12 Bending Moment, Shear Force and Normal Force Diagrams

In this chapter, we will look at how to calculate and draw M, V and N diagrams, with deformation symbols.

The formal approach using differential equations will often become far too laborious. Therefore, we will first present a number of rules in Sections 12.1 and 12.2 on how to draw the M, V and N diagrams more quickly. The rules follow directly from the differential equations for the equilibrium derived in Section 11.1.

In Section 12.3, we present a number of examples in which we calculate and draw the M, V and N diagrams for bent and compound bar structures loaded by concentrated forces and couples. We calculated the support reactions and/or interaction forces for these structures earlier in Chapter 5.

When calculating and drawing M, V and N diagrams, the influences of the various load contributions can be added. The individual contributions can often be found again from the shape of the M, V and N diagrams. We address this *principle of superposition* in Section 12.4.

Concentrated loads, couples and uniformly distributed loads are *models* of the actual load. We will be looking at the consequences of such modelling in Section 12.5. We also have a closer look at the shear force at a support, and investigate the effect of *eccentrically applied axial forces*.

12.1 Rules for drawing V and M diagrams more quickly

The formal approach for determining the variation of shear forces by solving differential equations is rather laborious, certainly if several fields within a member have to be distinguished.

We are often not so much interested in the precise variation of the section forces as a function of x, but rather in the extreme values and the locations where these occur, or the locations where the section forces change direction.

If we want to calculate the section forces in only a few locations, the direct method, whereby we look at the equilibrium of an isolated segment (*method of sections*) is far quicker.

The mathematical approach based on the differential equations for the equilibrium of a small member segment has the advantage that it leads to a number of generally-applicable rules for the relationship between load, shear force and bending moment that can be translated into properties of the V and M diagrams.[1]

In combination with the direct method, these rules allow us to sketch the V and M diagrams, and determine the relevant values quickly. Since the ability to sketch the V and M diagrams and indicate relevant locations and values is extremely important in engineering practice, the direct method is of great practical relevance.

We will look at the rules relating to the V and M diagrams below, using a number of examples. The rules often highlight various sides of one and the same property. In the examples, we will be using deformation symbols.

[1] The mathematical approach in Chapter 11 is also important at a later stage, when we further develop the theory to be able to determine deformations and displacements.

The calculations are not always performed in their entirety, and the reader is left to complete certain parts.

At the end of this section, we will also look at the properties of a *parabola*, the shape of the M diagram due to a uniformly distributed load.

12.1.1 Relationship between the variation of the distributed load q_z and the shape of V and M diagrams

In Section 11.3 we showed how to find the shear force V from the distributed load q_z by integration:

$$V = -\int q_z \, \mathrm{d}x$$

and the bending moment M by integrating again:

$$M = \int V \, \mathrm{d}x.$$

With a simple variation of the distributed load q_z we can show directly what the *variation* of the shear force and the bending moment will be, and which *shape* the V and M diagrams will have. This leads to the following three rules for an unloaded field ($q_z = 0$), a field with a uniformly distributed load (q_z constant and $\neq 0$) and a field with a linearly distributed load respectively:

• ***Rule 1***
In an unloaded field, the shear force V is constant and the bending moment M is linear. If the shear force is zero, the bending moment is constant.

$$q_z = 0 \Rightarrow V \text{ constant;} \quad V = 0 \Rightarrow M \text{ constant,}$$
$$V \neq 0 \Rightarrow M \text{ linear.}$$

Table 12.1 Relationship between the variation of the distributed load q_z normal to the member axis and the variation of the shear force V and the bending moment M.

Variation q_z ⇒	**variation V ⇒**	**Variation M**
constant $= 0$	constant $= 0$	constant
constant $= 0$	constant $\neq 0$	linear
constant $\neq 0$	linear	quadratic
linear	quadratic	cubic

• ***Rule 2***
In a field with a uniformly distributed load q_z, the shear force V varies linearly, and the bending moment M varies quadratically (parabolic).

$$q_z \text{ constant } (\neq 0) \Rightarrow V \text{ linear } \Rightarrow M \text{ quadratic.}$$

• ***Rule 3***
In a field with a linearly distributed load q_z, the shear force V varies quadratically, and the bending moment M is a cubic function.

$$q_z \text{ linear } \Rightarrow V \text{ quadratic } \Rightarrow M \text{ cubic.}$$

The three rules are summarised in Table 12.1. The correctness of the rules can be verified in Section 11.3, Examples 1 to 3.

Using these rules, it is often possible to draw the V and M diagrams more quickly by determining the values in a limited number of points and then to sketch the path between these points.

The rules are illustrated below by means of two examples.

Example 1
The simply supported beam AD in Figure 12.1a is loaded at B and C by two equal forces of 30 kN.

Question:
Determine the V and M diagrams.

Solution:
First the support reactions at A and D are calculated. Both support reactions turn out to be the same: upward forces of 30 kN (see Figure 12.1b).

V diagram
As far as the V diagram is concerned, it can be said that for each of the unloaded fields AB, BC and CD, the shear force is constant between the

concentrated loads (rule 1). To fully plot the shear force diagram, we have to calculate only three shear forces, namely V^{AB}, V^{BC} and V^{CD}, the shear forces in the fields AB, BC and CD,[1] respectively. These shear forces can be calculated as shown in Section 10.2.1, Example 2, etc.

Shear force V^{BC} is thus found by introducing a section at an arbitrary location in field BC. The magnitude and direction of V^{BC} then follows from the vertical force equilibrium of the isolated part to the right or to the left of the section. In this case the calculation leads to $V^{BC} = 0$. The calculation is left to the reader.

The shear force in the end fields AB and CD is calculated in the same way. The shear forces V^{AB} and V^{CD} in the end fields are of equal magnitude to the support reactions at A and D respectively. Only their directions are different. See Figure 12.1b, which shows the actual directions and the associated deformation symbols. V^{AB} and V^{CD} are therefore plotted in the V diagram on different sides of the member axis.

In the V diagram in Figure 12.1c, the three values calculated are shown by means of dots. Since the shear force is constant, the V diagram can now be completed by drawing a horizontal line in each field through these points.

M diagram
As far as the M diagram is concerned, we know that the bending moment is zero at the supports A and D, and that the bending moment in each field varies linearly (rule 1). To be able to plot the M diagram it is therefore only necessary to calculate the bending moments at the field joinings at B and C after which straight lines can be drawn between the values at A to D.

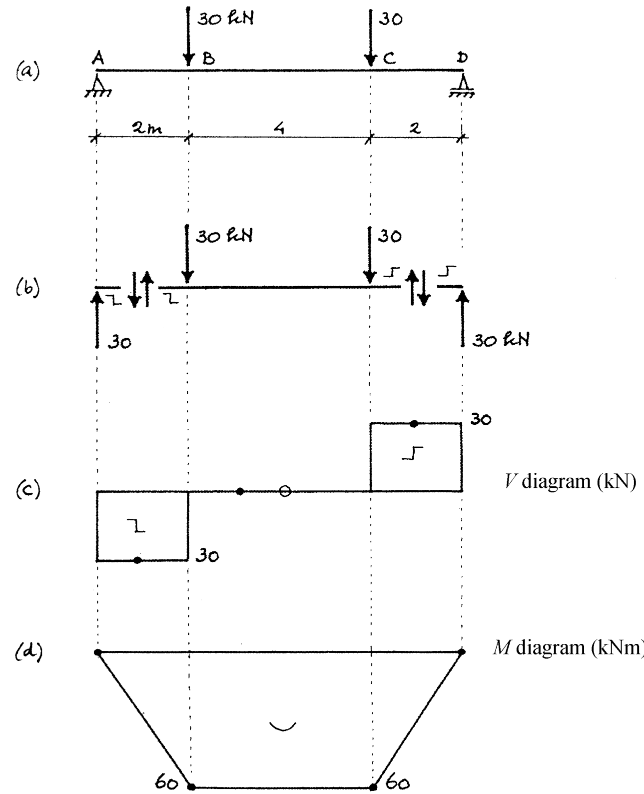


Figure 12.1 (a) Simply supported beam loaded symmetrically by two forces. (b) The support reactions at A and B, and the shear forces in fields AB and CD. (c) Shear force diagram. The shear force is constant in each field. (d) Bending moment diagram. This is fully determined by the values at the boundaries (ends and joinings of the fields).

[1] The upper index refers to the *segment* in which shear force V is acting.

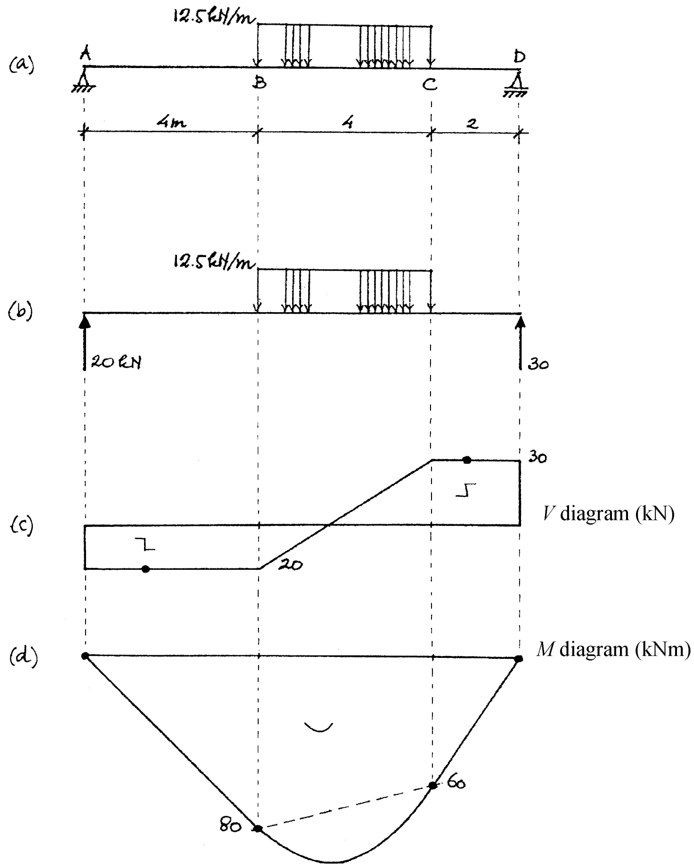


Figure 12.2 (a) Simply supported beam with a uniformly distributed load in field BC. (b) Support reactions. (c) Shear force diagram. The variation of the shear forces is linear under the uniformly distributed load. (d) Bending moment diagram. The bending moment is parabolic under the uniformly distributed load.

The bending moment M_B at B[1] is found from the moment equilibrium (about B) of the isolated part to the left or right of B. This bending moment is 60 kNm; the calculation is left to the reader. As this moment causes tension at the lower side of the beam, this value in the M diagram has to be plotted at the underside of the member axis. See the conventions discussed earlier in Sections 10.2.4 and 10.2.5 and see also Figure 10.40.

In the same way, the bending moment found at C is 60 kNm, also with tension at the lower side of the beam.

In the M diagram in Figure 12.1d, the two known and the two calculated values are shown by means of dots. The M diagram is completed by drawing straight lines between these values.

The M diagram shows that in field BC, where the shear force is zero, the bending moment is indeed constant (according to rule 1).

Example 2
In Figure 12.2a, the simply supported beam AD is subject to a uniformly distributed load of 12.5 kN/m in field BC.

Question:
Determine the V and M diagrams.

Solution:
The support reactions at A and D are 20 and 30 kN respectively, both upwards (see Figure 12.2b).

V diagram
We know that the shear force is constant in field AB (rule 1), varies linearly in field BC (rule 2) and is constant in field CD (rule 1). To draw the shear force diagram we have to calculate only the shear forces V^{AB} and V^{CD} in the end fields. Their magnitudes are the same as the support reactions at A

[1] The lower index refers to the *section* in which the bending moment M is acting.

and D respectively.

In the V diagram in Figure 12.2.c, the two calculated shear forces are shown by means of dots. Since the shear forces have a different shear symbol, they are plotted at different sides of the member axis. The V diagram is now completed by means of the horizontal lines in the fields AB and CD, after which the linear path in field BC can be drawn.

M diagram
The bending moment in the beam varies linearly in AB (rule 1), quadratically in BC (rule 2) and linearly in CD (rule 1). At A and D, the bending moment is zero. For the field joinings at B and C, we can calculate that the bending moment is 80 and 60 kNm respectively, both with tension at the lower side of the beam. In the M diagram in Figure 12.2d, these values are plotted downwards of the member axis.

On the M diagram, the four known values are shown by means of dots. The M diagram can now be completed by drawing straight lines between the values at A and B, respectively C and D (linear variation) and by drawing a parabola between the values at B and C (quadratic variation).

More detailed information is required to draw the parabola between B and C somewhat accurately. This is provided in the following subsections, in particular Section 12.1.6.

12.1.2 Slope of the V diagram and M diagram and extreme values of V and M

In Section 11.1, the differential equations for the equilibrium of an infinitesimally small member segment loaded normal to its axis were derived:

$$\frac{\mathrm{d}V}{\mathrm{d}x} + q_z = 0,$$

$$\frac{\mathrm{d}M}{\mathrm{d}x} - V = 0.$$

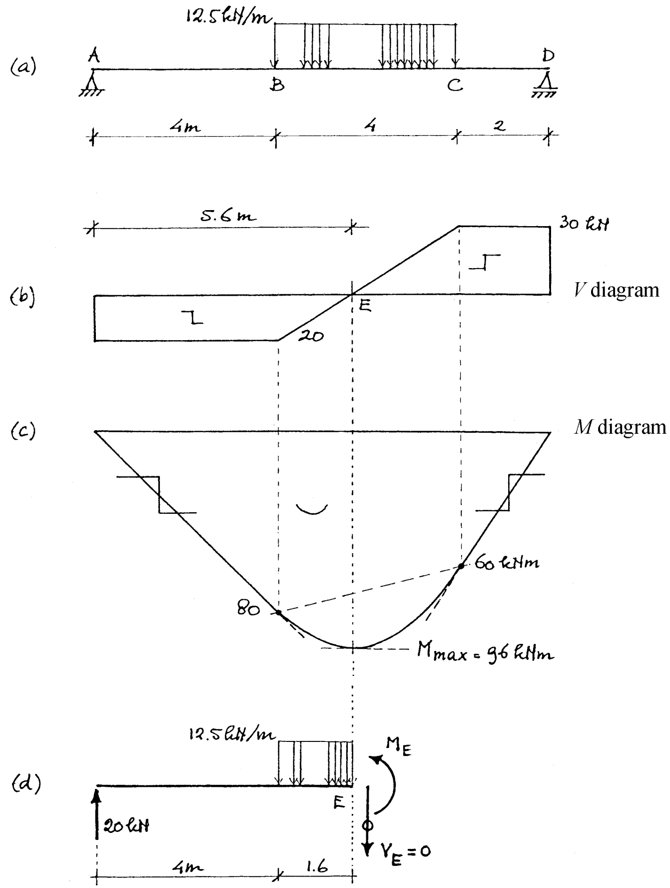


Figure 12.3 (a) Simply supported beam with a uniformly distributed load in field BC. (b) Shear force diagram. (c) Bending moment diagram. The steps in the M diagram agree with the deformation symbols in the V diagram. The bending moment is an extreme where the shear force is zero. (d) The isolated part AE for calculating the maximum bending moment at E.

These equations can also be written as

$$\frac{\mathrm{d}V}{\mathrm{d}x} = -q_z, \quad \text{(a)}$$

$$\frac{\mathrm{d}M}{\mathrm{d}x} = V. \quad \text{(b)}$$

Expressing the differential equations (a) and (b) in words gives the following two rules:

• ***Rule 4***
The slope of the V diagram ($\mathrm{d}V/\mathrm{d}x$) is equal to the distributed load q_z (but with an opposite sign).

• ***Rule 5***
The slope of the M diagram ($\mathrm{d}M/\mathrm{d}x$) is equal to the shear force V.

The correctness of these rules can be verified directly using the two examples in the previous section.

If working with deformation symbols, the direction of the slope of the V or M diagram can no longer be shown by means of plus and minus. We therefore have to work with the absolute values $|\mathrm{d}V/\mathrm{d}x|$ and $|\mathrm{d}M/\mathrm{d}x|$. The directions are deduced from the V and M diagrams respectively.

Example 1
You are given the beam in Figure 12.3a with its V and M diagram in respectively Figures 12.3b and 12.3c.

Questions:

a. Examine whether the M and V diagrams comply with rules 4 and 5.
b. Where is the bending moment an extreme, and how large is that moment?

Solution:
a. In the end fields AB and CD, where the distributed load is zero, the slope

of the V diagram is also zero. This is in line with rule 4.

In the middle field BC the slope of the V diagram is

$$\left|\frac{\Delta V}{\Delta x}\right| = \frac{(20 \text{ kN}) + (30 \text{ kN})}{4 \text{ m}} = 12.5 \text{ kN/m}.$$

This also agrees with rule 4, as it is exactly the value of the uniformly distributed load in field BC.

The slopes of the M diagram in the end fields AB and CD are

$$\left|\frac{\Delta M}{\Delta x}\right|^{(\text{AB})} = \frac{80 \text{ kNm}}{4 \text{ m}} = 20 \text{ kN} = V^{(\text{AB})},$$

$$\left|\frac{\Delta M}{\Delta x}\right|^{(\text{CD})} = \frac{60 \text{ kNm}}{2 \text{ m}} = 30 \text{ kN} = V^{(\text{CD})}.$$

In line with rule 5, the slopes of the M diagram are equal to the shear forces.

As a check for the directions of the shear force, we notice that each "*step*" in the M diagram (which stands for $|\Delta M/\Delta x|$) corresponds to the deformation symbol for the shear force. This check is possible only if one plots the M diagram in accordance with the convention in Section 10.2.4 which requires that the concave side of the bending symbol is faced to the member axis.

The shear forces directly to the left and right of B are equal:

$$V_{\text{B}}^{(\text{AB})} = V_{\text{B}}^{(\text{BC})} = 20 \text{ kN}.$$

So, in accordance with rule 5, the slopes of the M diagram directly to the left and right of B are equal. In other words, the straight M path in field AB is the tangent at B to the parabola in field BC. In the same way, the straight M path in field CD is the tangent to the parabola at C.

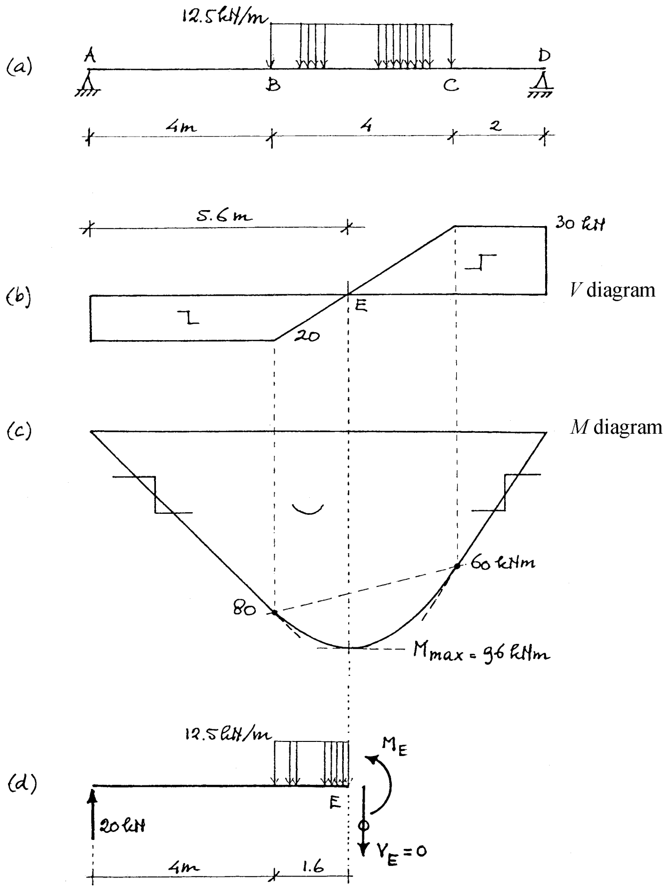


Figure 12.3 (a) Simply supported beam with a uniformly distributed load in field BC. (b) Shear force diagram. (c) Bending moment diagram. The steps in the M diagram agree with the deformation symbols in the V diagram. The bending moment is an extreme where the shear force is zero. (d) The isolated part AE for calculating the maximum bending moment at E.

b. At E, the shear force is zero and the parabola has a horizontal tangent. Here the bending moment is an extreme. The bending moment at E can be calculated from the moment equilibrium of the isolated segment AE or EB. Figure 12.3d shows segment AE. From the bending symbol in the M diagram, we know that the bending moment causes tension at the lower side of the beam; M_E in Figure 12.3d therefore has been plotted at the underside of the beam axis. For AE it holds that

$$\sum T|\text{E} \circlearrowleft = -(20\text{ kN})(5.6\text{ m}) + (1.6\text{ m})(12.5\text{ kN/m})(0.8\text{ m}) + M_\text{E} = 0 \qquad \text{(a)}$$

so that

$$M_\text{E} = (112\text{ kNm}) - (16\text{ kNm}) = 96\text{ kNm}.$$

Since no coordinate system is shown in Figure 12.3d, the positive direction of rotation of the moment about E in expression (a) has been depicted by means of a bent arrow. The positive direction of rotation chosen here is anti-clockwise.

The value of the maximum bending moment at E and the tangents at B, C and E are aids for sketching the parabolic M diagram in field BC (see also Section 12.1.6).

In general, the shear force V has an extreme value where $\mathrm{d}V/\mathrm{d}x = 0$. Also, the bending moment M is an extreme where $\mathrm{d}M/\mathrm{d}x = 0$. This leads to the following two rules.

• ***Rule 6***

The shear force V is an extreme where the distributed load q_z is zero (or changes sign). Per field, we have to take into account that the occurrence of values at the boundaries (e.g. at concentrated loads and supports).

- ***Rule 7***

The bending moment is an extreme where the shear force is zero (or changes sign). Per field, we have to take into account the occurrence of extreme values at the boundaries (e.g. where concentrated loads and/or couples act or at supports).

Example 2

The simply supported beam AB in Figure 12.4a carries a linearly distributed load of 30 kN/m upwards at A to 60 kN/m downwards at B.

Questions:

a. Determine the support reactions.
b. Make a good sketch of the V and M diagrams.
c. Determine the extreme values of V and M.

Solution:

a. To calculate the support reactions, the beam is divided into the fields AC and CB. Per field, the resultant of the triangle load is (see Figure 12.4b)

$$R^{(AC)} = \tfrac{1}{2}(3\text{ m})(30\text{ kN/m}) = 45\text{ kN},$$

$$R^{(CB)} = \tfrac{1}{2}(6\text{ m})(60\text{ kN/m}) = 180\text{ kN}.$$

The vertical support reaction at A ($A_v\uparrow$) follows from the moment equilibrium of AB about B:

$$\sum T|\text{B} \circlearrowleft \; = -A_v \times (9\text{ m}) - 45\text{ kN})(8\text{ m}) + (180\text{ kN})(2\text{ m}) = 0$$

$$\Rightarrow A_v = 0.$$

The vertical support reaction in B ($B_v\uparrow$) follows from the vertical force equilibrium of AB:

$$\sum F_{\text{vert}}\uparrow = (45\text{ kN}) - (180\text{ kN}) + B_v = 0 \Rightarrow B_v = 135\text{ kN}(\uparrow).$$

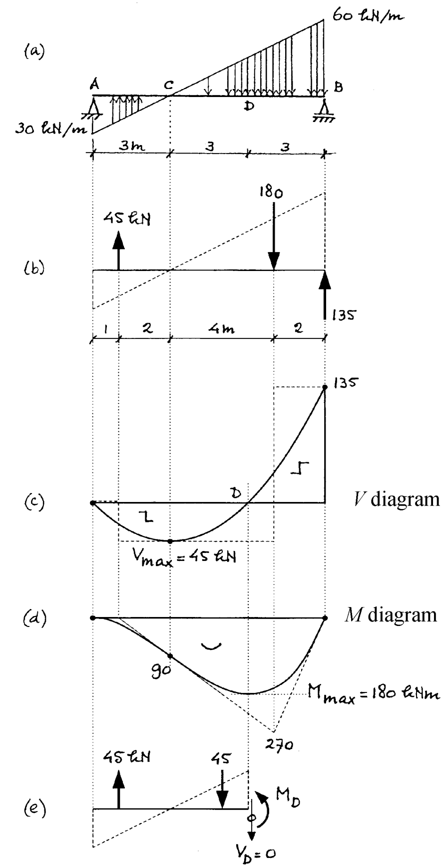


Figure 12.4 (a) Simply supported beam with linear distributed load. (b) The isolated beam with the distributed loads in field AC and CB replaced by their resultant. The support reaction at A is zero and at B is 135 kN. (c) Shear force diagram. (d) Bending moment diagram. (e) The isolated part AD for calculating the maximum bending moment at D.

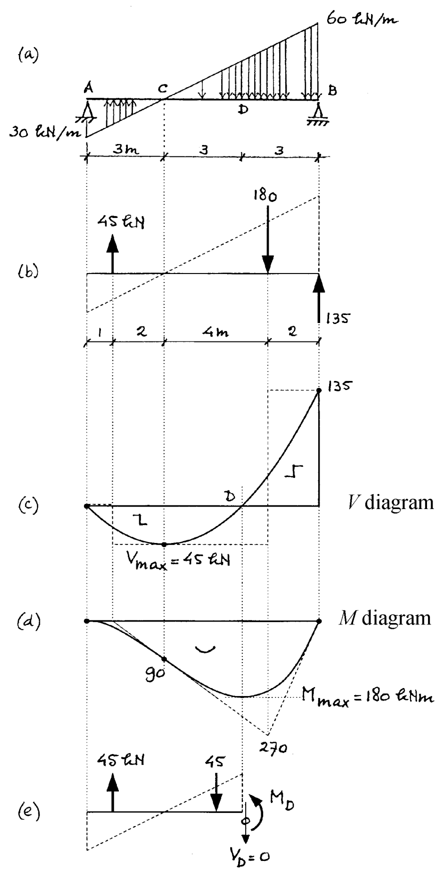


Figure 12.4 (a) Simply supported beam with linear distributed load. (b) The isolated beam with the distributed loads in field AC and CB replaced by their resultant. The support reaction at A is zero and at B is 135 kN. (c) Shear force diagram. (d) Bending moment diagram. (e) The isolated part AD for calculating the maximum bending moment at D.

b. For the beam in Figure 12.4b, loaded only by concentrated loads, calculating and drawing the V and M diagrams is relatively simple using rule 1 from the previous section. The calculation is left to the reader. The result is shown in Figures 12.4c and 12.4d by means of a dashed line.

The dashed lines do not give the correct V and M diagrams, but the values of V and M at the field boundaries are correct! They are shown in Figures 12.4c and 12.4d by means of dots. If we isolate a field to calculate the shear force and the bending moment in the field boundaries, it does not matter in the equilibrium equations whether we consider the actual load in the field or its resultant.

The actual V and M diagrams therefore pass through the dots at the field boundaries.

Since the shear forces at the field boundaries A, C and B in the dashed case have the correct values, the slopes of the M diagram in the dashed case are also correct. This means that the dashed M diagram in Figure 12.4d at A, C and B is tangent to the actual M diagram.

According to rule 3, the shear force in Figure 12.4c varies parabolically. Since at C the distributed load is zero, the tangent to the V diagram is horizontal there. The V diagram is now easy to draw.

According to rule 3, the M diagram in Figure 12.4d is cubic. It passes through the three black dots and has the dashed lines at the dots as *tangents*. After this it is not difficult to sketch the M diagram.

c. The shear force is an extreme where the distributed load is zero (rule 6). Isolating section AC or CB the vertical force equilibrium gives

$$V_{\max} = 45 \text{ kN}.$$

However, the largest shear force occurs at support B and is 135 kN (with opposite sign). This is an example of a *maximum at a field boundary*.

The maximum bending moment occurs where the shear force is zero, in D. The V diagram varies parabolically. The top of the parabola is at C. Since distances AC and CD are equal, the distance from D to A is therefore 6 m.

The maximum bending moment at D can be calculated from the moment equilibrium of isolated parts AD or DB. In Figure 12.4e, part AD has been isolated, and the two triangular loads have been replaced by their resultants. From the deformation symbol in the M diagram we observe tension at the underside of the beam due to the bending moment M_D at D. In Figure 12.4e, M_D is shown in accordance with this direction. For AD it holds that

$$\sum T|D \circlearrowleft = -(45 \text{ kN})(5 \text{ m}) + (45 \text{ kN})(1 \text{ m}) + M_D = 0$$

so that

$$M_D = M_{max} = 180 \text{ kNm}.$$

It is up to the reader to check that the same maximum bending moment is found from the equilibrium of part DB.

