2 Statics of a Particle

If several forces are exerted on a particle, they can be compounded as described in Sections 1.3.4 and 1.3.5 for vectors. The sum of all the forces is called the resultant force, or *resultant*. Since all the forces have the same point of application, namely the particle, the resultant also acts on that point. Section 2.1 addresses compounding and resolving forces on a particle in a plane. Section 2.2 looks at compounding and resolving forces on a particle in space. In Section 2.3 we show how to derive the equilibrium equation from the motion equation for a particle at rest.

2.1 Coplanar forces

We will first address compounding forces. Compounding is possible both analytically (Section 2.1.1) and graphically (Section 2.1.2). We will then show how to resolve a force into components with given directions. Here we can choose between an analytical approach (Section 2.1.3) and a graphical approach (Section 2.1.4).

2.1.1 Compounding forces analytically

We can compound forces analytically by adding together the respective components in each of the coordinate directions, see Section 1.3.5. This

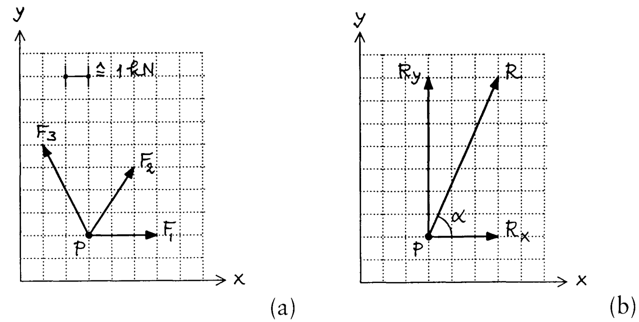


Figure 2.1 (a) Particle loaded by three forces and (b) the resultant R of the three forces.

is illustrated on the basis of an example.

Example
In Figure 2.1a, the three forces F_1, F_2 and F_3 located in the xy plane act on the particle P. Their magnitudes and directions can be derived using the squares from the figure.

Note: As you can see from the figure, here we use the *visual notation* for force vectors (see Section 1.3.6).

Question:
What is the magnitude and direction of the resultant R?

Solution: In the coordinate system shown, the components of the resultant R of the three forces are

$$\begin{aligned} R_x &= F_{x;1} + F_{x;2} + F_{x;3} = (3\text{ kN}) + (2\text{ kN}) + (-2\text{ kN}) = 3\text{ kN}, \\ R_y &= F_{y;1} + F_{y;2} + F_{y;3} = (0\text{ kN}) + (3\text{ kN}) + (4\text{ kN}) = 7\text{ kN}. \end{aligned}$$

Be careful with respect to the signs of the x and y components: they are related to the (not shown) unit vectors in the coordinate system!

The *magnitude* of the resultant is

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(3\text{ kN})^2 + (7\text{ kN})^2} = \sqrt{58}\text{ kN}.$$

From the angle α between the line of action of R and the x axis, it follows that

$$\tan\alpha = \frac{R_y}{R_x} = \frac{7\text{ kN}}{3\text{ kN}} = 2.33 \Rightarrow \alpha = 66.8^\circ + k \times 180^\circ.$$

The *direction* of R is determined by

$$\sin\alpha = \frac{R_y}{R} = 0.919 \quad \text{and} \quad \cos\alpha = \frac{R_x}{R} = 0.394 \Rightarrow \alpha = 66.8°.$$

The resultant R and its components are shown in Figure 2.1b.

2.1.2 Compounding forces graphically; force polygon

If all the forces are in the same plane, the vector addition can easily be performed graphically by repeatedly implementing the *parallelogram rule* (see Section 1.3.4) or by drawing a so-called *force polygon*. Two examples are given below.

Example 1
Figure 2.2 shows the result of the graphical approach for the example in the previous section. First, we determine the resultant of F_1 and F_2, after which we compound it with F_3. Using the square canvas the resultant R is

$$R = \sqrt{58}\ \text{kN}.$$

When compounding forces graphically, it is not necessary to draw all the parallelograms fully. A drawing in which all the forces with magnitudes and directions are drawn behind one another suffices, as shown in Figure 2.3. This type of drawing is called a *force polygon*.

If, as in the example, the starting point A (the tail of the arrow for the first force F_1) does *not* coincide with the end point B (the point of the arrow for the last force F_2), one refers to an *open force polygon*.

The starting point and the end point of the force polygon determine the direction and magnitude of the resultant R for all forces: the arrow for R runs from starting point A to end point B (see Figure 2.4). The magnitude and direction of R can be *measured* or *calculated* using the drawing. If you look closely, you will recognise the force polygon in Figure 2.2.

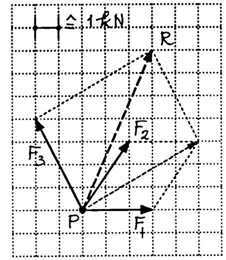


Figure 2.2 Graphical representation of the vector addition by repeatedly applying the parallelogram rule.

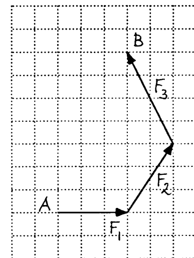


Figure 2.3 By placing the forces F_1 to F_3 head-tail behind each other you get an open force polygon.

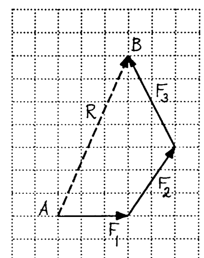


Figure 2.4 Graphical representation of the resultant using a force polygon.

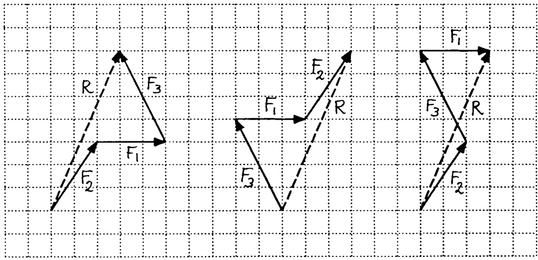


Figure 2.5 The end result is not influenced by the order in which the forces in a force polygon are exerted.

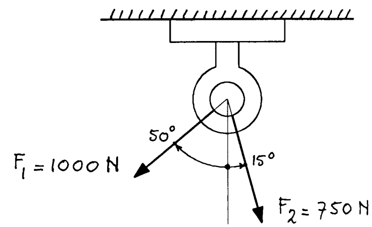


Figure 2.6 A ring subject to two forces.

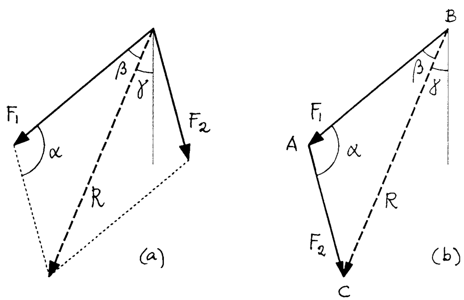


Figure 2.7 The forces drawn to scale in (a) a parallelogram and (b) force polygon.

Figure 2.5 shows all the forces in changing order in a force polygon. The order clearly does not influence the end result (the vector addition is *associative* and *commutative*, see Section 1.3.7).

Example 2
Two forces F_1 and F_2 are acting on the ring in Figure 2.6. Their directions are shown in the figure. The forces are not shown to scale.

Question:
Find the magnitude and direction of the resultant force on the ring if

$$F_1 = 1000 \text{ N},$$

$$F_2 = 750 \text{ N}.$$

Solution:
In Figure 2.7, the forces have been drawn to scale, with 1 cm $\hat{=}$ 250 N. Using the parallelogram rule in Figure 2.7a or the force polygon in Figure 2.7b, we can *construct* the resultant R. Through *measuring* we find that R has a length of approximately 5.95 cm, so that

$$R \approx 5.95 \times 250 \text{ N} = 1488 \text{ N}.$$

With a protractor, we find that the line of action of R makes an angle γ of approximately 22.5° with the vertical.

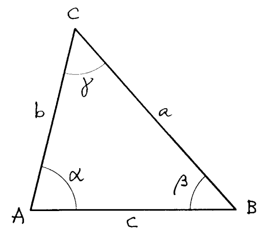
Check:
The magnitude and direction of the resultant R can also be *calculated* from the force triangle ABC. In doing so, we use the *cosine rule* and the *sine rule*, as shown in Figure 2.8.

