1 Introduction

This chapter provides a number of definitions and describes various concepts. Following a brief description of the field of mechanics in Section 1.1, Section 1.2 addresses the character of a number of important quantities in mechanics, and the units in which they are expressed. Quantities of a magnitude and direction that meet the conditions of the so-called parallelogram rule are called vectors, which are covered in Section 1.3.

Newton's three Laws of Motion and his Law of Gravitation were an important step forward in the development of mechanics. We look at these laws at the end of the chapter in Section 1.4.

1.1 Mechanics

1.1.1 Examples from the field of mechanics

Mechanics is the subdivision of physics which addresses equilibrium and the motion of matter.

Mechanics therefore includes for example:

- The description of the movement of natural and artificial heavenly bodies. Figure 1.1 is a schematic representation of our Galaxy. The vast majority of all stars are in a flat disk. Above and below this disk, there are some 200 globular star clusters that revolve in ellipsoidal orbits

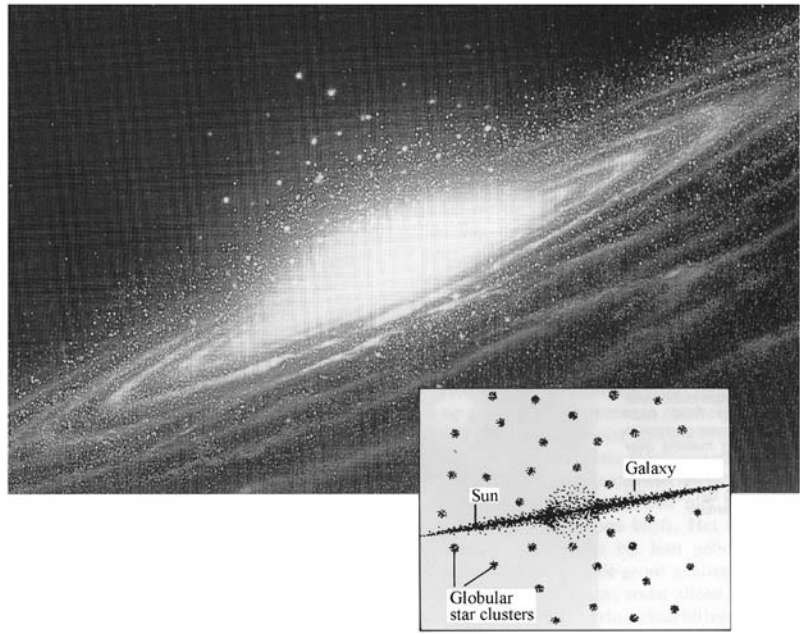

Figure 1.1 Model of our Galaxy and its globular star clusters. Source: *Natuur en Techniek* 89/10, p. 757.

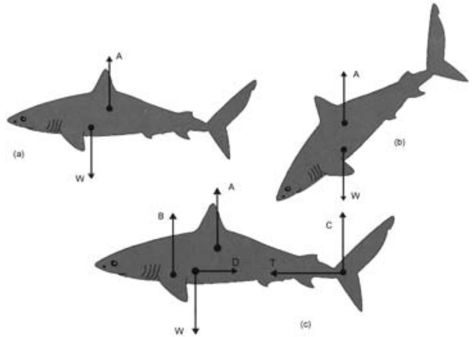

Figure 1.2 The forces exerted on a swimming shark. Source: *Natuur en Techniek* 90/02, p. 136.

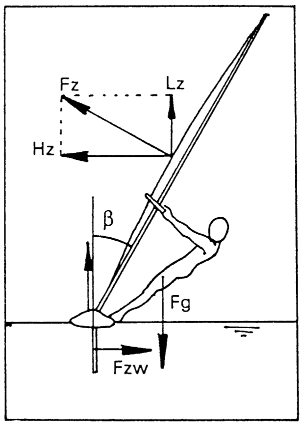

Figure 1.3 A surfer balancing on his board. Source: *Leidraad voor surfers*, Vereniging Zeilscholen Nederland.

around the centre of the Galaxy.

- A calculation of the forces exerted on a swimming shark. In Figure 1.2a, in which the shark is at rest, the shark is subject to forces resultant from its weight W and the upward force A caused by the water pressure. As a result, the animal tips over (see Figure 1.2b). If the shark's tail generates a thrust T, vertical forces are generated that keep the shark in vertical equilibrium (see Figure 1.2c).
- Balancing on a surfboard (see Figure 1.3).
- A calculation as to the deformation of an oil platform at sea subject to wave action. Figure 1.4 shows a concrete platform designed for the Norwegian Troll field with a water depth of 340 metres. The seabed consists of extremely weak clay. The sea conditions are extremely rough with waves over 10 metres in height. The mass of the deck is 60,000 tons (60×10^6 kg).
- The description of water currents in a river, estuary, or sea. Figure 1.5 represents a current model for the North Sea. The arrows indicate the direction and strength of the current for certain areas. This type of model can be used to investigate the distribution of toxic materials.
- The investigation of stresses in prostheses, such as an artificial hand, hip joint or knee joint. As shown in Figure 1.6, the attachment of the prosthesis in a knee joint is extremely important. Figure 1.7a shows the magnitude of the forces calculated by using an arithmic model. Major tensile stress occurs at the end of the prosthesis (black area). This stress can lead to fractures in the cement (adhesive) as shown in the X-ray in Figure 1.7b.
- Finding the right shape for a high tower by effectively transferring the loading by wind and its dead weight onto the foundation. Figure 1.8 shows the 300-metre Eiffel Tower, completed in 1889 and the first 1000-foot tower, built for the 1889 World Exhibition in Paris. The tower is constructed of wrought iron.