12.1.3 Tangents to the M diagram

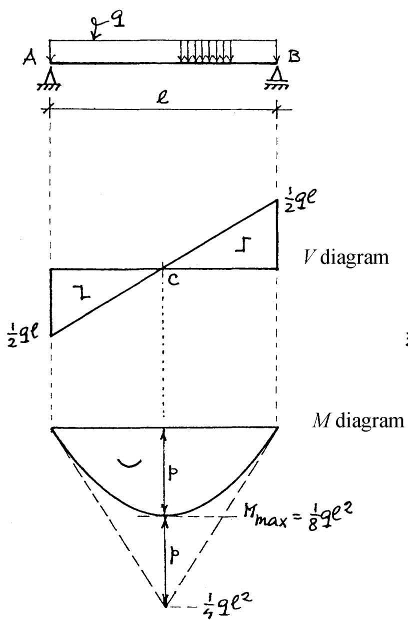
When drawing the V and M diagrams in Example 2 from the previous section, two new rules occur:

• ***Rule 8***

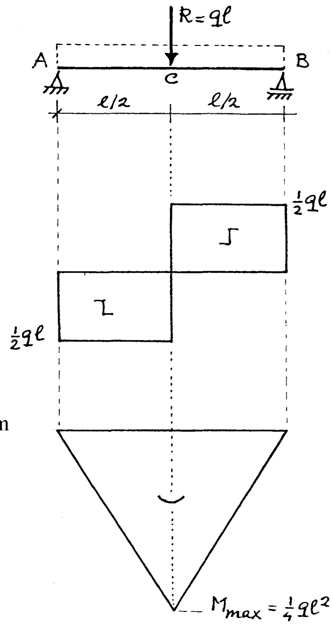
The tangents to the M diagram at the boundaries of a field intersect on the line of action of the load resultant in that field (for a distributed load this is at the centroid of the load diagram).

• ***Rule 9***

If we replace the load per field by its resultant and we draw the bending moment diagram due to these resultants, this bending moment diagram is tangent to the actual bending moment at the field boundaries.



(12.5)



(12.6)

Figure 12.5 Simply supported beam with V and M diagrams due to a uniformly distributed full load.

Figure 12.6 Simply supported beam with the V and M diagrams due to the resultant of a uniformly distributed full load.

We will illustrate this with three examples.

Example 1

The first example relates to a simply supported beam AB with length ℓ. In Figure 12.5, the beam is over its entire length subjected to a uniformly distributed load q. In Figure 12.6, the beam is subjected to a concentrated force $R = q\ell$ at midspan C, to be seen as the resultant of the previously mentioned uniformly distributed load q.

Question:
Verify the rules 8 and 9.

Solution:
In the case of Figure 12.5, the shear force varies linearly and the bending moment varies parabolically. At midspan C, it applies that

$$V = \frac{\mathrm{d}M}{\mathrm{d}x} = 0.$$

Here the M diagram has a horizontal tangent. This means that the bending moment M at C is a maximum (rule 7):

$$M_{\mathrm{max}} = \tfrac{1}{8}q\ell^2.$$

In the case in Figure 12.6, the shear force at C is not zero (as is often wrongly said), but rather changes sign. Here too the bending moment is a maximum (rule 7):

$$M_{\mathrm{max}} = \tfrac{1}{4}R\ell = \tfrac{1}{4}q\ell^2.$$

This maximum bending moment, twice as large as in the case of the uniformly distributed load, is a *boundary extreme* at the joining of fields AC and BC.

If the distributed load q in Figure 12.5 is replaced by its resultant R in Figure 12.6, the V and M diagrams change. The values at A and B do not change. Since the shear forces at A in both cases are equal, the slopes of the M diagram at A are also equal in both cases. The same holds for B.

It is now clear that the tangents at A and B for the distributed load can be found by drawing the M diagram due to the resultant R. Both tangents intersect, in accordance with rule 8, at the middle of AB, on the line of action R.

For the value p indicated in the M diagram,

$$p = \tfrac{1}{8} q \ell^2.$$

Note: This expression for p holds only if the load is *uniformly distributed*.

Example 2

Draw the V and M diagrams for the simply supported beam in Figure 12.7a, with the deformation symbols. See also Section 12.1.1, Example 2.

Solution:

If the distributed load $q = 12.5$ kN/m is replaced by its resultant $R = 50$ kN at the middle G of BC, then the V and M diagrams change only between B and C. The change over BC of V and M is shown by means of the dashed lines in Figures 12.7b and 12.7c.

The actual V diagram in field BC varies linearly (rule 2). This is shown in Figure 12.7b.

The resultant R gives the same shear force (20 kN) and bending moment (80 kNm) at B as the uniformly distributed load q. Since the shear force can be interpreted as the slope of the M diagram, this means that at B the M diagrams due to R and q have the same slope. The same applies at C.

The actual M diagram in field BC varies parabolically. At B and C the parabola is tangent to the dashed M diagram due to the resultant R (rule 9).

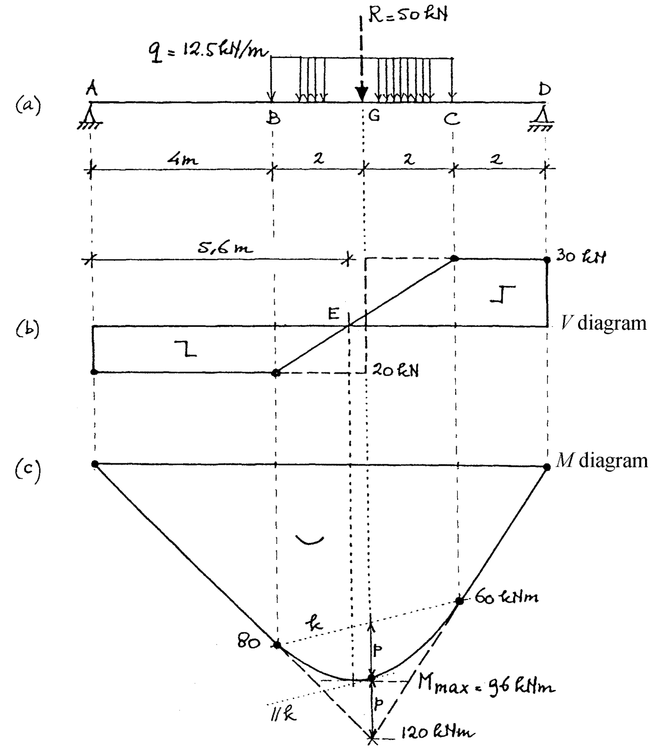


Figure 12.7 (a) Simply supported beam with uniformly distributed load in field BC. (b) Shear force diagram. The dashed V diagram in field BC corresponds with the resultant R of the distributed load. (c) Bending moment diagram. The dashed M diagram in field BC corresponds with the resultant R of the distributed load.

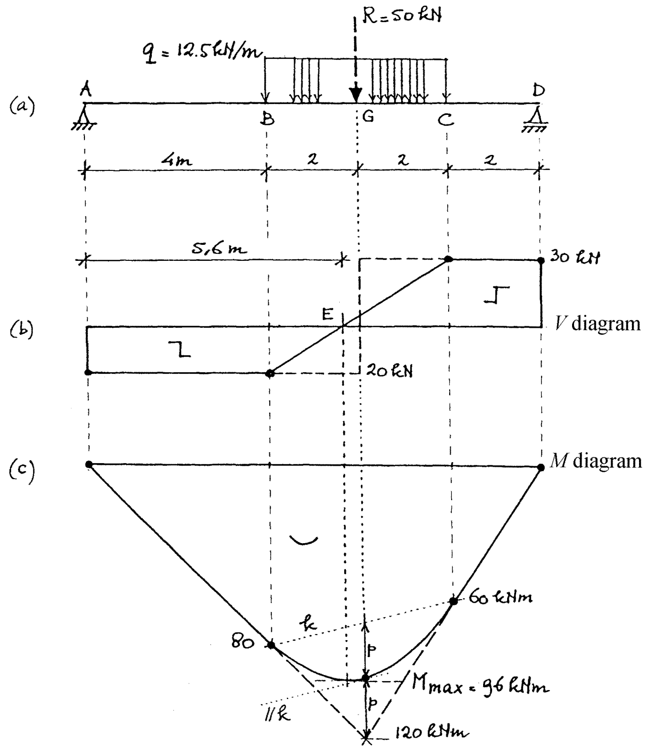


Figure 12.7 (a) Simply supported beam with uniformly distributed load in field BC. (b) Shear force diagram. The dashed V diagram in field BC corresponds with the resultant R of the distributed load. (c) Bending moment diagram. The dashed M diagram in field BC corresponds with the resultant R of the distributed load.

The tangents at B and C intersect on the line of action of resultant R, in the middle of BC (rule 8).

The parabola in field BC is the same parabola as in Example 1, except that it is now "*obliquely suspended*" between the values of 80 and 60 kNm at B and C respectively. The tangent midway of the parabola is parallel to chord k.

As in Example 1, in the middle of field BC, the value of p in the M diagram is

$$p = \tfrac{1}{8}q\ell^2 = \tfrac{1}{8}(12.5\text{ kN/m})(4\text{ m})^2 = 25\text{ kNm}.$$

Here ℓ represents the length of field BC.

It is up to the reader to check whether the distance p from the bend in the dashed M diagram to the actual M diagram, and from there to the dotted chord between the values at B and C is indeed 25 kNm.

It should be noted that the bending moment is not a maximum in the middle of BC, $x = 6$ m. The maximum occurs somewhat to the left of the middle ($x = 5.6$ m), where the shear force is zero (rule 7).

Example 3

A uniformly distributed load of 50 kN/m is acting on the simply supported beam AC in Figure 12.8a, downwards in field AB and upwards in field BC.

Questions:

a. Draw a good sketch of the V diagram, with the deformation symbols. Where is the shear force an extreme?
b. Draw a good sketch of the M diagram, with the deformation symbols. At relevant points, also draw the tangents to the M diagram. Where is the bending moment an extreme?

Solution:

a. Before drawing the shear force diagram and the bending moment diagram, we first have to determine the support reactions. The support reactions at A and C are respectively 170 kN (↑) and 70 kN (↓) (see Figure 12.8b).

The shear force varies linearly in both field AB and field BC (rule 2). To draw the V diagram we therefore have to calculate only the shear forces at A, B and C. The shear forces at A and C are equal to the support reactions at A and C. Beware of the deformation symbols! The shear force at B is found from the vertical force equilibrium of AB or BC.

Figure 12.8c shows the V diagram. The values calculated are shown by means of a dot.

At B, the distributed load changes sign, and the shear force is an extreme (rule 6). However, the maximum shear force occurs at the location of support A (a boundary maximum).

b. In order to draw the M diagram with its tangents, the distributed loads in fields AB and BC are replaced by their resultants (see Figure 12.8b). The bent M diagram due to these resultants is shown by means of the dashed path in Figure 12.8d. At A, B and C, the dashed M diagram is tangent to the actual M diagram (rule 9). Per field, the actual M diagram varies parabolically (rule 2).

If, in the middle of a field, one halves the distance between the maximum value of the dashed M diagram to the chord of the parabola, this gives an additional point on the parabola. At that point, the tangent to the parabola is parallel to the chord. Using this information, it is possible to make a very accurate free-hand sketch of the M diagram. The result is shown in Figure 12.8d.[1]

[1] The tangents in the middle of the fields (parallel to the chords) are not shown, to ensure the figure remains somewhat legible.

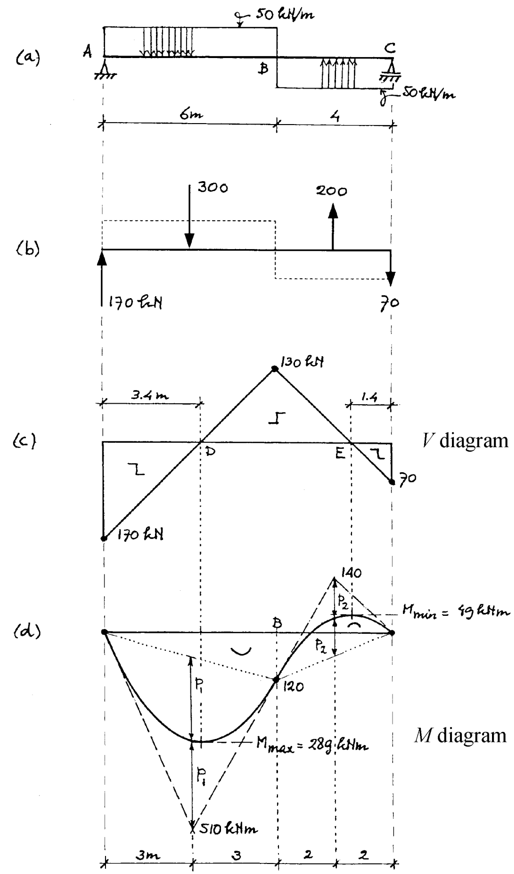


Figure 12.8 (a) Simply supported beam with an abruptly changing uniformly distributed load at B. (b) The isolated beam with its support reactions and the distributed loads in field AB and BC replaced by their resultants. (c) Shear force diagram. (d) Bending moment diagram. At the field boundaries A, B and C, this diagram is tangent to the dashed M diagram due to the load resultants.

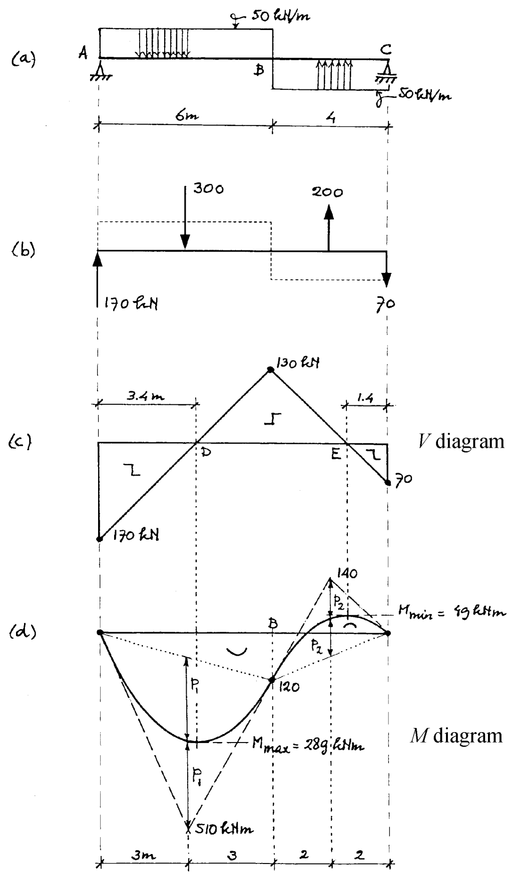


Figure 12.8 (a) Simply supported beam with an abruptly changing uniformly distributed load at B. (b) The isolated beam with its support reactions and the distributed loads in field AB and BC replaced by their resultants. (c) Shear force diagram. (d) Bending moment diagram. At the field boundaries A, B and C, this diagram is tangent to the dashed M diagram due to the load resultants.

The correctness of the M diagram can be checked using the values of p_1 and p_2:

$$p_1 = \tfrac{1}{8}(50\text{ kN/m})(6\text{ m})^2 = 225\text{ kNm},$$

$$p_2 = \tfrac{1}{8}(50\text{ kN/m})(4\text{ m})^2 = 100\text{ kNm}.$$

It is up to the reader to perform this check.

The bending moment is an extreme where the shear force is zero (rule 7), that is at D and E. These extreme values can be calculated from the moment equilibrium of the isolated part to the left or right of a section at respectively D and E.

In Section 12.1.4 we present an alternative method for calculating the maximum shear force and the maximum bending moment.

12.1.4 Interpreting the area of the load diagram and V diagram

The differential equations for the equilibrium of an infinitesimally small beam segment are

$$\frac{\mathrm{d}V}{\mathrm{d}x} = -q_z, \tag{a}$$

$$\frac{\mathrm{d}M}{\mathrm{d}x} = V. \tag{b}$$

This can also be written as

$$\mathrm{d}V = -q_z\,\mathrm{d}x, \tag{c}$$

$$\mathrm{d}M = V\,\mathrm{d}x. \qquad \text{(d)}$$

Integrating expression (c) over the interval between x_1 and x_2 gives

$$\int_{x_1}^{x_2} \mathrm{d}V = -\int_{x_1}^{x_2} q_z\,\mathrm{d}x$$

so that

$$\Delta V = V(x_2) - V(x_1) = -\int_{x_1}^{x_2} q_z\,\mathrm{d}x.$$

Expressed in words and ignoring the signs, this leads to rule 10:

• ***Rule 10***
Without concentrated forces,[1] the change in the shear force V over a certain length is equal to the area of the load diagram over that length.

Integrating expression (d) over the interval between x_1 and x_2 gives

$$\int_{x_1}^{x_2} \mathrm{d}M = \int_{x_1}^{x_2} V\,\mathrm{d}x$$

so that

$$\Delta M = M(x_2) - M(x_1) = \int_{x_1}^{x_2} V\,\mathrm{d}x.$$

Expressed in words and ignoring the signs, this leads to rule 11:

---

[1] If there are acting concentrated couples, the differential equation (a) is no longer valid.

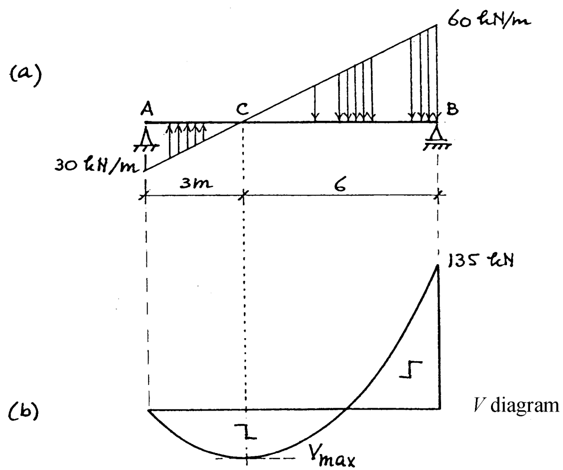


Figure 12.9 (a) Simply supported beam with linear distributed load and (b) the associated shear force diagram. The shear force is an extreme where the distributed load is zero – not taking into account the extremes at the field boundaries.

• ***Rule 11***

Without concentrated couples,[1] the change in the bending moment over a certain length equals the area of the V diagram over that length.

Rules 10 and 11 are demonstrated using two examples.

Example 1

The V diagram in Figure 12.9b for the beam in Figure 12.9a was previously calculated in Section 12.1.2, Example 2. The maximum shear force – boundary extremes not considered – occurs at C and can be calculated directly from the force equilibrium of AC or BC.

Question:

Determine the maximum shear force at C from the area of the load diagram.

Solution:

From the V diagram we can read that

$$\Delta V^{(AC)} = V_C - V_A = V_{max}.$$

$\Delta V^{(AC)}$ is equal to the area of the load diagram over AC:

$$V_{max} = \Delta V^{(AC)} = \tfrac{1}{2}(3\text{ m})(30\text{ kN/m}) = 45\text{ kN}.$$

Example 2

For the beam in Figure 12.10a, the V and M diagrams in Figures 12.10b and 12.10c were calculated earlier in Section 12.1.3, Example 3.

Question:

Calculate the extreme bending moments from the area of the shear force diagram.

1 If there are acting concentrated couples, the differential equation (b) is no longer true.

Solution:
The bending moments are extreme where the shear force is zero, that is at D and E.

The *maximum* bending moment occurs at D. From the M diagram, with $M_\mathrm{D} = M_\mathrm{max}$ and $M_\mathrm{A} = 0$, we can read that

$$\Delta M^{(\mathrm{AD})} = M_\mathrm{D} - M_\mathrm{A} = M_\mathrm{max}.$$

$\Delta M^{(\mathrm{AD})}$ is equal to the area of the shear force diagram over AD:

$$M_\mathrm{max} = \Delta M^{(\mathrm{AD})} = \tfrac{1}{2}(3.4\ \mathrm{m})(170\ \mathrm{kN}) = 289\ \mathrm{kNm}.$$

The *minimum* bending moment occurs at E, and is found in the same way from the area of the shear force diagram over EC:

$$M_\mathrm{min} = \Delta M^{(\mathrm{EC})} = \tfrac{1}{2}(1.4\ \mathrm{m})(70\ \mathrm{kN}) = 49\ \mathrm{kNm}.$$

Check:

$$\Delta M^{(\mathrm{DE})} = M_\mathrm{min} + M_\mathrm{max} = 338\ \mathrm{kNm}.$$

This must be equal to the area of the V diagram over DE:

$$\Delta M^{(\mathrm{DE})} = \tfrac{1}{2}(5.2\ \mathrm{m})(130\ \mathrm{kN}) = 338\ \mathrm{kNm},$$

which is indeed the case.

From the above, and taking into account rule 11, we discover another property:

• ***Rule 12***

a. For a beam without concentrated couples, the total area of the V diagram is zero. More generally

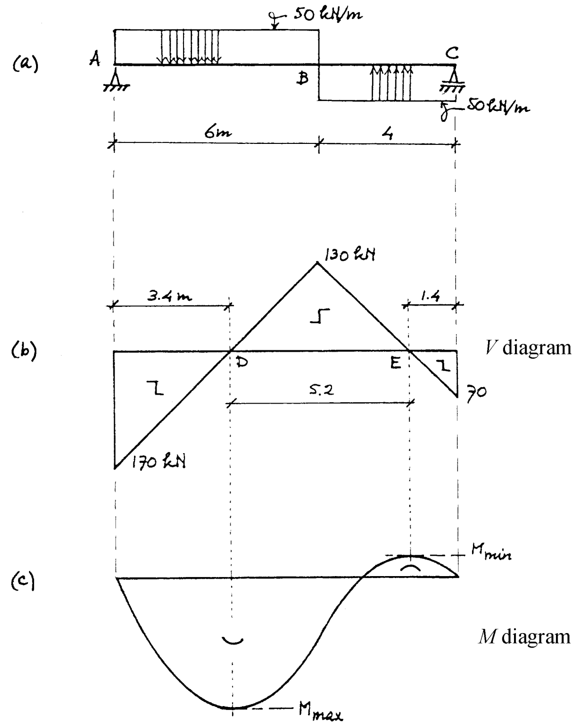


Figure 12.10 (a) Simply supported beam with a step change in the uniformly distributed load at B. (b) Shear force diagram. The shear force is an extreme where the distributed load changes sign. (c) Bending moment diagram. The bending moments are extreme where the shear force is zero.

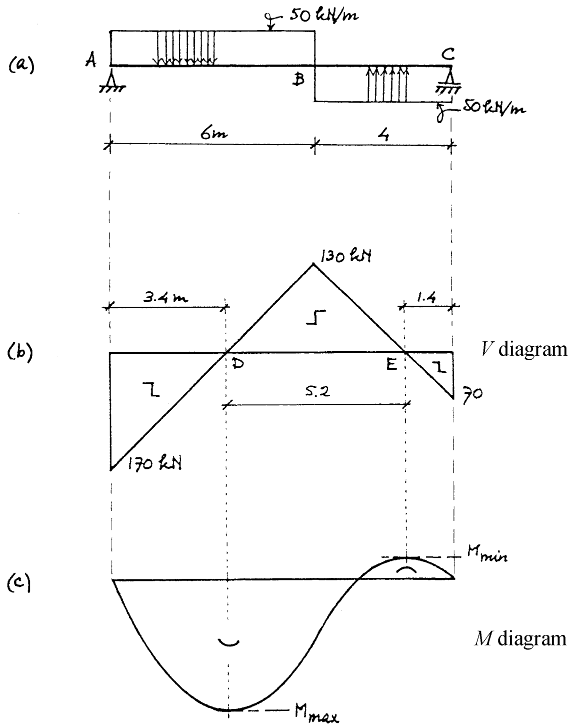


Figure 12.10 (a) Simply supported beam with a step change in the uniformly distributed load at B. (b) Shear force diagram. The shear force is an extreme where the distributed load changes sign. (c) Bending moment diagram. The bending moments are extreme where the shear force is zero.

b. The total area of the V diagram is equal to the sum of moments of all concentrated couples that act on the beam.

Example 3
Check rule 12a for the beam in Figure 12.10.

Solution:
For the beam simply supported at A and C, without concentrated couples, it applies that

$$\Delta M^{(\mathrm{AC})} = M_{\mathrm{C}} - M_{\mathrm{A}} = 0.$$

$\Delta M^{(\mathrm{AC})}$ is equal to the total area of the V diagram:

$$\tfrac{1}{2}(3.4 \text{ m})(170 \text{ kN}) - \tfrac{1}{2}(5.2 \text{ m})(130 \text{ kN}) + \tfrac{1}{2}(1.4 \text{ m})(70 \text{ kN}) = 0 \quad \text{(b)}$$

which is indeed zero.

Comment: When we calculate the area of the V diagram, the minus sign in expression (b) indicates that the deformation symbol for the shear force in field DE is opposite to that in the rest of the beam. The plus and minus signs alone here are arbitrary: they could just as well be interchanged one another. For this reason, we generally look at the *absolute value* of the area of the V diagram.

If there are (concentrated) couples acting on the beam, the (absolute value of the) total area of the V diagram is equal to the sum of the moments of the (concentrated) couples. This rule is closely bound up with the modelling of the couple as discussed in Section 12.5.3.

12.1.5 Step changes and bends in the V and M diagrams

• ***Rule 13***
A step change in the distributed load q_z gives a bend in the V diagram (and a point of inflection in the M diagram).

For rule 13, please refer to Figure 12.10. In both fields, the slope of the V diagram is 50 kN/m. The slopes are opposite as the distributed loads in the fields are opposite (rule 4). This causes a bend in the V diagram.

• ***Rule 14***

A (concentrated) force F normal to the member axis generates a step change in the V diagram, with magnitude F, and a bend in the M diagram.

• ***Rule 15***

A (concentrated) couple T gives a step change in the M diagram of magnitude T. The V diagram reveals no information about the point of application of the couple.

Two examples are given below to illustrate this.

Example 1

The simply supported beam in Figure 12.11a is loaded by a force of 40 kN and a (concentrated) couple of 80 kNm.

Questions:

a. Calculate and draw the V and M diagrams, with the deformation symbols.
b. Explain the step change in the V and M diagrams from the equilibrium of joints B and C respectively.
c. To what extent can rules 11 and 12 be applied here?

Solution:

a. The support reactions at A and B are both 20 kN (↑), see Figure 12.11b.

In principle, when drawing the V and M diagrams, we have to distinguish three fields: AB, BC and CD. There is no distributed load, so that the shear force is constant, and the bending moment varies linearly in all three fields (rule 1).

In the end fields AB and CD, the shear force is equal to the support reactions at A and D respectively. Beware of the deformation symbols!

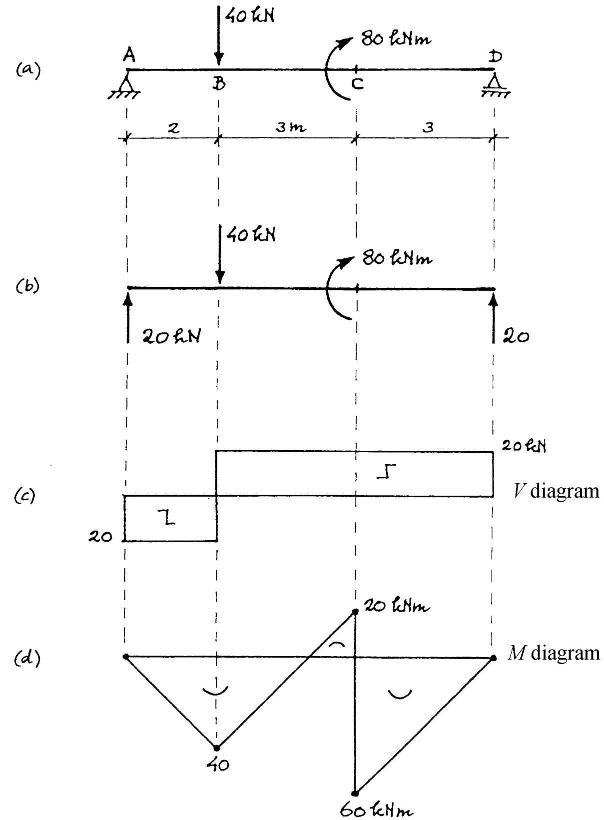


Figure 12.11 (a) Simply supported beam loaded by a concentrated force and a couple. (b) The isolated beam and its support reactions. (c) Shear force diagram. A step change occurs at the point of application of the concentrated force. The point of application of the couple cannot be derived from the V diagram. (d) Bending moment diagram. A bend occurs at the point of application of the concentrated force, and a step change occurs where the couple is applied.

