Attitude Determination & Control

1 Attitude Data

The know the **attitude** and **motion** of a spacecraft, we need to know its **position**, **velocity**, **orientation** and **rotational velocity**. The first two concern translations, while the latter two concern rotations.

Let's consider an orbiting satellite. The origin lies at the COM of the spacecraft. The x-axis points in the direction of the **velocity vector**. The z-axis points to the center of the earth (the **nadir vector**). Finally the y-axis is perpendicular to the past two, according to the right-hand rule (the **orbit normal vector**). Rotation about the x-axis is called **roll**, rotation about the y-axis is called **pitch** and rotation about the z-axis is called **yaw**.

2 Attitude Dynamics and Kinematics

The attitude of a spacecraft can be calculated using dynamics and kinematic equations. To do this, it is usually assumed that the spacecraft is a **rigid body**. It can now be derived that

$$I_{xx}\dot{\omega}_x + (I_{zz} - I_{yy})\,\omega_y\omega_z = M_{c_x} + M_{d_x}, \tag{2.1}$$
$$I_{yy}\dot{\omega}_y + (I_{xx} - I_{zz})\,\omega_z\omega_x = M_{c_y} + M_{d_y}, \tag{2.2}$$
$$I_{zz}\dot{\omega}_z + (I_{yy} - I_{xx})\,\omega_x\omega_y = M_{c_z} + M_{d_z}, \tag{2.3}$$

where ω is the rotational velocity of the spacecraft, I is the moment of inertia, M_c is the control torque and M_d is the disturbance torque, about the corresponding axes.

Let's define ϕ as the **roll angle** (the angle over which roll has taken place), θ as the **pitch angle** and ψ as the **yaw angle**. Now ϕ, θ and ψ are the **attitude angles**. In fact, the orientation of the spacecraft is given by the vector containing these angles. It can now also be shown that

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} + \omega_0 \begin{bmatrix} \psi \\ 1 \\ -\phi \end{bmatrix}, \tag{2.4}$$

where ω_0 is the angular velocity of the orbit. Note that the second part is necessary, because the satellite is rotating around Earth. Due to this, the z-axis continuously changes, as it is defined to point to the center of the Earth.

3 Disturbance Torques

Space isn't a perfect environment. There are many types of disturbances. First there are **aerodynamic disturbances**. This decreases as you are further away from earth (according to $e^{-\alpha r}$, where r is the distance from Earth). Earth's **magnetic field** can also cause disturbances. These also decrease as you are further away from earth (now according to r^{-3}). The same goes for **gravity gradients**, which are changes in Earth's gravitational field. The solar also influences the spacecraft attitude. **Solar radiation** can cause disturbances. On some places in the solar system also micrometeorites can play a role. And sometimes internal forces of the spacecraft can also causes disturbances.

How do we cope with these disturbances? One way to do this, is by using a **control wheel**. Let's suppose we have a spacecraft with rotational velocity Ω_v with respect to a reference axis. In that spacecraft is a control wheel, rotating with a velocity Ω_w with respect to the spacecraft. The wheel and the vehicle have moment of inertias of I_w and I_v and angular momentums of

$$H_w = I_w\left(\Omega_w + \Omega_V\right), \qquad \text{and} \qquad H_v = I_v\Omega_v. \tag{3.1}$$

If a disturbance torque M is acting on the spacecraft, an angular acceleration will take place. This happens according to

$$M = \frac{d}{dt}\left(H_w + H_v\right) = I_w\left(\dot{\Omega}_w + \dot{\Omega}_v\right) + I_v\dot{\Omega}_v = I_w\dot{\Omega}_w + \left(I_w + I_v\right)\dot{\Omega}_v. \tag{3.2}$$

If the angular acceleration of the spacecraft needs to be zero, then we find that $M = I_w\dot{\Omega}_w$. All the torque is then applied to the control wheel.

There are two types of disturbance torques that can be considered. There are **cyclic torques** which oscillate over an orbit. Since the torque is alternatingly positive and negative, the control wheel only needs to store the torque for a while. However, there are also **secular torques**. These torques build up angular momentum over time. So a control wheel eventually won't be enough. There should be some way to dump angular momentum (often by using thrusters).

