Appendix A

PHYSICAL CONSTANTS

q	electronic charge	1.602×10^{-19} coulomb
m_0	electronic rest mass	9.108×10^{-31} kg
c	velocity of light in vacuum	2.998×10^{8} m/s
ε_0	permittivity of free space	8.854×10^{-12} farad/m
h	Planck's constant	6.625×10^{-34} joule × s
k	Boltzmann's constant	1.380×10^{-23} joule/K
kT/q	thermal voltage	0.02586 V (at 300 K)
λ_0	wavelength in vacuum associated with photon of 1 eV energy	1.239×10^{-6} m

SELECTED PROPERTIE OF SILICON (at 300 K)

E_g	energy band gap	1.08 eV
N_C	effective density of states in conduction band	3.0×10^{25} m^{-3}
N_V	effective density of states in valence band	1.0×10^{25} m^{-3}
n_i	intrinsic concentration of carriers	1.5×10^{16} m^{-3}.
ε_r	relative permittivity	11.7
n	refractive index	3.5 (at 1.1×10^{-6} m wavelength)
μ_e	electron mobility	1350×10^{-4} m^2/Vs
μ_h	hole mobility	480×10^{-4} m^2/Vs
D_e	electron diffusion coefficient	$0.02586 \times \mu_e$
D_h	hole diffusion coefficient	$0.02586 \times \mu_h$

Appendix B

Chapter 2: SOLAR RADIATION

$$\nu = c/\lambda \tag{2.1}$$

$$h\nu = \frac{1}{q}\frac{hc}{\lambda} \tag{2.2}$$

$$\Phi(\lambda) = P(\lambda)\frac{\lambda}{hc} \tag{2.3}$$

$$Air\ mass = (\cos\theta)^{-1} \tag{2.4}$$

Chapter 3: SEMICONDUCTOR MATERIALS FOR SOLAR CELLS

$$E_G = E_C - E_V \tag{3.1}$$

$$g_C(E) = \left(\frac{4\sqrt{2}\,\pi m_n^*}{h^3}\right)^{3/2}(E - E_C)^{1/2} \tag{3.2a}$$

$$g_V(E) = \left(\frac{4\sqrt{2}\,\pi m_p^*}{h^3}\right)^{3/2}(E - E_V)^{1/2} \tag{3.2b}$$

$$f(E) = \frac{1}{1 + \exp\left[(E - E_F)/kT\right]} \tag{3.3}$$

$$n = \int_{E_C}^{E_{top}} g_C(E) f(E) dE \tag{3.4a}$$

$$p = \int_{E_{bottom}}^{E_V} g_V(E)\left[1 - f(E)\right] dE \tag{3.4b}$$

$$n = N_C \exp\left[(E_F - E_C)/kT\right] \qquad \text{for } E_C - E_F \geq 3kT \tag{3.5a}$$

$$p = N_V \exp\left[(E_V - E_F)/kT\right] \qquad \text{for } E_F - E_V \geq 3kT \tag{3.5b}$$

$$np = n_i^{\,2} = N_C N_V \exp\left[(E_V - E_C)/kT\right] = N_C N_V \exp\left[-E_g/kT\right], \tag{3.6}$$

$$n_i = N_C \exp\left[(E_i - E_C)/kT\right] = N_V \exp\left[(E_V - E_i)/kT\right] \tag{3.7}$$

$$E_i = \frac{E_C + E_V}{2} + \frac{kT}{2}\ln\left(\frac{N_V}{N_C}\right) = E_C - \frac{E_g}{2} + \frac{kT}{2}\ln\left(\frac{N_V}{N_C}\right) \tag{3.8}$$

