10 Section Forces

Section forces is the collective name for *interaction forces* or *joining forces* in a member axis. We make a distinction between *normal force*, *shear force*, *bending moment* and *torsional moment*. Section forces always occur in pairs and ensure *force transfer* in a member. This is addressed in further detail in Section 10.1.

The section forces can vary along the member axis. Here they are a function of the x coordinate, chosen along the member axis. Drawing these functions provides a graphic representation of the distribution of the section forces, known as diagrams.

In Section 10.2, we will determine the diagrams for the *normal force* (N), *shear force* (V) and *bending moment* (M) directly from the equilibrium.

For a correct interpretation of the signs in the M and V diagrams, we must always know the coordinate system in which we are working. For manual calculations, one often prefers the use of so-called *deformation symbols*. The deformation symbols are independent of the coordinate system. In Section 10.3, we will introduce the *bending symbol* for bending moments and the *shear symbol* for shear forces.

Section 10.4 summarises the sign conventions for N, V and M diagrams.

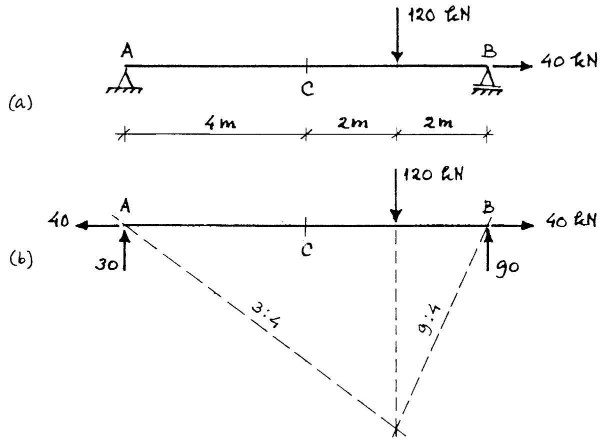

Figure 10.1 (a) A member modelled as a line element, loaded by two forces of 120 kN and 40 kN. (b) The isolated member with its support reactions.

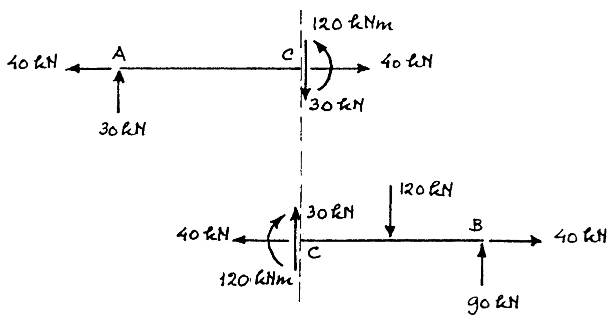

Figure 10.2 The section forces (interaction forces) that the member, modelled as a line, element has to transfer at C.

10.1 Force flow in a member

In mechanics, members in a frame are represented by means of lines. Each one-dimensional *line element* represents a three-dimensional member (see Section 4.3.2). All *member properties* are assigned to this single line. The *force flow* in the member is also assumed to occur along this line, which is known as the *member axis*.

Section forces is the collective name for *interaction forces* or *joining forces* in the member modelled as a line element. They always occur in pairs. An example is provided in Section 10.1.1.

In reality, the force transfer is not concentrated in the member axis, but is distributed over the *member cross-section*, and is the sum of a large number of small interactions between adjacent particles of matter. These interactions are described using the concept *stress*. We look at this in more detail in Section 10.1.2.

In Section 10.1.3, we discuss the *general definition* for the section forces, related to the *stresses in the cross-section*.

The *sign conventions* for section forces are closely related to those for stresses. They are summarised in Section 10.1.4.

10.1.1 Member axis and member cross-section; section forces

In Figure 10.1a, the load on beam AB in the one-dimensional model is transferred to the supports via the *member axis*. Figure 10.1b shows the support reactions. The lines of action of the resultant forces at A and B intersect in the line of action of the force of 120 kN (graphical check of the moment equilibrium for a body subjected to three forces, see Section 3.3.2).

Figure 10.2 shows the *interaction forces* that the member has to transfer at C. After introducing a section at C across the member, the interaction forces are found from the equilibrium of one of the isolated parts, to the right or left of C.

In reality, the member is not one-dimensional, but has cross-sectional dimensions. Figure 10.3 shows an arbitrary section across the member at C. The force transfer is not concentrated in the member axis, but varies over the section as the sum of a large number of very small interactions between adjacent particles of matter. Mathematically, we describe this phenomenon in the section by means of the concept *stress* (see Section 6.5).[1]

The distributions in magnitude and direction of the stresses in the section are as yet unknown. The equilibrium, however, shows that the stress resultant R, regardless of the shape of the section, must be 50 kN, and that the line of action of R must coincide with the line of action of the support reaction at A (see Figure 10.3).

We usually do not give the section across a member an arbitrary shape, but rather choose one that is straight and normal to the member axis, as shown in Figure 10.4. This type of section is called a *normal section* or simply *cross-section*. Hereafter, when we refer to a section, we always mean a normal section.

The intersection of the line of action of the stress resultant R and the cross-sectional plane is known as the *centre of force*.

The intersection of the member axis with the cross-sectional plane is the *normal force centre*, or *normal centre*, of the section.[2] The normal (force) centre is indicated by the two-character symbol NC (see Figure 10.4).

Consistent with the model used for a line element, it is usual to represent

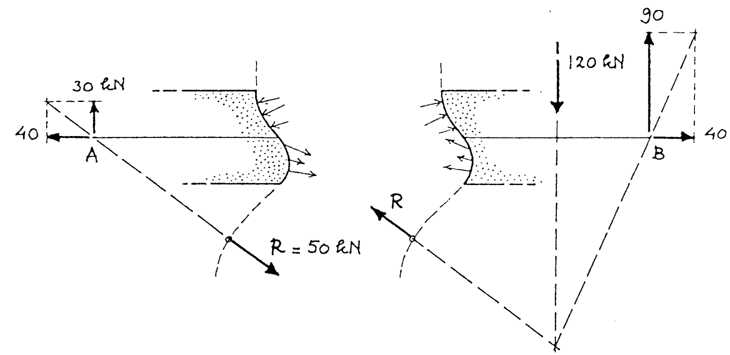

Figure 10.3 An arbitrarily shaped cross-section at C. The force transfer is not concentrated in the member axis but is distributed over the section as the sum of many very small interactions between adjacent particles of matter. The stress resultant R is the resultant force due to the stresses in the section.

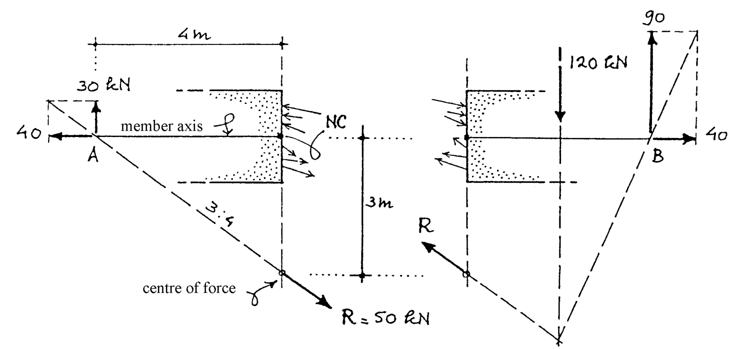

Figure 10.4 It is usual to choose the section across a member as a plane normal to the member axis. The centre of force is the intersection of the line of action of the stress resultant R with the plane of the cross-section.

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1 In Section 10.1.2, the definition of the concept stress, as introduced in Section 6.5, is adapted to describe the interaction between the particles of matter.

2 The member axis is by definition chosen through the normal centre NC of the cross-section. The location of the normal centre is covered in Volume 2 *Stresses, Deformations, Displacements*. In so-called *homogeneous cross-sections* (the whole cross-section consists of the same material) the normal centre coincides with the centroid of the cross-section.

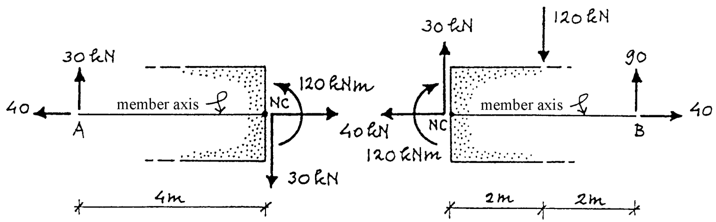


Figure 10.5 Linked up with the modelling as line element, the section forces (interaction forces) are said to act at the normal centre NC, the intersection of the member axis with the cross-sectional plane. Here there are three different section forces: a *normal force* of 40 kN, a *shear force* of 30 kN and a *bending moment* of 120 kNm.

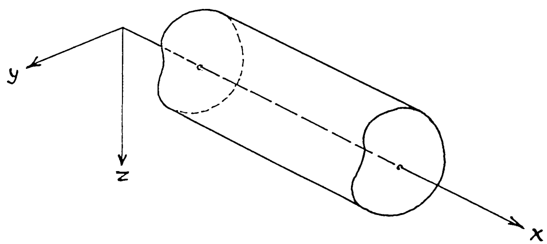


Figure 10.6 The sign of the section forces is related to a (local) coordinate system with the x axis along the member axis and the yz plane parallel to the cross-sections.

the forces in a section as acting in the member axis, or in other words, at the normal centre NC. The vertical component of the stress resultant R at section C is 30 kN and can be shifted directly along its line of action to the member axis. The horizontal component of R is 40 kN and has to be shifted 3 m in section C parallel to its line of action. This gives a moment of (40 kN)(3 m) = 120 kNm. The *section forces* in section C, acting at the member axis, are shown in Figure 10.5.

The section forces on the left- and right-hand sides of the section are equal and opposite. Section forces are *interaction forces* and always occur *in pairs*. You should always keep this in mind, even if you are drawing only one of the member segments to the right or left of the section.

In the case shown in Figure 10.5, we can distinguish between the following three section forces:

- A *normal force*: this is the pair of forces of 40 kN with their lines of action along the member axis; a normal force acts *normal to the cross-sectional plane*.
- A *shear force*: this is the pair of forces of 30 kN in the cross-sectional plane; a shear force acts *transverse to the member axis*.
- A *bending moment*: this is the pair of couples of 120 kNm in a plane normal to the cross-sectional plane.

For *normal force*, *shear force* and *bending moment*[1] we use the symbols N, V and M respectively.

Since section forces are interaction forces, their sign convention is somewhat more complicated than that for a force F or couple T. The sign of the section forces is related to a (*local*) *coordinate system* with the x axis along the member axis and the yz plane parallel to the member cross-sections (see Figure 10.6).

[1] These names used in practice in no sense reflect that we are talking about *interaction forces* (pair of forces).

After applying a section, there are two cross-sectional planes. To distinguish these from one another, we call the sectional plane *positive* where the x axis points outwards, and the sectional plane negative where the x axis points inwards. This is shown in Figure 10.7 where the sectional planes I are positive, and sectional planes II are negative.

More formally, we describe the position of a sectional plane using a so-called *unit normal vector* $\vec{n}$. This is a unit vector (a vector with length 1) pointing outwards from the matter and normal to the sectional plane that is considered. The position of a sectional plane in space is fully determined by the scalar components n_x; n_y; n_z of the unit normal vector $\vec{n}$. Since the cross-section is normal to the x axis as chosen along the member axis, $n_y = n_z = 0$. The sectional plane is now said to be *positive* if the unit normal vector $\vec{n}$ points in the positive x direction ($n_x = +1$), and *negative* if $\vec{n}$ is pointing in the negative x direction ($n_x = -1$). Again, see Figure 10.7.

Figure 10.8 shows the *positive directions* in the given xz coordinate system of the normal force N, the shear force V and the bending moment M. The *sign conventions* are as follows:

- A normal force N is positive when it acts on a positive sectional plane in the positive x direction and on a negative sectional plane in the negative x direction. To simplify: a normal force N is positive as a tensile force and negative as a compressive force. This sign convention has already been used in trusses (see Section 9.3).
- A shear force V is positive when it acts on a positive sectional plane in the positive z direction, and on a negative sectional plane in the negative z direction.
- A bending moment M is positive when it causes tension (tensile stresses) at the positive z side of the x axis, and causes compression (compressive stresses) at the negative z side.

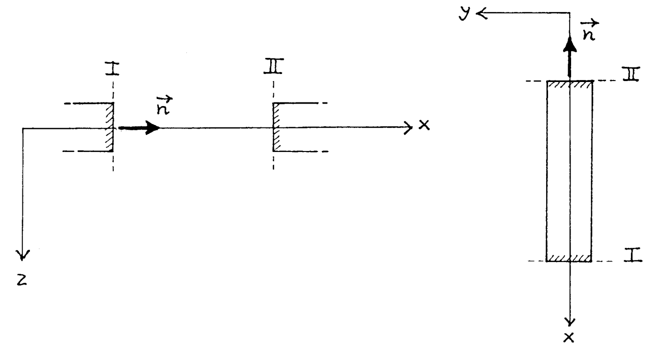


Figure 10.7 The sectional planes I are positive because the x axis points out of the matter and the unit normal vector $\vec{n}$ points in the positive x direction. The sectional planes II are negative because the x axis points into the matter and the unit normal vector $\vec{n}$ points in the negative x direction.

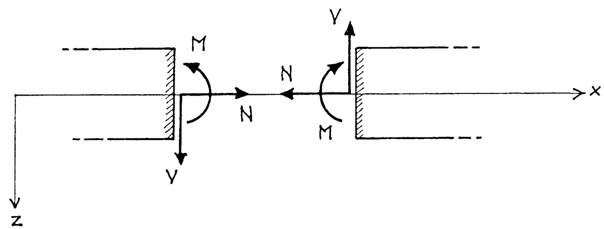


Figure 10.8 The positive directions of the normal force N, shear force V and the bending moment M in an xz coordinate system.

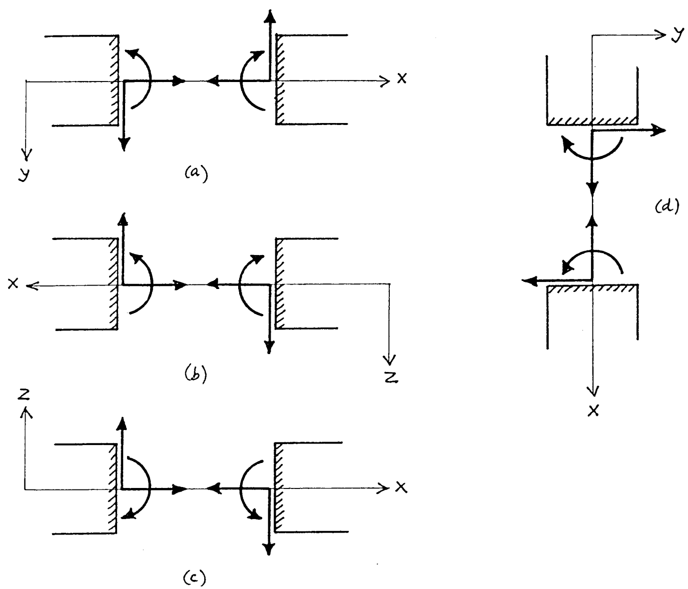


Figure 10.9 The positive directions of the section forces N, V and M in different coordinate systems.