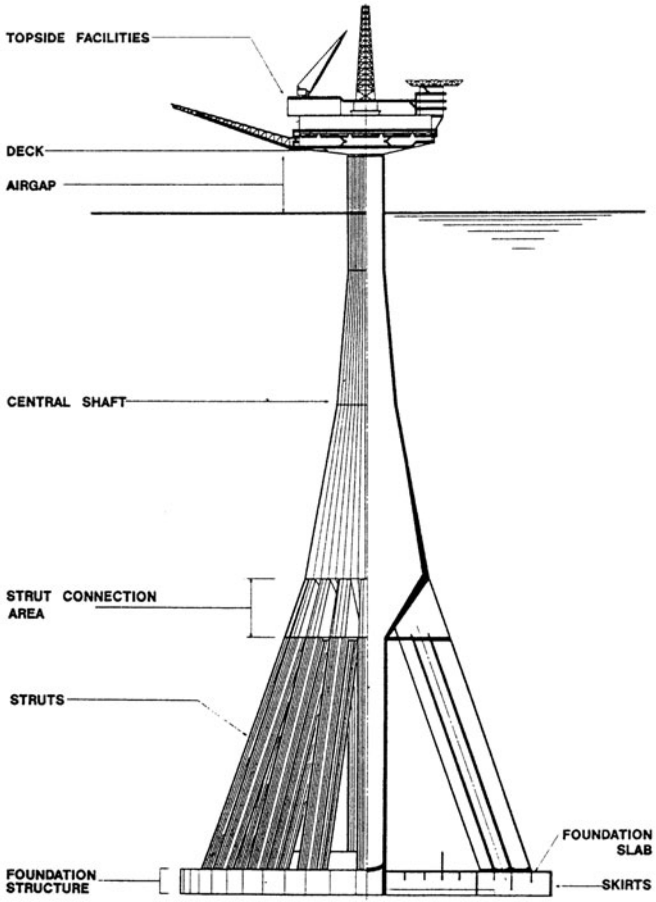

Figure 1.4 Design of a concrete platform in 340-metre deep water. Source: *Heron* 1986, no. 1, p. 86.

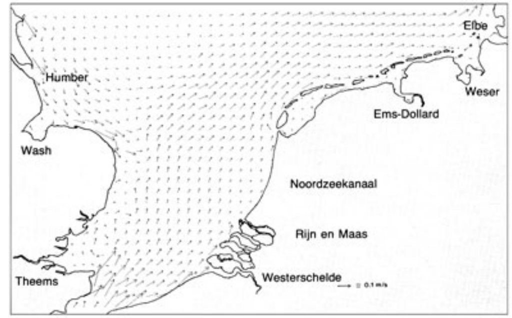

Figure 1.5 A current model for the North Sea. Source: *Natuur en Techniek* 90/04, p. 292.

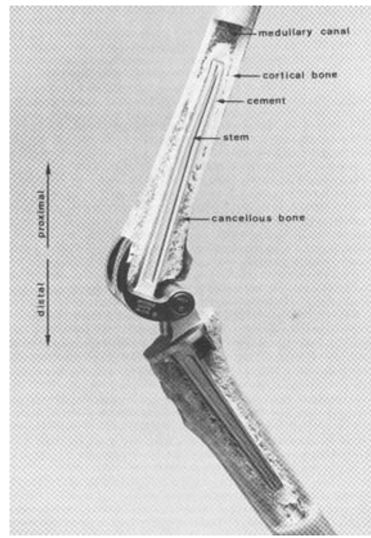

Figure 1.6 Open knee prosthesis. Source: *Heron* 1986, no. 1, p. 100.

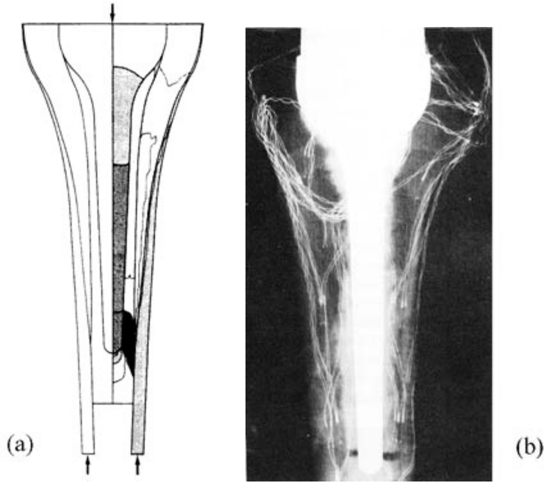

Figure 1.7 In the model of the knee prosthesis (a) the grey shades indicate the size of the stress according to an arithmetic model. The largest tensile force occurs in the black area near the end of the prosthesis. The X-ray (b) shows a fracture in the cement at that point. Source: *Heron* 1986, no. 1, p. 105.

- Water flow through a dam. In Figure 1.9, you can see stream lines and equipotential lines for a dam on an impermeable subsoil. They are perpendicular to one another and form a so-called flow net. The EF section of the slope is known as the seepage surface. Here the water leaves the dam and flows down along the slope.
- Closing the Maeslant barrier (see Figure 1.10).
  The Maeslant barrier, the storm barrier in the Nieuwe Waterweg in the west of the Netherlands, consists of two 22-metre high sector doors shaped like an arc with an arc length of 214 metres. The doors are turned towards each other afloat from docks. When closed, the doors are sunk onto a threshold by the inlet of water. The water pressure on the doors is diverted to foundation blocks by means of two 260-metre truss arms. The truss arm and the foundation block are joined by means of a ball-and-socket joint with a 10-metre diameter.

1.1.2 Subdivisions within mechanics

Mechanics' extensive field of operation can be subdivided in various ways.

A subdivision addressed in the given description of mechanics is based on the perspective of rest and movement:

- *Statics*, or the study of material at rest.
- *Dynamics*, or the study of moving material.

A subcomponent of dynamics is *kinematics*, the study that describes the displacement of bodies, without addressing the cause of the movement.

Another subdivision of mechanics is that which describes the degree of deformability of matter:

- *Theoretical mechanics*, the mechanics of particles and rigid (non-deformable) bodies.
- *Solid mechanics*, the mechanics of solid deformable bodies.
- *Fluid mechanics*.
- *Gas mechanics*.