4 Passive Control Systems

Let's take a look at the ways in which we can control the attitude of an spacecraft. In **spin stabilisation** the spacecraft is spinning about one axis. Due to gyroscopic stabilisation it will hardly rotate about other axes. Let's, for example, suppose it's spinning about its x-axis and a disturbance torque M_{d_y} appears. Because ω_x is big, ω_z will only get a small value (remember the equations of motion). Also it is the rotational velocity ω_z that will be nonzero (not the rotational acceleration $\dot{\omega}_y$). This means that the rotation caused by the disturbance torque will stop when the disturbance torque has disappeared. So we can conclude that the effect of the disturbance torque is limited.

In **dual spin stabilisation** only part of the spacecraft rotates. A spacecraft with a control wheel is an example of this. While the spacecraft has gyroscopic stabilisation, it is still possible to have a non-rotating platform on which equipment can be placed.

Some stabilisation methods use the environment. In **gravity gradient stabilisation** the spacecraft makes use of the fact that the gravity field of a planet decreases with increasing distance from that planet. The moment caused is given by

$$\mathbf{M_g} = \begin{bmatrix} M_{g_x} \\ M_{g_y} \\ M_{g_z} \end{bmatrix} = 3\omega_0^2 \begin{bmatrix} \left(I_{zz} - I_{yy}\right)\phi \\ \left(I_{zz} - I_{yy}\right)\theta \\ 0 \end{bmatrix}. \tag{4.1}$$

Let's now define the coefficients

$$k_x = \frac{I_{yy} - I_{zz}}{I_{xx}}, \qquad k_z = \frac{I_{yy} - I_{xx}}{I_{zz}}. \tag{4.2}$$

To have a gravitationally stable spacecraft, it must satisfy to

$$3k_x + k_x k_z + 1 > 0, \qquad 3k_x + k_x k_z + 1 > 4\sqrt{k_x k_z} \qquad \text{and} \qquad k_x k_z > 0. \tag{4.3}$$

If a spacecraft satisfies this condition, it doesn't automatically imply that it is gravitationally stable though.

Magnetic stabilisation is slightly similar to gravitational stabilisation. However, now the spacecraft uses the magnetic field of the earth to orient itself. In **aerodynamic stabilisation** aerodynamic forces are used to control the spacecraft orientation.

5 Active Control Systems

The previously discussed methods of stabilisation are all **passive** methods. Once a spacecraft is in space, it has no further control on its attitude. An example of an **active** stabilisation method is **three axis active attitude control**. Here the attitude of the spacecraft is controlled about all three axis. This can be done with for example reaction wheels and thrusters.

Let's once more consider an spacecraft with a control wheel. Let's suppose the control wheel is exerting a torque of M_c on the spacecraft. The spacecraft has control over this M_c. We can set

$$M_c = -K_p\Theta - K_d\dot{\Theta}, \tag{5.1}$$

where K_p and K_d are positive constants. Also Θ is the angle of the spacecraft with respect to some reference point (so $\dot{\Theta} = \Omega_v$). We now have programmed a **PD controller**. Here the P stands for the Proportional-term $K_p\Theta$ and the D stands for the Differentiation-term $K_d\dot{\Theta}$. In a **PID controller** also an Integration-term is included, which removes steady state errors due to disturbances.

Also thrusters can be used to provide three axis stabilisation. When thrusters are applied, it is important to consider the **moment arm** of the thruster with respect to the COM of the spacecraft.

6 Sensors

It's nice to be able to rotate. But you first need to know your orientation, before you can decide to change it. Therefore sensors are used. There are many types of sensors.

Some types use external reference points. **Sun sensors** use the sun as a reference point. **Earth sensors** use the Earth. Both are relatively inaccurate. **Star sensors** are more accurate and also aren't troubled by eclipses. However, they are heavier and use more power.

Gyroscopes do not use external reference points. Although they are initially very accurate, their accuracy decreases as time passes. Combining gyroscopes with other sensor systems could solve this problem. Now the gyroscope can be calibrated regularly.

Magnetometers measure Earth's magnetic field and calculate the attitude from the acquired data. Finally, if the satellite is close to the ground (low Earth orbits), **GPS** can be used to determine position.