In an xy coordinate system, the positive/negative section forces are defined in the same way. Figure 10.9 shows the positive directions of the section forces in various coordinate systems.[1]

The sign convention given here for the section forces N, V and M is associated with the sign convention for stresses in the cross-section. We look at this in more detail in Section 10.1.2.

10.1.2 Stresses in the cross-section

On a *positive sectional plane*, consider a small area ΔA. Let $\Delta\vec{F}$ be the resultant of all the small forces that are transferred by the matter via that small area. $\Delta\vec{F}$ is built up by the contributions of a large number of interactions between the particles of matter. Figure 10.10a shows the components $\Delta\vec{F}_x$; $\Delta\vec{F}_y$; $\Delta\vec{F}_z$ of the small force $\Delta\vec{F}$.

If ΔA is smaller, so is $\Delta\vec{F}$. It is assumed that the relationship between $\Delta\vec{F}$ and ΔA has a limit when ΔA approaches zero. This limit was defined in Section 6.5 as the stress vector $\vec{p}$:

$$\vec{p} = \lim_{\Delta A \to 0} \frac{\Delta\vec{F}}{\Delta A}.$$

The definition of the stress vector is based on the idealised model of continuous matter. Figure 10.10b shows the components p_x; p_y; p_z of the stress vector $\vec{p}$.

[1] Note: it is wrong to say that a bending moment is positive when the couple acts on the positive sectional plane in accordance with the positive sense of rotation and on the negative sectional plane in accordance with the negative sense of rotation. This is shown in Figures 10.9a and 10.9d.

If we look at the same small area ΔA on the *negative sectional plane*, there is an equal but opposite force, in accordance with the principle of action and reaction. The stress vectors $\vec{p}^{(\mathrm{I})}$ and $\vec{p}^{(\mathrm{II})}$ have the same magnitude at corresponding points ($\Delta A \to 0$) on the positive and negative sectional plane, but have opposite directions (see Figure 10.11):

$$\vec{p}^{(\mathrm{I})} = -\vec{p}^{(\mathrm{II})}.$$

The stress vector is defined in a particular point and for a particular sectional plane. If we want to indicate the *force transfer* (*interaction*) at a point of the cross-section, the stress vector $\vec{p}$ alone is not enough, as we also have to indicate the status of the sectional plane that is considered. This is done by means of the unit normal vector $\vec{n}$ on that plane.

To describe the action of the forces that the matter to the right of the section exerts on the matter to the left, and vice versa, we introduce the following quantities, which are known as *cross-sectional stresses* (see Figure 10.12):

$$\sigma_{xx} = \lim_{\Delta A \to 0} \frac{\Delta F_x}{\Delta A \cdot n_x},$$
$$\sigma_{xy} = \lim_{\Delta A \to 0} \frac{\Delta F_y}{\Delta A \cdot n_x},$$
$$\sigma_{xz} = \lim_{\Delta A \to 0} \frac{\Delta F_z}{\Delta A \cdot n_x}.$$

Here, n_x is the x component of the unit normal vector $\vec{n}$ on the sectional plane that is considered.

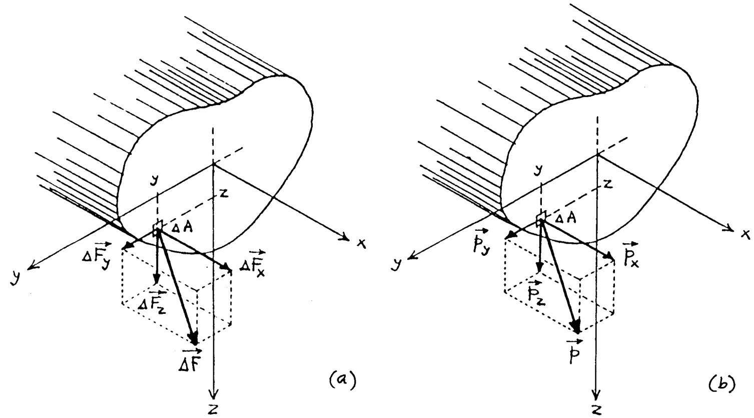


Figure 10.10 (a) The small force $\Delta\vec{F}$ is the resultant of all the small forces acting on a small but finite area ΔA. (b) The stress vector $\vec{p}$ is defined as the limit value of $\Delta\vec{F}/\Delta A$ for $\Delta A \to 0$.

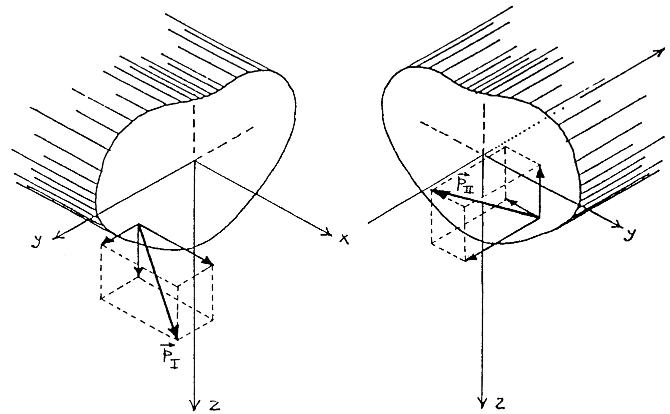


Figure 10.11 The stress vectors $\vec{p}^{(\mathrm{I})}$ and $\vec{p}^{(\mathrm{II})}$ in corresponding points on the positive and negative sectional plane are equal and opposite, so $\vec{p}^{(\mathrm{I})} = -\vec{p}^{(\mathrm{II})}$.

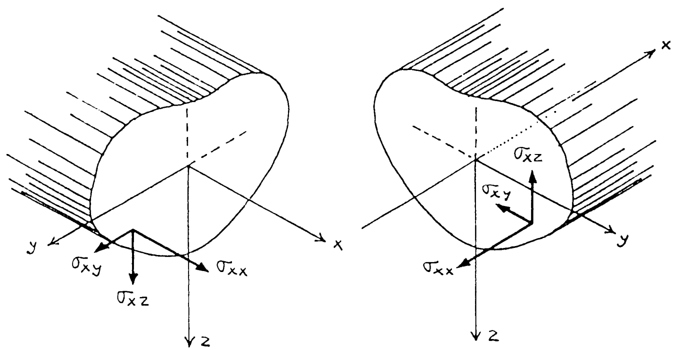


Figure 10.12 The stresses in the cross-section reflect the interaction through the area ΔA ($\Delta A \to 0$), of the right-hand part on the left-hand part, and vice versa. The normal stress σ_{xx} acts normal to the cross-sectional plane; the shear stresses σ_{xy} and σ_{xz} act in cross-sectional plane.

The kernel symbol σ for stress has two sub-indices. The first index relates to the *normal of the plane* on which the stress is acting; the second index relates to the *direction of the stress* (that is, the direction of the corresponding force component on that plane).

If we look at two corresponding equal areas ΔA to the right and to the left of the section, they are subject to two equal and opposite forces ΔF. Since the unit normal vectors also have opposite directions, the limit results for the negative sectional plane are the same as those for the positive sectional plane. The concept stress reflects the *interaction* through the small area ΔA ($\Delta A \to 0$), both for the right-hand part on the left-hand part, and vice versa.

The stress σ_{xx}, acting normal to the cross-sectional plane is known as the *normal stress*. The stresses σ_{xy} and σ_{xz}, that act in the cross-sectional plane are known as *shear stresses*.

The *sign convention for the stresses* results directly from their definition. The normal stress σ_{xx} is positive if n_x and ΔF_x are both positive or are both negative; the normal stress is negative if n_x and ΔF_x have different signs. In the same way, the shear stresses σ_{xy} and σ_{xz} are positive if n_x and ΔF_y, respectively n_x and ΔF_z are both positive or both negative; the shear stresses are negative if n_x and ΔF_y, respectively n_x and ΔF_z, have different signs.

The *sign convention* can be summarised as follows:

- A stress is positive when it acts on a positive plane in the positive direction or on a negative plane in the negative direction.
- A stress is negative when it acts on a positive plane in the negative direction or on a negative plane in the positive direction.

For more general cases, the *stress definition* can be summarised in short as follows:

$$\sigma_{ij} = \lim_{\Delta A \to 0} \frac{\Delta F_j}{\Delta A \cdot n_i} \quad (i, j = x, y, z)$$

in which both i and j can be replaced by x, y or z.[1]

Figure 10.13 shows the *positive stresses* acting on the sides of an (infinitesimally) small rectangular block. The block is bounded by six planes, of which three are positive and three are negative.

σ_{ij} is the stress

- on a small area with the unit normal vector parallel to the i axis (1st index),
- due to a force component parallel to the j axis (2nd index).

The stress σ_{ij} is a *normal stress* when the indices are the same ($i = j$) and a *shear stress* when the indices are different ($i \neq j$).

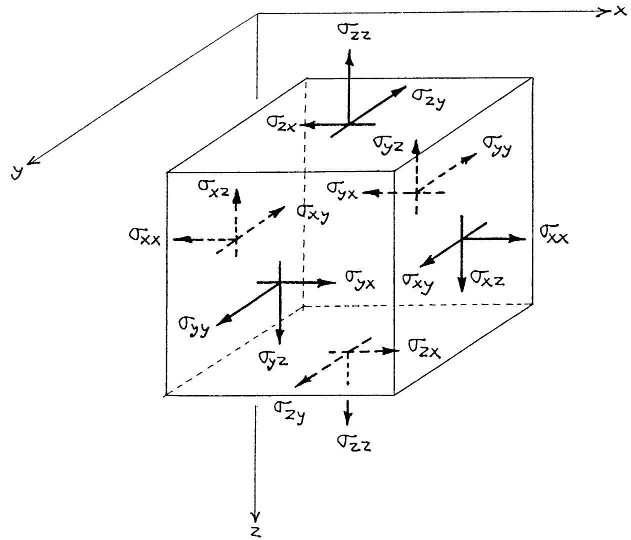


Figure 10.13 Positive stresses on the sides of a rectangular block. The kernel symbol σ for stress has two indices. The first index relates to the normal of the plane on which the stress is acting; the second index relates to the direction of the stress. The stress is a normal stress when both indices are equal, and a shear stress when both indices are different.

10.1.3 General definition of section forces

In a member cross-section, there are only normal stresses σ_{xx} and shear stresses σ_{xy} and σ_{xz} (see Figure 10.12). These stresses are as yet unknown functions of y and z, so that

$$\sigma_{xx} = \sigma_{xx}(y, z), \quad \sigma_{xy} = \sigma_{xy}(y, z) \quad \text{and} \quad \sigma_{xz} = \sigma_{xz}(y, z).$$

1 The stresses σ_{ij} $(i, j = x, y, z)$ are the components of a quantity (the so-called *stress tensor*) that in a certain point for each arbitrary plane links the components of the *stress vector* $\vec{p}$ and the components of the *unit normal vector* $\vec{n}$.

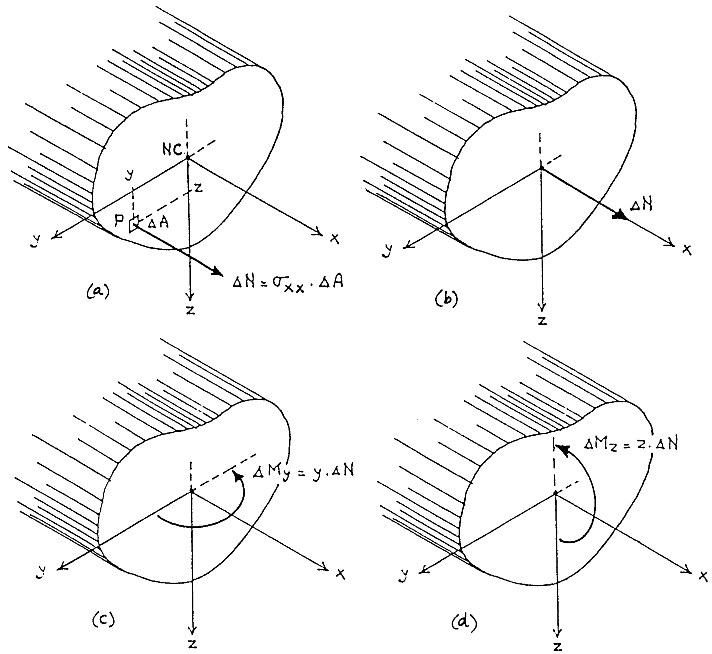


Figure 10.14 (a) The resultant of the normal stresses on a small area ΔA around a point P is a small force ΔN. This force in P is statically equivalent to (b) a small force ΔN in the normal force centre NC (the intersection of the member axis with the cross-sectional plane), together with (c) a small moment ΔM_y in the xy plane and (d) a small moment ΔM_z in the xz plane.

The resultant of the *normal stresses* on a small area ΔA around a point P is a small force ΔN:

$$\Delta N = \sigma_{xx} \Delta A.$$

This small force ΔN in P is statically equivalent to a small force ΔN in the member axis (the origin of the yz coordinate system), together with two small moments ΔM_y and ΔM_z, acting in the xy plane and the xz plane respectively (see Figure 10.14):

$$\begin{aligned} \Delta M_y &= y \cdot \Delta N = y \cdot \sigma_{xx} \Delta A, \\ \Delta M_z &= z \cdot \Delta N = z \cdot \sigma_{xx} \Delta A. \end{aligned}$$

If we sum up the contributions of all the forces ΔN for the entire cross-section, this gives:

$$\begin{aligned} N &= \int_A \sigma_{xx} \, \mathrm{d}A, \\ M_y &= \int_A y\sigma_{xx} \, \mathrm{d}A, \\ M_z &= \int_A z\sigma_{xx} \, \mathrm{d}A. \end{aligned}$$

- N is the resulting force (or rather: the resulting pair of forces) due to the normal stresses in the cross-section, and is by definition known as *normal force* when it acts at the *normal centre* NC of the cross-section (the intersection of the member axis with the cross-sectional plane).
- M_y is a moment (or rather: a pair of moments) that acts in the xy plane. M_y is known as the *bending moment in the xy plane*.
- M_z is a moment (or rather: a pair of moments) that acts in the xz plane. M_z is known as the *bending moment in the xz plane*.

Note that indices y and z in M_y and M_z also occur under the integral symbol. This makes the formulas easy to memorise. In addition, y and z reoccur in the indication of the planes in which the bending moments act: M_y in the $x\underline{y}$ plane and M_z in the $x\underline{z}$ plane.

The *normal force* N is positive as a tensile force and negative as a compressive force.

The *bending moments* M_y and M_z are positive when a tensile stress ($\sigma_{xx} > 0$) on a small elemental area ΔA for $y > 0$ makes a positive contribution to M_y or for $z > 0$ makes a positive contribution to M_z.

Figure 10.15 shows the positive directions of N, M_y and M_z. These are the section forces that are transferred via *normal stresses* in the member cross-section.