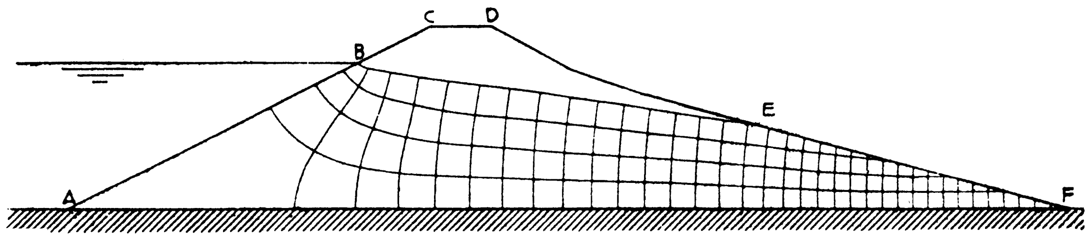

Figure 1.9 Stream lines and equipotential lines in a dam on an impermeable subsoil.

Figure 1.8 The Eiffel tower (1889) was the world's first 1000-foot tower. Photograph: Hans Welleman.

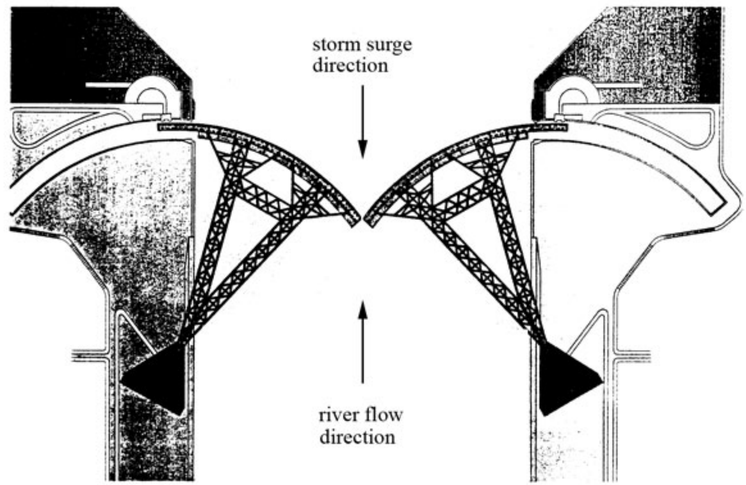

Figure 1.10 The Maeslant barrier – a storm barrier in the Nieuwe Waterweg near Rotterdam in the Netherlands.

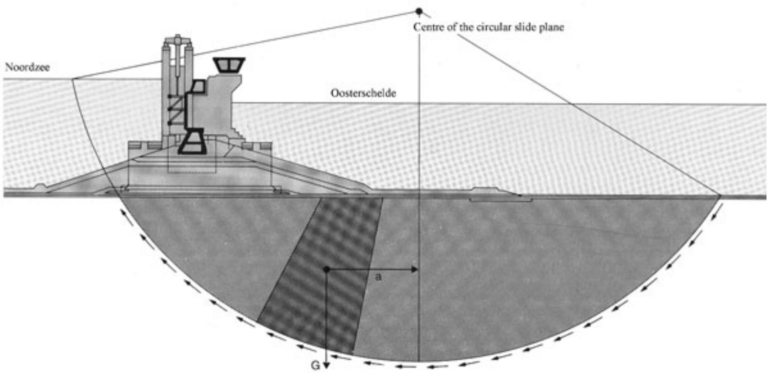

Figure 1.11 The Oosterschelde barrier; the subsoil has to be able to bear the structure.

1.1.3 Applied mechanics

In principle, the *mechanics of structures* addresses both the statics and dynamics of structures. This book solely covers the statics of structures.

Mechanics allows us to investigate to what degree a structure, both in its entirety and with respect to its individual components, is effective and reliable regarding strength, stiffness, and stability.[1]

For structures made of solid, deformable materials (concrete, wood, synthetics, or metals such as steel or aluminium), the field of mechanics is also known as *applied mechanics*.

The part of applied mechanics which focusses on calculating the forces in a structure is known as *structural mechanics*. The part in which the focus is on stress and deformation (strength and stiffness) is known as *mechanics of materials*. The division between structural mechanics and the mechanics of materials is only effective for so-called *statically-determinate structures*,[2] or structures in which the force flow can be determined directly from the equilibrium. For calculations relating to structures other than those that are statically-determinate (so-called *statically-indeterminate structures*) one has to use elements from both structural mechanics and the mechanics of materials.

The behaviour of a structure must be investigated "*beyond the base*". For example, it is important that the slides in the Oosterschelde barrier in Figure 1.11 are sufficiently strong, but it is equally important that the structure can be properly carried by the subsoil. Since the behaviour of soil clearly

---

1 *Stability* is defined as the *reliability of the equilibrium*. Since the stability of the equilibrium depends on the stiffness of the structure, the stability demand can also be interpreted as a stiffness demand.

2 The concepts *statically-determinate* and *statically-indeterminate* are covered in more detail in Chapter 4.

differs from that of regular solid material, the investigation into the forces and deformations in soil is part of a separate field of expertise known as *soil mechanics*.

1.1.4 Theory and experiment

Mechanics, as a part of physics, is a science that addresses the determination of laws and patterns that can be used to describe natural phenomena and, more importantly, that can be used to predict them. As such, mechanics is an *empirical science*: it aims to formulate the phenomena investigated and their mutual relationship as accurately as possible. In doing so, it is not the results from the calculations that are decisive, but rather their agreement with what we learn from observation and experimentation. After all, we want to be able to predict with a certain degree of accuracy whether a satellite we launch will end up in its orbit, or whether a bridge is sufficiently strong and rigid.

Reality is however far too complex to be described fully. For this reason, one always has to work with a *model*, a simplified representation of reality, and one which addresses only a limited number of factors concurrently. Which aspects are addressed and which *schematisations* (simplifications) are used, depends on the objective in question.

1.1.5 Schematisation

Schematisation is an abstraction that at the same time includes all the relevant issues. When investigating the movement of the earth around the sun, the dimensions of the earth are of subsidiary importance, and we schematize the earth as a particle. If, however, one is looking to investigate the rotation of the earth around its axis, we do have to take the dimensions of the earth into consideration.

