Calculating M, V and N Diagrams

13

In this chapter, we will look at a number of examples for calculating M, V and N diagrams. In the presentation we distinguish between the self-contained structures in Section 13.1, the somewhat more complex compound and associated structures in Section 13.2 and the statically indeterminate structures in Section 13.3. For some of the calculations we will, to prevent repetition, make only a start, and leave it to the reader to work out the answer further. Should you decide to work out the questions yourself, you will notice that there are several ways to arrive at the answer.

13.1 Self-contained structures

In this section we will be determining the M, V, and sometimes the N diagrams for self-contained structures subject to distributed loads.

13.1.1 Beam with triangular load (lintel)

The lintel in Figure 13.1a is supporting the part of the brickwork shown above a door opening. The load on the lintel modelled as a line element is the triangular load in Figure 13.1b, with top value $\hat{q}$. The brick wall is $d = 240$ mm thick. The mass density of the brickwork is $\rho = 1800$ kg/m^3.

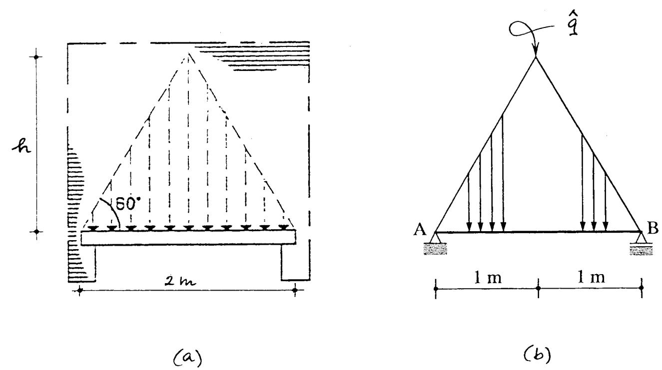


Figure 13.1 (a) A lintel carrying the triangular part of the brickwork. (b) Modelling of lintel and load as a beam with triangular load.

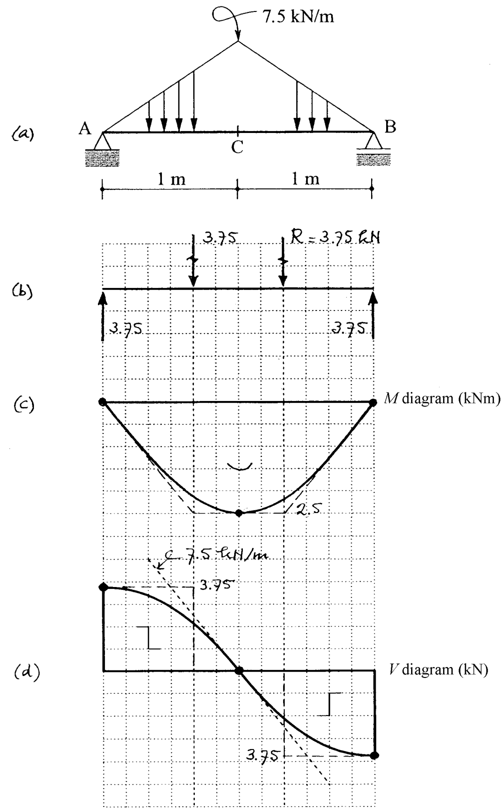


Figure 13.2 (a) Beam with triangular load. (b) The isolated beam with the field loads on AC and BC replaced by their resultants, and the associated support reactions. (c) Bending moment diagram and (d) shear force diagram.

Questions:

a. Determine the top value $\hat{q}$ of the triangular load.

b. For AB, determine and draw the M and V diagrams. At A and B also draw the tangents to the M diagram. How large is the maximum bending moment?

Solution:

a. The height h of the brickwork in Figure 13.1a is

$$h = (1 \text{ m}) \times \tan 60^\circ = 1.732 \text{ m}.$$

With a gravitational field strength of $g = 10$ N/kg, the top value $\hat{q}$ of the triangular load on the lintel is

$$\hat{q} = \rho g h d = (1800 \text{ kg/m}^3)(10 \text{ N/kg})(1.732 \text{ m})(0.240 \text{ m}) \approx 7.5 \text{ kN/m}$$

(see Figure 13.2a).

b. In Figure 13.2b, the distributed loads on AC and BC have been replaced by their resultants R:

$$R = \tfrac{1}{2} \times (1 \text{ m})(7.5 \text{ kN/m}) = 3.75 \text{ kN}.$$

The support reactions are also shown.

In Figures 13.2c and 13.2d, the M and V diagrams due to these resultants are shown by means of dashed lines. This way, we can find the correct values for M and V at A, B and C (shown by means of dots) and the correct slopes of the M diagram.

We can now draw the actual M diagram, a cubic, see the solid line in Figure 13.2c. The maximum bending moment occurs at midspan and is 2.5 kNm.

The actual V diagram is parabolic (see the solid line in Figure 13.2d). The slope of the V diagram is equal to the magnitude of the distributed load. At A and B, the distributed load is zero, and the V diagram has horizontal tangents. At C, the distributed load is largest, and the slope of the V diagram is steepest. The slope is 7.5 kN/m, and is shown separately in Figure 13.2d.

13.1.2 Beam with parabolic distributed load

Beam ABC in Figure 13.3 is supported by a hinge at A and on a roller at B. The beam is loaded by a parabolic distributed load in field AB and a point load of 25 kN at end C of cantilever BC. The longitudinal dimensions of the beam are shown in the figure. The parabolic distributed load can be represented with

$$q(x) = -30\left(\frac{x}{\ell}\right)^2 + 30\left(\frac{x}{\ell}\right)\ \text{kN/m}.$$

Here, $\ell = 10$ m is the length of AB. The dead weight of the beam is not considered in the calculation.

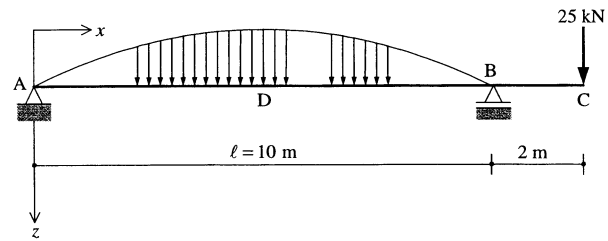


Figure 13.3 Parabolic load over AB on beam ABC.

Questions:

a. Replace the distributed load over AB by its resultant, and draw the M and V diagrams for the entire beam ABC.
b. Draw a (rough) sketch of the actual M and V diagrams for AB. In addition to the deformation symbols in the M and V diagrams, also include the plus and minus signs in the given xz coordinate system.
c. For AB, through consecutive integration, determine the shear force V and the bending moment M as a function of x. Determine the values of V and M at A and B and at the middle D of field AB. At D draw the tangent to the M diagram.
d. Where in AB is the field moment a maximum? It is enough to give a rough indication of the location. Using the M diagram estimate the

value of the maximum field moment. This value need not be determined accurately.

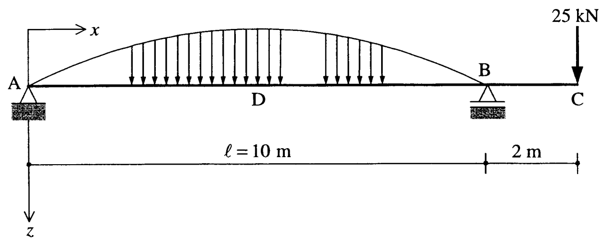


Figure 13.3 Parabolic load over AB on beam ABC.

Solution (units kN and m):
a. With $\ell = 10$ m, for the parabolically distributed load on AB it applies that

$$q(x) = (-0.3x^2 + 3x) \text{ kN/m}$$

if x is expressed in metres. The top of the parabola is at the middle of AB. This is derived from

$$\frac{\mathrm{d}q(x)}{\mathrm{d}x} = -0.6x + 3 = 0 \Rightarrow x = 5 \text{ m}.$$

On the basis of symmetry, the resultant R of the distributed load is acting here. The magnitude of R is equal to the area of the load diagram, and is found by integrating the distributed load:

$$R = \int_0^{10} q(x)\,\mathrm{d}x = \int_0^{10} (-0.3x^2 + 3x)\,\mathrm{d}x = (-0.1x^3 + 1.5x^2)\Big|_0^{10}$$
$$= 50 \text{ kN}.$$

Figure 13.4a shows the resultant R, together with the support reactions at A and B. In Figures 13.4b and 13.4c, the M and V diagrams due to this (concentrated) force R are shown (with dashed lines for AB).

b. The M and V diagrams are correct for the cantilever BC. In field AB, only the values at A and B (shown by means of dots) are correct. In addition, at A and B the dashed M diagram B is tangent to the actual M diagram. There are no other handholds to sketch the M diagram, but we can now certainly make a rough sketch (see the solid line in Figure 13.4b).

The actual V diagram has horizontal tangents at A and B because the distributed load is zero there. This allows us to make a pretty good sketch of the V diagram (see the solid line in Figure 13.4c).

c. With

$$q(x) = -0.3x^2 + 3x$$

integrating gives

$$V = -\int q(x)\,\mathrm{d}x = +0.1x^3 - 1.5x^2 + C_1.$$

Beware of the signs!

After integrating again we find

$$M = \int V\,\mathrm{d}x = +0.025x^4 - 0.5x^3 + C_1 x + C_2.$$

The integration constants C_1 and C_2 follow from the boundary conditions. Because the M and V diagrams are roughly known, we have a free choice here. Below we have selected the boundary conditions relating to the bending moments at A and B:

$$x = 0;\ M = 0 \Rightarrow C_2 = 0,$$

$$x = 10;\ M = -50 \Rightarrow C_1 = +20\ \text{kN}.$$

For the variation of the shear force and the bending moment we find

$$V = (0.1x^3 - 1.5x^2 + 20)\ \text{kN}, \qquad \text{(a)}$$

$$M = (+0.025x^4 - 0.5x^3 + 20x)\ \text{kNm}. \qquad \text{(b)}$$

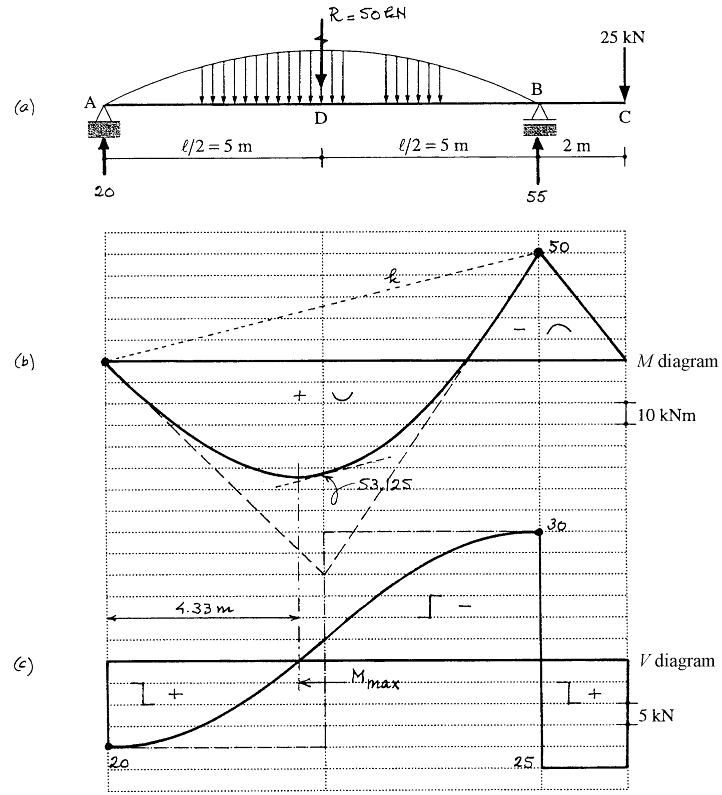


Figure 13.4 (a) Support reactions, (b) bending moment diagram and (c) shear force diagram.

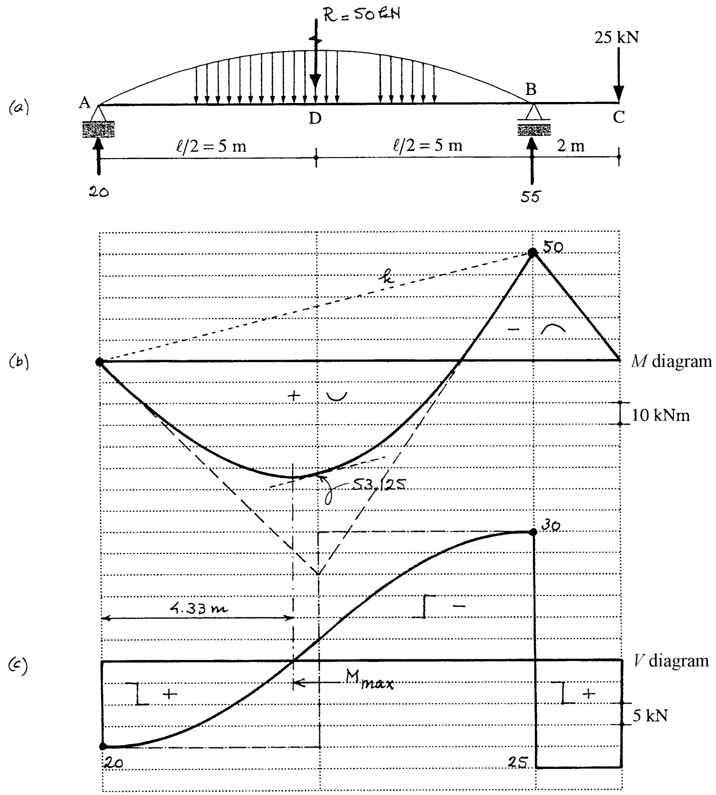


Figure 13.4 (a) Support reactions, (b) bending moment diagram and (c) shear force diagram.

Check: With $x = 0$ and $x = 10$ expression (a) must give the shear force at A, and that to the left of B, respectively:

$$x = 0;\ V = +20 \text{ kN (correct)},$$
$$x = 10;\ V = +100 - 150 + 20 = -30 \text{ kN (correct)}.$$

At D ($x = 5$):

$$V = +0.1 \times 5^3 - 1.5 \times 5^2 + 20 = -5 \text{ kN},$$
$$M = +0.025 \times 5^4 - 0.5 \times 5^3 + 20 \times 5 = +53.125 \text{ kNm}.$$

At D, the middle of span AB, the tangent to the M diagram is parallel to the chord k (see Figure 13.4.b).

d. The maximum bending moment in AB will occur slightly to the left of the middle D. Looking at the M diagram in Figure 13.4b, we can estimate the magnitude of that moment as approximately 55 kNm.

Accurate determination:
If we are looking for the root of the V diagram, (a) gives

$$x = 4.33 \text{ m}.$$

Substituting this value in (b) leads to an accurate value of the maximum bending moment:

$$M_{\text{max}} = 54.8 \text{ kNm } (\smile).$$

13.1.3 Beam on three bar supports with a uniformly distributed load

The structure in Figure 13.5 consists of a beam supported by three bars. Dimensions and loads are given in the figure.

Questions:

a. Determine the support reactions at P, Q and R. Draw them as they act in reality, and include their values.
b. For ABCD, draw the V and M diagrams, with the deformation symbols. Include relevant values. At A, B and E also draw the tangents to the M diagram.
c. Determine the location and magnitude of the maximum field moment in BC.

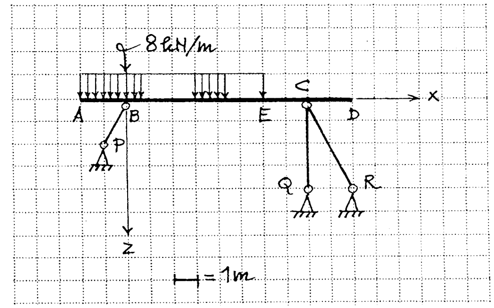


Figure 13.5 Beam on three bar supports.

Solution:

a. In Figure 13.6, the distributed load over AE has been replaced by its resultant of (8 kN/m)(8 m) = 64 kN. This simplifies the calculation for the support reactions. From the moment equilibrium about S we can find the vertical support reaction at Q:

$$\sum T_y|\mathrm{S} = 0 \Rightarrow Q_\mathrm{v} = 32\ \mathrm{kN}\ (\downarrow).$$

From the moment equilibrium about T we can find the vertical support reaction at R:

$$\sum T_y|\mathrm{T} = 0 \Rightarrow R_\mathrm{v} = 48\ \mathrm{kN}\ (\uparrow).$$

From the slope of bar support RC we find

$$R_\mathrm{h} = \tfrac{1}{2} R_\mathrm{v} = 24\ \mathrm{kN}\ (\leftarrow).$$

Finally, the horizontal and vertical force equilibrium gives

$$\sum F_z = 0 \Rightarrow P_\mathrm{v} = 48\ \mathrm{kN}\ (\uparrow),$$

$$\sum F_x = 0 \Rightarrow P_\mathrm{h} = 24\ \mathrm{kN}\ (\rightarrow).$$

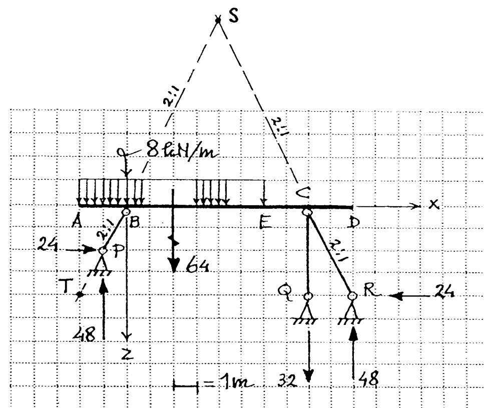


Figure 13.6 Support reactions.

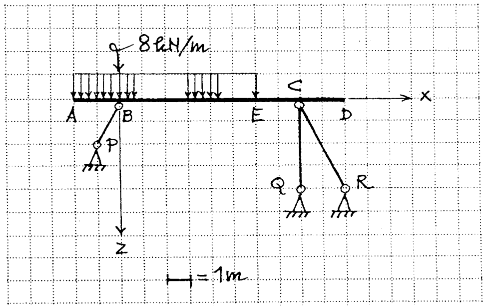


Figure 13.5 Beam on three bar supports.

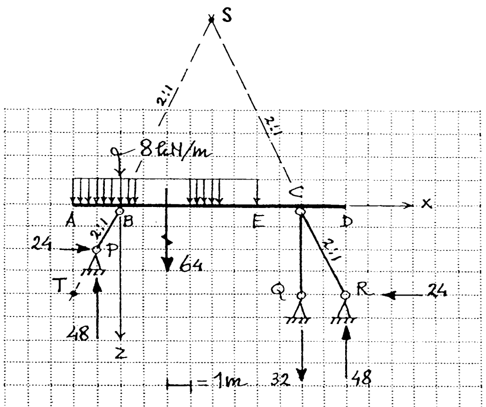


Figure 13.6 Support reactions.

Optional solution question a:
With $P_h = \frac{1}{2} P_v$ the moment equilibrium about C gives

$$P_v = 48 \text{ kN } (\uparrow),$$

$$P_h = 24 \text{ kN } (\rightarrow).$$

The horizontal force equilibrium gives

$$R_h = 24 \text{ kN } (\leftarrow).$$

The slope of the bar support RC gives

$$R_v = 2R_h = 48 \text{ kN } (\uparrow).$$

Finally, the vertical force equilibrium gives:

$$Q_v = 32 \text{ kN } (\downarrow).$$

b. Figure 13.7a shows the isolated beam AD with all the forces acting on it. To simplify the calculation and drawing of the V and M diagrams for the fields AB and BC, the resultants of the distributed loads are also shown.

In Figures 13.7b and 13.7c, the dashed line shows the V and M diagrams due to the concentrated forces. These diagrams have to be adjusted in fields AB and BE. Here the shear force is linear and the bending moment is parabolic. The parabolic bending moment diagram "hangs" between the values at A, B and E. The definitive V and M diagrams are shown as solid lines.

Checking the M diagram for field BE:
In the middle of the field, the parabola bisects the distance between the chord (8 kNm) and the top value due to the load resultant (80 kNm). From

Figure 13.7c we can deduce:

$$p = \frac{(80 - 8)\ \text{kNm}}{2} = 36\ \text{kNm}.$$

This value of p, the rise of the parabola, must be equal to $\frac{1}{8}q\ell^2$, in which ℓ is the length of field BE:

$$p = \tfrac{1}{8}(8\ \text{kN/m})(6\ \text{m})^2 = 36\ \text{kNm}.$$

This is the case.

c. From the V diagram we can deduce that the shear force in field BC is zero at G, 4 m to the right of B. Here the maximum field moment occurs. We can determine the magnitude from the moment equilibrium of beam segments AG or GD, or from the area of the V diagram for beam segments AG or GD. From the area of the V diagram for beam segment AG we find

$$M_{\text{max}} = \left|\tfrac{1}{2}(2\ \text{m})(16\ \text{kN}) - \tfrac{1}{2}(4\ \text{m})(32\ \text{kN})\right| = 48\ \text{kNm}\ (\smile).$$

If we look at beam segment GD this must off course give the same value:

$$M_{\text{max}} = \left|\tfrac{1}{2}(2\ \text{m})(16\ \text{kN}) + (2\ \text{m})(16\ \text{kN})\right| = 48\ \text{kNm}\ (\smile).$$

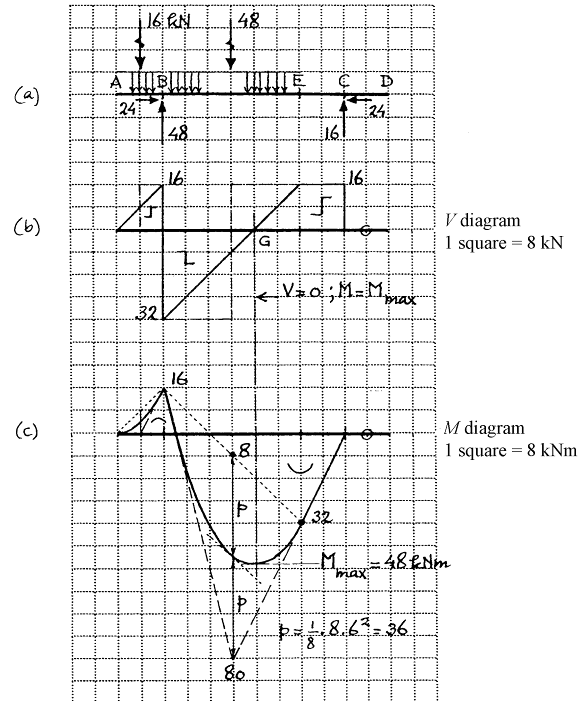


Figure 13.7 (a) The isolated beam AD with its (b) shear force diagram and (c) bending moment diagram.

13.1.4 Pile (cantilever beam)

A concrete pile with length $\ell = 20.5$ m and square cross-section of $0.35 \times 0.35\ \text{m}^2$ is supported as shown in Figure 13.8. The mass density ρ of concrete is assumed $\rho = 2500\ \text{kg/m}^3$.

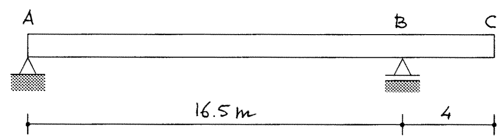


Figure 13.8 A pile that is picked up to be driven, can be seen as a simply supported beam with overhang.

Questions:

a. Determine and draw the M and V diagrams. At A, B and C also draw the tangents to the M diagram.

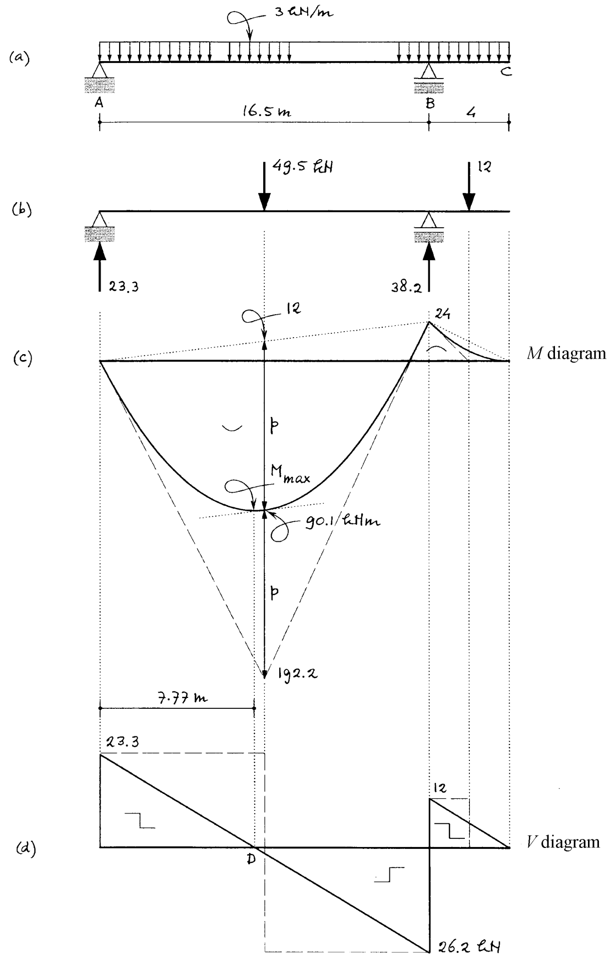


Figure 13.9 (a) Model for the pile subject to its dead weight. (b) The load resultants on AB and BC and the associated support reactions. (c) Bending moment diagram and (d) shear force diagram.

b. Determine the extreme value(s) of the bending moment.
c. Where should the support B be placed to minimise the bending moment? Draw the associated M and V diagrams.

Solution:
a. The dead weight of the pile is $q = \rho g A$, in which $g = 10$ N/kg is the gravitational field strength and A is the cross-sectional area of the pile:

$$q = \rho g A = (2500\ \text{kg/m}^3)(10\ \text{N/kg})(0.35\ \text{m})^2 = 3062.5\ \text{N/m}.$$

Hereafter, assume $q = 3$ kN/m.

Figure 13.9a shows the model for the pile. In Figure 13.9b, the distributed loads in fields AB and BC have been replaced by their resultants, and the support reactions are shown. In Figure 13.9c, the M diagram due to the load resultants is shown by means of dashed lines. At A, B and C the dashed diagram gives the correct values for the actual M diagram and the correct tangents. The actual M diagram is shown by means of a solid line.

Checking the M diagram in field AB (see Figure 13.9c):

$$p = \tfrac{1}{8} q \ell^2 = \tfrac{1}{8}(3\ \text{kN/m})(16.5\ \text{m})^2 = 102.1\ \text{kNm} = (12 + 192.2)/2\ \text{kNm}.$$

In Figure 13.9d, the V diagram due to the resultants is shown by dashed lines. This V diagram gives the correct values in A, B and C. The actual V diagram is linear, and is shown by means of a solid line.

Checking the V diagram:
The slope of the V diagram is equal to the distributed load, and is the same in both fields.

b. From the V diagram in Figure 13.9d we find that the shear force in field AB is zero at D. This is where the bending moment in the field is an extreme. The distance from D to A is

$$\ell^{AD} = \frac{23.3}{23.3 + 26.2} \times (16.5 \text{ m}) = 7.77 \text{ m}.$$

The bending moment at D can be found from the moment equilibrium of the isolated segment AD or, as shown below, from the area of the V diagram for AD:

$$M_{max} = \tfrac{1}{2}(7.77 \text{ m})(23.3 \text{ kN}) = 90.5 \text{ kNm}.$$

Another extreme bending moment is the *support moment*[1] at B. Note that this moment can be found from the area of the V diagram for segment BC (see Figure 13.9d):

$$M_{min} = \tfrac{1}{2}(4 \text{ m})(12 \text{ kN}) = 24 \text{ kNm}.$$

c. Let the total length of the pile be ℓ and the length of the overhang be a (see Figure 13.10a). Figure 13.10b shows a sketch of the V diagram. The shear force to the right of B is equal to qa. The slope of the V diagram is the same everywhere. The extreme bending moments occur at E and B. The bending moment at B is equal to the hatched area of the V diagram between B and C:

$$M_B = \tfrac{1}{2}qa^2.$$

The bending moment at E is equal to the hatched area of the V diagram between A and E. The bending moment in the pile is least when the extreme bending moments at E and B are equal:

$$M_E = M_B = \tfrac{1}{2}qa^2.$$

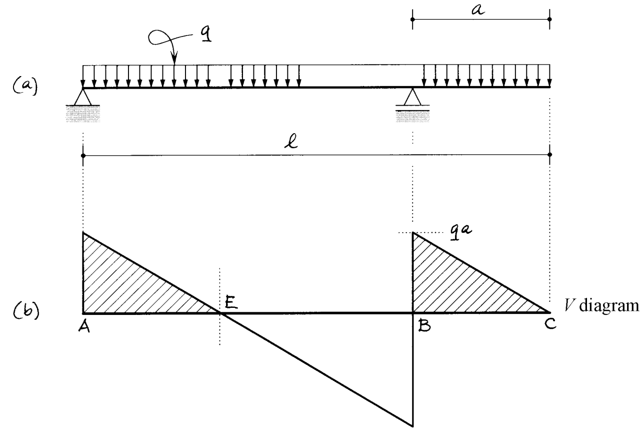


Figure 13.10 (a) Simply supported beam with total length ℓ and overhang of length a. (b) If the maximum bending moment at E is equal to the bending moment at B, the hatched areas in the V diagram are also equal.

[1] A support moment is the bending moment in the beam at a support.

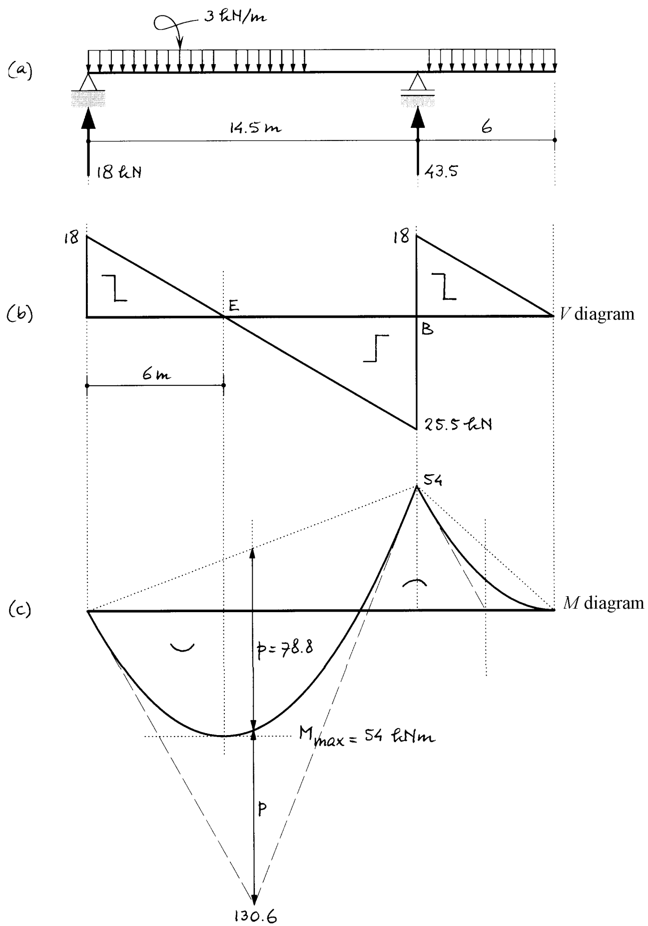


Figure 13.11 (a) The pile supported in such a way that the maximum field moment in AB and the support moment at B are of equal magnitude. (b) Associated shear force diagram and (c) bending moment diagram.

