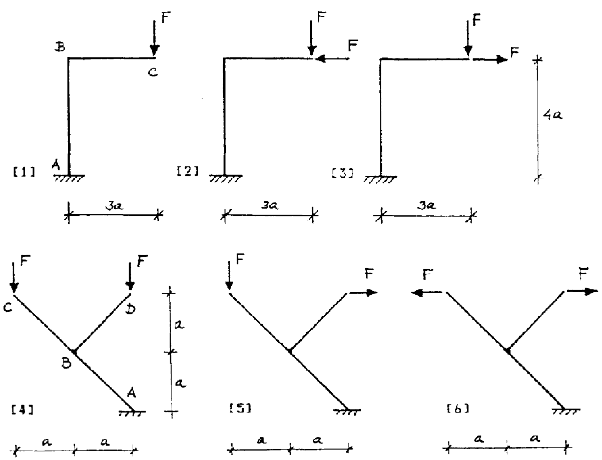
***Self-contained structures*** (Section 5.1)

**5.1: 1–4** A block is supported on a roller at A and a hinge at B. The block is loaded by a force $F = 20\sqrt{2}$ kN. Length scale: 1 square $\equiv$ 1 m.

*Question*:
Determine the support reactions at A and B:
a. analytically;
b. graphically.

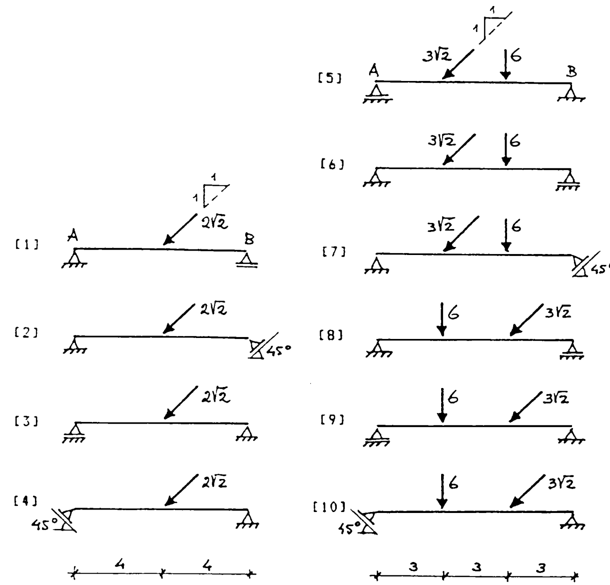



**5.2: 1–6** Given a number of fixed structures.




*Questions*:
a. In which directions would you expect the support reactions at A to act?
b. Determine the support reactions at A, working with the directions assumed in (a).
c. For which support reactions did you assume the wrong direction?
d. Draw all the support reactions as they act in reality.

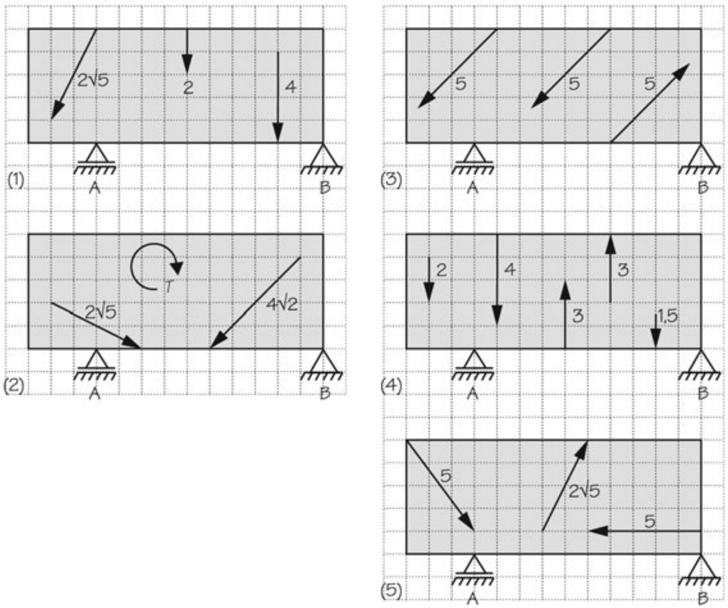
**5.3: 1–10** A number of beams are supported on a hinge and a roller. The dimensions are given in m, the forces are in kN.




*Questions*:
a. Determine the support reactions analytically.
b. Check the answers graphically (if possible).

**5.4: 1–5** A block is supported on a roller at A and a hinge at B. A number of forces act on the block. In case 2, a couple $T = 36$ kNm also acts on the block. Force scale: 1 square $\equiv$ 1 kN; length scale: 1 square $\equiv$ 1 m.

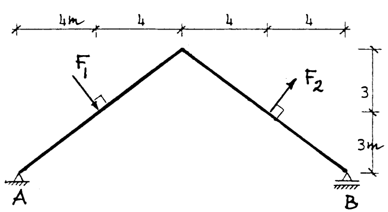
*Question*: Determine the support reactions at A and B.



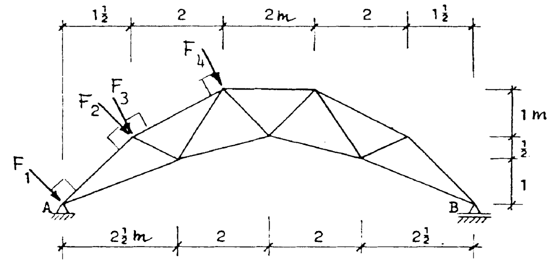

**5.5** A roof structure is loaded by wind forces:

$$F_1 = 5.6 \text{ kN},$$
$$F_2 = 2.8 \text{ kN}.$$

*Question*:
Determine the support reactions at A and B.