When calculating the force flow in a framework, it is common practice to schematize the columns and beams as so-called line elements, and to

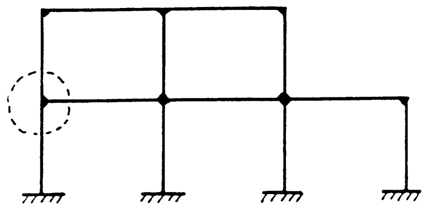

Figure 1.12 When performing calculations for a building, the columns and beams are generally modelled as so-called line elements; the column-beam joints are thereby reduced to particles, with negligible dimensions.

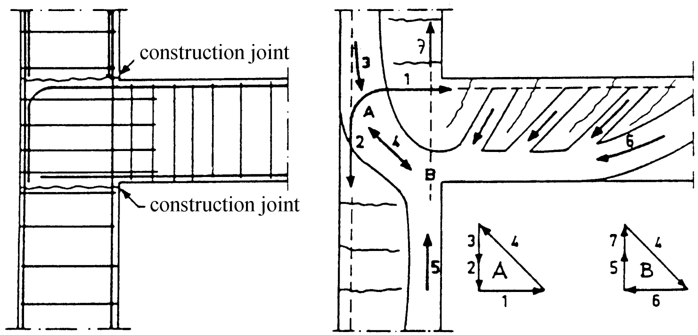

Figure 1.13 When detailing the column-beam joints, like here in reinforced concrete, the dimensions are no longer negligible.

represent the joints between the columns and beams as particles (see Figure 1.12). If we want to find out more about the interaction of forces in a column-beam joint, for example in the case of a concrete structure to be able to determine where the reinforcement has to be placed, the dimensions of the joint can no longer be ignored, and one has to use another schematisation for the joint (see Figure 1.13).

Mechanics uses a range of concepts that offer a schematisation of reality. For example, dimensions are ignored for *particles*, while for *rigid bodies* deformation is ignored. Another concept is that of *stress*, in which a continuous structure of matter is assumed, while in reality (on a micro level) this is discrete, with molecules and atoms. You should be aware of these schematisations.

Much knowledge within mechanics is set down using mathematical formulas, based on certain schematisations and modelling. Mathematics subsequently offers a language that enables us to formulate and solve the problems, and interpret the solution unambiguously. We can then use the findings to make predictions relating to the behaviour of a structure. It is from this predictive capacity that the science of mechanics derives its practical use.

Within a given schematisation, mathematical models are used, and the results are exact; for this reason, mechanics is often called an *exact science*.

1.2 Quantities, units, dimensions

1.2.1 Quantities and their units

Mechanics involves measurable, physical quantities. A *quantity* X is generally characterised by a *numerical value* $\{X\}$ and a *unit* $[X]$. This can be symbolically described as

$$\text{quantity} = \text{numerical value} \times \text{unit}$$
$$X = \{X\} \times [X].$$

The unit $[X]$ is the degree to which the quantity X is measured.

In mechanics, one uses the International Units System (Système International d'Unité), abbreviated in all languages to SI. The SI includes

- seven basic units (Table 1.1);
- two supplementary units (Table 1.2);
- and a large number of derived units.

Basic units, critical in the structural mechanics, are *length*, *mass* and *time*.

Length (ℓ)
A measure for measuring distances in space. Space is defined as the geometric area in which people live and work and in which they build their structures. The basic unit of length is the metre [m].

Mass (m)
A measure for the characteristic of a body that it resists a change in its movement. This characteristic is known as the *inertia* of the body. The basic unit for mass is the kilogram [kg] (not grams!).

Time (t)
A measure for the sequence of events. The fundamental unit for time is the second [s].

SI derivative units are obtained from the definitions of the derived basic quantities as products and quotients of powers of basic units. A number

Table 1.1 Basic quantities and basic units.

Basic quantity		**Basic unit**	
Name	**Symbol**	**Name**	**Symbol**
length	ℓ	metre	m
mass	M	kilogram	kg
time	T	second	s
electric current	I	amp	A
thermodynamic temperature	T	Kelvin	K
amount of material	N	mol	mol
luminosity	I	candela	cd

Table 1.2 Supplementary quantities and units.

Supplementary quantity		**Supplementary unit**	
Name	**Symbol**	**Name**	**Symbol**
(plane) angle	α	radian	rad
solid angle	Ω	steradian	sr

of derived units have their own names and their own symbols. You will find a number of these units in Table 1.3.

Table 1.3 Derived units with their own name and symbol.

Derived quantity	Derived unit	
	Name	Symbol
area	square metre	m^2
volume, content	cubic metre	m^3
frequency	Hertz	$Hz = s^{-1}$
force	Newton	$N = kgm/s^2$
pressure, tension	Pascal	$Pa = N/m^2$
work, energy, amount of warmth	Joule	$J = Nm$
capacity, energy flow	Watt	$W = J/s$

1.2.2 Prefixes

If numbers are either very large or very small, you can use a prefix for the unit. Frequently used prefixes are shown in Table 1.4.

Example:

$$1\ \text{MPa} = 10^6\ \text{Pa} = 10^6\ \text{N/m}^2 = 1\ \text{N/mm}^2.$$

Table 1.4 Common SI prefixes.

Prefix	Symbol	Factor
giga	G	10^9
mega	M	10^6
kilo	k	10^3
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}

For derived units as a product of a number of units, we can join up the symbol group, unless this gives rise to confusion. In the latter case, place a multiplication point between the units. In this vein, Nms could either be Newton-metre-second or Newton-milliseconds. Depending on what one is trying to say, you should therefore write Nm·s or N·ms.

1.2.3 Dimensions

Besides the unit $[X]$ in which a quantity X is expressed and the associated numerical value $\{X\}$, a quantity also has a dimension $\dim(X)$. The *dimension* indicates the type of quantity without saying anything about the choice of unit or the magnitude of the numerical value.

The dimensions of the basic quantities are called the basic dimensions. For the basic dimensions of length (ℓ), mass (m), and time (t) one writes

$$\begin{aligned} \dim(\ell) &= L, \\ \dim(m) &= M, \\ \dim(t) &= T. \end{aligned}$$

You will find a number of examples of derived quantities and their dimensions in Table 1.5.