In that case, the shear force diagrams for AE and BD must be equal. From this it follows that the shear force at A is equal to qa, and the length of AE is equal to a. From the linear variation of the shear force along AB, it follows that the shear force to the left of B is equal to $q(\ell - 2a)$. The total area of the M diagram is zero as there are no concentrated couples acting. The hatched area of the V diagram must therefore be equal to the non-hatched area:

$$2 \times \tfrac{1}{2}qa^2 = \tfrac{1}{2}q(\ell - 2a)^2.$$

This leads to the following quadratic equation in a:

$$2a^2 - 4\ell a + \ell^2 = 0.$$

The solution is

$$a = \frac{-(-4\ell) \pm \sqrt{(-4\ell)^2 - 4 \times 2 \times \ell^2}}{2 \times 2} = \left(1 \pm \tfrac{1}{2}\sqrt{2}\right)\ell.$$

Since $a < \ell$ the solution with the plus sign is invalid, so that

$$a = \left(1 - \tfrac{1}{2}\sqrt{2}\right)\ell = 0.293\ell.$$

With $\ell = 20.5$ m this gives

$$a = 0.293 \times (20.5 \text{ m}) = 6 \text{ m}.$$

Figure 13.11 shows the associated M and V diagrams. The extreme bending moments are

$$M_E = M_B = \tfrac{1}{2}(3 \text{ kNm})(6 \text{ m})^2 = 54 \text{ kNm}.$$

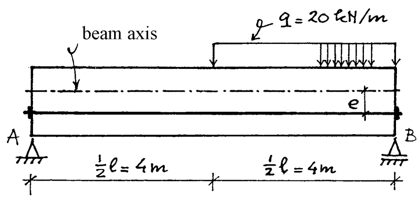


Figure 13.12 Simply supported prestressed beam with a uniformly distributed load on the right-hand side. The straight single bar tendon has eccentricity e.

13.1.5 Prestressed beam

The simply supported beam AB in Figure 13.12 has a length $\ell = 8$ m and is prestressed with a straight single bar tendon. The tendon is at a distance $e = 0.2$ m under the beam axis. The prestressing force is $F_p = 200$ kN. The right-hand half of the beam is loaded by a uniformly distributed load $q = 20$ kN/m.

Questions:

a. Model beam AB as a line element and draw all the forces acting on it.
b. Draw the V and M diagrams with the deformation symbols. Include relevant values.
c. Determine the location and magnitude of the maximum bending moment in the beam.

Solution:

a. Via the anchorages the tensile force $F_p = 200$ kN in the tendon exerts equal compressive forces $F_p = 200$ kN on the beam ends (see Figure 13.13a). In Figure 13.13b, the beam has been modelled as a line element. By definition, the line element coincides with the beam axis. So the force flow is assumed to occur via the beam axis. All the forces are therefore shifted to the beam axis. The eccentric compressive forces on the

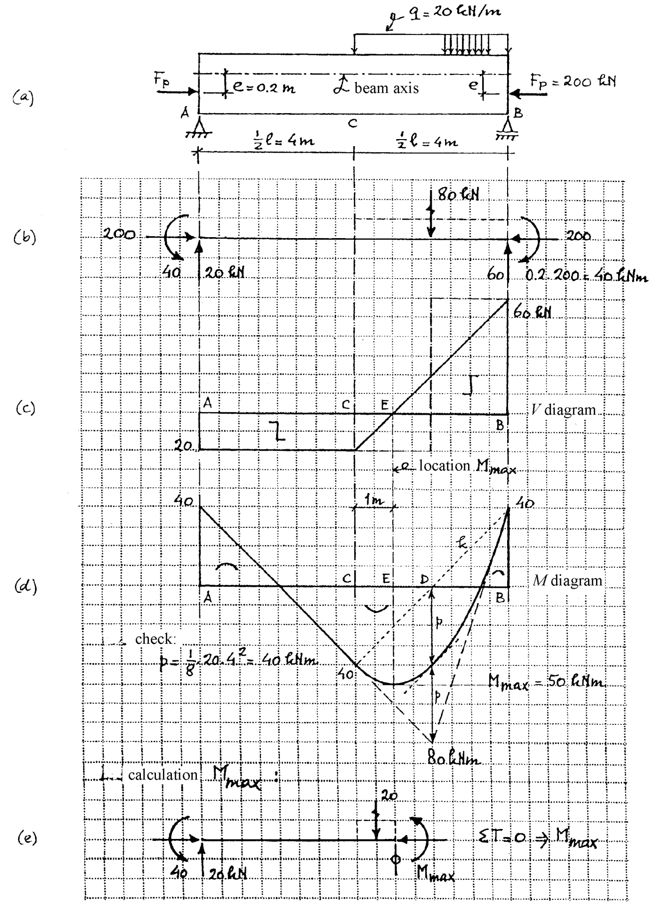


Figure 13.13 (a) Due to the prestressing, the eccentric compressive forces F_p are exerted on the ends of the beam. (b) The beam modelled as a line element. The eccentric compressive forces on the beam ends cause couples. (c) Shear force diagram and (d) bending moment diagram. (e) The maximum bending moment at the shear force zero E can be found from the moment equilibrium of AE.

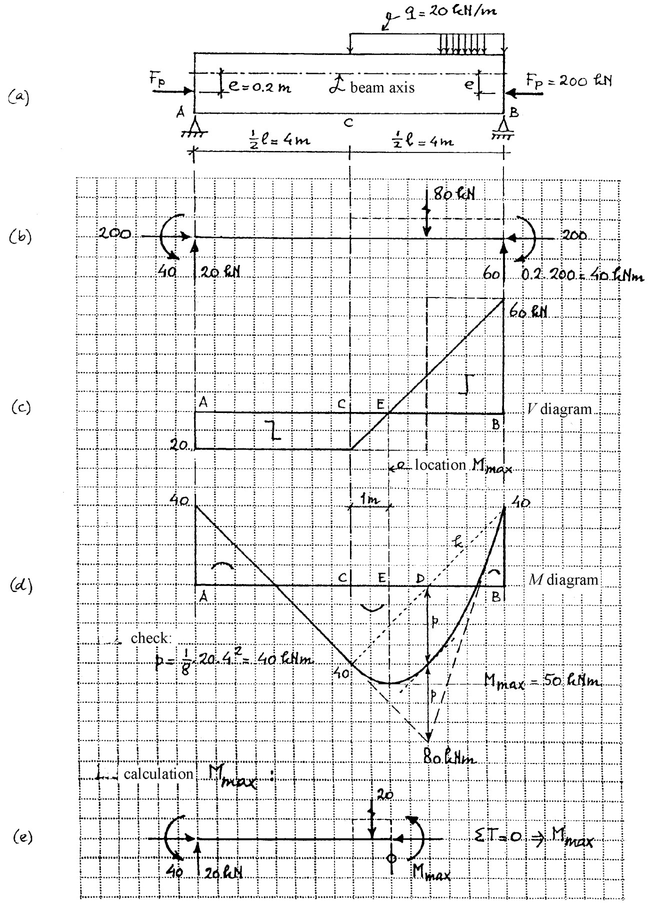


Figure 13.13 (a) Due to the prestressing, the eccentric compressive forces F_p are exerted on the ends of the beam. (b) The beam modelled as a line element. The eccentric compressive forces on the beam ends cause couples. (c) Shear force diagram and (d) bending moment diagram. (e) The maximum bending moment at the shear force zero E can be found from the moment equilibrium of AE.

beam ends are statically equivalent with centric compressive forces (forces acting in the beam axis) $F_p = 200$ kN and additional couples T:

$$T = F_p e = (200 \text{ kN})(0.2 \text{ m}) = 40 \text{ kNm}.$$

To simplify the calculation, the uniformly distributed load over BC has been replaced by its resultant R in Figure 13.13b:

$$R = q \times \tfrac{1}{2}\ell = (20 \text{ kN/m})(4 \text{ m}) = 80 \text{ kN}.$$

The support reactions follow from the moment equilibrium about supports A and B.

b. Figure 13.13c shows the V diagram. We can first draw the V diagram for all the concentrated forces (dashed line), and then adapt them for field CB by drawing a linear path between the values at C and B.

Figure 13.13d shows the M diagram. We first draw the M diagram due to the concentrated forces (dashed line) and then adjust the variation for field CB by sketching a parabola between the values at C and B, where it is tangent to the M diagram due to the resultant R of the distributed load. At the middle D of field CB the distance p between chord k and the parabola is

$$p = \tfrac{1}{8} q \left(\tfrac{1}{2}\ell\right)^2 = \tfrac{1}{8}(20 \text{ kN/m})(4 \text{ m})^2 = 40 \text{ kNm}.$$

Here the tangent is parallel to chord k.

For the bending moment at the middle D of field CB, we can read from the M diagram in Figure 13.13d:

$$M_D = p = 40 \text{ kNm } (\smile)$$

Check: The tangents at C and B intersect at D at a distance $2p$ under chord k.

c. The maximum bending moment in field AB occurs where the shear force $V = \mathrm{d}M/\mathrm{d}x$ is zero. This is at E, 1 m to the right of the middle C of beam AB. The magnitude of this maximum can, for example, be determined from the moment equilibrium about E of beam segment AE (see Figure 13.13e). This gives

$$M_{\max} = 50 \text{ kNm } (\smile).$$

Note that this maximum moment is not equal to the area of the V diagram for beam segment AE or BE, while the total area of the V diagram certainly is zero. It is left to the reader to explain this.

13.1.6 Slice from a long floating barge

A transverse slice has been isolated from the long floating concrete trough in Figure 13.14a, and has been modelled as the line element in Figure 13.14b. The dead weight of the slice is uniformly distributed over walls and bottom and is 12 kN/m. The dimensions and depth can be read from the figure. Note: The width of the slice is unknown.

Questions:

a. From the equilibrium of the slice modelled as a line element, determine the water pressure on the bottom AB. Draw the water pressure on both the bottom and the walls. Include the values.
b. Isolate bottom AB, and draw all the forces acting on it. Include the values.
c. For the entire slice, draw the M, V and N diagrams, with the deformation symbols. Include relevant values.
d. Determine the maximum bending moment. Where does it occur?

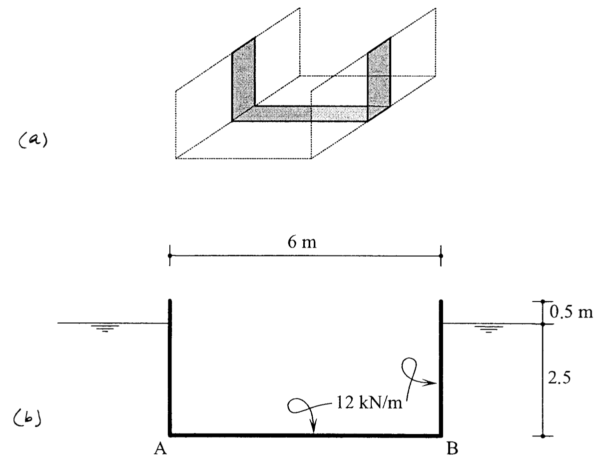


Figure 13.14 (a) A slice from a long floating concrete barge. (b) The slice modelled as a line element.

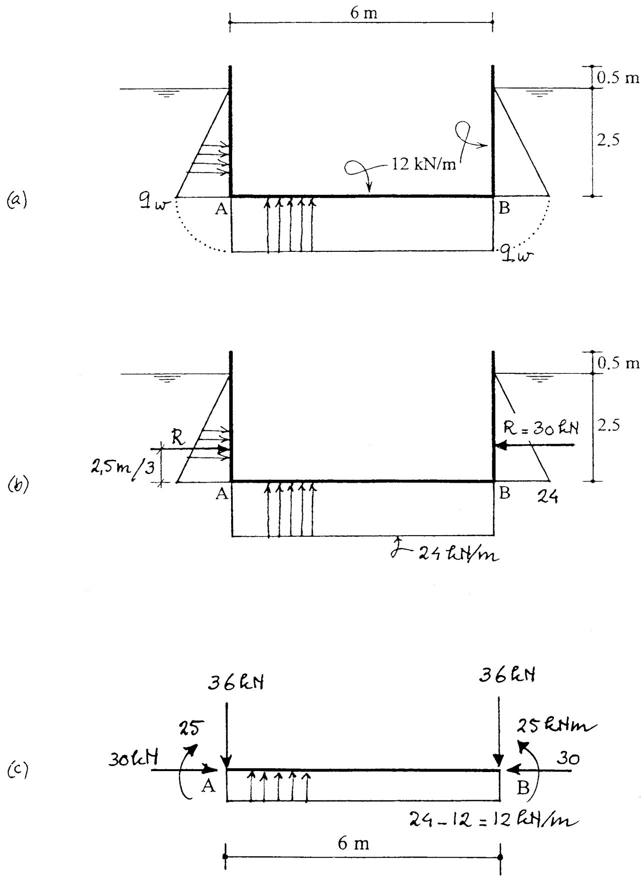


Figure 13.15 (a) The distribution of the water pressure and (b) the resulting water pressure on the walls. (c) The isolated bottom with all the forces and couples acting on it.

Solution:

a. Figure 13.15a shows the water pressure on the slice modelled as a line element. The water pressure on the bottom is constant. Let the water pressure there be q_w. The water pressure on the walls varies linearly from zero at the water level to q_w at the bottom. The upward water pressure q_w on the bottom must be in equilibrium with the dead weight of the bottom and walls of the strip:

$$(6 \text{ m}) \times q_w \,(\uparrow) = \{(6 \text{ m}) + 2 \times (3 \text{ m})\} \times (12 \text{ kN/m}) \,(\downarrow)$$
$$= 144 \text{ kN} \,(\downarrow).$$

This gives $q_w = 24$ kN/m.

b. The resulting water pressure R on the walls is (see Figure 13.15b):

$$R = \tfrac{1}{2}(24 \text{ kN/m})(2.5 \text{ m}) = 30 \text{ kN}.$$

The forces R, which pass through the centroid of the load diagram and therefore act (2.5 m)/3 above bottom AB, exert horizontal forces of 30 kN on AB and couples of (30 kN)(2.5 m)/3 = 25 kNm. The bottom AB can be seen as an eccentrically compressed beam. In addition, at A and B, the vertical forces due to the dead weight of the walls are (3 m)(12 kN/m) = 36 kN. In Figure 13.15c, the base AB has been isolated, and all the forces are shown. The resulting (uniformly) distributed load q on base AB is equal to the difference between the upward water pressure $q_w = 24$ kN/m (↑) and the dead weight $q_w = 12$ kN/m (↓):

$$q = q_w - q_{dw} = (24 \text{ kN/m}) - (12 \text{ kN/m}) = 12 \text{ kN/m}) \,(\uparrow).$$

c. Figures 13.16a to 13.16c shows the M, V and N diagrams for the entire structure.

Walls: Due to the linearly distributed water pressure, the M diagram is a cubic, and the V diagram is a parabola. In order to find the tangents to the M diagram at A and B, the M diagram due to resultant R has been shown by means of dashed lines. The slope of the V diagram is zero, where the water pressure is zero and increases downwards. Due to the uniformly distributed dead weight, the normal force in the wall is linear.

Bottom: Due to the uniformly distributed (upward) load, the bending moment is parabolic and the shear force is linear. The moments at A and B "go round the corner". Between A and B a parabola is "hanging" with a rise p in the middle:

$$p = \tfrac{1}{8}q\ell^2 = \tfrac{1}{8} \times (12\ \text{kN/m})(6\ \text{m})^2 = 54\ \text{kNm}.$$

Because the distributed load is acting upwards, the parabolic M diagram is turned upwards. The normal force in the bottom is a constant compressive force of 30 kN.

d. In the middle of field AB, the shear force is zero and the bending moment is an extreme. The maximum field moment is an upward bending moment and can be determined from the M diagram in Figure 13.16b:

$$M_{\text{max}} = (54\ \text{kNm}) - (25\ \text{kNm}) = 29\ \text{kNm}\ (\frown).$$

In addition, there are boundary extremes at A and B of 25 kNm ($\smile$).

13.1.7 Floating tunnel segment

A tunnel segment is afloat, waiting to be towed to the location where it will be sunk. The tunnel segment can be seen as a rigid beam and has a freeboard of 0.09 m (see Figure 13.17). The length ℓ, width b and height h of the tunnel segment are respectively 80, 9.5 and 6 m.

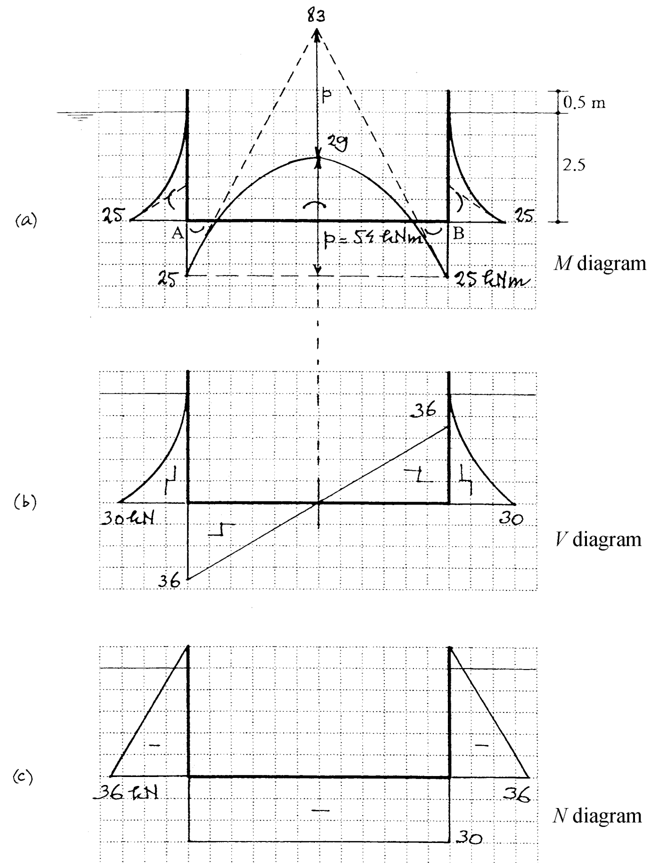


Figure 13.16 (a) Bending moment diagram, (b) shear force diagram and (c) normal force diagram for the slice from the floating concrete barge modelled as a line element.

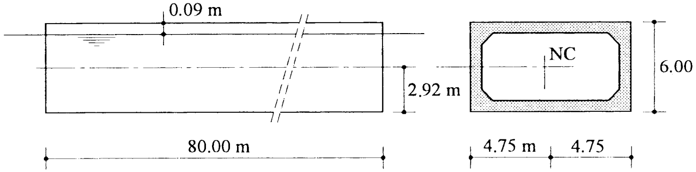


Figure 13.17 Longitudinal view and cross-section of a floating tunnel segment.

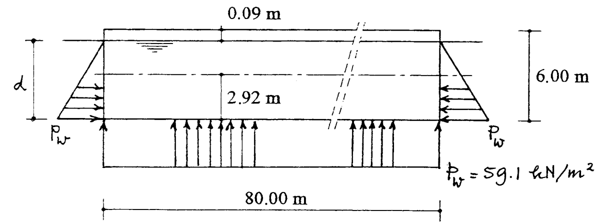


Figure 13.18 Distribution of the water pressure on the tunnel segment.

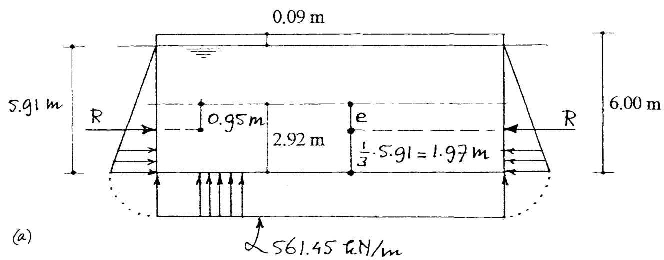


Figure 13.19 (a) Water pressure on the tunnel segment modelled as a plane element.

Figure 13.17 also shows the place of the normal force centre NC of the tunnel segment. The dead weight of the tunnel is 554 kN/m. The two temporary bulkheads both have a dead weight of 298 kN. The mass density of water is $\gamma_w = 10$ kN/m^3.

Questions:

a. Determine and draw the water pressure on the base of the tunnel. Write down the units.
b. Draw the variation of the water pressure on a bulkhead. Write down the units.
c. Model the tunnel segment as a line element. Draw all the forces (distributed or not) (and/of couples) acting on it. Include the values.
d. Draw the *N* diagram, *V* diagram and *M* diagram for the tunnel segment including the deformation symbols. Include relevant values. Determine the maximum bending moment in the tunnel segment.

Solution:

a. The water pressure p_w on the base of the tunnel is

$$p_w = \gamma_w d$$

in which $d = (6.00 \text{ m}) - (0.09 \text{ m}) = 5.91$ m is the depth of water at the base of the tunnel (see Figure 13.18), so that

$$p_w = (10 \text{ kN/m}^3)(5.91 \text{ m}) = 59.1 \text{ kN/m}^2.$$

b. The horizontal water pressure on the bulkheads varies linearly over the height (see Figure 13.18).

c. In Figure 13.19a, the tunnel has been modelled as a plane element. The water pressure on the base is

$$q_w = bp_w = (9.5 \text{ m})(59.1 \text{ kN/m}^2) = 561.45 \text{ kN/m}.$$

The resulting water pressures R on the bulkheads are

$$R = \tfrac{1}{2}q_w d = \tfrac{1}{2} \times (561.45\text{ kN/m})(5.91\text{ m}) = 1659\text{ kN}.$$

The forces R act at a distance $d/3 = 5.91\text{ m})/3 = 1.97$ m from the base of the tunnel segment. The eccentricity e with respect to the tunnel axis (through the normal force centre NC) is

$$e = (2.92\text{ m}) - (1.97\text{ m}) = 0.95\text{ m}.$$

In Figure 13.19b, the tunnel segment has been modelled as a line element. The force flow is assumed to take place along the tunnel axis, through the normal centre NC. All the forces are therefore shifted to the tunnel axis. By shifting the eccentric water pressures R on the bulkheads to the tunnel axis, couples T are generated at the ends of the line element:

$$T = Re = (1659\text{ kN})(0.95\text{ m}) = 1576\text{ kNm}.$$

In addition to the upward water pressure $q_w = 561.45$ kN/m (↑), there is also the dead weight of the tunnel segment $q_{dw} = 554$ kN/m (↓). The resulting distributed load is an upward load q:

$$q = q_w - q_{dw} = (561.45\text{ kN/m}) - (554\text{ kN/m}) = 7.45\text{ kN/m})\ (\uparrow).$$

Check: The resulting upward load q, shown in Figure 13.20a, must be in equilibrium with the dead weight of the bulk heads:

$$\sum F_{\text{vert}}\ (\downarrow) = 2 \times (298\text{ kN}) - (7.45\text{ kN/m})(80\text{ m}) = 0.$$

d. Figure 13.20b to d shows the N, V and M diagrams. The normal force is a constant compressive force. The shear force varies linearly. The bending moment varies parabolically. Since the distributed load is acting upwards the parabolic M diagram is also aimed upwards. The maximum bending

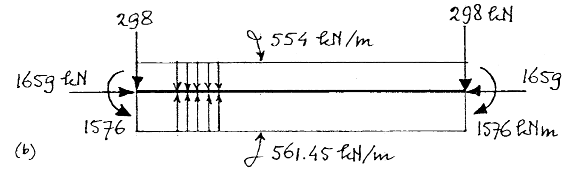


Figure 13.19 (b) Forces (and couples) on the tunnel section modelled as a line element.

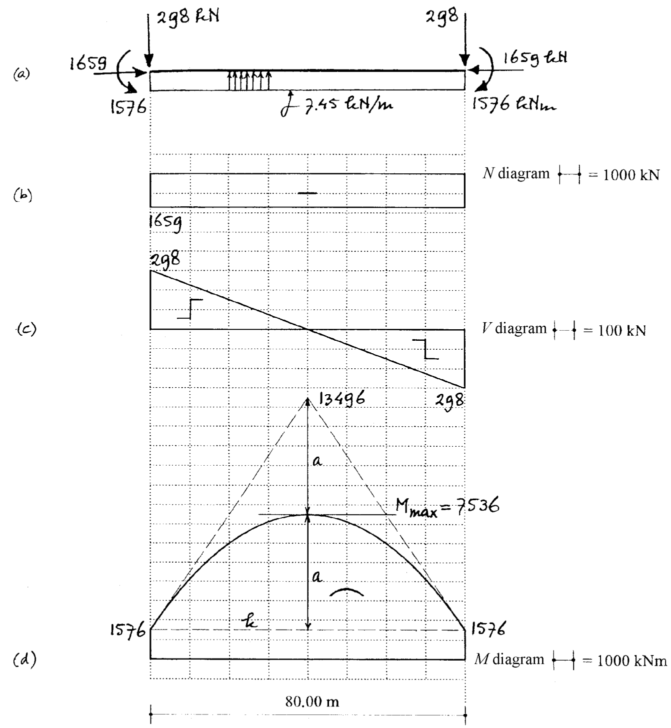


Figure 13.20 (a) The tunnel segment modelled as a line element with its (b) normal force diagram (c) shear force diagram and (d) bending moment diagram.

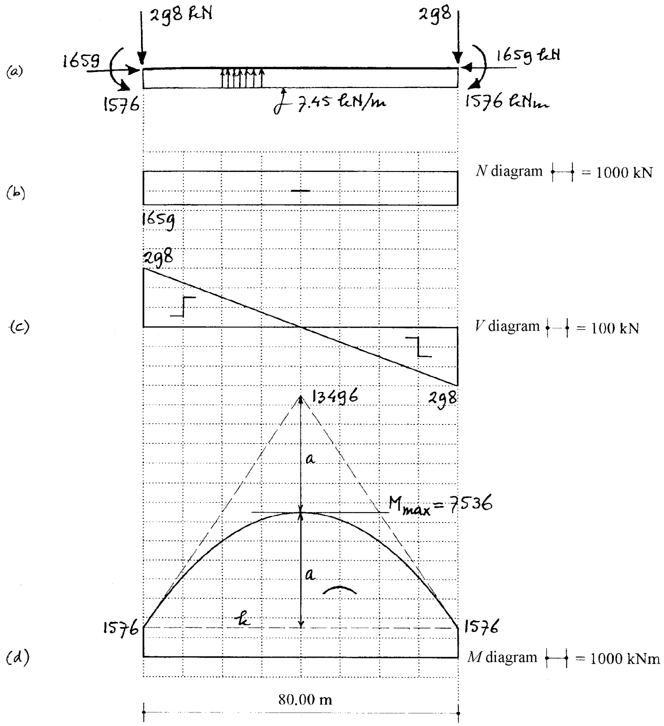


Figure 13.20 (a) The tunnel segment modelled as a line element with its (b) normal force diagram (c) shear force diagram and (d) bending moment diagram.

moment occurs in the middle of the tunnel segment. This can be determined from the moment equilibrium of half a tunnel segment or directly from the M diagram. In the middle of the tunnel segment, the distance a from the chord k to the parabola is

$$a = \tfrac{1}{8}q\ell^2 = \tfrac{1}{8} \times (7.45\ \text{kN/m})(80\ \text{m})^2 = 5960\ \text{kNm}$$

with which we can find the maximum bending moment:

$$M_{\text{max}} = (1576\ \text{kNm}) + (5960\ \text{kNm}) = 7536\ \text{kNm}\ (\frown).$$

Figure 13.20d shows the end tangents to the M diagram.

13.1.8 Oblique roof beam on bar supports with triangular load

The structure in Figure 13.21 is subject to a linear distributed load normal to ABC, varying from from 18 kN/m at A to zero at C.

Questions:

a. Determine the support reactions at A, E and D. Draw them as they act in reality and include their values.
b. Isolate beam ABC, and draw all the forces acting on it.
c. For ABC draw a clear sketch of the V and M diagrams, with the deformation symbols and the plus and minus signs in the given (local) xz coordinate system. Include relevant values, and at A, B and C draw the tangents to the M diagram.
d. For AB, determine the shear force V and the bending moment M as a function of x. Use the given xz coordinate system.

Solution:

a. The support reactions are shown in Figure 13.22. To determine the support reactions, the triangular load on ABC is replaced by its resultant

R^{ABC}:

$$R^{\mathrm{ABC}} = \tfrac{1}{2} \times (18\ \mathrm{kN/m})(6\sqrt{2}\ \mathrm{m}) = 54\sqrt{2}\ \mathrm{m}.$$

The vertical support reaction $A_\mathrm{v}\,(\uparrow)$ at A is found from the moment equilibrium about G, the intersection of the two-force members BE and CD:

$$\sum T|G \circlearrowleft\ = (4\sqrt{2}\ \mathrm{m})(54\sqrt{2}\ \mathrm{kN}) - (4\ \mathrm{m}) \times A_\mathrm{v}\,(\uparrow) = 0$$

$$\Rightarrow A_\mathrm{v} = 108\ \mathrm{kN}\ (\uparrow).$$

The vertical support reaction $E_\mathrm{v}\,(\uparrow)$ in E is then found from the moment equilibrium about C:

$$\sum T|C \circlearrowleft\ = (4\sqrt{2}\ \mathrm{m})(54\sqrt{2}\ \mathrm{kN}) - (6\ \mathrm{m})(108\ \mathrm{kN}) - (2\ \mathrm{m}) \times E_\mathrm{v}\,(\uparrow) = 0$$

so that

$$E_\mathrm{v}\,(\uparrow) = -108\ \mathrm{kN}$$

or in other words

$$E_\mathrm{v} = 108\ \mathrm{kN}\ (\downarrow).$$

Finally, the support reactions at D follow from the horizontal and vertical force equilibrium of the structure:

$$D_\mathrm{h} = 54\ \mathrm{kN}\ (\leftarrow),$$

$$D_\mathrm{v} = 54\ \mathrm{kN}\ (\uparrow).$$

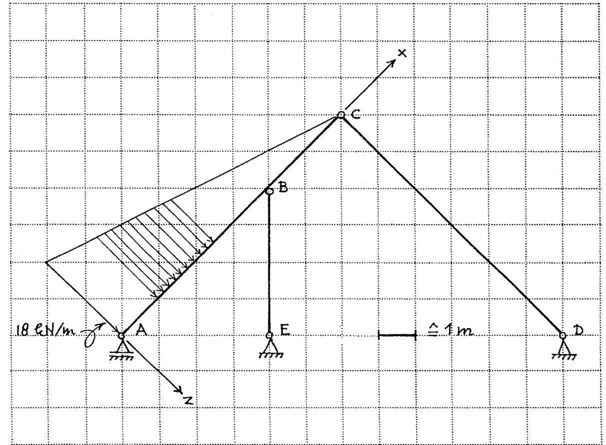


Figure 13.21 Oblique roof beam with triangular load.

