Hooke's Law

$$\sigma = E \cdot \varepsilon$$

Shear Stresses

$$\tau_{avg} = \frac{V}{A} \qquad \tau = G\gamma \qquad G = \frac{E}{2(1+\nu)}$$

$$\varepsilon_x = \frac{\sigma_x - \nu(\sigma_y + \sigma_z)}{E}$$

Poisson's Ratio

$$\nu = -\frac{\varepsilon_{lat}}{\varepsilon_{long}}$$

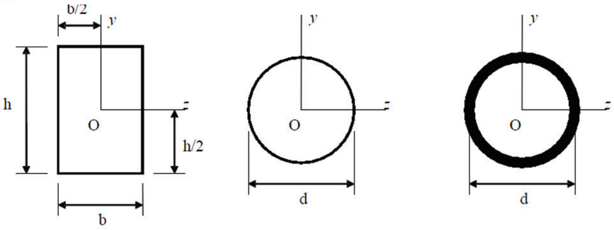
$$I_x = I_{x'} + A \cdot d_y^2$$

$$\delta = \sum \frac{PL}{EA}$$

volumetric strain

$$e = \frac{\Delta V}{V} = \varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{1-2\nu}{E}(\sigma_x + \sigma_y + \sigma_z)$$

$$e = -\frac{3(1-2\nu)}{E}p = -\frac{p}{K} \qquad \text{with } K = \frac{E}{3(1-2\nu)}.$$



$$I_z = \frac{bh^3}{12} \qquad I_z = \frac{\pi d^4}{64} \qquad I_z = \frac{\pi d^3}{4}$$

General bending eq.

$$\sigma_z = \left(\frac{M_y I_{xx} - M_x I_{xy}}{I_{xx}I_{yy} - I_{xy}^2}\right)x + \left(\frac{M_x I_{yy} - M_y I_{xy}}{I_{xx}I_{yy} - I_{xy}^2}\right)y = \left(\frac{I_{yy}y - I_{xy}x}{I_{xx}I_{yy} - I_{xy}^2}\right)M_x + \left(\frac{I_{xx}x - I_{xy}y}{I_{xx}I_{yy} - I_{xy}^2}\right)M_y$$

If $I_{xy}=0$

$$\sigma_z = \frac{M_x}{I_{xx}}y + \frac{M_y}{I_{yy}}x$$

Neutral axis

$$\alpha = \arctan\left(\frac{M_y I_{xx} - M_x I_{xy}}{M_x I_{yy} - M_y I_{xy}}\right)$$

non-warping Neuber beams

$$p_R G t = \text{constant}$$

Torsion Formulas (Circular shafts)

$$\tau(\rho) = \frac{T \cdot \rho}{J} \qquad \phi = \frac{TL}{GJ} \qquad \phi = \int_0^L \frac{T(x)}{J(x)G}dx$$

Flexure Formula

$$\sigma = -\frac{My}{I}$$

Torsion Formulas (Thin-walled shafts)

$$\tau_{avg} = \frac{T}{2tA_m} \qquad q = \frac{T}{2A_m} \qquad \phi = \frac{TL}{4A_m^2 G} \cdot \int_0^{L_m} \frac{1}{t}ds$$

Shear Formula

$$\tau = \frac{VQ}{It} \qquad Q = \bar{y}' \cdot A'$$

Airy

$$\sigma_x = \frac{\partial^2\phi}{\partial y^2}, \quad \sigma_y = \frac{\partial^2\phi}{\partial x^2} \qquad \tau_{xy} = -\frac{\partial^2\phi}{\partial x \partial y}$$

Open cross sections

$$\tau_{zs} = \frac{2n}{J}T = 2Gn\frac{d\theta}{dz}$$

at edges $n = t/2$

$$J = \frac{1}{3}\int t^3\,ds \qquad \text{where also} \qquad T = GJ\frac{d\theta}{dz}$$