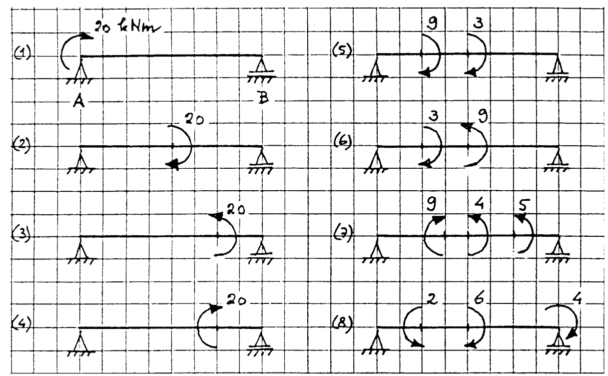



**5.6** A truss arch is loaded by wind forces: $F_1 = F_2 = 750\sqrt{2}$ kN,
$F_3 = F_4 = 500\sqrt{5}$ kN.




*Question*:
Determine the support reactions at A and B.

**5.7: 1–8** The simply supported beam AB is loaded in various ways by couples. The magnitude of the couples is shown in kNm. Length scale: 1 square $\equiv$ 1 m.

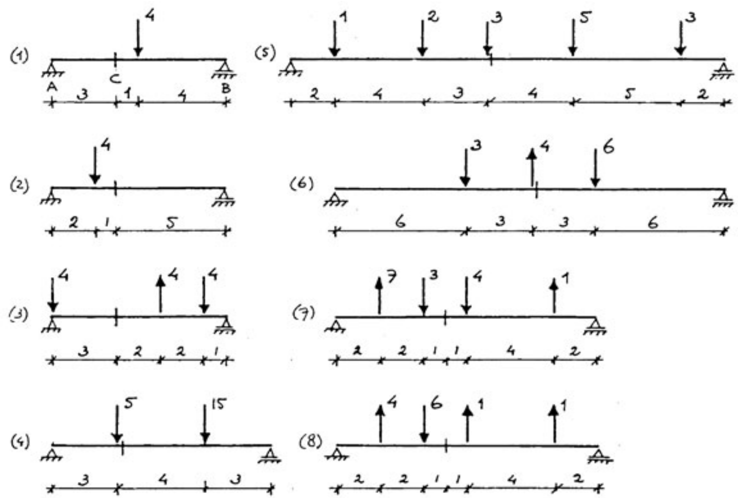



*Question*:
Find the support reactions at A and B.

**5.8: 1–8** A number of beams simply supported at A and B are composed of the segments AC and BC that are rigidly connected at C. The location of joint C is shown in the figure by means of a vertical dash. The forces are given in kN, the lengths in m.

*Questions*:

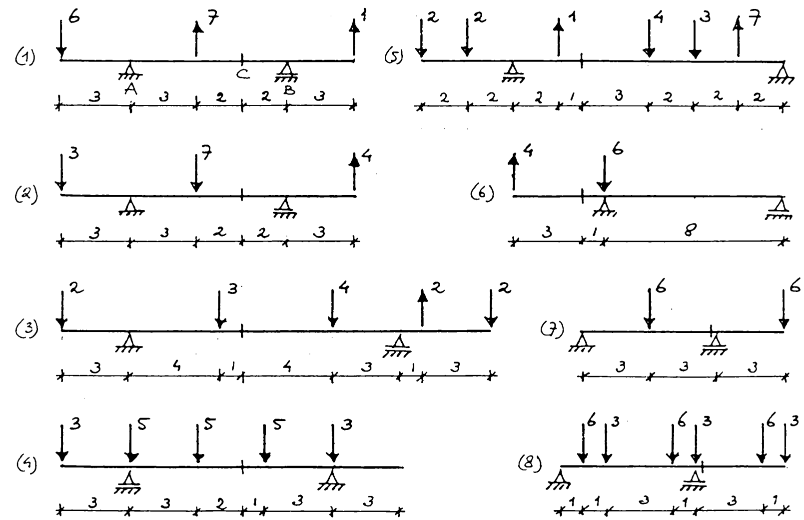
a. Determine the support reactions.
b. Determine the interaction forces at C; draw these forces as they act at C on segments AC and BC.
c. Draw the forces as they really act on joint C.



**5.9: 1–6** A number of cantilever beams, simply supported at A and B, are composed of two segments that are rigidly connected at C. The location of joint C is shown in the figure by means of a vertical dash. The forces are given in kN, the lengths are in m.

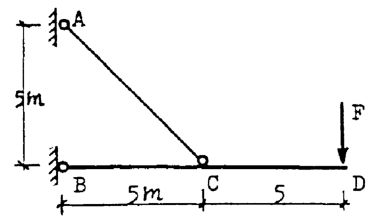
*Questions*:

a. Determine the support reactions.
b. Determine the interaction forces at C; draw these forces as they act at C on the segments AC and BC.
c. Draw the forces as they really act on joint C.



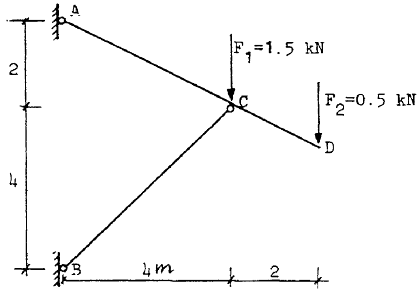
**5.10** *Question*:
Determine the support reactions at A and B.

a. graphically;
b. analytically.