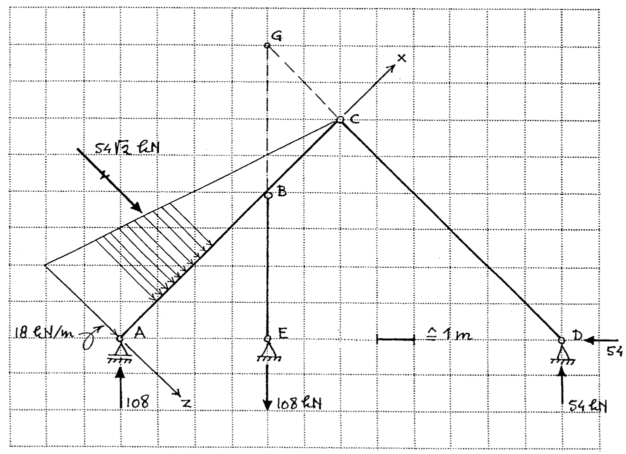


Figure 13.22 Support reactions.

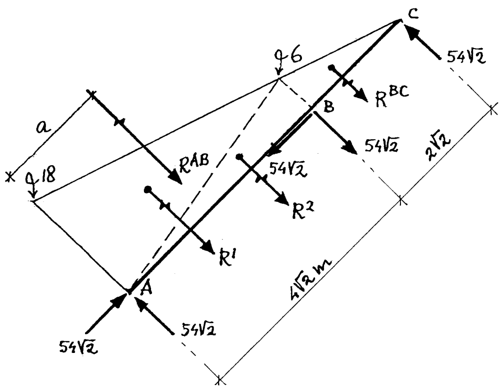


Figure 13.23 Isolated beam ABC with the load resultants for the fields AB and BC.

This final calculation is left to the reader.

b. In Figure 13.23, the beam ABC has been isolated, and all forces acting on it at A, B and C are shown. To draw the M and V diagrams, the distributed loads in fields AB and BC have been replaced by their resultants R^{AB} and R^{BC}. For the triangular load on BC

$$R^{\mathrm{BC}} = \tfrac{1}{2} \times (6 \text{ kN/m})(2\sqrt{2} \text{ m}) = 6\sqrt{2} \text{ kN}.$$

The trapezoidal load on AB is divided into triangular loads (1) and (2); their resultants can be calculated easier:

$$R^{(1)} = \tfrac{1}{2} \times (18 \text{ kN/m})(4\sqrt{2} \text{ m}) = 36\sqrt{2} \text{ kN},$$

$$R^{(2)} = \tfrac{1}{2} \times (6 \text{ kN/m})(4\sqrt{2} \text{ m}) = 12\sqrt{2} \text{ kN},$$

$$R^{\mathrm{AB}} = R^{(1)} + R^{(2)} = 48\sqrt{2} \text{ kN}.$$

The location of the line of action of R^{AB} is found from the moment about A:

$$a R^{\mathrm{AB}} = \tfrac{1}{3} \times 4\sqrt{2} \times R^{(1)} + \tfrac{2}{3} \times 4\sqrt{2} \times R^{(2)} = 160 \text{ kNm}$$

so that

$$a = \frac{160 \text{ kNm}}{48\sqrt{2} \text{ kN}} = \tfrac{5}{3}\sqrt{2} \text{ m}.$$

c. Figure 13.24a shows all the forces acting normal to the beam axis. They generate shear forces and bending moments in the beam. The distributed loads on AB and BC have been replaced by their load resultants. In Figures 13.24b and 13.24c, the V and M diagrams due to the load resultants are shown by means of dashed lines. The solid lines are the actual V and M diagrams.

The actual V diagram has a parabolic variation with a step change at B. At C, the distributed load is zero, and the V diagram has a "horizontal" tangent. To the left and to the right of B, the V diagram has the same slope. In other words: the slope of the V diagram is continuous at B.

The actual M diagram is a cubic, with a bend at B. The M diagram due to the load resultants gives the tangents to the actual M diagram at A, B and C.

d. The load q on ABC varies linearly:

$$q = c_1 x + c_2.$$

The coefficients c_1 and c_2 follow from the values $q = +18$ for $x = 0$ and $q = 0$ for $x = 6\sqrt{2}$:

$$q = -\tfrac{3}{2}x\sqrt{2} + 18.$$

The units used are kN and m, and are omitted in this part of the answer.

By integrating, we can find the variation of the shear force and the bending moment from the distributed load:

$$V = -\int q \, \mathrm{d}x = \tfrac{3}{4}x^2\sqrt{2} - 18x + C_1,$$

$$M = \int V \, \mathrm{d}x = \tfrac{1}{4}x^3\sqrt{2} - 9x^2 + C_1 x + C_2.$$

The integration constants C_1 and C_2 can be found from the boundary conditions at A:

$$x = 0 : V = +54\sqrt{2} \Rightarrow C_1 = +54\sqrt{2},$$

$$x = 0 : M = 0 \qquad \Rightarrow C_2 = 0.$$

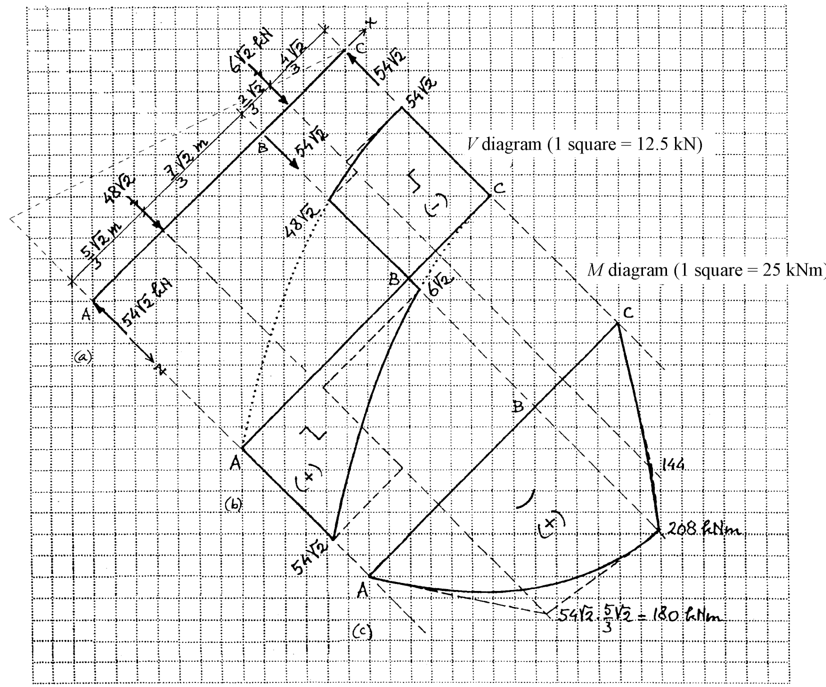


Figure 13.24 (a) Isolated beam ABC with (b) shear force diagram and (c) bending moment diagram.

The functional forms of the shear force V and the bending moment M in AB are

$$V = +\tfrac{3}{4}x^2\sqrt{2} - 18x + 54\sqrt{2},$$
$$M = +\tfrac{1}{4}x^3\sqrt{2} - 9x^2 + 54\sqrt{2}.$$

These expressions can be verified by substituting $x = 4\sqrt{2}$ m to obtain the previously determined values of V and M at B:

$$x = 4\sqrt{2}\ \text{m} : V = +6\sqrt{2}\ \text{kN}\ \ \text{(correct)}$$
$$x = 4\sqrt{2}\ \text{m} : M = +208\ \text{kNm}\ \ \text{(correct).}$$

13.1.9 Railway sleeper in a ballast bed

Figure 13.25a shows a railway sleeper in a ballast bed. In Figure 13.25b the sleeper is modelled as a line element. The sleeper is loaded across the width of the rail by a uniformly distributed load $q = 15$ kNm. It is assumed that the ballast bed exerts a uniformly distributed counter-pressure p (kN/m) on the entire length of the sleeper. The dimensions are shown in the figure.

Questions:

a. Determine the counter-pressure p exerted by the ballast bed.
b. Draw the V diagram with the deformation symbols, and include relevant values.
c. Draw the M diagram with the deformation symbols and the tangents at A, B and C.
d. Determine the extreme bending moments in the railway sleeper. Where do they occur?

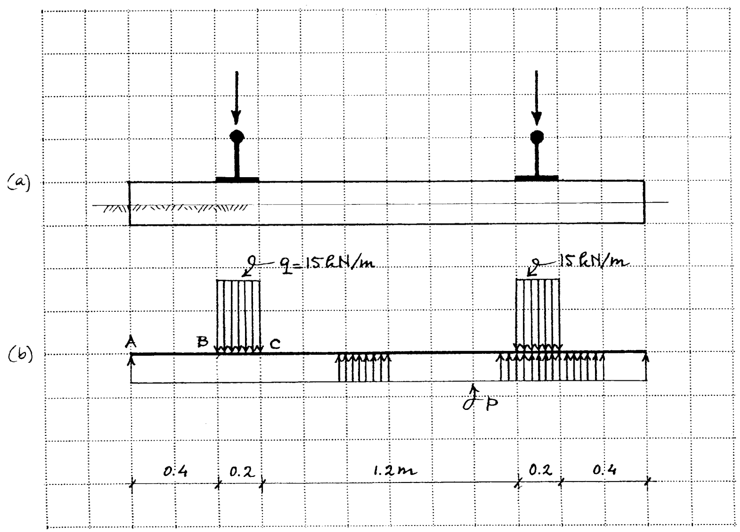


Figure 13.25 (a) Railway sleeper in a ballast bed with (b) the schematic representation.

Solution:

a. The magnitude of p follows from the vertical force equilibrium of the railway sleeper (see Figure 13.25b):

$$\sum F_{\text{vert}}\ (\downarrow) = 2 \times (0.2\text{ m})(15\text{ kN/m}) - (2.4\text{ m}) \times p = 0$$

so that

$$p = 2.5\text{ kN/m}.$$

b. In Figure 13.26a, the railway sleeper is split into the fields AB, BC, etc. Figure 13.26b shows the resultants of the field loads. In Figure 13.26c, the V diagram due to the load resultants is shown by means of dashed lines. The values denoted by a dot at the field boundaries A, B, C, and so forth, are the correct values. The actual (solid) V diagram varies linearly per field between the values indicated by means of dots.

c. Figure 13.26d shows the M diagram due to the load resultants by means of dashed lines. The values on the field boundaries A, B, C, and so forth, indicated by a dot, are the correct values. They can be determined directly from the dashed M diagram. The dashed M diagram also gives the tangents to the actual (solid) M diagram at A, B, C and so forth. In each field, the actual M diagram varies parabolically. In the fields AB, CD and EG, the parabola is turned upwards (the distributed load is acting upwards), in the fields BC and DE the parabola is turned downwards (the resulting distributed load is acting downwards).

Per field, the parabolic variation can be drawn in the standard manner. For field CD it holds that

$$p_1 = \tfrac{1}{8} q \ell^2 = \tfrac{1}{8} \times (2.5\text{ kN/m})(1.2\text{ m})^2 = 0.45\text{ kNm}$$

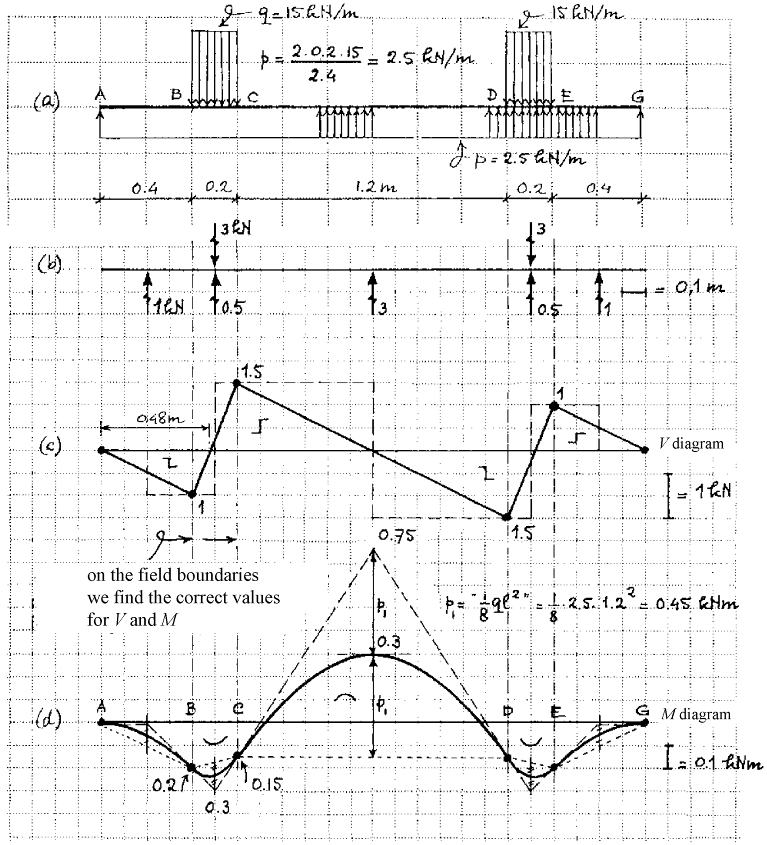


Figure 13.26 (a) The distributed loads on the railway sleeper replaced by (b) their resultants. (c) Shear force diagram and (d) bending moment diagram.

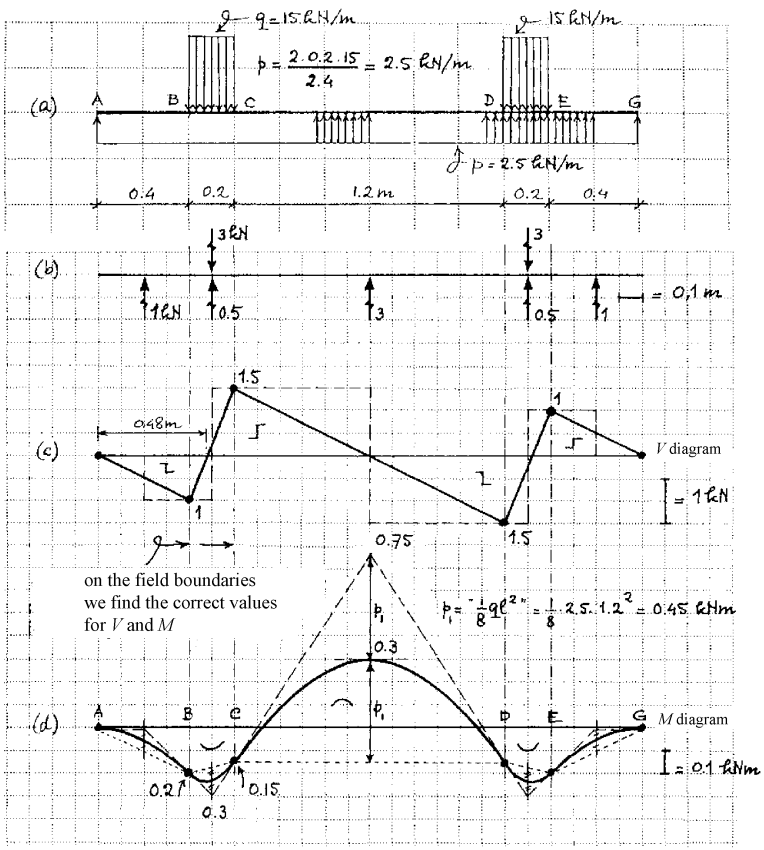


Figure 13.26 (a) The distributed loads on the railway sleeper replaced by (b) their resultants. (c) Shear force diagram and (d) bending moment diagram.

so that

$$M_1 = p_1 - (0.15\text{ kNm}) = 0.3\text{ kNm }(\frown).$$

In the middle of the fields BC and DE it applies that

$$p_2 = \tfrac{1}{8}q\ell^2 = \tfrac{1}{8} \times (12.5\text{ kN/m})(0.2\text{ m})^2 = 0.0625\text{ kNm}$$

so that

$$M_2 = (0.3\text{ kNm}) - p_2 = 0.2375\text{ kNm }(\smile).$$

The values p_2 and M_2 are not shown in Figure 13.26d.

d. The M diagram has three extreme values, as the V diagram has three zeros (not including the end zeros). The largest extreme value M_{max} occurs in the middle of the railways sleeper. This maximum was determined in question c, and can be read directly from the M diagram:

$$M_{max} = M_1 = 0.3\ kNm\ (\frown).$$

The two other extreme values are equal, and occur in the fields BC and DE. Here we will determine the extreme value M_{min} for field BC. In the shear force diagram the distance from A to the zero in BC is (see Figure 13.26c)

$$(0.4\text{ m}) + \frac{1\text{ kN}}{(1\text{ kN}) + (1.5\text{ kN})} \times (0.2\text{ m}) = 0.48\text{ m}.$$

The magnitude of M_{min} is now found most easily from the area of the V diagram:

$$M_{min} = \tfrac{1}{2} \times (0.48\text{ m})(1\text{ kN}) = 0.24\text{ kNm }(\smile).$$

13.1.10 Beam on the ground

Figure 13.27 shows a beam AB lying on the ground, of which the dead weight can be ignored. A uniformly distributed load q is acting on the right-hand side of the beam over a length a. Due to this load, the earth pressure on the underside of the beam varies linearly, from 0 at A to 48 kN/m at B.

Questions:

a. From the equilibrium of the beam, determine length a and load q.
b. Make a good sketch of the V diagram and the M diagram for the beam.
c. At which cross-section is the shear force an extreme? Write down the extreme values for the V diagram. For these cross-sections, also include the tangents to the M diagram.
d. At which cross-section is the bending moment an extreme? Determine this value, and include it with the M diagram.

Solution:

a. The resultant of the earth pressure and the resultant of the q load must have the same line of action (moment equilibrium of a body subject to two forces). The distance from B to the line of action of both resultants is (see Figure 13.27)

$$\tfrac{1}{2}a = \tfrac{1}{3} \times (1.80\ \text{m}) \Rightarrow a = 1.20\ \text{m}.$$

On the basis of the vertical force equilibrium, both resultants must be of equal magnitude:

$$qa = \tfrac{1}{2} \times (1.80\ \text{m})(48\ \text{kN/m}) = 43.2\ \text{kN}$$

so that

$$q = \frac{43.2\ \text{kN}}{1.20\ \text{m}} = 36\ \text{kN/m}.$$

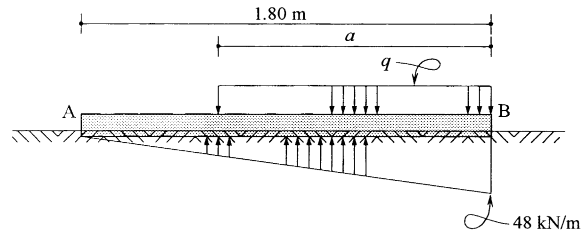


Figure 13.27 Beam on the ground. Due to the uniformly distributed load q, the ground pressure is distributed linearly.

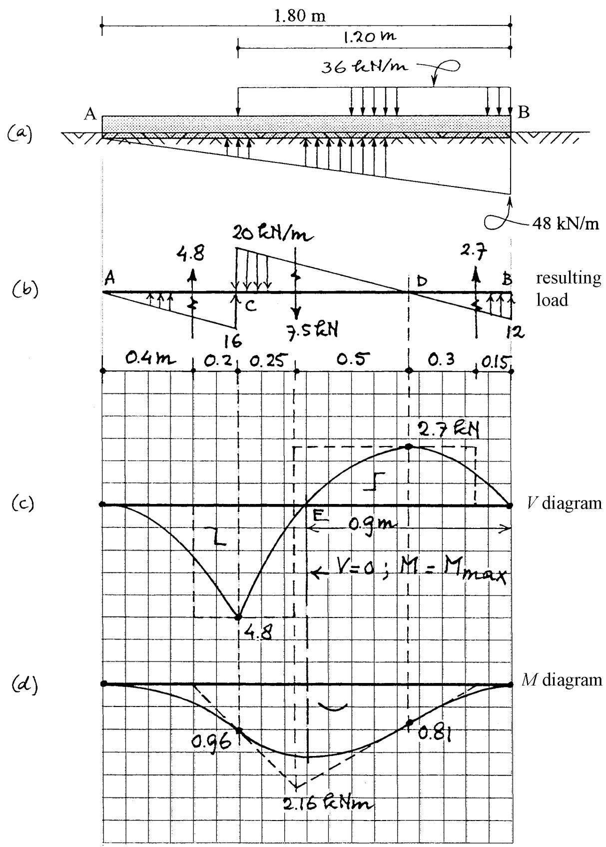


Figure 13.28 (a) Beam with load and earth pressure. (b) Resulting distributed load on the beam and the resultants per field. (c) Shear force diagram and (d) bending moment diagram.

b. In Figure 13.28a, all the loads on beam AB are shown. In Figure 13.28b, the beam has been modelled as a line element, and the resulting load is shown. Three fields with a triangular load can be distinguished. The figure also shows the resultants of these triangular loads.

In Figures 13.28c and 13.28d, the V and M diagrams due to the three load resultants are shown by means of dashed lines. They give the correct values on the field boundaries, in both figures shown by dots. In the field boundaries, the dashed M diagram also gives the tangents to the actual M diagram. The actual V and M diagrams are shown by means of solid lines.

c. A linearly distributed load produces a parabolic V diagram. The shear force is an extreme where the (resulting) distributed load is zero or changes sign, so at A, C and D. At A and D, the V diagram has a horizontal tangent (the distributed load is zero here) and the parabolas have their top. At C, the step change in the distributed load gives a bend in the V diagram. The extreme values can be read off directly from the V diagram.

d. The bending moment is a cubic and is relatively simple to draw using the tangents at the field boundaries. The maximum bending moment M_{max} occurs where the shear force is zero. This is at E, 0.9 m to the left of B, see the V diagram in Figure 13.28c. The magnitude of M_{max} can be determined from the moment equilibrium of the isolated part EB in Figure 13.29:

$$M_{max} = (2.7 \text{ kN})(0.6 \text{ m}) = 1.62 \text{ kNm } (\smile).$$

M_{max} can also be determined from the area of the parabolic V diagram for EB. To do so we have to know that the area of the parabola is equal to two-thirds of the area of the rectangle with a width of 0.9 m and a height of 2.7 kN. This then gives the same value:

$$M_{max} = \tfrac{2}{3} \times (0.9 \text{ m})(2.7 \text{ kN}) = 1.62 \text{ kNm } (\smile).$$

13.1.11 Lean-to subject to dead weight, wind and snow loads

In Figure 13.30, the lean-to ACD is modelled as a line element. We want to determine the N diagram and the extreme values of the bending moment due to the three uniformly distributed loads:

a. A wind pressure of $q_\text{w} = 5$ kN/m (force per length measured along ACD).
b. A dead weight of $q_\text{dw} = 5$ kN/m (force per length measured along ACD).
c. A snow load of $q_\text{sn} = 5$ kN/m (force per length measured along the projection of ACD on the horizontal ground plane).

Solution:
Since the dead weight and the snow load have components transverse to the beam axis (q_tr) and parallel to the beam axis (q_pa), the N, V and M diagrams are first determined due to the separate loads $q_\text{tr} = 1$ kN/m (Figure 13.31a) and $q_\text{pa} = 1$ kN/m (Figure 13.31b). By means of superposition, we then determine the final N diagram for each of the given loads and the extreme values of the bending moments.

In preparation, the dimensions given in Figure 13.30 are first used to determine the angles α, β and γ and the lengths ℓ^AC, ℓ^CD and ℓ^ACD of AC, CD and ACD respectively. The angles α, β and γ we find from

$$\tan\alpha = 2/5 \Rightarrow \alpha = 21.8°,$$

$$\tan\beta = 4/5 \Rightarrow \beta = 38.7°,$$

$$\gamma = \alpha + \beta = 60.5°.$$

The lengths of AC, CD and ACD are

$$\ell^\text{AC} = (5 \text{ m})/\cos\alpha = 5.385 \text{ m},$$

$$\ell^\text{CD} = (2 \text{ m})/\cos\alpha = 2.154 \text{ m},$$

$$\ell^\text{ACD} = \ell^\text{AC} + \ell^\text{CD} = 7.539 \text{ m}.$$

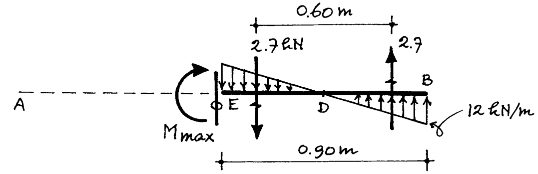


Figure 13.29 The maximum bending moment at E can be derived from the moment equilibrium of EB.

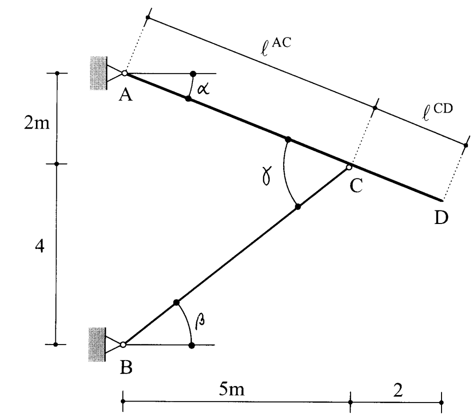


Figure 13.30 Lean-to.

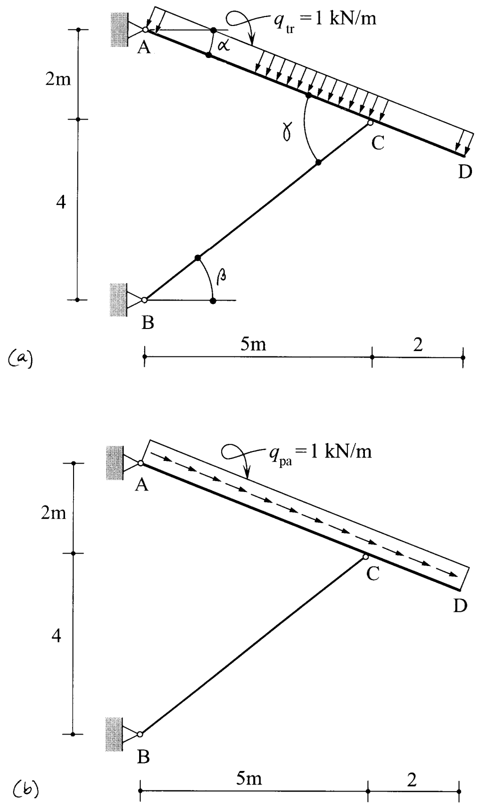


Figure 13.31 Lean-to loaded by a uniformly distributed load of 1 kN/m (a) normal to and (b) parallel to roof plane ACD.

• ***N, V and M diagrams due to q_{tr} = 1 kN/m*** (Figure 13.31a)
In Figure 13.32, ACD has been isolated. The distributed load $q_{tr} = 1$ kN/m has been replaced by its resultant R_{tr}. In addition, the joining forces acting at A and C on ACD are also shown. The indices "*pa*" and "*tr*" point to the directions "*pa*rallel to the beam axis" respectively "*tr*ansverse to the beam axis".

Since BC is a two-force member, the resultant of C_{tr} and C_{pa} must be along BC:

$$C_{pa} = C_{tr}/\tan\gamma.$$

The resultant of the uniformly distributed load is

$$R_{tr} = q_{tr}\ell^{ACD} = (1\text{ kN/m})(7.539\text{ m}) = 7.539\text{ kN}.$$

The moment equilibrium about A gives C_{tr}:

$$\sum T|A \circlearrowright = R_{tr} \times \tfrac{1}{2}\ell^{ACD} - C_{tr} \times \ell^{AC}$$
$$= (7.539\text{ kN}) \times \tfrac{1}{2} \times (7.539\text{ m}) - C_{tr} \times (5.385\text{ m}) = 0$$

so that

$$C_{tr} = 5.277\text{ kN}.$$

C_{pa} is found from the direction of two-force member BC:

$$C_{pa} = C_{tr}/\tan\gamma = \frac{5.277\text{ kN}}{\tan 60.5^\circ} = 2.986\text{ kN}.$$

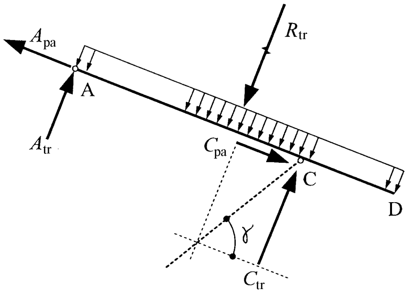


Figure 13.32 Isolated beam ACD with the support reactions at A and C due to a uniformly distributed load normal to the roof plane.

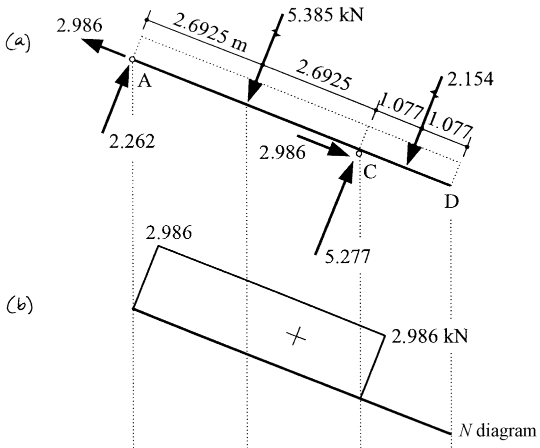


Figure 13.33 (a) Isolated beam ACD with a uniformly distributed load of 1 kN/m normal to the roof plane and associated (b) normal force diagram.

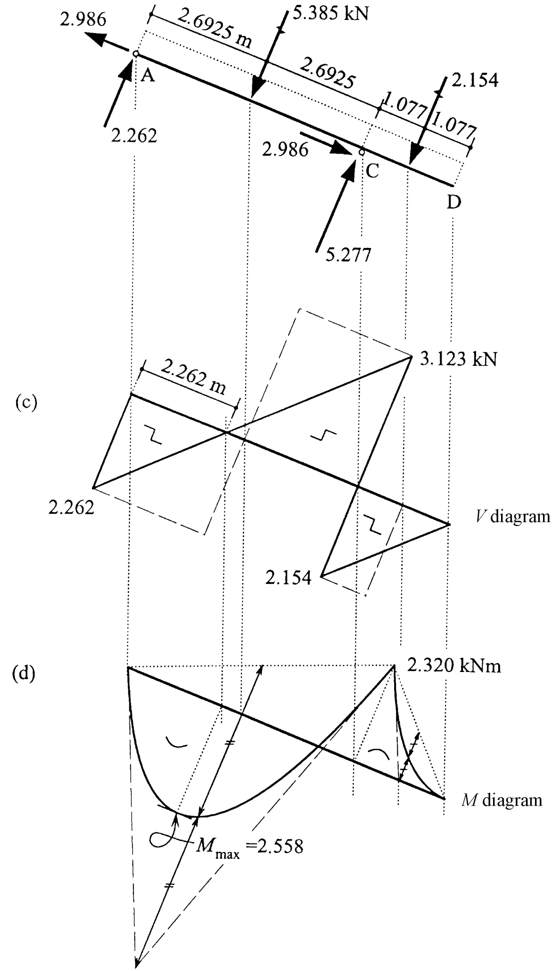


Figure 13.33 (c) Shear force diagram and (d) bending moment diagram due to a uniformly distributed load of 1 kN/m normal to the roof plane.