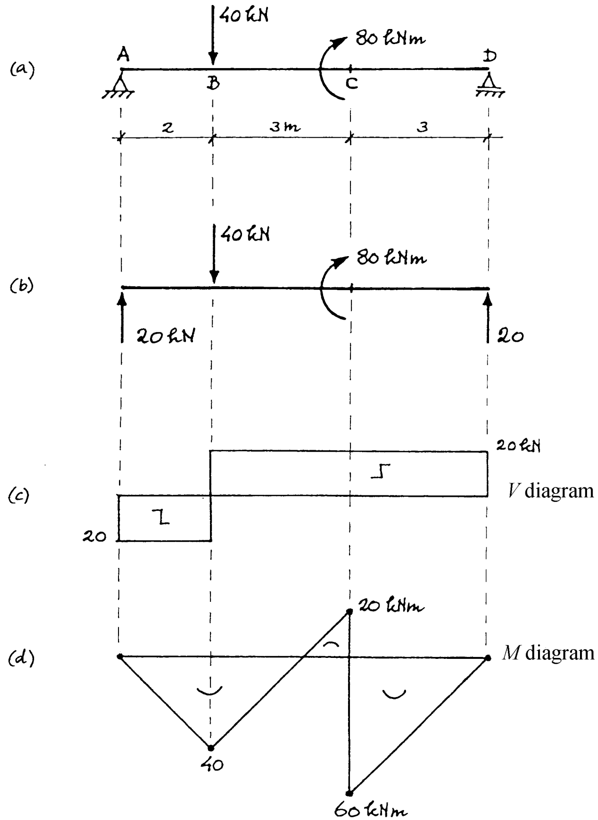


Figure 12.11 (a) Simply supported beam loaded by a concentrated force and a couple. (b) The isolated beam and its support reactions. (c) Shear force diagram. A step change occurs at the point of application of the concentrated force. The point of application of the couple cannot be derived from the V diagram. (d) Bending moment diagram. A bend occurs at the point of application of the concentrated force, and a step change occurs where the couple is applied.

To find the shear force in field BC, we have to introduce a section somewhere in field BC and investigate the vertical force equilibrium of the isolated part to the left or to the right of the section. If we write down the expression for the force equilibrium of the right-hand part, the couple of 80 kNm plays no part in this expression! The shear force in BC is therefore equal to the shear force in CD and is 20 kN.

Figure 12.11c shows the V diagram. The V diagram reveals no information about the point of application of the couple (rule 15). At B, at the location of the 40 kN load, a step change occurs in the V diagram. The magnitude of the step change is equal to the magnitude of the concentrated load.

To draw the M diagram, we have to know the values at the field boundaries A, B, C and D. Between them, the bending moment varies linearly. At A and D the bending moment is zero.

The bending moment at B is calculated from the moment equilibrium of the isolated part AB or BD. The result is 40 kNm with tension at the underside.

In the same way, the bending moment at C is calculated from the moment equilibrium of the isolated part AC or CD. But beware: due to the 80 kNm couple it makes a difference whether we take the section directly to the left or directly to the right of C. In the section directly to the left of C, we find 20 kNm with tension at the upper side, and in the section directly to the right of C we find 60 kNm with tension at the underside.

Figure 12.11d shows the M diagram. The values calculated at the field boundaries are shown by means of dots. Between these values, the moment varies linearly. At C, the point of application of the 80 kNm couple, there is a step change in the M diagram. The magnitude of the step change is equal to the magnitude of the (concentrated) couple.

Note that because the shear force over BCD is constant, the M diagram to the left and to the right of C have the same slope (rule 5).

At B, the shear force changes sign, and the M diagram is an extreme (rule

7). We cannot however see from the V diagram that the M diagram also has extreme values directly to the left and to the right of C, the point where the couple is applied. The maximum bending moment in an absolute sense occurs in the section directly to the right of C and is 60 kNm.

b. In Figure 12.12a, joint B has been isolated. In the sections, only the shear forces are shown. The shear forces directly to the left and right of B have to be in equilibrium with the vertical force on the joint. From this, it follows that the step change in the V diagram must be equal to the force on the joint.

In Figure 12.12b, joint C has been isolated. In the sections, only the bending moments are shown. From the moment equilibrium of joint C it follows that the bending moments directly to the left and to the right of C have to be in equilibrium with the couple at C. The magnitude of the step change in the M diagram must therefore be equal to the magnitude of the couple.

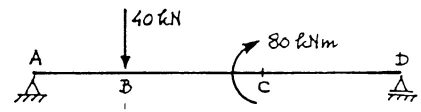
c. Rule 11, states that the change ΔM of the bending moment M is equal to the area of the V diagram. This rule is still valid as long as no couples are acting in the field that is considered.

If one looks at part AC of the beam, to the left of the couple, we could, for example, determine M_B and $M_{C;left}$ from the area of the V diagram. With $M_A = 0$ we find

$$M_B = \Delta M^{(AB)} = (2 \text{ m})(20 \text{ kN}) = 40 \text{ kNm},$$

$$M_{C;left} = \Delta M^{(AC)} = (2 \text{ m})(20 \text{ kN}) - (3 \text{ m})(20 \text{ kN}) = -20 \text{ kNm}.$$

In the latter case, because the V diagrams over AB and BC have different deformation symbols, the area of the V diagram is equal to the difference in the areas over AB and BC. Here, the opposite signs of M_B and $M_{C;left}$ indicate that the deformation symbols of M_B and $M_{C;left}$ are opposite. The plus and minus signs alone are however arbitrary and can be interchanged. For this reason, we generally work with absolute values and use for ΔM



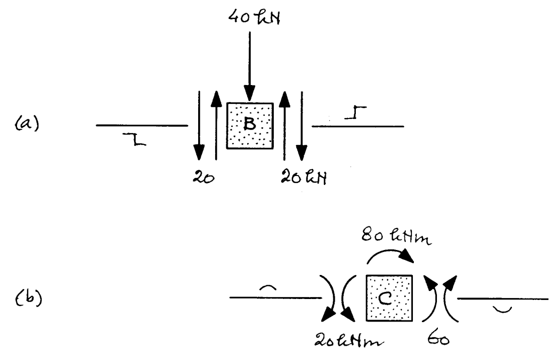


Figure 12.12 (a) The shear forces directly to the left and right of joint B are in equilibrium with the point load of 40 kN on joint B. (b) The bending moments directly to the left and right of joint C are in equilibrium with the load due to the concentrated couple of 80 kNm on joint C.

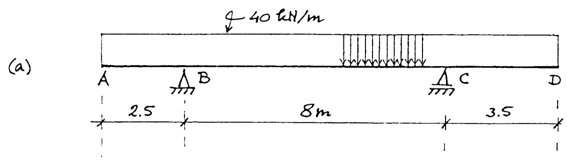


Figure 12.13 (a) Simply supported cantilever beam with unequal overhangs and a uniformly distributed full load.

the *absolute value* of the area of the V diagram.

Since a couple is acting on the beam, the area of the V diagram between A and D is not zero but rather

$$\left|(2 \text{ m})(20 \text{ kN}) - (6 \text{ m})(20 \text{ kN})\right| = 80 \text{ kNm}.$$

In accordance with rule 12, this value is exactly equal to the magnitude of the couple at C.

Example 2

The simply supported cantilever beam in Figure 12.13a has two overhangs and is carrying a uniformly distributed load of 40 kN/m over its entire length.

Questions:

a. Draw the M diagram.
b. Derive the V diagram from the M diagram.
c. Verify the step changes in the V diagram.
d. Determine the maximum bending moments in field BC and at the supports.

Solution:

The support reactions at B and C are respectively 245 kN and 315 kN, both aimed upwards (see Figure 12.13b). Three fields, AB, BC and CD, are distinguished for drawing the M and V diagrams. In each of these fields, the shear force varies linearly and the bending moment varies parabolically (rule 2).

a. Per field, we replace the distributed load by its resultant, and draw the moment due to these resultants. We find the bent M diagram shown by the dashed lines in Figure 12.13c. At the field boundaries A, B, C and D, the actual parabolic M diagram is tangent to the dashed M diagram. Extra points on the parabolic M diagram are found in the middle of each field by halving the distance between the chord and the bend in the dashed M diagram. At these points, the tangent to the M diagram is parallel to the chord.[1] With three values and three tangents per field, it is now easy to make a free-hand sketch of the M diagram, see the solid line in Figure 12.13c. To keep the image legible, the tangents in the field middles are not shown.

The correctness of the M diagram can be checked using the values $p = \frac{1}{8}q\ell^2$, where ℓ stands for field length:

$$p_1 = \tfrac{1}{8}(40\ \text{kN/m})(2.5\ \text{m})^2 = 31.25\ \text{kNm},$$

$$p_2 = \tfrac{1}{8}(40\ \text{kN/m})(8\ \text{m})^2 = 320\ \text{kNm},$$

$$p_3 = \tfrac{1}{8}(40\ \text{kN/m})(3.5\ \text{m})^2 = 61.25\ \text{kNm}.$$

The value $2p$ is the distance between the chord and the bend in the dashed M diagram. This distance can also be derived from the M diagram. It is up to the reader to check whether the calculated values of p fit on the M diagram shown.

b. The V diagram varies linearly with step changes at B and C where the support reactions act. At the field boundaries, the shear forces can be derived from the slope of the dashed M diagram (rule 5). This gives

$$V_{\text{A}} = 0,$$

$$V_{\text{B;left}} = \frac{125\ \text{kNm}}{1.25\ \text{m}} = 100\ \text{kN} \quad (\_\urcorner),$$

$$V_{\text{B;right}} = \frac{(125 + 455)\ \text{kNm}}{4\ \text{m}} = 145\ \text{kN} \quad (\ulcorner\_),$$

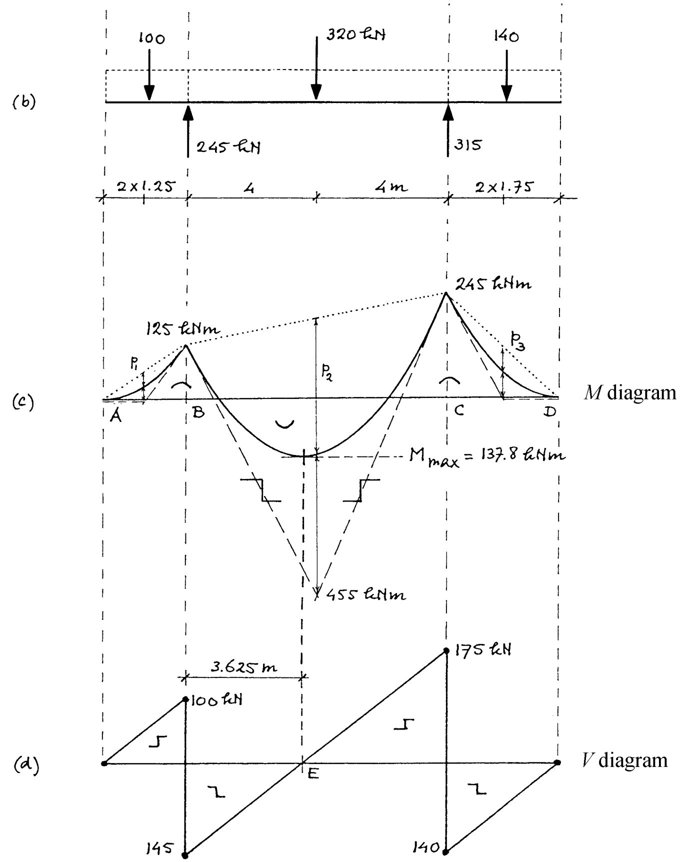


Figure 12.13 (b) The isolated beam with the resultants of the distributed loads in the fields AB, BC and CD, and the support reactions. (c) Bending moment diagram. At the field boundaries A to D, this diagram is tangent to the dashed bending moment diagram due to the load resultants. (d) Shear force diagram. The bending moments are extreme where the shear force is zero or changes sign.

[1] See Section 12.1.6, "Properties of parabolic M diagrams".

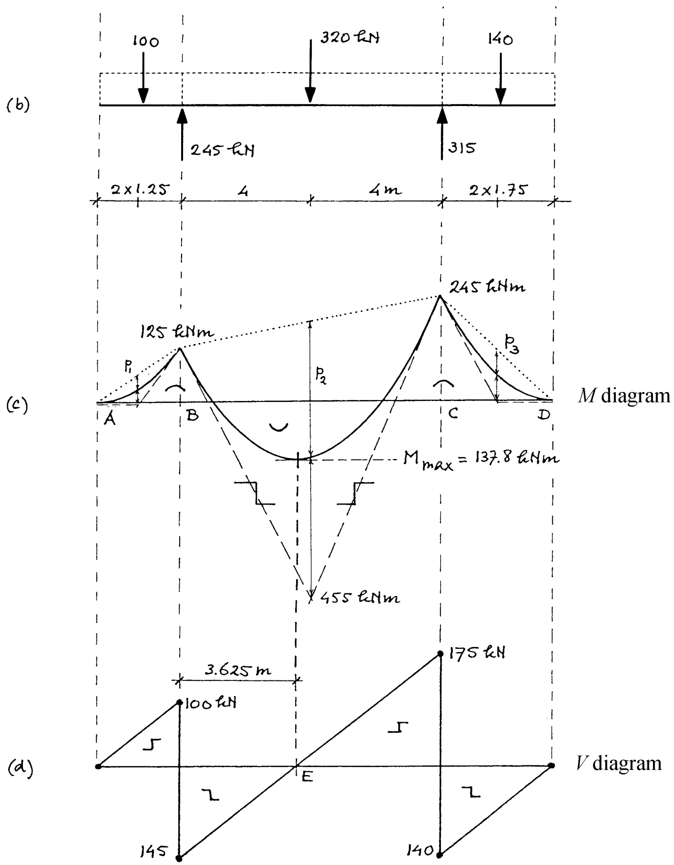


Figure 12.13 (b) The isolated beam with the resultants of the distributed loads in the fields AB, BC and CD, and the support reactions. (c) Bending moment diagram. At the field boundaries A to D, this diagram is tangent to the dashed bending moment diagram due to the load resultants. (d) Shear force diagram. The bending moments are extreme where the shear force is zero or changes sign.

$$V_{\text{C;left}} = \frac{(245 + 455)\text{ kNm}}{4\text{ m}} = 175\text{ kN},$$

$$V_{\text{C;right}} = \frac{245\text{ kNm}}{1.75\text{ m}} = 140\text{ kN},$$

$$V_{\text{D}} = 0.$$

The deformation symbols also follow from the slope of the M diagram; they are equal to the "steps" in the M diagram. These "steps" are shown explicitly only for $V_{\text{B;right}}$ and $V_{\text{C;left}}$ in the M diagram in Figure 12.13c.

Figure 12.13d shows the V diagram. The V values derived from the M diagram are shown by means of dots. Between these values the shear force varies linearly. Since the same uniformly distributed load acts over the entire length of the beam, the slopes of the V diagram are equal in all the fields, and are 40 kN/m (rule 4).

c. The step changes in the V diagram at the supports at B and C are 245 kN and 315 kN respectively, and are equal in magnitude to the support reactions (rule 14).

d. The bending moments at the supports at B and C are referred to as *support moments*.[1] The maximum support moment occurs at C.

The largest moment in field BC is known as the maximum *field moment*. This occurs at E, where the shear force is zero (rule 7). Here the tangent is horizontal. The magnitude can be derived from the moment equilibrium of the isolated part AE or ED, but also from the area of the V diagram (rule 11). With $M_{\text{E}} = M_{\text{max}}$ and $M_{\text{A}} = 0$,

$$\Delta M^{(\text{AE})} = M_{\text{E}} - M_{\text{A}} = M_{\text{max}}.$$

---

1 For a fixed-end, the *support moment* is called a *fixed-end moment*.

$\Delta M^{(\mathrm{AE})}$ is equal to the (absolute value of the) area of the V diagram over AE:

$$\begin{aligned} M_{\max} &= \Delta M^{(\mathrm{AE})} \\ &= \left| \tfrac{1}{2}(2.5\ \mathrm{m})(100\ \mathrm{kN}) - \tfrac{1}{2}(3.625\ \mathrm{m})(145\ \mathrm{kN}) \right| = 137.8\ \mathrm{kNm}. \end{aligned}$$

Because the V diagrams over AB and BE have different deformation symbols, the total area is equal to the difference in the areas over AB and BE. And because the sign is not so important, we look at the absolute value.

Of course we can also look at the right-hand part ED:

$$\Delta M^{(\mathrm{ED})} = M_{\mathrm{E}} - M_{\mathrm{D}} = M_{\max}$$

so that

$$\begin{aligned} M_{\max} &= \Delta M^{(\mathrm{ED})} \\ &= \left| \tfrac{1}{2}(4.375\ \mathrm{m})(175\ \mathrm{kN}) - \tfrac{1}{2}(3.5\ \mathrm{m})(140\ \mathrm{kN}) \right| = 137.8\ \mathrm{kNm}. \end{aligned}$$

Note that the total area of the V diagram is zero (rule 12).

If in this example (as well as in other examples) we further investigate the M diagram, we notice a correlation between the shape of the M diagram and the shape that a cable or cord assumes under the same load. This leads to the following statement:

• ***Rule 16***
If a beam is exclusively loaded by forces normal to its axis, the M diagram has the same shape as a cable (or cord) on which one lets the same forces act.

In addition to the shape of the M diagram, this rule also allows us to easily check whether the M diagram has been drawn at the correct side and there-

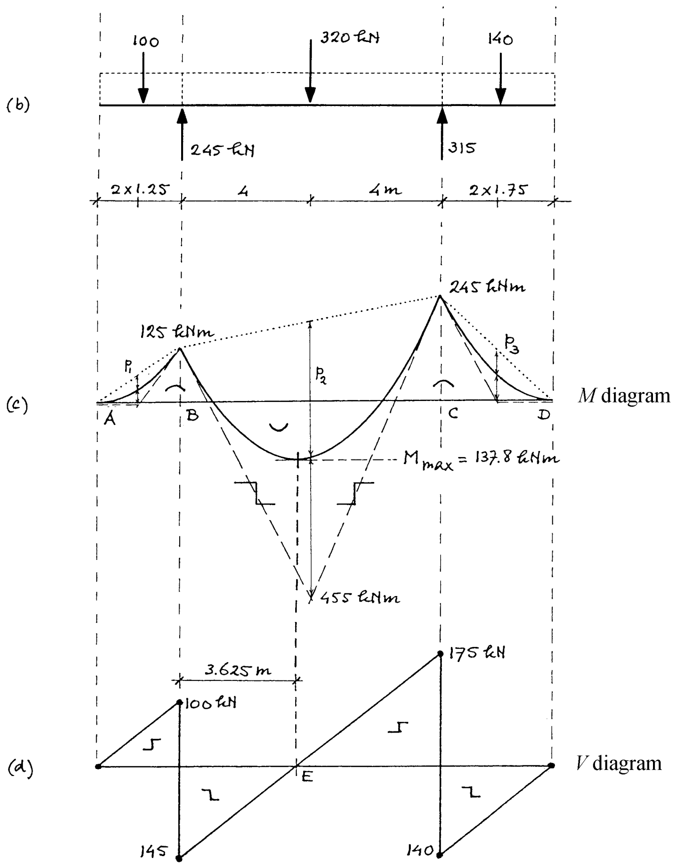


Figure 12.13 (b) The isolated beam with the resultants of the distributed loads in the fields AB, BC and CD, and the support reactions. (c) Bending moment diagram. At the field boundaries A to D, this diagram is tangent to the dashed bending moment diagram due to the load resultants. (d) Shear force diagram. The bending moments are extreme where the shear force is zero or changes sign.

fore with the correct deformation symbol. A downward force generates a downward bend in a cable, and also in the M diagram. An upward force lifts the cable, and therefore also lifts the M diagram.

In the M diagram in Figure 12.13c, we can recognise a cable AD hanging (as a parabola) under the influence of the uniformly distributed full load, and pushed upwards by the support reactions at B and C.

The general validity of rule 16 is explained in Chapter 14.

12.1.6 Properties of parabolic M diagrams

Due to a uniformly distributed load, the M diagram has the shape of a parabola. Since uniformly distributed loads occur frequently in practice we will discuss a number of the striking properties of parabolas below. They can be used to sketch a parabolic M diagram quickly.

We will use the isolated beam segment in Figure 12.14a as starting point, with length ℓ and a uniformly distributed load q over the full length. In addition to shear forces, the section planes at A and B are subject to bending moments M_A and M_B, both causing tension at the underside. Figure 12.14b shows the associated M diagram.

Properties of the parabolic M diagram include the following:

- The tangents to the M diagram at A and B are found by drawing the M diagram for the resultant of the distributed load. The tangents at A and B intersect in the middle C of AB.
- The vertically measured distance p between chord k and the parabola is

$$p = \tfrac{1}{2}qab \quad \text{(in which } a + b = \ell\text{)}.$$

- In the middle C ($a = b = \frac{1}{2}\ell$) this distance is

$$p_C = \tfrac{1}{8}q\ell^2.$$

- The intersection of the tangents at A and B is at a distance $2p_C$ under the chord k.

- In the example, with bending moments M_A and M_B, both causing tension at the underside, the bending moment in the middle C is:

$$M_C = \frac{1}{2}(M_A + M_B) + \frac{1}{8}q\ell^2.$$

- The tangent in the middle C is parallel to chord k.
- The field moment in AB is an extreme where the tangent is horizontal. This is generally not in the middle of AB.

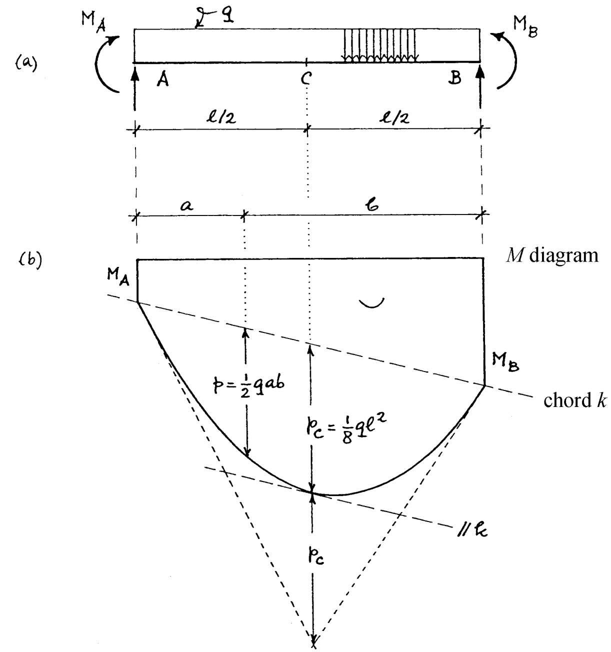


Figure 12.14 (a) An isolated beam segment with uniformly distributed full load and (b) the associated parabolic bending moment diagram.

12.1.7 Summary of all the rules relating to M and V diagrams

This section includes a summary of all the rules discussed in Sections 12.1.1 to 12.1.5.

• *Rule 1*

In an unloaded field, the shear force V is constant, and the bending moment M varies linearly. If the shear force is zero, the bending moment is constant.

$$q_z = 0 \Rightarrow V \text{ constant;} \quad V = 0 \Rightarrow M \text{ constant}$$
$$V \neq 0 \Rightarrow M \text{ linear.}$$

• *Rule 2*

In a field with a uniformly distributed load q_z, the shear force V varies linearly, and the bending moment M varies quadratically (parabolic).

$$q_z \text{ constant } (\neq 0) \Rightarrow V \text{ linear } \Rightarrow M \text{ quadratic.}$$

• *Rule 3*

In a field with a linearly distributed load q_z, the shear force V varies quadratically, and the bending moment M is a cubic function.

$$q_z \text{ linear } \Rightarrow V \text{ quadratic } \Rightarrow M \text{ cubic.}$$

- ***Rule 4***

The slope of the V diagram $(\mathrm{d}V/\mathrm{d}x)$ is equal to the distributed load q_z (but with an opposite sign).

- ***Rule 5***

The slope of the M diagram $(\mathrm{d}M/\mathrm{d}x)$ is equal to the shear force V.

- ***Rule 6***

The shear force V is an extreme where the distributed load q_z is zero (or changes sign). Per field, we have to take into account the occurrence of extreme values at the boundaries (e.g. at concentrated loads and supports).

- ***Rule 7***

The bending moment is an extreme where the shear force is zero (or changes sign). Per field, we have to take into account the occurrence of extreme values at the boundaries (e.g. where concentrated loads and/or couples are applied or at supports).

- ***Rule 8***

The tangents to the M diagram at the boundaries of a field intersect on the line of action of the load resultant in that field (for a distributed load this is at the centroid of the load diagram).

- ***Rule 9***

If we replace the load per field by its resultant, and draw the bending moment diagram due to these resultants, this bending moment diagram is tangent to the actual bending moment diagram at the field boundaries.

- ***Rule 10***

Without concentrated forces, the change of the shear force V over a certain length is equal to the area of the load diagram over that length.

- ***Rule 11***

Without concentrated couples, the change in the bending moment over a certain length is equal to the area of the V diagram over that length.

• ***Rule 12***
The total area of the V diagram is equal to the sum of moments of all concentrated couples that act on the beam. For a beam without concentrated couples, the total area of the V diagram is zero.

• ***Rule 13***
A step change in the distributed load q_z gives a bend in the V diagram and a point of inflection in the M diagram.

• ***Rule 14***
A (concentrated) force F normal to the member axis generates a step change in the V diagram, of magnitude F, and a bend in the M diagram.

• ***Rule 15***
A (concentrated) couple T gives a step change in the M diagram of magnitude T. The V diagram reveals no information about the point of application of the couple.

• ***Rule 16***
If a beam is exclusively loaded by forces normal to its axis, the M diagram has the same shape as a cable (chord) on which one lets the same forces act.

12.2 Rules for drawing the N diagram more quickly

The differential equations for the force equilibrium of an infinitesimal member segment in Section 11.1 are

$$\frac{\mathrm{d}N}{\mathrm{d}x} + q_x = 0 \quad \text{(extension)}, \tag{a}$$

$$\frac{\mathrm{d}V}{\mathrm{d}x} + q_z = 0 \quad \text{(bending)}. \tag{b}$$

Table 12.2 Relationship between the distributed axial load q_x and the variation of the normal force N.

Variation q_x	**Variation N**
constant $= 0$	constant
constant $\neq 0$	linear
linear	quadratic

Based on the analogy of the differential equations (a) and (b), we can say that, for the relationship between N and q_x, the same rules apply as derived in the previous section for the relationship between V and q_z. Without any further commentary, and set down in the same order, the rules for the relationship between N and q_x are presented.

• ***Rule 1***
In an unloaded field, the normal force N is constant:

$$q_x = 0 \Rightarrow N \text{ constant.}$$

• ***Rule 2***
In a field with a uniformly distributed load q_x, the normal force N varies linearly:

$$q_x \text{ constant } (\neq 0) \Rightarrow N \text{ linear.}$$

• ***Rule 3***
In field with a linearly distributed load q_x, the normal force N varies quadratically (parabolic).

$$q_x \text{ linear } \Rightarrow N \text{ quadratic.}$$

Rules 1 to 3 are summarised in Table 12.2.

• ***Rule 4***
The slope of the N diagram ($\mathrm{d}N/\mathrm{d}x$) is equal to the distributed load q_x (but with an opposite sign).

• ***Rule 6***
The normal force N is an extreme where the distributed load q_x is zero (or changes sign). Per field, we have to take into account the occurrence of extreme values at the boundaries (e.g. at concentrated loads and supports).

- ***Rule 10***

Without concentrated forces, the change in the normal force N over a certain length is equal to the area of the load diagram over that length.

- ***Rule 13***

A step change in the distributed load q_x gives a bend in the N diagram.

- ***Rule 14***

A (concentrated) axial member force F generates a step change in the N diagram of magnitude F.

12.3 Bent and compound bar type structures

With bent and compound bar type structures, the force flow can be found by dividing the structure into all its (straight) members and by calculating all the support reactions and joining forces as shown in Chapter 5. It is then possible to draw the M, V and N diagrams for each separate member. These diagrams are then linked together to form the M, V and N diagrams for the structure as a whole. Only when the diagram becomes illegible should you draw part of the structure with its M, V and N diagrams separately.

Below we determine and draw the M, V and N diagrams for a number of structures for which we previously calculated the support reactions and/or joining forces in Chapter 5. The load consists of concentrated forces and couples. Distributed loads are covered in detail in Chapter 13.

Example 1

The support reactions were calculated for the lighting mast in Figure 12.15a in Section 5.1, Example 1.

Question:

Determine the M, V and N diagrams.