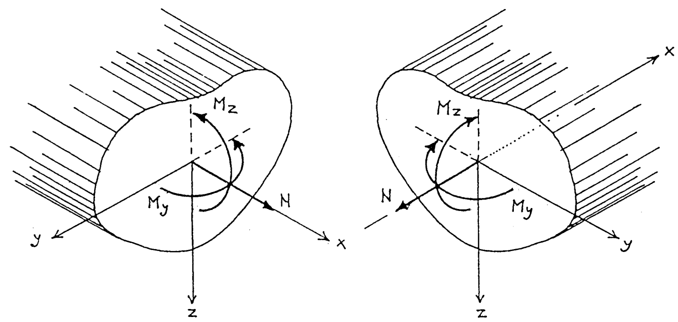


Figure 10.15 The positive directions of the section forces that are transferred via normal stresses. N is the normal force, M_y is the bending moment in the xy plane and M_z is the bending moment in the xz plane.

The resultant of the *shear stresses* on a small area ΔA around a point P is a small shear force ΔV, with components ΔV_y and ΔV_z:

$$\Delta V_y = \sigma_{xy} \Delta A,$$
$$\Delta V_z = \sigma_{xz} \Delta A.$$

When assuming these small forces act in the member axis (by shifting them to the origin of the yz coordinate system), we have to add a small moment ΔM_t *in the cross-sectional plane* (see Figure 10.16):

$$\Delta M_\mathrm{t} = y \cdot \Delta V_z - z \cdot \Delta V_y = (y\sigma_{xz} - z\sigma_{xy})\Delta A.$$

Summation of the contributions of all the forces ΔV_y and ΔV_z for the entire cross-section results in

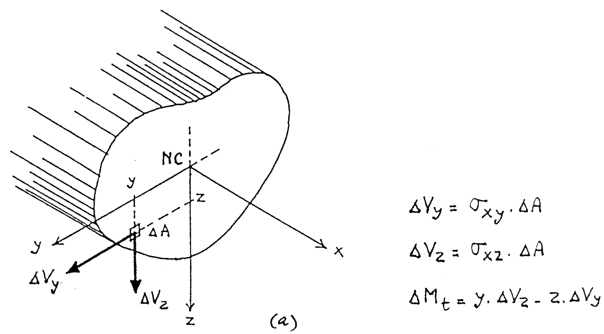


Figure 10.16 (a) The resultant of the shear stresses on a small area ΔA around a point P is a small shear force, with components ΔV_y and ΔV_z.

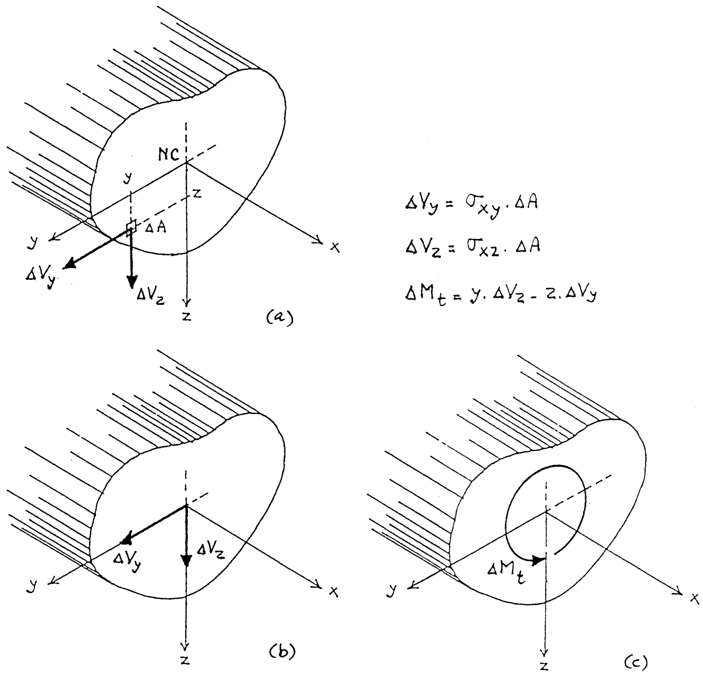


Figure 10.16 (a) The resultant of the shear stresses on a small area ΔA around a point P is a small shear force, with components ΔV_y and ΔV_z. The forces ΔV_y and ΔV_z in P are statically equivalent to (b) the small forces ΔV_y and ΔV_z in the normal force centre NC (the intersection of the member axis with the cross-sectional plane), together with (c) a small moment ΔM_t in the cross-sectional plane.

$$V_y = \int_A \sigma_{xy}\,\mathrm{d}A,$$

$$V_z = \int_A \sigma_{xz}\,\mathrm{d}A,$$

$$M_\text{t} = \int_A (y\sigma_{xz} - z\sigma_{xy})\,\mathrm{d}A.$$

- V_y and V_z are the components of the *shear force* V, that are the resultant forces (or rather: pair of forces) due to the shear stresses in the cross-section.
- M_t is a moment (or rather: pair of moments) that acts in the cross-sectional plane (the yz plane). M_t is known as a *torsional moment.*

The components V_y and V_z of the *shear force* V are (in accordance with the sign convention for the shear stresses σ_{xy} and σ_{xz}) positive when they act on a positive plane in the positive coordinate direction and on a negative plane in the negative coordinate direction.

The *torsional moment* M_t is positive when the couple acts on the positive sectional plane in the positive sense of rotation about the x axis and when the couple acts on the negative plane in the negative direction of rotation.

Figure 10.17 shows the positive directions of V_y, V_z and M_t. These are the section forces that are transferred in the cross-section via *shear stresses.*

Note: The expression given for the torsional moment is not universally applicable. Sometimes, to determine the torsional moment, we do not move the lines of action of the shear forces V_y and V_z to the *normal centre* NC (or the member axis, where we selected the origin of the yz coordinate system), but to another point in the cross-section that we refer to as the *shear force* *centre*, or *shear centre*, SC.[1] With (y_{SC}, z_{SC}) as the coordinates of the shear force centre, the expression for the torsional moment in that case is

$$M_t = \int_A [(y - y_{SC})\sigma_{xz} - (z - z_{SC})\sigma_{xy}]\,dA.$$

The expression given earlier,

$$M_t = \int_A (y\sigma_{xz} - z\sigma_{xy})\,dA,$$

applies only when $y_{SC} = 0$; $z_{SC} = 0$, or in other words, when the shear centre SC coincides with the normal centre NC. This occurs for cross-sections that have *rotational symmetry*.

A cross-section is said to have rotational symmetry when we rotate the cross-section n times $(n > 1)$ with an angle of $\alpha = 360°/n$ about the member axis, and the rotated cross-section coincides with the original, un-rotated cross-section.

Figure 10.18 gives a number of examples of cross-sections with rotational symmetry; the angle of rotation α is given for each of the cross-sectional shapes.

10.1.4 Summary of the sign conventions for stresses and section forces

We use a (*local*) *coordinate system* with the x axis along the member axis.

A *cross-section* is straight and normal to the member axis. The location of a cross-section is determined by the x coordinate.

[1] Volume 2, *Stresses, Deformations, Displacements*, addresses the location of the shear force centre SC in more detail.

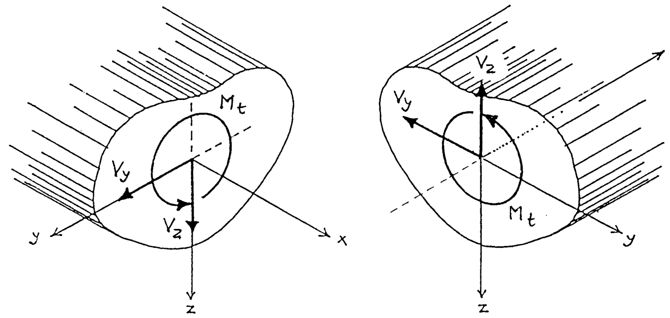


Figure 10.17 The positive directions of the section forces that are transferred via shear stresses. V_y and V_z are the components of the shear force V in respectively the y and z direction. M_t is the torsional moment and acts in the plane of the cross-section.

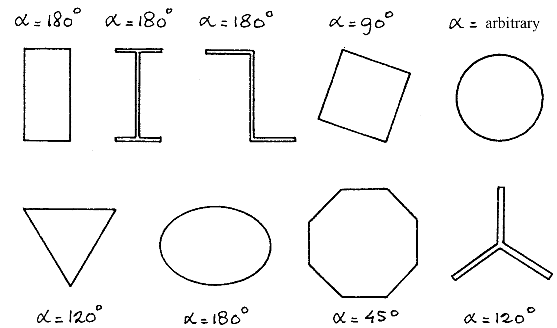


Figure 10.18 Examples of cross-sections with rotational symmetry. The angle of rotation α is mentioned for each cross-sectional shape.

The *unit normal vector* is a unit vector pointed outwards from matter, and normal to the sectional plane that is considered.

A *sectional plane* is
- positive when the unit normal vector is pointing in the positive coordinate direction;
- negative when the unit normal vector is pointing in the negative coordinate direction.

This can also be formulated as follows, without the unit normal vector.

A *sectional plane* is
- positive when the coordinate axis points out of the matter;
- negative when the coordinate axis points into the matter.

A *stress* is
- positive when it acts on a positive plane in the positive coordinate direction or on a negative plane in the negative coordinate direction.
- negative when it acts on a positive plane in the coordinate negative direction or on a negative plane in the positive coordinate direction.

In general, stress σ_{ij} acts
- on a plane with the unit normal vector parallel to the i axis (1st index),
- due to a force component, parallel to the j axis (2nd index).

The stress σ_{ij} is a *normal stress* when the indices are the same ($i = j$) and a *shear stress* when the indices are different ($i \neq j$).

The section forces transferred by *normal stresses* are
- the *normal force* N;
- the *bending moment* M_y, acting in the xy plane;
- the *bending moment* M_z, acting in the xz plane.

The section forces transferred by *shear stresses* are
- the *shear force* V_y in y direction;
- the *shear force* V_z in z direction;
- the *torsional moment* M_{t}, acting in the yz plane.

The following sign conventions apply for *section forces*:

- A *normal force* N is positive when it acts on a positive cross-sectional plane in the positive x direction. In other words, a normal force N is positive as a tensile force and negative as a compressive force.
- A *shear force* V_y (V_z) is positive when it acts on a positive cross-sectional plane in the positive y direction (z direction) and on a negative cross-sectional plane in the negative y direction (z direction).
- A *bending moment* M_y (M_z) is positive when it causes tensile stresses at the positive y side (z side) of the x axis and compressive stresses at the negative y side (z side).
- A *torsional moment* M_t is positive when the couple on the positive cross-sectional plane acts in accordance with the positive direction of rotation about the x axis and the couple on the negative cross-sectional plane acts in accordance with the negative direction of rotation.

10.2 Diagrams for the normal force, shear force and bending moment

The section forces in a member are in general not constant, but may vary along the member axis. They are then a function of the x coordinate chosen along the member axis. By drawing these functions, we get a graphical representation of the distribution of the section forces. These types of *diagrams* are extremely useful to see at a glance where the section forces change sign (direction) and where they are at largest.

In this section, we cover examples of diagrams for the *normal force*, *shear force* and *bending moment*. Section 10.2.1 covers members subject to concentrated forces and couples, while Sections 10.2.2 and 10.2.3 look at members subject to a uniformly distributed load.

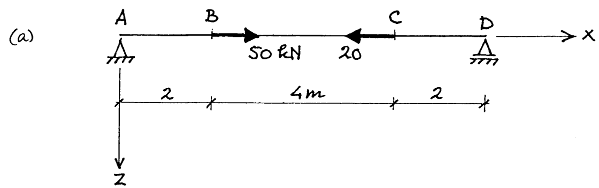


Figure 10.19 (a) A simply supported member loaded by two forces of which the lines of action coincide with the member axis.

10.2.1 Members subject to concentrated forces/couples

We look at three examples:

1. a simply supported member with forces in the direction of the member axis;
2. a simply supported member with forces normal to the member axis;
3. a simply supported member subject to a couple.

Example 1
The simply supported member AD in Figure 10.19a is loaded at B and C by two forces of respectively 50 and 20 kN, of which the lines of action coincide with the member axis.

Question:
Determine the diagrams for the section forces.

Solution:
In Figure 10.19b, the member has been isolated from its supports and the support reactions are shown; the vertical support reactions are zero.

The interaction forces in a section (the section forces) can be determined from the equilibrium of the isolated member segments to the left or to the right of the section. Figure 10.19c shows the member segment to the left of a section located between A and B. In the section, both segments are rigidly joined. The section must therefore be able to transfer a normal force N, shear force V and bending moment M. In Figure 10.19c, the unknown section forces are shown with their positive directions in the given xz axis system.[1]

Actually, the shear force and the bending moment in this xz coordinate system should be shown as respectively V_z and M_z. In obvious situations, the indices are generally omitted to simplify the writing.

---

[1] Remember that you should always include quantities shown as symbols to which a sign is linked positively in the calculation.

From the force and moment equilibrium of the segment to the left of the section it follows that

$$\sum F_x = -(30\text{ kN}) + N = 0 \Rightarrow N = +30\text{ kN},$$

$$\sum F_z = 0 \Rightarrow V = 0,$$

$$\sum T_y|\text{section} = 0 \Rightarrow M = 0.$$

The normal force N is a tensile force of 30 kN, while the shear force V and the bending moment M are zero. These values are independent of the location of the section between A and B and therefore apply for $(0\text{ m}) \le x < (2\text{ m})$.

The shear force and bending moment are not only zero in AB, but also in the rest of the member. This follows from equilibrium of each member segment to the left or right of a (arbitrarily chosen) section. For this reason, we will look only at the distribution of the normal force.

Figure 10.19d shows the isolated member segment to the left of a section between B and C. The equation for the force equilibrium in the x direction now also includes the force of 50 kN at B:

$$\sum F_x = -(30\text{ kN}) + (50\text{ kN}) + N = 0 \Rightarrow N = -20\text{ kN}.$$

This result, a compressive force of 20 kN, is independent of the location of the section between B and C and therefore applies for $(2\text{ m}) < x \le (6\text{ m})$.

Of course, instead of the equilibrium for the part to the left of the section, we can also determine the equilibrium for the part to the right of the section (see Figure 10.19e):

$$\sum F_x = -N - (20\text{ kN}) = 0 \Rightarrow N = -20\text{ kN}.$$

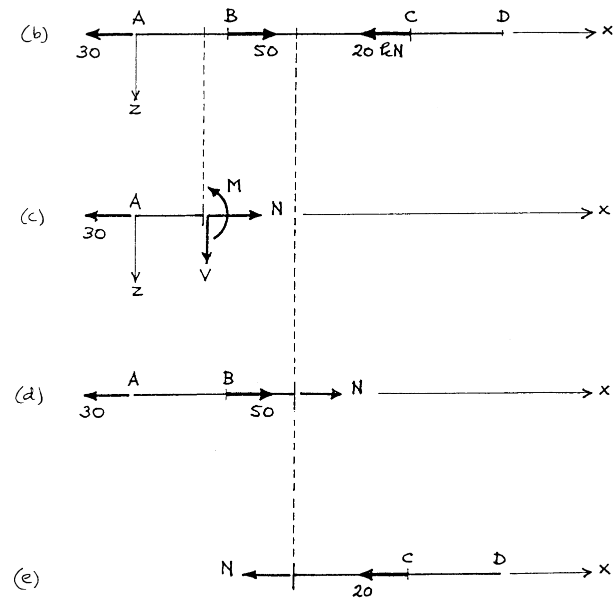


Figure 10.19 (b) The isolated member with its support reactions. (c) The isolated part of the member to the left of a section between A and B. The section can transfer a normal force N, shear force V and bending moment M. The unknown section forces are shown in accordance with their positive directions in the coordinate system. (d) The isolated part of the member to the left of a section between B and C. In the section, only the unknown normal force N is shown as it was determined earlier that the shear force V and the bending moment M are zero throughout the member. (e) The isolated part of the member to the right of a section between B and C, with the unknown normal force N.