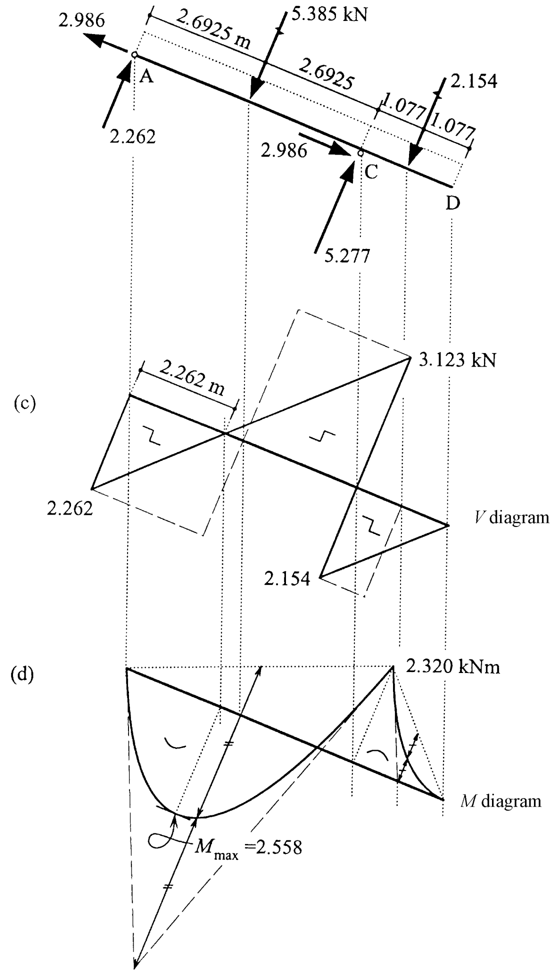


Figure 13.33 (c) Shear force diagram and (d) bending moment diagram due to a uniformly distributed load of 1 kN/m normal to the roof plane.

The force equilibrium in the longitudinal direction of ACD gives

$$A_{pa} = C_{pa} = 2.986 \text{ kN}$$

and the force equilibrium normal to ACD

$$A_{tr} = R_{tr} - C_{tr} = (7.539 \text{ kN}) - (5.277 \text{ kN}) = 2.262 \text{ kN}.$$

Figure 13.33a again shows ACD with the forces determined at A and C, and the resultants of the load on fields AC and CD. In Figures 13.33b to 13.33d, the N, V and M diagrams are shown. It is assumed that the reader is familiar with the necessary calculation. The V and M diagrams due to the load resultants are shown by means of dashed lines.

The M diagram has two extreme values: the (in an absolute sense) smallest moment M_{min} is the bending moment at C and the (in an absolute sense) largest moment M_{max} is the bending moment in field AC:

$$M_{min} = 2.320 \text{ kNm } (\frown).$$

The maximum moment in field AC is found 2.262 m from A. The magnitude can be determined from the area of the V diagram:

$$M_{max} = \tfrac{1}{2}(2.262 \text{ m})(2.262 \text{ kN}) = 2.558 \text{ kNm } (\smile).$$

• ***N, V and M diagrams due to q_{pa} = 1 kN/m*** (Figure 13.31b)
In Figure 13.34, ACD has been isolated and the joining forces acting on ACD are shown. The distributed load $q_{pa} = 1$ kN/m has been replaced by its resultant R_{pa}:

$$R_{pa} = q_{pa}\ell^{ACD} = (1\ \text{kN/m})(7.539\ \text{m}) = 7.539\ \text{kN}.$$

The moment equilibrium about A gives $C_{tr} = 0$ and so $C_{pa} = 0$. The force equilibrium of ACD gives $A_{tr} = 0$ and $A_{pa} = R_{pa} = 7.539$ kN.

In Figure 13.35a, ACD is shown again with the forces determined. In Figure 13.35b, the associated N diagram is shown: due to a uniformly distributed load the normal force is linear. With this load there are no bending moments and shear forces.

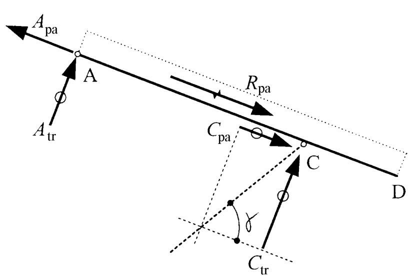


Figure 13.34 Isolated beam ACD with the support reactions in A and C due to a uniformly distributed load parallel to the roof plane.

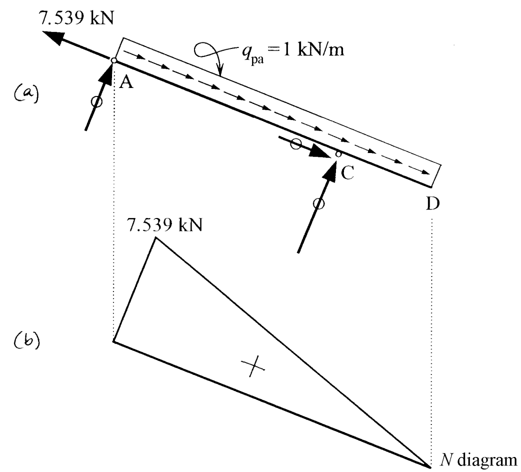
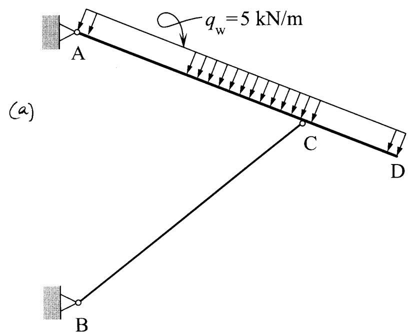


Figure 13.35 (a) Isolated beam ACD with a uniformly distributed load of 1 kN/m normal to the roof plane and associated (b) normal force diagram.



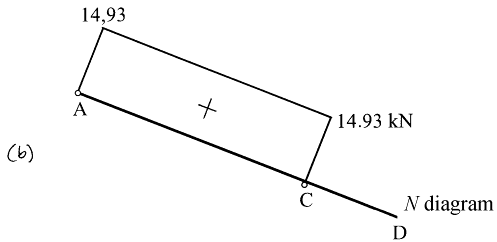


Figure 13.36 (a) Lean-to with wind load of 5 kN/m and (b) associated normal force diagram.

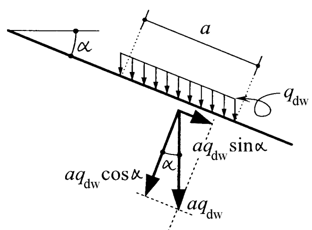


Figure 13.37 The components of the dead weigh q_{dw} of a member segment with length a.

a. *Wind load*

The wind load $q_w = 5$ kN/m in Figure 13.36a is normal to the roof plane ACD. The associated N, V and M diagrams are equal to those in Figure 13.33b to d, but with values that are five times as large:

$$M_{w;max} = (2.558 \text{ kNm}) \times 5 = 12.79 \text{ kNm},$$

$$M_{w;min} = (2.320 \text{ kNm}) \times 5 = 11.60 \text{ kNm},$$

$$N^{AC} = (2.986 \text{ kNm}) \times 5 = 14.93 \text{ kNm}.$$

The N diagram for ACD is shown in Figure 13.36b.

b. *Dead weight*

In Figure 13.37 we take a closer look at a member segment with length a. The dead weight of this member segment is aq_{dw} with components $aq_{dw}\cos\alpha$ and $aq_{dw}\sin\alpha$ respectively normal to and parallel to the beam axis. For the components of the *distributed load* normal to and parallel to the beam axis we find

$$q_{dw;tr} = \frac{aq_{dw}\cos\alpha}{a} = q_{dw}\cos\alpha = (5 \text{ kN/m})\cos 21.8° = 4.642 \text{ kN/m},$$

$$q_{dw;pa} = \frac{aq_{dw}\sin\alpha}{a} = q_{dw}\sin\alpha = (5 \text{ kN/m})\sin 21.8° = 1.857 \text{ kN/m}.$$

The distributed load $q_{dw} = 5$ kN/m due to the dead weight (Figure 13.38a) has components of 4.642 kN/m normal to the beam axis (Figure 13.38b) and 1.857 kN/m parallel to the beam axis (Figure 13.38c).

The bending moment in ACD is caused by the load of 4.642 kN/m normal to the beam axis. The M diagram is equal to that in Figure 13.33d, but 4.462 as large, so that the extreme values of the bending moments are

$$M_{dw;max} = (2.558 \text{ kNm}) \times 4.642 = 11.87 \text{ kNm},$$

$$M_{dw;min} = (2.320 \text{ kNm}) \times 4.642 = 10.77 \text{ kNm}.$$

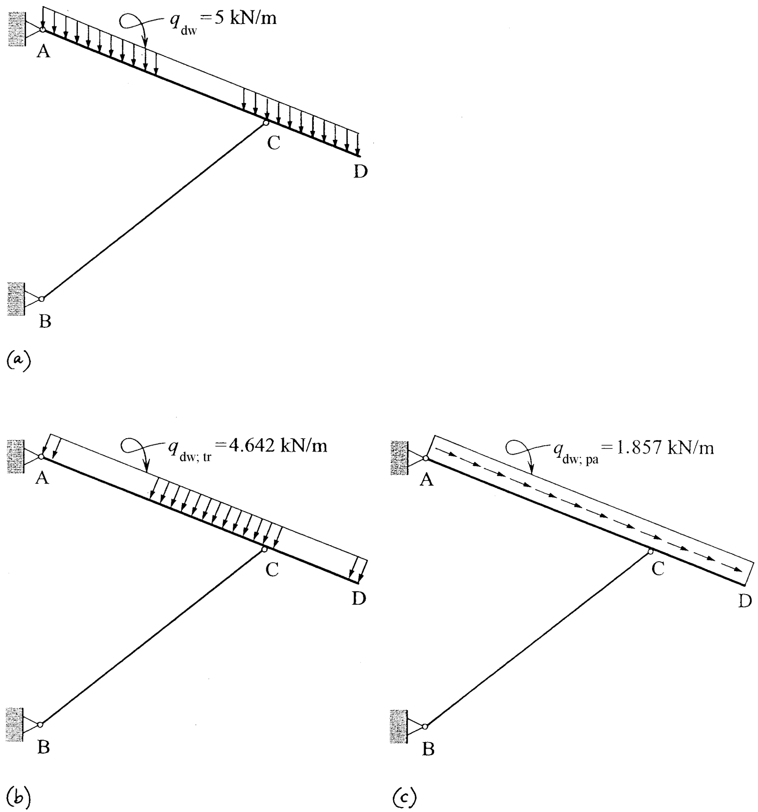


Figure 13.38 (a) The dead weight of beam ACD of 5 kN/m, resolved into components (b) normal to the roof plane and (c) parallel to the roof plane.

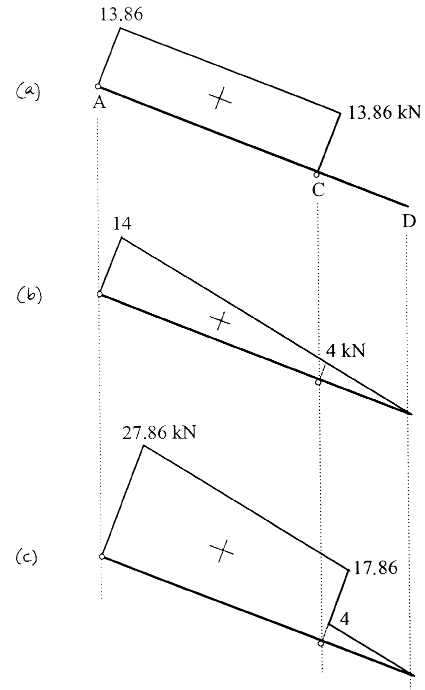


Figure 13.39 (a) The *N* diagram due to the component normal to the roof plane, superposed on (b) the *N* diagram due to the component parallel to the roof plane gives (c) the requested *N* diagram due to the dead weight.

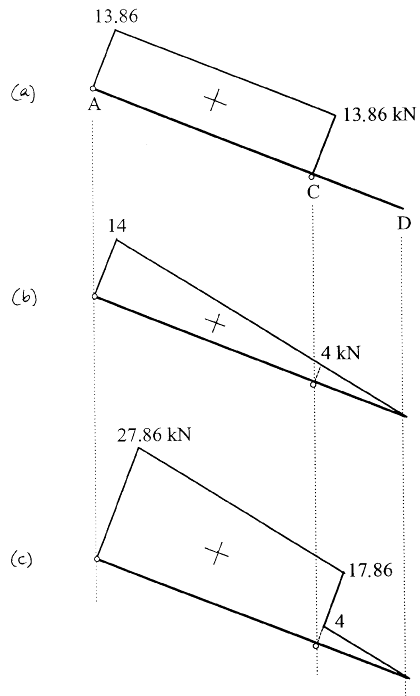


Figure 13.39 (a) The N diagram due to the component normal to the roof plane, superposed on (b) the N diagram due to the component parallel to the roof plane gives (c) the requested N diagram due to the dead weight.

The N diagram in Figure 13.39a due to the 4.642 kN/m load normal to the beam axis is equal to the N diagram in Figure 13.33b, but with values that are 4.642 times as large. The N diagram in Figure 13.39b due to the load of 1.857 kN/m parallel to the beam axis is equal to the N diagram in Figure 13.35b, but then with values that are 1.857 times as large. Superposing the N diagrams in Figures 13.39a and 13.39b gives the N diagram in Figure 13.39c. This is the requested N diagram due to the dead weight $q_{\text{dw}} = 5$ kN/m.

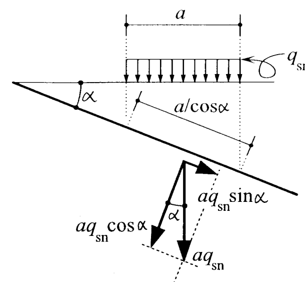


Figure 13.40 The components of the snow load q_{sn} on a member segment with length a measured horizontally.

c. *Snow load*

Over a length a measured horizontally, the resultant of the snow load is aq_{sn} (see Figure 13.40). The components of this force normal to and parallel to the axis are respectively $aq_{\text{sn}}\cos\alpha$ and $aq_{\text{sn}}\sin\alpha$. They act on a member segment with length $a/\cos\alpha$. For the components of the distributed load normal to and parallel to the beam axis we now find

$$q_{\text{sn;tr}} = \frac{aq_{\text{sn}}\cos\alpha}{a/\cos\alpha} = q_{\text{sn}}\cos^2\alpha,$$

$$= (5\ \text{kN/m})\cos^2 21.8^\circ = 4.310\ \text{kN/m},$$

$$q_{\text{sn;pa}} = \frac{aq_{\text{sn}}\sin\alpha}{a/\cos\alpha} = q_{\text{sn}}\sin\alpha\cos\alpha,$$

$$= (5\ \text{kN/m})\sin 21.8^\circ\cos 21.8^\circ = 1.724\ \text{kN/m}.$$

The distributed load $q_{\text{sn}} = 5$ kN/m due to the snow (Figure 13.41a) has components of 4.310 kN/m normal to the beam axis (Figure 13.41b) and 1.724 kN/m parallel to the beam axis (Figure 13.41c).

The bending moment in ACD is caused by the load of 4.310 kN/m normal to the beam axis. The M diagram is equal to that in Figure 13.33d, but now 4.310 times as large, so that the extreme values of the bending moment are:

$$M_{\text{sn;max}} = (2.558\ \text{kNm}) \times 4.310 = 11.02\ \text{kNm}\ (\smile),$$

$$M_{\text{sn;min}} = (2.320\ \text{kNm}) \times 4.310 = 10\ \text{kNm}\ (\frown).$$

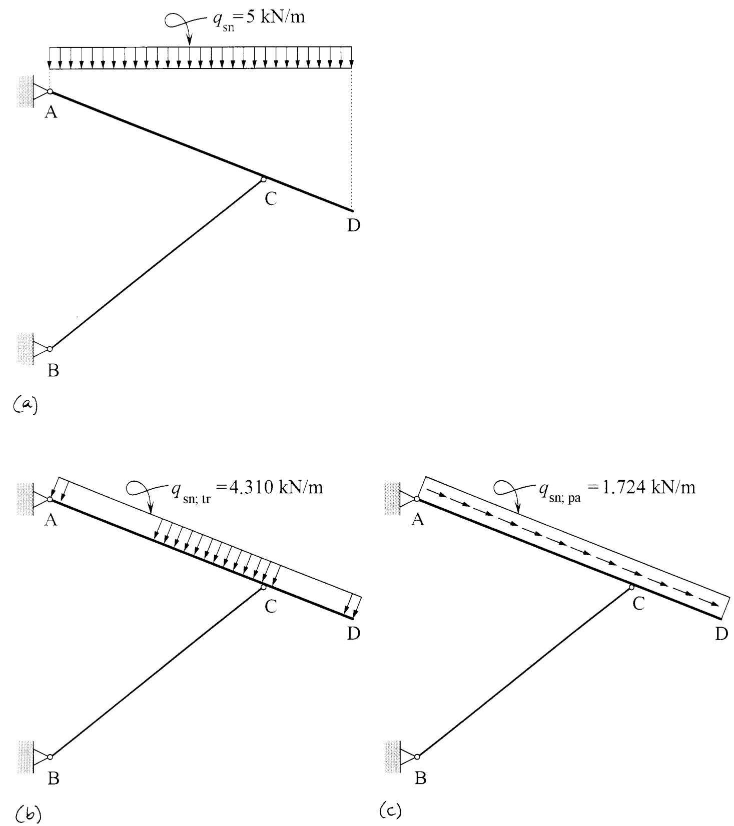


Figure 13.41 (a) The snow load of 5 kN/m on the lean-to, resolved into components (b) normal to the roof plane and (c) parallel to the roof plane.

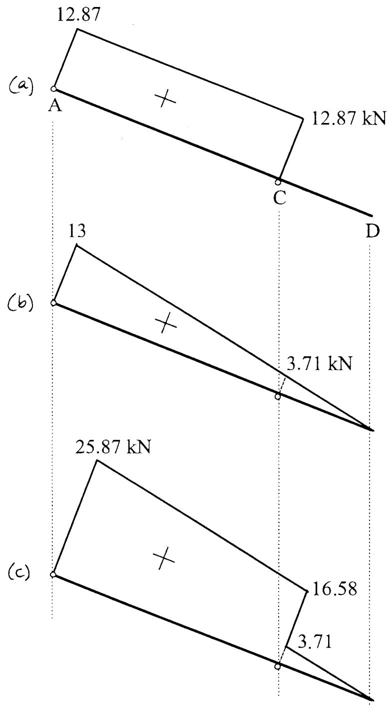


Figure 13.42 (a) The N diagram due to the component normal to the roof plane, superpositioned on (b) the N diagram due to the component parallel to the roof plane gives (c) the requested N diagram resulting from the snow load.

The N diagram in Figure 13.42a due to the load of 4.310 kN/m normal to the beam axis is equal to the N diagram in Figure 13.33b, but with values that are 4.310 times as large. The N diagram in Figure 13.42b due to the load of 1.724 kN/m parallel to the beam axis is equal to the N diagram in Figure 13.35b, but with values that are 1.724 times as large. By superposing the N diagrams in Figures 13.42a and 13.42b on one another we get the N diagram in Figure 13.42c. This is the requested N diagram due to the snow loading $q_{\text{sn}} = 5$ kN/m.

13.1.12 Indirectly loaded beam

With indirectly loaded beams, the load does not act on the beam directly, but is rather transferred to the beam by means of a system of stringer beams and cross beams.

Figure 13.43 shows a schematic representation of a bridge constructed as an indirectly loaded beam. *Main beam* (mb) AB is carrying *cross beams* (cb) at regular distances which in turn are carrying *stringer beams* (sb).

The main beam is divided into a number of fields by the cross beams, five in Figure 13.43. It is assumed that the lengths of the stringer beams are equal to the field lengths, and that the stringer beams are simply supported at the cross beams.

Since the main beam is loaded only by forces exerted by the cross beams, the shear force in each field is constant, and the bending moment in each field is linear (excluding the dead weight of the main beam).

For the indirectly loaded beam in Figure 13.43, the M and V diagrams are determined for the following two loading cases:

1. a concentrated load,
2. a uniformly distributed full load.

Example 1

In Figure 13.44a, the main beam AB is indirectly loaded by a point load of 60 kN in field CD. The dimensions can be read from the figure.

Question:

Determine the *M* and *V* diagrams for the indirectly loaded main beam and for the stringer beams.

Solution:

The support reactions at A and B are 40 kN and 20 kN respectively (see Figure 13.44a).

In Figure 13.44b, the *M* diagram is shown for the *directly loaded* beam. The dashed line indicates how this deviates from the requested *M* diagram for the *indirectly loaded* beam.

The force of 60 kN in field CD exerts forces on the main beam of 20 kN and 40 kN via the cross beams in C and D respectively (see Figure 13.44f). The other cross beams do not exert any forces on the main beam. The *M* diagram due to these forces of 20 and 40 kN at C and D is equal to the *M* diagram due to the (resulting) force of 60 kN (this is the *M* diagram for the directly loaded beam), with the exception of field CD. In field CD, the *M* diagram varies linearly between the values of 120 kNm at C and 180 kNm at D. The *M* diagram of the indirectly loaded beam can therefore be found by snipping the *M* diagram of the directly loaded beam over field CD.

The snipped part of the *M* diagram is equal to the *M* diagram of the simply supported stringer beam CD (see Figure 13.44c).

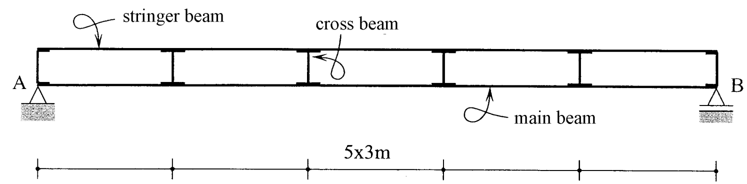


Figure 13.43 A bridge constructed as an indirectly loaded beam. The load on the stringer beams is transferred to the main beams via crossbeams.

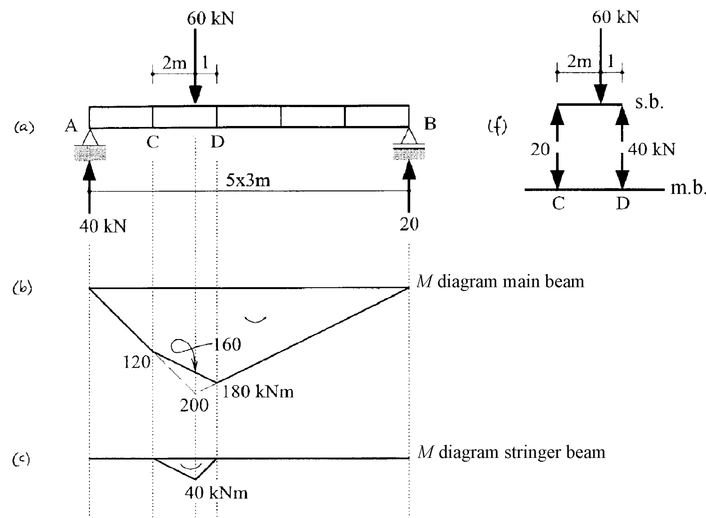


Figure 13.44 (a) Indirectly loaded beam AB, loaded in field CD by a point load. (b) The bending moment diagram of the indirectly loaded beam can be found from the dashed bending moment diagram of the directly loaded beam by snipping it between C and D. (c) The bending moment diagram of stringer beam CD is equal to the difference between the bending moment diagrams for the directly and indirectly loaded beam.

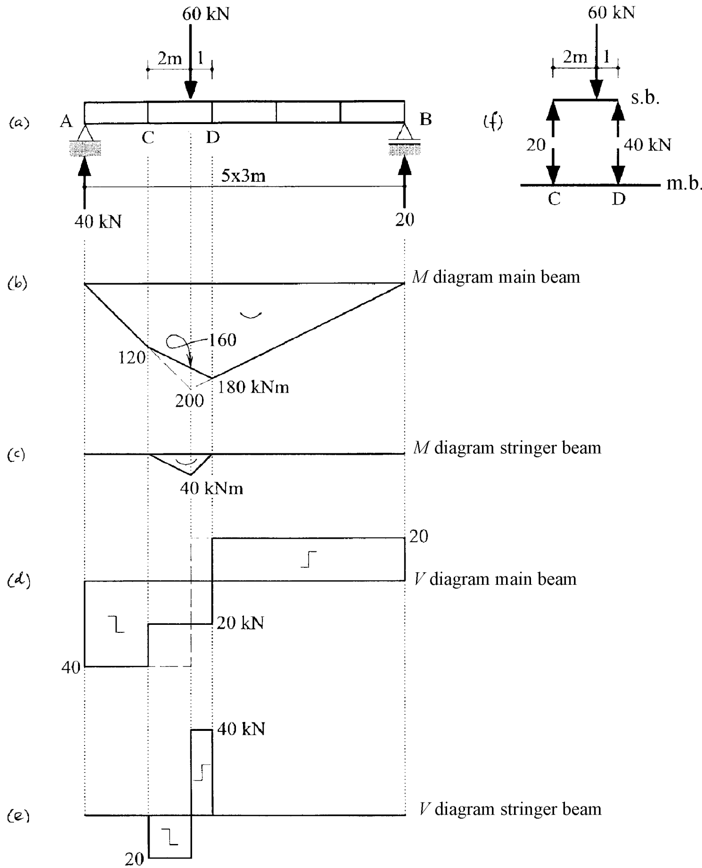


Figure 13.44

The V diagram is found from the slope of the M diagram (see Figure 13.44d). The V diagram for the *indirectly loaded* beam deviates from the dashed V diagram for the *directly loaded* beam only in the field CD.

The area enclosed in field CD between dashed and solid lines (the V diagrams for the directly and indirectly loaded beam respectively) is exactly the same as the V diagram for the simply supported stringer beam CD (see Figure 13.44e). The V diagram for the indirectly loaded beam can therefore be found by reducing the shear force of the directly loaded beam in field CD by the shear force in the stringer beam.

Example 2

In Figure 13.45a the indirectly loaded beam AB is carrying a uniformly distributed load of 16 kN/m. The dimensions are found in the figure.

Question:

Determine the M and V diagrams for the indirectly loaded main beam and for the stringer beams.

Figure 13.44 (a) Indirectly loaded beam AB, loaded in field CD by a point load. (b) The bending moment diagram of the indirectly loaded beam can be found from the dashed bending moment diagram of the directly loaded beam by snipping it between C and D. (c) The bending moment diagram of stringer beam CD is equal to the difference between the bending moment diagrams for the directly and indirectly loaded beam. (d) The shear force diagram of the indirectly loaded beam can be found from the slopes of the associated bending moment diagram. (e) The shear force diagram of the stringer beams can be found from the slopes of the associated bending moment diagram and is equal to the difference between the shear force diagrams for the directly and indirectly loaded beam. (f) The forces exerted via the cross beams in C and D on the main beam are found from the equilibrium of the stringer beam CD.

Solution:
The support reactions at A and B are 120 kN (see Figure 13.45a).

In Figure 13.45b, the *M* diagram for the *directly loaded* beam is shown. The values at the cross beams can be determined directly from the equilibrium or (because the *M* diagram is parabolic) by means of the formula $M = \frac{1}{2}qab$, in which a is the distance to A and b is the distance to B (see Section 12.1.6). By *snipping* the *M* diagram for the directly loaded beam over the fields we can find the *M* diagram for the *indirectly loaded* beam.

The snipped part of the *M* diagram is equal to the *M* diagram of the simply supported stringer beams (see Figure 13.45c).

The *V* diagram is found from the slope of the *M* diagram (see Figure 13.45d). The *V* diagram for the directly loaded beam is shown by means of a dashed line. The difference between both *V* diagrams is equal to the *V* diagram of the simply supported stringer beams (see Figure 13.45e).

Note that the shear forces at the end fields are not equal to the support reactions. Half of the load on the end fields is not carried by the main beam but is transferred by the end cross beams directly to the supports.

Figure 13.45 (a) Indirectly loaded beam AB with a uniformly distributed load. (b) The bending moment diagram of the indirectly loaded beam is found by snipping the dashed (parabolic) bending moment diagram of the directly loaded cross beams. (c) The bending moment diagram of the stringer beams is equal to the difference between the bending moment diagrams for the directly and indirectly loaded beam. (d) The shear force diagram of the indirectly loaded main beam can be found from the slopes of the associated bending moment diagram. (e) The shear force diagram of the stringer beams can be found from the slopes of the associated bending moment diagram and is equal to the difference between the shear force diagrams for the directly and indirectly loaded beam.

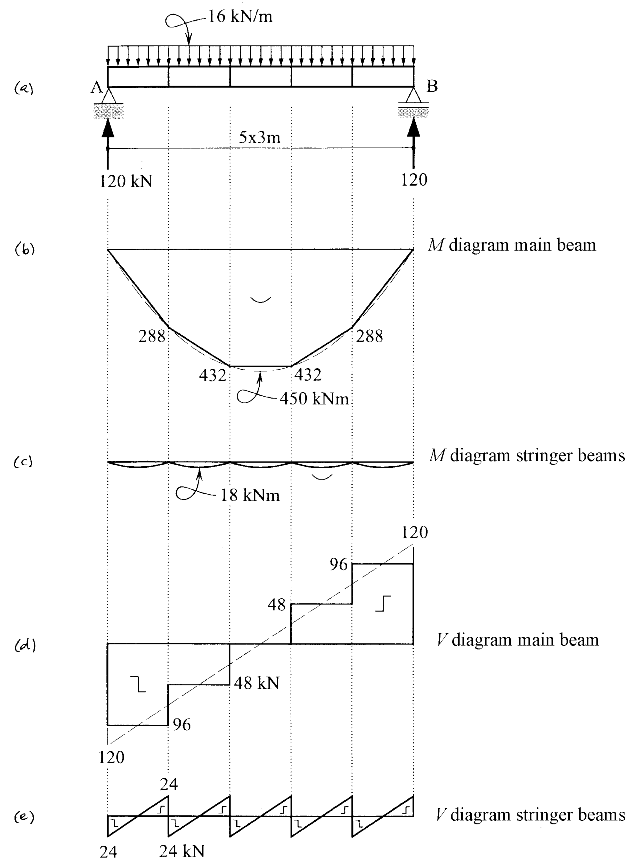


Figure 13.45

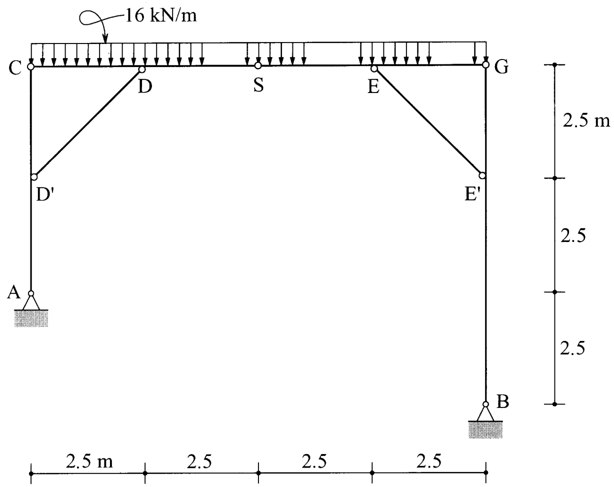


Figure 13.46 Three-hinged shored frame.