$$\rho = q\left(p + N_D^+ - n - N_A^-\right) \tag{3.9}$$

$$p + N_D^+ - n - N_A^- = 0 \tag{3.10}$$

$$p + N_D - n - N_A = 0 \tag{3.11}$$

$$p + N_D - n = 0\,. \tag{3.12}$$

$$N_D \approx N_D^+ \approx n \tag{3.13}$$

$$p = \frac{n_i^2}{n} = \frac{n_i^2}{N_D} << n \qquad (3.14)$$

$$p - n - N_A = 0\,. \qquad (3.15)$$

$$N_A \approx N_A^- \approx p \qquad (3.16)$$

$$n = \frac{n_i^2}{p} = \frac{n_i^2}{N_A} << p \qquad (3.17)$$

$$E_C - E_F = kT \ln\left(N_C / N_D\right) \quad \text{for } n\text{-type} \qquad (3.18a)$$

$$E_F - E_V = kT \ln\left(N_V / N_A\right) \quad \text{for } p\text{-type} \qquad (3.18b)$$

$$\mathbf{v}_{dn} = -\mu_n \boldsymbol{\xi} \qquad (3.19a)$$

$$\mathbf{v}_{dp} = \mu_p \boldsymbol{\xi} \qquad (3.19b)$$

$$\mathbf{J}_{N|drift} = -q\, n\, \mathbf{v}_{dn} = q\, n\, \mu_n \boldsymbol{\xi} \qquad (3.20a)$$

$$\mathbf{J}_{P|drift} = q\, p\, \mathbf{v}_{dp} = q\, p\, \mu_p \boldsymbol{\xi} \qquad (3.20b)$$

$$\mathbf{J}_{drift} = q\,(p\,\mu_p + n\,\mu_n)\boldsymbol{\xi} \qquad (3.21)$$

$$\mathbf{J}_{N|diff} = q\, D_N \nabla n \qquad (3.22a)$$

$$\mathbf{J}_{P|diff} = -q\, D_P \nabla p \qquad (3.22b)$$

$$\mathbf{J}_{diff} = q\left(D_N \nabla n - D_P \nabla p\right) \qquad (3.23)$$

$$\mathbf{J} = \mathbf{J}_{drift} + \mathbf{J}_{diff} = q\,(p\,\mu_p + n\,\mu_n)\boldsymbol{\xi} + q\left(D_N \nabla n - D_P \nabla p\right) \qquad (3.24)$$

$$\frac{D_N}{\mu_n} = \frac{kT}{q} \qquad (3.25a)$$

$$\frac{D_P}{\mu_p} = \frac{kT}{q} \qquad (3.25b)$$

$$\left.\frac{\partial n}{\partial t}\right|_{\text{light}} = \left.\frac{\partial p}{\partial t}\right|_{\text{light}} = G_L \qquad (3.26)$$

$\Delta n << p_0\,, \quad p \cong p_0$ in a p-type material

$\Delta p << n_0\,, \quad n \cong n_0$ in an n-type material

$$\left.\frac{\partial n}{\partial t}\right|_{\substack{\text{thermal}\\ \text{R-G}}} = -c_n\, N_T\, \Delta n \quad \text{for electrons in a } p\text{-type material} \qquad (3.27a)$$

$$\left.\frac{\partial p}{\partial t}\right|_{\substack{\text{thermal}\\ \text{R-G}}} = -c_p\, N_T\, \Delta p \quad \text{for holes in an } n\text{-type material} \qquad (3.27b)$$

$$\tau_n = \frac{1}{c_n N_T} \tag{3.28a}$$

$$\tau_p = \frac{1}{c_p N_T} \tag{3.28b}$$

$$\left.\frac{\partial n}{\partial t}\right|_{\substack{\text{thermal}\\ \text{R-G}}} = -\frac{\Delta n}{\tau_n} \quad \text{for electrons in a } p\text{-type material} \tag{3.29a}$$

$$\left.\frac{dp}{dt}\right|_{\substack{\text{thermal}\\ \text{R-G}}} = -\frac{\Delta p}{\tau_p} \quad \text{for holes in an } n\text{-type material} \tag{3.29b}$$

$$L_N = \sqrt{D_N \tau_n} \quad \text{for electrons in a } p\text{-type material} \tag{3.30a}$$

$$L_P = \sqrt{D_P \tau_p} \quad \text{for holes in an } n\text{-type material} \tag{3.30b}$$