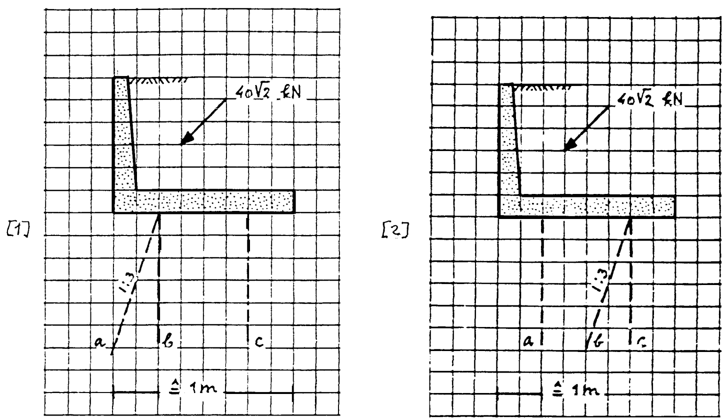


**5.11** *Questions*:
For porch ACD determine the support reactions at A and B due to:

a. only $F_1$;
b. only $F_2$;
c. both $F_1$ and $F_2$.

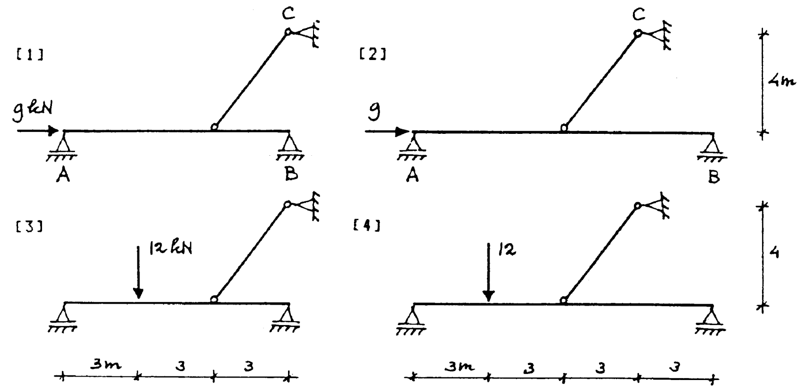



**5.12: 1–2** You are given a retaining wall on piles. Assume the piles are only transferring forces in their longitudinal direction. The resultant of all the loads that the piles have to bear is a force of $40\sqrt{2}$ kN. The direction and line of action are given in the figure. Length scale: 2 squares ≡ 1 m.




*Question*:
Find the pile forces with the correct signs for tension and compression (a tensile force is positive and a compression force is negative).

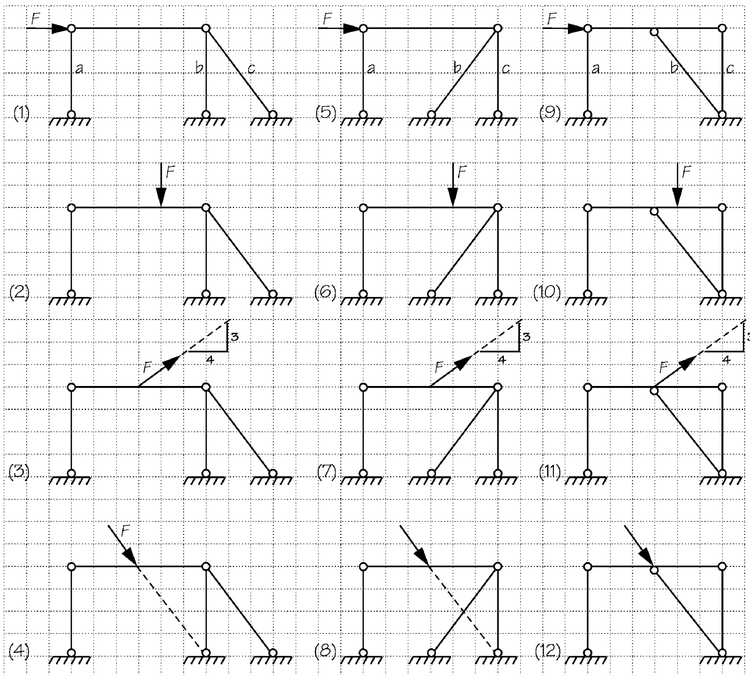
**5.13: 1–4**




*Questions*:

a. Make a realistic assumption about the directions of the support reactions at A, B and C.
b. Determine these support reactions.
c. Draw the support reactions as they really act and include relevant values.
d. If possible, check the calculated support reactions graphically.

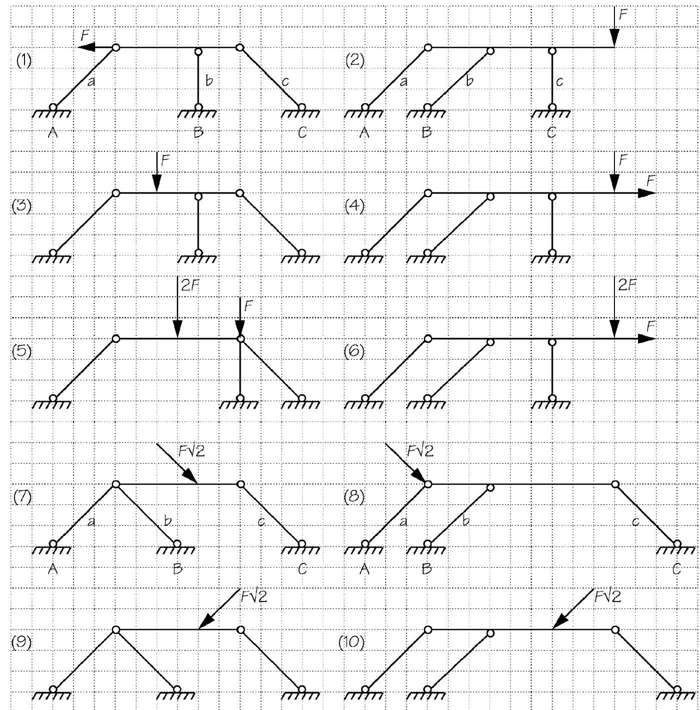
**5.14: 1–12** A beam, loaded by a force $F = 30$ kN, is supported by the three bars a, b and c. Length scale: 1 square $\equiv$ 1 m.