13.2 Compound and associated structures

To be able to draw the M, V and N diagrams for compound and associated structures, it is first necessary to determine the support reactions and the joining forces between the compound sections. Subsequently, the M, V and N diagrams can be determined and drawn for the constituent parts, in the same way as for the self-contained structures in Section 13.1. By then adding together the M, V and N diagrams of the constituent parts we can determine the requested M, V and N diagrams for the entire structure.

13.2.1 Three-hinged shored frame

The three-hinged shored frame ASB in Figure 13.46 is loaded over CDSEG by a uniformly distributed load of 16 kN/m. The dimensions are shown in the figure.

Questions:

a. Determine the support reactions at A and B.
b. Determine the forces in shores DD′ and EE′, with the correct signs for tension and compression.
c. Isolate CDSEG, and draw all the forces acting on it.
d. For CDSEG, draw the N, V and M diagrams with the deformation symbols and the tangents at C, D, S, E and G to the M diagram. Include relevant values.

Solution (units kN and m):

a. Determining the support reactions is left to the reader (see Section 5.3, Example 1).

b. In Figure 13.47, all parts of the frame have been isolated, and all the joining forces are shown. The support reactions at A and B are also shown. Both shores DD′ and EE′ are at 45°:

$$D_h = D_v = \tfrac{1}{2}N^{DD'}\sqrt{2},$$

$$E_h = E_v = \tfrac{1}{2}N^{EE'}\sqrt{2}.$$

D_h can be determined from the moment equilibrium of post AD′C about C:

$$\sum T|C \circlearrowleft = +32 \times 5 + D_h \times 2.5 = 0 \Rightarrow D_h = D_v = -64 \text{ kN}$$

so that

$$N^{DD'} = -64\sqrt{2} \text{ kN (a compressive force).}$$

In the same way, E_h can be determined from the moment equilibrium of post BE′G about G:

$$\sum T|G \circlearrowleft = -32 \times 7.5 - E_h \times 2.5 = 0 \Rightarrow E_h = E_v = -96 \text{ kN}$$

so that

$$N^{EE'} = -96\sqrt{2} \text{ kN (a compressive force).}$$

The forces in C and G follow from the force equilibrium of AC and BG:

$$C_h = -32 - D_h = +32 \text{ kN},$$

$$C_v = +72 + D_v = +8 \text{ kN},$$

$$G_h = -32 - E_h = +64 \text{ kN},$$

$$G_v = +88 + E_v = -8 \text{ kN}.$$

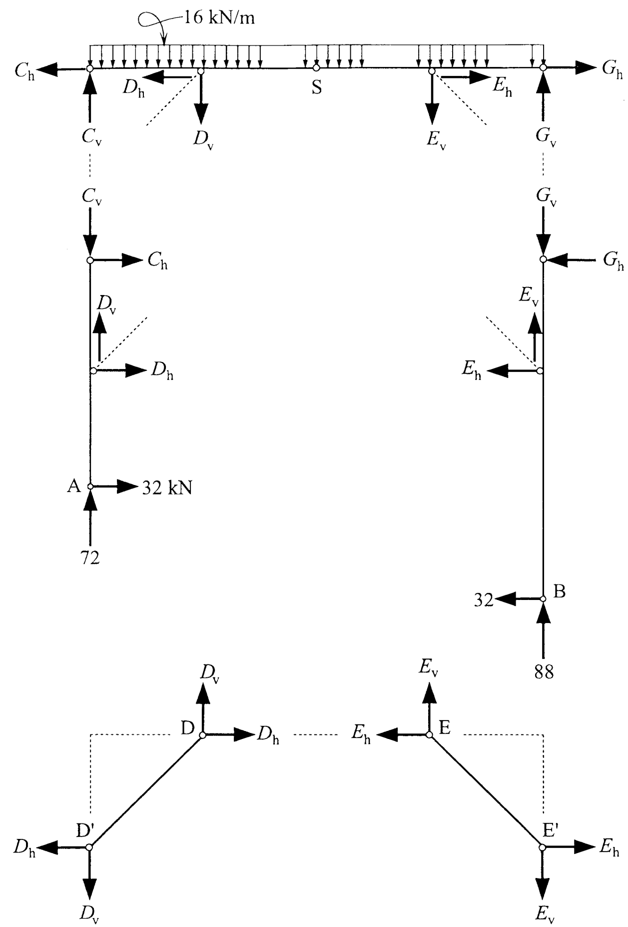


Figure 13.47 Girder, posts and shores isolated from the three-hinged shored frame, with joining forces and support reactions.

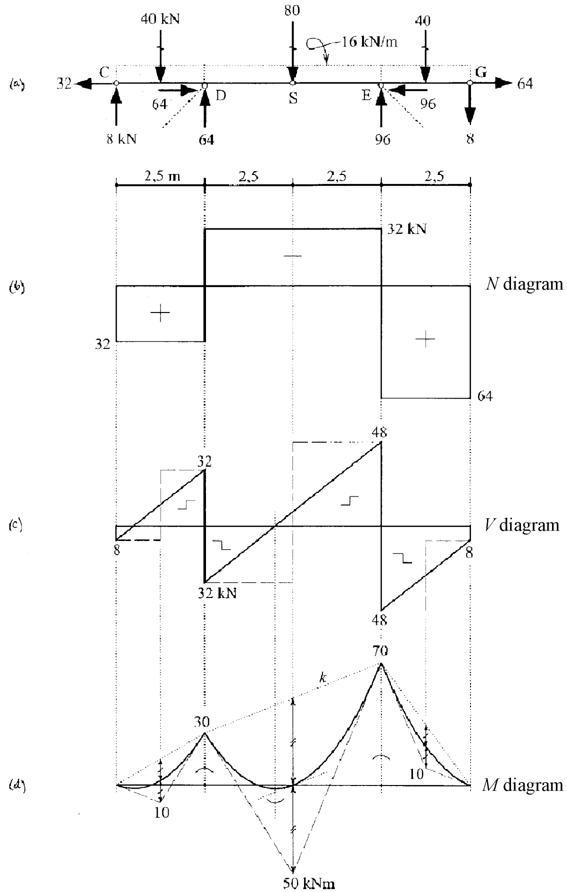


Figure 13.48 (a) Isolated girder with the associated (b) normal force diagram, (c) shear force diagram and (d) bending moment diagram.

c. In Figure 13.48a, CDSEG has been isolated, and all the forces are shown. The force and moment equilibrium of CDSEG can be used to check the correctness of the forces determined above.

d. In Figures 13.48b to d the N, V and M diagrams are shown for CDSEG. Three fields are distinguished: CD, DSE and EG.

In each field the normal force is constant. In CD and EG, the normal force is a tensile force, while in DSE it is a compressive force.

The shear force is linear in each field, and the bending moment is parabolic. The V and M diagrams due to the resultants of the field loads are shown in Figure 13.48c and d by means of dashed lines.

The V diagram has the same slope in all fields, equal to the distributed load of 16 kN/m.

The dashed M diagram due to the load resultants gives the tangents at A, D, E and G. The parabola in field DE passes through hinge S, since $M = 0$. Here, in the middle of field DE, the tangent is parallel to the chord k between the M values at D and E.

Note that the M diagram at C and G has no horizontal tangents as the shear force is not zero.

Check M diagram:
Per field, $p = \frac{1}{8}q\ell^2$ applies for the rise p of the parabolic M diagram.

13.2.2 Three-hinged frame with tie rod

Figure 13.49 shows a three-hinged frame ASB with tie rod AB. Girder CSD is carrying a uniformly distributed full load of 25 kN/m. The dimensions are shown in the figure.

Questions:

a. Determine the support reactions at A and B and the force in tie rod AB.
b. Isolate frame ASB, and draw all the forces acting on it at A and B.
c. For ASB, draw the M, V and N diagrams, with the deformation symbols. Include relevant values. At D, S and C, draw the tangents to the M diagram.
d. Determine the maximum bending moment in field CSD.

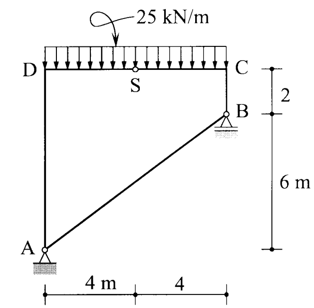


Figure 13.49 Three-hinged frame with a tie rod.

Solution (units kN and m):

a. The support reactions at A and B are forces of 100 kN aimed upwards. The calculation is left to the reader. In Figure 13.50, frame ASB has been isolated at A and B. In addition to the support reactions of 100 kN, there are also joining forces exerted by the tie rod AB. With a tensile force N in the tie rod, the horizontal forces exerted on ASB at A and B are $\frac{4}{5}N$, and the vertical forces are $\frac{3}{5}N$, as shown in Figure 13.50. N can be found from the moment equilibrium about S of ADS or BCS. The moment equilibrium of BCS about S gives:

$$\sum T|S \circlearrowright = +100 \times 2 + \tfrac{4}{5}N \times 2 + \tfrac{3}{5}N \times 4 - 100 \times 4 = 0$$

$$\Rightarrow N = +50 \text{ kN}.$$

In bar AB, there is therefore a tensile force of 50 kN.

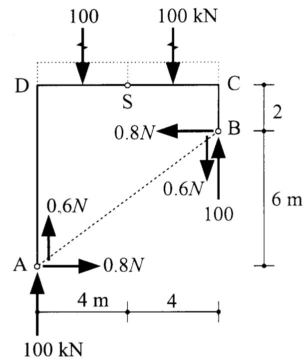


Figure 13.50 Three-hinged frame ASB with the support reactions and the forces exerted by tie rod AB at A and B.

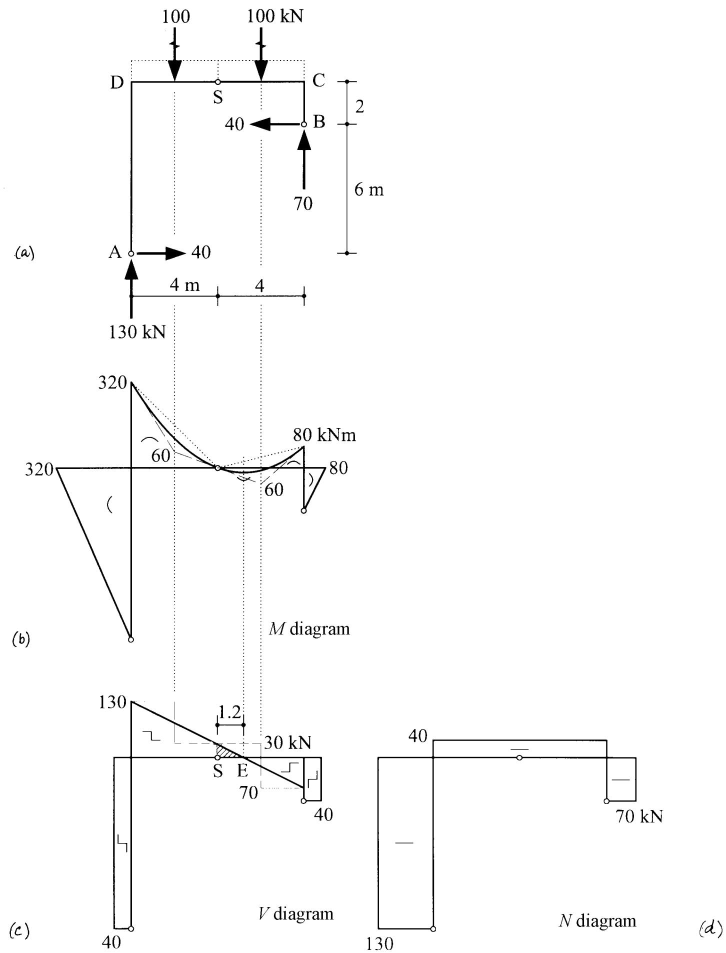


Figure 13.51 (a) Isolated three-hinged frame ASB with the resulting forces at A and B and the associated (b) bending moment diagram, (c) shear force diagram and (d) normal force diagram.

b. In Figure 13.51a, the resulting forces on the isolated frame ASB are shown, and the associated M, V and N diagrams are shown in Figures 13.51b to 13.51d.

c. To draw the M, V and N diagrams due to the forces at A and B, and the resultants of the field loads on CS and DS please refer to Section 5.3, Example 5.

The M and V diagrams due to the resultants of the field loads on CS and DS are not correct for girder CSD. They are therefore shown by means of dashed lines in Figures 13.51b and 13.51c. For the uniformly distributed load, the shear force over DSC is linear and the bending moment is parabolic. The dashed M and V diagrams give the correct values of V and M at C, S and D, and the correct tangents to the M diagram.

d. The zero shear force in CSD is found at E, $\frac{30}{30+70} \times 4 = 1.2$ m to the right of S (see Figure 13.51c). Here the bending moment is an extreme. Because the bending moment at S is zero, the bending moment at E is equal to the (hatched) area of the V diagram for SE:

$$M_{\mathrm{E}} = \tfrac{1}{2} \times 1.2 \times 30 = 18 \text{ kNm } (\smile).$$

This maximum field moment, which is significantly smaller than the boundary moments at C and D, can of course also be found from the moment equilibrium of the frame part ADSE to the left of E, or ECB to the right of E.

13.2.3 Trussed beam

The trussed beam ABSC in Figure 13.52 is carrying a uniformly distributed load of 12 kN/m. The dimensions of the structure are shown in the figure.

Questions:

a. Determine the forces in the members AD, BD and CD with the correct signs for tension and compression. Include the force polygon for joint D.
b. Isolate ABSC, and draw all the forces acting on it.
c. For ABSC, draw the M, V and N diagrams, with the deformation symbols. At A, B, S and C draw the tangents to the M diagram.
d. Determine the maximum bending moment in ABSC.

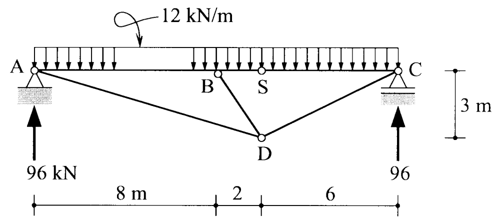


Figure 13.52 Trussed beam ABC with uniformly distributed load.

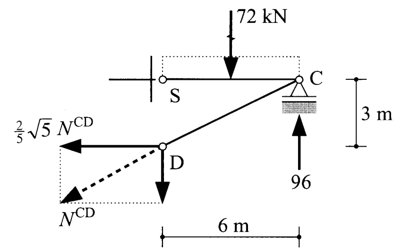


Figure 13.53 Normal force N^{CD} in two-force member CD is found from the moment equilibrium of part SCD about S.

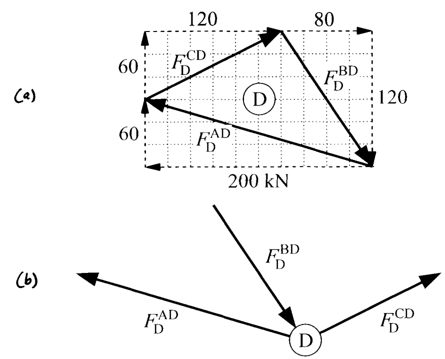


Figure 13.54 (a) Force polygon for the equilibrium of joint D. (b) Joint D with the forces exerted on it by members AD, BD and CD.

Solution (units kN and m):

a. In Figure 13.52 the support reactions are already shown. The calculation is left to the reader. In order to determine the normal forces in the two-force members AD, BD and CD, part SCD has been isolated in Figure 13.53. At D, the (normal) force N^{CD} has been resolved into its components. The force N^{CD} is found from the moment equilibrium of the isolated part about S:

$$\sum T|S \circlearrowright = +72 \times 3 - 96 \times 6 + \tfrac{2}{5}\sqrt{5} \times N^{CD} \times 3 = 0$$

so that:

$$N^{CD} = +60\sqrt{5}\ \text{kN}\ (= +134.2\ \text{kN}).$$

The normal forces in the members AD and BD can now be determined from the force equilibrium of joint D. To do so, we have created the closed force polygon in Figure 13.54a. We first set down the tensile force $F_D^{CD} = N^{CD}$, which is the force member CD exerts on joint D. Next, the force polygon is closed with two lines parallel to AD and BD.

The magnitude of the forces F_D^{AD} and F_D^{BD} can be found from the force polygon. In order to interpret them as normal forces N, with the correct signs (positive as tensile force and negative as compressive force), we first have to check whether the forces F_D^{AD} and F_D^{BD} from the force polygon

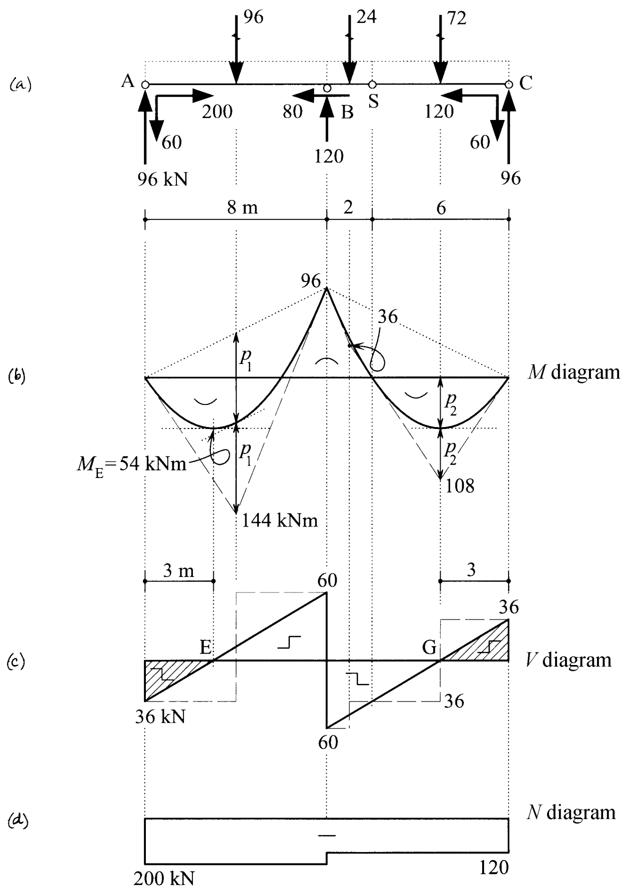


Figure 13.55 (a) Isolated beam ASC with (b) bending moment diagram, (c) shear force diagram and (d) normal force diagram.

exert tension or compression on joint D (see Figure 13.54b). We find

$$N^{AD} = +F_D^{AD} = +20\sqrt{109}\text{ kN } (= +208.8\text{ kN}),$$

$$N^{BD} = -F_D^{BD} = -40\sqrt{13}\text{ kN } (= -144.2\text{ kN}).$$

There is therefore tension in AD and CD and compression in BD.

b. In Figure 13.55a, the beam ABSC has been isolated. At A and C, in addition to the support reactions, the components of the tensile forces in AD and CD are also active. At B the components of the compressive force in BD are acting. In the figure, the distributed loads in the fields AB, BS and SC have been replaced by their resultants.

c. In Figures 13.55b to 13.55d the M, V and N diagrams are shown. The M and V diagrams due to the resultants of the field loads are shown by means of dashed lines. They give the correct values in the field boundaries. Here the dashed M diagram also gives the tangents.

The final M diagram shown in Figure 13.55b with a solid line, can be checked using the rise p of the parabolas for both fields:

$$p_1 = \tfrac{1}{8} \times 12 \times 8^2 = 96\text{ kNm},$$

$$p_2 = \tfrac{1}{8} \times 12 \times 6^2 = 54\text{ kNm}.$$

These values of p fit in the M diagram shown.

Note that in Figure 13.55c the shear forces at the supports A and C are not equal to the support reactions there. This is caused by the vertical components of the member forces in AD and CD.

Also note that the shear force in all fields has the same slope, equal to the distributed load of 12 kN/m.

There is a compressive force over the entire length of beam ABSC (see Figure 13.55d). At B, a step change in the N diagram occurs due to the horizontal component of the member force in BD.

d. The largest bending moment in an absolute sense is the *support moment* at B:

$$M_B = 96 \text{ kNm } (\frown).$$

In addition, there are extreme *field moments* at E and G three metres from the supports, where the shear force is zero (see Figure 13.55c). The easiest way to find their magnitudes is from the hatched area of the V diagram:

$$M_E = M_G = \tfrac{1}{2} \times 3 \times 36 = 54 \text{ kNm } (\smile).$$

M_G is also equal to the maximum bending moment in the simply supported beam SC with uniformly distributed full load:

$$M_G = \tfrac{1}{8} \times 12 \times 6^2 = 54 \text{ kNm } (\smile).$$

Note that the M diagram has mirror symmetry about B.

13.2.4 Sideways-supported mast

The mast ABC in Figure 13.56a is supported sideways by a number of bars. Dimensions and load are shown in the figure.

Questions:

a. Determine the support reactions at A and D.
b. Determine the forces in the bars 1 to 3, with the correct signs for tension and compression.
c. Isolate beam ABC, and draw all the forces acting on it.
d. For beam ABC draw the M, V and N diagram, with the deformation

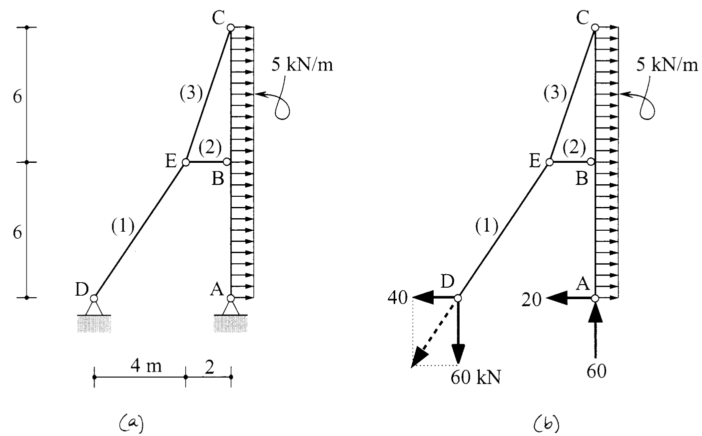


Figure 13.56 (a) Sideways-supported mast ABC with (b) its support reactions.

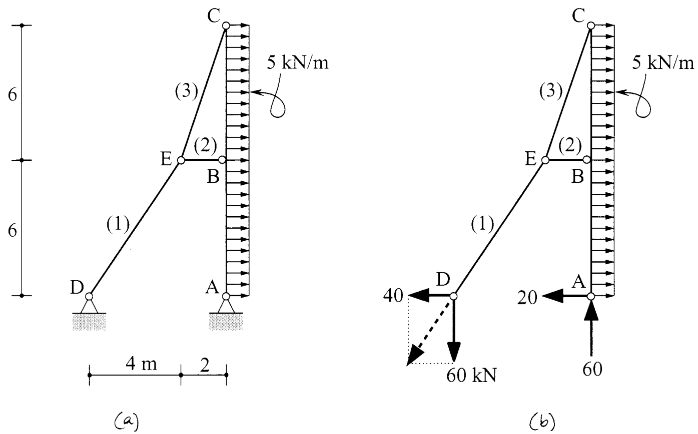


Figure 13.56 (a) Sideways-supported mast ABC with (b) its support reactions.

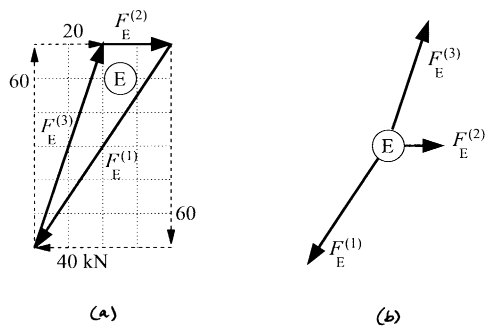


Figure 13.57 (a) Force polygon for the equilibrium of joint E. (b) Joint E with the forces exerted on it by bars (1), (2) and (3).

symbols. At A, B and C, also draw the tangents to the M diagram.

e. Determine the extreme moments in ABC.

Solution (units kN and m):

a. The vertical support reaction D_v (↓) at D follows from the moment equilibrium of the entire structure about A:

$$\sum T|A \circlearrowleft = -(12 \times 5) \times 6 + D_v \times 6 = 0 \Rightarrow D_v = 60 \text{ kN } (\downarrow).$$

Bar (1) is a two-force member so that the line of action of the support reaction at D coincides with DE. The horizontal component D_h is therefore

$$D_h = \tfrac{4}{6} \times D_v = 40 \text{ kN } (\leftarrow).$$

The support reactions at A follow from the force equilibrium of the structure as a whole:

$$A_h = 20 \text{ kN } (\leftarrow),$$

$$A_v = 60 \text{ kN } (\uparrow).$$

The support reactions are shown in Figure 13.56b.

b. The support reactions at D show that there is a tensile force in bar (1):

$$N^{(1)} = +\sqrt{40^2 + 60^2} = +20\sqrt{13} \text{ kN } (= +72.11 \text{ kN}).$$

The (normal) forces in bars (2) and (3) can now be determined from the force equilibrium of joint E. To do so we have to draw the force polygon for joint E (see Figure 13.57a). The force $F_E^{(1)} = N^{(1)} = 20\sqrt{13}$ kN, which bar (1) exerts on joint E, is known. We close the force polygon with the forces $F_E^{(2)}$ and $F_E^{(3)}$, parallel to the two-force members (2) and (3). Figure 13.57b shows that all the forces in the force polygon are tensile forces:

$$N^{(2)} = +F_{\mathrm{E}}^{(2)} = +20\ \mathrm{kN},$$

$$N^{(3)} = +F_{\mathrm{E}}^{(3)} = +20\sqrt{10}\ \mathrm{kN}\ (= +63.24\ \mathrm{kN}).$$

c. In Figure 13.58a, ABC has been isolated and all the forces acting on it have been shown. For the fields AB and BC, the resultants of the distributed load are also shown.

d. In Figure 13.58b to d the M, V and N diagrams are shown. The dashed M and V diagrams, which are determined first, are an important tool for drawing the actual M and V diagrams. The answer is left to the reader. The value p can be used to check the M diagram shown:

$$p = \tfrac{1}{8} \times 5 \times 6^2 = 22.5\ \mathrm{kNm}.$$

Note that the M diagram has mirror symmetry about B.

e. The bending moment is an extreme at G and H, where the shear force is zero, and at B, where the shear force changes sign (see Figure 13.58c).

The M diagram in Figure 13.58b gives

$$M_{\mathrm{B}} = 30\ \mathrm{kNm}\ (\,)\,).$$

The actual maximum bending moment occurs at G and H, 4 m from the ends A and B, and is most easily determined from the area of the V diagram:

$$M_{\mathrm{G}} = M_{\mathrm{H}} = M_{\mathrm{max}} = \tfrac{1}{2} \times 4 \times 20 = 40\ \mathrm{kNm}\ (\,)\,).$$

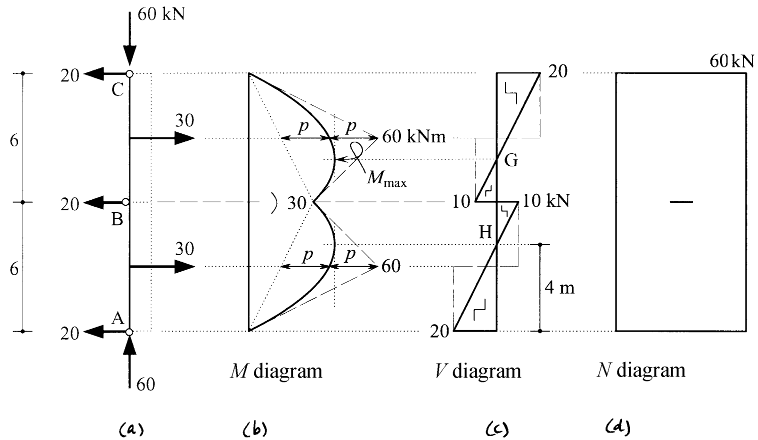


Figure 13.58 (a) Isolated mast ABC with (b) bending moment diagram, (c) shear force diagram and (d) normal force diagram.

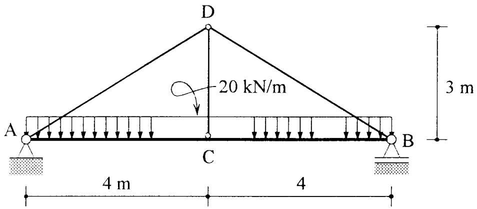


Figure 13.59 Statically indeterminate trussed beam with uniformly distributed load.

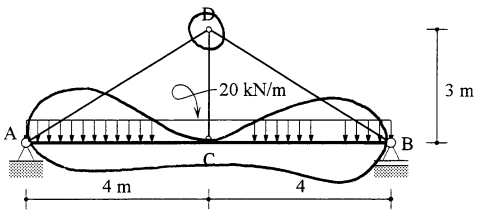


Figure 13.60 Three joining forces are acting between joint D and beam ACB, namely the normal forces in the three two-force members.

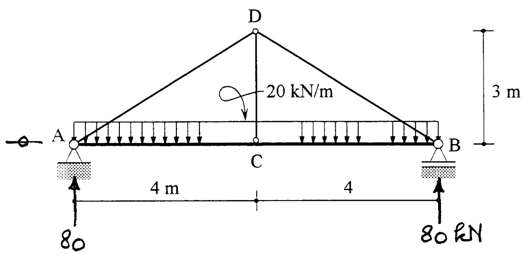


Figure 13.61 Support reactions.

13.3 Statically indeterminate structures

With statically indeterminate structures, it is not possible to determine all the support reactions and joining forces directly from the equilibrium, as there are too few equilibrium equations. In this section, sufficient support reactions and/or joining forces are given in magnitude and direction for a statically indeterminate structure so that all the other support reactions and joining forces can be determined with the available number of equilibrium equations. Thereafter, it is possible to determine and draw the M, V and N diagrams for the entire structure.

13.3.1 Trussed beam with a given normal force

The dimensions and load for the trussed beam ACB can be found in Figure 13.59. For the given load, there is a tensile force of 60 kN in member CD.

Questions:

a. Determine the degree of static indeterminacy of the structure.
b. Isolate beam ACB, and draw all the forces acting on it.
c. For ACB, draw the N, V and M diagrams, with the deformation symbols. At A, B and C also draw the tangents to the M diagram.

Solution:

a. In Figure 13.60, beam ACB and joint D have been isolated from one another. There are $v = 3$ unknown joining forces acting between beam ACB and joint D: the normal forces N^{AD}, N^{BD} and N^{CD}. In addition, there are $r = 3$ support reactions, namely A_h, A_v and B_v. That makes a total of $r + v = 6$ unknowns. Beam ACB provides three equilibrium equations (force equilibrium and moment equilibrium); joint D provides two (force equilibrium). In total, there are therefore $e = 5$ equilibrium equations available. The degree of static indeterminacy n is equal to the difference between the number of unknowns and the number of available equilibrium equations:

$$n = (r + v) - e = 6 - 5 = 1.$$

The structure is therefore statically indeterminate to the first degree.

b. In Figure 13.61 the support reactions are shown; they follow from the equilibrium of the structure as a whole.