Dimension formulas can be used to determine whether inaccuracies have occurred in deriving a physical relationship. This is known as a *dimension check*. They can be used to check that the expressions to the left and the right of the equals sign have the same dimensions. The same can be achieved by checking whether the products of all the units, expressed in terms of the basic units, are the same on both sides.

The radian and solid angle are considered dimensionless quantities. When performing a dimension check, we must assign the symbols for rad and sr the dimension 1.

Table 1.5 Examples of derived quantities and their dimensions.

Type of quantity	Definition	Dimension formula	SI unit
velocity	$v = \mathrm{d}u/\mathrm{d}t$	LT^{-1}	m/s
force	$F = m \cdot a$	LMT^{-2}	$\mathrm{N} = \mathrm{kgm/s}^2$
energy, work	$E = A = F \cdot \ell$	$\mathrm{L}^2\mathrm{MT}^{-2}$	$\mathrm{J} = \mathrm{kgm}^2/\mathrm{s}^2$

1.3 Vectors

1.3.1 Scalars and vectors

Certain physical quantities are fully determined by a numerical value with the associated unit. These include length, mass, time, temperature, work and energy, and are referred to as scalar quantities, or *scalars*. Other physical quantities can be fully described only if, in addition to the magnitude (determined by a number and a unit), one also defines in which direction in space the quantity is oriented. If these quantities with a magnitude and direction meet the conditions of the so-called *parallelogram rule* (see Section 1.3.4), they are known as *vectors*. Vectors include motion, velocity, impulse, acceleration, and force.

In order to distinguish vectors from scalars, the symbols for a vector are printed in bold (**a**) or we place an arrow over the symbol ($\vec{a}$).

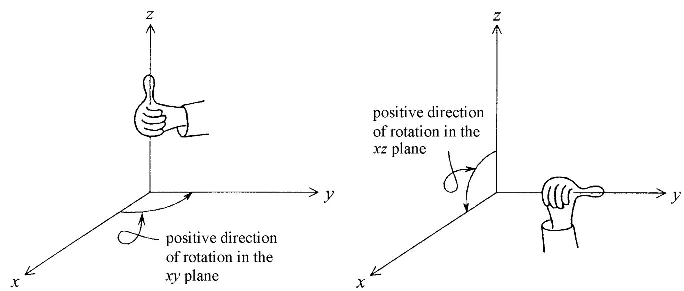

Figure 1.14 A right orthogonal coordinate system.

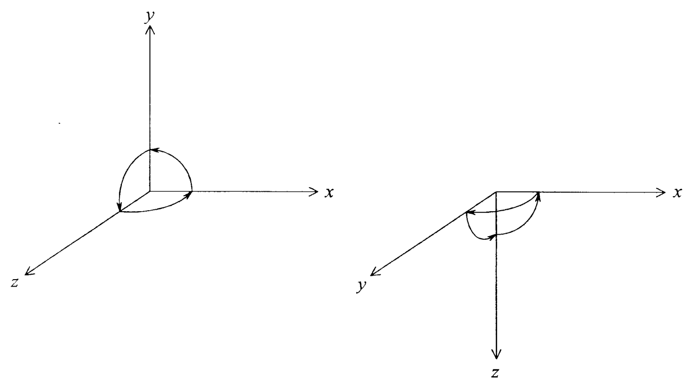

Figure 1.15 A right orthogonal coordinate system in two other positions, with the positive direction of rotation in the coordinate planes.

Other physical quantities besides scalars and vectors are *tensors*[1] (of the second order and above). Tensors are not covered in this book.

1.3.2 Coordinate system

Vectors are quantities with a direction in space. Space is seen as three dimensional and *Euclidian* (after Euclid[2]). When describing phenomena in space, one uses a *right orthogonal coordinate system*. This is a system of three mutually perpendicular axes x, y and z, that are oriented in such a way that they meet the conditions of the so-called *right-hand rule*: if you make a fist with the fingers of your right hand, as shown in Figure 1.14, and you point the free thumb in the z direction, the bent fingers in your fist have to point in the direction of a rotation with the smaller angle of the x axis to the y axis. This direction of the rotation with the smaller angle of the x axis to the y axis is called the positive *direction of rotation* in the xy plane (about the z axis). In this description, x, y and z can be exchanged cyclically (see Figure 1.14).

Figure 1.15 shows two more examples of such coordinate systems, with the positive directions of rotation in the various coordinate planes.

An orthogonal coordinate system is called *Cartesian* (after Descartes[3]) if equal units are chosen along the coordinate axes.

---

1 Scalars and vectors can be seen as members of the family of tensors. Vectors are also known as tensors of the first order. Scalars are tensors of the zero order. Second order tensors in mechanics include strain tensor, stress tensor, and bending stiffness tensor. Tensors can be recognised by the transformation rules for their components when rotating the coordinate system.

2 Euclid (approx. 300 BC), Greek mathematician in Alexandria.

3 René Descartes (Cartesius) (1596–1650), French mathematician and philosopher. Main work: “Discours de la méthode” (1637).

If equal unit vectors are chosen along the axes, this is referred to as an *orthonormal* coordinate system (contraction of orthogonal and normalised).

1.3.3 Types of vectors

In a diagram, a vector in space can be represented by an arrow. The direction of the arrow represents the direction of the vector. The length of the arrow (in a particular scale) can be drawn to represent the magnitude of the vector.

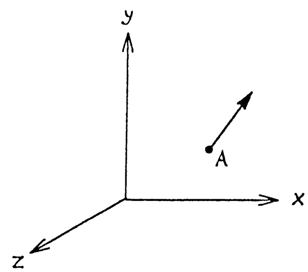

Figure 1.16 Fixed vector with point of application A.

There are three types of vector:

- *Fixed vectors*
  Fixed vectors, in addition to their magnitude and direction, also have a *point of application* (see Figure 1.16).
  Example: a force on a deformable body.
- *Sliding vectors*
  Sometimes the location of the point of application is of no importance and may be moved in the direction of the vector. This is called a sliding vector. Sliding vectors do not have a fixed point of action, but have only a *line of action* (see Figure 1.17).
  Example: the force on a rigid body.
- *Free vectors*
  When the place of the line of action of a vector is not important either, one refers to a free vector.
  Example: The translation of a rigid body. All points of the body are subject to the same displacement. The free vector stands for the entire collection of displacement vectors (see Figure 1.18).