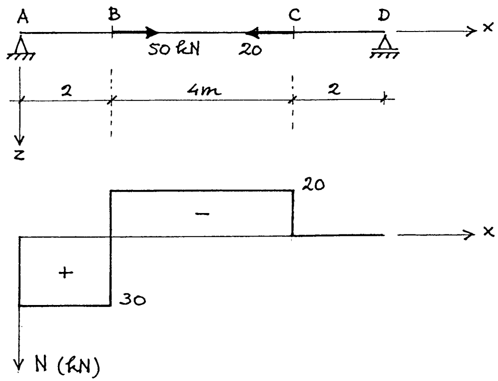


Figure 10.20 The normal force diagram (N diagram) for the simply supported member loaded by two forces of which the lines of action coincide with the member axis. Step changes occur in the N diagram at the location of the point loads at B and C.

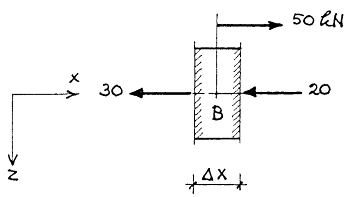


Figure 10.21 A step change in the N diagram can be found from the equilibrium of a small member segment. In this way, the normal forces on both sectional planes of a small member segment are in equilibrium with the load of 50 kN (shown eccentrically for the sake of clarity).

Note that the positive direction of N on a cross-sectional plane is by definition always that of a tensile force.

For a section between C and D, the equilibrium of the part to the right of the section gives

$$N = 0.$$

To summarise, for normal force N applies:

$$\begin{aligned} N &= +30\,\text{kN} \quad \text{for} \quad (0\text{ m}) \leq x < (2\text{ m}),\\ N &= -20\,\text{kN} \quad \text{for} \quad (2\text{ m}) < x < (6\text{ m}),\\ N &= 0 \qquad\qquad \text{for} \quad (6\text{ m}) < x \leq (8\text{ m}). \end{aligned}$$

Figure 10.20 shows the distribution of the normal force N graphically in a diagram. This is called the *normal force diagram*, or *N diagram*. Positive values of N (tensile forces) are plotted at the positive side of the z axis and negative values (compressive forces) are plotted at the negative side of the z axis. We usually place the sign of N ("+" for tension and "−" for compression) *within the diagram* and write down the relevant values *without a sign*.

At $x = 2$ m and $x = 6$ m there is a *step change* in the N diagram equal to the forces acting there. In these sections, the value of N is undetermined. This is a result of modelling the load into concentrated forces (acting in a particular point).

The *step change in the normal force diagram* can be found from the equilibrium of a small member segment with length Δx ($\Delta x \to 0$), at the point load. Figure 10.21 shows "*joint*" B between the member segments AB and

BC. From the *N diagram* we can read off that there is a tensile force of 30 kN directly to the left of B and a compressive force of 20 kN directly to the right of B. Both forces are in equilibrium with the 50 kN load (which is shown eccentrically for clarity).

Example 2

The simply supported member AD in Figure 10.22a is loaded at B normal to the member axis by a force of 60 kN.

Question:
Determine the distribution of the section forces.

Solution:
The units used are m and kN. To simplify the writing, the units have been omitted from the calculation.

In Figure 10.22b, the member has been isolated and the support reactions are shown; the horizontal support reaction at A is zero.

Figure 10.22c shows the member segment to the left of a section between A and B, with all the forces acting on it. The section forces N (normal force), V (shear force) and M (bending moment) follow from the equilibrium.[1]

For a length x of the isolated member segment it holds that

$$\sum F_x = 0 \Rightarrow N = 0,$$

$$\sum F_z = -45 + V = 0 \Rightarrow V = +45 \text{ kN},$$

$$\sum T_y|\text{section} = -45x + M = 0 \Rightarrow M = 45x \text{ kNm}.$$

[1] The sub-index z is again omitted from the symbols for shear force (V_z) and bending moment (M_z); see also Example 1.

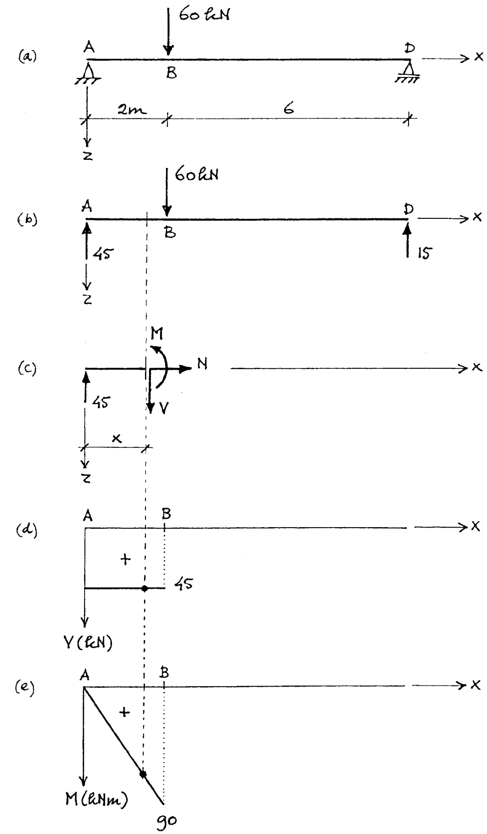


Figure 10.22 (a) A simply supported member that is loaded in B normal to the member axis by a force of 60 kN. (b) The isolated member with its support reactions. (c) The isolated part of the member to the left of a section between A and B. The section can transfer a normal force N, shear force V and a bending moment M. The unknown section forces are shown in accordance with their positive directions in the coordinate system. (d) The shear force diagram (V diagram) for AB. (e) The moment diagram (M diagram) for AB.

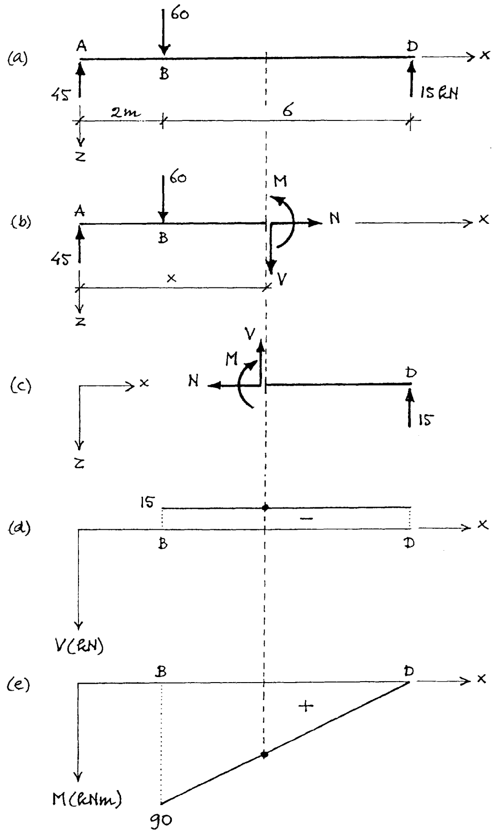


Figure 10.23 (a) The isolated member with its support reactions. (b) The isolated part of the member to the left of a section between B and D. The section can transfer a normal force N, shear force V and a bending moment M. The unknown shear forces are shown in accordance with their positive directions in the coordinate system. (c) The isolated part of the member to the right of a section between B and D. (d) The shear force diagram (V diagram) for BD. (e) The moment diagram (M diagram) for BD.

The normal force N is not only zero in AB, but also in the rest of the member. This follows from the force equilibrium in the x direction of each member segment to the left or right of a (arbitrarily chosen) section. We will therefore only look at the shear force and the bending moment.

The shear force is constant between A and B: $V = +45$ kN. The bending moment M varies linearly, from 0 at A ($x = 0$ m) to $+90$ kNm at B ($x = 2$ m).

Figures 10.22d and 10.22e show the variation for AB of the shear force and the bending moment with a so-called *shear force diagram* (*V diagram*), respectively a *bending moment diagram* (*M diagram*).

Positive values of V and M are plotted at the positive side of the z axis, and negative values are plotted at the negative side. The sign is shown *within the diagram*; relevant values are written down *without a sign*.

In Figure 10.23b, the member segment to the left of a section located between B and D has been isolated. The equations for the force equilibrium in the z direction and the moment equilibrium now includes the load of 60 kN:

$$\sum F_z = -45 + 60 + V = 0 \rightarrow V = -15 \text{ kN},$$

$$\sum T_y|\text{section} = -45x + 60 \times (x - 2) + M = 0$$

$$\Rightarrow M = (-15x + 120) \text{ kNm}.$$

These values can also be found from the equilibrium of the member segment to the right of the section, as shown in Figure 10.23c. It should be noted that the sectional plane in the coordinate system shown is negative, and that the positive directions of N, V and M are therefore opposite to those on a positive sectional plane.

$$\sum F_z = -V - 15 = 0 \Rightarrow V = -15 \text{ kN},$$

$$\sum T_y|\text{section} = 15 \times (8 - x) - M = 0 \Rightarrow M = (-15x + 120) \text{ kNm}.$$

The shear force between B and D is constant: $V = -15$ kN. The bending moment M decreases linearly, from +90 kNm at B ($x = 2$ m) to 0 at D ($x = 8$ m).

Figures 10.23d and 10.23e show the distribution for BD of respectively the shear force V and the bending moment M.

The *shear force diagram* (V diagram) and *bending moment diagram* (M diagram) for the entire member AD are shown in Figure 10.24.

At B, the point of application of the concentrated force of 60 kN, there is an *abrupt change in slope of the bending moment diagram*. Here the bending moment is at its largest.

The shear force in B is undetermined; this is the result of modelling the load as a point load. This finds expression in the shear force diagram as a *step change*: the shear force is +45 kN directly to the left of B and −15 kN directly to the right of B. The magnitude of the step change equals the magnitude of the point load at B.

The *step change in the shear force diagram* can be found from the force equilibrium in z direction of a small member segment at B, with length Δx ($\Delta x \to 0$) (see Figure 10.25). The 60 kN point load is kept in equilibrium by both shear forces in the sectional planes (the bending moments are not shown).

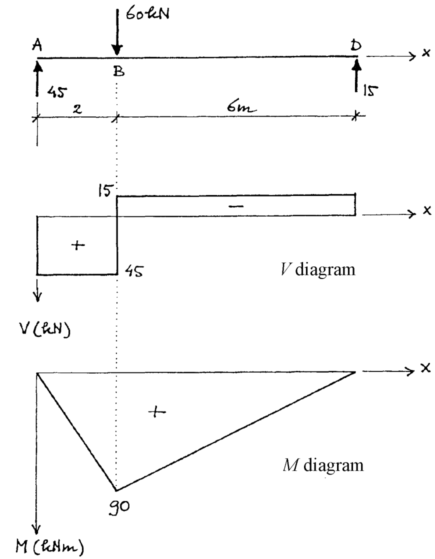


Figure 10.24 A simply supported member that is loaded at B normal to the member axis by a force of 60 kN, with its shear force diagram (V diagram) and bending moment diagram (M diagram). A step change occurs at the location of the point load in the V diagram and an abrupt change in slope occurs in the M diagram.

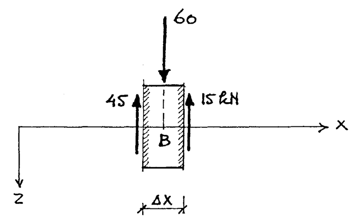


Figure 10.25 A step change in the V diagram can be found from the equilibrium of a small member segment. In this way, the shear forces on both sectional planes at B are in equilibrium with the load of 60 kN.

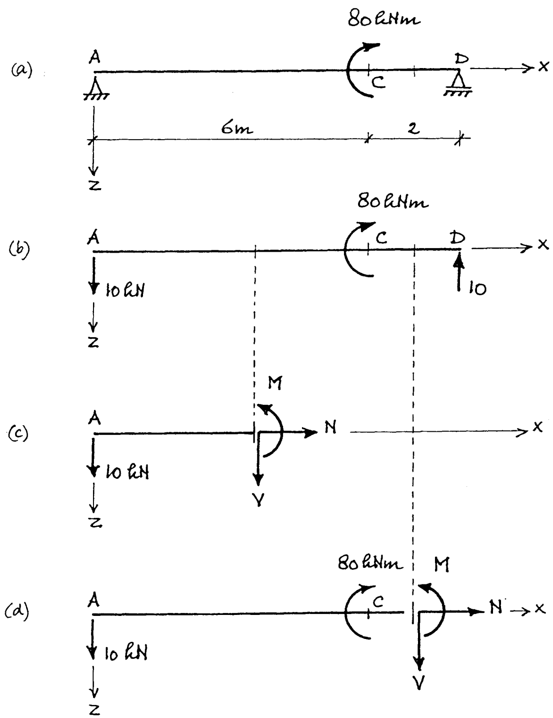


Figure 10.26 (a) A simply supported member AD, which is loaded in C by a couple of 80 kNm. (b) The isolated member with its support reactions. (c) The isolated part of the member to the left of a section between A and C. The section can transfer a normal force N, shear force V and a bending moment M. The unknown shear forces are shown in accordance with their positive directions in the coordinate system. (d) The isolated part of the member to the left of a section between C and D.

Example 3

The simply supported member in Figure 10.26a is loaded at C by a couple of 80 kNm.

Question:
Determine the distribution of the section forces.

Solution:
The units used are m and kN.

In Figure 10.26b, the member has been isolated and the support reactions are shown.

Figure 10.26c shows the isolated member segment to the left of a section between A and C, with all the forces acting on it. From the equilibrium we find:

$$\sum F_x = N = 0 \quad \Rightarrow N = 0,$$

$$\sum F_z = 10 + V = 0 \quad \Rightarrow V = -10 \text{ kN},$$

$$\sum T_y|\text{section} = 10x + M = 0 \quad \Rightarrow M = -10x \text{ kNm}.$$

Figure 10.26d shows the member segment to the left of a section between C and D, with all the forces acting on it. The equation for the moment equilibrium now includes the load from the couple of 80 kNm:

$$\sum F_x = N = 0 \quad \Rightarrow N = 0,$$

$$\sum F_z = 10 + V = 0 \quad \Rightarrow V = -10 \text{ kN},$$

$$\sum T_y|\text{section} = 10x - 80 + M = 0 \quad \Rightarrow M = (-10x + 80) \text{ kNm}.$$

Figures 10.26e and 10.26f show the *shear force diagram* (V diagram) and the *bending moment diagram* (M diagram) for member AD. Since the normal force is zero everywhere, the normal force diagram has been omitted. The shear force is constant across the entire length of the member: $V = -10$ kN. The bending moment varies linearly, from 0 at A ($x = 0$ m) to -60 kNm directly to the left of C ($x = 6$ m) and from $+20$ kNm directly to the right of C ($x = 6$ m) to 0 at D ($x = 8$ m).