If it is known that a tensile force of 60 kN is acting in CD, the normal forces in two-force members AD and BD can be determined from the equilibrium of joint D. In Figure 13.62a the closed force polygon is shown for the equilibrium of the forces acting on joint D. Figure 13.62b shows how these forces are acting on the joint. This figure also shows whether the forces are tensile or compressive. The normal forces are

$$N^{AD} = -F_D^{AD} = -50 \text{ kN},$$
$$N^{BD} = -F_D^{BD} = -50 \text{ kN},$$
$$N^{CD} = +F_D^{CD} = +60 \text{ kN (given)}.$$

Figure 13.63a shows the isolated beam ACB. At A and B there are not only support reactions, but also (the components of) the compressive forces exerted by the members AD and BD.

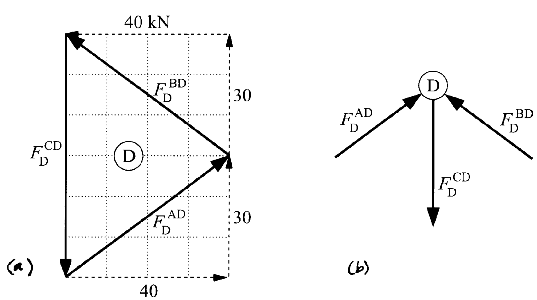


Figure 13.62 (a) Force polygon for the equilibrium of joint D. (b) Joint D with the forces exerted on it by members AD, BD and CD.

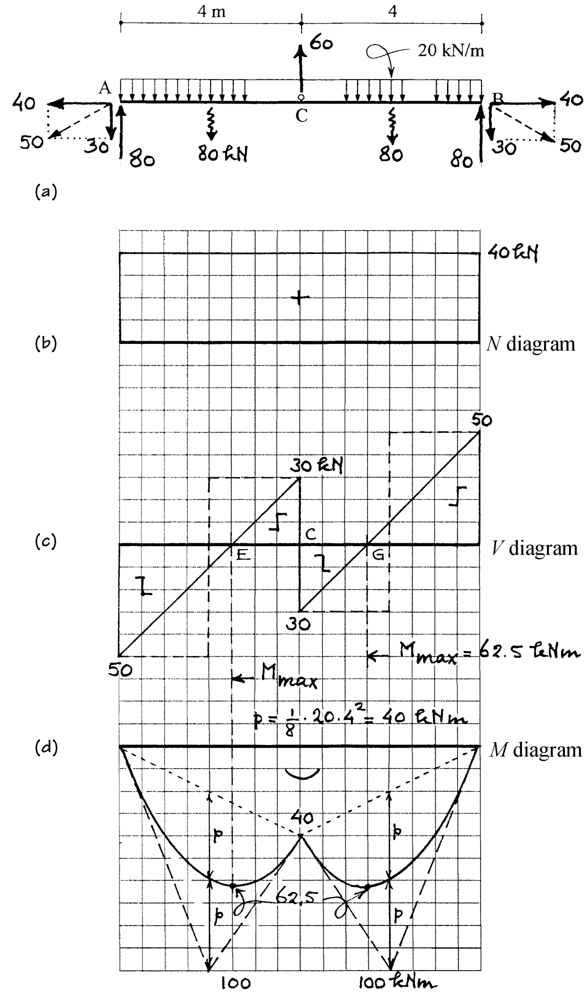


Figure 13.63 (a) Isolated beam ACB with (b) normal force diagram, (c) shear force diagram and (d) bending moment diagram.

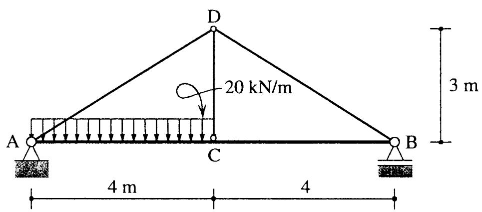


Figure 13.64 Statically indeterminate trussed beam with a uniformly distributed load on the left-hand side. We are given a zero bending moment at C.

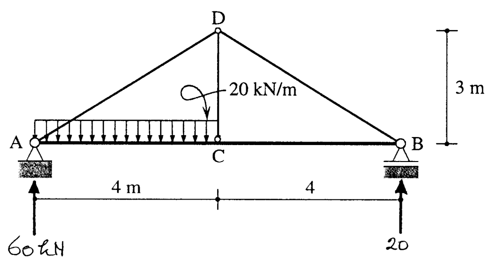


Figure 13.65 Support reactions.

c. The N, V and M diagrams are shown in Figures 13.63b to 13.63d. In beam ACD there is a tensile force of 40 kN. When drawing the V and M diagrams, we used the dashed V and M diagram associated with the load resultants of 80 kN in the fields AC and BC. The bending moment is an extreme at E and G, where the shear force is zero, and at C where the shear force changes sign:

$$M_{\text{min}} = M_{\text{C}} = 40 \text{ kNm } (\smile),$$

$$M_{\text{max}} = M_{\text{E}} = M_{\text{G}} = \tfrac{1}{2}(2.5 \text{ m})(50 \text{ kN}) = 62.5 \text{ kNm } (\smile).$$

13.3.2 Trussed beam with a given bending moment

The trussed beam ACB in Figure 13.64 is the same as that in the previous section, except it now has a different load. Further, we are given a zero bending moment at C.

Question:
For ACB draw the N, V and M diagrams, with the deformation symbols. At A, B and C also draw the tangents to the M diagram.

Solution (units kN and m):
The support reactions follow from the equilibrium of the structure as a whole, and are shown in Figure 13.65.

In the unloaded field BC, the bending moment (dependent on the shear force) is constant or linear. Since the bending moment is zero at both B and C (given), the bending moment must be zero throughout field BC.

Due to the uniformly distributed load, the bending moment in field AC is parabolic. In addition, the bending moment is zero at both A and C. This allows us to directly draw the M diagram for AB (see Figure 13.66d). At the middle of AC:

$$M = M_{\max} = p = \tfrac{1}{8} \times 20 \times 4^2 = 40 \text{ kNm}.$$

The V diagram can be determined from the M diagram. In field BC, the shear force is zero, in field AC it varies linearly. The shear forces at A and to the left of C are

$$\frac{2p}{\frac{1}{2}\ell^{\text{AC}}} = \frac{2 \times 40}{\frac{1}{2} \times 4} = 40 \text{ kN}.$$

Their deformation symbols follow from the slope of the M diagram (see Figure 13.66c).

Check: The shear forces found must agree with the support reactions of the simply supported beam AC.

The vertical force equilibrium of joint C gives (see Figure 13.67)

$$N^{\text{CD}} = +40 \text{ kN}.$$

Using the equilibrium of joint D, we can now find the normal forces in AD and BD. They turn out to be compressive forces:

$$N^{\text{AD}} = N^{\text{BD}} = -100/3 \text{ kN}.$$

The calculation is left to the reader.

In Figure 13.66a, beam ACB has been isolated, and all forces acting on it are shown. At A and B, there are acting support reactions and (components of the) compressive forces exerted by members AD and BD.

Check: By reducing the support reactions (pointed upwards) at A and B by the vertical component (pointed downwards) of these member forces we

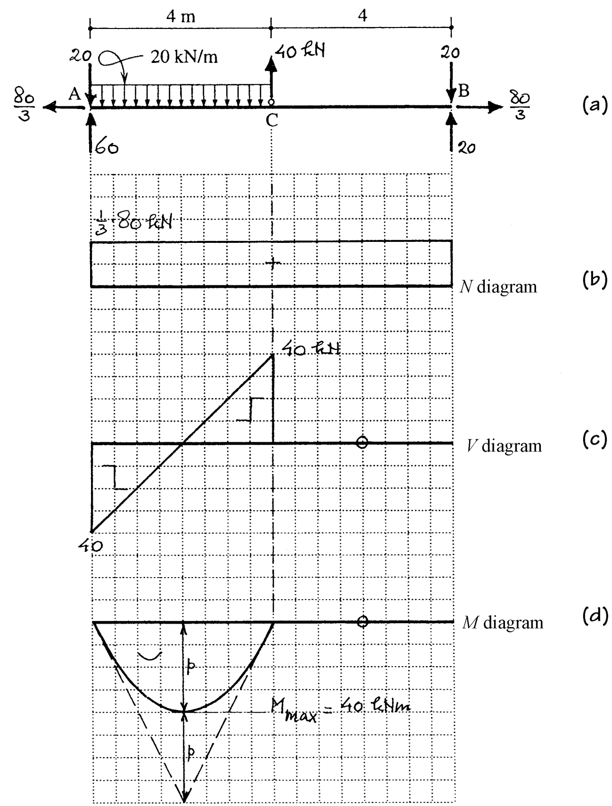


Figure 13.66 (a) Isolated beam ACB with (b) normal force diagram, (c) shear force diagram and (d) bending moment diagram.

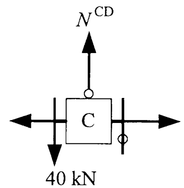


Figure 13.67 The normal force N^{CD} in member CD follows from the vertical force equilibrium of joint C.

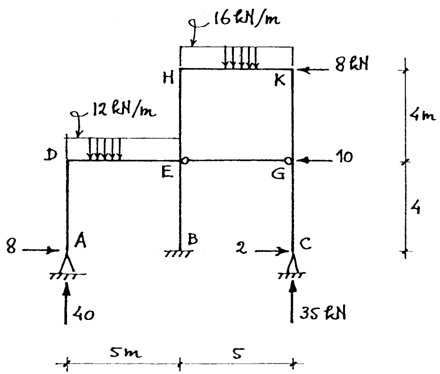


Figure 13.68 Statically indeterminate portal structure with the support reactions at A and C.

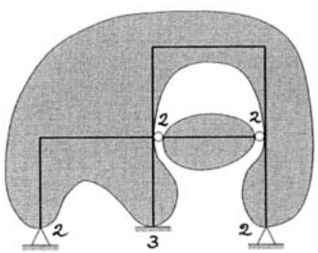


Figure 13.69 The portal structure consists of two singly-cohesive sub-structures.

find the same shear forces as from the V diagram.

The horizontal component of the compressive member forces at A and B results in a tensile force of 80/3 kN in ACB, see the N diagram in Figure 13.66b.

13.3.3 Portal structure with a number of given support reactions

With the load given, the support reactions at A and C for the structure are given in Figure 13.68.

Questions:

a. Determine the degree of static indeterminacy of the structure.
b. Determine the support reactions at B.
c. Isolate ADEBH, and draw all the forces acting on it, with the additional information of a compressive force of 6 kN acting in EG.
d. For ADE and BH, draw the M, V and N diagrams with the deformation symbols. At D and E, draw the tangents to the M diagram.

Solution (units kN and m):
a. The structure consists of two singly-cohesive sub-structures (see Figure 13.69). There are $v = 2 + 2 = 4$ unknown joining forces acting in the hinged joints between both sub-structures. In addition, there are $r = 2 + 3 + 2 = 7$ unknown support reactions. In total, that makes $r + v = 11$ unknowns. Each sub-structure offers three equilibrium equations, making a total of $e = 2 \times 3 = 6$ equilibrium equations available. The degree of static indeterminacy n is equal to the number of unknown joining forces and support reactions minus the number of available equilibrium equations:

$$n = (r + v) - e = 11 - 6 = 5.$$

The structure is therefore statically indeterminate to the fifth degree.

b. The support reactions at B follow from the equilibrium of the structure as a whole. For the assumed directions of B_h, B_v and B_m in Figure 13.70a, we find

$$\sum F_x = 8 + B_h + 2 - 10 - 8 = 0 \Rightarrow B_h = 8 \text{ kN},$$

$$\sum F_y = 40 + B_v + 35 - 80 - 60 = 0 \Rightarrow B_v = 65 \text{ kN},$$

$$\sum T_z|B = -40 \times 5 + B_m + 35 \times 5 + 10 \times 4$$
$$+ 8 \times 8 - 80 \times 2.5 + 60 \times 2.5 = 0 \Rightarrow B_m = -29 \text{ kN}.$$

The fixed-end moment reaction B_m is acting opposite to the direction assumed in Figure 13.70a. In Figure 13.70b, the support reactions at B are shown as they act in reality.

c. In Figure 13.71a, ADEBH has been isolated and all forces acting on it are shown. Additional information given is that member EG exerts a horizontal compressive force of 6 kN on joint E. The three joining forces at H can now be determined from the equilibrium of ADEBH, or (less laboriously) from the equilibrium of CGKH. The calculation is left to the reader.

d. In Figures 13.71b to 13.71d, the *N*, *M* and *V* diagrams for ADEBH are shown. When determining and drawing these lines, it is best to work from the member ends A, B and H to joint E. To verify the calculation, the force and moment equilibrium of joint E can be investigated. One could also first isolate all the members and determine all the joining forces at D and E. See for example Section 5.3, Example 3.

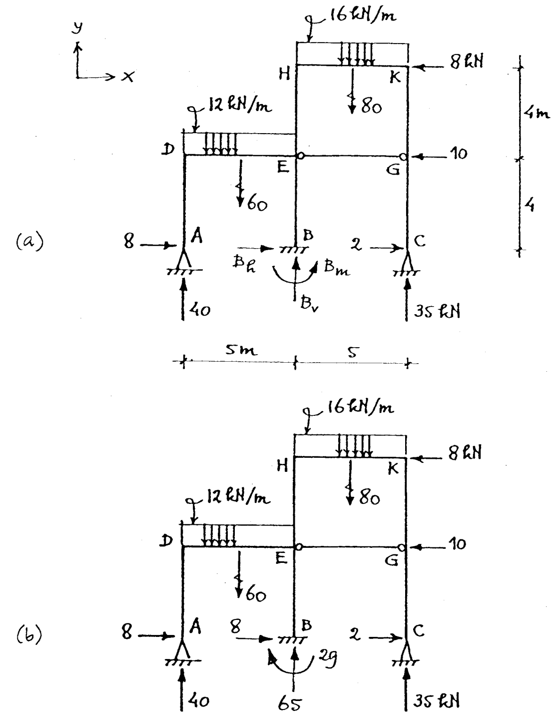


Figure 13.70 (a) Assumed and (b) calculated support reactions at B.

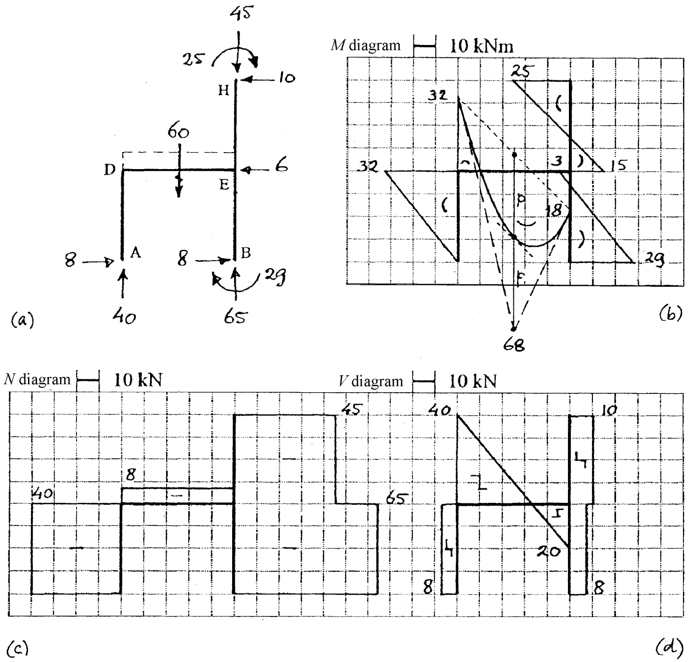


Figure 13.71 (a) Isolated part ADEBH with (b) bending moment diagram, (c) normal force diagram and (d) shear force diagram.

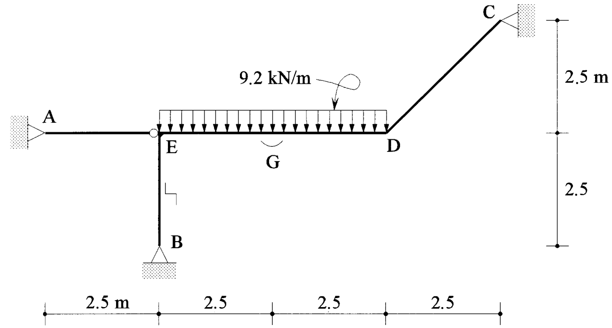


Figure 13.72 Statically indeterminate frame for which the bending moment at G, and the shear force in BE are given.

13.3.4 Frame with given shear force and bending moment

For the statically indeterminate structure in Figure 13.72 we are given the following:

- the bending moment at the middle G of DE: $M_G = 12.5$ kNm, and
- the shear force in BE: $V^{BE} = 5$ kN.

The associated deformation symbols are given in the figure, as are the measurements and the load.

Questions:

a. Determine the degree of static indeterminacy of the structure.
b. Determine the support reactions. Draw them as they are acting in reality on the structure.
c. Determine the normal force in CD.
d. Determine the N, V, and M diagrams for the entire structure, with the deformation symbols. At D, G and E draw the tangents to the M diagram.

Solution (units kN and m):

a. The structure BEGD is statically indeterminate to the second degree. The structure has five unknown support reactions while there are only three equilibrium equations.

b. The horizontal support reaction at B follows from the shear force in BE.

$$B_h = 5 \text{ kN } (\rightarrow).$$

Bar AE is a two-force member so that

$$A_v = 0.$$

Introduce a cut at G, and investigate the moment equilibrium of ABEG about G (see Figure 13.73):

$$\sum T|G \circlearrowright = +12.5 + 23 \times 1,25 + 5 \times 2.5 - B_v \times 2.5 = 0$$

$$\Rightarrow B_v = 21.5 \text{ kN } (\uparrow).$$

The support reactions at A and C are found from the equilibrium of the structure as a whole:

$$\sum F_{vert} = 0 \Rightarrow C_v = 24.5 \text{ kN } (\uparrow),$$

$$\sum T|A = 0 \Rightarrow C_h = 32.5 \text{ kN } (\rightarrow),$$

$$\sum F_{hor} = 0 \Rightarrow A_h = 37.5 \text{ kN } (\leftarrow).$$

In Figure 13.74, the support reactions are shown as they act in reality.

c. By resolving the horizontal and vertical support reaction at C into components parallel to and normal to CD we find the normal force N^{CD} and the

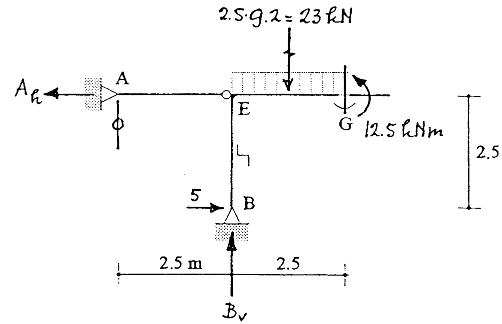


Figure 13.73 The vertical support reaction at B follows from the moment equilibrium of ABEG about G.

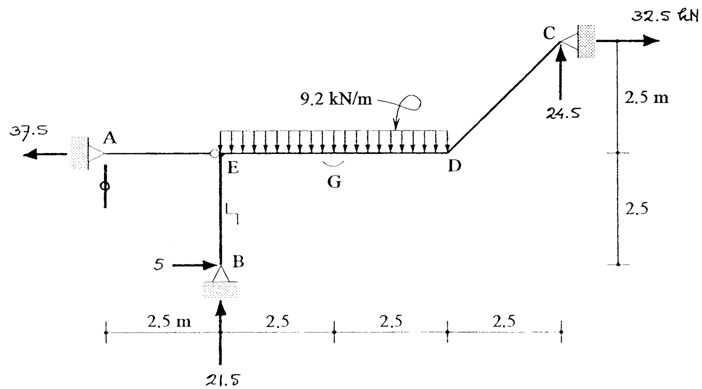


Figure 13.74 Support reactions.

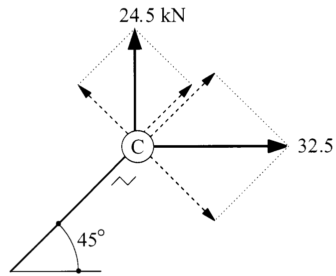


Figure 13.75 To determine the normal force and shear force in CD, the support reactions at C can be resolved into components normal to and parallel to member CD.

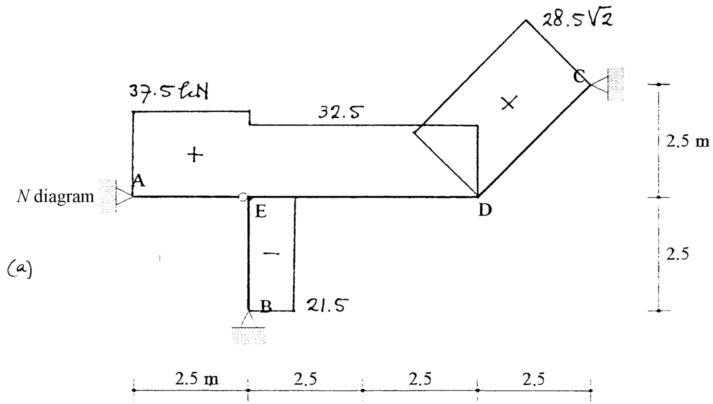


Figure 13.76 (a) Normal force diagram.

shear force V^{CD} (see Figure 13.75):

$$N^{CD} = \tfrac{1}{2}\sqrt{2} \times (32.5 + 24.5) = 28.5\sqrt{2}\text{ kN},$$

$$V^{CD} = \tfrac{1}{2}\sqrt{2} \times (32.5 - 24.5) = 4\sqrt{2}\text{ kN}.$$

The normal force is a tensile force; the deformation symbol for the shear force is given in Figure 13.75.

d. In Figures 13.76a to 13.76c, the N, V and M diagrams are shown. We provide a number of comments about the M and V diagrams below. At E and D, the bending moment "goes round the corner". At G, the tangent to the M diagram is parallel to the chord k of the parabola. The tangents at E and D are formed by the dashed M diagram due to the resultant of the distributed load on DE. The slope of this dashed M diagram gives the magnitude and the deformation symbol for the shear forces at E and D. The shear force in DE varies linearly between the values at D and E. The slope of the V diagram can be used as a check: it is equal to the distributed load. The maximum bending moment in DE is slightly to the left of the middle G of ED, and will be only marginally larger than M_G. From the area under the V diagram we find

$$M_{\max} = \tfrac{1}{2} \times 21.5 \times \tfrac{21.5}{21.5+24.5} \times 5 - 12.5 = 12.62\text{ kNm } (\smile).$$

13.3.5 Frame with two given shear forces

The statically indeterminate structure in Figure 13.77 has a hinged joint at S. All other joints are rigid. Dimensions and loads are given in the figure. The shear forces directly next to joint C are given:

$$V_C^{BC} = 2.5\sqrt{2}\text{ kN},$$

$$V_C^{CS} = 10\text{ kN}.$$

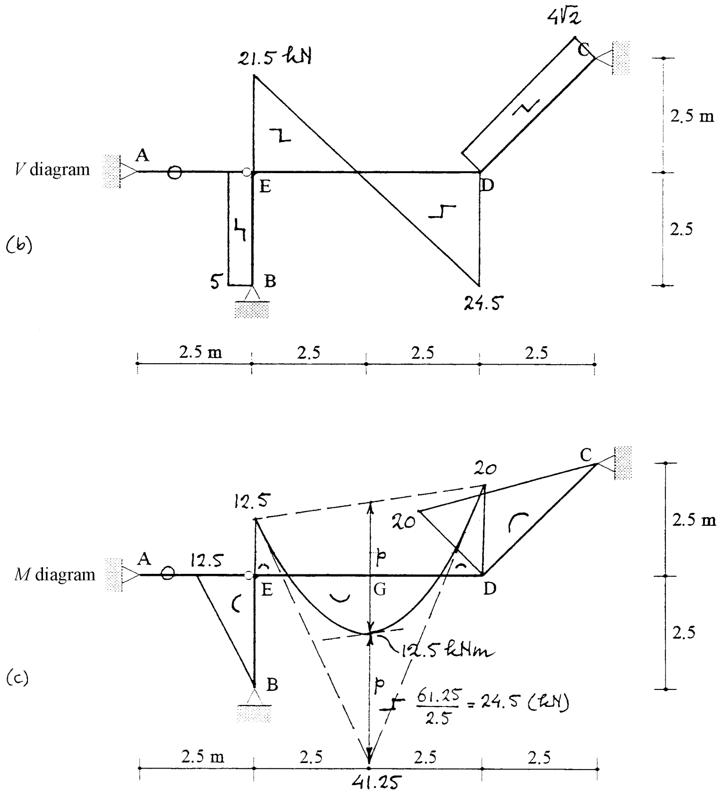


Figure 13.76 (b) Shear force diagram and (c) bending moment diagram.

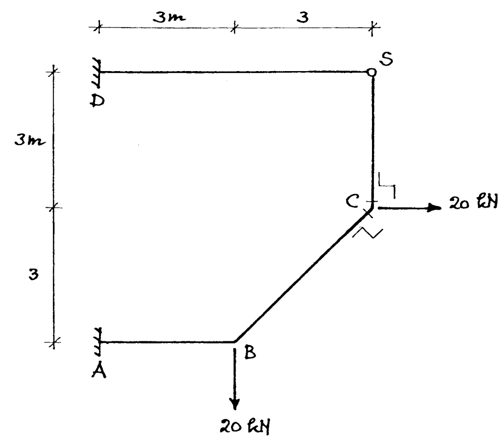


Figure 13.77 Statically indeterminate frame with two given shear forces directly next to joint C.

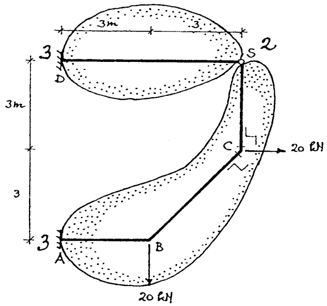


Figure 13.78 The frame consists of the two singly-cohesive sub-structures ABCS and DS.

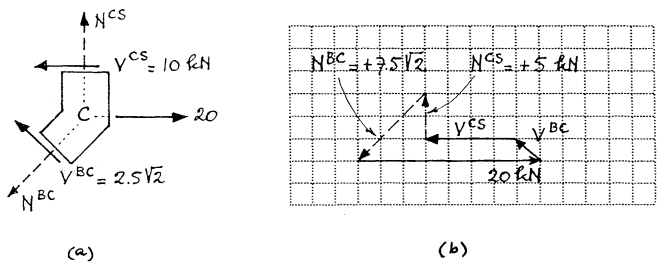


Figure 13.79 (a) The forces on joint C and (b) the closed force polygon for the force equilibrium (scale: 1 square = 2.5 kN).

The directions follow from the deformation symbols given in the figure.

Questions:

a. Determine the degree of static indeterminacy of the structure.
b. Draw the force polygon for the force equilibrium of joint C.
c. Determine the support reactions at A and D.
d. Determine the M, V and N diagrams for the entire structure, with the deformation symbols.

Solution (units kN and m):
a. The two parts ABCS and DS provide $e = 2 \times 3 = 6$ equilibrium equations (see Figure 13.78). The number of unknown support reactions at A and D is $r = 3 + 3 = 6$. The number of unknown joining forces at S is $v = 2$. The degree of static indeterminacy is:

$$n = r + v - e = 6 + 2 - 6 = 2.$$

The structure is therefore statically indeterminate to the second degree.

b. In Figure 13.79a, joint C has been isolated and all forces acting on it are shown. The bending moments acting on the joint are not shown! Figure 13.79b shows the closed force polygon for the force equilibrium of the joint (scale: 1 square = 2.5 kN). The force polygon gives

$$N^{BC} = +7.5\sqrt{2}\ \text{kN},$$

$$N^{CS} = +5\ \text{kN}.$$

c. With $N^{CS} = +5$ kN the vertical equilibrium of CSD gives

$$D_v = 5\ \text{kN}\ (\uparrow).$$

With $V^{CS} = 10$ kN the horizontal equilibrium of ABC gives

$$A_h = 10 \text{ kN } (\leftarrow).$$

From the force equilibrium of the structure as a whole follows

$$D_h = 10 \text{ kN } (\leftarrow),$$

$$A_v = 15 \text{ kN } (\uparrow).$$

Finally, the fixed-end moment reactions at A and D follow from the moment equilibrium about S of DS and ABCS respectively. In Figure 13.80, the support reactions are shown as they act in reality.

d. In Figures 13.81a to 13.81c the M, V and N diagrams are shown. At B and C, the bending moment "*goes round the corner*". The slopes of the M diagram are in line with the magnitudes and the deformations symbols of the shear forces.

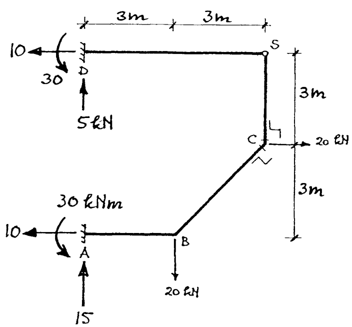


Figure 13.80 Support reactions.

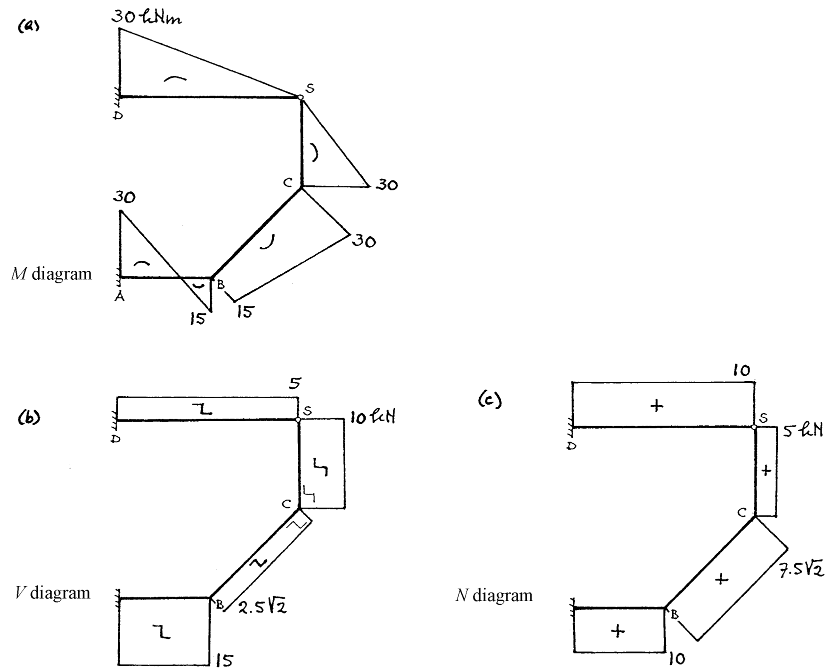


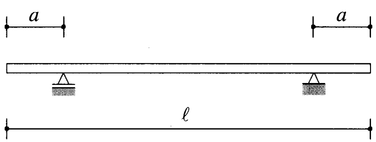
Figure 13.81 (a) Bending moment diagram, (b) shear force diagram and (c) normal force diagram.