*Questions*:

a. Determine the support reactions at A, B and C.
b. Determine the forces in the beams, with the correct sign.
c. Isolate the beam, draw all the forces as they really act on it, and check the equilibrium.

**5.15: 1–10** A beam is supported by the bars a, b and c. The load of the beam is expressed in the force $F = 30$ kN. Length scale: 1 square $\equiv$ 1 m.

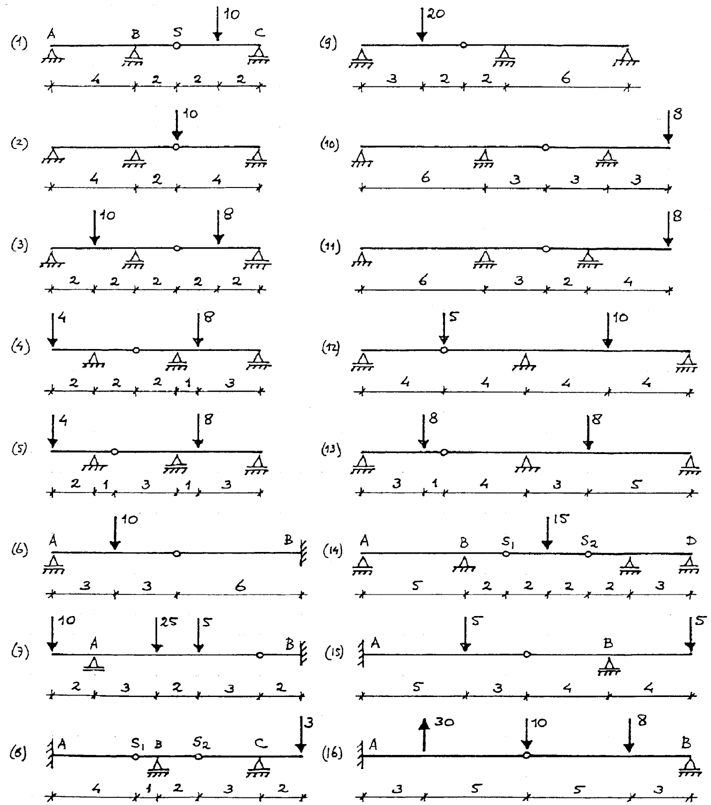


*Questions*:

a. Determine the support reactions at A, B and C.
b. Determine the forces in the bars, with the correct sign.
c. Isolate the beam, draw all the forces as they really act on it, and check the equilibrium.

***Hinged beams*** (Section 5.2)

**5.16: 1–16** The dimensions of the hinged beams are given in m, the forces are in kN.



*Questions*:

a. Determine the support reactions.
b. Isolate all the beam segments and draw the forces as they really act on these segments.
c. Check the force and moment equilibrium of the structure as a whole.

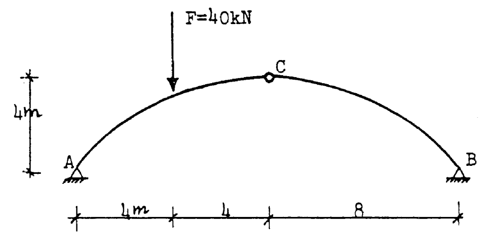
***Three-hinged frames*** (Section 5.3)

**5.17** Three-hinged arch ACB is loaded by a force $F = 40$ kN.

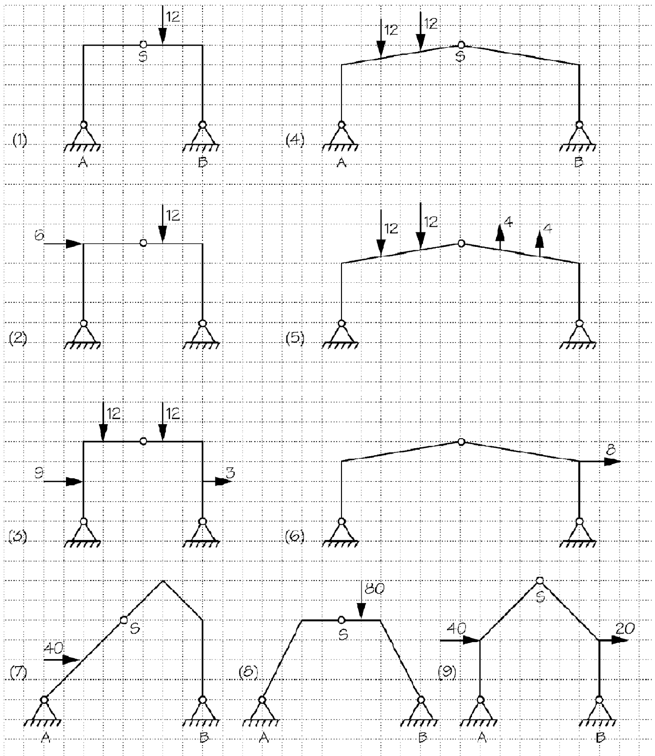
*Questions*:

Determine the support reactions at A and B:

a. graphically;
b. analytically.