In C, where the couple acts, the bending moment is undetermined. This is a result of modelling the load as a couple that is concentrated in a single point. This finds expression in the bending moment diagram as a *step change* equal to the magnitude of the couple.

The *step change in the bending moment diagram* can be found from the moment equilibrium of a small member segment at C, with a length of Δx ($\Delta x \to 0$) (see Figure 10.27; the shear forces are not shown). The bending moments on both sectional planes are in equilibrium with the couple of 80 kNm.

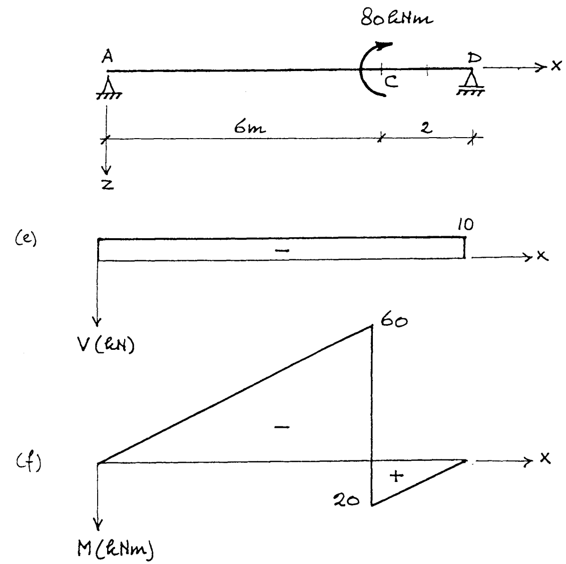


Figure 10.26 (e) The shear force diagram (V diagram) for AD. (f) The bending moment diagram (M diagram) for AD. The bending moment varies linearly and makes a step change at C, the point of application of the couple. The magnitude of the step change is equal to the magnitude of the couple.

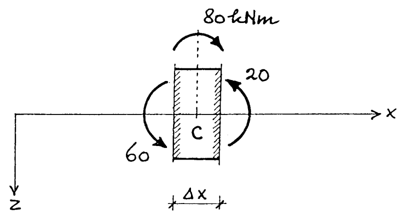


Figure 10.27 A step change in the M diagram can be found from the equilibrium of a small member segment. In this way, the bending moments on both sectional planes of a small member segment are in equilibrium with the couple of 80 kNm.

10.2.2 Members with a uniformly distributed load in the direction of the member axis

In straight members a (distributed) longitudinal load does not produce bending moments or shear forces. In these cases, there are only normal forces. The variation of the normal force is elaborated for two examples:

1. a column subject to its dead weight;
2. a simply supported member subject to a uniformly distributed axial load over three-quarters of its length.

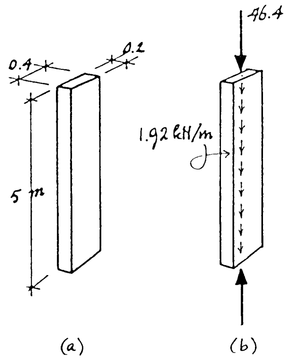


Figure 10.28 (a) Dimensions of a concrete column of the building used in Section 6.4. (b) The column is loaded on top by a force of 46.4 kN. The dead weight is to be considered as a uniformly distributed (line)load along the member axis of 1.92 kN/m.

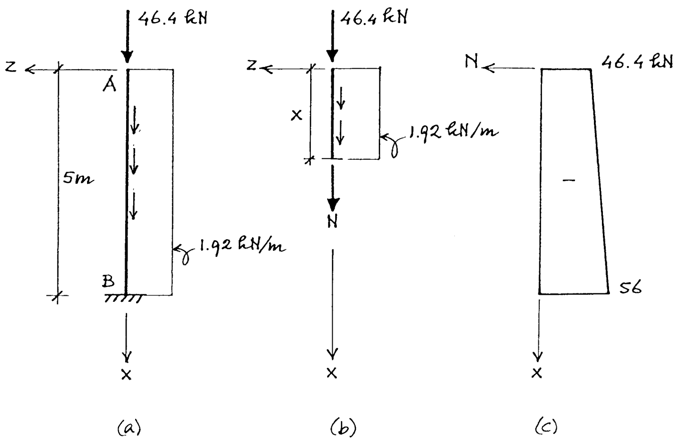


Figure 10.29 (a) The model for the column and the load. (b) To determine the normal force N we look at the equilibrium of the part above the section. (c) The normal force diagram.

Example 1

For the example in Figure 10.28, we will use the concrete column from the building that we looked at in Section 6.4.

Question:
Determine the N diagram.

Solution:
The column is loaded on top by a force of 46.4 kN (see Figure 10.28b). The dead weight is a uniformly distributed (line) load along the member axis. With a specific weight of concrete of 24 kN/m^3, and the cross-sectional dimensions given in Figure 10.28a, the dead weight is

$$(0.4\text{ m})(0.2\text{m})(24\text{kN/m}^3) = 1.2\text{kN/m}.$$

The model for the column and load is shown in Figure 10.29a.

In Figure 10.29b, a segment with length x has been isolated at the top of the column. In the section, the as yet unknown normal force N is shown according to its positive direction (that of a tensile force). For this segment, the equation for the force equilibrium in the x direction is

$$\sum F_x = (46.4\text{ kN}) + (1.92\text{ kN/m})(x\text{ m}) + N = 0$$

from which it follows that (x expressed in m)

$$N = (-46.4 - 1.92x)\text{ kN}.$$

In Figure 10.29c, the normal force N is shown as a function of x. The normal force is a compressive force everywhere and varies linearly, from 46.4 kN at A ($x = 0$ m) to 56 kN at B ($x = 5$ m).

As expected, the compressive force increases downwards due to the column's dead weight. The compressive force of 56 kN at B is in conformity with the previously determined support reaction (in Section 6.3) from the equilibrium of the column as a whole.

Example 2
In Figure 10.30a, the simply supported member AC is subject to a uniformly distributed axial load q along segment BC.

Question:
Determine the normal force distribution.

Solution:
There is only one support reaction not equal to zero, namely the horizontal support reaction at A. Figure 10.30b shows the isolated member, with all the forces acting on it.

The variation of the normal force can be determined from the force equilibrium in the x direction for the member segment to the left of a section at a distance x from A. Here, we have to distinguish between two parts, or *fields*:

- AB ($0 < x < a$),
- BC ($a < x < 3a$).

For $0 < x < a$ (the section is within AB) the equation for the force equilibrium of the left-hand member segment is (see Figure 10.30c)

$$\sum F_x = -2qa + N = 0$$

from which it follows that

$$N = 2qa.$$

The normal force in field AB is a constant tensile force.

For $a < x < 3a$ (the section is within BC) the equilibrium equation for the left-hand member segment is (see Figure 10.30d)

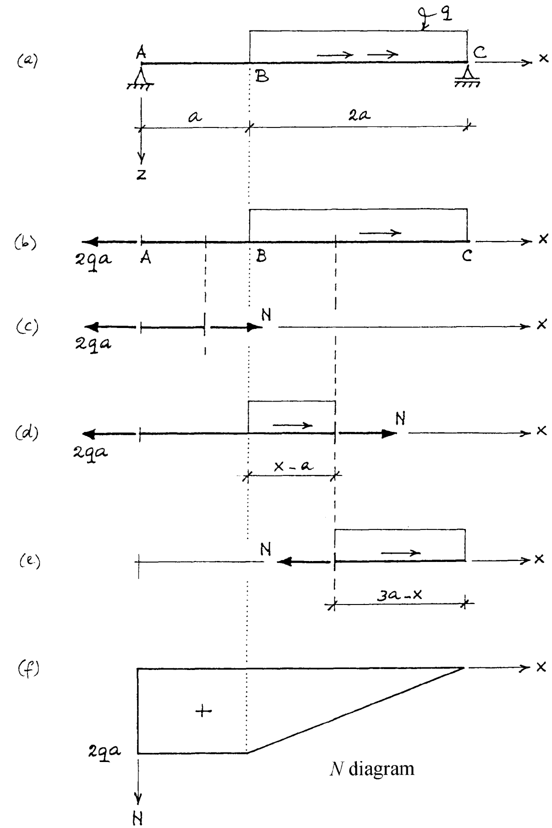


Figure 10.30 (a) A simply supported member AC, with a uniformly distributed load q over part BC along the member axis. (b) The isolated member with its support reactions. (c) The isolated part to the left of a section between A and B. (d) The isolated part to the left of a section between B and C. (e) The isolated part to the right of a section between B and C. (f) The normal force diagram. It has an abrupt change in slope at the joining of fields AB and BC.

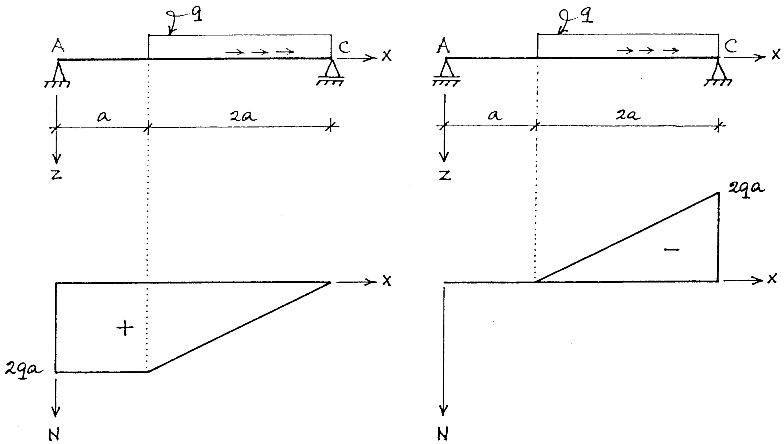


Figure 10.31 The simply supported member AC is subject to a uniformly distributed axial load q along part BC. If we switch the hinged support and roller support, the N diagram changes.

$$\sum F_x = -2qa + q(x-a) + N = 0.$$

Here $q(x-a)$ is the resultant of the distributed load on the isolated left-hand segment. This leads to

$$N = q(-x+3a).$$

Of course we find the same result if we look at the segment to the right of the section (see Figure 10.30e).

In field BC the normal force is a tensile force that decreases linearly, from $2qa$ at $x = a$ to zero at $x = 3a$.

Figure 10.30f shows the entire *normal force diagram*. This gives a bend (an abrupt change of slope) at the joining of the fields AB and BC.

It should be noted that the normal force variation changes if you swap the hinged and roller support at A and C (see Figure 10.31). It is up to you to check this.

10.2.3 Members with a uniformly distributed load normal to the member axis

This section looks at two examples:

1. a simply supported member;
2. a member fixed at one side and free at the other.

Example 1
The simply supported beam AB in Figure 10.32a carries a uniformly distributed load q over its entire length ℓ.

Question:
Determine the distribution of all the section forces.

Solution:
In Figure 10.32b, the beam has been isolated and the support reactions are shown. To determine the variation of the section forces, we will look at the equilibrium of the part to the left of the section (see Figure 10.32c):

$$\sum F_x = N = 0,$$
$$\sum F_z = -\tfrac{1}{2}q\ell + qx + V = 0,$$
$$\sum T_y|\text{section} = -\tfrac{1}{2}q\ell \cdot x + qx \cdot \tfrac{1}{2}x + M = 0$$

so that

$$N = 0$$

$$V = -qx + \tfrac{1}{2}q\ell, \qquad \text{(a)}$$

$$M = -\tfrac{1}{2}qx^2 + \tfrac{1}{2}q\ell x = \tfrac{1}{2}qx(\ell - x). \qquad \text{(b)}$$

The normal force is zero everywhere, and therefore not interesting.

The shear force varies linearly from $+\frac{1}{2}q\ell$ in A ($x = 0$) to $-\frac{1}{2}q\ell$ in B ($x = \ell$). The *shear force diagram* is shown in Figure 10.32d.

The bending moment varies quadratically in x and is positive everywhere. The bending moment diagram is shown in Figure 10.32e and is shaped like a (second degree) *parabola*. In A and B, the tangents of the parabola are also shown; both tangents intersect at the middle of AB.[1]

[1] It is assumed that the reader is familiar with plotting graphical functions, drawing tangents, and calculating extreme values.

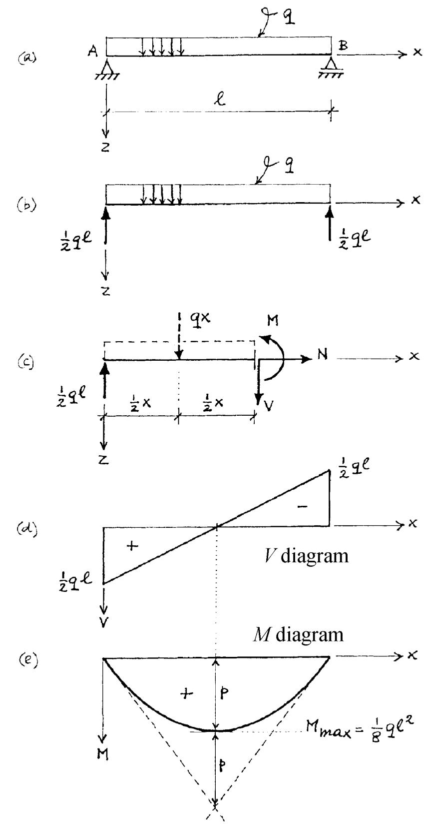


Figure 10.32 (a) A simply supported beam AB is bearing a uniformly distributed load q over its entire length ℓ. (b) The isolated member with its support reactions. (c) The isolated part of the member to the left of a section. The section can transfer a normal force N, shear force V and a bending moment M. The unknown section forces are shown in accordance with their positive directions in the coordinate system. (d) The shear force diagram for AB. (e) The bending moment diagram for AB, with the tangents in A and B.

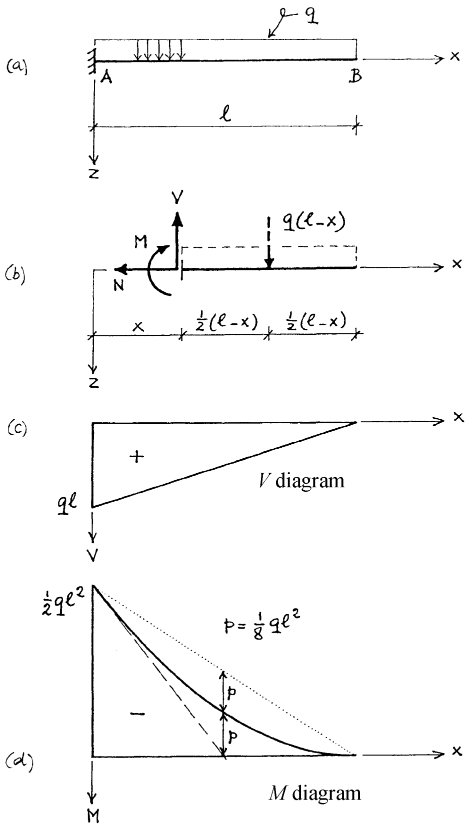


Figure 10.33 (a) A member AB fixed at A carries a uniformly distributed load q over its entire length ℓ. (b) To determine the section forces, we look at the equilibrium of the part to the right of the section. (c) The shear force diagram for AB. (d) The bending moment diagram for AB, with the tangents at A and B.