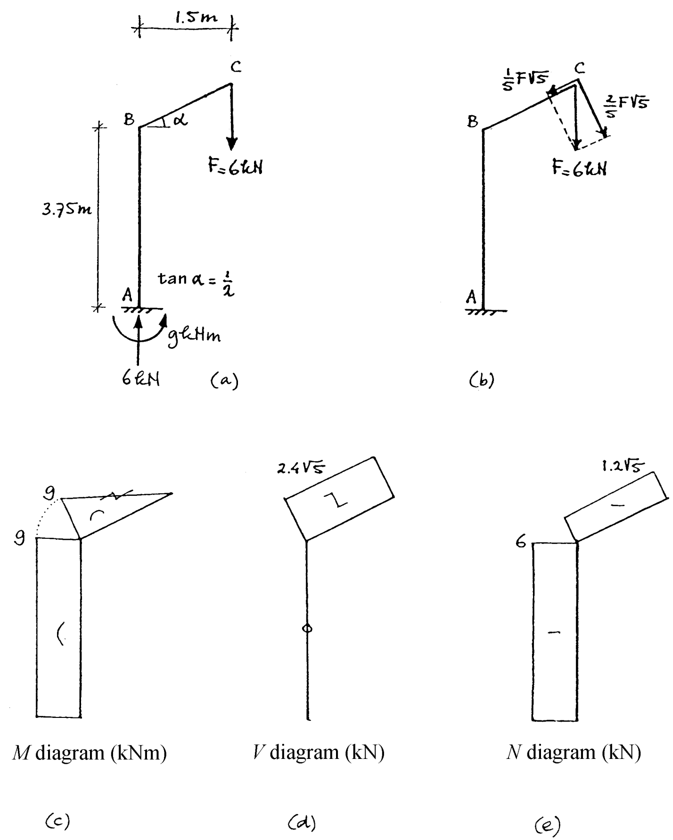


Figure 12.15 (a) The support reactions at A of a lighting mast loaded by a vertical force at C. (b) The force at C resolved into components normal to and parallel to member axis BC. (c) Bending moment diagram. The bending moment "*goes round the corner*" at B. (d) Shear force diagram. (e) Normal force diagram.

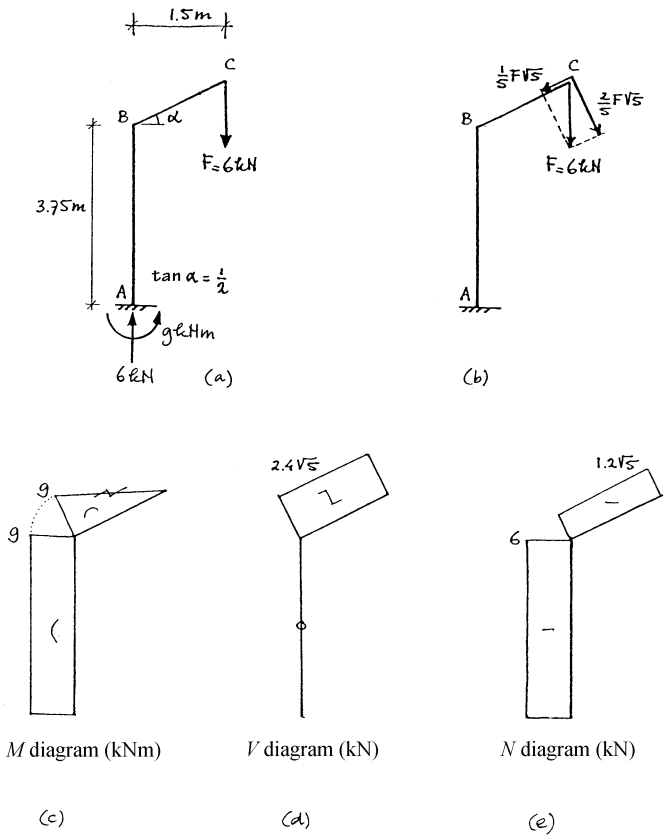


Figure 12.15 (a) The support reactions at A of a lighting mast loaded by a vertical force at C. (b) The force at C resolved into components normal to and parallel to member axis BC. (c) Bending moment diagram. The bending moment "*goes round the corner*" at B. (d) Shear force diagram. (e) Normal force diagram.

Solution:
Neither of the segments AB or BC is subject to a distributed load, so that the shear force in each segment is constant, and the bending moment varies linearly (rule 1).

M diagram
We can draw the M diagram as soon as we know the bending moments at A, B and C. The fixed-end moment at A and the bending moment at the free end C are known:

$$M_{\mathrm{A}} = 6\ \mathrm{kNm},$$

$$M_{\mathrm{C}} = 0.$$

We now have to determine only the bending moment at B. Without resolving F, we can calculate M_{B} from the moment equilibrium of the isolated segment BC:

$$M_{\mathrm{B}} = (6\ \mathrm{kN})(1.5\ \mathrm{m}) = 9\ \mathrm{kNm}.$$

Of course it is also possible to determine M_{B} from the equilibrium of the isolated segment AB.

The M diagram is shown in Figure 12.15c. All values are plotted normal to the member axis.

The bending moment at joint B is the same magnitude on both sides of the joint and is also plotted at the same side. This follows directly from the moment equilibrium of joint B (see Figure 12.16, which shows only the bending moments). It is said that the bending moment at B "*goes round the corner*", which is further emphasised in Figure 12.15c by the dotted arc (normally not drawn).

V diagram
The shear forces can be calculated directly from the slopes of the M diagram (rule 5):

$$V^{(AB)} = \frac{\Delta M^{(AB)}}{\ell^{(AB)}} = \frac{0 \text{ kNm}}{3.75 \text{ m}} = 0,$$

$$V^{(BC)} = \frac{\Delta M^{(BC)}}{\ell^{(BC)}} = \frac{9 \text{ kNm}}{0.75\sqrt{5} \text{ m}} = 2.4\sqrt{5} \text{ kN}.$$

Since we are concerned here with the magnitude (and not the direction) of the shear force, we use the absolute value of ΔM in the calculation.

The V diagram is shown in Figure 12.15d. The deformation symbol for the (direction of the) shear force is found from the "*steps*" in the M diagram.

The shear force in AB is zero; this is in agreement with the horizontal support reaction at A.

In Figure 12.15b, the force at C has been resolved into components parallel to and normal to the member axis. The component normal to the member axis corresponds with the magnitude and direction of the shear force in BC as calculated earlier from the M diagram.

Comment: It is often useful, particularly for *oblique members*, to draw the bending moment diagram first and then use it to calculate the shear forces. In that case it is not necessary to resolve forces into their components.

N diagram
For determining the normal force in BC, we cannot escape from resolving the 6 kN force at C into components parallel to and normal to BC (see Figure 12.15b). The N diagram is shown in Figure 12.15e. The normal force is a constant compressive force in both members.

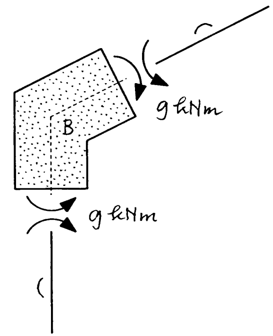


Figure 12.16 From the moment equilibrium of joint B it follows that the bending moment in B "*goes round the corner*".

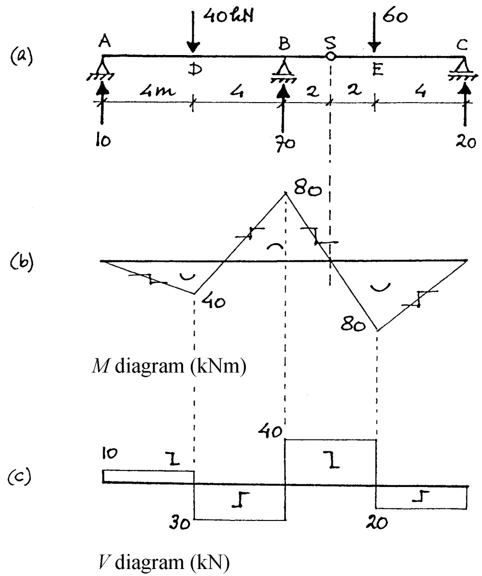


Figure 12.17 (a) Hinged beam with load and support reactions. (b) Bending moment diagram (M diagram) with the step changes for the deformation symbols in the shear force diagram (V diagram). (c) Shear force diagram.

Example 2

The support reactions and joining forces for the hinged beam in Figure 12.17a were calculated in Section 5.2, Example 1.

Question:
Determine the M and V diagrams.

Solution:
M diagram
The M diagram is shown in Figure 12.17b. The bending moment is zero at A, S and C; furthermore, the M diagram is linear with bends at the points of application of the forces at D, B and E. To be able to draw the M diagram, we have to calculate only the bending moments at D and E. The value at B is found by drawing a straight line through the values at E and S.

V diagram
The shear forces can be found from the slopes of the M diagram. The V diagram is shown in Figure 12.17c. The deformation symbols must correspond to the "*steps*" in the M diagram. The step changes in the V diagram must correspond with the forces on the beam (including the support reactions).

For a straight (continuous) beam, the shape of the V diagram is also found easily by plotting the successive step changes due to the concentrated loads one behind the other. These step changes are shown from left to right in Figure 12.18a. The values of the step changes are included in the figure. To draw the V diagram, the deformation symbols need to be included, as do the values of the shear forces. The values of the step changes are usually not included. Here this has been done only to illustrate the method.

Instead of going from left to right, we can plot the successive step changes from right to left (see Figure 12.18b). The result is the same figure again (Figure 12.18a), but now in reverse. If we include the correct deformation symbols, this V diagram is also correct. If we use the deformation symbols, it does not make a difference for the V diagram at which side of the member axis we plot the values.

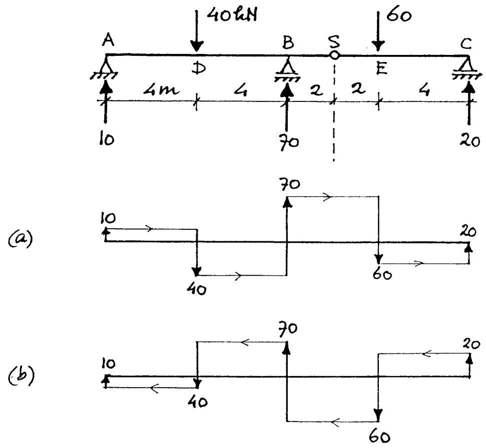


Figure 12.18 The shape of the V diagram can also be found by plotting the successive step changes due to the point loads at the beam: (a) from left to right or (b) from right to left.

Note that, between the two successive zero moments, in AS and SC, ΔM is zero, and therefore the corresponding area of the V diagram is also zero (rule 12). It is left to the reader to check this.

Example 3

The support reactions and the interaction forces at joint C for the structure in Figure 12.19a were calculated in Section 5.1, Example 5 (see Figure 12.20).

Question:

Determine the M, V and N diagrams.

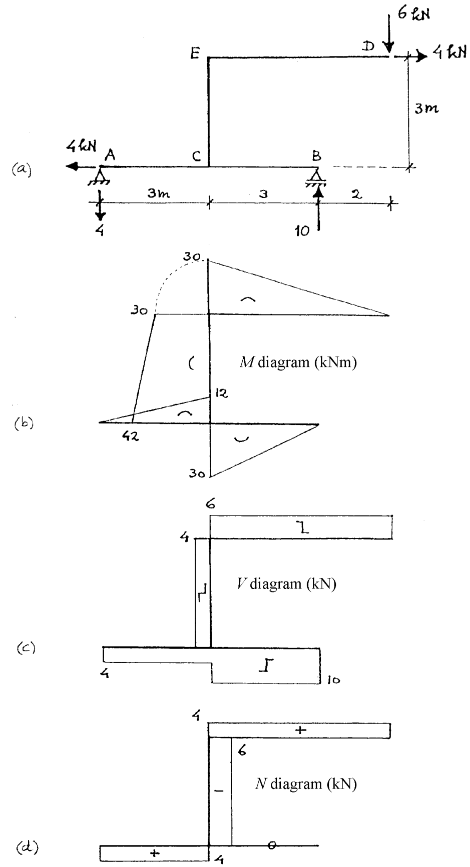


Figure 12.19 (a) A structure of which parts AC, BC and DC are rigidly joined at C. (b) Bending moment diagram. (c) Shear force diagram. (d) Normal force diagram.

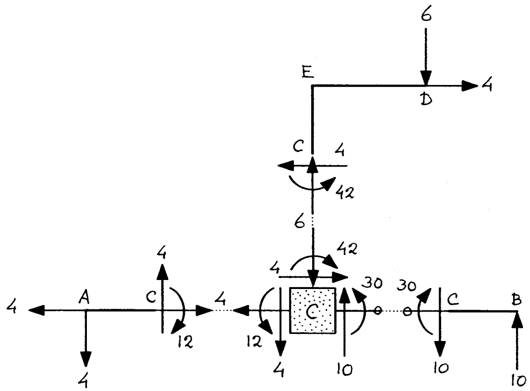


Figure 12.20 The interaction forces between joint C and the isolated parts AC, BC and DC.

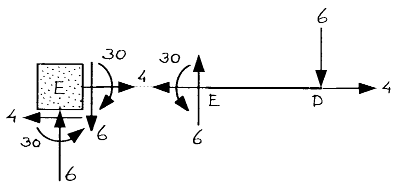


Figure 12.21 Joint E and member ED isolated.

Solution:
If we determine the interaction forces at joint E, it is possible to plot the M, V and N diagrams for the entire structure. As there are no distributed loads, the bending moment varies linearly along all members, and the shear force and normal force are constant in all members (rule 1).

M diagram
The M diagram is shown in Figure 12.19b. Since the variation of M along all members is linear, it is sufficient to determine the bending moments at the member ends to get the M diagram.

The bending moments $M_{\mathrm{C}}^{(\mathrm{CA})}$, $M_{\mathrm{C}}^{(\mathrm{CB})}$ and $M_{\mathrm{C}}^{(\mathrm{CE})}$ at joint C have already been calculated[1] (see Figure 12.20).

$M_{\mathrm{E}}^{(\mathrm{ED})}$ follows from the moment equilibrium of the isolated part ED (see Figure 12.21):

$$M_{\mathrm{E}}^{(\mathrm{CE})} = M_{\mathrm{E}}^{(\mathrm{ED})}$$

with tension on the upper side of ED. This value in the M diagram is therefore plotted at the upper side.

The moment equilibrium of joint E in Figure 12.21 gives

$$M_{\mathrm{E}}^{(\mathrm{CE})} = M_{\mathrm{E}}^{(\mathrm{ED})}.$$

The bending moment "*goes round the corner*". This is emphasised in the M diagram in Figure 12.19b by means of a dotted arc at joint E.

---

[1] The upper index refers to the member in which the bending moment acts and the lower index refers to the location.

V diagram
The magnitude of the shear force and the associated deformation symbol follow from the slope of the *M* diagram.[1] The *V* diagram is shown in Figure 12.19c.

N diagram
The *N* diagram is shown in Figure 12.19d.

Example 4
We are given the trussed beam in Figure 12.22a. All the forces acting on the isolated hinged beam ASB shown in Figure 12.22b were calculated in Section 5.6.

The *N*, *V* and *M* diagrams are shown in Figures 12.22c to 12.22e.

Determining and drawing the *V* and *M* diagrams is done in the same way as for the hinged beam in Example 2. To draw the *M* diagram, we have to calculate only the bending moments at C and D. The bending moment varies linearly between D and E, so that the *M* diagram must pass through S where the bending moment is zero. This also fixes the value at E. The *V* diagram can subsequently be calculated from the *M* diagram.

We can change the order: first draw the *V* diagram and calculate the values at C, E and D from the areas of the *V* diagram.

Note: The shear forces at A and B are not equal to the support reactions at A and B! Why not?

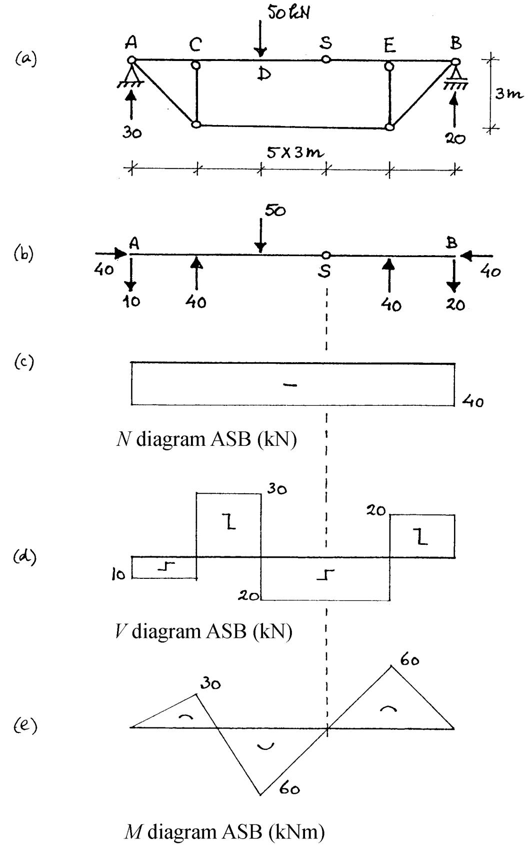


Figure 12.22 (a) Trussed beam with load and support reactions. (b) Isolated beam ASB. (c) Normal force diagram. (d) Shear force diagram. The shear forces at A and B are not equal to the support reactions at A and B. (e) Bending moment diagram.

1 In Figure 12.19b, the "steps" in the *M* diagrams are no longer shown.

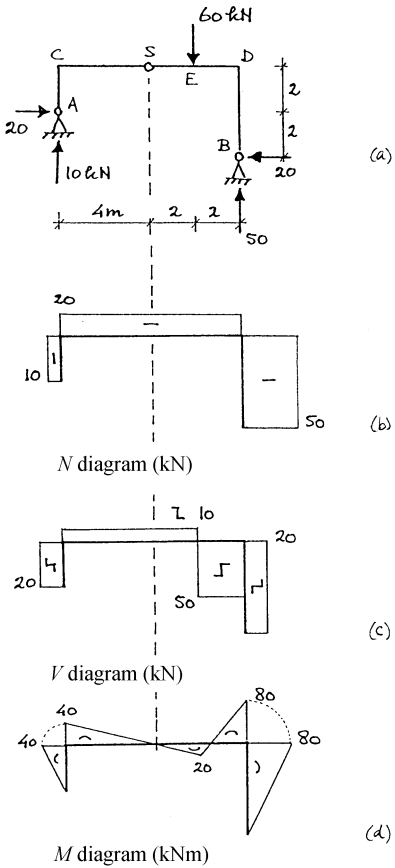


Figure 12.23 (a) Three-hinged portal frame with load and support reactions. (b) Normal force diagram. (c) Shear force diagram. (d) Bending moment diagram.

Example 5

The support reactions for the three-hinged portal frame in Figure 12.23a were calculated in Section 5.3, Example 1.

As there are no distributed loads, the normal forces and shear forces in each field are constant, and the bending moment varies linearly (rule 1).

N and *V diagrams*

To draw the N and V diagrams, we have to investigate the force equilibrium of the separate parts. The necessary calculations are left to the reader. The result is shown in Figures 12.23b and 12.23c.

Due to the concentrated load at E, a step change of 60 kN occurs in the V diagram (rule 14).

M diagram

To draw the M diagram, we have to know only the bending moments at C and D.

The bending moment at C follows from the moment equilibrium of the isolated part AC:

$$M_{\mathrm{C}}^{(\mathrm{AC})} = 40 \text{ kNm}$$

with tension at the "*outside*" of the frame.

From the moment equilibrium of joint C, where the two members AC and CS are rigidly joined to one another, it follows that the bending moments $M_{\mathrm{C}}^{(\mathrm{AC})}$ in column AC and $M_{\mathrm{C}}^{(\mathrm{CS})}$ in beam CS are of equal magnitude, and that both cause tension at the "*outside*" of the frame (see Figure 12.24, which shows only the bending moments). Both moments are plotted "*outside*". In the M diagram, we can see the bending moment in C "*going round the corner*".

For the bending moment at D we find

$$M_{\text{D}}^{(\text{BD})} = M_{\text{D}}^{\text{d}(DS)} = 80 \text{ kNm}$$

also with tension at the "outside" of the frame.

The M diagram for AC and BD varies linearly, from 0 to 40 and 80 kNm respectively. The M diagram for CSD consists of two straight lines that have a bend at the concentrated load at E. In addition, the M diagram passes through hinge S where the bending moment is zero. We therefore have to draw a straight line from the value of 40 kNm at C, through S, up to 20 kN on the opposite side at E. From there, we continue with a straight line to the value of 80 kNm at D.

The M diagram is shown for the entire three-hinged portal frame in Figure 12.23d.

Check 1:
We can read from the M diagram that the bending moment at the position of the point load is 20 kNm, with tension at the underside of the beam. This can be checked using the moment equilibrium of the isolated part BDE.

Check 2:
Note that the magnitude of the shear forces and the deformation symbols agree with the slopes of the M diagram. This relationship between the M and V diagram represents a simple and fast way of checking their correctness.

Example 6
All the forces on the isolated members ACE and BDE for the structure in Figure 12.25a were calculated in Section 5.5, Example 2.

M diagram
To draw the M diagram we have to calculate only the bending moment at the three points C, D and G. To do so, it is sufficient to know the support reactions and the normal force in member CD (see Figure 12.25b):

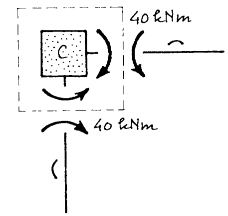


Figure 12.24 From the moment equilibrium of joint C it follows that the bending moment "*goes round the corner*" at C (see Figure 12.23).

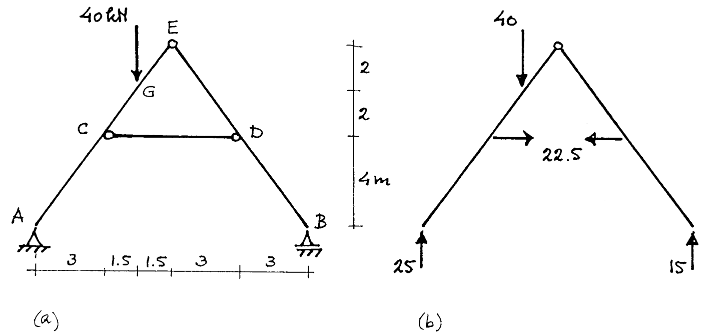


Figure 12.25 (a) A structure loaded by a vertical force on the left frame leg. (b) The isolated member AEB. (c) Bending moment diagram.

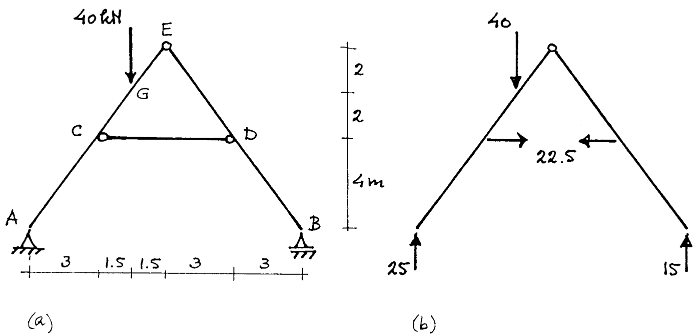


Figure 12.25 (a) A structure loaded by a vertical force on the left frame leg. (b) The isolated member AEB. (c) Bending moment diagram.

$$M_{\rm C} = (25 \text{ kN})(3 \text{ m}) = 75 \text{ kNm},$$

$$M_{\rm G} = (25 \text{ kN})(4.5 \text{ m}) - (22.5 \text{ kN})(2 \text{ m}) = 67.5 \text{ kNm},$$

$$M_{\rm D} = (15 \text{ kN})(3 \text{ m}) = 45 \text{ kNm}.$$

The M diagram is shown in Figure 12.25c. Since there is only a normal force in the two-force member CD, this member has been omitted to simplify the figure.

V diagram

The shear forces can be determined directly from the slopes of the M diagram. For example:

$$V^{(\rm AC)} = \frac{\Delta M^{(\rm AC)}}{\ell^{(\rm AC)}} = \frac{75 \text{ kNm}}{5 \text{ m}} = 15 \text{ kN},$$

$$V^{(\rm CG)} = \frac{\Delta M^{(\rm CG)}}{\ell^{(\rm CG)}} = \frac{(75 \text{ kNm}) - (67.5 \text{ kNm})}{2.5 \text{ m}} = 3 \text{ kN},$$

$$V^{(\rm GE)} = \frac{\Delta M^{(\rm GE)}}{\ell^{(\rm GE)}} = \frac{67.5 \text{ kNm}}{2.5 \text{ m}} = 27 \text{ kN}.$$

The associated deformation symbols follow from the "*steps*" in the M diagram (they are not shown here). The complete V diagram is shown in Figure 12.25d. Here too the two-force member CD has been omitted.

N diagram

The N diagram is shown in Figure 12.25e. Determining the N diagram is relatively laborious as we have to resolve all the forces on the members into components normal to the member axis (step changes in the V diagram) and components parallel to the member axis (step changes in the N diagram).

For example, for the horizontal force of 22.5 kN at C, the component normal to the axis of member ACE is

$$\frac{4}{5} \times (22.5 \text{ kN}) = 18 \text{ kN}$$

and the component parallel to the member axis is

$$\frac{3}{5} \times (22.5 \text{ kN}) = 13.5 \text{ kN}.$$

At C we observe a step change of 18 kN in the V diagram, and a step change of 13.5 kN in the N diagram (rule 14).

In the same way, the component normal to ACE of the vertical force of 40 kN in G is

$$\frac{3}{5} \times (40 \text{ kN}) = 24 \text{ kN}$$

and the component parallel to ACE is

$$\frac{4}{5} \times (40 \text{ kN}) = 32 \text{ kN}.$$

At G we observe a step change of 24 kN in the V diagram, and a step change of 32 kN in the N diagram (rule 14).

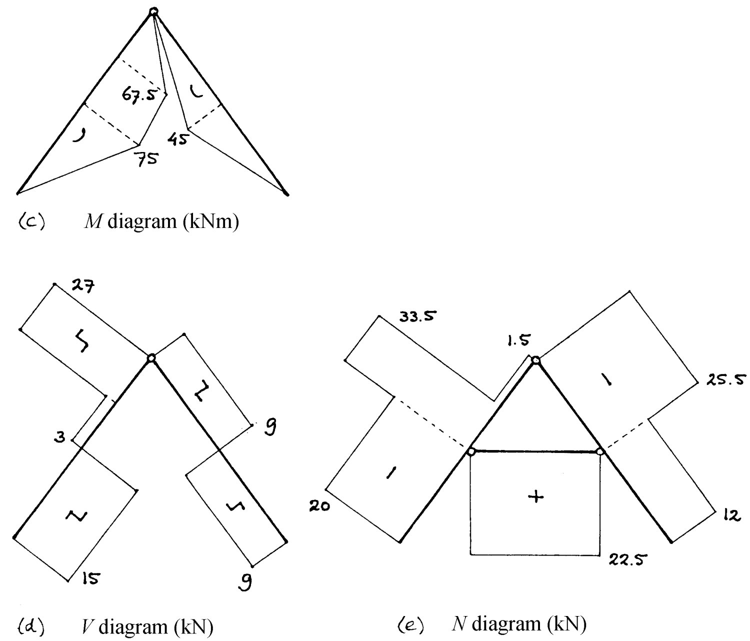


Figure 12.25 (c) Bending moment diagram. (d) Shear force diagram. (e) Normal force diagram.

12.4 Principle of superposition

If several loads are acting on a structure, the separate influences of the various loads on the support reactions and section forces can be added together.[1]

The validity of this so-called *principle of superposition* is a result of the linear relationships between the loads, section forces and support reactions.

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1 In Section 6.3.1, we showed that distributed loads can be split and that the individual influences on the support reactions can be added together.

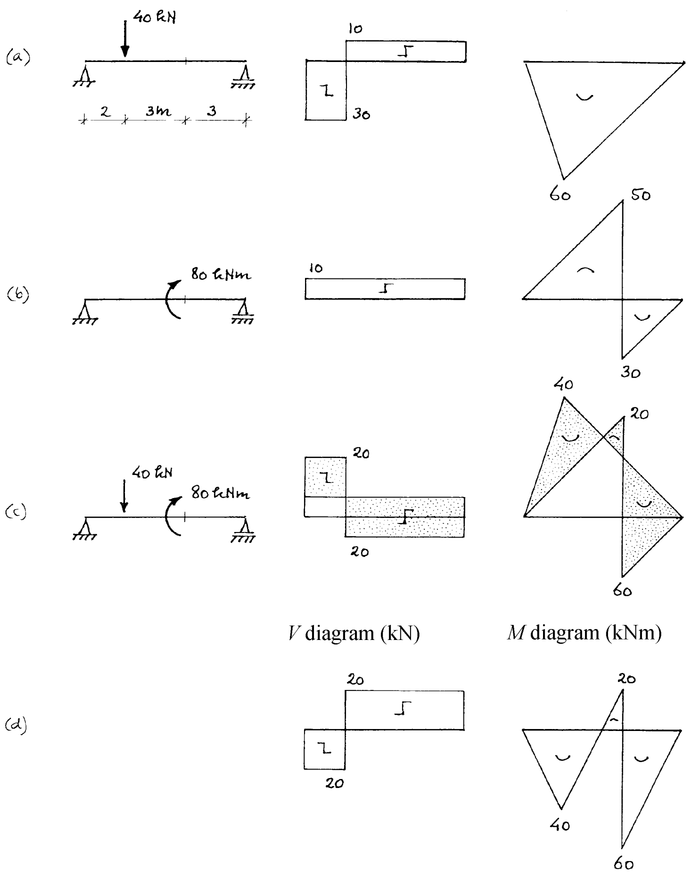


Figure 12.26 Simply supported beam with the V and M diagrams due to (a) the force and (b) the couple. (c) The V and M diagrams due to the force and the couple together, found by superposing the V and M diagrams from (a) and (b). (d) The V and M diagrams from (c) transferred to a horizontal axis.