In the triangle ABC in Figure 2.7b

$$\alpha = 180° - (50° + 15°) = 115°.$$

Using the cosine rule we find that

$$
\begin{aligned}
R &= \sqrt{F_1^2 + F_2^2 - 2F_1F_2\cos\alpha} \\
&= \sqrt{(1000\ \mathrm{N})^2 + (750\ \mathrm{N})^2 - 2 \times (1000\ \mathrm{N})(750\ \mathrm{N})\cos 115^\circ} \\
&= \sqrt{2.196 \times 10^6\ \mathrm{N}^2} = 1482\ \mathrm{N}.
\end{aligned}
$$



cosine rule: $c = \sqrt{a^2 + b^2 - 2ab\cos\gamma}$

sine rule: $\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}$

Figure 2.8 The cosine and sine rules.

The angle β in triangle ABC can be calculated using the sine rule:

$$
\frac{R}{\sin\alpha} = \frac{F_2}{\sin\beta},
$$

$$
\sin\beta = \frac{F_2}{R}\sin\alpha = \frac{750\ \mathrm{kN}}{1482\ \mathrm{kN}}\sin 115^\circ = 0.459
$$

so that

$$
\beta = 27.3^\circ.
$$

The angle γ that the resultant R makes with the vertical is therefore

$$
\gamma = 50^\circ - \beta = 22.7^\circ.
$$

The values measured in Figure 2.7 correspond well with the values we have calculated.

2.1.3 Resolving a force in two given directions analytically

In a plane, we can resolve a force F into two components with given lines of action.

Example
The force $F = \sqrt{34}$ kN in Figure 2.9a has to be resolved into two forces F_a and F_b with the given lines of action a and b.

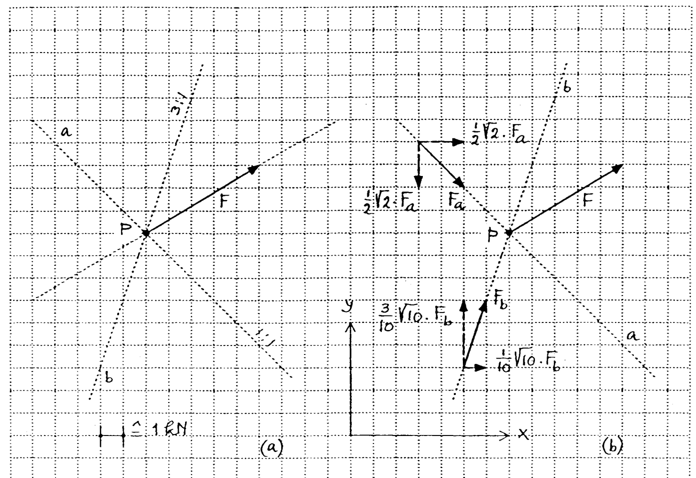


Figure 2.9 The force F in P has to be resolved into two components with given lines of action a and b.

Solution:
Figure 2.9b shows the forces F_a and F_b on the lines of action a and b. Along a and b the directions of the forces can be chosen freely. In the analytical approach, we calculate F_a and F_b on the basis of the condition that the sum of the components from F_a and F_b is equal to the corresponding component of F in each of the coordinate directions

$$\begin{aligned} F_{x;\mathrm{a}} + F_{x;\mathrm{b}} &= F_x, \\ F_{y;\mathrm{a}} + F_{y;\mathrm{b}} &= F_y. \end{aligned} \tag{a}$$

For the components of F in the coordinate system shown in Figure 2.9b

$$F_x = 5\ \mathrm{kN}; \quad F_y = 3\ \mathrm{kN}$$

and for the components of F_a and F_b respectively:

$$F_{x;\mathrm{a}} = \tfrac{1}{2}\sqrt{2} \times F_\mathrm{a}; \qquad F_{y;\mathrm{a}} = -\tfrac{1}{2}\sqrt{2} \times F_\mathrm{a},$$

$$F_{x;\mathrm{b}} = \tfrac{1}{10}\sqrt{10} \times F_\mathrm{b}; \qquad F_{y;\mathrm{b}} = +\tfrac{3}{10}\sqrt{10} \times F_\mathrm{b}.$$

Substitution in (a) gives two equations with F_a and F_b as unknowns:

$$\tfrac{1}{2}\sqrt{2} \times F_\mathrm{a} + \tfrac{1}{10}\sqrt{10} \times F_\mathrm{b} = 5\ \mathrm{kN},$$

$$-\tfrac{1}{2}\sqrt{2} \times F_\mathrm{a} + \tfrac{3}{10}\sqrt{10} \times F_\mathrm{b} = 3\ \mathrm{kN}.$$

The solution is

$$F_\mathrm{a} = 3\sqrt{2}\ \mathrm{kN},$$

$$F_\mathrm{b} = 2\sqrt{10}\ \mathrm{kN}.$$

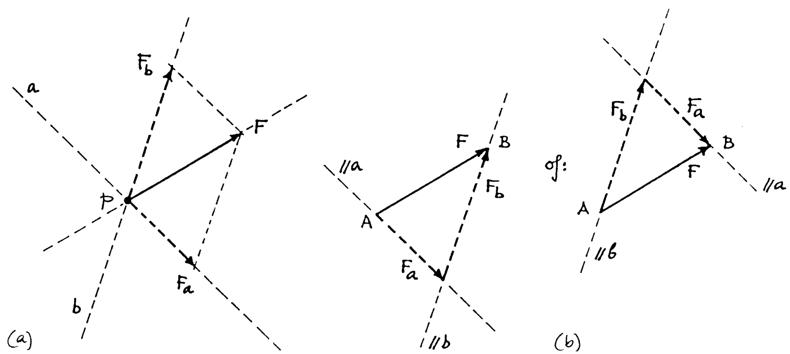


Figure 2.10 Resolving a force in two given directions graphically using (a) the parallelogram rule and (b) a force polygon.

The fact that F_a and F_b are both positive means that they act in the directions we chose in Figure 2.9b. If we had chosen the directions of F_a and F_b in the opposite sense, we would have found that F_a and F_b were negative.

2.1.4 Resolving a force in two given directions graphically

A force F can be resolved graphically into two components F_a and F_b, with given lines of action a en b using the *parallelogram rule* in Figure 2.10a or the *force polygon* in Figure 2.10b. The graphical approach has the advantage that you can at once see in which directions the components F_a and F_b are working.

Example
A force $F = 30$ kN acts on the trestle in Figure 2.11a. This has to be resolved into the components F_a and F_b with lines of action a and b.

Solution:
In Figure 2.12, the force has been resolved using a force polygon. The force scale is 1 cm $\hat{=}$ 5 kN. By *measuring*, you find (in the force polygon) F_a and F_b have lengths 5.7 cm and 6.3 cm respectively, so that

$$F_a \approx 5.7 \times 5 \text{ kN} = 28.5 \text{ kN},$$

$$F_b \approx 6.3 \times 5 \text{ kN} = 31.5 \text{ kN}.$$

In Figure 2.11b forces F_a and F_b are shown as they act on the trestle. It is clear that force F_a is pulling on the beam while force F_b is pushing against it. Together, they exert the same load as the single force F. The forces F_a and F_b are *statically equivalent* (equal in an equilibrium consideration) to their resultant F.

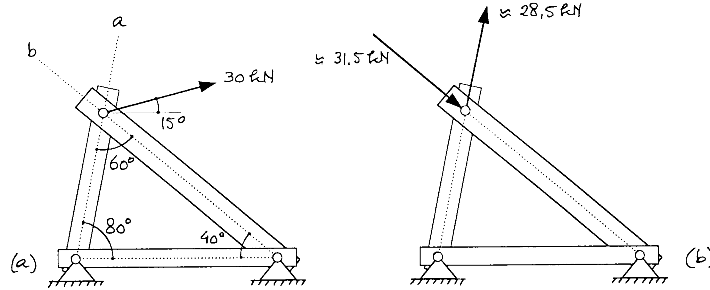


Figure 2.11 A trestle is loaded by a (tensile) force of 30 kN that has to be resolved into components along the lines a and b.

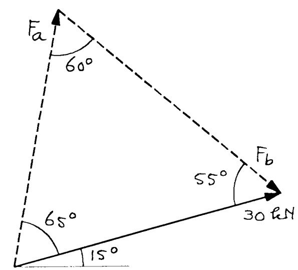


Figure 2.12 The forces drawn to scale in a force polygon.