The bending moment is an *extreme* (*maximum* or *minimum*) when the derivative of M with respect to x is zero:

$$\frac{\mathrm{d}M}{\mathrm{d}x} = -qx + \tfrac{1}{2}q\ell = 0 \Rightarrow x = \tfrac{1}{2}\ell \text{ and } M_{\max} = \tfrac{1}{8}q\ell^2.$$

If we differentiate expression (b) for the bending moment with respect to x we find the expression (a) for the shear force. The derivative of the bending moment M is therefore equal to the shear force V:

$$\frac{\mathrm{d}M}{\mathrm{d}x} = V.$$

Consequently: *the gradient of the moment diagram is equal to the shear force*. In Chapter 11 we will demonstrate that this property is generally applicable. It is up to you to check the property for Examples 2 and 3 in Section 10.2.1.

Example 2

In Figure 10.33a, the member AB is fixed at A and carries a uniformly distributed load q over its entire length ℓ.

Question:
Determine the variation of the section forces.

Solution:
To determine the section forces, we will look at the equilibrium of the segment to the right of the section. In this case, it is not necessary to previously determine the support reactions at A (see Figure 10.33b):

$$\sum F_x = N = 0 \Rightarrow N = 0,$$

$$\sum F_z = -V + q(\ell - x) = 0 \Rightarrow V = q(\ell - x),$$

$$\sum T_y|\text{section} = -M - q(\ell - x)\cdot \tfrac{1}{2}(\ell - x) = 0 \Rightarrow M = -\tfrac{1}{2}q(\ell - x)^2.$$

The normal force is zero everywhere, and therefore not of interest.

The shear force is positive everywhere and varies linearly, from $q\ell$ at A ($x = 0$) to zero at B ($x = \ell$). The *shear force diagram* is shown in Figure 10.33c.

The bending moment is quadratic in x and negative everywhere. It varies from $-\frac{1}{2}q\ell^2$ at the fixed end A ($x = 0$) to zero at the free end B ($x = \ell$).

Figure 10.33d shows the *bending moment diagram*: a parabola with its apex at B. The tangents at A and B are also shown. The tangent at B is horizontal. Both tangents intersect at the middle of AB. The values p are equal to $\frac{1}{8}q\ell^2$.

The *maximum*[1] bending moment occurs at the fixed support in A:

$$|M|_{\max} = \tfrac{1}{2}q\ell^2.$$

Note that at A: $\mathrm{d}M/\mathrm{d}x = V \neq 0$; here it concerns a maximum at a field boundary.

The support reactions at A can be derived according to magnitude and direction from the shear force diagram and the bending moment diagram:

$$V_\mathrm{A} = +q\ell,$$
$$M_\mathrm{A} = -\tfrac{1}{2}q\ell^2.$$

Since the support reactions at A act on a negative sectional plane, they have the directions shown in Figure 10.34. Whether this is correct can easily be checked by looking at the equilibrium of the structure as a whole.

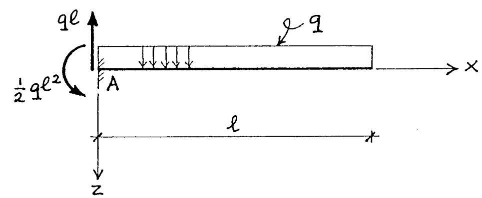
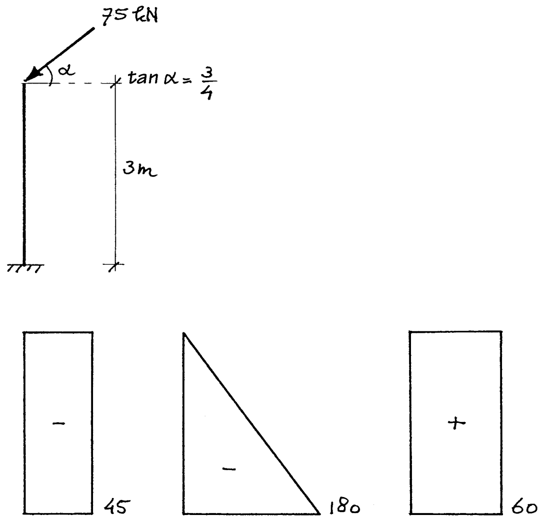


Figure 10.34 The magnitude and direction of the support reactions in A can be derived from the shear force diagram and the bending moment diagram.

1 With "*maximum*" we often mean "the largest value in an *absolute sense*"; we call this the global maximum.



N diagram (kN) *M* diagram (kNm) *V* diagram (kN)

Figure 10.35 A fixed member, loaded at its free end by a force of 75 kN, with its *N* diagram, *M* diagram and *V* diagram. The magnitude of the section forces follows directly from these diagrams. The direction is determined by the sign. To do so, we have to know the coordinate system in which we are working. Since that is not given, the signs in the *M* and *V* diagrams have here lost their meaning.

10.3 Deformation symbols for shear forces and bending moments

Figure 10.35 shows a fixed member that is loaded at its free end by a force of 75 kN. The same figure also shows the *N* diagram, *M* diagram and the *V* diagram. The magnitude of the section forces can be read directly from these diagrams. The direction is determined by the plus or minus sign.

The normal force *N* can be read directly from the *N* diagram without coordinate system. The direction of the normal force follows directly from the plus or minus sign. The *N* diagram shows that the normal force is negative and therefore a compressive force.

Other than for the normal force *N*, we have to know the coordinate system in which we are working to interpret the signs in the *M* and *V diagrams*. In Figure 10.35, without the coordinate system, the signs in the *M* and *V diagrams* have lost their meaning.

Assume we were working in a *xz* coordinate system with, of course, the *x* axis along the member axis. In order to determine the direction of the bending moment *M* from the sign, we have to know the direction of the *z* axis.[1] In order to determine direction of the shear force *V* from the sign, we have to know the direction of the *z* axis and also the *x* axis.[2]

---

1 A bending moment is positive if it causes tension at the positive *z* side of the member axis and compression at the negative *z* side. The direction of the *x* axis is not important here.

2 A shear force is positive if it acts in the positive *z* direction on a positive sectional plane, and in the negative direction on a negative sectional plane. Now you also have to know the direction of the *x* axis to determine whether a sectional plane is positive or negative.

The signs in the M and V diagrams are in accordance only with the correct directions for the bending moment M and the shear force V for the coordinate system given in Figure 10.36.

To interpret the signs in the M and V diagrams correctly, we therefore have to know the coordinate system. So far, in all the examples showing the M and V *diagrams*, the structure has consisted of a single straight member, and the coordinate system was always shown. When you have to deal with bent members or structures consisting of several members, you have to introduce a local coordinate system along each straight member segment if you want to indicate the directions of M and V using the plus and minus signs. For the simple structure in Figure 10.37, this already leads to three local coordinate systems: one for AB, one for BC and one for CD.

This soon becomes cumbersome and cluttered. In manual calculations, we will therefore use *deformation symbols*: the *bending symbol* for bending moments, and the *shear symbol* for shear forces.

The bending symbol and shear symbol symbolise the deformation of the member axis due to a bending moment and a shear force respectively. These deformation symbols can be used to set the direction of the section forces unequivocally, regardless of a coordinate system.

We always use the plus and minus sign for normal forces.

The bending symbol and shear symbol will be explained in more detail below.

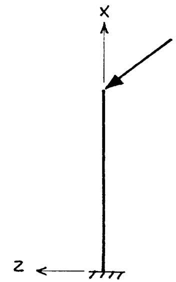


Figure 10.36 The fixed member with the coordinate system used.

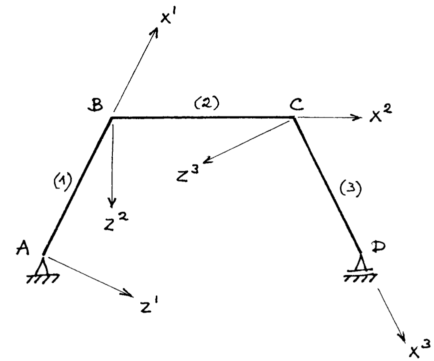


Figure 10.37 When you have to deal with bent members or structures consisting of several members, you have to introduce a local coordinate system along each straight member segment if you want to indicate the directions of M and V using the plus and minus signs. This soon becomes cumbersome and cluttered in manual calculations.

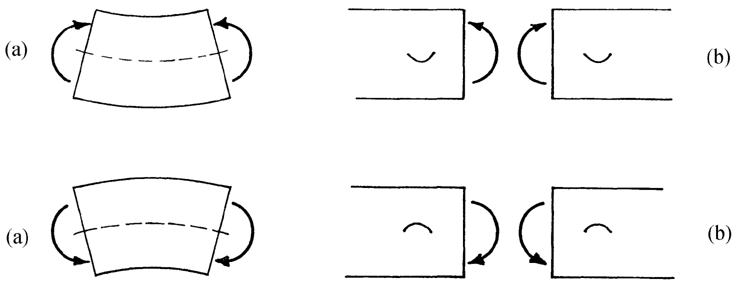


Figure 10.38 Bending symbols.

Figure 10.38 (a) A small member segment subject to bending moments will lengthen at the tension side and shorten at the compression side. The member segment will bend. (b) The small arc that represents the deformation is used as deformation symbol for the bending moment and is known as the bending symbol.

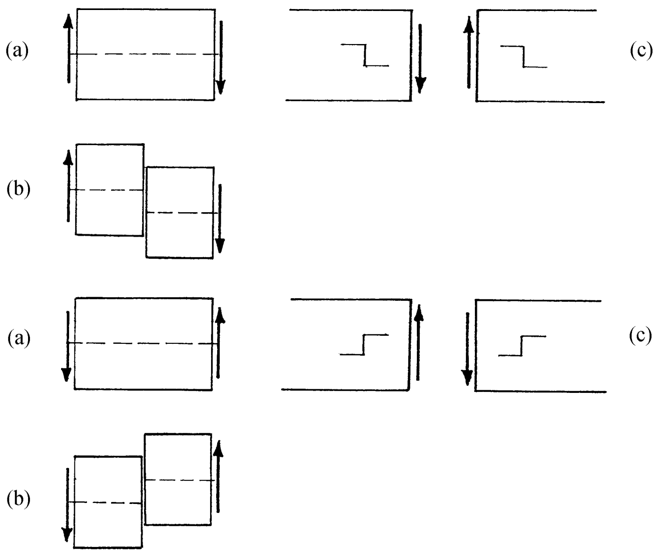


Figure 10.39 Shear symbols.

Figure 10.39 (a) In a member segment subject to shear forces, one sectional plane will try to shift with respect to the other. (b) This effect can be visualised by introducing an (imaginary) slide joint in the segment, so that both sectional planes can move with respect to one another. (c) The step formed by the moved member axes is used as the deformation symbol for shear forces and is known as the shear symbol.

- *Bending symbol* (deformation symbol for *bending moments*)
  Figure 10.38a shows a small member segment subject to bending moments. The member segment will lengthen at the side being pulled, and shorten at the side being compressed. The member segment will bend. Since it is possible to determine the bending moment from the bent shape of the member axis, we use the small arc as deformation symbol for the bending moment (see Figure 10.38b).
- *Shear symbol* (deformation symbol for *shear forces*)
  Figure 10.39a again shows a small member segment, but now with shear forces. When subject to shear forces, one sectional plane will try to shift with respect to the other. This effect can be visualised by applying an (imaginary) slide joint within the segment, so that both parts can move with respect to one another (see Figure 10.39b). The step change formed by the moved member axes is used as the deformation symbol for shear forces (see Figure 10.39c).

Figure 10.40 shows the V diagram and the M diagram with the deformation symbols for a simply supported beam, loaded by a single point load.

Since there is no coordinate system, we are in principle free to choose on which side of the member axis we plot the bending moment and the shear force, as long as we use the correct deformation symbols. See the shear force diagrams in Figures 10.40b and 10.40c, which are both correct.

We make an exception for the bending moment. It is agreed that the M values are plotted at the side where the bending moment causes tension, so at the convex side of the member axis (see Figure 10.40d). The open side of the deformation symbol is therefore always faced towards the member axis.[1]

The advantage of this is that the gradient of the M diagram ($\Delta M/\Delta x$), shown in Figure 10.40d as a "step", now corresponds directly with the shear symbol for the shear force. This allows us to easily and directly check the relationship between the moment diagram and the shear force diagram.

In Figure 10.40e, the M diagram has been plotted at the wrong side of the member axis. Although you will come across this often in books, we strongly recommend that you do not draw the moment diagram in this way, as the relationship with the deformation symbol for the shear force (the "step" in the M diagram) is lost.

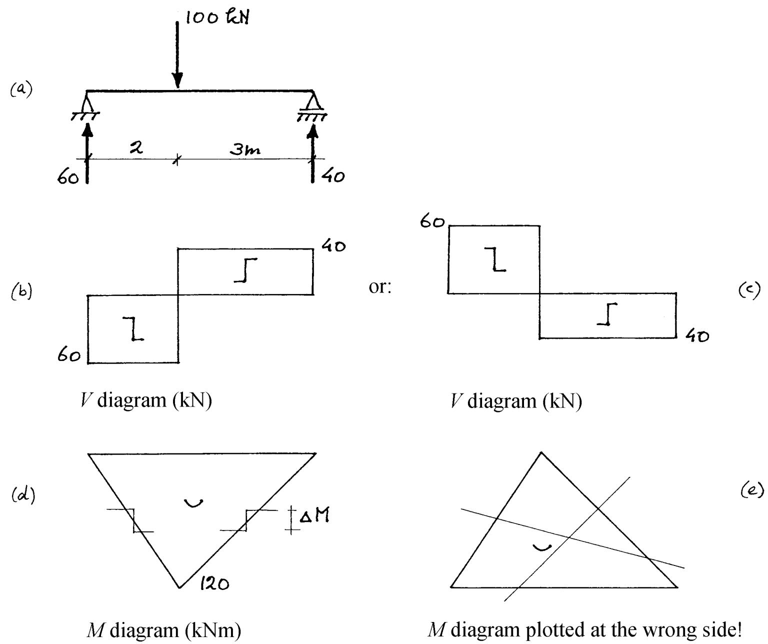


Figure 10.40 (a) A simply supported beam, loaded by a single point load. (b) and (c) When using deformation symbols, we are free to choose at which side of the member axis the shear force is plotted. (d) The bending moment is plotted at the side where the bending moment causes tension, so at the convex side of the member axis. The open side of the bending symbol therefore always faces towards the member axis. The gradient of the M diagram ($\Delta M/\Delta x$), shown as a "step", now corresponds directly to the shear symbol for the shear force. (e) If however the M diagram is plotted at the wrong side of the member axis, the relationship with the deformation symbol for the shear force (the "step" in the M diagram) is lost.