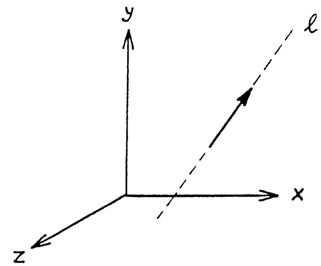

Figure 1.17 Sliding vector with line of action ℓ.

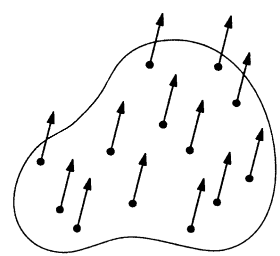

Figure 1.18 Free vector.

Comment:
If we want to investigate the equilibrium (or the motion) of a body as a whole, the body can often be considered a rigid (non-deformable) body, with the forces as sliding vectors. After all, it does not make a difference for rigid bodies whether it is kept in equilibrium by a force from above or

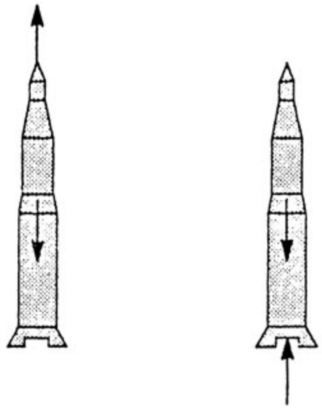

Figure 1.19 For the equilibrium of a rigid body, it is not relevant whether a force is moved along its line of action. It certainly makes a difference with respect to internal phenomena.

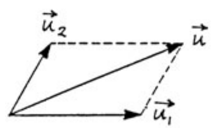

Figure 1.20 Vector addition using the parallelogram rule.

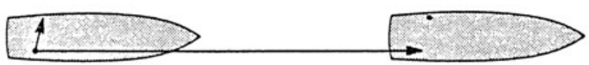

Figure 1.21 The vector addition illustrated using a sailor walking on a moving ship.

below (see Figure 1.19). On the other hand, if one is looking to investigate deformations or internal phenomena within the body, the points of application of the forces do play a role and the forces must be considered fixed vectors. For phenomena inside bodies (such as human bodies), it certainly makes a difference whether the body is hung from above or is supported from below!

1.3.4 Parallelogram rule

We can add two vectors with the same point of application into a single vector using the so-called *parallelogram rule* in Figure 1.20. The parallelogram rule is easy to understand if one imagines, as in Figure 1.21, the movement of a sailor walking on a moving ship. The displacement $\vec{u}$ of the sailor with respect to the earth consists of the sum of the displacement $\vec{u}_1$ of the ship with respect to the earth and his own displacement $\vec{u}_2$ with respect to the ship. In the same way, one can also add up velocity vectors and forces.

For the vector addition, as shown in Figure 1.20, one writes

$$\vec{u} = \vec{u}_1 + \vec{u}_2.$$

In reverse, we say that $\vec{u}_2$ is the difference between $\vec{u}$ and $\vec{u}_1$, or

$$\vec{u}_2 = \vec{u} - \vec{u}_1.$$

1.3.5 Vector components and scalar components

We often describe a vector by means of its so-called *components*. If $\vec{e}_x$, $\vec{e}_y$ and $\vec{e}_z$ are the unit vectors along respectively the x, y and z axis (vectors directed along the axes and with a length equal to 1), the vector can also be defined as the vector sum of its three components (see Figure 1.22):

$$\vec{a} = \vec{a}_x + \vec{a}_y + \vec{a}_z = a_x\vec{e}_x + a_y\vec{e}_y + a_z\vec{e}_z.$$

The vector quantities $\vec{a}_x$, $\vec{a}_y$ and $\vec{a}_z$ are known as the *vector components* of vector $\vec{a}$. The scalar quantities a_x, a_y and a_z are the *scalar components*[1] of vector $\vec{a}$.

In this book we will usually take the word *component* to mean *scalar component*.

The *magnitude* or *norm*[2] of the vector $\vec{a}$ is:

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2} \qquad (a \geq 0).$$

To add two vectors given by their components we add the respective components. The sum of two vectors $\vec{a}$ and $\vec{b}$ with components a_x, a_y and a_z, respectively b_x, b_y and b_z, is:

$$\vec{a} + \vec{b} = (a_x + b_x)\vec{e}_x + (a_y + b_y)\vec{e}_y + (a_z + b_z)\vec{e}_z.$$

This is illustrated in Figure 1.23 for two vectors in the xy plane (with $a_z = b_z = 0$).

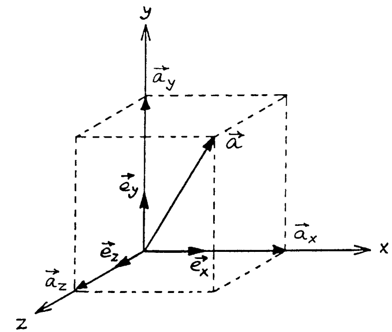

Figure 1.22 The vector components $\vec{a}_x$, $\vec{a}_y$, $\vec{a}_z$ of vector $\vec{a}$.

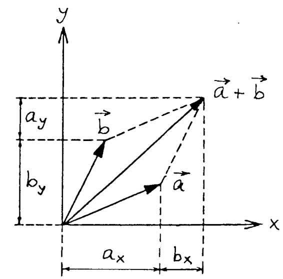

Figure 1.23 It is possible to add two vectors by adding their associated scalar components.

1.3.6 Formal and visual notation of a vector

So far in the figures, the arrow for a vector included the vector symbol (letter with an arrow above). In addition to this *formal notation* there is also a *visual notation*. Both notations are shown in Figure 1.24.

Figure 1.24 Vector notation (a) and visual notation (b).

[1] The scalar components a_x; a_y; a_z of vector $\vec{a}$ are not scalars: they depend on the coordinate system that is used.

[2] The magnitude or norm a of vector $\vec{a}$ is a scalar: it is independent of the coordinate system that is used.