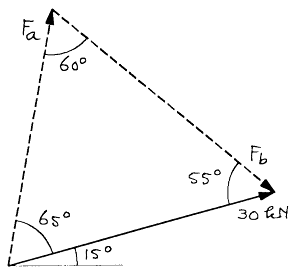


Figure 2.12 The forces drawn to scale in a force polygon.

Check: The magnitude of F_a and F_b can also be *calculated* from the force triangle by using the *sine rule* (see Figure 2.12):

$$\frac{F_\mathrm{a}}{\sin 55^\circ} = \frac{F_\mathrm{b}}{\sin 65^\circ} = \frac{F}{\sin 60^\circ}$$

so that:

$$F_\mathrm{a} = F \cdot \frac{\sin 55^\circ}{\sin 60^\circ} = 28.4 \text{ kN},$$

$$F_\mathrm{b} = F \cdot \frac{\sin 65^\circ}{\sin 60^\circ} = 31.4 \text{ kN}.$$

The values *measured* in Figure 2.12 correspond closely to the *calculated* values.

2.2 Forces in space

If not all the forces are in the same plane, the analytical approach is generally simpler than the graphical approach. For example, for the components R_x, R_y and R_z of the resultant $\vec{R}$:

$$R_x = \sum F_x,$$

$$R_y = \sum F_y,$$

$$R_z = \sum F_z.$$

The summation symbol means that all the forces exerted on the particle have to be added together.

For the magnitude R of $\vec{R}$ therefore

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}.$$

The direction of $\vec{R}$ is determined by the angles α_x, α_y and α_z which $\vec{R}$ makes with respectively the x, y and z axis, see Figure 2.13:

$$\cos\alpha_x = \frac{R_x}{R}; \quad \cos\alpha_y = \frac{R_y}{R}; \quad \cos\alpha_z = \frac{R_z}{R}.$$

The quantities $\cos\alpha_x$, $\cos\alpha_y$ and $\cos\alpha_z$ are called the *direction cosines*. Regardless of the direction of $\vec{R}$, the angles α_x, α_y and α_z are always between 0° and 180°.

When defining a vector, such as the force $\vec{R}$, we need three numbers (and a unit). The three numbers could be the values of the three components R_x, R_y and R_z, or, for example, the magnitude of $\vec{R}$ and two of the three direction cosines. In the latter case, the third direction cosine is given by the other two as

$$\cos^2\alpha_x + \cos^2\alpha_y + \cos^2\alpha_z = 1.$$

Working with forces in space is illustrated using two examples. The first example relates to resolving a force into its components (Section 2.2.1). The second example relates to compounding forces (Section 2.2.2).

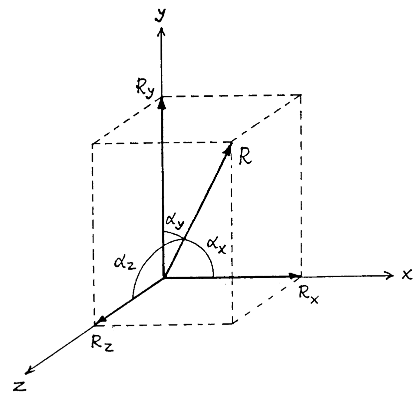


Figure 2.13 The resultant R, with its components R_x, R_y, R_z and the angles α_x, α_y, α_z that R makes with the coordinate axes.

2.2.1 Resolving a force into its components

In order to be able to resolve a force into its x, y and z component, we first have to calculate the direction cosines. This is illustrated by means of an example.

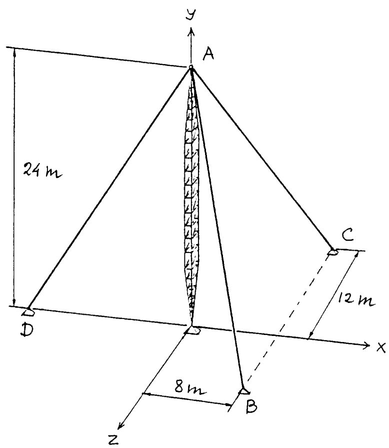


Figure 2.14 A secured mast.

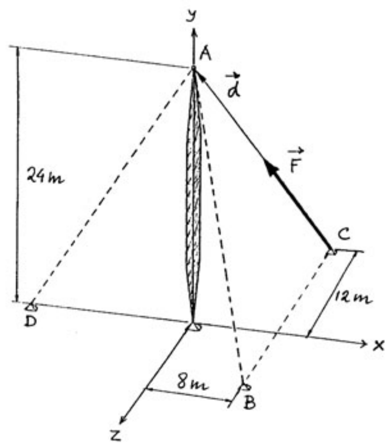


Figure 2.15 The (tensile) force $\vec{F}$ that the rope AC exerts on the foundation block in C.

Example
In Figure 2.14, a mast is being kept upright by three ropes. Rope AC is subject to a tensile force of 35 kN.

Question:
Find the x, y and z component of force $\vec{F}$ that the rope exerts on the foundation block in C (see Figure 2.15).

Note that here we use the *formal vector notation*.

Solution:
The force $\vec{F}$ that is working on the foundation block has the same direction as the vector **CA** (directed from C to A). This vector, which indicates the direction of $\vec{F}$ is hereby referred to as $\vec{d}$,[1] see Figure 2.15. $\vec{F}$ and $\vec{d}$ have the same direction cosines, so that

$$\cos\alpha_x = \frac{F_x}{F} = \frac{d_x}{d},$$

$$\cos\alpha_y = \frac{F_y}{F} = \frac{d_y}{d},$$

$$\cos\alpha_z = \frac{F_z}{F} = \frac{d_z}{d}.$$

Figure 2.15 shows that

$$d_x = -8 \text{ m},$$

$$d_y = +24 \text{ m},$$

$$d_z = +12 \text{ m}.$$

The x component of $\vec{d}$ is negative as it is pointing in the negative x

[1] Remember the *d* of *direction*.

direction. The magnitude (length) d of $\vec{d}$ is

$$d = \sqrt{d_x^2 + d_y^2 + d_z^2} = \sqrt{(-8\text{ m})^2 + (24\text{ m})^2 + (12\text{ m})^2} = 28\text{ m}.$$

Using this information, we can calculate the components of $\vec{F}$:

$$F_x = F\frac{d_x}{d} = (35\text{ kN}) \times \frac{-8\text{ m}}{28\text{ m}} = -10\text{ kN},$$

$$F_y = F\frac{d_y}{d} = (35\text{ kN}) \times \frac{24\text{ m}}{28\text{ m}} = +30\text{ kN},$$

$$F_z = F\frac{d_z}{d} = (35\text{ kN}) \times \frac{12\text{ m}}{28\text{ m}} = +15\text{ kN}.$$

Figure 2.16 shows the components of $\vec{F}$ as they are working on the foundation block.

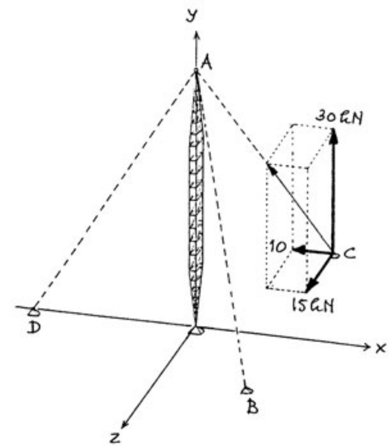


Figure 2.16 The components of $\vec{F}$.

2.2.2 Compounding forces

In order to determine the resultant of the forces on a particle in space, we first resolve all the forces into their x, y and z component, and then add all the associated components together. This is illustrated in an example.

Example

Figure 2.17 shows the schematised situation in a salvage operation. A shows the wreckage of a crashed lorry on a slope. People are trying to salvage the wreckage using cables AB and AC and winches in B and C. Cable AB is pulling on the wreckage with a force of magnitude $F_1 = 7.5$ kN; cable AC is pulling on the wreckage with force of magnitude $F_2 = 10$ kN.

Question:

Find the resultant force being exerted by the cables on the wreckage.