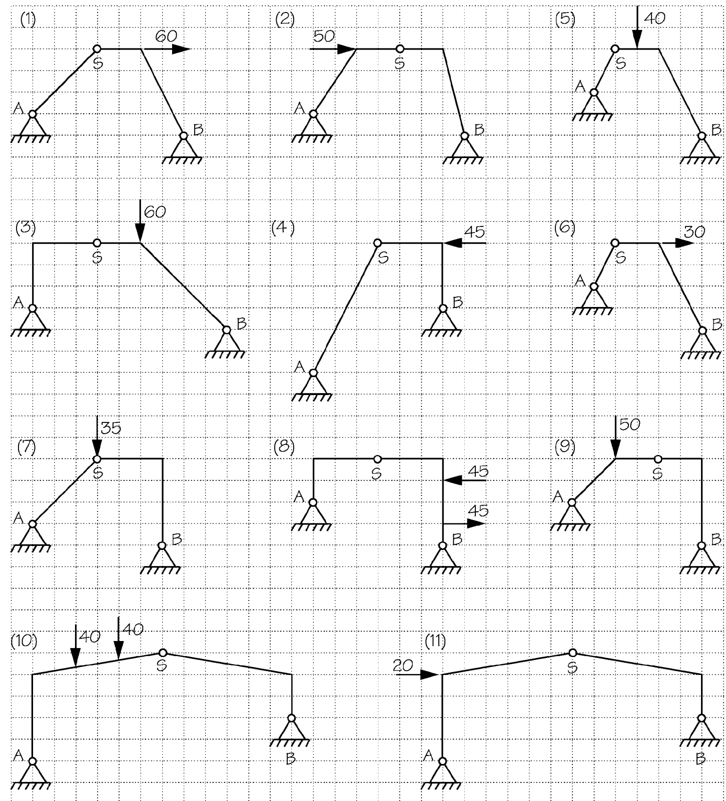
**5.18: 1–9** The figure shows a number of three-hinged frames with loads expressed in kN. Length scale: 1 square ≡ 1 m.




*Questions*:

a. Determine the support reactions.
b. Isolate both frame halves and draw all the forces as they really act on them.

**5.19: 1–11** The figure shows a number of three-hinged frames with the loading expressed in kN. Length scale: 1 square ≡ 1 m.

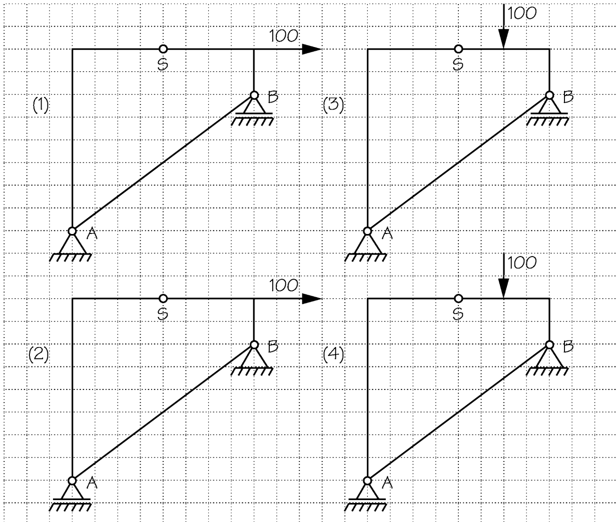



*Questions*:

a. Determine the support reactions.
b. Graphically check the support reactions (if possible).
c. Isolate both frame halves and draw all the forces as they really act on them.

***Three-hinged frames with tie rods*** (Section 5.4)

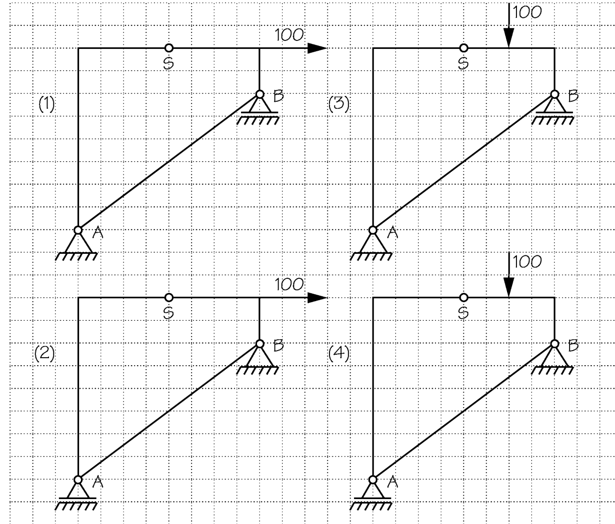
**5.20** The structural parts AB, AS and BS are connected by hinges at A, B and S. The load is in kN; length scale: 1 square ≡ 1 m.




*Questions*:

a. Determine the support reactions.
b. Determine the force in rod AB, with the correct sign.
c. Determine the forces acting on the isolated parts AB, AS and BS.
d. Determine the forces acting on the isolated joints A, B, and S.

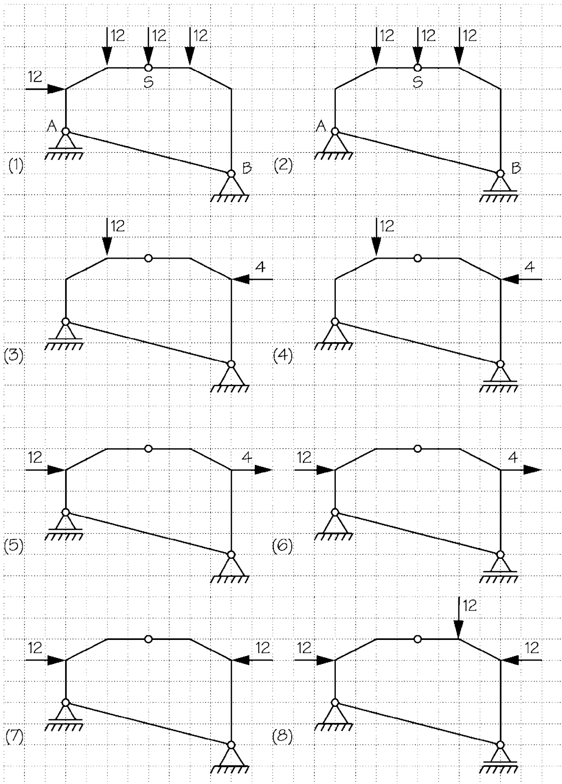
**5.21: 1–4** The structural parts AB, AS and BS are connected by hinges at A, B and S. The load is given in kN; length scale: 1 square ≡ 1 m.