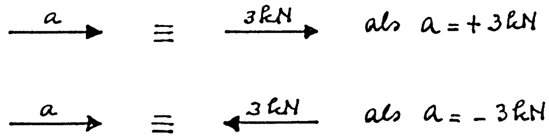

Figure 1.25 In visual notation, the arrow depicted should be seen as a unit vector that has to be multiplied by the depicted value.

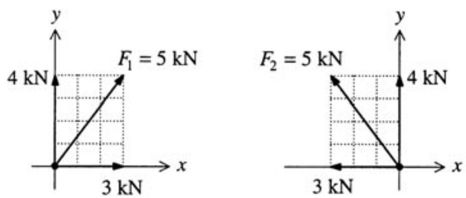

Figure 1.26 The forces $\vec{F}_1$ and $\vec{F}_2$ resolved into their components.

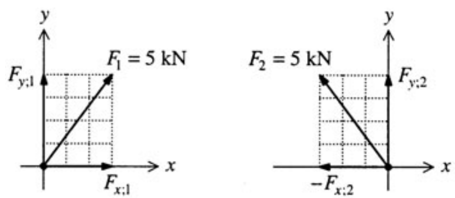

Figure 1.27 If we want to define the components of force $\vec{F}_2$ as $F_{x;2}$ and $F_{y;2}$ in a visual model, we have to place a minus sign next to $F_{x;2}$, the x component of $\vec{F}_2$.

In the visual notation, each arrow shown reflects a unit vector, which has to be multiplied by the value shown with the arrow. If this value is negative, the vector works in the direction opposite to the one shown (see Figure 1.25). Since in visual notation the emphasis lies on "*seeing what is happening*", it is preferred to not include a negative value alongside a vector arrow.

The visual notation is frequently used in mechanics for manual calculations. When setting up manual calculations, the visual aspect plays an important role as one generally links the calculation to a "*picture*" on the basis of which one can better imagine what is happening.

In Figure 1.26, forces $\vec{F}_1$ and $\vec{F}_2$ have been resolved into components along the x and y axis. All the forces have been drawn in the directions in which they operate and include their magnitude.

If one wants to name the components in the xy coordinate system shown, one has to imagine that F_x and F_y relate to the (not shown) unit vectors in the coordinate system, respectively $\vec{e}_x$ and $\vec{e}_y$.

Therefore[1]

$$F_{x;1} = +3 \text{ kN}; \quad F_{y;1} = +4 \text{ kN};$$
$$F_{x;2} = -3 \text{ kN}; \quad F_{y;2} = +4 \text{ kN}.$$

The x component of force $\vec{F}_2$ opposes the x direction (is opposite to the direction of the unit vector $\vec{e}_x$) and is therefore negative. If one wants to denote the components of $\vec{F}_1$ and $\vec{F}_2$ by F_x and F_y in a visual representation, as in Figure 1.27, one must place a minus sign next to $F_{x;2}$, the x

[1] The *direction indices* x and y always precede the other indices. It is common practice to separate the indices by a semicolon. Sometimes the *separator* is omitted.

component of force $\vec{F}_2$.

Forces are vectors. In the formal notation they are indicated with an arrow over the symbol: $\vec{F}$. In structural mechanics the visual notation is generally used. In that case it is usual to indicate a force by its magnitude $F = |\vec{F}|$. In this book we will principally use the visual notation for a force.

1.3.7 Vector properties

Quantities can be imagined as vectors if they meet the calculation rules for vectors. These rules include the *commutative* property for addition:

$$\vec{a} + \vec{b} = \vec{b} + \vec{a}$$

and the *associative* property for addition:

$$(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c}).$$

These properties indicate that the vector sum is independent of the order in which the vectors are added.

Not every quantity that is defined by a magnitude and a direction is a vector. For example, the rotation of a body, a quantity with a magnitude and a direction, is not a vector, as the quantity does not meet the commutative and associative properties of the addition. This can be checked for a book in the xz plane by first rotating it through 90° about the y axis and then through 90° about the z axis. As shown in Figure 1.28, the final position changes if the rotation is performed in a different order.

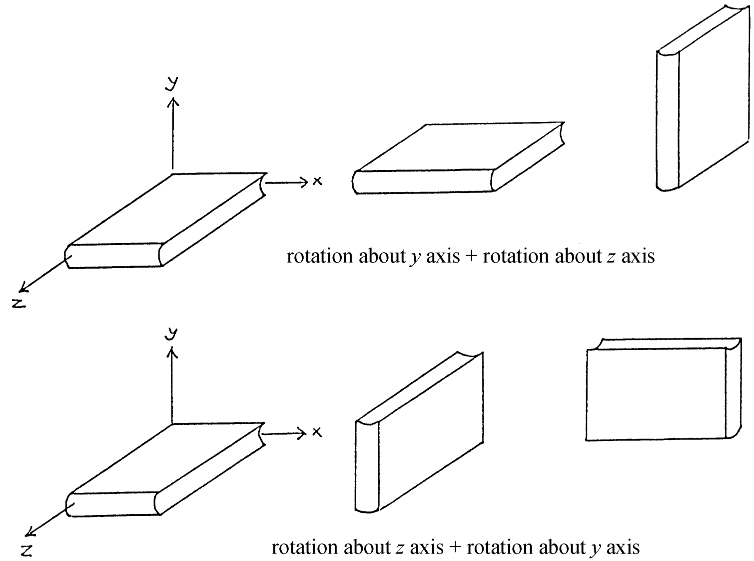

Figure 1.28 If the order changes when adding (finite) rotations, the end result also changes.

1.4 Newton's Laws

1.4.1 Basic laws

The basic laws for the displacement of a particle (a body with negligibly small dimensions but with some mass) were first formulated by Newton (1687).[1] Newton's three laws are as follows:

- First law or *law of inertia*.
  Every particle persists in its state of rest or uniform motion in a straight line unless it is compelled to change that state by forces imposed on it.
- Second law or *law of motion*.
  The rate of change in *momentum* of a particle (the product of mass and velocity) is equal to the force applied to it, and has the direction of that force.
- Third law or the *law of action and reaction*.
  If particle (1) exerts a force on particle (2), particle (2) will exert an equal and opposite force on particle (1).