13.4 Problems

General comment: When asked to draw an M, V or N diagram, please draw the diagrams including the (deformation) symbols (or plus and minus signs) and the values at relevant points.

Self-contained structures (Section 13.1)

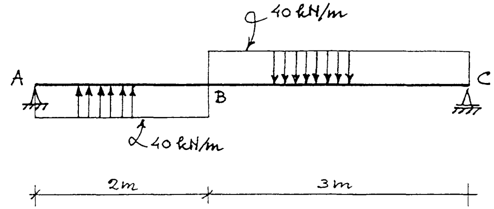
13.1 A beam with length $\ell = 16.90$ m is supported as shown. The dead weight of the beam is uniformly distributed and is 4 kN/m.



Questions:
a. How do you choose distance a to minimise the bending moment in the beam due to the dead weight (in an absolute sense)?
b. How large is this bending moment?
c. Draw the M and V diagrams.

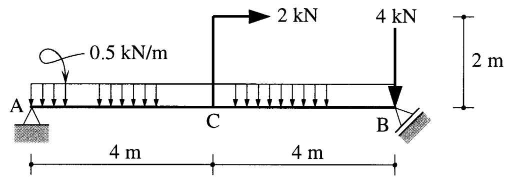
13.2 Beam ABC is simply supported at A and B. A uniformly distributed load of 40 kN/m acts upwards over AB, and downwards over BC.

Questions:
a. Determine the support reactions and draw them as they act on the beam.
b. For ABC, draw the bending moment diagram with the tangents at A, B and C. Clearly show where these tangents intersect.
c. For ABC, draw the shear force diagram.



d. Determine the maximum and minimum bending moment in the beam and indicate where these moments occur.

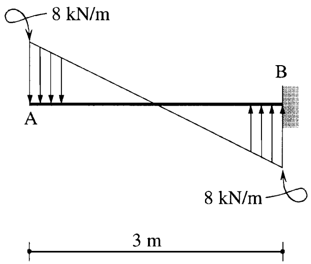
13.3 Beam ACB is supported by a hinge at A, and on a roller at B. The roller track at B is on a slope of 45°. A uniformly distributed load of 0.5 kN/m acts over the entire length ACB. At B, the beam is loaded by a vertical force of 4 kN. At C there is an eccentric axial force of 2 kN.



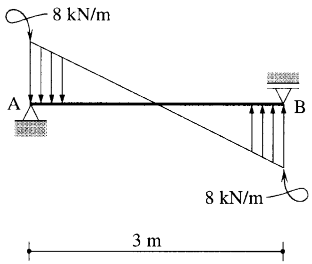
Questions:
a. Determine and draw the support reactions at A and B.
b. For ACB draw the N diagram.
c. For ACB draw the V diagram.
d. For ACB draw the M diagram. At A, C and B, draw the tangents to the M diagram, and clearly indicate where they intersect.

13.4: 1–2 Beam AB is supported in two different ways and carries a linearly distributed load.

(1)



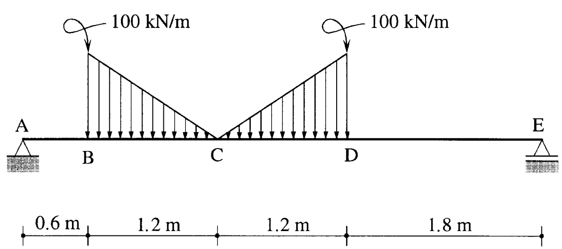
(2)



Questions:

a. Determine and draw the support reactions.
b. Make a clear sketch of the *V* and *M* diagrams. At A, B and the middle of AB, also draw the tangents to the *M* diagram.
c. Where are *V* and *M* an extreme and how large are these extreme values?

13.5 The simply supported beam AE is loaded in the fields BC and CD by two equally large triangular loads. The top value of the distributed load is 100 kN/m.



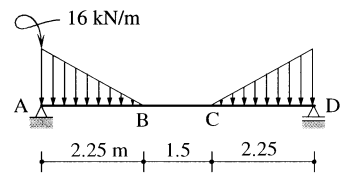
Questions:

a. Draw the *M* and *V* diagrams. At B, C and D draw the tangents to the *M* and *V* diagrams, and clearly show where they intersect.
b. Where is the bending moment an extreme? Using the bending moment diagram drawn, estimate the value of this moment.
c. Make an accurate calculation of the maximum bending moment.

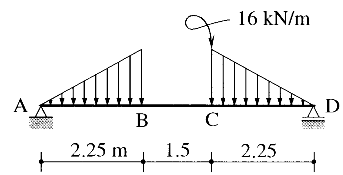
13.6: 1–3 The simply supported beam AD is loaded in three different ways by triangular loads with a top value of 16 kN/m.

Questions:

a. Draw the *M* and *V* diagrams. At A to D, also draw the tangents to the *M* and *V* diagrams, and clearly show where they intersect.
b. Where is the bending moment an extreme? Determine this moment.



(1)



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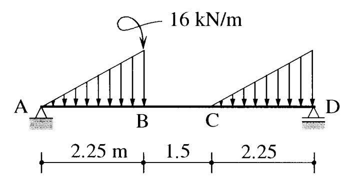
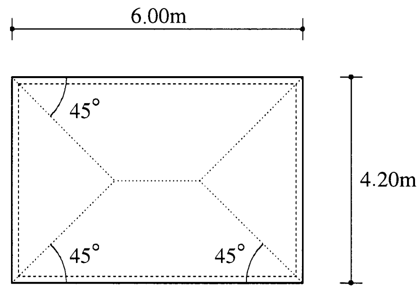


(3)

13.7 A rectangular slab rests on four edge beams, of which it is assumed that they are simply supported on columns at the corners of the slab. A uniform full load on the slab of 4 kN/m^2 is transferred in accordance with the envelope pattern shown to the edge beams.

Questions:

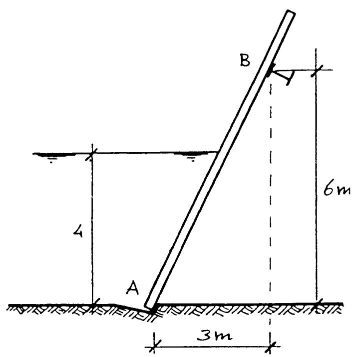
a. For each of the edge beams, draw the loading diagram.
b. Drawn the M and V diagrams for the short edge beam. How large is the maximum bending moment?
c. Draw the M and V diagrams for the long edge beam. How large is the maximum bending moment?



13.8 A barrage is composed of 1.5-metre-wide bulkheads that on the underside rest in a groove at A and on top rest against an I-beam. The I-beam is supported by the barrage walls.

Questions:

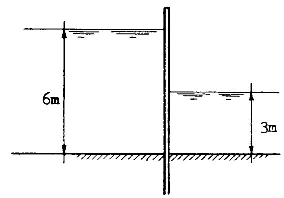
a. Draw the distribution of the water pressure on the bulkheads.
b. Draw a model of a bulkhead with a width of 1.5 m as a line element, and determine the support reactions at A and B.
c. Draw the M and V diagrams for bulkhead AB.
d. How large is the maximum bending moment, and where does it occur?



13.9 A steel sheet-pile wall is fixed in a concrete floor with 6 metres of water on one side, and 3 metres on the other side. The mass density of water is 1000 kg/m^3.

Questions:

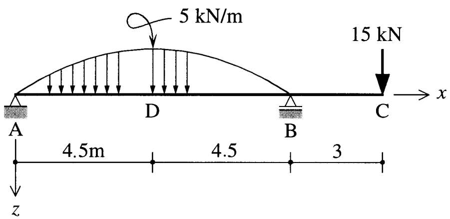
a. Schematize a 1-metre wide vertical strip from the sheet-pile wall as a line element, and draw the load diagram.
b. Determine the support reactions for the strip.
c. Draw the M and V diagrams for the strip. In a number of places, draw the tangents to the M and V diagrams.



13.10 Beam ABC is supported by a hinge at A and on a roller at B. In field AB the beam carries a parabolically distributed load, and at the end C of the overhang BC a point load of 15 kN. In the given coordinate system, the parabolically distributed load is represented by

$$q(x) = -20\left(\frac{x}{\ell}\right)^2 + 20\left(\frac{x}{\ell}\right) \text{ kN/m.}$$

Here $\ell = 9$ m is the length of AB. The dead weight of the beam is not considered here.



Questions:

a. Substitute the distributed load over AB by its resultant, and for the entire beam ABC draw the M and V diagrams.
b. Now give a (rough) sketch of the actual M and V diagrams for AB, with the deformation symbols, and also the plus and minus signs in the given xz coordinate system.
c. For AB, through successive integration, determine the shear force V and the bending moment M as a function of x. Determine the values of V and M at A and B and in the middle D of field AB. At D draw the tangent to the M diagram.
d. Where in AB is the field moment an extreme? It is enough to indicate the location of this maximum roughly. On the basis of the M diagram, estimate the value of the maximum field moment. This value need not be calculated accurately.

13.11: 1–3 A simply supported beam AB with length ℓ is loaded for bending by three different distributed loads with the same top value $\hat{q}$:

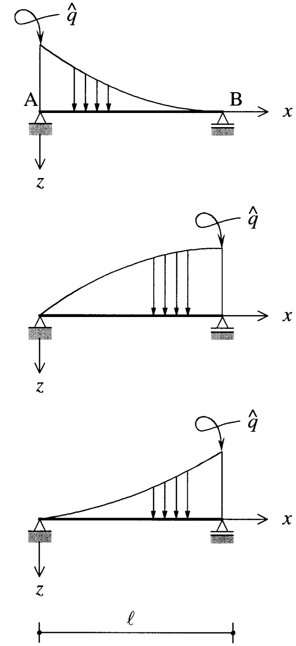
$$(1)\ q(x) = \hat{q} \cdot \left(\frac{x^2}{\ell^2} - 2\frac{x}{\ell} + 1 \right),$$

$$(2)\ q(x) = \hat{q} \cdot \left(-\frac{x^2}{\ell^2} + 2\frac{x}{\ell} \right),$$

$$(3)\ q(x) = \tfrac{1}{2}\hat{q} \cdot \left(\frac{x^2}{\ell^2} + \frac{x}{\ell} \right).$$

For the numerical calculation, assume $\ell = 4$ m and $\hat{q} = 48$ kN/m.

(1) (2) (3)



Questions:

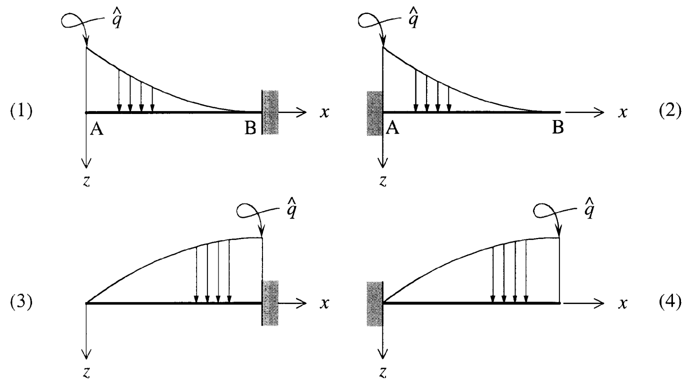
a. Determine M and V as a function of x.
b. Draw the M and V diagrams with the *deformation symbols*.
c. Determine the location and magnitude of the maximum bending moment.
d. Determine the support reactions at A and B and draw them as they actually act on the beam.

13.12: 1–4 Two cantilever beams AB with length ℓ are subject to bending by two different distributed loads with the same top value $\hat{q}$:

$$\text{(1) and (2)} \quad q(x) = \hat{q} \cdot \left(\frac{x^2}{\ell^2} - 2\frac{x}{\ell} + 1 \right),$$

$$\text{(3) and (4)} \quad q(x) = \hat{q} \cdot \left(-\frac{x^2}{\ell^2} + 2\frac{x}{\ell} \right).$$

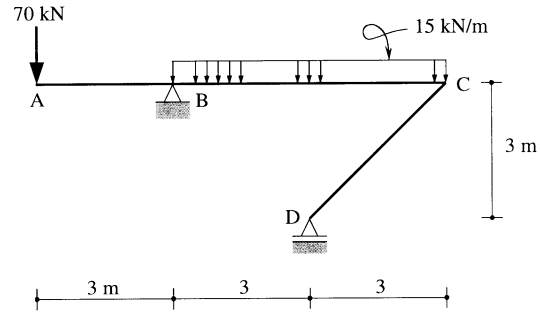
For the numerical calculation, assume $\ell = 4$ m and $\hat{q} = 48$ kN/m.



Questions:

a. Determine M and V as a function of x.
b. Draw the M and V diagrams with the *deformation symbols*.
c. Determine the support reactions as they act on the beam.

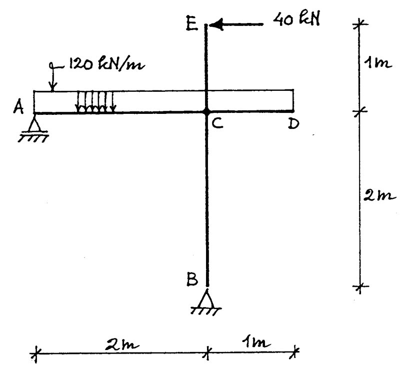
13.13 The bent beam ABCD is supported by a hinge at A and on a roller at D. The structure is loaded by a uniformly distributed load in field BC and a point load at A.



Questions:

a. Determine the support reactions. Draw them in the directions in which they act.
b. For the entire construction, draw the M, V and N diagrams with the deformation symbols. At B and C, draw the tangents to the M diagram.
c. Indicate in which cross-section of BC the field moment is an extreme. Determine this extreme value.

13.14 The structure consists of the members ACD and BCE that are rigidly joined to one another at C.



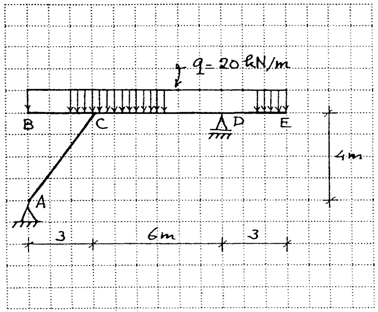
Questions:

a. Determine the support reactions and draw them as they act in reality.
b. Isolate ACD, and draw all the forces acting on it.
c. For ACD, draw the M and V diagrams. At A, C and D draw the tangents to the M diagram.
d. Determine the maximum bending moment in field AC. In which cross-section does this occur?
e. Draw the M and V diagrams for BCE.

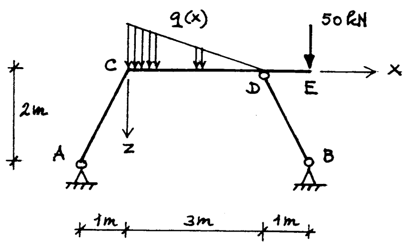
13.15 The structure in Figure 13.21 consists of the members AC and BCDE that are rigidly joined to one another at C.

Questions:

a. Determine the support reactions at A and D.
b. For the entire structure draw the M and V diagrams. At B to E, draw the tangents to the M diagram, and clearly indicate where they intersect.
c. Where in field CD is the bending moment an extreme? Determine this moment.
d. Draw the N diagram for the entire structure.



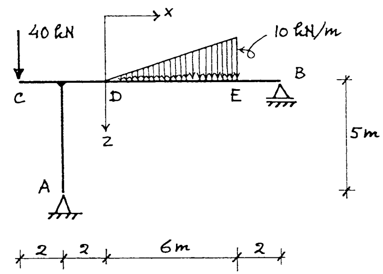
13.16 The structure is subject to a force of 50 kN at E, and a linearly distributed load $q(x)$ in field CD. The following applies in the given xz coordinate system: $q(x) = (-10x + 30)$ kN/m with x expressed in metres.



Questions:

a. Determine and draw the support reactions at A and B.
b. Isolate member CDE, and draw all the forces acting on it.
c. Write down the shear force in CD as a function of x. Verify the function values at C and D.
d. Write down the bending moment in CD as a function of x. Check the function values at C and D.
e. For CDE, draw the V and M diagrams with the deformation symbols. At C and D, draw the tangents to the V and M diagrams.
f. Where in field CD is the bending moment an extreme, and how large is this moment?

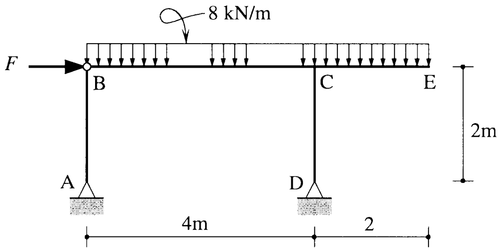
13.17 The structure is supported by a hinge at A and on a roller at B. At C, the overhang is subject to a vertical force of 40 kN. A triangular load acts between D and E, with a top value of 10 kN/m at E.



Questions:

a. Determine and draw the support reactions.
b. Draw the M, V and N diagrams for the entire structure with the deformation symbols. At D and E draw the tangents to the M and V diagrams.
c. Determine M and V in field DE as a function of x. Use the given xz coordinate system. Check the values (including the signs) of M and V at both D and E.
d. Determine the location and magnitude of the maximum bending moment in field DE.

13.18 The structure is subject to a uniformly distributed vertical load of 8 kN/m and a horizontal force $F = 3$ kN at B. The joint at B is hinged, the joint at C is rigid.



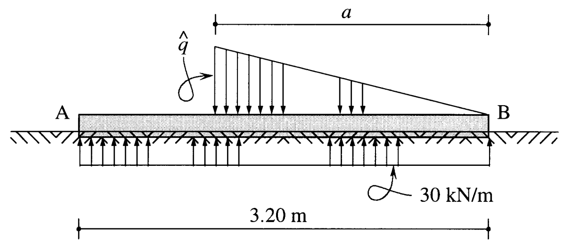
Questions:

a. Determine the support reactions at A and D.
b. Draw the M, V and N diagrams for the entire structure.
c. Determine the location and magnitude of the maximum field moment in BC.
d. Find the value of F for which the normal force in post AB is zero (with the given uniformly distributed load of 8 kN/m on BCE).

13.19 A structure, modelled as beam AB, is lying on the ground. Its dead weight can be ignored. On the right-hand side, the beam is subject to a linearly distributed load over length a with a top value $\hat{q}$. Due to this load, the earth pressure on the underside of the beam is constant, and is 30 kN/m.

Questions:

a. From the equilibrium of the beam determine length a and the top value $\hat{q}$.
b. Draw the resulting distributed load on the beam (the load diagram).
c. For the beam, draw good sketches of the V diagram and the M diagram (with their tangents at relevant points).
d. In which cross-section(s) is the shear force an extreme? At these cross-sections also draw the tangents to the M diagram.

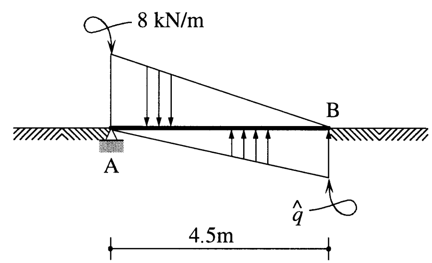


e. In which cross-section is the bending moment an extreme? Determine this value.

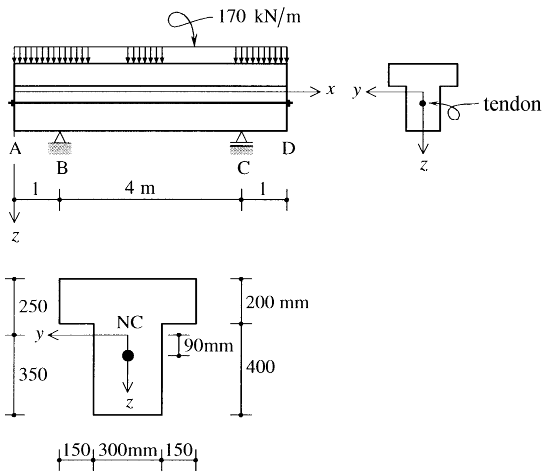
13.20 A weightless rigid beam AB is resting on a hinge at A, while the remainder is resting on the ground, which provides a linearly distributed counter-pressure with top value $\hat{q}$ at B. The load on the beam consists of a triangular load with a top value of 8 kN/m at A.

Questions:

a. From the equilibrium of the beam, determine the top value $\hat{q}$ of the earth pressure.
b. Draw the resulting load on the beam (the resultant load diagram).
c. For the beam, draw good sketches of the V diagram and the M diagram (with their tangents at relevant points).
d. At which cross-section(s) is the shear force an extreme? Draw the tangents to the M diagram at these cross-sections.
e. At which cross-section is the bending moment an extreme? Determine this value.



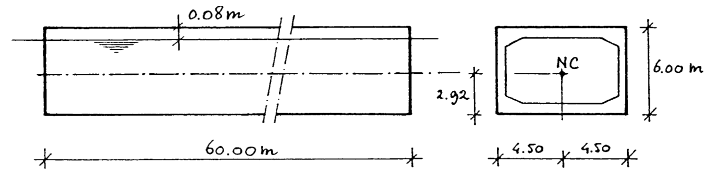
13.21 Given an eccentrically prestressed T-beam with overhangs and a uniformly distributed full load. The straight single bar tendon is 90 mm under the beam axis. The prestressing force is 1200 kN.



Questions:

a. Determine the support reactions.
b. Determine the N, V and M diagrams, with the deformation symbols. At A to D, draw the tangents to the M diagram.
c. In which cross-section(s) is the bending moment an extreme? Determine this/these extreme value(s).

13.22 A tunnel segment is afloat, ready to be moved to its final location where it will be sunk. The tunnel segment, which can be seen as a rigid body, has a freeboard of 0.08 m. The dead weight of the tunnel is 525 kN/m. The weight of each of the two temporary bulkheads is 234 kN. The specific weight of water is 10 kN/m^3. The dimensions of the tunnel segments are shown in the figure. The figure also shows the location of the normal centre NC in the cross-section.



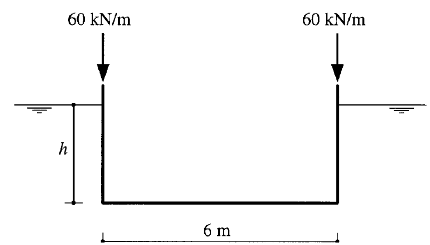
Questions:

a. How large is the water pressure on the underside of the tunnel?
b. Draw the distribution of the water pressure on a bulkhead, and determine the magnitude and location of the resultant.
c. Model the tunnel segment as a line element, and draw all (distributed and non-distributed) forces (and/or couples) acting on it.
d. For the tunnel segment, draw the M, V and N diagrams, with the deformation symbols. How large is the maximum bending moment?

13.23 A long weightless barge is loaded on its walls by a distributed load of 60 kN/m.

Questions:

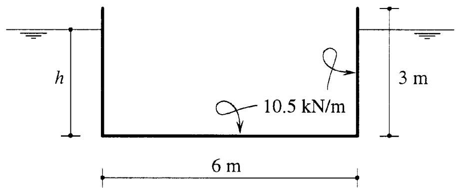
a. Determine the draught h of the barge.
b. Determine the distribution of the water pressure on the walls and bottom of the barge.
c. Draw the M, V and N diagrams for a 1-metre wide strip from the wall.
d. Isolate a 1-metre strip from the bottom of the barge and draw all the forces acting on it.
e. Draw the M, V and N diagrams for this 1-metre strip out of the bottom.



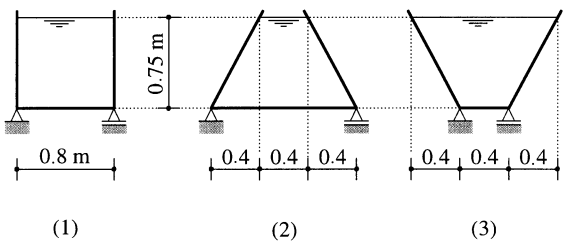
13.24 A 1-metre strip has been isolated from a long barge and is modelled as a bent line element. The dead weight of line element (walls and bottom) is 10.5 kN/m. The width of the strip is not given.

Questions:

a. Determine the draught h of the barge.
b. Determine the distribution of the water pressure on the walls and bottom.
c. Draw the M, V and N diagrams for the wall.
d. Isolate the bottom of the barge and draw all the forces acting on it.
e. Draw the M, V and N diagrams for the bottom.



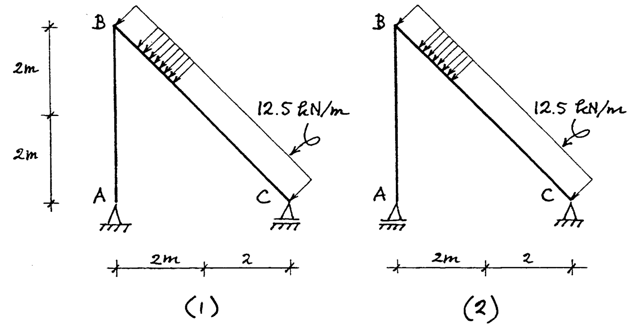
13.25: 1–3 A 1-metre strip has been isolated from a long trough filled with water and is modelled as a line element. There are three different trough shapes.



Questions:

a. Determine the support reactions.
b. Draw the distribution of the water pressure on the walls and the bottom.
c. Isolate the bottom and draw all the forces acting on it.
d. Draw the M, V and N diagrams for the bottom.
e. Determine the maximum field moment in the bottom.

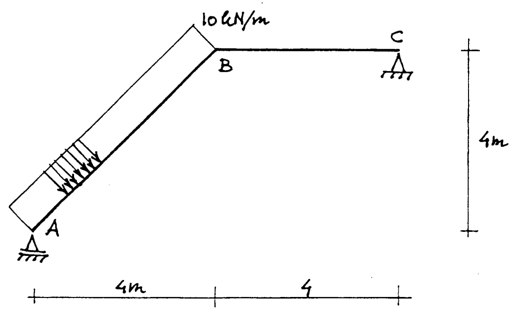
13.26: 1–2 The two structures shown differ only in their method of support.



Question:

Draw the M, V and N diagrams for the entire structure. At B and C, draw the tangents to the M diagram, and clearly indicate where they intersect.

13.27: 1–2 The two structures given differ only in their method of support.



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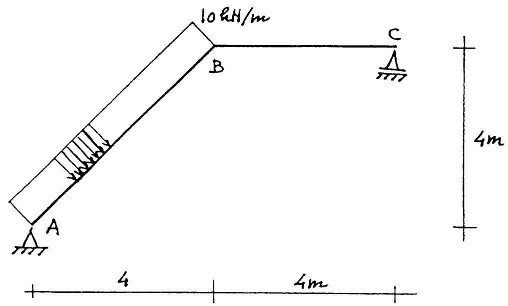

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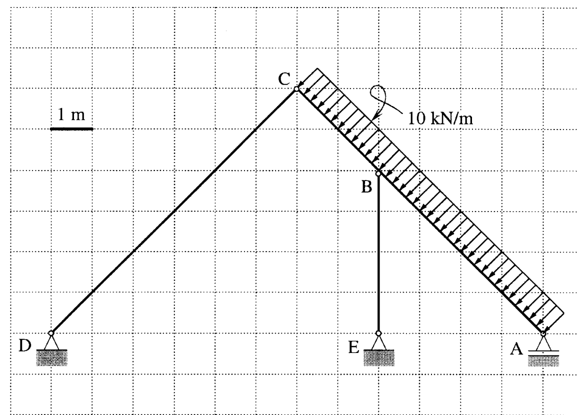
Question:
Draw the M, V and N diagrams for the entire structure. At A and B, draw the tangents to the M diagram, and clearly indicate where they intersect.

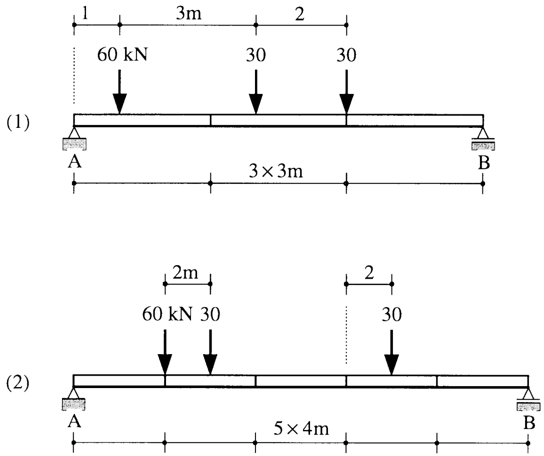
13.28 The structure consists of the members ABC, BE and CD that are joined together by hinges. A uniformly distributed load of 10 kN/m acts normal to ABC.



Questions:

a. Determine the support reactions at A, D and E. Draw them as they act on the structure.
b. Isolate ABC, and draw all the forces acting on it.
c. For ABC determine and draw the M and V diagram, with the deformation symbols. At A, B and C, draw the tangents to the M diagram, and clearly indicate where they intersect.

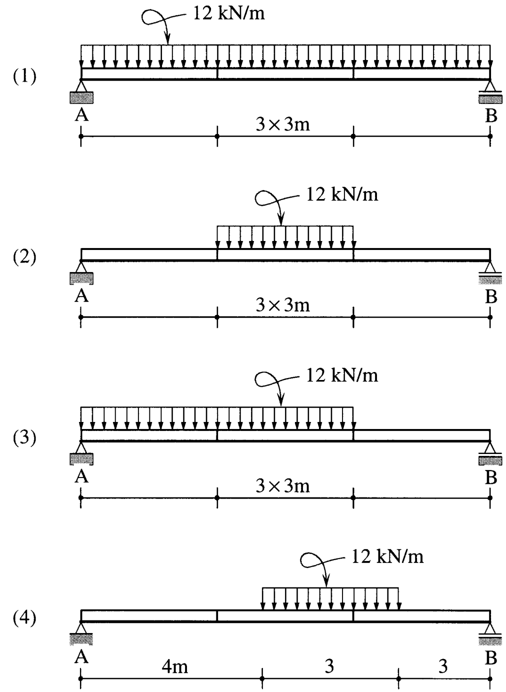
13.29: 1–2 Two different beams AB are indirectly loaded by a number of point loads.



Questions:

a. Determine the support reactions at A and B.
b. Determine the M and V diagrams for the indirectly loaded (main) beam.
c. Determine the M and V diagrams for the directly loaded (stringer) beams.
d. Explain any difference in magnitude between the support reactions at A and B, and the shear force in the main beam at those places.

13.30: 1–4 Four different distributed loads act on the same indirectly loaded beam AB.

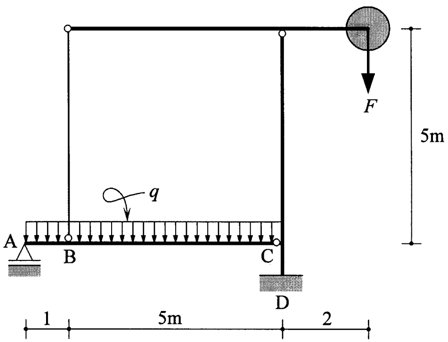


Questions:

a. Determine the support reactions at A and B.
b. Determine the M and V diagrams for the indirectly loaded (main) beam.
c. Determine the M and V diagrams for the directly loaded (stringer) beams.
d. Explain any difference in magnitude between the support reactions at A and B, and the shear force in the main beam at those places.