1 Thanks to this agreement, the deformation symbol in the M diagram is actually unnecessary. The deformation symbol is nevertheless always shown for clarity.

10.4 Summary sign convention for the *N*, *V* and *M* diagrams

When drawing the N, V and M diagrams, you can use:

- plus and minus signs, related to a (local) coordinate system with the x axis along the member axis;
- deformation symbols (only for the V diagram and the M diagram).

If working with plus and minus signs, in a xz coordinate system:

- positive section forces are plotted at the positive side of the z axis, and negative values at the negative side of the z axis;
- the sign is placed *within the diagram*; relevant values are included *without their sign*.

If working with deformation symbols:

- You use plus and minus signs for the normal force N, and you use the bending symbol for the bending moment M and the shear symbol for the shear force V.
- The bending moment is plotted at the side where the bending moment causes tension, this is at the convex side of the member axis. The open side of the deformation symbol therefore always faces the member axis. The gradient of the M diagram, shown as a "step", now corresponds directly to the shear symbol in the V diagram.
- It does not matter at which side of the member axis you plot the values for the normal force and the shear force.
- Plus and minus signs for the normal force and deformation symbols for the bending moment and shear force are placed within the diagram; relevant values in the diagram are written without sign.

10.5 Problems

Member axis and member cross-section; section forces (Section 10.1.1)

10.1 The sign of the section forces N, V and M can be related to a coordinate system with the x axis along the member axis and the yz plane parallel to the member cross-sections.

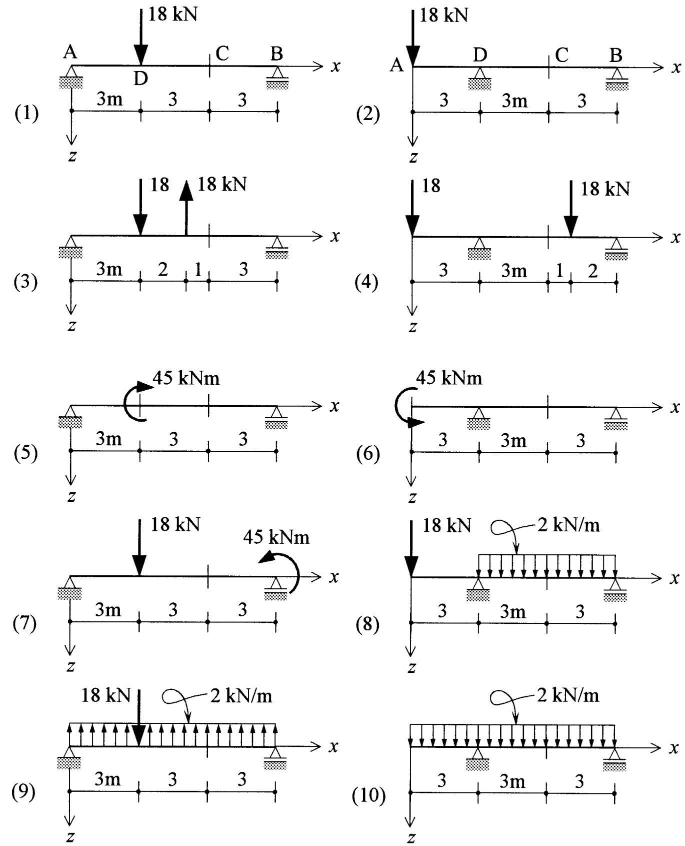
Questions:

a. When is a section plane positive, and when is it negative?
b. When is a normal force positive, and when is it negative?
c. When is a shear force positive, and when is it negative?
d. When is a bending moment positive, and when is it negative?

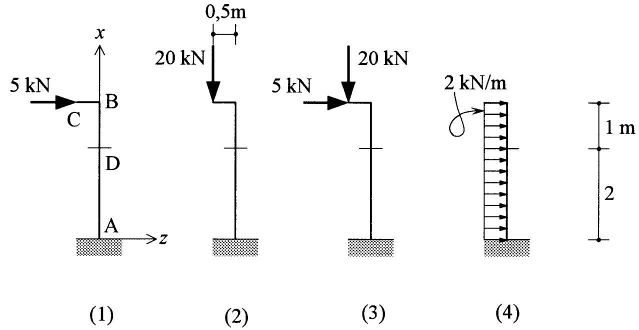
10.2: 1–10 You are given two beams loaded in five different ways.

Question:
Determine the bending moment and the shear force, with the correct sign in the given coordinate system, in the following cross-sections:

a. directly to the right of A.
b. directly to the left of B.
c. in C.
d. directly to the left of D.
e. directly to the right of D.



10.3: 1–4 You are given a column AB fixed at A, with console BC, loaded in four different ways.



Question:
Determine the sections forces, with the correct sign in the given coordinate system, in the following cross-sections:

a. directly below the console at B.
b. at D, one meter below B.
c. directly above fixed support A.

Stresses (Section 10.1.2)

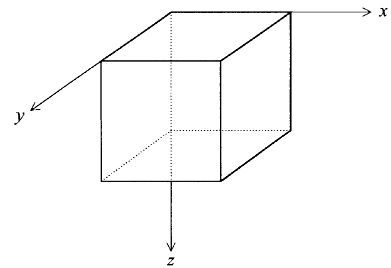
10.4: 1–4 You are given a block with the following four states of stress:

(1)
$$\begin{aligned} \sigma_{xx} &= -25\ \text{N/mm}^2 \\ \sigma_{xy} = \sigma_{yx} &= -30\ \text{N/mm}^2 \\ \sigma_{yz} = \sigma_{zy} &= +15\ \text{N/mm}^2 \\ \sigma_{zz} &= +10\ \text{N/mm}^2 \end{aligned}$$

(2)
$$\begin{aligned} \sigma_{xy} = \sigma_{yx} &= +15\ \text{N/mm}^2 \\ \sigma_{xz} = \sigma_{zx} &= +10\ \text{N/mm}^2 \\ \sigma_{yy} &= -5\ \text{N/mm}^2 \\ \sigma_{zz} &= +20\ \text{N/mm}^2 \end{aligned}$$

(3)
$$\begin{aligned} \sigma_{xx} &= +30\ \text{N/mm}^2 \\ \sigma_{xz} = \sigma_{zx} &= +15\ \text{N/mm}^2 \\ \sigma_{yz} = \sigma_{zy} &= -25\ \text{N/mm}^2 \\ \sigma_{zz} &= -5\ \text{N/mm}^2 \end{aligned}$$

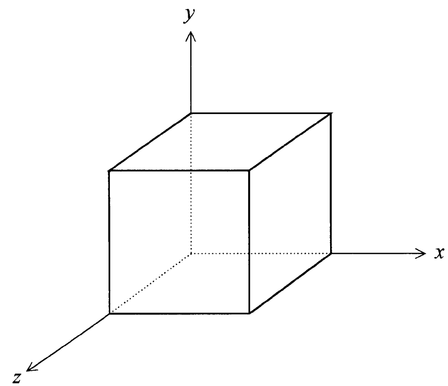
(4)
$$\begin{aligned} \sigma_{xx} &= -6\ \text{N/mm}^2 \\ \sigma_{xy} = \sigma_{yx} &= -8\ \text{N/mm}^2 \\ \sigma_{yy} &= +5\ \text{N/mm}^2 \\ \sigma_{yz} = \sigma_{zy} &= +12\ \text{N/mm}^2 \end{aligned}$$



Question:
Draw (for each case) the stresses on the block in the directions in which they act and include their values:

a. for the planes shown.
b. for the planes not shown.

10.5: 1–4 You are given a block with the following four states of stress:



(1)
$$\begin{aligned} \sigma_{xx} &= +10\ \text{N/mm}^2 \\ \sigma_{xy} = \sigma_{yx} &= +15\ \text{N/mm}^2 \\ \sigma_{xz} = \sigma_{zx} &= -30\ \text{N/mm}^2 \\ \sigma_{yy} &= -5\ \text{N/mm}^2 \end{aligned}$$

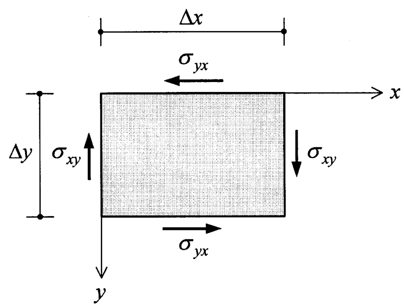
(2)
$$\begin{aligned} \sigma_{xy} = \sigma_{yx} &= +10\ \text{N/mm}^2 \\ \sigma_{yy} &= -3\ \text{N/mm}^2 \\ \sigma_{yz} = \sigma_{zy} &= -8\ \text{N/mm}^2 \\ \sigma_{zz} &= +14\ \text{N/mm}^2 \end{aligned}$$

(3) $\sigma_{xx} = -12\ \text{N/mm}^2$
$\sigma_{xz} = \sigma_{zx} = +5\ \text{N/mm}^2$
$\sigma_{yz} = \sigma_{zy} = -8\ \text{N/mm}^2$
$\sigma_{zz} = -15\ \text{N/mm}^2$

(4) $\sigma_{xx} = +4\ \text{N/mm}^2$
$\sigma_{xz} = \sigma_{zx} = +10\ \text{N/mm}^2$
$\sigma_{yy} = +7\ \text{N/mm}^2$
$\sigma_{yz} = -16\ \text{N/mm}^2$

Question:
Draw (for each case) the stresses on the block in the directions in which they act and include their values:
a. for the planes shown.
b. for the planes not shown.

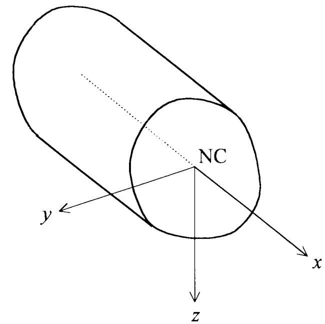
10.6 The dimensions of a rectangular block are Δx; Δy; Δz and are so small that three stresses on opposite planes are equal. The figure shows the top view of the block with only the shear stresses σ_{xy} and σ_{yx} on two planes. The other stresses are not shown.



Questions:
a. Draw all the other stresses that act parallel to the xy plane. Show that, from the moment equilibrium of the block in the xy plane, it follows that $\sigma_{xy} = \sigma_{yx}$.
b. Also show that $\sigma_{xz} = \sigma_{zx}$.
c. Show that $\sigma_{yz} = \sigma_{zy}$.

General definition section forces (Section 10.1.3)

10.7 A normal force N and the bending moments M_y and M_z act in a cross-section. If $\sigma = \sigma(x, y)$ is the normal stress in the cross-section, then the normal force is: $N = \int_A \sigma\, \mathrm{d}A$.



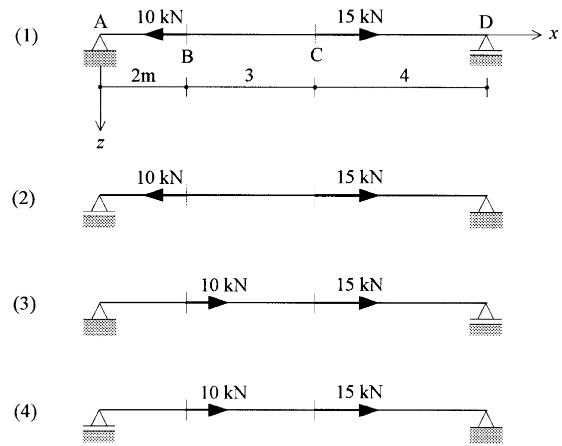
Questions:
a. Draw the (positive) normal force N in the cross-section.
b. Draw the (positive) bending moments M_y and M_z in the cross-section.
c. How can we express the bending moments M_y and M_z in the normal stress σ?

10.8 The shear stresses $\sigma_{xy} = \sigma_{xy}(x, y)$ and $\sigma_{xz} = \sigma_{xz}(x, y)$ in a cross-section lead to the shear forces V_y and V_z and a torsional moment M_t.

Questions:
a. Draw the (positive) shear forces V_y and V_z in the cross-section.
b. How can the shear forces V_y and V_z be expressed in the shear stresses σ_{xy} and σ_{xz}?
c. Draw the (positive) torsional moment M_t in the cross-section.
d. How can the torsional moment M_t be expressed in the shear stresses σ_{xy} and σ_{xz}?

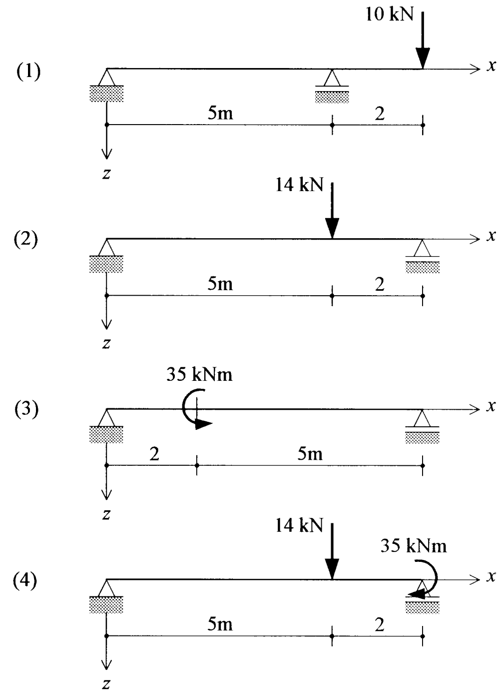
Diagrams for the normal force, shear force and bending moment (Section 10.2)

10.9: 1–4 Member AD is supported in two different ways and is loaded at B and C by forces of respectively 10 and 15 kN.



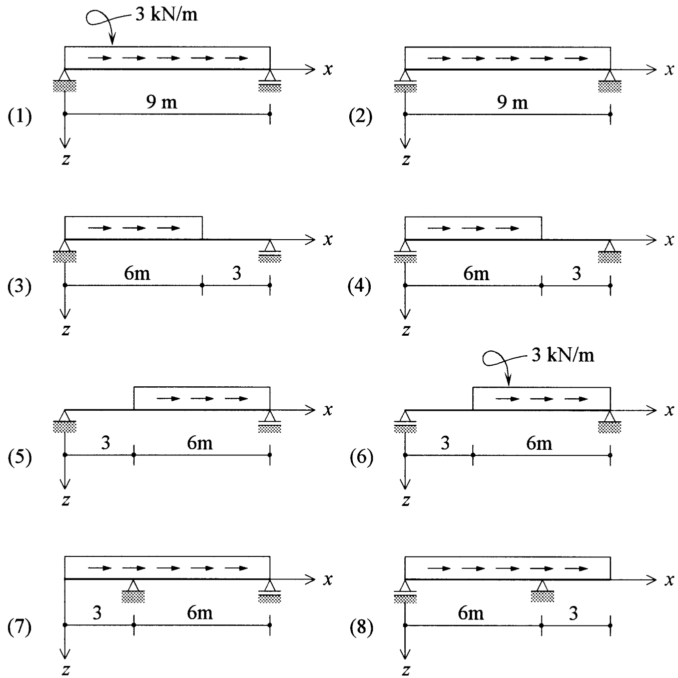
Questions:
a. From the equilibrium determine the normal force N as a function of x.
b. Draw the N diagram.