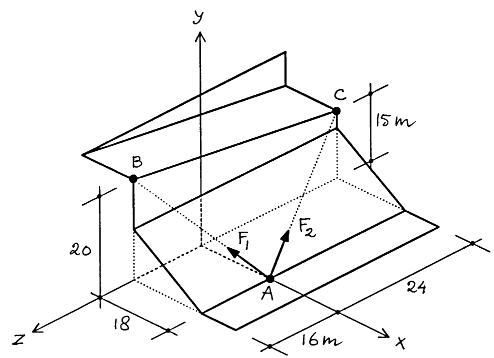


Figure 2.17 Schematisation of the situation surrounding a salvage operation. The wreckage of a crashed lorry is located on a slope at A, and is being salvaged with the cables AB and AC and winches in B and C.

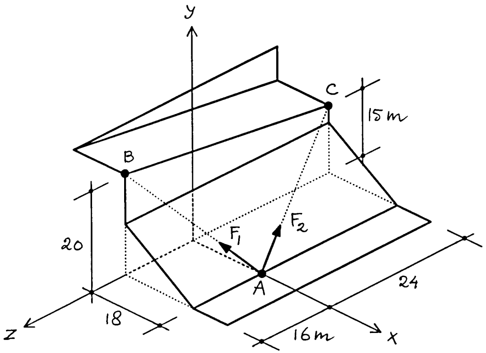


Figure 2.17 Schematisation of the situation surrounding a salvage operation. The wreckage of a crashed lorry is located on a slope at A, and is being salvaged with the cables AB and AC and winches in B and C.

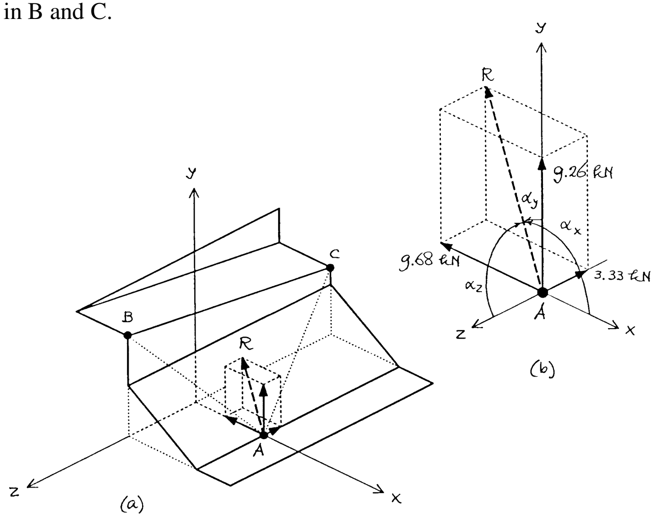


Figure 2.18 (a) The components of the resultant $\vec{R}$ that the cables exert on the wreckage and (b) the angles α_x, α_y, α_z that this force makes with the coordinate axes.

Table 2.1 Calculation of the components of the resultant $\vec{R}$.

	d_x	d_y	d_z	d	F	F_x	F_y	F_z
$\vec{d}_1 = \mathbf{AB}, \vec{F}_1$	–18	+20	+16	31.30	7.5	–4.31	+4.79	+3.83
$\vec{d}_2 = \mathbf{AC}, \vec{F}_2$	–18	+15	–24	33.54	10	–5.37	+4.47	–7.16
					$\sum =$	–9.68	+9.26	–3.33

Solution:
First, we have to resolve the forces $\vec{F}_1$ and $\vec{F}_2$ into their components in the same way as described in the previous section: $F_x = F d_x/d$, etc. The calculation is shown in Table 2.1 (d in m and F in kN).

The components of the resultant force $\vec{R}$ on the wreckage are therefore

$$R_x = \sum F_x = -9.68\ \text{kN},$$

$$R_y = \sum F_y = +9.26\ \text{kN},$$

$$R_z = \sum F_z = -3.33\ \text{kN},$$

so that

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

$$= \sqrt{(-9.68\ \text{kN})^2 + (9.26\ \text{kN})^2 + (-3.33\ \text{kN})^2} = 13.80\ \text{kN}.$$

Figure 2.18a shows the components of the resultant $\vec{R}$ as they act on the wreckage. Figure 2.18b shows the angles that the resultant makes with the coordinate axes. The angles are calculated as follows:

$$\cos\alpha_x = \frac{R_x}{R} = \frac{-9.68\ \text{kN}}{13.80\ \text{kN}} = -0.701 \Rightarrow \alpha_x = 134.5°,$$

$$\cos\alpha_y = \frac{R_y}{R} = \frac{+9.26 \text{ kN}}{13.80 \text{ kN}} = +0.671 \Rightarrow \alpha_y = 47.8°,$$

$$\cos\alpha_z = \frac{R_z}{R} = \frac{-3.33 \text{ kN}}{13.80 \text{ kN}} = -0.241 \Rightarrow \alpha_z = 104.0°.$$

2.3 Equilibrium of a particle

According to Newton's first law, if the resultant of all the forces on a particle is zero, it will remain at rest if it was at rest originally. This means that the particle is in *equilibrium*.

If the particle is to be in equilibrium, then

$$\sum \vec{F} = \vec{0}.$$

This (vector) equation is called the *equilibrium condition for the particle*. The summation symbol means that all the forces acting on the particle have to be added together.

2.3.1 Graphical: closed force polygon

In a force polygon, the equilibrium condition means that all the forces acting on the particle have to form a *closed force polygon*: the resulting force is then zero.

An example of this is given in Figure 2.19. The four coplanar forces acting on the particle form a closed force polygon; the particle is therefore in equilibrium.

In Figure 2.20a, the particle is subject to the three coplanar forces F_1, F_2 and F_3. Together they form an open force polygon. The particle is not

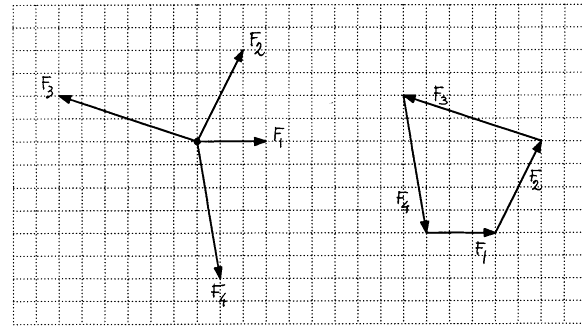


Figure 2.19 If the forces on a particle form a closed force polygon then the particle is in equilibrium.

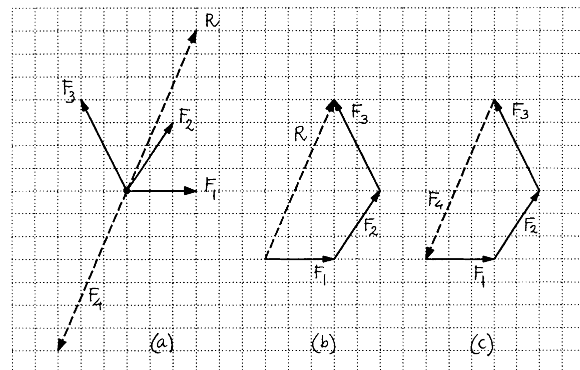


Figure 2.20 The force F_4 that closes the force polygon from F_1 to F_3 – and ensures equilibrium – is equal and opposite to the resultant R from F_1 to F_3.

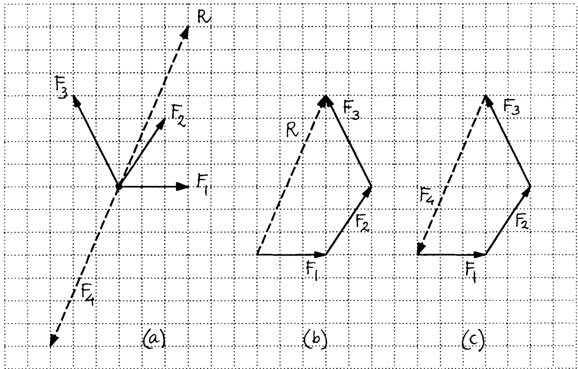


Figure 2.20 The force F_4 that closes the force polygon from F_1 to F_3 – and ensures equilibrium – is equal and opposite to the resultant R from F_1 to F_3.

in equilibrium. Due to the resultant force R (Figure 2.20b), a change in momentum occurs (Newton's second law): the resultant R on the particle will cause its velocity to change.

The force F_4 that closes the open force polygon (Figure 2.20c) is the force that brings the given forces into equilibrium. F_4 has the same magnitude, but opposite direction to the resultant R of the forces F_1, F_2 and F_3.

2.3.2 Analytical: equilibrium equations

A particle is and stays at rest if the resultant of all the forces acting on the particle is zero.