$$L_N = \sqrt{D_N \tau_n} \quad \text{(for electrons in a } p\text{-type material)}$$

$$L_P = \sqrt{D_P \tau_p} \quad \text{(for holes in an } n\text{-type material)}$$

$$E_{ph} = hc/\lambda \tag{3.31}$$

$$k = \frac{\alpha \lambda}{4\pi} \tag{3.32}$$

$$h\nu = \frac{h}{q}\frac{c}{\lambda} \tag{3.33}$$

$$\tilde{r} = \frac{\tilde{n}_0 - \tilde{n}_1}{\tilde{n}_0 + \tilde{n}_1} \tag{3.34}$$

$$\tilde{t} = \frac{2\tilde{n}_0}{\tilde{n}_0 + \tilde{n}_1} \tag{3.35}$$

$$R = \left|\tilde{r}\right|^2 = \left|\frac{\tilde{n}_0 - \tilde{n}_1}{\tilde{n}_0 + \tilde{n}_1}\right|^2 \tag{3.36}$$

$$T = \left|\frac{\tilde{n}_1}{\tilde{n}_0}\right| \left|\tilde{t}\right|^2 = \frac{4\left|\tilde{n}_0 \tilde{n}_1\right|}{\left|\tilde{n}_0 + \tilde{n}_1\right|^2} \tag{3.37}$$

$$\Phi(x,\lambda) = \Phi^0(\lambda)\exp(-\alpha(\lambda)x), \tag{3.38}$$

$$\Phi^0(\lambda) = P(\lambda)\frac{\lambda}{hc} \tag{3.39}$$

$$g_{sp}(x,\lambda) = \eta_g \Phi^0(\lambda)\, \alpha(\lambda)\, e^{-\alpha(\lambda)x} \tag{3.40}$$

$$G_L(x) = \int_{\lambda_1}^{\lambda_2} g_{sp}(x,\lambda)\, d\lambda \tag{3.41}$$

$$G_L(x) = \eta_g A(x) \tag{3.42}$$

$$A(x) = \int_{\lambda_1}^{\lambda_2} \Phi^0(\lambda)\, \alpha(\lambda)\, e^{-\alpha(\lambda)x}\, d\lambda \tag{3.43}$$

$$n = N_C \exp\left[\left(E_{FC} - E_C\right)/kT\right] \quad (3.44a)$$

$$p = N_V \exp\left[\left(E_V - E_{FP}\right)/kT\right] \quad (3.44b)$$

$$n\,p = N_C\,N_V \exp\left[\frac{E_V - E_C}{kT}\right]\exp\left[\frac{E_{FC} - E_{FP}}{kT}\right] = n_i^2 \exp\left[\frac{E_{FC} - E_{FP}}{kT}\right] \quad (3.45)$$

$$\mathbf{J_N} = n\,\mu_n \nabla E_{FC} \quad (3.46a)$$

$$\mathbf{J_P} = p\,\mu_p \nabla E_{FV} \quad (3.46b)$$

$$\frac{\partial n}{\partial t} = \left.\frac{\partial n}{\partial t}\right|_{\text{drift}} + \left.\frac{\partial n}{\partial t}\right|_{\text{diff}} + \left.\frac{\partial n}{\partial t}\right|_{\substack{\text{thermal}\\ \text{R-G}}} + \left.\frac{\partial n}{\partial t}\right|_{\substack{\text{other processes}\\ \text{(photogeneration)}}} \quad (3.47a)$$

$$\frac{\partial p}{\partial t} = \left.\frac{\partial p}{\partial t}\right|_{\text{drift}} + \left.\frac{\partial p}{\partial t}\right|_{\text{diff}} + \left.\frac{\partial p}{\partial t}\right|_{\substack{\text{thermal}\\ \text{R-G}}} + \left.\frac{\partial p}{\partial t}\right|_{\substack{\text{other processes}\\ \text{(photogeneration)}}} \quad (3.47b)$$