10.10: 1–4 You are given four different loaded beams.



Questions:
a. From the equilibrium, determine the shear force V as a function of x.
b. Draw the shear force diagram.
c. From the equilibrium, determine the bending moment M as a function of x.
d. Draw the bending moment diagram.

10.11: 1–8 A number of members are loaded for extension by a uniformly distributed load of 3 kN/m.

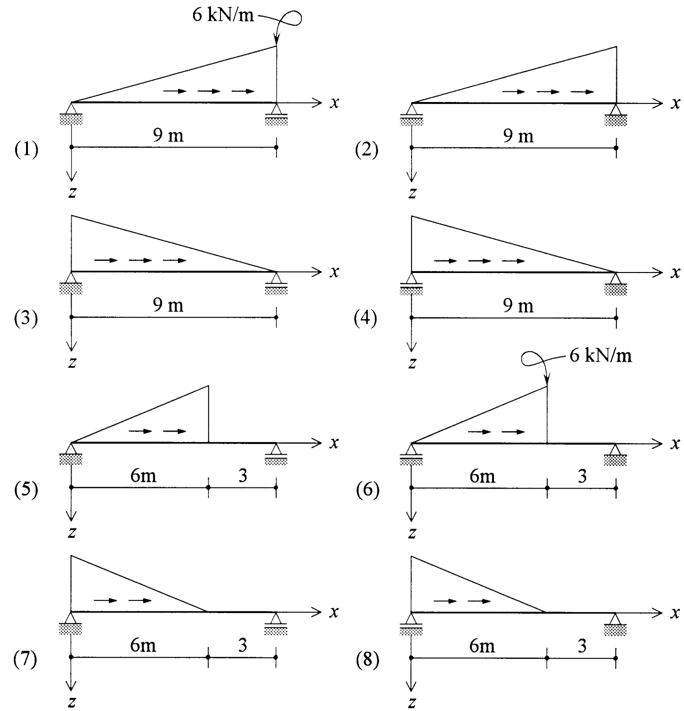


Questions:

a. From the equilibrium, determine the normal force as a function of x.

b. Draw the normal force diagram.

10.12: 1–8 The same member is loaded in two different ways and is loaded for extension in four different ways by a linearly distributed load. In all cases, the top value of the distributed load is 6 kN/m.

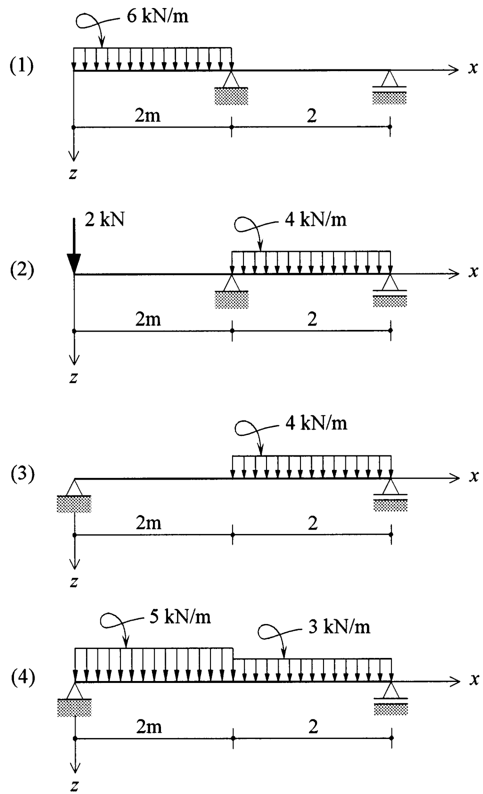


Questions:

a. From the equilibrium, determine the normal force as a function of x.

b. Draw the normal force diagram.

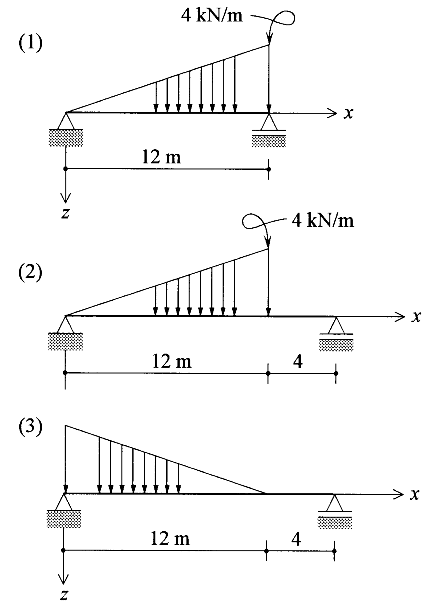
10.13: 1–4 You are given four differently loaded beams.



Questions:

a. From the equilibrium, determine the shear force V as a function of x.
b. Draw the shear force diagram.
c. From the equilibrium determine the bending moment M as a function of x.
d. Draw the bending moment diagram.

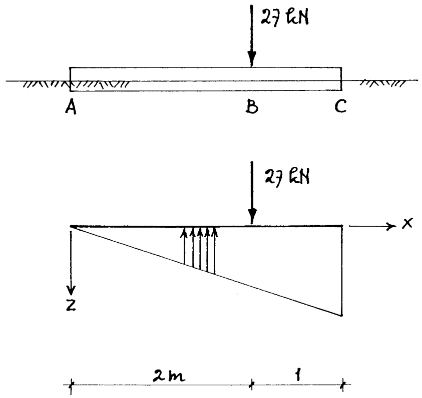
10.14: 1–3 You are given three beams with a linearly distributed load normal to the member axis. The top value for the distributed load in all three cases is 4 kN/m.



Questions:

a. From the equilibrium, determine the shear force V as a function of x.
b. Draw the V diagram.
c. Form the equilibrium, determine the bending moment M as a function of x.
d. Draw the M diagram.

10.15 The figure shows a foundation beam on sand and loaded by a force of 27 kN. For this load, the earth pressure is linearly distributed, as shown in the load diagram. The dead weight of the beam is ignored.



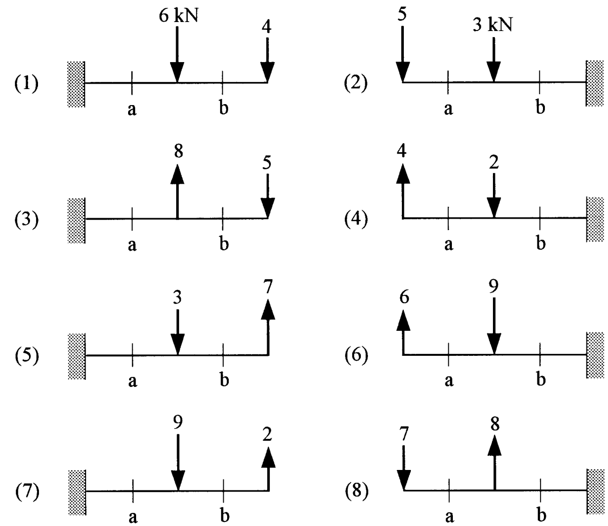
Questions:

a. Determine the top value of the earth pressure.
b. From the equilibrium, determine the shear force and the bending moment for AB as a function of x ($0 \le x < 2$ m).
c. From the equilibrium, determine the shear force and the bending moment for BC as a function of x ($2 \text{ m} < x \le 4$ m).
d. For ABC, draw the shear force diagram and the bending moment diagram.

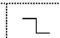
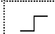
Deformation symbols for shear force and bending moment (Section 10.3)

10.16 *Question*: Explain the *shape* of the deformation symbols that are used for shear force and bending moment.

10.17: 1–8 A fixed beam is loaded in various ways.

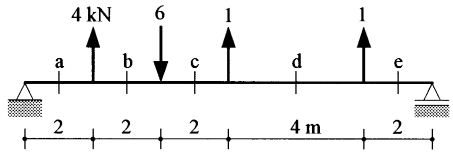


Question:
Which deformation symbol belongs to the shear force in cross-sections a and b respectively?

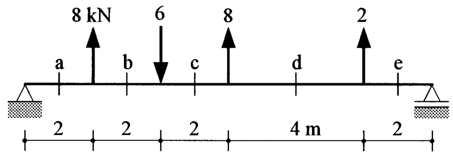
a.	
b.	
c.	0

10.18: 1–4 You are given four different beams.

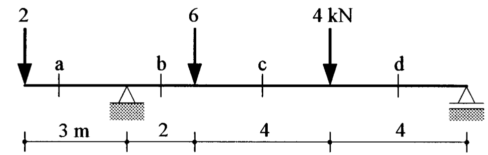
(1)



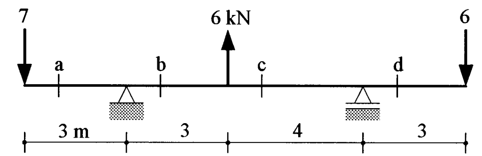
(2)



(3)



(4)

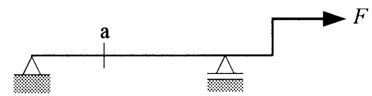


Question:
Which deformation symbol belongs to the shear force in cross-sections a to e respectively?

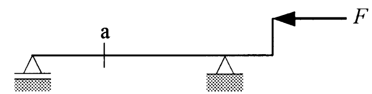
a.	‾|_
b.	_|‾
c.	0

10.19: 1–2 Two beams are loaded by an eccentrically applied normal force.

(1)



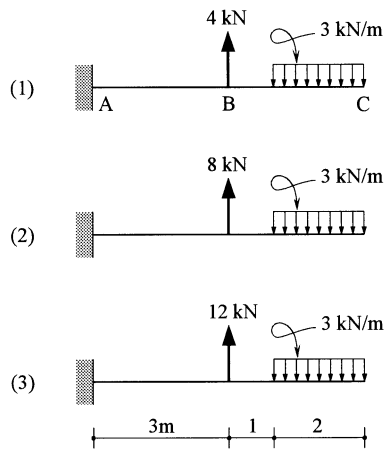
(2)



Question:
Which combination of deformation symbols belongs to the shear force and the bending moment in cross-section a?

	V	M
a.	‾|_	∪
b.	_|‾	∪
c.	‾|_	∩
d.	_|‾	∩

10.20: 1–3 You are given the same cantilever beam with three different loads.

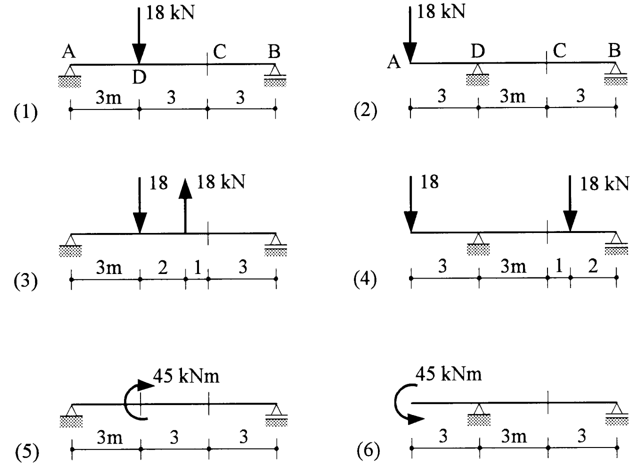


Question:
Determine the combination of deformation symbols that belong to the shear force and the bending moment:

a. in the cross-section directly to the right of B.
b. in the cross-section directly to the left of B.
c. in the cross-section directly next to the fixed support A.

	V	M
a.		
b.		
c.		
d.		

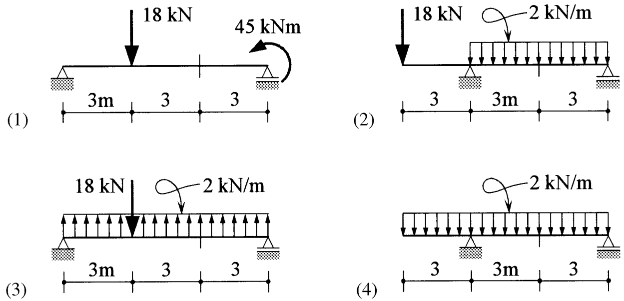
10.21: 1–6 You are given two beams loaded in three different ways.



Question:
Determine the bending moment and the shear force, with the correct deformation symbol, in the following cross-sections:

a. directly to the right of A.
b. directly to the left of B.
c. at C.
d. directly to the left of D.
e. directly to the right of D.

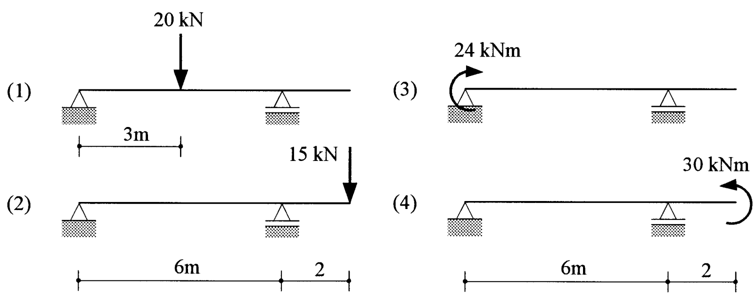
10.22: 1–4 You are given two beams loaded in different ways.



Question:
Determine the bending moment and the shear force, with the correct deformation symbol, in the following cross-sections:

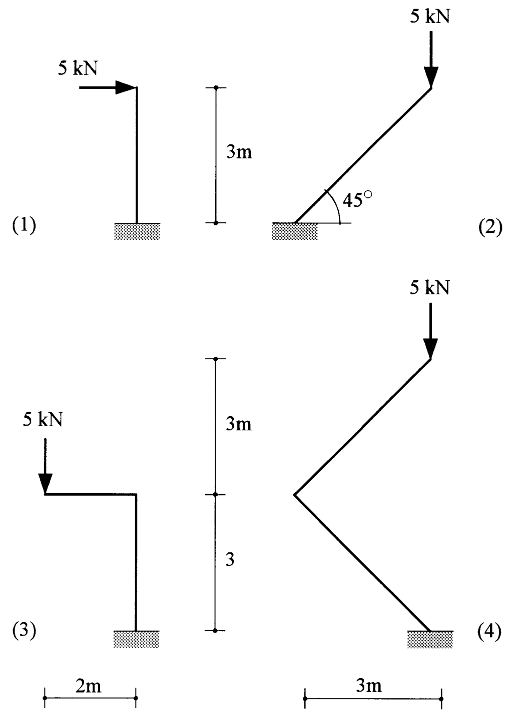
a. directly to the right of A.
b. directly to the left of B.
c. at C.
d. directly to the left of D.
e. directly to the right of D.

10.23: 1–4 You are given a cantilever beam and four different loads.



Question:
For the entire beam, draw the shear force diagram and the bending moment diagram, with the deformation symbols. Include their values at relevant points.

10.24: 1–4 You are given four different structures.



Question:
For the entire structure, draw the V diagram and the M diagram, with the deformation symbols. Include the values.