Example 1

In Figure 12.26, the principle of superposition has been applied to determine the V and M diagrams for an 8-metre beam that is loaded by a force of 40 kN and a couple of 80 kNm.

In Figure 12.26a, the V and M diagrams have been calculated due to the force only. In Figure 12.26b, the V and M diagrams have been calculated due to the couple only. The final V and M diagrams with concurrent loading by the force and the couple is shown in Figure 12.26c. To draw these V and M diagrams, one of the two diagrams to be superposed has been reflected with respect to the horizontal axis to simplify the graphics. In areas with opposite deformation symbols that overlap one another, the combined contribution to the section force is zero. The remaining areas have been filled and form the final V and M diagrams.

In Figure 12.26d, these diagrams have been transferred to a horizontal axis, but this is generally not necessary.

Of course the superposition can also be performed by determining the ordinates at a number of points and adding them together.

Example 2

The second example relating to the principle of superposition concerns further analysis of the force flow in segment BC of beam AD in Figure 12.27, with a uniformly distributed load over BC. The V and M diagrams for this beam were calculated in Section 12.1.3, Example 2.

For BC, we find the same V and M diagrams if we isolate segment BC from beam AD, support it simply at its ends B and C, and there load it by couples of 80 and 60 kNm respectively (see Figure 12.28).

We can distinguish three loads on beam BC in Figure 12.28:
(a) a couple of 80 kNm at end B;
(b) a couple of 60 kNm at end C;
(c) a uniformly distributed full load of 12.5 kN/m.

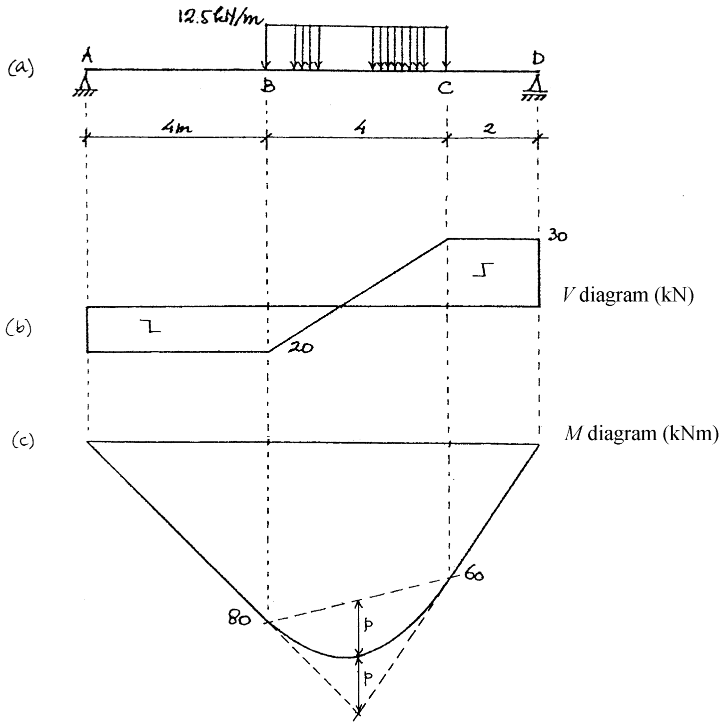


Figure 12.27 (a) Simply supported beam with a uniformly distributed load in field BC. (b) Shear force diagram. (c) Bending moment diagram.

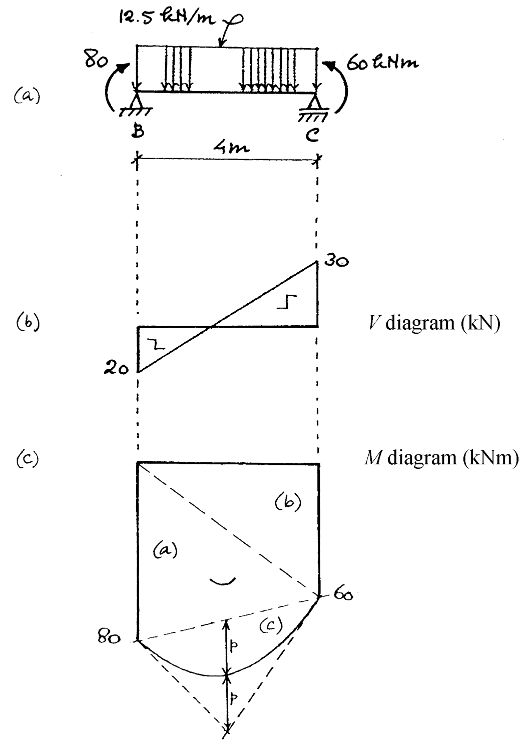


Figure 12.28 For BC in Figure 12.27, the same V and M diagrams are found if BC is isolated from AD, simply supported at its ends B and C, and loaded there by couples of 80 and 60 kNm respectively.

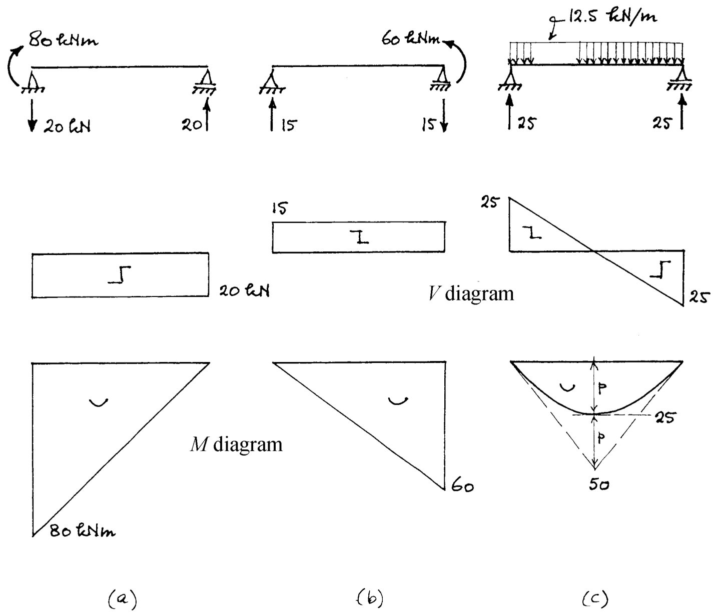


Figure 12.29 The V and M diagrams in Figure 12.28 can be found by superposing the V and M diagrams due to (a) a couple of 80 kNm in B, (b) a couple of 60 kNm in C and (c) a uniformly distributed full load of 12.5 kN/m.

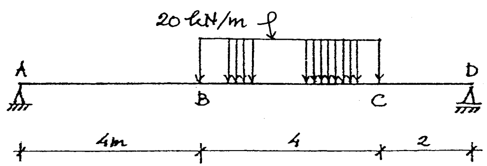


Figure 12.30 Simply supported beam with a uniformly distributed load in field BC.

In Figure 12.29, the V and M diagrams are shown for each of these loads.

The V and M diagrams in Figure 12.28 can be found by superposing the V and M diagrams from Figure 12.29. In the M diagram in Figure 12.28c we can clearly recognise the contributions (a) to (c) of the separate loads.

The principle of superposition can be used also to compare the effects of similar loads of different magnitudes. Since the entire system behaves linearly, we can say that if a load leads to certain values for the section forces, a similar load that is n times as large causes section forces that are in turn n times as large.

Example 3
If the maximum bending moment in Figure 12.27c is 96 kNm, how large is the maximum bending moment for the beam in Figure 12.30?

Solution:
The load in Figure 12.30 is similar to that in Figure 12.27a and is $20/12.5 = 1.6$ times as large. In both cases, the V and M diagrams have the same shape, except that the values for the beam in Figure 12.30 are now 1.6 times as large as those for the beam in Figure 12.27. This applies also for the maximum bending moment. Therefore, for the beam in Figure 12.30,

$$M_{\text{max}} = 1.6 \times (96 \text{ kNm}) = 153.6 \text{ kNm}.$$

12.5 Schematisations and reality

Forces, couples and uniformly distributed loads are schematisations of the real loads. In this section we will look at the consequences of these schematisations. We will also look at the shear forces at a support and the influence of eccentric axial forces.

12.5.1 Point load

The simply supported beam in Figure 12.31a is loaded at the middle of span ℓ by a force F. Figures 12.31b and 12.31c show the associated V and M diagrams.

The force F, modelled as a point load, is a load concentrated at one single point, or in other words, a load that applies over a length zero. This does not exist in reality. In fact, F is the resultant of a distributed load over a small yet finite length $\xi\ell$. Figure 12.32 shows the V and M diagrams for the case in which load F is uniformly distributed over length $\xi\ell$:

$$q = \frac{F}{\xi\ell}.$$

For a point load, ξ approaches zero and at the same time the force intensity q increases such that $q \cdot \xi\ell = F$ remains constant.

Over the (small) area $\xi\ell$ in Figure 12.32, the shear force varies (steeply) from $+\frac{1}{2}F$ to $-\frac{1}{2}F$ and the M diagram is parabolic. The maximum bending moment occurs at midspan:

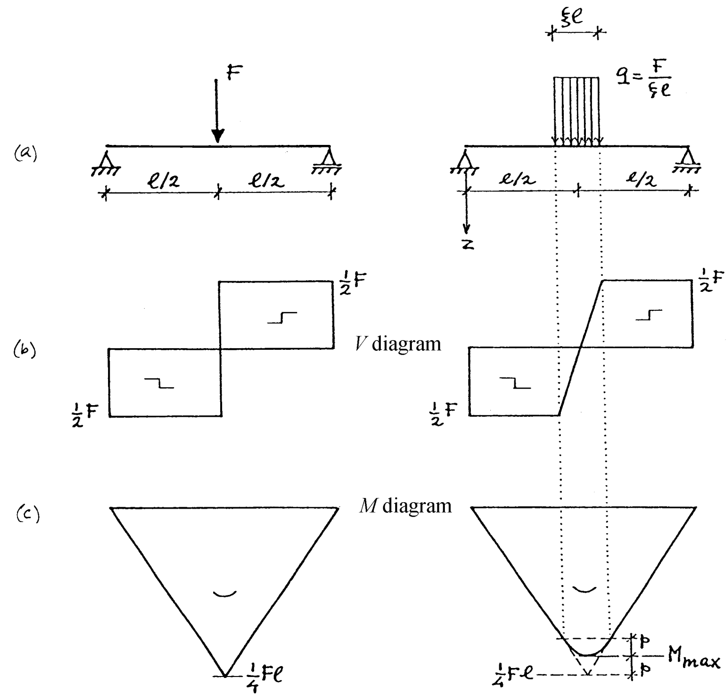
$$M_{\max} = \tfrac{1}{4}F\ell - p = \tfrac{1}{4}F\ell - \tfrac{1}{8}\frac{F}{\xi\ell}(\xi\ell)^2 = \tfrac{1}{4}F\ell\left(1 - \tfrac{1}{2}\xi\right).$$

For calculating this moment, we used the property that

$$p = \text{“}\tfrac{1}{8}q\ell^2\text{”} = \tfrac{1}{8}\frac{F}{\xi\ell}(\xi\ell)^2 = \tfrac{1}{8}F\xi\ell.$$

If we allow ξ to approach zero, the slope of the V diagram gets increasingly steep and eventually changes into a step change of magnitude F. The shear force, and therefore the slope of the M diagram, changes increasingly rapidly as length $\xi\ell$ gets smaller. In other words, the parabola curves more and more. In the limiting case $\xi \to 0$ the area of the parabola is reduced to a point: the M diagram gets bent under the concentrated load.

For uniformly distributed loads over a finite length $\xi\ell$, the maximum bend-



(12.31) (12.32)

Figure 12.31 The V and M diagrams due to a point load F at midspan.

Figure 12.32 The V and M diagrams in the case that load F is uniformly distributed over a small length $\xi\ell$.

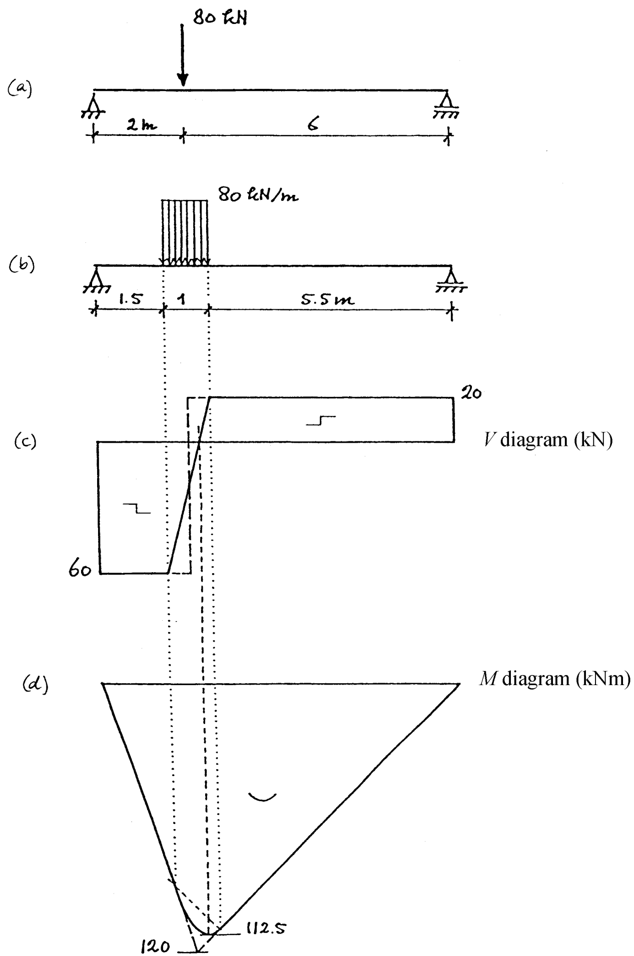


Figure 12.33 The V and M diagrams due to a point load (dashed line) to replace a uniformly distributed load over a small length (solid line).

ing moment is smaller than when the load is concentrated at a single point.

The difference is

$$\frac{p}{\frac{1}{4}F\ell} = \frac{\frac{1}{8}F\xi\ell}{\frac{1}{4}F\ell} \times 100\% = \tfrac{1}{2}\xi \times 100\%.$$

We can show that this value applies also when the load is not acting at midspan.

To illustrate, we compare the two loads in Figures 12.33a and 12.33b. With

$$\xi = \tfrac{1}{8}$$

we find

$$\tfrac{1}{2}\xi \times 100\% = \tfrac{1}{2} \times \tfrac{1}{8} \times 100\% = 6.25\%.$$

The maximum bending moment due to the distributed load will be 6.25% smaller than the maximum bending moment due to the point load.

A calculation (which is left to the reader) shows that the maximum bending moment due to the concentrated load is 120 kNm, and due to the distributed load is 112.5 kNm, which indeed is 6.25% less than before.

In Figures 12.33c and 12.33d, the V and M diagrams are shown for both loads. The V and M diagrams due to the concentrated loads are shown by dashed lines, insofar they deviate.

Note: The maximum values of 120 and 112.5 kNm do not occur in the same cross-section!

12.5.2 Uniformly distributed load

If a large number of almost equal point loads are acting on a beam at regular distances, such as a bridge with a traffic jam or a train crossing, the point loads can often be replaced by a uniformly distributed load to simplify the calculation. The uniformly distributed load is then a schematisation of the actual load.

To gain a picture of the consequences of this type of modelling, we will look at the simply supported beam in Figure 12.34, which is subject to a system of equal point loads at mutually equal distances. With n point loads F at mutually equal distances a on a span of length ℓ it applies that

$$\ell = n \cdot a.$$

The substitute uniformly distributed load is

$$q = \frac{F}{a} = \frac{nF}{\ell}.$$

The point loads generate a stepped shear force diagram and a bent bending moment diagram. The uniformly distributed load causes a linear shear force diagram and a parabolic bending moment diagram. Both V and M diagrams have the same value midway between two successive point loads. Because the shear force V is equal to the slope of the M diagram, the parabola is tangent to the bent M diagram there.

With an even number of point loads, both moment diagrams have the same maximum bending moment at midspan, see the M diagram in Figure 12.34b:

$$M_{\max} = \tfrac{1}{8} q\ell^2 = \tfrac{1}{8}\frac{nF}{\ell}\ell^2 = \tfrac{1}{8} nF\ell.$$

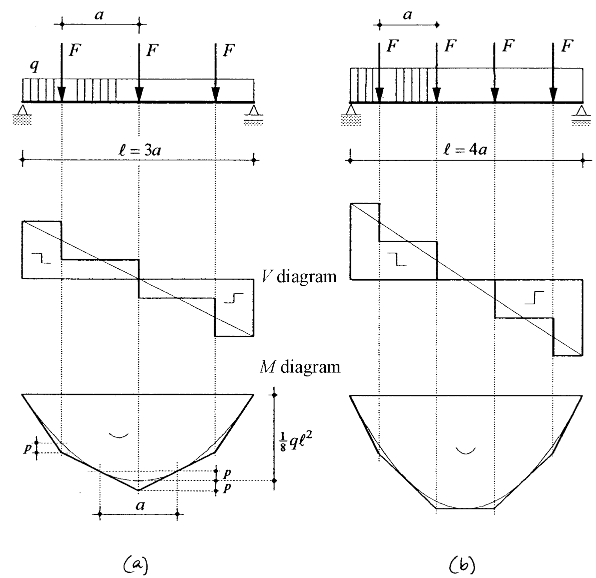


Figure 12.34 The V and M diagrams due to a number of point loads, respectively a substitute uniformly distributed load. (a) With an odd number of point loads the substitute distributed load gives a somewhat smaller maximum bending moment at midspan. (b) With an even number of point loads, both bending moment diagrams have the same maximum bending moment.

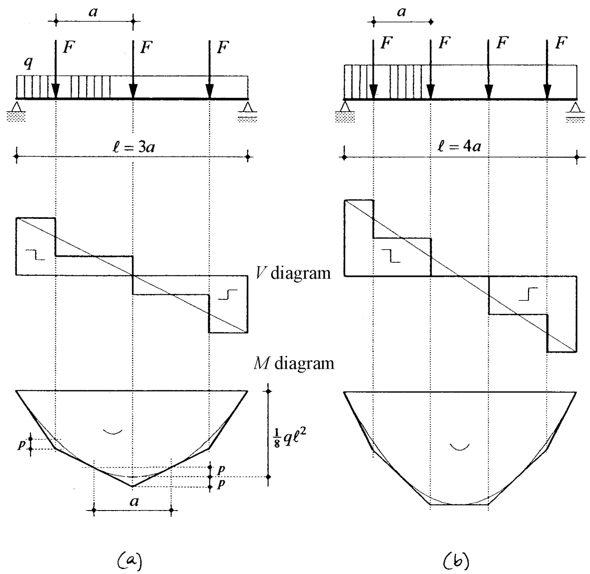


Figure 12.34 The V and M diagrams due to a number of point loads, respectively a substitute uniformly distributed load. (a) With an odd number of point loads the substitute distributed load gives a somewhat smaller maximum bending moment at midspan. (b) With an even number of point loads, both bending moment diagrams have the same maximum bending moment.

With an odd number of point loads, the maximum bending moment at midspan due to the point loads is a value of p larger than the bending moment due to the substitute uniformly distributed load, see the M diagram in Figure 12.34a:

$$M_{\max} = \tfrac{1}{8}q\ell^2 + p$$

in which

$$p = \tfrac{1}{8}qa^2.$$

The actual maximum bending moment is therefore larger. The difference is

$$\frac{p}{\frac{1}{8}q\ell^2} \cdot 100\% = \frac{a^2}{\ell^2} \cdot 100\% = \frac{100}{n^2}\%.$$

With one concentrated load ($n = 1$) the difference is 100%,[1] but this reduces rapidly for more loads. With $n = 3$ the difference is 11% and for $n = 5$ we are already down to a difference of 4%. It now does not make much difference whether we use in calculations point loads or a substitute uniformly distributed load.

12.5.3 Couple

The load on the beam in Figure 12.35a consists of two parallel and opposite forces F. If the distance a between these forces is small as compared to the length of the beam, the load can also be modelled as a concentrated couple $T = F \cdot a$ (see Figure 12.35b).

1 See Section 12.1.3, Example 1, with Figures 12.5 and 12.6.

Figure 12.35 shows the M and V diagrams for

$$F = 120 \text{ kN},$$

$$a = 0.5 \text{ m}.$$

Thus

$$T = F \cdot a = 60 \text{ kNm}.$$

The differences in the M diagrams are minor: the maximum bending moments due to the forces are a fraction smaller than those due to the substitute couple.

On the other hand, the differences in the V diagrams are far larger. The shear force is equal to the slope of the M diagram. At the concentrated couple the slope becomes infinitessimally large over an infinitely small length. Since infinitessimally small and infinitessimally large do not exist in physical reality, the dashed part of the V diagram in Figure 12.35b is omitted.

For the simply supported beam in Figure 12.35a, the total area of the V diagram is equal to zero (rule 12):

$$\Delta M^{(\mathrm{AB})} = M_{\mathrm{B}} - M_{\mathrm{A}} = \int_0^{\ell} V \, \mathrm{d}x = 0.$$

It is clear that the area of the V diagram is no longer zero when a concentrated couple acts on the beam. By omitting the dashed part in the V diagram in Figure 12.35b at the concentrated couple (an infinitely large value over an infinitessimally small length, but with a finite area), the total area of the V diagram changes. This is no longer zero, but is now equal to the magnitude of the couple.

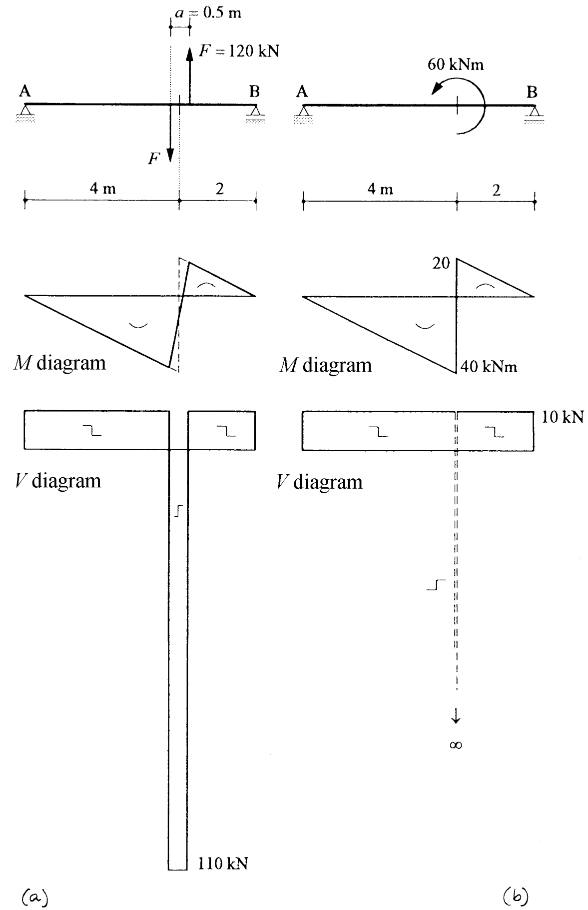


Figure 12.35 The M and V diagrams (a) due to a couple of 60 kNm formed by two parallel opposite forces and (b) due to a concentrated couple of 60 kNm.

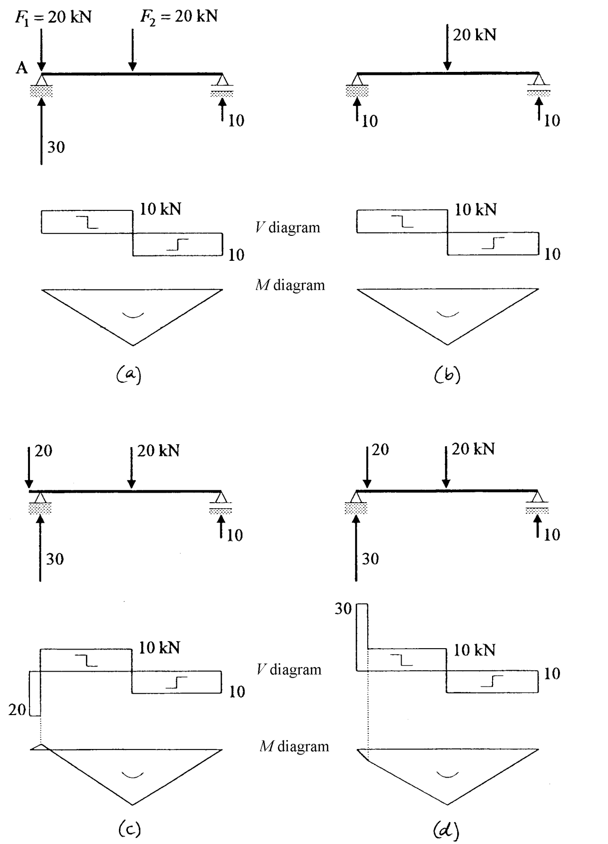


Figure 12.36 Various loads with associated V and M diagrams. In cases (a) and (b) the V and M diagrams are the same, but the support reactions differ due to the load at A. If this load is not directly above the support, but slightly to the left or to the right, the situations in respectively (c) and (d) occur.

12.5.4 Shear forces at a support

The simply supported beam in Figure 12.36a is loaded by two forces F_1 and F_2. The force F_1 at support A is directly transferred to the foundation and does not influence the force flow in the beam. The V and M diagrams are equal to those for the beam in Figure 12.36b, without force F_1. Only the support reactions differ.

Note: In Figure 12.36a, the shear force directly to the right of A is not equal to the support reaction at A!

The fact that force F_1 is exactly above support A is theoretically possible. However, it is more likely that F_1 acts slightly to the left (Figure 12.36c) or slightly to the right (Figure 12.36d) of A. In both cases the maximum bending moment changes only by a small amount, but there are major differences in the V diagram at A.

The differences noted in the V diagram may be less serious than sketched here, for concentrated forces do not exist in reality. Also, members idealised as line elements (the member axis) in reality have cross-sectional dimensions in which the shear force is a model for the transfer of forces normal to the member axis. One should always keep in mind that there are differences between an idealised and the real situation.

12.5.5 Eccentric axial forces

Line elements are structural elements for which the cross-sectional dimensions are considerably smaller than the length. Through simplifying assumptions in the smaller directions (those of the cross-section) the properties of such a structural element can be ascribed to a single line. This line, the so-called member axis, is a one-dimensional model of a structural element that in reality is three-dimensional. In mechanical diagrams, we usually represent a line element by its axis, and draw it without its cross-sectional dimensions.

We have not yet discussed the position of the member axis within the cross-section. This location is not important as long as all the forces are acting normal to the member axis. It does make a difference if there are also forces (with components) parallel to the member axis.

As an example, we use the simply supported beam in Figure 12.37a, for which we assume that the member axis is halfway down the depth of the beam.

The support reactions follow directly from the equilibrium of the beam. If we model the beam as a line element, none of the forces on the beam (including the support reactions) have their point of application on the member axis (see Figure 12.37b).

Forces (force components) normal to the member axis can be shifted along their line of action to the member axis.