The *vector equation* for the force equilibrium

$$\sum \vec{F} = \vec{0}$$

resolves into three *algebraic equations* in space:

$$\sum F_x = 0,$$
$$\sum F_y = 0,$$
$$\sum F_z = 0.$$

These are referred to as the three equations for the *force equilibrium* of the particle in the x, y, and z direction respectively. If a particle is to be in equilibrium, each of the three components of the resultant must be zero.

If all the forces are coplanar, the number of equations for the force equilibrium is reduced to two. This is illustrated by two examples.

Example 1
Investigate whether the particle P in Figure 2.21, subject to the forces F_1 to F_4 in the xy plane is in equilibrium.

Solution (forces in kN):

$$R_x = \sum F_x = F_{x;1} + F_{x;2} + F_{x;3} + F_{x;4} = +6 - 2 - 1 - 3 = 0,$$

$$R_y = \sum F_y = F_{y;1} + F_{y;2} + F_{y;3} + F_{y;4} = -1 - 4 + 6 - 1 = 0.$$

The particle is in equilibrium since the resultant is zero: the forces F_1 to F_4 therefore together form an *equilibrium system.*[1]

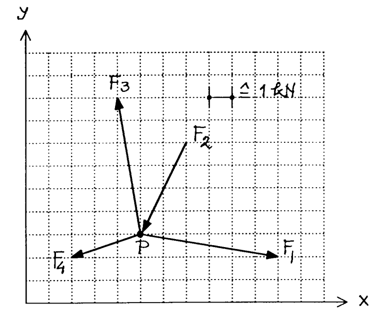


Figure 2.21 Is particle P in equilibrium?

Example 2
In Figure 2.22, a container with mass 880 kg is being unloaded. Forces F_1, F_2 and F_3, act on joint A. Here, F_3 stands for the weight of the container. In the figure, the system is in equilibrium. The gravitational field strength is $g = 10$ N/kg.

Question:
How large are F_1 and F_2?

Solution:
The weight of the container is

$$F_3 = mg = (880\ \text{kg})(10\ \text{N/kg}) = 8800\ \text{N}.$$

The unknown forces F_1 and F_2 are obtained from the two equations for the force equilibrium in the x and y directions:

$$\sum F_x = F_1 \cdot \sin 3^\circ - F_2 \cdot \cos 20^\circ = 0,$$

$$\sum F_y = F_1 \cdot \cos 3^\circ - F_2 \cdot \sin 20^\circ - (8800\ \text{N}) = 0.$$

Here we have two equations with two unknowns, namely the magnitude of

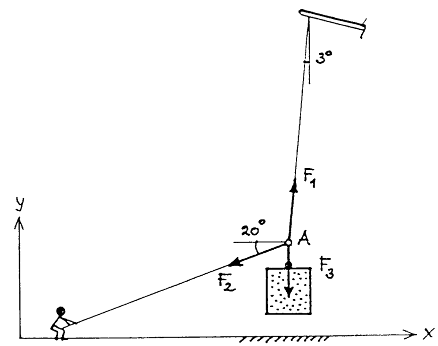


Figure 2.22 The forces acting on joint A when unloading a container.

1 It is incorrect to say that "the forces neutralise one another" as the forces continue to exist.

the forces F_1 and F_2. We can write the equations as

$$0.0523 \times F_1 - 0.9397 \times F_2 = 0,$$

$$0.9986 \times F_1 - 0.3420 \times F_2 = 8800 \text{ N},$$

so that:

$$F_1 = 8984 \text{ N},$$

$$F_2 = 500 \text{ N}.$$

Alternative solution:
We also can calculate the forces on the basis of the closed force polygon for the equilibrium of junction A. A rough sketch of the force polygon, such as that in Figure 2.23, suffices.

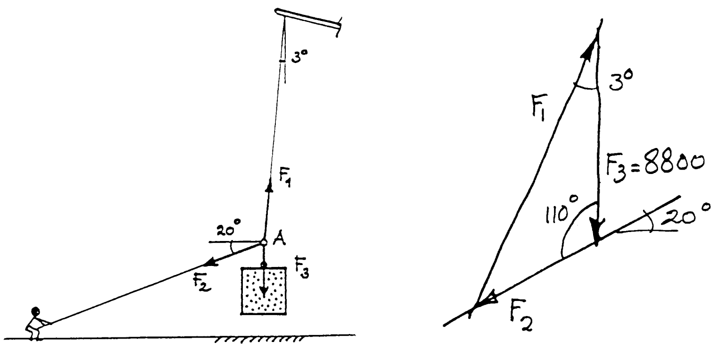


Figure 2.23 A rough sketch of the closed force polygon for the force equilibrium of joint A.

According to the sine rule this gives

$$\frac{F_1}{\sin 110^\circ} = \frac{F_2}{\sin 3^\circ} = \frac{F_3}{\sin(180^\circ - 110^\circ - 3^\circ)} = \frac{F_3}{\sin 67^\circ}.$$

With $F_3 = 8800$ N this means that

$$F_1 = F_3 \cdot \frac{\sin 110^\circ}{\sin 67^\circ} = 8983 \text{ N},$$

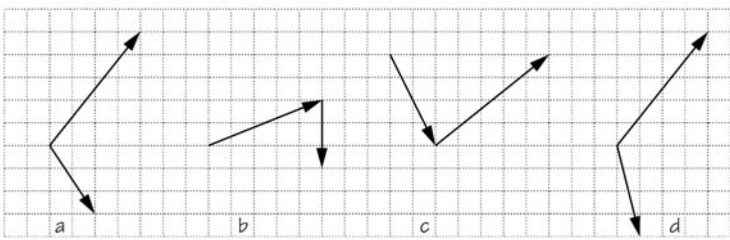
$$F_2 = F_3 \cdot \frac{\sin 3^\circ}{\sin 67^\circ} = 500 \text{ N}.$$

The fact that F_1 is 1 N less than before is the result of rounding off the goniometric function values (to four decimal places) in the previous solution.

2.4 Problems

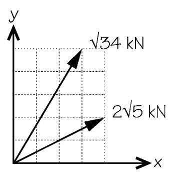
Compounding coplanar forces (Sections 2.1.1 and 2.1.1)

2.1 Which combination of forces has the smallest resultant?



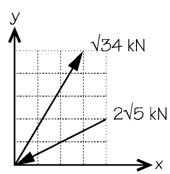
2.2 Two forces are acting on a particle, of which the magnitude and direction are shown in the figure.

Question:
Determine the magnitude and direction of the resultant for both forces:
a. graphically (choose a scale of 5 mm ≡ 1 kN);
b. analytically.



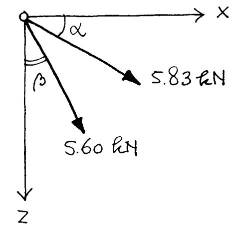
2.3 Two forces are acting on a particle, of which the magnitude and direction are shown in the figure. The resultant is R.

Question:
Calculate the components R_x and R_y:
a. analytically;
b. graphically (choose a scale of 5 mm ≡ 1 kN).



2.4 Two forces are acting on a particle. The values are included in kN. The directions are: $\tan\alpha = 3/5$ and $\tan\beta = 1/2$.

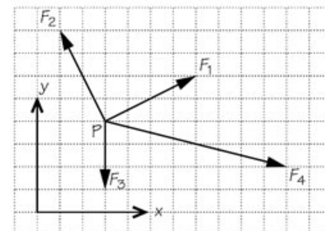
Question:
a. Calculate the resultant of the two forces graphically.
b. Check the result analytically.



2.5 Four forces are acting on particle P. The directions of the forces are shown in the figure. The forces are not drawn to scale.

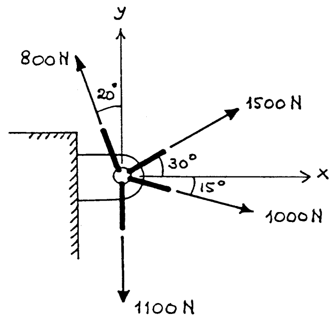
$$F_1 = 15\sqrt{5}\text{ kN},\ \ F_2 = 10\sqrt{5}\text{ kN},\ \ F_3 = 30\text{ kN and } F_4 = 5\sqrt{17}\text{ kN}.$$

Question:
Determine the magnitude and direction of the resultant of these four forces:
a. graphically (choose a scale of 1 mm ≡ 1 kN);
b. analytically.