Compound and associated structures (Section 13.2)

13.31 The scheme of a (weightless) draw bridge is given. A uniformly distributed load q acts on the bridge deck ABC. The weight of the balance is F. Assume that $q = 12$ kN and $F = 90$ kN.



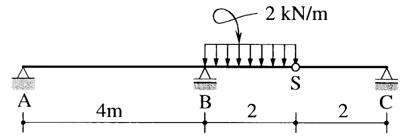
Questions:

a. Isolate ABC, and draw all the forces acting on it.
b. Draw the M and V diagram for ABC. At A, B and C, draw the tangents to the M diagram.
c. How large are the support reactions at D?
d. Calculate F in order to obtain a zero support reaction at A due to the given load.

13.32 As problem 13.31, but now with $q = 12$ kN and $F = 60$ kN.

13.33: 1–4 A number of hinged beams are given.

(1)



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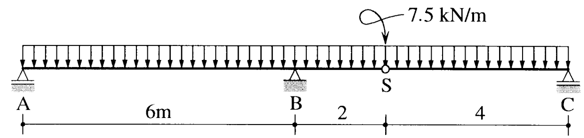

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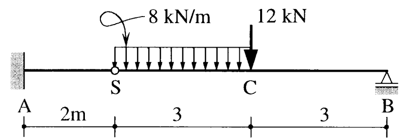

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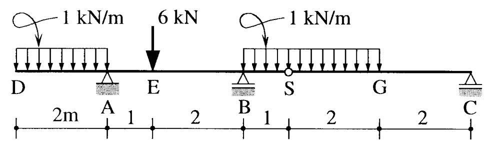
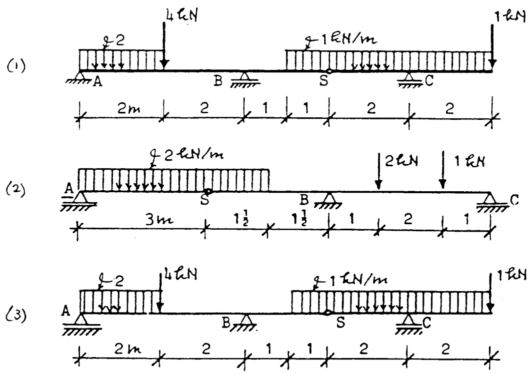


Questions:

a. Determine the support reactions.
b. Determine the V diagram.
c. Determine the M diagram, with the tangents at a number of points.
d. Determine the location and magnitude of the extreme bending moments.

13.34: 1–5 A number of hinged beams are given.



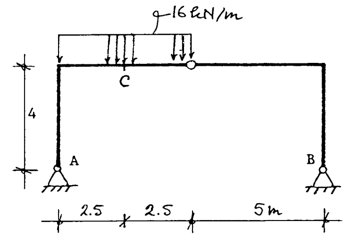
Questions:

a. Determine the support reactions.
b. Determine the V diagram.
c. Determine the M diagram, with the tangents at a number of points.
d. Determine the location and magnitude of the extreme bending moments.

13.35 A three-hinged portal frame with a uniformly distributed vertical load of 16 kN/m on the left-hand side of the girder is given.

Questions:

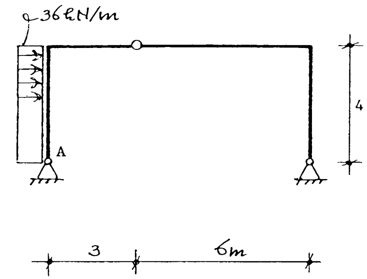
a. Determine the support reactions.
b. Determine the M diagram for the entire structure with the tangents at a number of points.
c. Determine the V diagram for the entire structure.
d. Determine the location and magnitude of the maximum field moment in the girder.
e. Determine the N diagram for the entire structure.



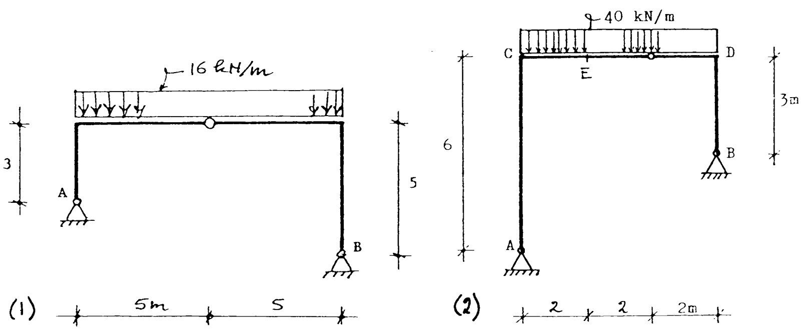
13.36 A uniformly horizontal distributed load of 36 kN/m acts on the left-hand column of a three-hinged portal frame.

Questions:

a. Determine the support reactions.
b. Determine the M diagram for the entire structure with the tangents at a number of points.
c. Determine the V diagram for the entire structure.
d. Determine the location and magnitude of the maximum field moment in the left-hand column.
e. Determine the N diagram for the entire structure.



13.37: 1–2 Two three-hinged frames with unequal column lengths and a uniformly distributed full load on the beam are given.



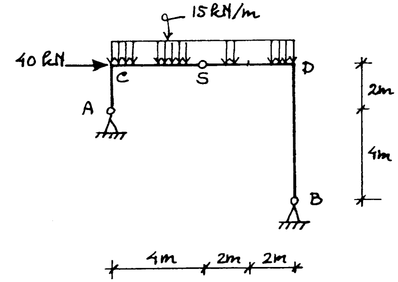
Questions:

a. Determine the support reactions.
b. Determine the *M* diagram for the entire structure with the tangents at a number of points.
c. Determine the *V* diagram for the entire structure.
d. Determine the location and magnitude of the maximum field moment in the beam.
e. Determine the *N* diagram for the entire structure.

13.38 The girder of a three-hinged frame with unequal posts is loaded by a uniformly distributed vertical load and a horizontal force.

Questions:

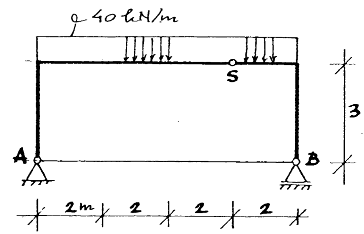
a. Determine the support reactions.
b. Determine the *M* diagram for the entire structure with the tangents at a number of points.
c. Determine the *V* diagram for the entire structure.
d. Determine the location and magnitude of the maximum field moment in the girder.
e. Determine the *N* diagram for the entire structure.



13.39 A three-hinged frame with a tie rod is carrying a uniformly distributed load of 40 kN/m.

Questions:

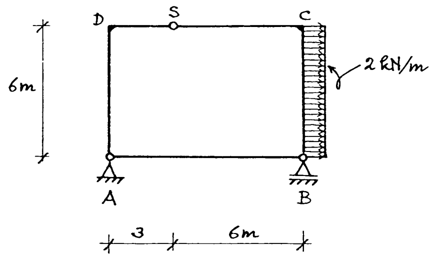
a. Determine the support reactions.
b. Determine the force in the tie rod.
c. Determine the *M* diagram for the entire structure with the tangents at a number of points.
d. Determine the *V* diagram for the entire structure.
e. Determine the location and magnitude of the maximum field moment in the girder.
f. Determine the *N* diagram for the entire structure.



13.40 A uniformly horizontal distributed load of 2 kN/m acts on the right-hand post of a three-hinged frame with tie rod.

Questions:

a. Determine the support reactions.
b. Determine the force in the tie rod.
c. Determine the *M* diagram for the entire structure with the tangents at a number of points.
d. Determine the *V* diagram for the entire structure.
e. Determine the location and magnitude of the maximum field moment of the loaded post.
f. Determine the *N* diagram for the entire structure.



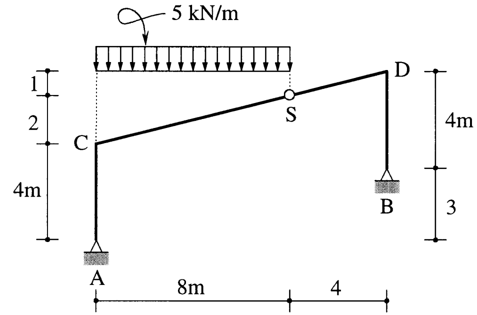
13.41 As problem 13.40, but now with the distributed load acting on the left-hand post.

13.42: 1–3 The same three-hinged frame is loaded in three different ways by a uniformly distributed load on CS.

Questions:

a. Determine the support reactions.
b. For the entire frame, draw the *M* diagram with the tangents at C and S.
c. Draw the *V* diagram for the entire frame.
d. Draw the *N* diagram for the entire frame.
e. Determine the location and magnitude of the maximum bending moment in field CS.

(1)



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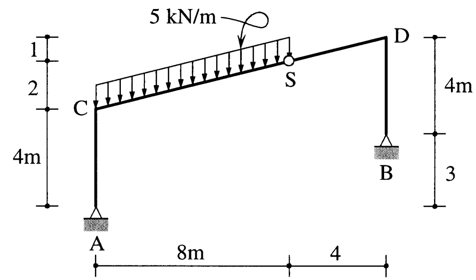

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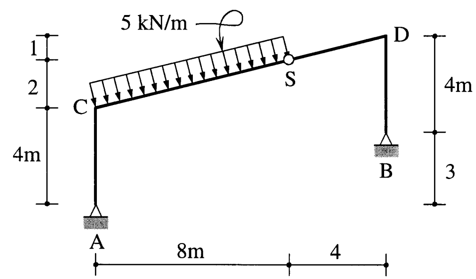
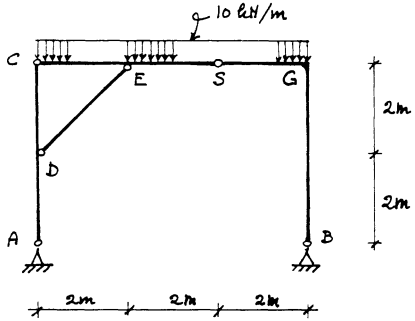


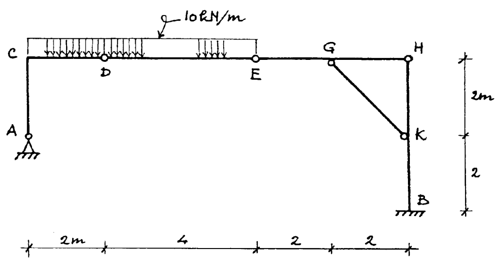
13.43 A shored three-hinged frame with a uniformly distributed full load of 10 kN/m on the girder is given.



Questions:

a. Determine the support reactions.
b. Determine the force in shore DE, with the correct sign.
c. Isolate parts ADC, CES and SGB and draw all the forces acting on them.
d. Draw the *N* diagram for the entire structure.
e. Draw the *V* diagram for the entire structure.
f. Draw the *M* diagram for the entire structure, with the tangents at C, E and G.
g. Determine the location and magnitude of the extreme moments in CESG.

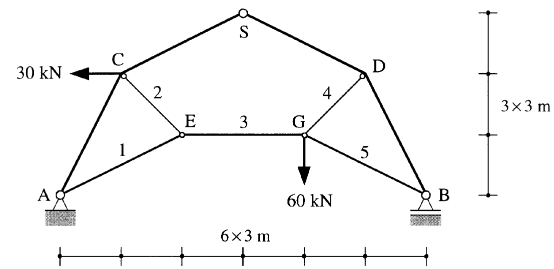
13.44 The structure is supported by a hinge at A and is fixed at B. GK is a shore with hinged connections at G and K. The structure also has hinged connections at D, E and H. A uniformly distributed load of 10 kN/m acts on CDE.



Questions:

a. For ACDEGH draw the *M* diagram, with the tangents at C, D and E.
b. For ACDEGH draw the *V* diagram.
c. Determine the support reactions.
d. Determine the force in shore GK, with the correct sign.
e. For HKB draw the *M* and *V* diagram.
f. Draw the *N* diagram for the entire structure.

13.45 The structure shown consists of the bent members ACS and BDS and the straight members 1 to 5, all joined by hinges. The structure is supported by a hinge at A, and on a roller at B. The load consists of a horizontal force of 30 kN at C and a vertical force of 60 kN at G.



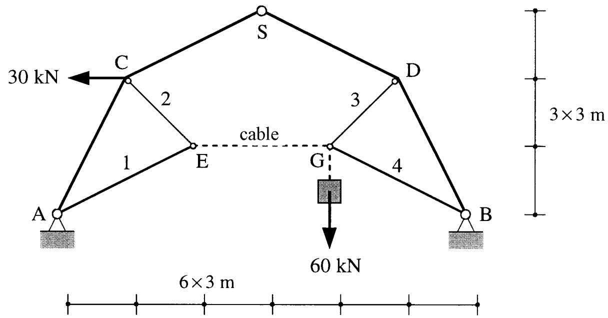
Questions:

a. Determine the support reactions at A and B.
b. Determine the forces in members 1 to 5 (with the correct sign). Draw the force polygons for joints E and G.
c. Isolate part ACS and draw all the forces acting on it.
d. For ACS draw the *M* and *V* diagram.
e. For ACS draw the *N* diagram.
f. Isolate part BDS and draw all the forces acting on it.
g. For BDS draw the *M* and *V* diagram.
h. For BDS draw the *N* diagram.

13.46 As problem 13.45, but now without the horizontal force at C.

13.47 As problem 13.45, but now without the vertical force at G.

13.48 The structure shown consists of the bent members ACS and BDS and the straight bars 1 to 4, all joined at hinges. The structure is supported by hinges at A and B. At E, a cable is fixed that at G runs through a frictionless pulley. A weight of 60 kN is attached to the cable. At C there is a horizontal force of 30 kN.



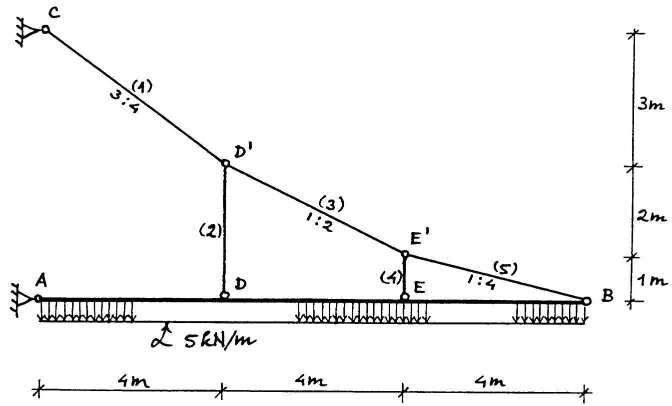
Questions:

a. Determine the support reactions at A and B.
b. Determine the forces in members 1 to 4 (with the correct sign). Draw the force polygons for joints E and G.
c. Isolate part ACS and draw all the forces acting on it.
d. For ACS draw the M and V diagram.
e. For ACS draw the N diagram.
f. Isolate part BDS and draw all the forces acting on it.
g. For BDS draw the M and V diagram.
h. For BDS draw the N diagram.

13.49 As problem 13.48, but now without the horizontal force at C.

13.50 As problem 13.48, but now without the weight of 60 kN at G.

13.51 The cantilever beam AB, with a uniformly distributed full load of 5 kN/m, is supported by means of a cable structure.



Questions:

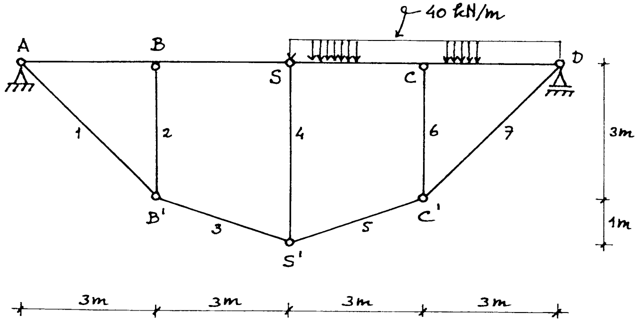
a. Determine the support reactions.
b. Determine the forces in cables 1 to 5.
c. Isolate beam AB and draw all the forces acting on it.
d. Draw the M, V and N diagrams for AB. At A, D, E and B, draw the tangents to the M diagram.
e. Determine the magnitude and location of the extreme bending moments in AB.

13.52 The trussed beam ASD is carrying a uniformly distributed load of 40 kN/m over SD.

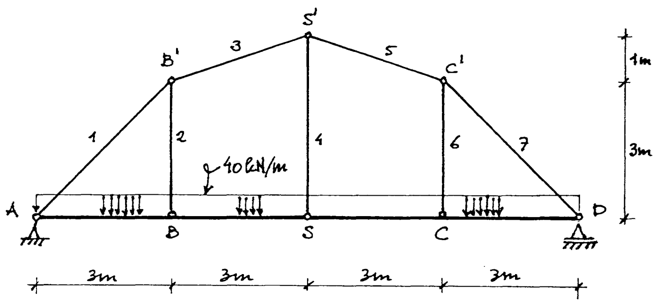
Questions:

a. Determine the support reactions.
b. Determine the forces in members 1 to 7.
c. Isolate beam ASD and draw all the forces acting on it.

d. Draw the M, V and N diagrams for ASD. In S, C and D also draw the tangents to the M diagram.
e. Determine the location and magnitude of the extreme bending moments in ASD.

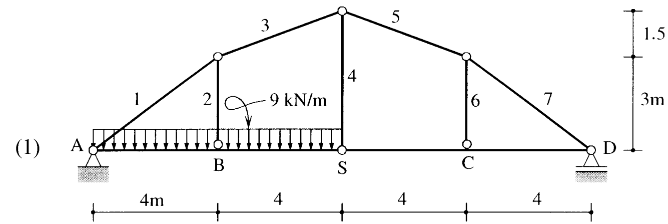


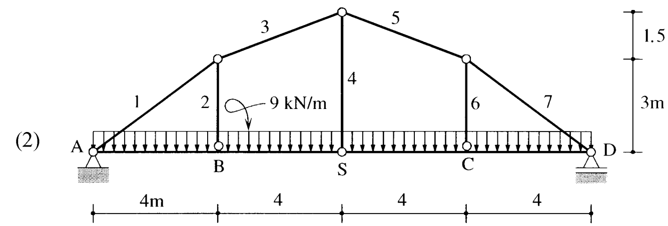
13.53 The trussed beam ASD is carrying a uniformly distributed full load of 40 kN/m.



Questions:
a. Determine the support reactions.
b. Determine the forces in members 1 to 7.
c. Isolate beam ASD and draw all the forces acting on it.
d. Draw the M, V and N diagrams for ASD. At A, B, S, C and D, draw the tangents to the M diagram.
e. Determine the location and magnitude of the extreme bending moments in ASD.

13.54: 1–2 The same trussed beam ASD is loaded in two different ways.

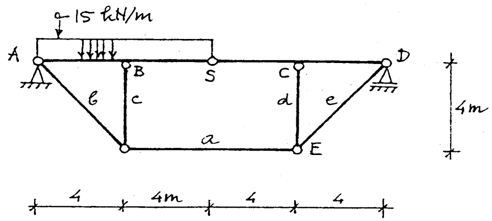




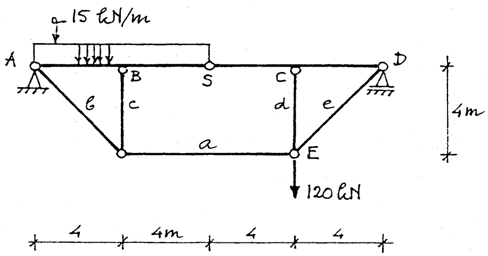
Questions:
a. Determine the support reactions.
b. Determine the forces in members 1 to 7.
c. Isolate beam ASD and draw all the forces acting on it.
d. Draw the M, V and N diagrams for ASD. Also draw at relevant points the tangents to the M diagram.
e. Determine the location and magnitude of the extreme bending moments in ASD.

13.55: 1–2 The same trussed beam ASD is loaded in two different ways.

(1)



(2)



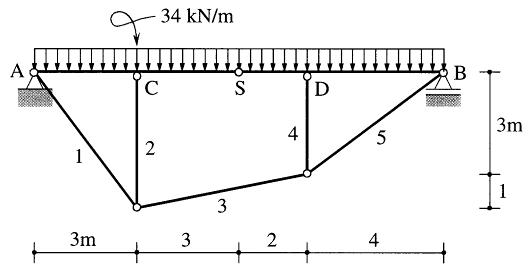
Questions:

a. Determine the support reactions.
b. Determine the forces in members a to e.
c. Isolate beam ASD, and draw all the forces acting on it.
d. Draw the M, V and N diagrams for ASD. At A, B and S, draw the tangents to the M diagram.

13.56 The trussed beam ASD carries a uniformly distributed full load of 34 kN/m.

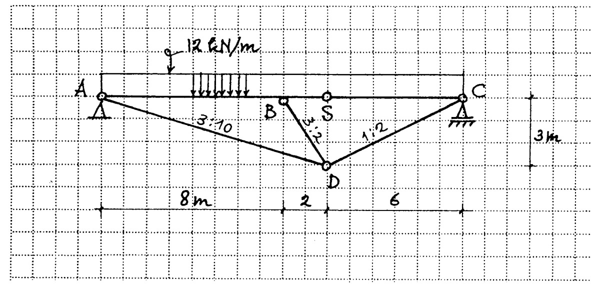
Questions:

a. Determine the forces in members 1 to 5, with the correct sign.
b. Isolate beam ASB and draw all the forces acting on it.
c. For ASB draw the N diagram.
d. For ASB draw the V diagram.
e. For ASB draw the M diagram with the tangents at A, C, D and B.



f. Determine the location and magnitude of the extreme bending moments in beam ASB.

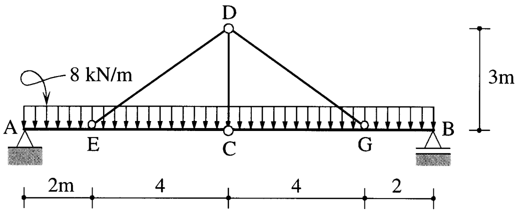
13.57 The trussed beam ASC carries a uniformly distributed full load of 12 kN/m.



Questions:

a. Determine the forces in members AD, BD and CD. Draw the force polygon for joint D. Use a force scale of 1 cm ≡ 40 kN.
b. Isolate beam ASC, and draw all the forces acting on it.
c. For ASC draw the N diagram.
d. For ASC draw the V diagram.
e. For ASC draw the M diagram, with the tangents at A, B and C.
f. Determine the location and magnitude of the extreme bending moments in beam ASC.

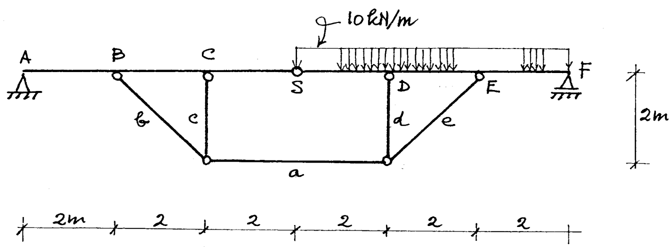
13.58 The trussed beam ACB carries a uniformly distributed full load of 8 kN/m.



Questions:

a. Determine the forces in members DC, DE and DG.
b. Isolate beam ASC and draw all the forces acting on it.
c. For ACB draw the *N* diagram.
d. For ACB draw the *V* diagram.
e. For ACB draw the *M* diagram, with the tangents at A, E and C.
f. Determine the location and magnitude of the extreme bending moments in beam ACB.

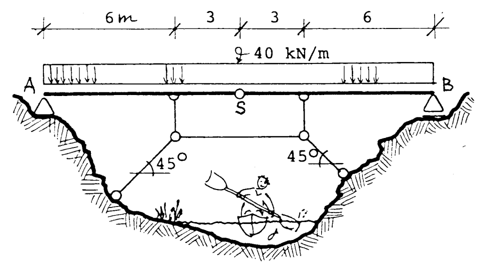
13.59 The trussed beam ASF carries a uniformly distributed load of 10 kN/m over SF.



Questions:

a. Isolate beam ASF, and draw all the forces acting on it.
b. For ASF draw the *N* diagram.
c. For ASF draw the *V* diagram.
d. For ASF draw the *M* diagram with the tangents at S, D, E and F.
e. Determine the location and magnitude of the extreme bending moments in beam ASF.

13.60 A queen post truss with a uniformly distributed full load of 40 kN/m is given.

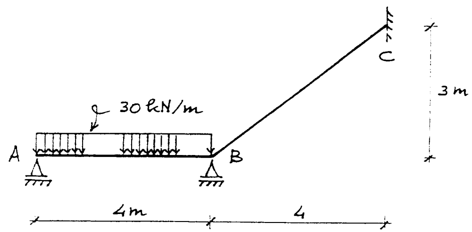


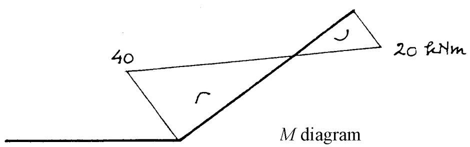
Questions:

a. Determine the *M* and *V* diagrams for beam ASB. Also draw at relevant points the tangents to the *M* diagram.
b. Determine the location and magnitude of the extreme bending moments in beam ASB.

Statically indeterminate structures (Section 13.3)

13.61 The *M* diagram for BC is given for the structure shown.



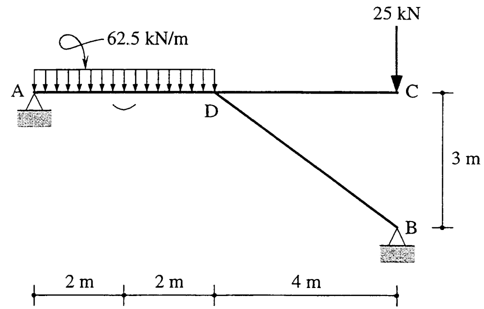


M diagram

Questions:

a. To what degree is the structure statically indeterminate? Substantiate your answer.
b. Draw the *M* diagram for AB, with the tangents at A and B.
c. Draw the *V* diagram for the entire structure.
d. How large is the maximum field moment in AB, and where does it occur?
e. Draw the *N* diagram for the entire structure.
f. Determine the support reactions at A, B and C. Draw them as they act on the structure.

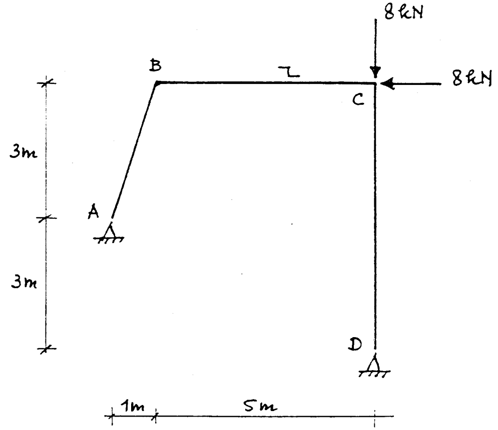
13.62 The structure is supported by hinges at A and B. At joint D all the members are rigidly joined to one another. With the given load, the bending moment in the middle of field AD is 25 kNm. The associated deformation symbol is given in the figure.



Questions:

a. To what degree is the structure statically indeterminate? Substantiate your answer.
b. Determine the support reactions, and draw them as they act on the structure.
c. For the entire structure, determine and draw the *M* and *V* diagrams. At A and D and the middle of field AD, draw the tangents to the *M* diagram.
d. Determine and draw the *N* diagram for the entire structure.

13.63 A two-hinged frame is loaded at C by the forces of 8 kN as shown. With this load, the shear force in girder BC is 6 kN. The associated deformation symbol is shown in the figure.



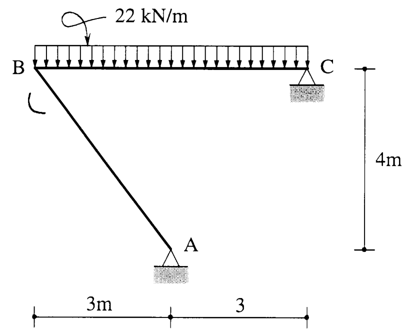
Questions:

a. Determine the support reactions. Draw them as they act in reality.
b. Draw the bending moment diagram for the entire structure.
c. Draw the shear force diagram for the entire structure.
d. Draw the normal force diagram for the entire structure.

13.64 The bend beam is supported by hinges at A and C. The joint at B is entirely rigid. With the given load the bending moment in member BA, directly under joint B, is 60 kNm. The deformation symbol is given in the figure.

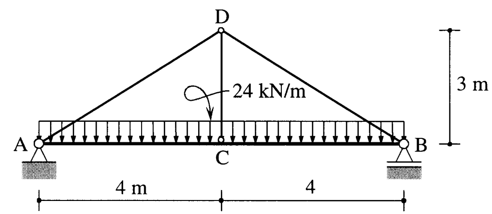
Questions:

a. To what degree is the structure statically indeterminate? Substantiate your answer.
b. Determine the support reactions and draw them as they act on the structure.



c. Determine and draw the *M* and *V* diagrams for the entire structure. At B and C draw the tangents to the *M* diagram.
d. Determine and draw the *N* diagram for the entire structure.

13.65 In the trussed beam ACB the bending moment at C is zero for the given load.



Questions:

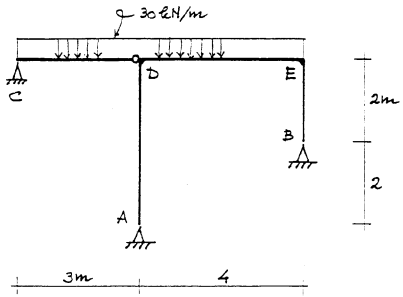
a. Determine the degree of static indeterminacy for the structure.
b. For ACB draw the *M* and *V* diagrams. At A, B and C also draw the tangents to the *M* diagram.
c. Determine the normal forces in bars AD, BD and CD.
d. Draw the *N* diagram for ACB.

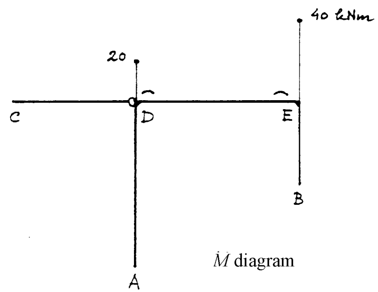
13.66 In the trussed beam ACB from problem 13.65 the bending moment at C with the given load is 36 kNm.

Questions:

a. Draw the M and V diagram for ACB (there are two possibilities). At A, B and C, draw the tangents to the M diagram.
b. Determine the normal forces in bars AD, BD and CD.
c. Draw the N diagram for ACB.

13.67 The structure consists of a two-hinged frame ADEB that is supported horizontally by member CD. The link between member CD and frame ADEB is a hinge. With the given uniformly distributed load of 30 kN/m on CDE, the bending moments with deformation symbols at D and E are given for DE.





M diagram

Questions:

a. Draw the M diagram for the entire structure. At D and E, draw the tangents to the M diagram.
b. Draw the V diagram for the entire structure.
c. Draw the N diagram for the entire structure.
d. Draw all the support reactions in the directions in which they act.