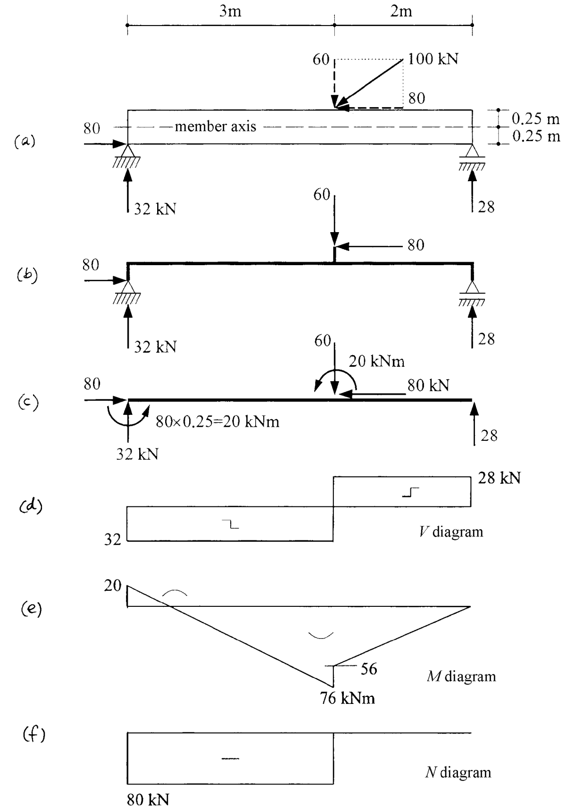
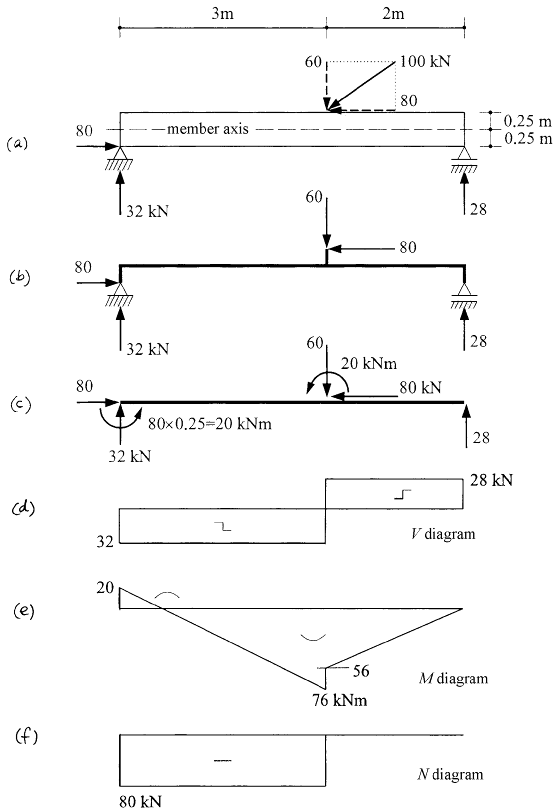


Figure 12.37 (a) Simply supported beam with load and support reactions. The member axis is at half the depth of the beam. (b) The beam modelled as a line element. None of the forces have their point of application on the member axis. (c) Forces (force components) normal to the member axis can be shifted along their line of action to the member axis. Forces (force components) acting eccentrically and parallel to the member axis can be shifted provided that a couple is added concurrently. The magnitude of the couple is equal to the product of force and eccentricity. (d) Shear force diagram. (e) Bending moment diagram. Note that the bending moment at the hinged support is not zero. (f) Normal force diagram.



Forces (force components) acting eccentrically and parallel to the member axis can be shifted to the member axis as long as we concurrently add couples that are equal in magnitude to the product of force and eccentricity (see Section 3.1.5). In this example, the two couples are:

$$(80\ \text{kN})(0.25\ \text{m}) = 20\ \text{kNm}.$$

In Figure 12.37c, all the loads are acting on the member axis. Figures 12.37d to 12.37f show the associated V, M and N diagrams.

Eccentric axial forces exert moments on the beam modelled as a line element and cause step changes in the M diagram.

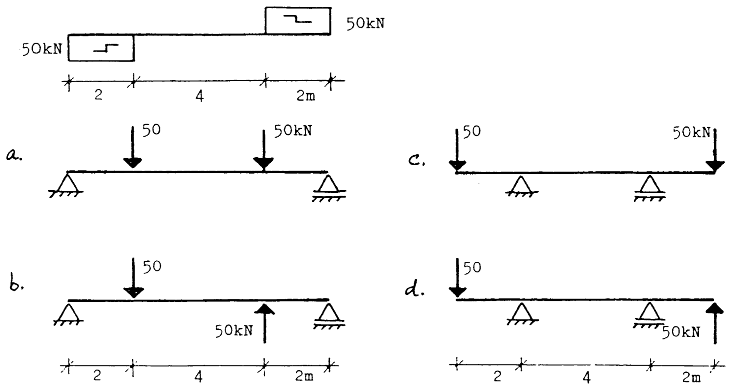
Note that the bending moment at the hinged support is not zero. Also note that the slope of the M diagram agrees with the shear force, and that the total area of the V diagram is no longer zero, but is equal to the sum of the concentrated couples on the line element.

Figure 12.37 (a) Simply supported beam with load and support reactions. The member axis is at half the depth of the beam. (b) The beam modelled as a line element. None of the forces have their point of application on the member axis. (c) Forces (force components) normal to the member axis can be shifted along their line of action to the member axis. Forces (force components) acting eccentrically and parallel to the member axis can be shifted provided that a couple is added concurrently. The magnitude of the couple is equal to the product of force and eccentricity. (d) Shear force diagram. (e) Bending moment diagram. Note that the bending moment at the hinged support is not zero. (f) Normal force diagram.

12.6 Problems

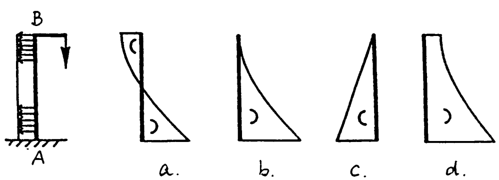
Shape of the V and M diagrams (Section 12.1.1)

12.1 Given a shear force diagram and four loaded beams.



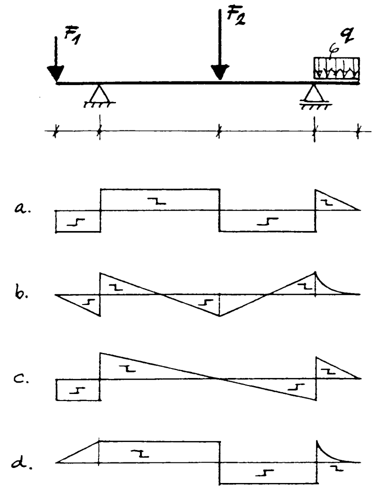
Question :
Which beam matches the shear force diagram?

12.2 Four bending moment diagrams are given for column AB.



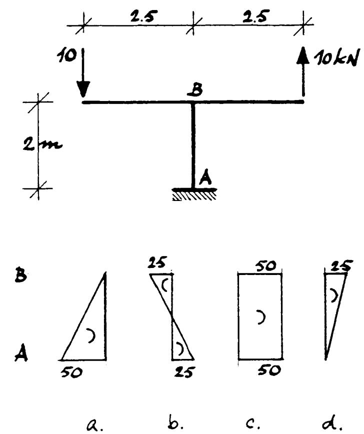
Question :
Which bending moment diagram matches the given load?

12.3 Given a loaded beam and four shear force diagrams.



Question :
Which shear force diagram matches the loaded beam?

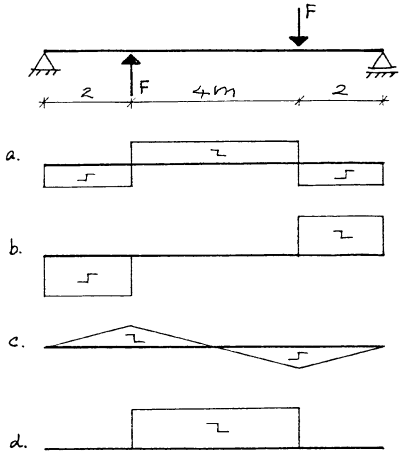
12.4 Four bending moment diagrams are given for column AB (values in kNm).



Question :
Which bending moment diagram matches the given load?

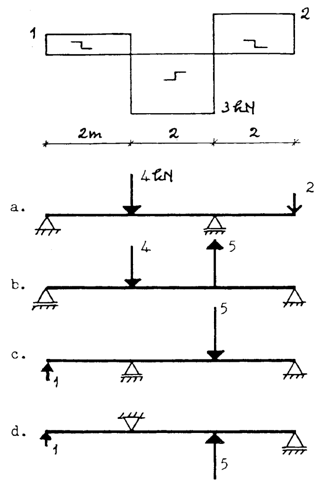
12.5 A loaded beam and four shear force diagrams are given.

Question :
Which shear force diagram matches the loaded beam?



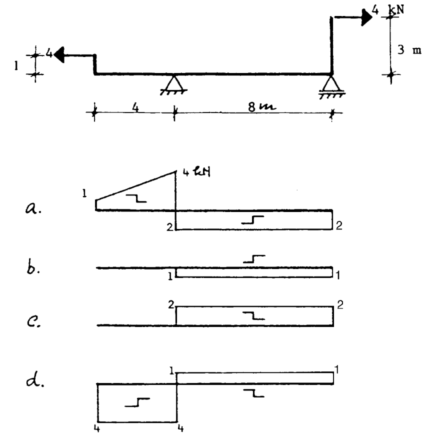
12.6 A shear force diagram and four loaded beams are given.

Question :
Which loaded beam matches the shear force diagram?



12.7 Four shear force diagrams are shown for beam ABC, loaded by two eccentric tensile forces.

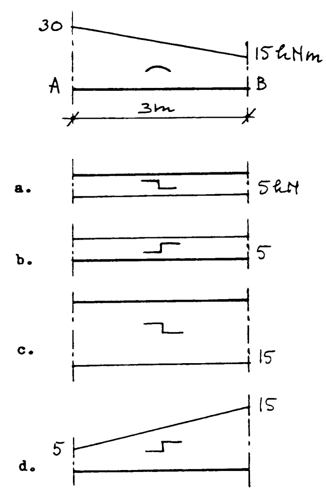
Question :
Which shear force diagram is correct?



Slope of the V and M diagrams and extreme values (Section 12.1.2)

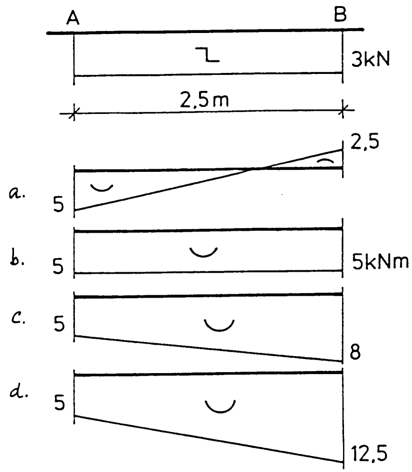
12.8 Given the bending moment diagram for beam segment AB and four shear force diagrams.

Question:
Which shear force diagram is correct?



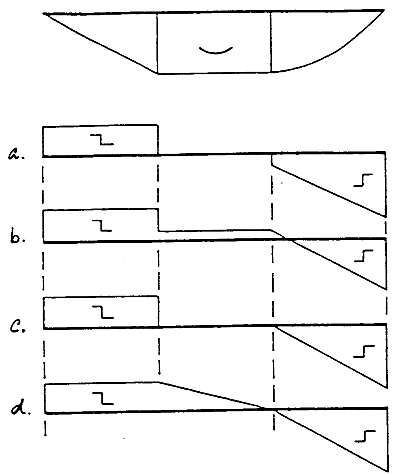
12.9 The shear force diagram for beam segment AB and four bending moment diagrams are given.

Question:
Which bending moment diagram may be correct?



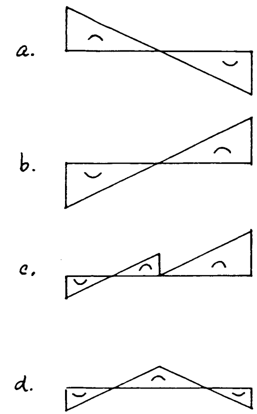
12.10 A bending moment diagram and four shear force diagrams are given.

Question:
Which shear force diagram matches the bending moment diagram?



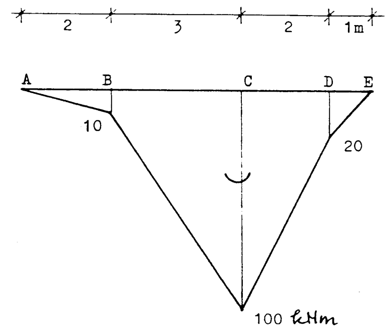
12.11 Four bending moment diagrams are drawn to the same scale.

Question:
Which two bending moment diagrams match the same shear force diagram?



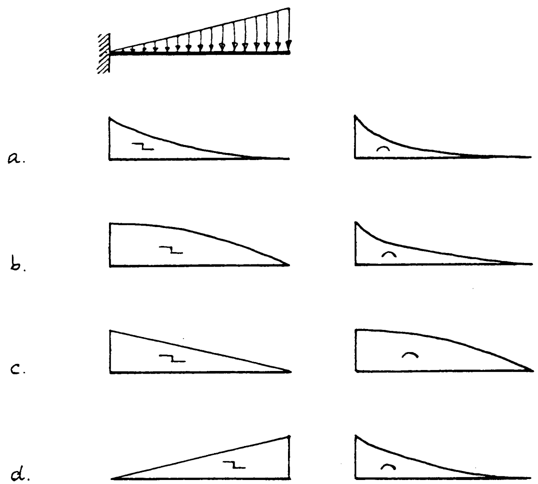
12.12 The bending moment diagram for beam AE is given.

Question:
Where in the beam is the shear force an extreme?



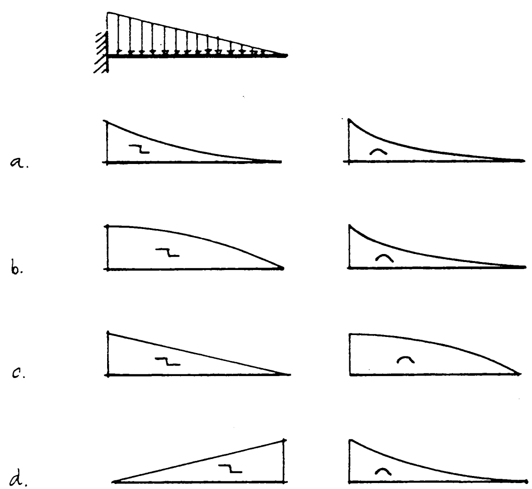
12.13 A cantilever beam with a linear distributed load is given.

Question:
Which combination of V and M diagrams matches this loading case?

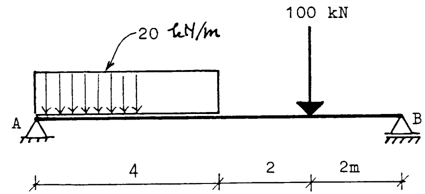


12.14 A cantilever beam with a linear distributed load is given.

Question:
Which combination of V and M diagrams matches this loading case?

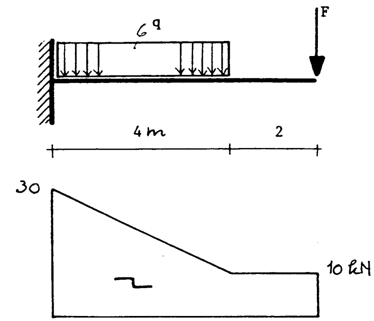


12.15 A beam with its load is given.



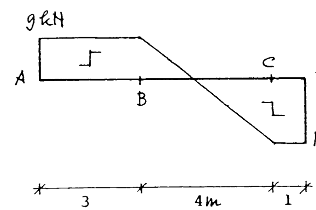
Question:
Determine at A the slope of the tangent to the M diagram (in kNm/m).

12.16 A rigidly supported beam with its shear force diagram is given.



Question:
Determine the magnitude of the uniformly distributed load q.

12.17 The shear force diagram for beam AD is given.

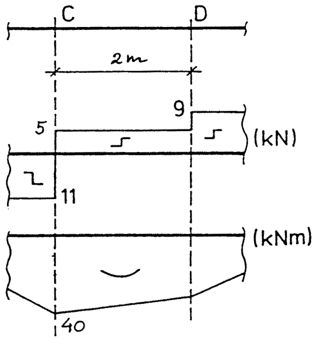


Question:
Determine the magnitude and direction of the uniformly distributed load in field BC.

12.18 Given the shear force diagram for beam segment CD and the bending moment at C.

Questions:

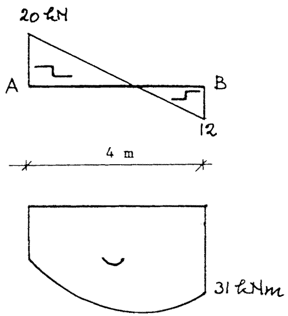
a. Isolate segment CD and draw all the (section) forces acting on it.
b. From the equilibrium of CD determine the bending moment at D.
c. Check for CD that the slope of the *M* diagram is equal to the shear force.



12.19 Given the shear force diagram for beam segment AB and a sketch of the bending moment diagram. The bending moment at B is 31 kNm.

Questions:

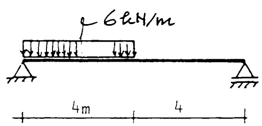
a. Isolate beam segment AB and draw all the forces acting on it.
b. From the equilibrium of AB determine the bending moment at A.
c. Where is the bending moment an extreme?
d. Determine this moment.



12.20 Given a simply supported beam with a uniformly distributed load on the left-hand half.

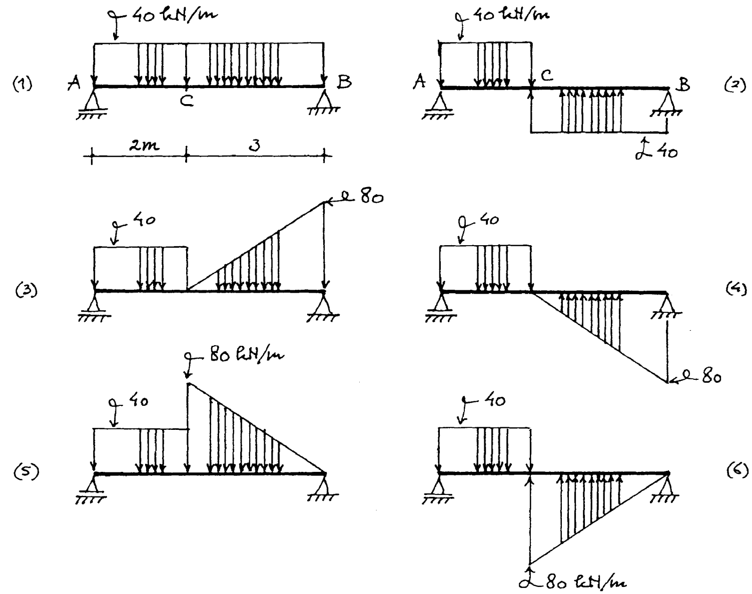
Questions:

a. Draw the shear force diagram.
b. Where is the bending moment an extreme?
c. Determine this bending moment.



Tangents to the M diagram (Section 12.1.3)

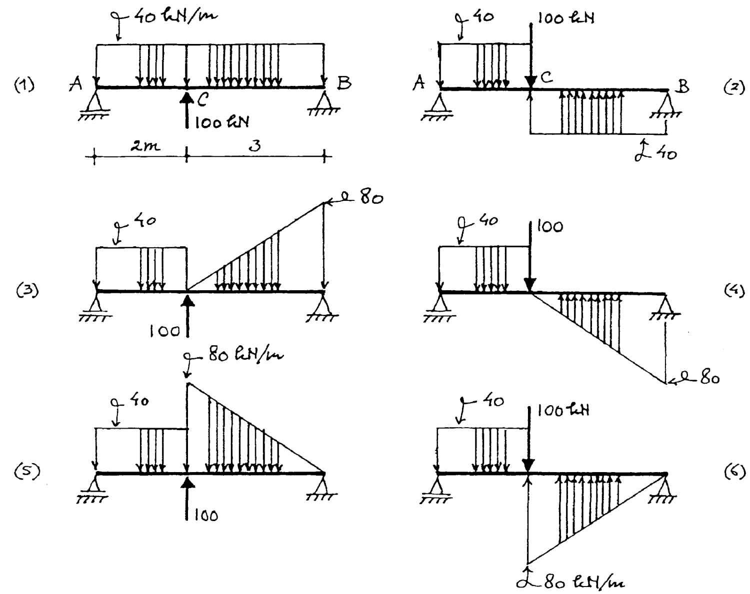
12.21: 1–6 The same beam ACB is loaded in six different ways.



Questions:

a. Draw the bending moment diagram due to the load resultants in the fields AC and CB.
b. Use this bending moment diagram to determine the bending moment and the shear force at C, the join of the fields.
c. Sketch the bending moment diagram due to the distributed load.

12.22: 1–6 The same beam ACB is loaded in six different ways. The difference to the previous problem is an additional force of 100 kN at C.



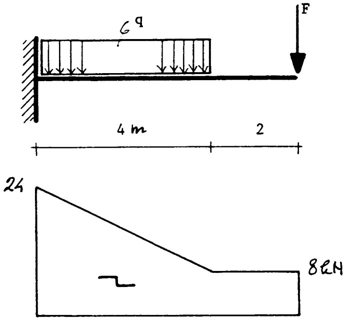
Questions:

a. Draw the bending moment diagram due to the load resultants in the fields AC and CB and the force of 100 kN at C.
b. Sketch the bending moment diagram due to the load actually present.

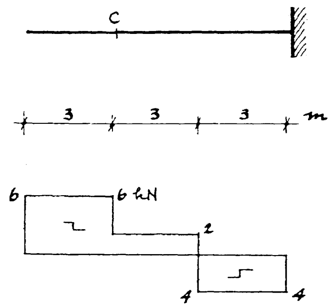
Interpreting the area of the load diagram and V diagram (Section 12.1.4)

12.23 Given a fixed beam and its shear force diagram.

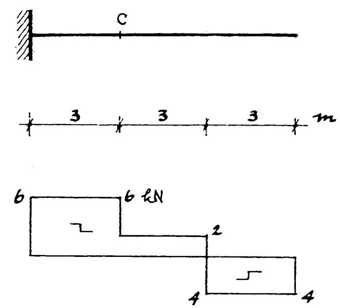
Question:
Determine the magnitude of the fixed-end moment.



12.24: 1–2 Given the shear force diagram for a fixed beam, loaded by three point loads.



(1)

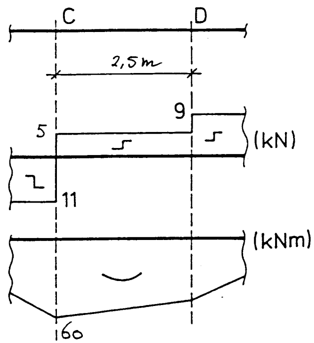


(2)

Question:
How large is the bending moment at C?

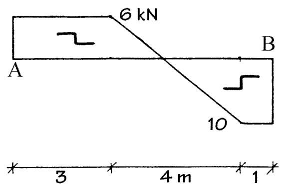
12.25 Given the shear force diagram for a beam segment CD and a sketch of the bending moment diagram. The bending moment at C is 60 kNm.

Question:
Determine the bending moment at D.



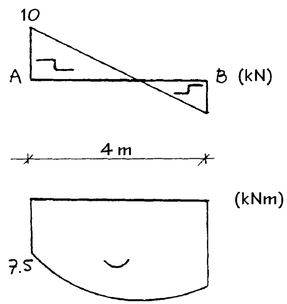
12.26 Given the shear force diagram for a simply supported beam AB. No (concentrated) couples are acting on the beam.

Question:
Determine the maximum bending moment.



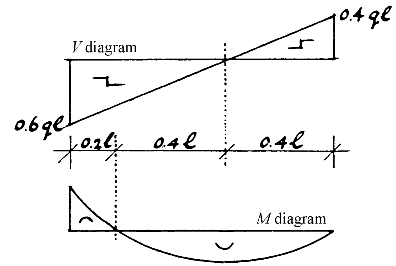
12.27 Given the shear force diagram for beam segment AB and a sketch of the bending moment diagram. The bending moment at A is 7.5 kNm.

Questions:
a. Determine the maximum bending moment.
b. Determine the bending moment at B.



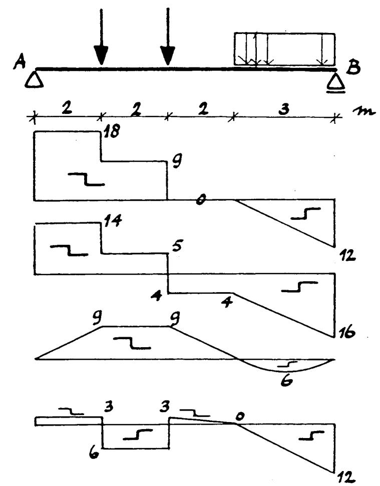
12.28 Given a shear force diagram and the associated bending moment diagram.

Question:
Determine the extreme values of the bending moment.

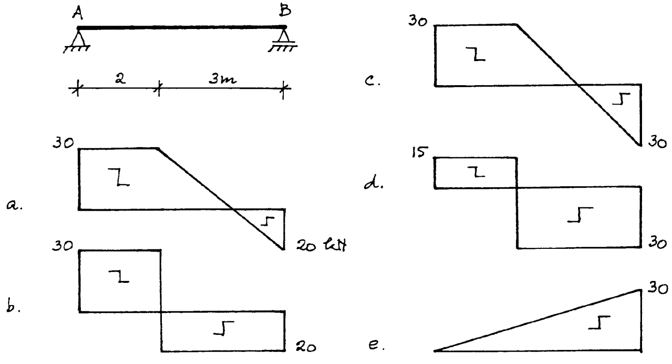


12.29 Given a loaded beam and four shear force diagrams.

Question:
Which shear force diagram could be the correct one?



12.30 Given beam AB with five shear force diagrams. In addition to (distributed) forces normal to the beam axis, the beam may also be subject to a (concentrated) couple.

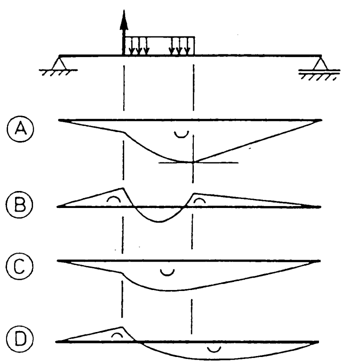


Question:
In which cases is a (concentrated) couple acting on the beam? In those cases determine the magnitude of the couple and the direction in which it is acting.

Step changes and bends in the V and M diagrams (Section 12.1.5)

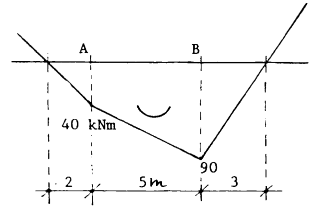
12.31 Given a loaded beam and four bending moment diagrams.

Question:
Which bending moment diagram has the right shape?



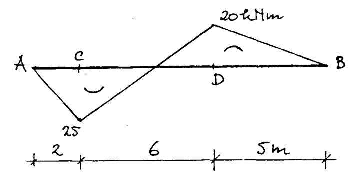
12.32 Given the bending moment diagram for a beam segment subject to forces at A and B.

Question:
The magnitude and direction of these forces.



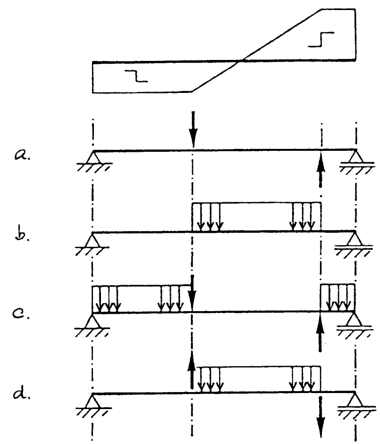
12.33 Given the bending moment diagram for beam AB.

Question:
The magnitude and direction of the forces at C and D.

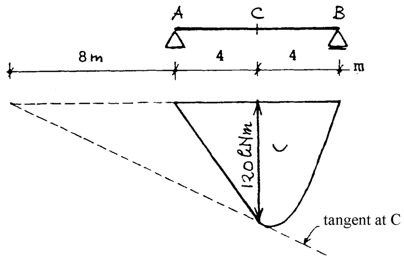


12.34 Given a shear force diagram for four different loaded beams.

Question:
Which load matches the shear force diagram?

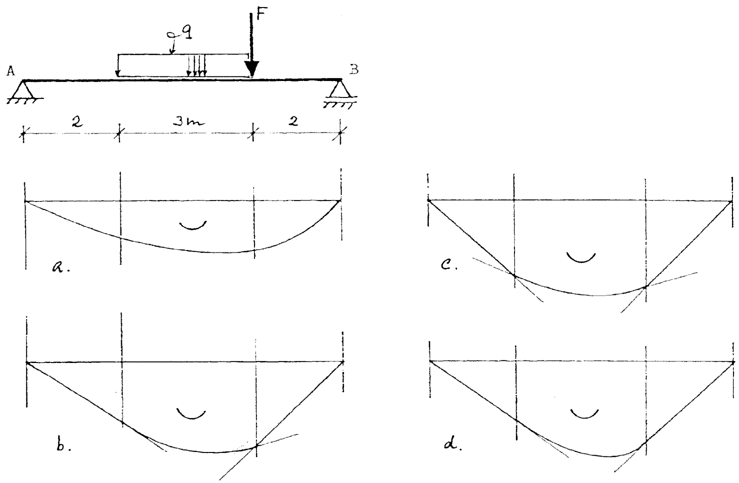


12.35 The bending moment diagram of beam AB varies linearly along the left-hand side and is curved on the right-hand side. The tangent to the curved part of the M diagram in the middle C is also shown.