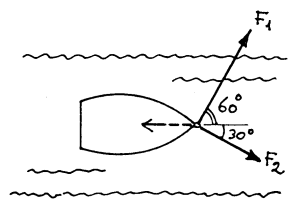
2.6 A number of coplanar cables are attached to a console. They exert the forces as shown in the figure.

Question:
Determine the magnitude and direction of the resultant force on the console.



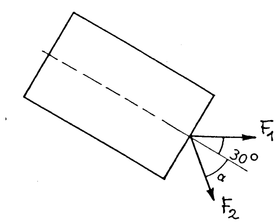
2.7 A boat is kept in the middle of a river by means of two cables. The direction in which the boat pulls on both cables is shown by the dashed arrow.

Question:
Calculate F_2 if $F_1 = 1000$ N.



2.8 A vessel is being pulled in the direction of its longitudinal axis by two tugs with a force of 20 kN. The directions of the forces F_1 and F_2 as exerted by the tow lines on the vessel are shown in the figure.

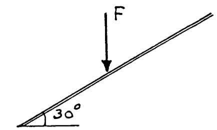
Question:
a. Determine F_1 and F_2 if $\alpha = 45°$.
b. The value of α whereby F_2 is minimal. How large in this case are F_1 and F_2?



Resolving coplanar forces (Sections 2.1.3 and 2.1.4)

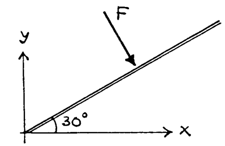
2.9 A vertical force F on a sloping roof has to be resolved into components perpendicular to and parallel to the surface of the roof.

Question:
Determine these components.

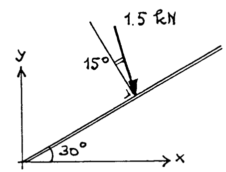


2.10 A force F perpendicular to a sloping roof has to be resolved into the components F_x and F_y, parallel to the x and y axis respectively.

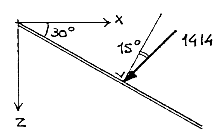
Question:
Determine these components.



2.11 Resolve the force of 1.5 kN on the surface of the roof into:
a. components perpendicular to and parallel to the surface of the roof;
b. components in the x and y direction.

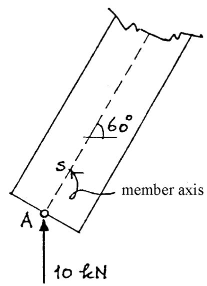


2.12 Resolve the force of 1414 N on the surface of the roof into:
a. components perpendicular to and parallel to the surface of the roof;
b. components in the x and z direction.



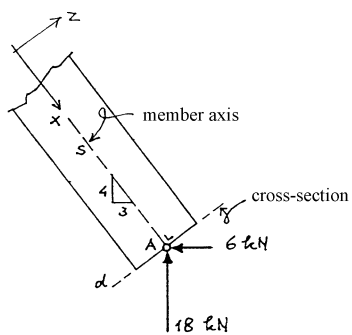
2.13 Resolve the force of 10 kN shown at A into components perpendicular to and parallel to the bar axis s:

a. graphically (mention the scale used for the forces);
b. analytically.

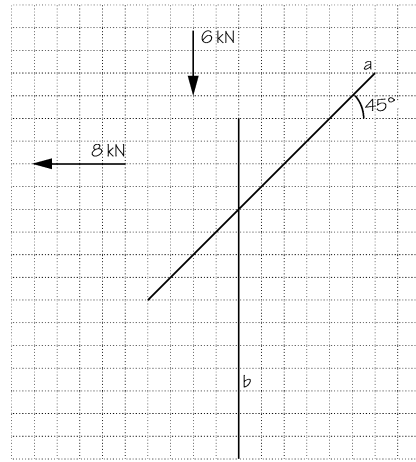


2.14 At the end A of the bar, the forces of 6 and 18 kN as shown in the diagram are exerted on the cross-section d.

Question:
Resolve the resulting force at A into:
a. components perpendicular to and parallel to cross-section d;
b. the components F_x and F_y.



2.15 The resultant R of the two forces shown is resolved into the components R_a and R_b, parallel to the directions a and b.

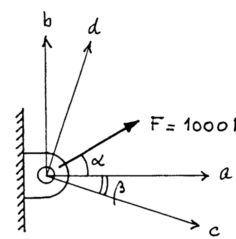


Question:
a. Draw the lines of action of R_a and R_b.
b. Determine R_a and R_b.
c. Draw R_a and R_b on their lines of action (in the directions in which they are acting) and record their values next to them.

2.16 A console is subject to a force $F = 1000$ N.

In addition: $\alpha = \beta = 30°$.

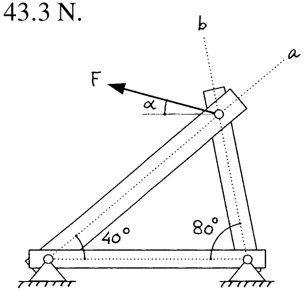
Question:
Resolve these forces into components in respectively:
a. the a and b direction;
b. the c and d direction;
c. the a and c direction;
d. the b and c direction.



2.17 As 2.16 but now with $\alpha = 20°$ and $\beta = 30°$.

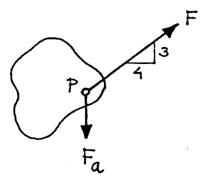
2.18 A trestle is subject to a tensile force F with an angle α.

Question:
Resolve this force into the components F_a and F_b along the lines of action a and b, for:
a. $\alpha = 20°$ and $F = 38.0$ N;
b. $\alpha = 35°$ and $F = 36.8$ N;
c. $\alpha = 50°$ and $F = 43.3$ N.



2.19 A force F at P is resolved into the components F_a and F_b.

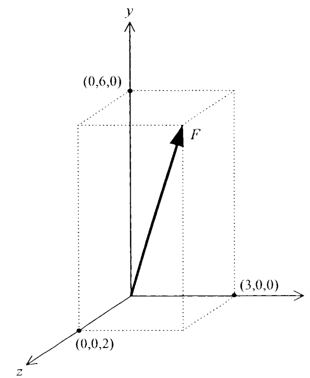
Question:
Determine the magnitude and direction of F_b if $F = 5$ kN and $F_a = 1$ kN:
a. graphically (choose a scale of 5 mm ≡ 1 kN);
b. analytically.



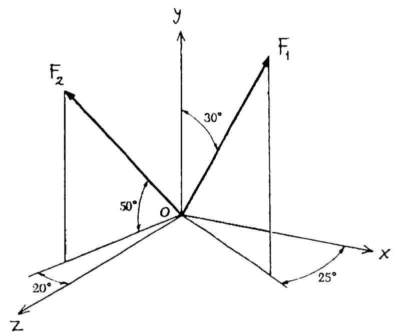
2.20 As 2.19, but now with $F = F_a = 5$ kN.

Resolving a force in space into its components (Section 2.2.1)

2.21 *Question*:
a. Determine the components in the x, y and z direction for force F shown when $F = 35$ kN.
b. Calculate the angles α_x, α_y and α_z that F makes with respectively the x, y and z axis.

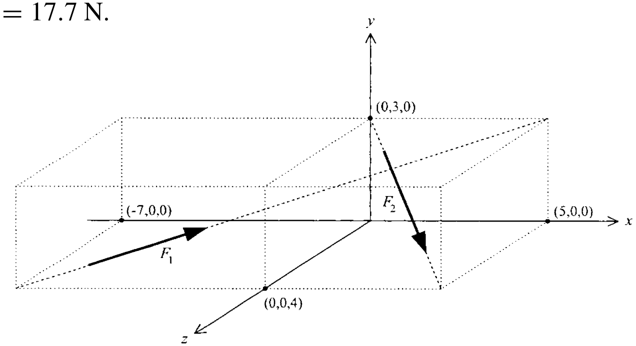


2.22 *Question*:
For the forces shown, determine the components in the x, y and z direction and the angles α_x, α_y and α_z that they make with respectively the x, y and z axis:
a. $F_1 = 1250$ N.
b. $F_2 = 1500$ N.



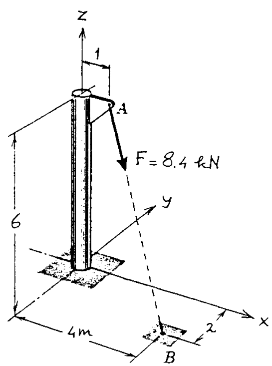
2.23 *Question*:
For the forces shown, determine the components in the x, y and z direction and the angles α_x, α_y and α_z that they make with respectively the x, y and z axis:

a. $F_1 = 6.5$ N;
b. $F_2 = 17.7$ N.