*Questions*:

a. Determine the support reactions.
b. Determine the forces in rod AB, with the correct sign.
c. Determine the forces acting on the isolated parts AB, AS and BS.
d. Determine the forces acting on the isolated joints A, B and S.

**5.22: 1–12** The structural parts AB, AS and BS are connected by hinges at A, B and S. The load is given in kN; length scale: 1 square ≡ 1 m.

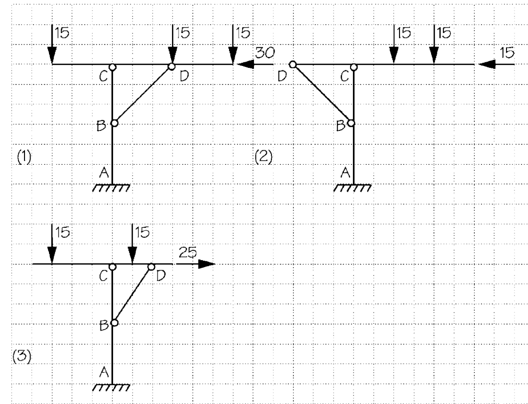



*Questions*:

a. Determine the support reactions.
b. Determine the forces in rod AB, with the correct sign.
c. Determine the forces acting on the isolated parts AB, AS and BS.
d. Determine the forces acting on the isolated joints A, B and S.

***Shored structures*** (Section 5.5)

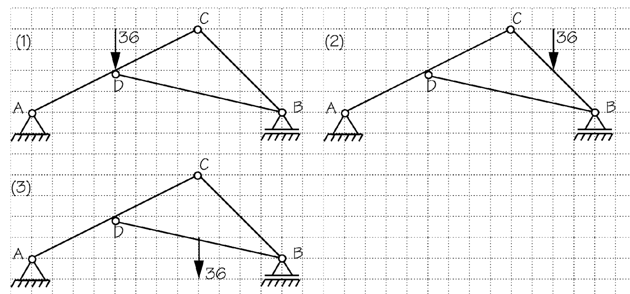
**5.23: 1–3** The forces are given in kN; length scale: 1 square ≡ 1 m.




*Questions*:

a. Determine the support reactions.
b. Determine the force in the shoring bar, with the correct sign.
c. Determine the forces acting on the isolated joints B and C.

**5.24: 1–3** The load is given in kN; length scale: 1 square ≡ 1 m.

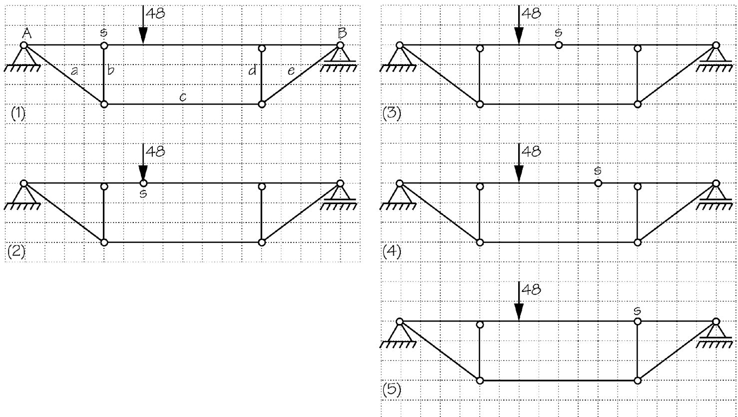


*Questions*:

a. Determine the support reactions at A and B.
b. Determine the force in bar BD, with the correct sign.
c. Isolate all bars and draw all the forces really acting on them.

***Trussed beams*** (Section 5.6)

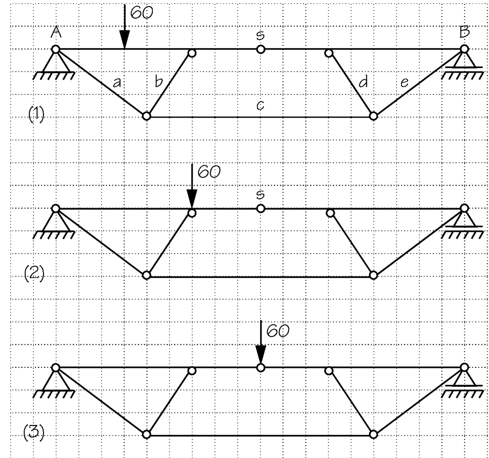
**5.25: 1–5** The forces are given in kN; length scale: 1 square ≡ 1 m.




*Questions*:

a. Determine the support reactions at A and B.
b. Determine the forces in bars a to e, with the correct sign.
c. Isolate the beam sections AS and BS and draw all the forces acting on them.
d. Isolate joint B, draw all the forces really acting on it, and check the force equilibrium using a force polygon.

**5.26: 1–3** The forces are given in kN; length scale: 1 square ≡ 1 m.



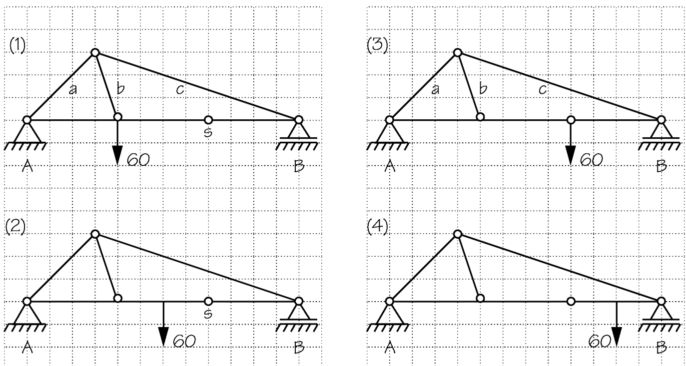

*Questions*:

a. Determine the support reactions at A and B.
b. Determine the forces in bars a to e, with the correct signs.
c. Isolate beam segments AS and BS and draw all the forces really acting on them.
d. Isolate joint B, draw all the forces really acting on it, and check the force equilibrium using a force polygon.