$$\left.\frac{\partial n}{\partial t}\right|_{\substack{\text{thermal}\\ \text{R-G}}} = -R_N\,; \qquad \left.\frac{dp}{dt}\right|_{\substack{\text{thermal}\\ \text{R-G}}} = -R_P \quad (3.48a,b)$$

$$\left.\frac{\partial n}{\partial t}\right|_{\text{other processes}} = G_N\,; \qquad \left.\frac{\partial p}{\partial t}\right|_{\text{other processes}} = G_P \quad (3.49a,b)$$

$$\left.\frac{\partial n}{\partial t}\right|_{\text{drift}} + \left.\frac{\partial n}{\partial t}\right|_{\text{diff}} = \frac{1}{q}\nabla\cdot\mathbf{J}_{\text{N}} \quad (3.50a)$$

$$\left.\frac{\partial p}{\partial t}\right|_{\text{drift}} + \left.\frac{\partial p}{\partial t}\right|_{\text{diff}} = -\frac{1}{q}\nabla\cdot\mathbf{J}_{\text{P}} \quad (3.50b)$$

$$\frac{\partial n}{\partial t} = \frac{1}{q}\nabla\cdot\mathbf{J_N} - R_N + G_N \quad (3.51a)$$

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\nabla\cdot\mathbf{J_P} - R_P + G_P \quad (3.51b)$$

$$\mathbf{J}_{\text{N}} = q\,n\,\mu_n\,\boldsymbol{\xi} + q\,D_N \nabla n \quad (3.52a)$$

$$\mathbf{J}_{\text{P}} = q\,p\,\mu_p\,\boldsymbol{\xi} - q\,D_p \nabla p \quad (3.52b)$$

$$\nabla\cdot\boldsymbol{\xi} = \frac{\rho}{\varepsilon_r \varepsilon_0} \quad (3.53)$$

Chapter 4: SEMICONDUCTOR MATERIALS FOR SOLAR CELLS

$$n = n_{n0} \approx N_D \quad (4.1a)$$

$$p = p_{n0} \approx n_i^2 / N_D\,. \quad (4.1b)$$

$$p = p_{p0} \approx N_A \quad (4.2a)$$

$$n = n_{p0} \approx n_i^2 / N_A\,. \quad (4.2b)$$

$$\rho(x) = q\,N_D \qquad \text{for } -l_n \le x \le 0 \quad (4.3a)$$

$$\rho(x) = -q\,N_A \qquad \text{for } 0 \le x \le l_p \quad (4.3b)$$

$$\frac{d^2\psi}{dx^2} = -\frac{d\,\xi}{dx} = -\frac{\rho}{\varepsilon_r\,\varepsilon_0}\,. \tag{4.4}$$

$$\xi = \frac{1}{\varepsilon_r\,\varepsilon_0}\int \rho\, dx \tag{4.5}$$

$$\xi(-l_n) = \xi(l_p) = 0\,, \tag{4.6}$$

$$\xi(x) = \frac{q}{\varepsilon_r\,\varepsilon_0} N_D\,(l_n + x) \qquad \text{for } -l_n \le x \le 0 \tag{4.7a}$$

$$\xi(x) = \frac{q}{\varepsilon_r\,\varepsilon_0} N_A\,(l_p - x) \qquad \text{for } 0 \le x \le l_p \tag{4.7b}$$

$$N_A\, l_p = N_D\, l_n \tag{4.8}$$

$$\psi = -\int \xi\, dx \tag{4.9}$$

$$\psi(l_p) = 0\,. \tag{4.10}$$

$$\psi(x) = -\frac{q}{2\varepsilon_r\,\varepsilon_0} N_D\,(x + l_n)^2 + \frac{q}{2\varepsilon_r\,\varepsilon_0}\left(N_D l_n^2 + N_A l_p^2\right) \qquad \text{for } -l_n \le x \le 0 \tag{4.11a}$$

$$\psi(x) = \frac{q}{2\varepsilon_r\,\varepsilon_0} N_A\,(x - l_p)^2 \qquad \text{for } 0 \le x \le l_p \tag{4.11b}$$