2.24 Cable AB exerts a tensile force $F = 8.4$ kN at A on the console.

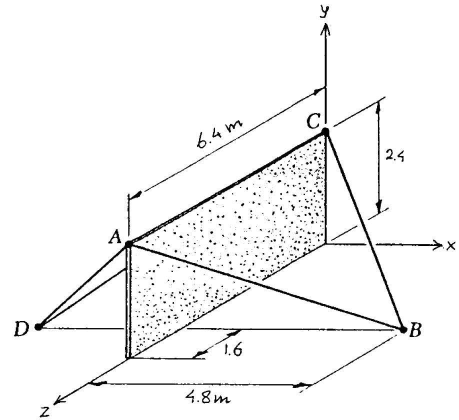
Question:
Determine the x, y and z component of the force at A.



2.25 A prefabricated concrete wall is temporarily kept in place by cables.

Question:

a. If there is a tensile force of 3.5 kN in cable AB, determine the components of the force that cable AB exerts on the wall in A.
b. If there is a tensile force of 4.5 kN in cable BC, determine the components of the force that cable BC exerts on the wall in C.

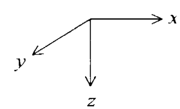


Resultant of the forces on a particle in space (Section 2.2.2)

2.26 The components of a force are

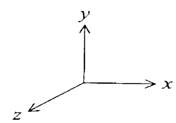
$$F_x = +7.5 \text{ kN}, \quad F_y = +17.5 \text{ kN} \quad \text{and} \quad F_z = -10 \text{ kN}.$$

Question:
Determine the magnitude and direction of this force.



2.27 The components are given in the table below for three forces F_1, F_2 and F_3.

	F_x (kN)	F_y (kN)	F_z (kN)
F_1	–60	20	100
F_2	30	50	–80
F_3	90	20	–60



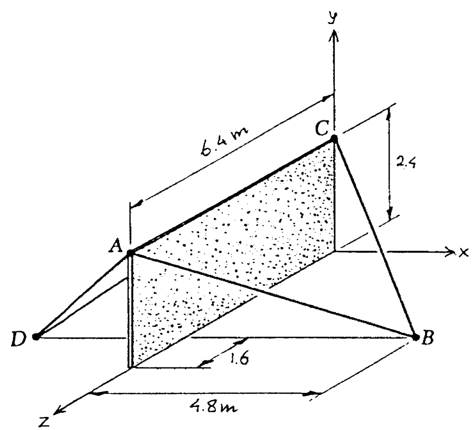
Question:
Determine the magnitude and direction of the resultant of these three forces.

2.28 Three forces F_a, F_b and F_c in the origin O of the xyz coordinate system are aimed respectively at the points A$(-1, 2, 4)$, B$(3, 0, -3)$ en C$(2, -2, 4)$.

Question:
Determine the magnitude and direction of the resultant of these three forces if $F_a = 200$ N, $F_b = 50$ N and $F_c = 150$ N.

2.29 A prefabricated concrete wall is kept in place temporarily by cables. There is a tensile force in the cables AB and BC of respectively 7.0 kN and 6.0 kN.

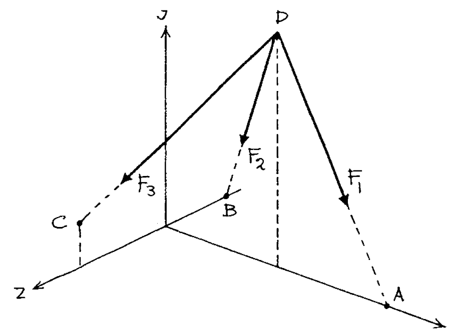
Question:
Determine the magnitude and direction of the force that the cables AB and BC exert jointly on anchor B.



2.30 Three forces are exerted on D$(5, 10, 0)$, namely $F_1 = 3$ kN, $F_2 = 4$ kN and $F_3 = 5$ kN. Coordinates in m.

F_1 is aimed at A$(10, 0, 0)$,
F_2 is aimed at B$(0, 0, -3)$ and
F_3 is aimed at C$(0, 2, 4)$.

Question:
Determine the magnitude and direction of the resultant of these three forces.



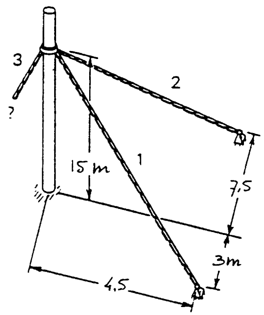
2.31 A force F is acting on the origin O of the xyz coordinate system. The force has an angle of 150° with respect to the z axis. The components in the x and y direction are respectively $F_x = 4$ kN and $F_y = 3$ kN.

Question:
a. Determine F.
b. Determine the component F_z.
c. Determine the direction cosines for F.

2.32 A narrow steel mast is supported at its top by three tight cables. There is a tensile force of 5.9 kN in cables 1 and 2. Cable 3 makes an angle of 30° with the mast. The forces that the three cables exert on the mast have a vertical resultant. Assume that all the forces are aimed at a single point.

Question:

a. Determine the magnitude of the tensile force in cable 3.
b. Determine where cable 3 is anchored at ground level.

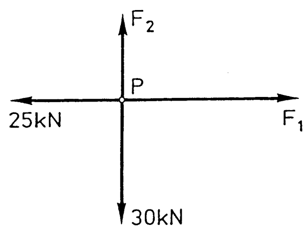


Equilibrium of a particle in a plane (Sections 2.3.1 and 2.3.2)

2.33 Particle P is subject to five forces, four of which are shown. The particle is in equilibrium. $F_1 = 40$ kN and $F_2 = 20$ kN.

Question:
The fifth force is acting:

a. top right;
b. top left;
c. bottom right;
d. bottom left.

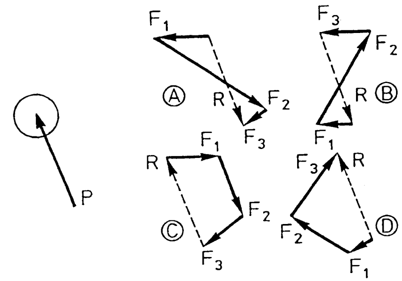


2.34 As 2.33, but now with $F_1 = F_2 = 35$ kN.

2.35 As 2.33, but now with $F_1 = 10$ kN and $F_2 = -15$ kN.

2.36 A particle is subject to three forces F_1, F_2 and F_3 with resultant R. The body is kept in equilibrium by an additional force P.

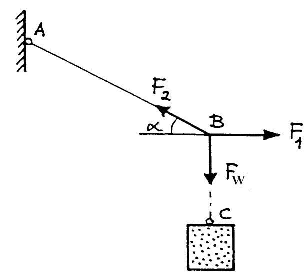
Question:
Which of the force polygons for F_1, F_2 and F_3 and R is correct?



2.37 Cable ABC is carrying a block in C with mass $m = 50$ kg. At point B of the cable the forces F_1, F_2 and F_w are exerted. F_w represents to the weight of the block. The system is in equilibrium.

Question:

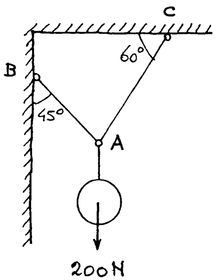
a. Determine F_1 if $\tan\alpha = 0.5$.
b. Determine F_1 as a function of α; represent this in a graph.



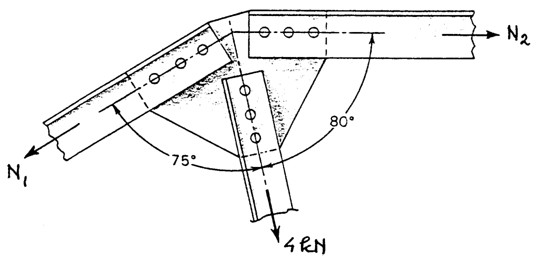
2.38 A lamp with mass 200 N is hung from two wires.

Question:

a. Determine and draw all the forces exerted on joint A.
b. Draw the force polygon for the equilibrium in joint A.