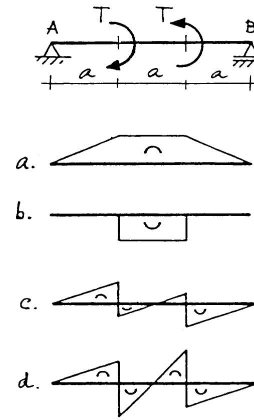
Question:
Determine the magnitude and direction of the point load at C.

12.36 The simply supported beam AB is loaded by a uniformly distributed load q and a force F. Four bending moment diagrams are shown.



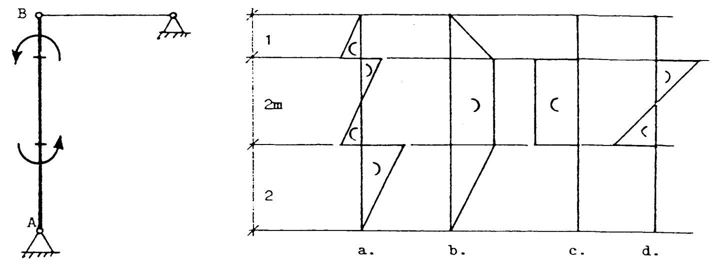
Question:
Which bending moment diagram matches the loaded beam?

12.37 Given four bending moment diagrams for beam AB.



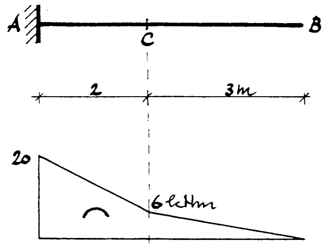
Question:
Which bending moment diagram matches the given load?

12.38 Given four bending moment diagrams for post AB.



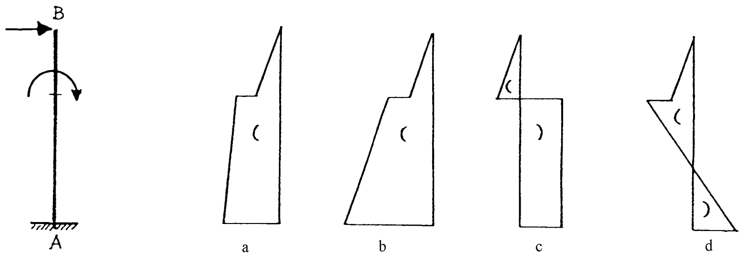
Question:
Which bending moment diagram could match the given load?

12.39 Given the bending moment diagram for the cantilever beam AB.



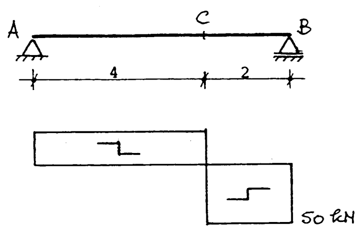
Question:
Determine the magnitude and direction of the force at C.

12.40 Given four bending moment diagrams for column AB.



Question:
Which bending moment diagram could match the given load?

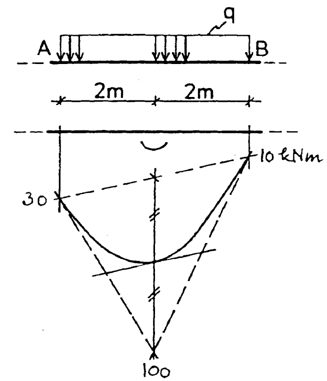
12.41 A sketch is given of the shear force diagram of the simply supported beam ACB, loaded by a force F at C. The magnitude and direction of the shear force in field CB are also given.



Question:
Determine the magnitude and direction of F.

Properties of parabolic M diagrams (Section 12.1.6)

12.42 Given the bending moment diagram for an isolated beam segment AB with a uniformly distributed load q. At A and B the tangents to the M diagram are shown.



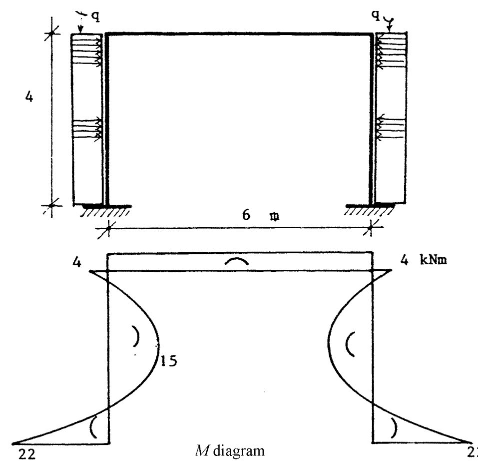
Questions:

a. The magnitude of the uniformly distributed load q.
b. The shear force in the middle of AB.
c. The shear forces at A and B.
d. Isolate beam segment AB, draw all the forces acting on it and check the equilibrium.

12.43 Given a statically indeterminate portal frame with its bending moment diagram due to the given load. The bending moment half-way up the column is 15 kNm.

Questions:

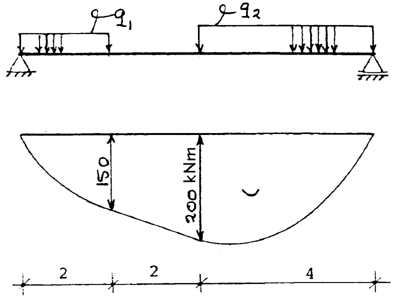
a. The magnitude of the uniformly distributed load q on column AB.
b. The shear force diagram for column AB.
c. The location and magnitude of the maximum bending moment in column AB.



12.44 Given a sketch of the bending moment diagram of a simply supported beam with uniformly distributed loads q_1 and q_2.

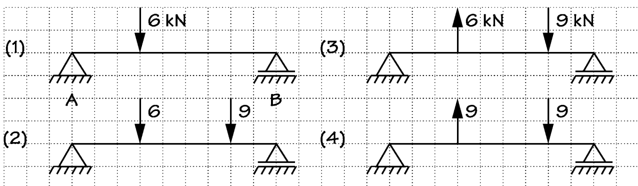
Questions:

a. From the shape of the M diagram determine the magnitude of the distributed loads.
b. Draw the shear force diagram for the entire beam.
c. Determine the location and magnitude of the maximum bending moment in the beam.



Mixed problems (Section 12.1)

12.45: 1–4 The same simply supported beam is loaded in various ways. Length scale: 1 square ≡ 1 m; forces in kN.



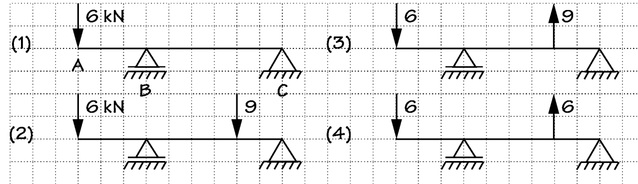
Questions:

a. Determine the bending moment diagram.
b. Determine the shear force diagram.

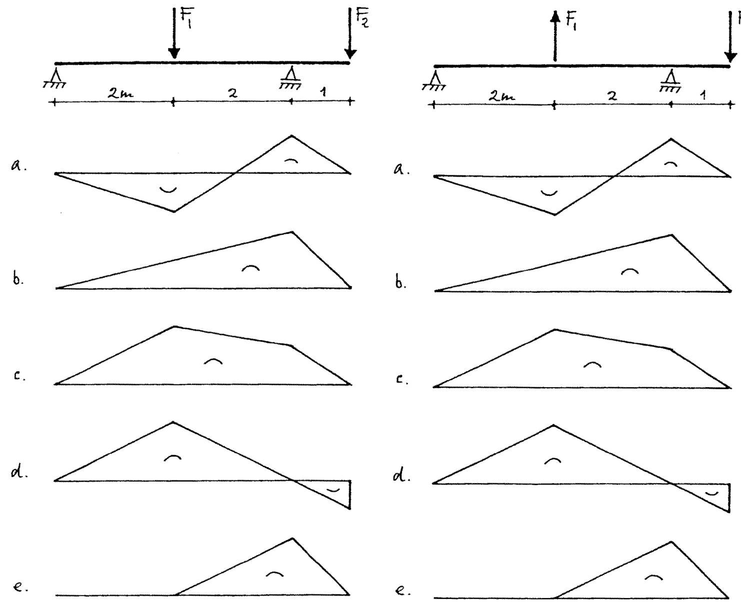
12.46: 1–4 The same beam with overhang is loaded in various ways. Length scale: 1 square ≡ 1 m; forces in kN.

Questions:

a. Determine the bending moment diagram.
b. Determine the shear force diagram.



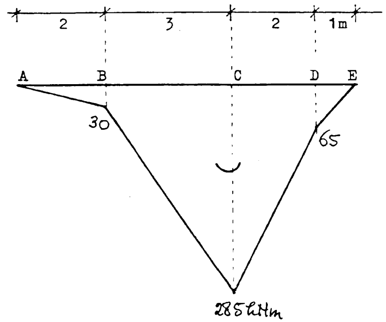
12.47: 1–2 Two loading cases and five bending moment diagrams are given.



(1) (2)

Question:
Which bending moment diagram(s) in no way matches (match) the loading case?

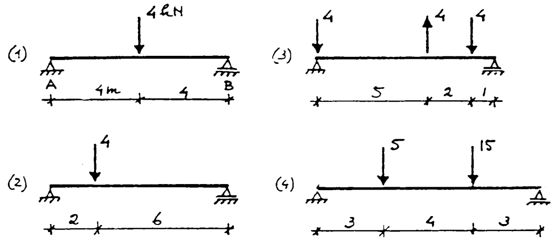
12.48 Given the bending moment diagram for beam AE. Five forces are acting on the isolated beam.



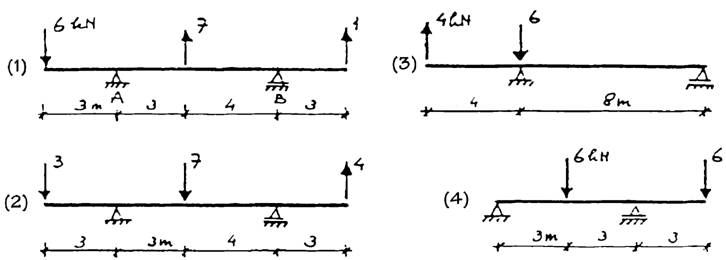
Question:
Determine the magnitude and direction of these forces.

12.49: 1–4 Given a number of beams loaded in various ways.

Questions:
a. Determine the bending moment diagram.
b. Determine the shear force diagram.

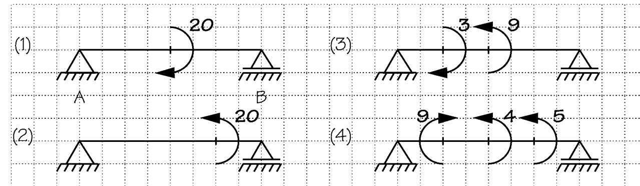


12.50: 1–4 Given a number of cantilever beams are loaded in various ways.



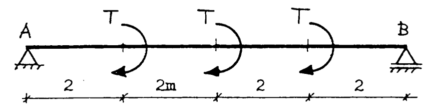
Questions:
a. Determine the bending moment diagram.
b. Determine the shear force diagram.

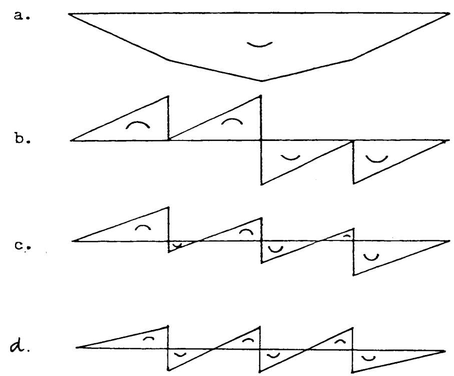
12.51: 1–4 The same simply supported beam is loaded in various ways by only couples. Length scale: 1 square ≡ 1 m; couples in kNm.



Questions:
a. Determine the bending moment diagram.
b. Determine the shear force diagram.

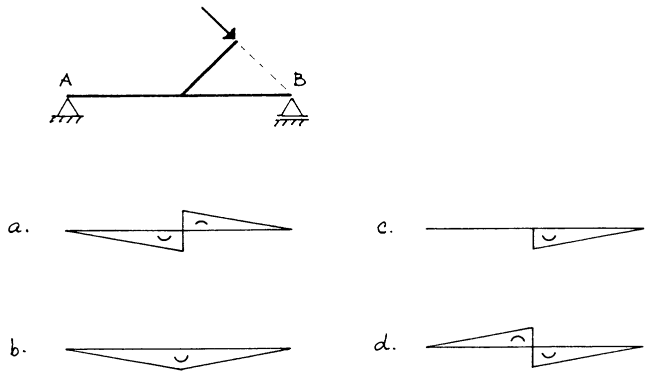
12.52 Given four bending moment diagrams for beam AB.





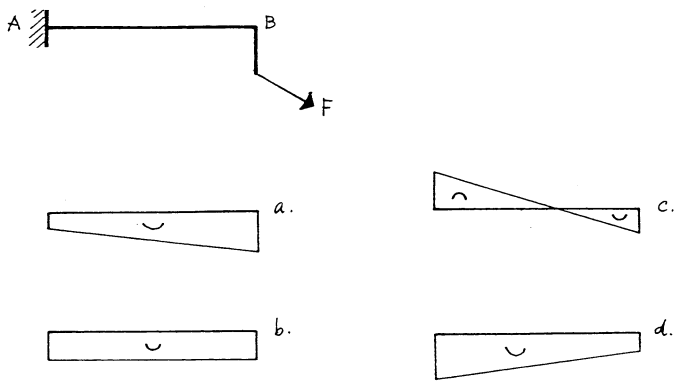
Question:
Which bending moment diagram matches the given load?

12.53 Given four bending moment diagrams for beam AB.



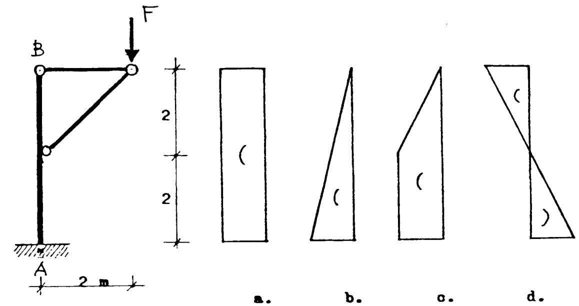
Question:
Which bending moment diagram matches the given load?

12.54 Given four bending moment diagrams for beam AB.



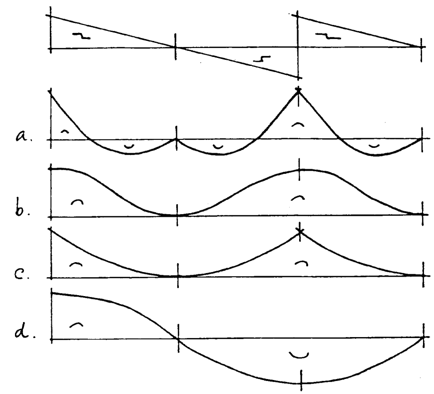
Question:
Which bending moment diagram matches the given load?

12.55 Four bending moment diagrams for column AB are given.



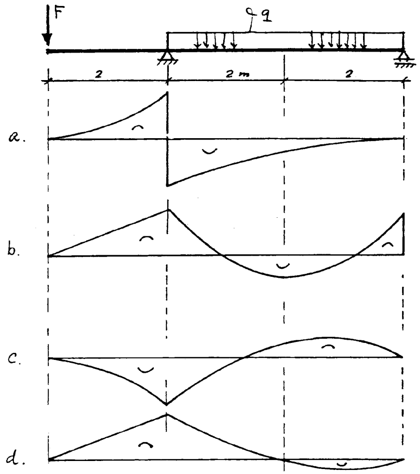
Question:
Which bending moment diagram is correct?

12.56 A shear force diagram and four bending moment diagrams are given.



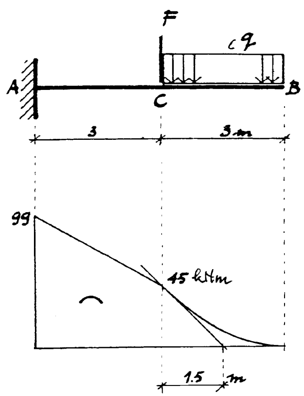
Question:
Which bending moment diagram matches the shear force diagram?

12.57 Given a loaded beam and four bending moment diagrams.



Question:
Which bending moment diagram matches the loaded beam?

12.58 Given the bending moment diagram for beam AB due to a force F at C and a uniformly distributed load q along field CB. The direction of F is not given.

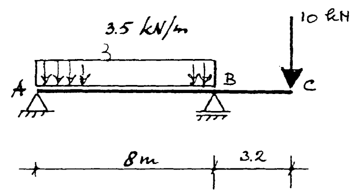


Questions:

a. Determine the magnitude of q.

b. Determine the magnitude and direction of F.

12.59 A beam with overhang is loaded as shown.



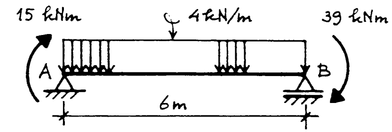
Questions:

a. Determine the bending moment diagram with the tangents at A and B.

b. Determine the shear force diagram.

c. Determine the maximum field moment in AB.

12.60 Given the simply supported beam AB loaded as shown.



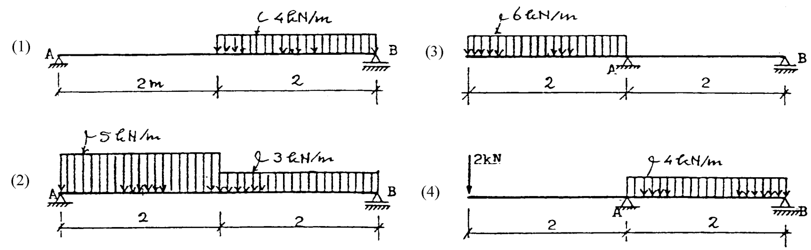
Questions:

a. Determine the bending moment diagram with the tangents at A and B.

b. Determine the shear force diagram.

c. Determine the maximum field moment.

12.61: 1–4 Given four different beams with a uniformly distributed load.



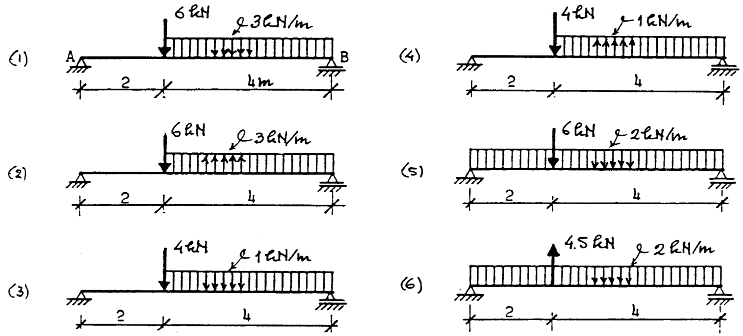
Questions:

a. Determine the M diagram with its tangents at the field boundaries.

b. Determine the V diagram.

c. Determine the location and magnitude of the extreme bending moments.

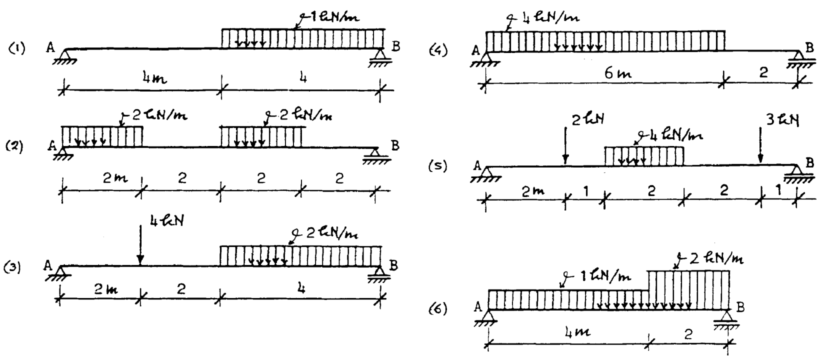
12.62: 1–6 The same beam AB is loaded in six different ways.



Questions:

a. Determine the *M* diagram with the tangents at relevant points.
b. Determine the *V* diagram.
c. Determine the location and magnitude of the maximum/minimum bending moment.

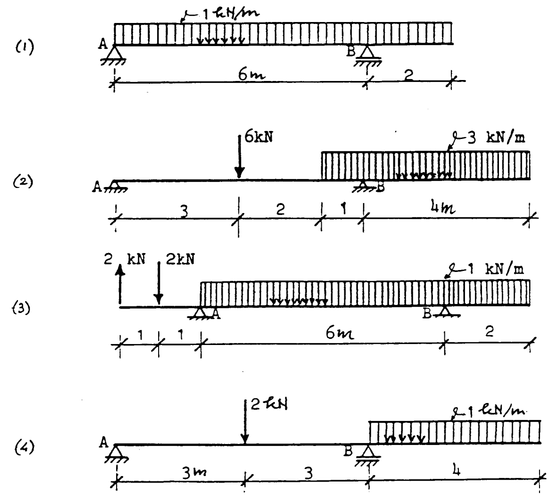
12.63: 1–6 Six different loaded beams are given.



Questions:

a. Determine the *M* diagram with the tangents at relevant points.
b. Determine the *V* diagram.
c. Determine the location and magnitude of the maximum/minimum bending moment.

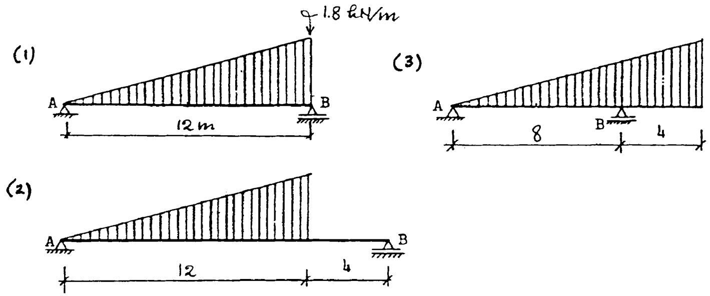
12.64: 1–4 Four different loaded beams with overhang are given.



Questions:

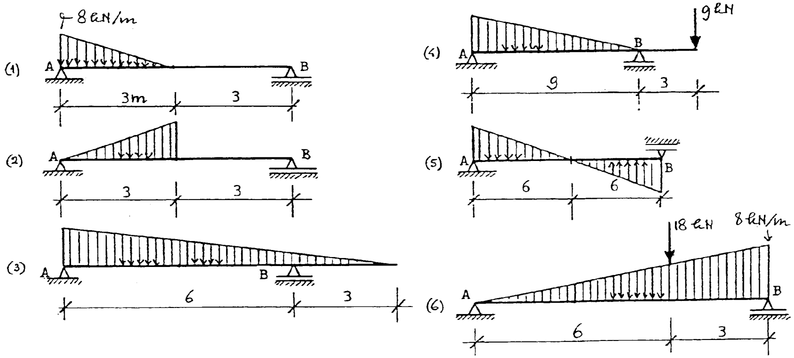
a. Determine the *M* diagram with the tangents at relevant points.
b. Determine the *V* diagram.
c. Determine the location and magnitude of the maximum/minimum bending moment.

12.65: 1–3 Three beams with a linear distributed load are given. The top value of the distributed load is in all cases 1.8 kN/m.



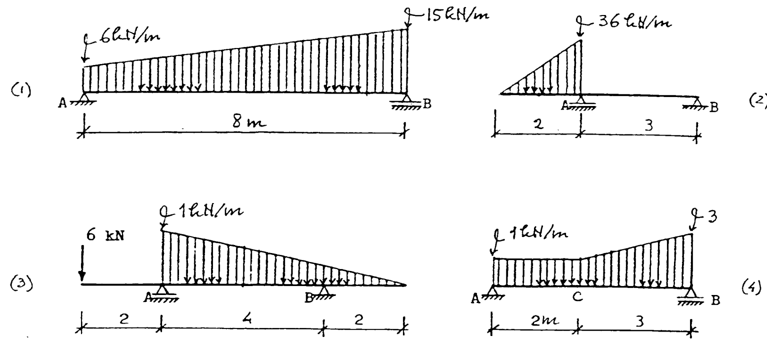
Questions:
a. Sketch the *M* diagram with its tangents at the field boundaries.
b. Sketch the *V* diagram.

12.66: 1–6 A number of beams with linear distributed load and in two cases also a point load are given. The figures are not all drawn to the same scale. The top value of the linear distributed load is in all cases 8 kN/m. The magnitude of the two point loads can be read off from the figure.



Questions:
a. Sketch the *M* diagram with its tangents at the field boundaries.
b. Sketch the *V* diagram.

12.67: 1–4 Four different loaded beams are given.

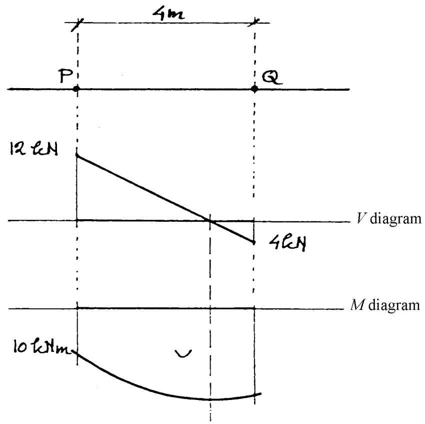


Question:
Sketch the bending moment diagram with the tangents at A, B and C.

12.68 For the segment PQ of a beam the shear force diagram (without deformation symbols), the bending moment at P and a sketch of the bending moment diagram are given.

Questions:

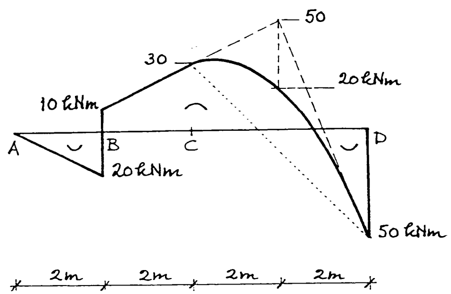
a. Place the deformation symbols in the *V* diagram.
b. Which load is acting on PQ? Draw the load.
c. Determine the bending moment at Q.
d. Determine the maximum bending moment at PQ.
e. Draw PQ with all the forces (loads and section forces at the edges) acting on it and include the values.



12.69 Member ABCD is loaded normal to its axis. The bending moment diagram is given with the tangents in C and D. The bending moment between C and D is a parabola.

Questions:

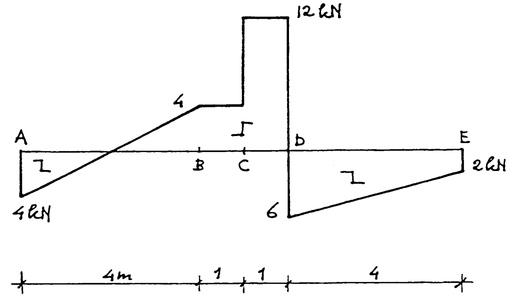
a. Draw the associated shear force diagram with the deformation symbols. Include the relevant values.
b. Draw all the forces (couples) acting on the isolated member. Include the relevant values.
c. Check the equilibrium of member ABCD.



12.70 The *V* diagram of beam ABCDE is given. There are no couples acting on the beam.

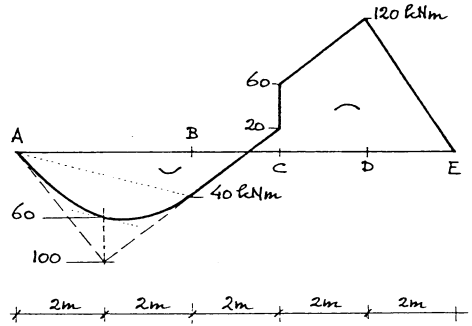
Questions:

a. Draw all the (distributed) forces associated with this *V* diagram acting on beam ABCDE.



b. For the entire beam, draw the *M* diagram with the deformation symbols. Include the relevant values. Draw in relevant points the tangents to the *M* diagram.

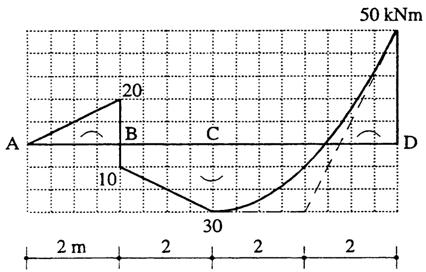
12.71 Member ABCDE is loaded normal to its member axis. The bending moment diagram is given, as are the tangents in A and B. The bending moment between A and B is a second-degree curve (parabola).



Questions:

a. Draw the associated shear force diagram with deformation symbols. Include the relevant values.
b. Draw all the forces (couples) acting on the isolated member. Include the relevant values.
c. Check the equilibrium of the member.