**5.27: 1–4** The forces are given in kN; length scale: 1 square ≡ 1 m.

*Questions*:

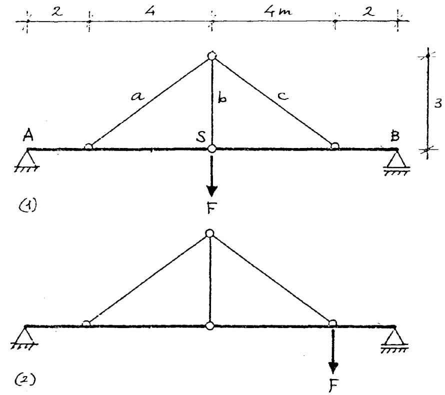
a. Determine the support reactions at A and B.
b. Determine the forces in bars a to c, with the correct signs.
c. Isolate beam segments AS and BS and draw all the forces really acting on it.
d. Isolate joint A, draw all the forces really acting on it, and check the force equilibrium using a force polygon.




**5.28: 1–2** Trussed beam ASB is loaded by a vertical force $F = 48$ kN.

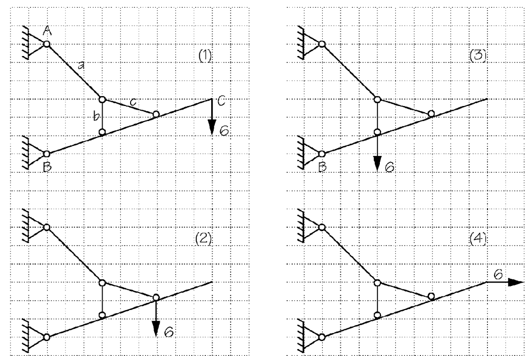
*Questions*:

a. Determine the support reactions at A and B.
b. Determine the forces in bars a to e, with the correct signs.
c. Isolate beam segments AS and BS, and draw all the forces really acting on them.




***Strengthened beams*** (Section 5.7)

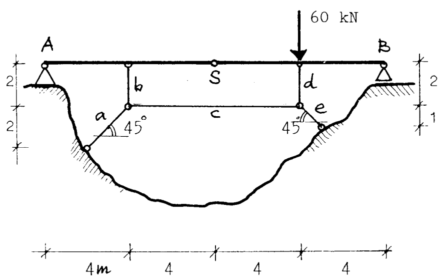
**5.29: 1–4** The forces are given in kN; length scale: 1 square ≡ 1 m.




*Questions*:

a. Determine the support reactions at A and B.
b. Determine the forces in bars a to c, with the correct signs.
c. Isolate beam BC, draw all the forces really acting on it, and check the force and moment equilibrium.

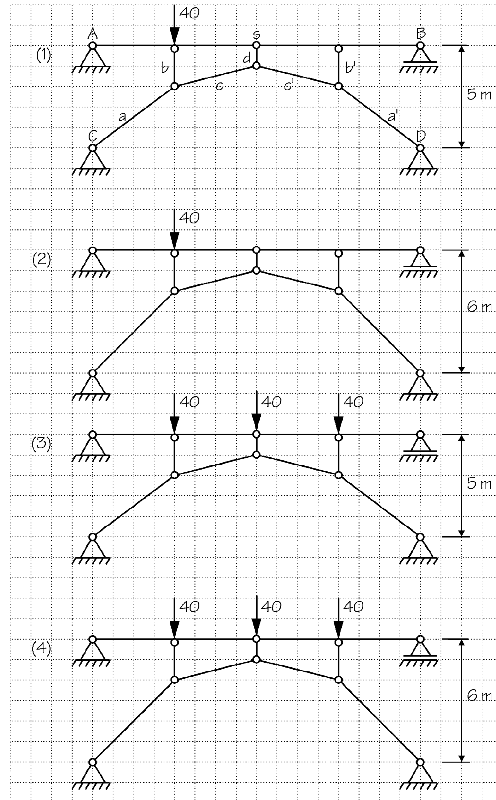
**5.30** The queen post truss is loaded at stay d by a vertical force of 60 kN.




*Questions*:

a. Determine the forces in bars a to e, with the correct signs.
b. Isolate beam segments AS and BS and draw all the forces really acting on them.
c. Determine the support reactions at A and B.

**5.31: 1–4** The forces are given in kN; length scale: 1 square ≡ 1 m.

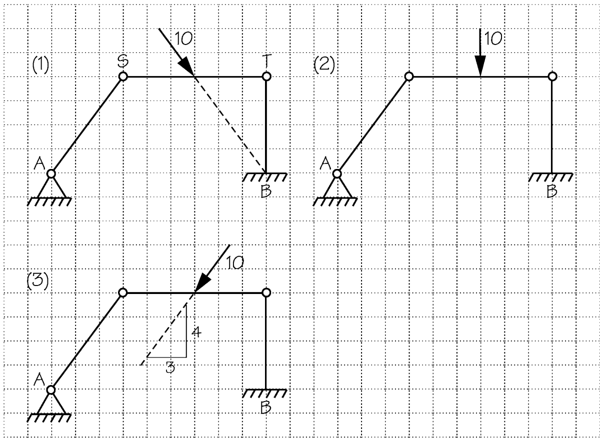



*Questions*:

a. Determine the forces in bars a to d and a′ to c′, with the correct signs.
b. Isolate beam segments AS and BS and draw all the forces really acting on them.
c. Determine the support reactions at A, B, C and D.