Prandtl

$$\tau_{zy} = -\frac{\partial\phi}{\partial x} \quad \text{and} \quad \tau_{zx} = \frac{\partial\phi}{\partial y}$$

$$GJ_i = \frac{4A^2G}{\oint \frac{1}{t}ds} \quad \text{for closed sections, and} \quad GJ_i = \frac{G}{3}\int t^3\,ds \quad \text{for open sections.}$$

Torsion over several boxes

$$T_R = 2A_R q_R \quad \text{with} \quad \sum_{R=1}^{N} T_R = T$$

$$\frac{d\theta}{dz} = \frac{1}{2A_R G}\oint_R \frac{q}{t}ds$$

Simplifying a piece of skin into two beams

$$B_1 = \frac{t_D b}{6}\left(2 + \frac{\sigma_2}{\sigma_1}\right) \quad \text{and} \quad B_2 = \frac{t_D b}{6}\left(2 + \frac{\sigma_1}{\sigma_2}\right)$$

Trusses

kin. indet. m+r<2n

stat. indet. m+r=2n

degree of stat. indet. d=m+r-2n

n=joints

m=members

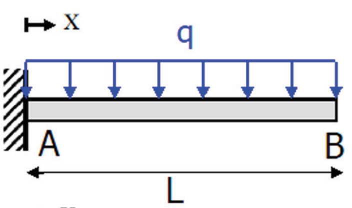
r=reaction forces

Castigliano's theorem

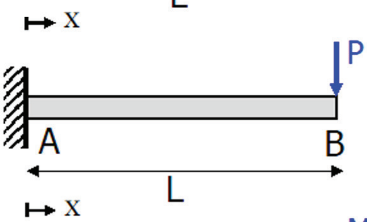
$$U = \int_0^l P\,d\delta = \int_0^l \frac{P}{k}dP = \frac{P^2}{2k} = \frac{P^2L}{2AE}$$

Maxwell

$$f_{BA} = f_{AB}$$



$$\theta_B^{ccw+} = \frac{-qL^3}{6EI} \qquad v_B^{\uparrow+} = \frac{-qL^4}{8EI}$$



$$\theta_B^{ccw+} = \frac{-PL^2}{2EI} \qquad v_B^{\uparrow+} = \frac{-PL^3}{3EI}$$

$$\theta_B^{ccw+} = \frac{M_0 L}{EI} \qquad v_B^{\uparrow+} = \frac{M_0 L^2}{2EI}$$

Von Misses stress

$$Y = \sqrt{\frac{1}{2}\left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2\right] + 3\tau_{xy}^2 + 3\tau_{yz}^2 + 3\tau_{xz}^2}$$

Type	Displacement	Energy
Tension/Compression Bar	$d\delta = \frac{F\,dx}{AE}$	$dU = \frac{1}{2}F\,d\delta = \frac{F^2\,dx}{2AE}$
Torsion Bar	$d\phi = \frac{T\,dx}{GJ}$	$dU = \frac{1}{2}T\,d\phi = \frac{T^2\,dx}{2GJ}$
Bending Beam	$d\theta = \frac{M\,dx}{EI}$	$dU = \frac{1}{2}M\,d\theta = \frac{M^2\,dx}{2EI}$
Shear Beam	$\gamma = \frac{VQ}{ItG} \approx \frac{V}{AG}$	$dU = \frac{1}{2}V\gamma dx = \frac{V^2Q\,dx}{2ItG} \approx \frac{V^2dx}{2AG}$

Mohr

$$\tan 2\theta_{m\sigma} = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\tan 2\theta_{m\tau} = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

$$\tau_{max} = R = \frac{\sigma_I - \sigma_{II}}{2}$$

$$\sigma_I = \sigma_{av} + R \quad \text{and} \quad \sigma_{II} = \sigma_{av} - R$$

Statics

tbv diagrams

$$q = EIu''''$$

$$V = EIu'''$$

$$M = EIu''$$

$$\theta = u'$$

$$\delta = u$$