2.39 Three forces are exerted on the joint of a truss. The system is in equilibrium.



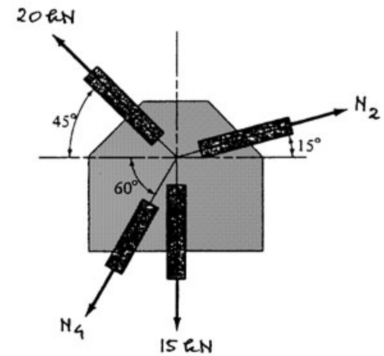
Question:

a. Draw the force polygon for the equilibrium in the joint.
b. Determine the forces N_1 and N_2. Are they tensile or compressive forces?

2.40 Four forces act on the joint of a truss. The system is in equilibrium.

Question:

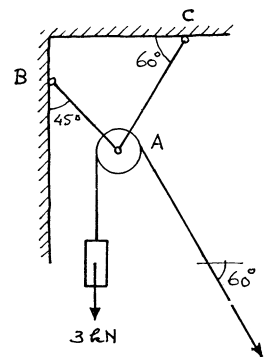
a. Draw the force polygon for the equilibrium of the joint.
b. Determine forces N_2 and N_4. Are they tensile or compressive forces?



2.41 A pulley is hanging on two bars. A weight of 3 kN is hanging on a cable that is fed over the pulley. The tensile force in the cable is equal on both sides of the pulley (this can be shown with knowledge from Chapter 3).

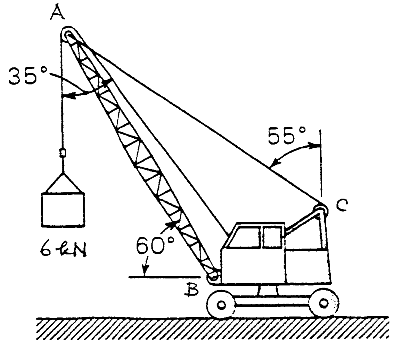
Question:

From the force equilibrium of the pulley, determine the forces that the bars exert on the pulley in the situation shown.



2.42 Top A of the crane is schematised as a particle. The tensile force in the cable is equal on both sides of the pulley (this can be shown with knowledge from Chapter 3). The force that jib AB exerts on particle A has a line of action along AB.

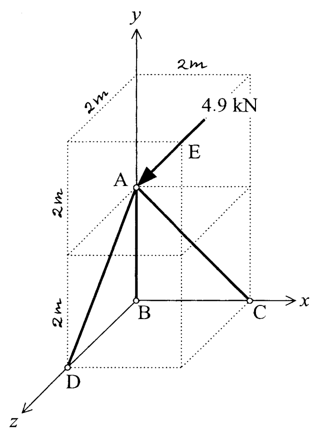
Question:
Draw the particle and determine and draw all the forces that are acting on it. Also draw the force polygon for the equilibrium of the particle.



Equilibrium of a particle in space (Section 2.3.2)

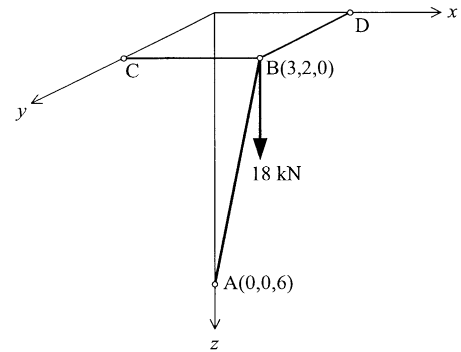
2.43 Three bars joined at A can transfer forces only in the direction of their axes. Joint A is loaded by the force shown of 4.9 kN, with its line of action through E.

Question:
Determine the forces that the bars exert on joint A. Are the forces in the members tensile or compressive forces?



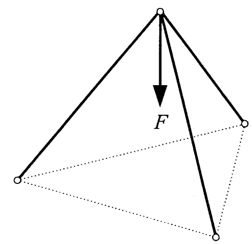
2.44 A derrick AB is mounted as a hinge in A(0, 0, 6) and is supported in B(3, 2, 0) by two horizontal wires BC and BD, parallel to the x and y axis respectively. The vertical loading in B is 18 kN. Coordinates in m.

Question:
Determine the forces exerted by AB, BC and BD on joint B if their lines of action are along the axes shown.



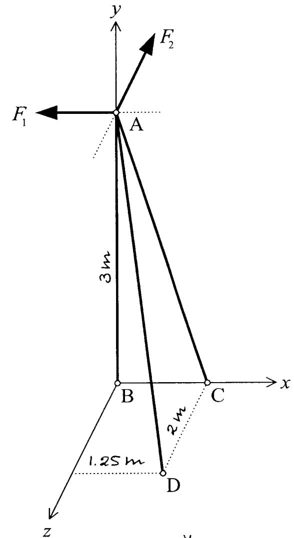
2.45 The hoisting device consists of three 4.5-metre poles. The three feet are in an equilateral triangle with sides of 3.9 metres. The device bears a vertical load of $F = 13$ kN. The poles can transfer forces only in the direction of their axes.

Question:
Determine the compressive forces in the poles.



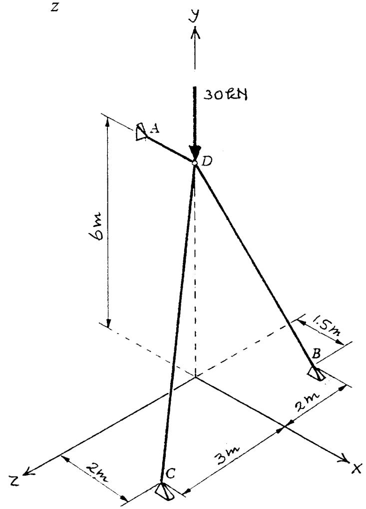
2.46 Three bars linked at A can transfer forces only in the direction of their axes. Joint A is loaded by the two forces shown $F_1 = 3.75$ kN and $F_2 = 5.25$ kN, parallel to the x and z axis respectively.

Question:
Determine the forces that the bars exert on joint A. Are the forces in the members tensile or compressive?



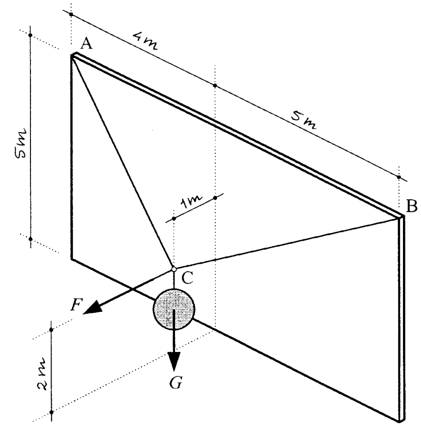
2.47 Three bars linked at D can transfer forces only in the direction of their axes. Joint D is loaded by a vertical force of 30 kN.

Question:
Determine the forces that the bars exert on joint D. Are they tensile or compressive?



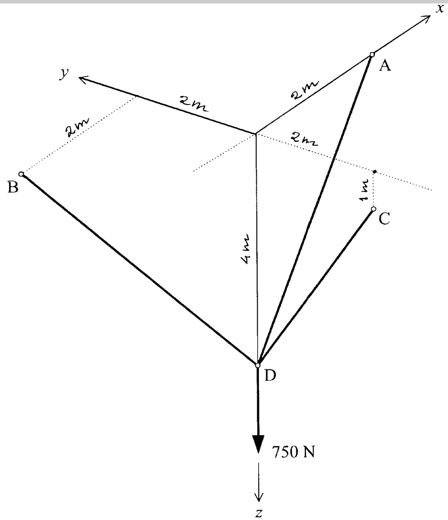
2.48 A load of weight $G = 9$ kN is hanging at C on the cables AC and BC. The cables are joined to the corners of a vertical wall. A horizontal force F, perpendicular to the wall keeps the block in the position shown.

Question:
a. Determine the magnitude of force F on the basis of the equilibrium in joint C.
b. Determine the forces in the cables.



2.49 At D, a weight of 750 N is hanging from three wires. The points of attachment A and B are in the horizontal xy plane; point of attachment C is 1 metre below.

Question:
Determine the forces in the wires.



2.50 The hoisting device consists of three bars joined at A that can transfer forces only in the direction of their axes. Joint A is loaded by a vertical force F. The compressive force may be no larger than 12 kN in any of the bars.

Question:

a. Determine the maximum load the device can bear.
b. Determine the forces in the bars under this maximum load; are they tensile or compressive?