12.72 Member ABCD is loaded normal to its axis. The bending moment diagram is given, as are the tangents at C and D. The bending moment between C and D is a second-degree curve (parabola).



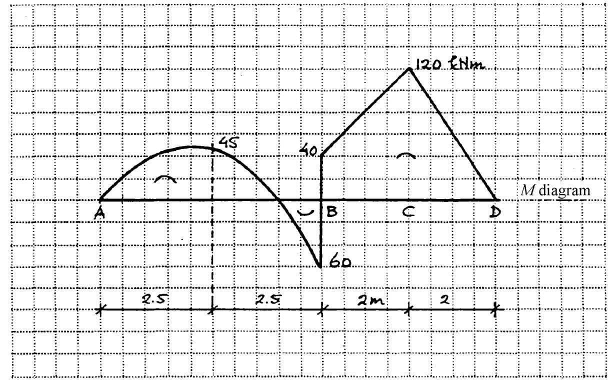
Questions:

a. Draw the associated shear force diagram with deformation symbols. Include the relevant values.
b. Draw all the forces (couples) acting on the isolated member. Include the relevant values.
c. Check the equilibrium of the member. Clearly indicate how you performed this check.

12.73 The bending moment diagram is given for member ABCD. The bending moment between A and B is a parabola (second-degree curve).

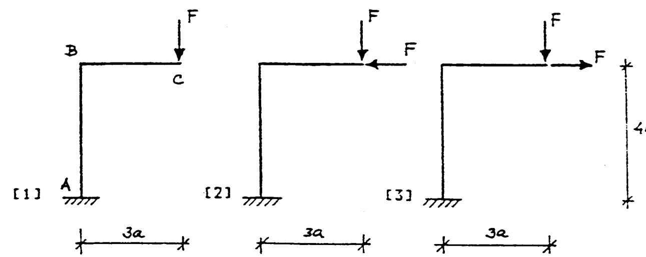
Questions:

a. Draw the associated shear force diagram with deformation symbols. Include the relevant values.
b. Draw all the forces (couples) acting on the isolated member. Include the relevant values.
c. Check the equilibrium of the member.



Bent and compound bar type structures (Section 12.3)

12.74: 1–3 The same bent cantilever is loaded in three different loading ways.

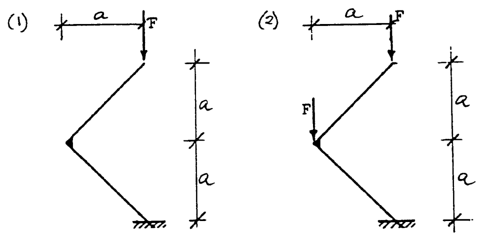


Question:

For the entire structure determine:

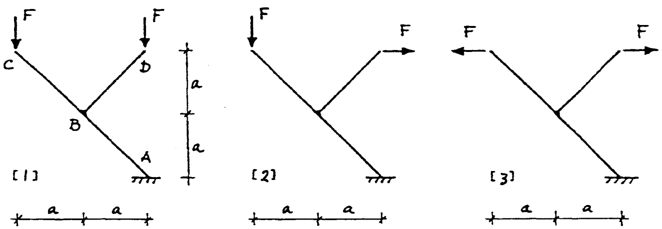
a. the M diagram.
b. the V diagram.
c. the N diagram.

12.75: 1–2 The same bent cantilever is loaded in two different ways.



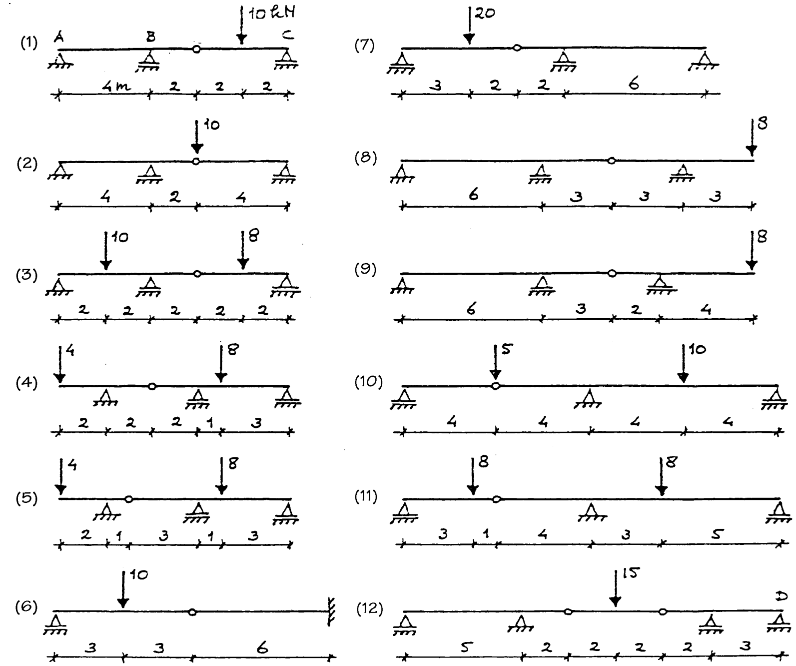
Question:
For the entire structure determine:
a. the *M* diagram.
b. the *V* diagram.
c. the *N* diagram.

12.76: 1–3 The same structure is loaded in three different ways.



Question:
For the entire structure determine:
a. the *M* diagram.
b. the *V* diagram.
c. the *N* diagram.

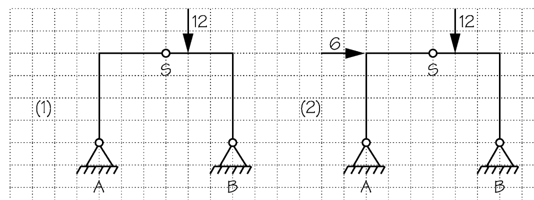
12.77: 1–12 A number of hinged beams with load are given. Dimensions are in m; forces in kN.



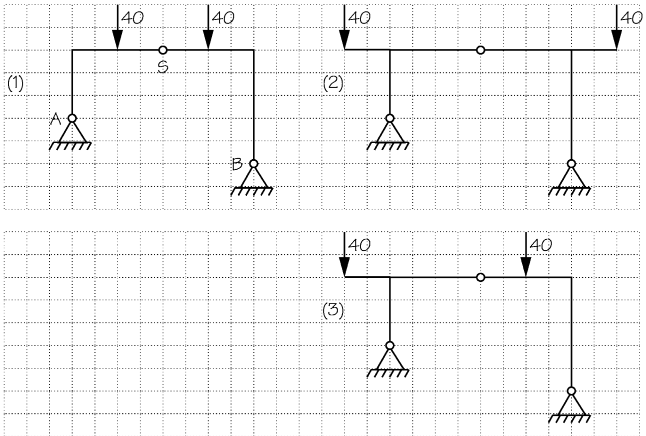
Question:
Determine the bending moment diagram and shear force diagram.

12.78: 1–2 The same three-hinged portal frame is loaded in two ways. Length scale: 1 square ≡ 1 m; forces in kN.

Question:
For the entire structure determine:
a. the *M* diagram.
b. the *V* diagram.
c. the *N* diagram.

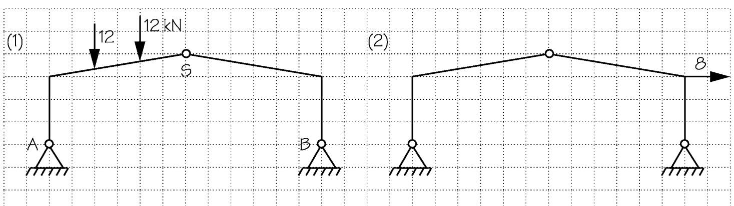


12.79: 1–3 Given three three-hinged portal frames with their load. Length scale: 1 square ≡ 1 m; forces in kN.



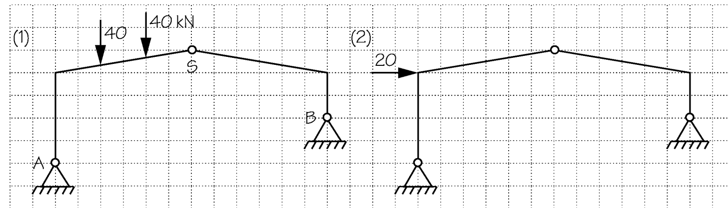
Question:
For the entire structure determine:
a. the M diagram.
b. the V diagram.
c. the N diagram.

12.80: 1–2 The same pitched roof portal frame is loaded in two different ways. Length scale: 1 square ≡ 1 m; forces in kN.



Question:
For the entire structure determine:
a. the M diagram.
b. the V diagram.

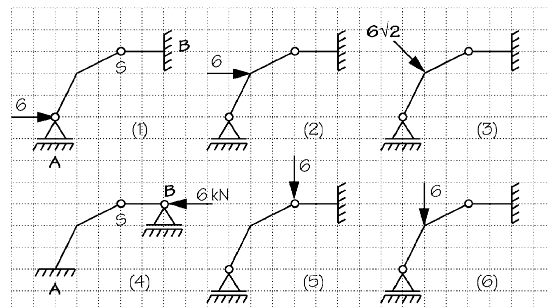
12.81: 1–2 The same three-hinged portal frame is loaded in two different ways. Length scale: 1 square ≡ 1 m; forces in kN.



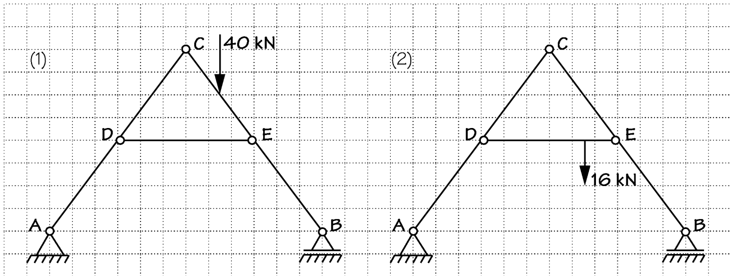
Question:
For the entire structure determine:
a. the M diagram.
b. the V diagram.

12.82: 1–6 Given six bent structures. Length scale: 1 square ≡ 1 m; forces in kN.

Question:
For the entire structure determine:
a. the M diagram.
b. the V diagram.
c. the N diagram.

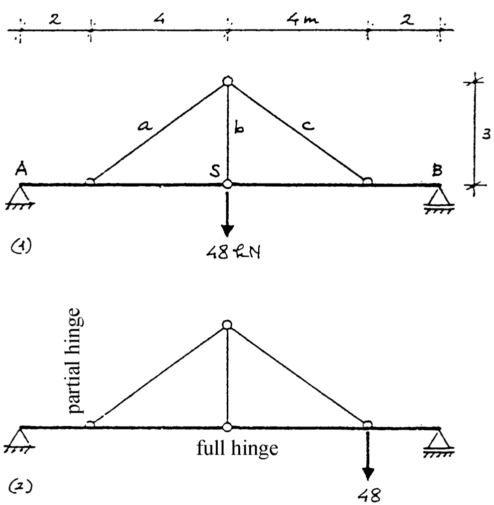


12.83: 1–2 The same structure is loaded in two different ways.



Question:
For the entire structure determine:
a. the M diagram.
b. the V diagram.
c. the N diagram.

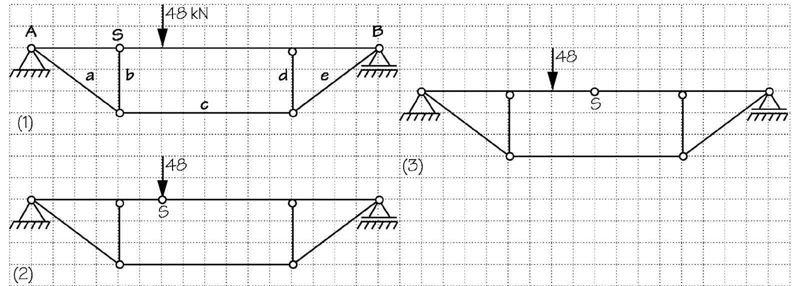
12.84: 1–2 A trussed beam ASB is loaded in two different ways.



Questions:
a. Determine the M and V diagram for ASB.
b. Determine the N diagram for ASB.

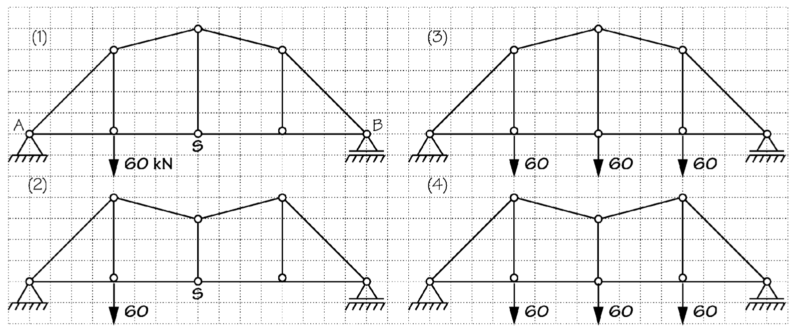
Mind the difference between full hinges and partial hinges.

12.85: 1–5 Hinge S in trussed beam ASB is located in different places. Length scale: 1 square ≡ 1 m; forces in kN.



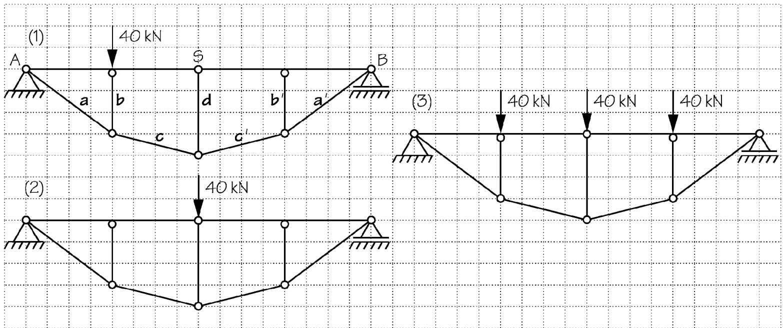
Questions:
a. Determine the M and V diagram for ASB.
b. Determine the N diagram for ASB.

12.86: 1–4 Two trussed beams ASB are loaded in two different ways. Length scale: 1 square ≡ 1 m; forces in kN.



Questions:
a. Determine the M and V diagrams for ASB.
b. Determine the N diagram for ASB.

12.87: 1–3 The same trussed beam ASB is loaded in three different ways. Length scale: 1 square ≡ 1 m; forces in kN.



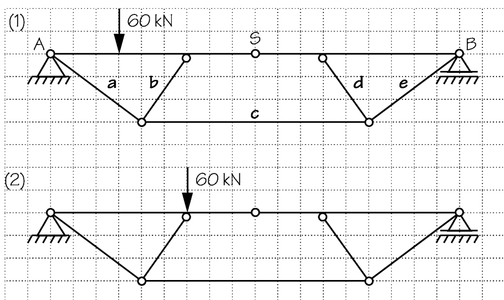
Questions:

a. Determine the M and V diagrams for ASB.
b. Determine the N diagram for ASB.

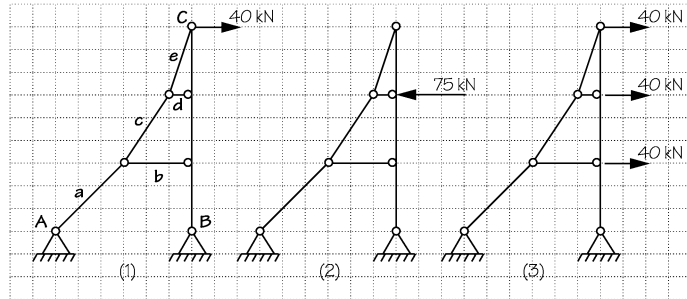
12.88: 1–2 The same trussed beam ASB is loaded in two different ways. Length scale: 1 square ≡ 1 m; forces in kN.

Questions:

a. Determine the M diagram for ASB.
b. Determine the V diagram for ASB.
c. Determine the N diagram for ASB.



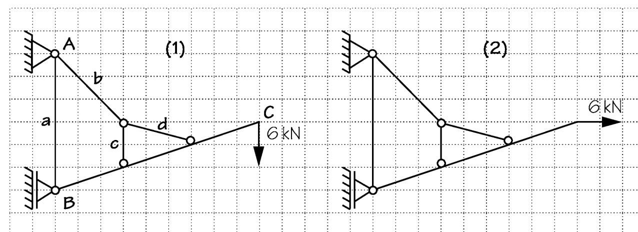
12.89: 1–3 Given a trussed beam with three different loads. Length scale: 1 square ≡ 1 m; forces in kN.



Questions:

a. Determine the M and V diagrams for BC.
b. Determine the N diagram for BC.

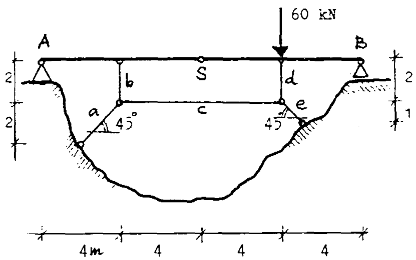
12.90: 1–2 Given a trussed beam with two different loads. Length scale: 1 square ≡ 1 m; forces in kN.



Questions:

a. Determine the M and V diagrams for BC.
b. Determine the N diagram for BC.

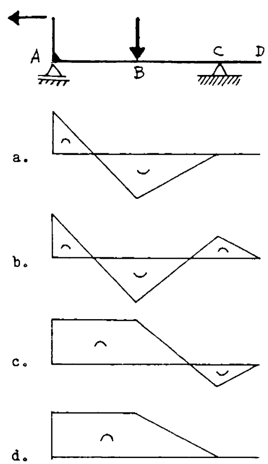
12.91 Given a queen post truss.



Question:
Determine the M and V diagrams for ASB.

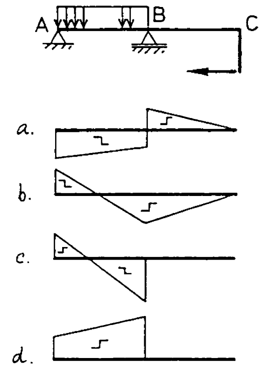
Principle of superposition (Section 12.4)

12.92 Given four bending moment diagrams for beam ABCD.



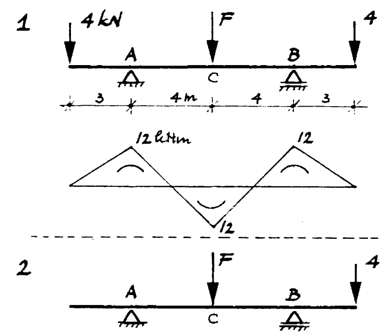
Question:
Which bending moment diagram matches the given load?

12.93 Given four shear force diagrams for beam ABC.



Question:
Which shear force diagram matches the given load?

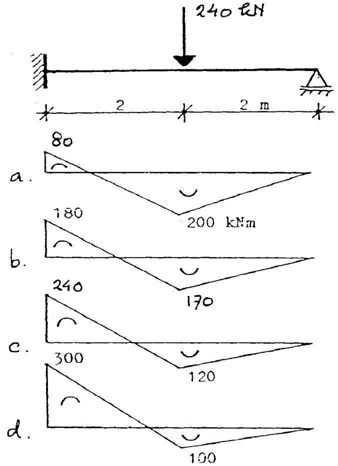
12.94 Given loading case 1 with the associated bending moment diagram and loading case 2 without bending moment diagram. In loading case 2 there is no force on the left-hand overhang.



Question:
Determine the bending moment at C for loading case 2.

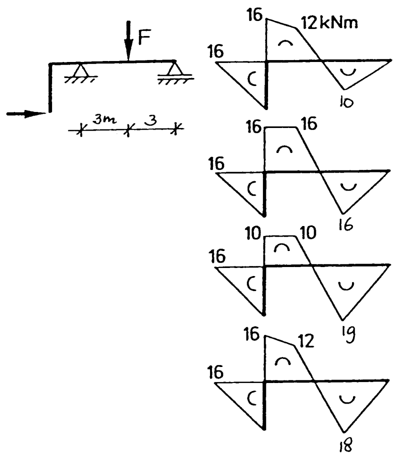
12.95 Given a statically indeterminate beam with four bending moment diagrams.

Question:
Which bending moment diagram(s) may be correct?



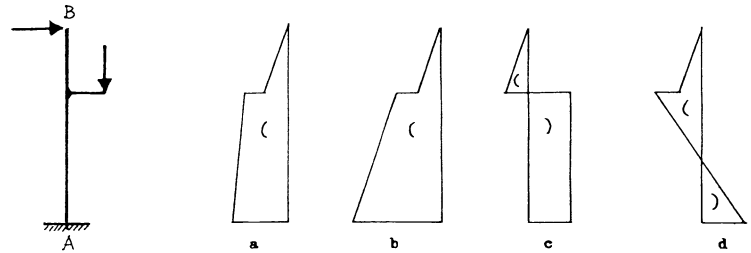
12.96 Four bending moment diagrams are shown for the loaded structure, of which only one is correct.

Question:
Using the correct bending moment diagram determine the magnitude of the force F.



Eccentric axial forces (Section 12.5.5)

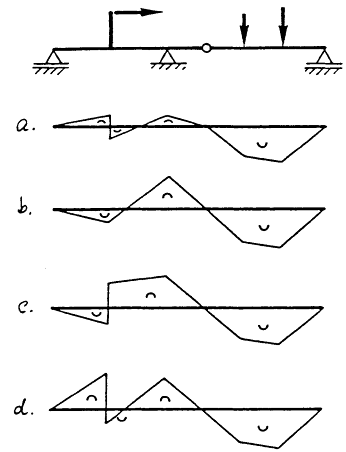
12.97 Given four bending moment diagrams for column AB.



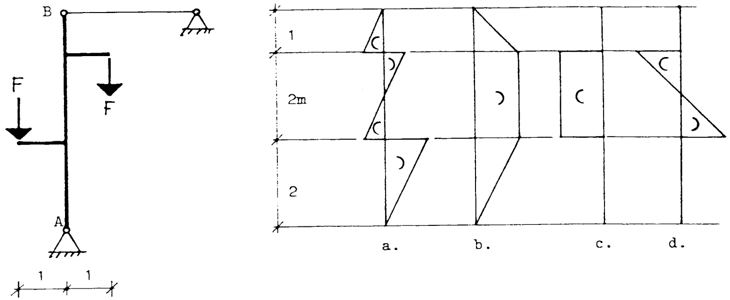
Question:
Which bending moment diagram, with the given load, could be correct?

12.98 Given a loaded beam with four bending moment diagrams.

Question:
Which bending moment diagram has the right shape?

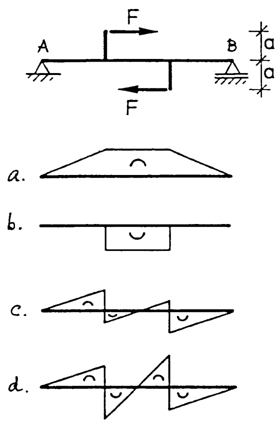


12.99 Given four bending moment diagrams for post AB.



Question:
Which bending moment diagram matches the given load?

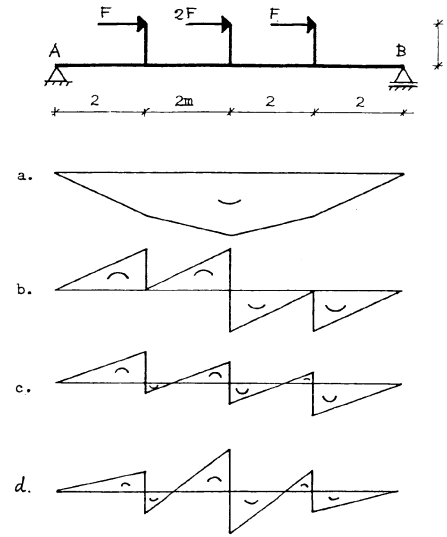
12.100 Given four bending moment diagrams for beam AB.



Question:
Which bending moment diagram matches the given load?

12.101 Given four bending moment diagrams for beam AB.

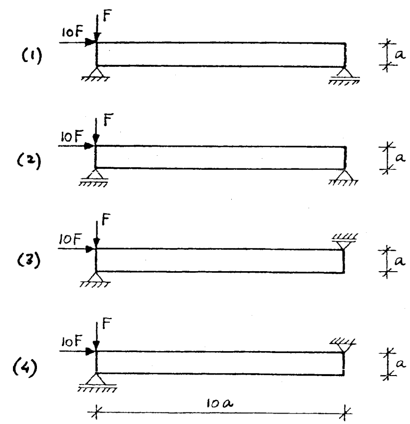
Question:
Which bending moment diagram matches the given load?



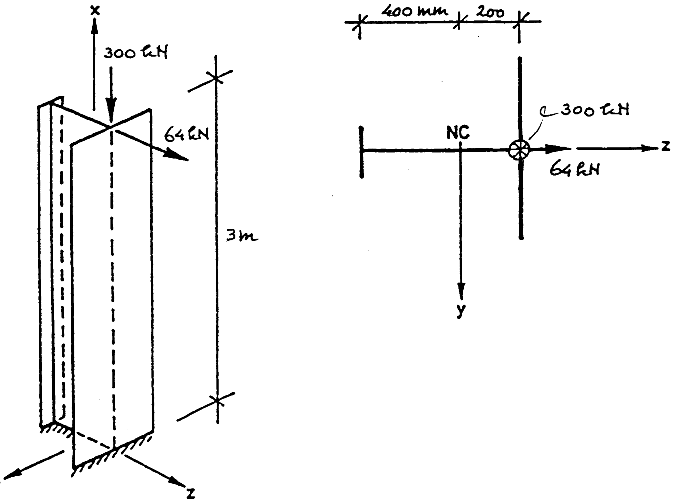
12.102: 1–4 A beam with rectangular cross-section is supported in four different ways. The beam axis is half-way up.

Questions:

a. Indicate how the load is acting on the beam modelled as a line element.
b. Draw the N, V and M diagrams.

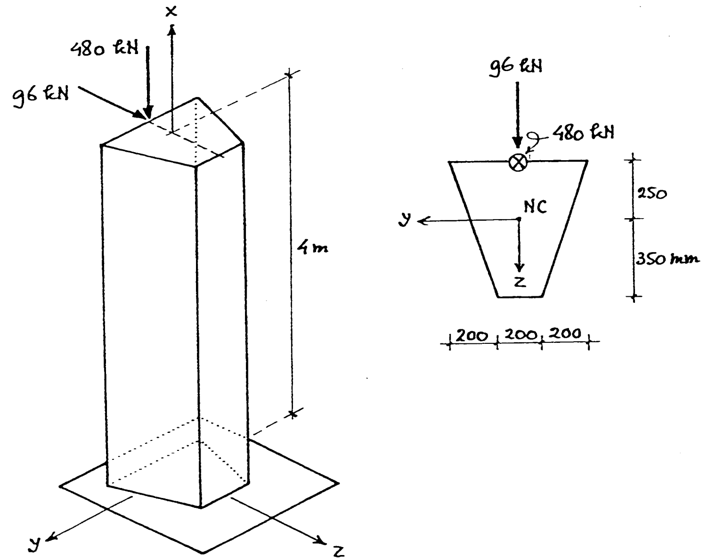


12.103 A fixed column is loaded at its free end as shown. The column axis passes through the normal centre NC and coincides with the x axis shown.



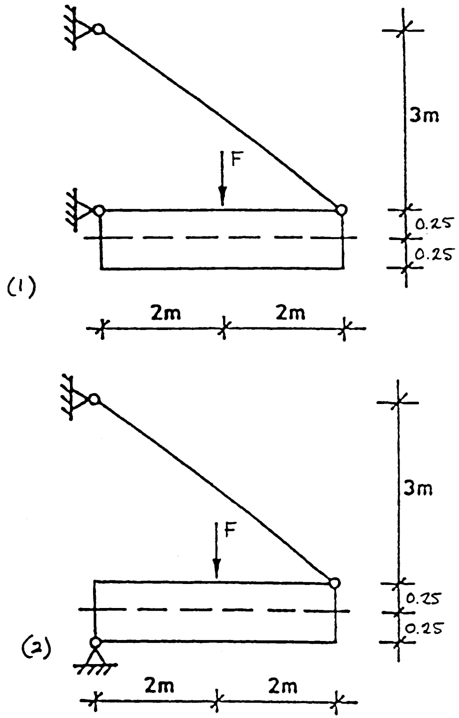
Question:
Draw the N, V and M diagrams for the column modelled as a line element.

12.104 A fixed column fixed is loaded at its free end as shown. The column axis passes through the normal centre NC and coincides with the x axis shown.



Question:
Draw the N, V and M diagrams for the column modelled as a line element.

12.105: 1–2 In the figure, the beam axis is shown by means of a dashed line.



Question:
Schematise the beam as a line element and draw the N, V and M diagrams due to the force $F = 84$ kN.