$$\psi_0 = \psi(-l_n) - \psi(-l_p) = \psi(-l_n) \tag{4.12}$$

$$\psi_0 = \frac{q}{2\varepsilon_r\,\varepsilon_0}\left(N_D l_n^2 + N_A l_p^2\right). \tag{4.13}$$

$$q\,\psi_0 = E_G - E_1 - E_2 \tag{4.14}$$

$$q\,\psi_0 = E_G - kT\ln\left(\frac{N_V}{N_A}\right) - kT\ln\left(\frac{N_C}{N_D}\right) = E_G - kT\ln\left(\frac{N_V\,N_C}{N_A\,N_D}\right) \tag{4.15}$$

$$\psi_0 = \frac{kT}{q}\ln\left(\frac{N_A\,N_D}{n_i^2}\right). \tag{4.16}$$

$$l_n = \sqrt{\frac{2\varepsilon_r\,\varepsilon_0}{q}\psi_0\;\frac{N_A}{N_D}\left(\frac{1}{N_A + N_D}\right)} \tag{4.17a}$$

$$l_p = \sqrt{\frac{2\varepsilon_r\,\varepsilon_0}{q}\psi_0\;\frac{N_D}{N_A}\left(\frac{1}{N_A + N_D}\right)} \tag{4.17b}$$

$$W = l_n + l_p = \sqrt{\frac{2\varepsilon_r\,\varepsilon_0}{q}\psi_0\,\left(\frac{1}{N_A} + \frac{1}{N_D}\right)} \tag{4.18}$$

$$\xi_{\max} = \sqrt{\frac{2q}{\varepsilon_r\,\varepsilon_0}\psi_0\,\left(\frac{N_A\,N_D}{N_A + N_D}\right)} \tag{4.19}$$

$$J = J_{rec} - J_{gen} = 0 \qquad \text{for } V_a = 0\ \text{V} \tag{4.20}$$

$$J_{rec}(V_a) = J_{rec}(V_a = 0)\,\exp\left(\frac{qV_a}{kT}\right) \tag{4.21}$$

$$J_{gen}(V_a) \approx J_{gen}(V_a = 0) \tag{4.22}$$

$$J(V_a) = J_{rec}(V_a) - J_{gen}(V_a) = J_0\left[\exp\left(\frac{qV_a}{kT}\right) - 1\right], \tag{4.23}$$

$$J_0 = J_{gen}(V_a = 0) \tag{4.24}$$

$$J_0 = q\, n_i^2\left(\frac{D_N}{L_N\, N_A} + \frac{D_P}{L_P\, N_D}\right). \tag{4.25}$$

$$J(V_a) = -J_0\,, \tag{4.26}$$

$$J(V_a) = J_{rec}(V_a) - J_{gen}(V_a) - J_{ph} = J_0\left[\exp\left(\frac{qV_a}{kT}\right) - 1\right] - J_{ph} \tag{4.27}$$

$$J_{ph} = q\, G\left(L_N + W + L_P\right), \tag{4.28}$$

$$V_{oc} = \frac{kT}{q}\ln\left(\frac{J_{ph}}{J_0} + 1\right) \tag{4.29}$$

$$FF = \frac{J_{mp}\, V_{mp}}{J_{sc}\, V_{oc}} \tag{4.30}$$

$$FF = \frac{v_{oc} - \ln(v_{oc} + 0.72)}{v_{oc} + 1} \qquad v_{oc} = V_{oc}/(kT/q) \tag{4.31}$$

$$\eta = \frac{P_{\max}}{P_{in}} = \frac{J_{mp}\, V_{mp}}{P_{in}} = \frac{J_{sc}\, V_{oc}\, FF}{P_{in}} \tag{4.32}$$