### *Various compound structures*

**5.32: 1–3** The forces are given in kN. Length scale: 1 square ≡ 1 m.



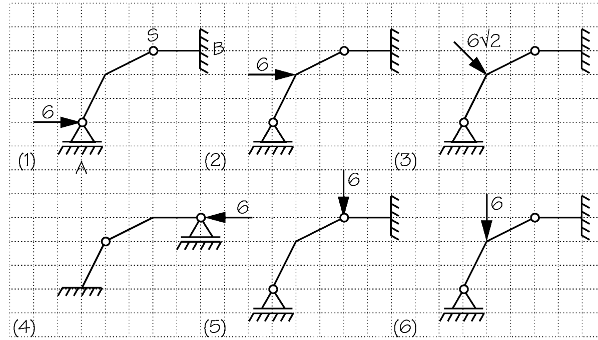

*Questions*:

a. Determine the support reactions.
b. Isolate all the structural members at the supports and hinged joints, and draw all the forces really acting on them.

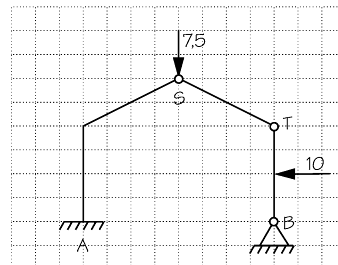
**5.33: 1–6** The forces are given in kN. Length scale: 1 square ≡ 1 m.

*Questions*:

a. Determine the support reactions.
b. Isolate all the structural members at the supports and hinged joints, and draw all the forces really acting on them.



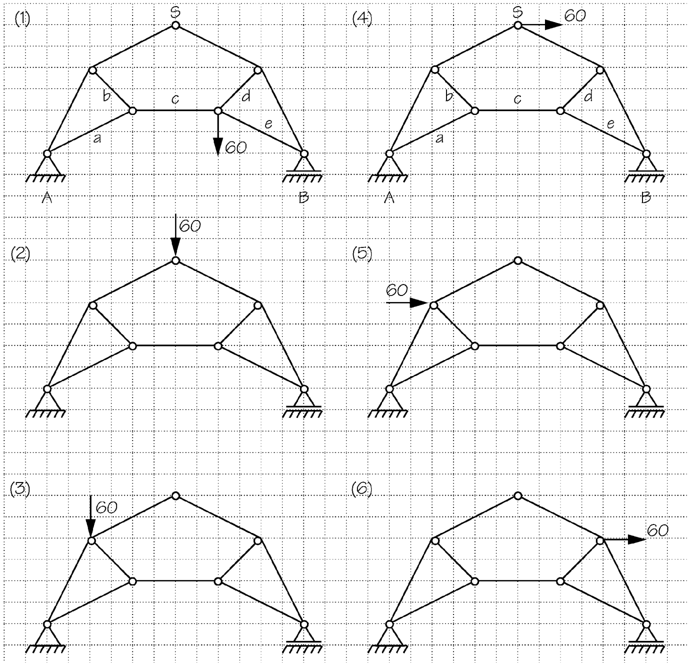

**5.34** Given a frame with the loads in kN. Length scale: 1 square ≡ 1 m.




*Questions*:

a. Determine the support reactions at A and B.
b. Isolate all the structural members at the supports and hinged joints, and draw all the forces really acting on them.

**5.35: 1–6** The forces are given in kN; length scale: 1 square ≡ 1 m.



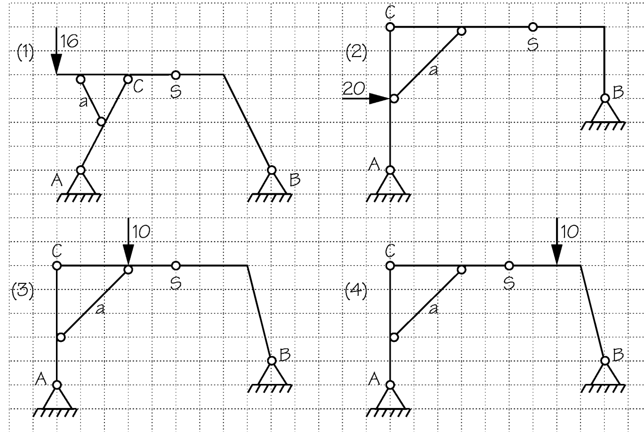

*Questions*:

a. Determine the support reactions at A and B.
b. Determine the forces in bars a to e, with the correct signs.
c. Determine the interaction forces at S, as they act on AS and BS.

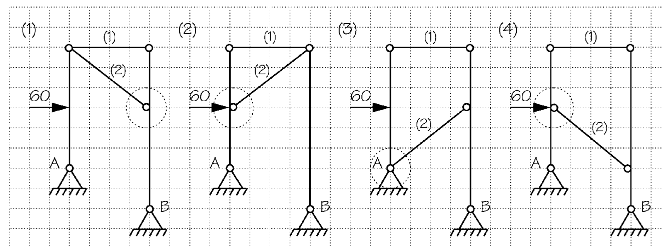
**5.36: 1–4** The forces are given in kN; length scale: 1 square ≡ 1 m.

*Questions*:

a. Determine the support reactions.
b. Graphically check the support reactions.
c. Determine the force in shoring bar a, with the correct sign.
d. Determine the hinge forces at S, as they act on CS and BS.




**5.37: 1–4** The forces are given in kN; length scale: 1 square ≡ 1 m.




*Questions*:

a. Determine the support reactions.
b. Determine the force in bars 1 and 2, with the correct signs.
c. Isolate the circled joint and draw all the forces acting on it.