Law of inertia

The first law states that a particle at rest will remain at rest if no force is exerted on it, and that a particle that is in motion in a straight line at a constant speed, will continue that movement at that same speed in the same straight line if no forces are exerted on it. The property with which a particle resists a change in its state of rest or movement is called its *inertia*. Newton's first law is therefore also known as the law of inertia.

---

[1] Sir Isaac Newton (1642–1727), an English mathematician and physicist, published his laws at the age of 44 in his book "*Philosophiae naturalis principia mathematica*", also known as "*Principia*". In his laws, Newton uses the word *body*. Later developments in mechanics showed that it must relate to a body without dimensions, here referred to as a *particle*. A body with finite dimensions can still perform rotations, which are not mentioned by Newton.

Law of motion

The second law is defined by the following formula:

$$\vec{F} = \frac{\mathrm{d}(m\vec{v})}{\mathrm{d}t}.$$

Here, $\vec{F}$ is the force on the particle, m its mass, and $\vec{v}$ its velocity.[1] The notation with vectors shows that the change in momentum has the same direction as the force.

If the mass of the particle does not change during the motion, the second law can also be written as

$$\vec{F} = m\vec{a},$$

in which $\vec{a}$ represents the acceleration of the particle:

$$\vec{a} = \frac{\mathrm{d}\vec{v}}{\mathrm{d}t}.$$

It should be noted that the first law is actually a special case of the second law: if the force on the particle is zero, its acceleration is also zero.

By not including a proportionality constant in the mathematical formulation of the second law (the formulation in words only refers to proportionality between force and a change in momentum), we actually define the unit of force as the force that gives a mass of 1 kilogram an acceleration of 1 metre per second squared. This unit of force is the Newton (symbol N,

---

1 One of the essential distinctions in mechanics is between *speed* and *velocity*: speed is a scalar and velocity is a vector. The speed v is the magnitude of the velocity $\vec{v}$: $v = |\vec{v}|$. If a particle traverses, say, a circle, with constant speed v, then its velocity $\vec{v}$ will change, because its direction is changing.

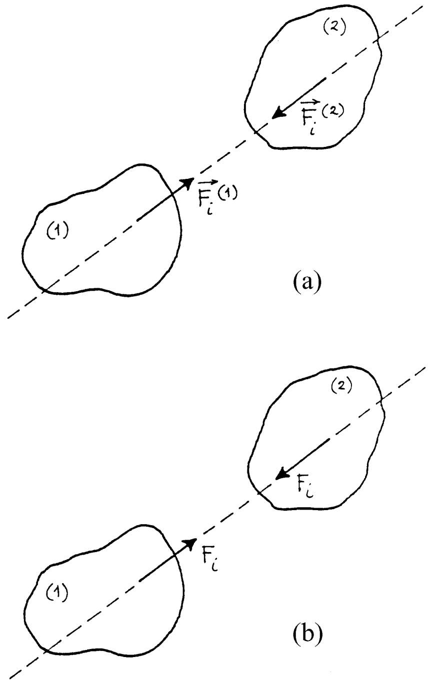

Figure 1.29 Newton's law of action and reaction in (a) vector notation ("*action* = –*reaction*") and (b) visual notation ("*action* = *reaction*").

see Section 1.2.2):

$$1\ \text{N} = 1\ \text{kg} \cdot \text{m/s}^2.$$

Law of action and reaction

The third law is the so-called law of action and reaction. If one defines the force that body (1) exerts on body (2) by $\vec{F}_\text{i}^{(2)}$ and the force which body (2) exerts on body (1) by $\vec{F}_\text{i}^{(1)}$, the third law states that[1]

$$\vec{F}_\text{i}^{(1)} = -\vec{F}_\text{i}^{(2)}.$$

According to the law of action and reaction, forces always act in pairs of equal and opposite forces. The law of action and reaction is depicted in Figure 1.29 in both vector notation and visual notation.

In the vector notation in Figure 1.29a one would say

"*action* = −*reaction*".

In the visual notation in Figure 1.29b one would rather say

"*action* = *reaction*".

In both cases, the meaning is the same. In the visual notation it can clearly be seen that the *interaction* between both bodies occurs between the *pair of forces* F_i.

1 The upper index denotes the body on which the force is exerted, the lower index is the i of *interaction*.

1.4.2 Law of gravitation

Newton also formulated the law that describes the attraction between two bodies. This *Law of Universal Gravitation* states that the force between two particles with masses m_1 and m_2 at a distance r apart is an attraction that operates along the joining line of the two particles, with magnitude

$$F = G\frac{m_1 m_2}{r^2}.$$

Here, G is a universal constant, which is the same for all pairs of particles. The value of G, the *gravitation constant*, has been experimentally determined as

$$G = 66.71 \times 10^{-12}\ \mathrm{Nm^2/kg^2}.$$

In general, all attractive forces on earth between bodies are dominated by the attractive force of the Earth on those bodies, as the mass of the Earth is so much greater (5.975×10^{24} kg) than that of any other body.

On the basis of Newton's second law and the law of gravitation, it follows that in the event of a free fall near the surface of the earth, all masses (in the absence of friction) are subject to the same acceleration (denoted by g, the *gravitational acceleration*).

Assuming that one can imagine the mass of the earth as concentrated in its centre, this gives

$$g = \frac{GM}{R^2},$$

whereby M is the mass of the earth and R is the distance from the particle to the centre.

Since the Earth is flattened at the poles, the exact value of g depends on the location on earth. At the equator, g is approximately $9.790\ \mathrm{m/s^2}$, at the poles it is approximately 9.832 m/s^2, while in the Netherlands (52° lattitude) it is 9.813 m/s^2.

For simplicity, in building practice, we assume

$$g = 10 \text{ m/s}^2.$$

The 2% error is minor if one considers all the uncertainties in, for example, the magnitudes and points of application of the loads, the dimensions of the structural elements, and the properties of the materials.

Since 1 N = 1 kg·m/s^2, we can also say:

$$g = 10 \text{ N/kg}.$$

In the gravitational field, a mass of 1 kg therefore weighs 10 N. The gravitational acceleration g is also known as the *gravitational field strength*.