Chapter 5: SOLAR CELL CONVERSION-EFFICIENCY LIMITS

$$P_{in} = \int_0^\infty P(\lambda)d\lambda \tag{5.1}$$

$$P_{in} = \int_0^\infty \Phi(\lambda)\frac{hc}{\lambda}d\lambda \tag{5.2}$$

$$p_{abs} = \frac{\int_0^{\lambda_G} \Phi(\lambda)\frac{hc}{\lambda}d\lambda}{\int_0^\infty \Phi(\lambda)\frac{hc}{\lambda}d\lambda} \tag{5.3}$$

$$p_{use} = \frac{E_G \int_0^{\lambda_G} \Phi(\lambda)d\lambda}{\int_0^{\lambda_G} \Phi(\lambda)\frac{hc}{\lambda}d\lambda} \tag{5.4}$$

$$\eta = p_{abs}\, p_{use} = \frac{\int_0^{\lambda_G} \Phi(\lambda)\frac{hc}{\lambda}d\lambda}{\int_0^\infty \Phi(\lambda)\frac{hc}{\lambda}d\lambda}\,\frac{E_G \int_0^{\lambda_G} \Phi(\lambda)d\lambda}{\int_0^{\lambda_G} \Phi(\lambda)\frac{hc}{\lambda}d\lambda} \tag{5.5}$$

$$QE(\lambda) = (1 - R^*)\, QE_{op}(\lambda)\eta_g(\lambda)\, QE_{el}(\lambda), \tag{5.6}$$

$$J_{\max} = q \int_0^{\lambda_G} \Phi(\lambda)\ d\lambda \tag{5.7}$$

$$J_{sc} = J_{\max} \left(1 - R^*\right) QE_{opt}\, \eta_G\, QE_{el}\, \frac{A_f}{A_{tot}} \tag{5.8}$$

$$J_{sc} = q \left(1 - R^*\right) QE_{opt}\, \eta_G\, QE_{el}\, \frac{A_f}{A_{tot}} \int_0^{\lambda_G} \Phi(\lambda)\ d\lambda \tag{5.9}$$

$$\eta = \frac{q \int_0^{\lambda_G} \Phi^0(\lambda)\ d\lambda}{\int_0^{\infty} \Phi^0(\lambda) \frac{hc}{\lambda} d\lambda} \left(1 - R^*\right) QE_{opt}\, \eta_G\, QE_{el}\, \frac{A_f}{A_{tot}} V_{oc}\, FF \tag{5.10}$$

$$V_{oc} = \frac{qV_{oc}}{E_G} \frac{E_G}{q} \tag{5.11}$$

$$\eta = \frac{q \int_0^{\lambda_G} \Phi^0(\lambda)\ d\lambda}{\int_0^{\infty} \Phi^0(\lambda) \frac{hc}{\lambda} d\lambda} \left(1 - R^*\right) QE_{opt}\, \eta_G\, QE_{el}\, \frac{A_f}{A_{tot}} \frac{qV_{oc}}{E_G} \frac{E_G}{q} FF \tag{5.12}$$

$$\eta = \frac{\int_0^{\lambda_G} \Phi^0(\lambda) \frac{hc}{\lambda} d\lambda}{\int_0^{\infty} \Phi^0(\lambda) \frac{hc}{\lambda} d\lambda} \quad \frac{E_G \int_0^{\lambda_G} \Phi^0(\lambda) d\lambda}{\int_0^{\lambda_G} \Phi^0(\lambda) \frac{hc}{\lambda} d\lambda} \quad \left(1 - R^*\right) \quad QE^*_{opt}\ \eta^*_G\ QE^*_{el}\ \frac{A_f}{A_{tot}} \frac{qV_{oc}}{E_G} FF \tag{5.13}$$

$$J = J_0 \left[\exp\left(\frac{q\left(V - AJR_s\right)}{kT} \right) - 1 \right] + \frac{V - AJR_s}{R_p} - J_{ph} \tag{5.14}$$

$$J = J_{01} \left[\exp\left(\frac{q\left(V - AJR_s\right)}{n_1 kT} \right) - 1 \right] + J_{02} \left[\exp\left(\frac{q\left(V - AJR_s\right)}{n_2 kT} \right) - 1 \right] + \frac{V - AJR_s}{R_p} - J_{ph} \tag{5.15}$$