Continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0$$

Two-Dimensional streamline condition

$$v\,dx = u\,dy$$

Vorticity

$$\xi = \nabla \times \mathbf{V}$$

Circulation

$$\Gamma = -\oint_C \mathbf{V} \cdot \mathbf{ds}$$

condition of irrotationality for two-dimensional flow

$$\frac{\partial v}{\partial x} = \frac{\partial u}{\partial y}$$

Stream function

$$\rho u = \frac{\partial \bar{\psi}}{\partial y}, \qquad \rho v = -\frac{\partial \bar{\psi}}{\partial x}$$

$$u = \frac{\partial \psi}{\partial y}, \qquad v = -\frac{\partial \psi}{\partial x}$$

$$V_r = \frac{1}{r}\frac{d\psi}{d\theta}, \qquad V_\theta = -\frac{d\psi}{dr}$$

Velocity potential

$$u = \frac{\partial \phi}{\partial x}, \qquad v = \frac{\partial \phi}{\partial y}, \qquad w = \frac{\partial \phi}{\partial z}$$

$$V_r = \frac{d\phi}{dr}, \qquad V_\theta = \frac{1}{r}\frac{d\phi}{d\theta}$$

Continuity & Bernoulli $\quad V_1 = \sqrt{\frac{2(p_1 - p_2)}{\rho\left(\frac{A_1}{A_2} - 1\right)}}$

Flow type	Velocity	Velocity potential	Stream function
Uniform flow in x-direction	$u = V_\infty$ $v = 0$	$\phi = V_\infty x$	$\psi = V_\infty y$
Source/Sink	$V_r = \frac{\Lambda}{2\pi r}$ $V_\theta = 0$	$\phi = \frac{\Lambda}{2\pi}\ln r$	$\psi = \frac{\Lambda}{2\pi}\theta$
Doublet	$V_r = -\frac{\kappa}{2\pi}\frac{\cos\theta}{r^2}$ $V_\theta = -\frac{\kappa}{2\pi}\frac{\sin\theta}{r^2}$	$\phi = \frac{\kappa}{2\pi}\frac{\cos\theta}{r}$	$\psi = -\frac{\kappa}{2\pi}\frac{\sin\theta}{r}$
Vortex	$V_r = 0$ $V_\theta = -\frac{\Gamma}{2\pi r}$	$-\frac{\Gamma}{2\pi}\theta$	$\psi = \frac{\Gamma}{2\pi}\ln r$

Pressure coefficient

$$C_p = \frac{p - p_\infty}{q_\infty} = 1 - \left(\frac{V}{V_\infty}\right)^2$$

Energy equation

$$\nabla \cdot \left(\rho\left(e + \frac{V^2}{2}\right)\mathbf{V}\right) = -\nabla \cdot (p\mathbf{V})$$

Substantial derivative

$$\frac{Dx}{Dt} = \frac{\partial x}{\partial t} + (\nabla \cdot \mathbf{V})\,x$$

Divergence of velocity

$$\nabla \cdot \mathbf{V} = \frac{1}{\nu}\frac{D\nu}{Dt}$$

Kutta-Joukowski Theorem

$$L' = c_l q_\infty c = \frac{\Gamma}{RV_\infty}\frac{1}{2}\rho_\infty V_\infty^2 2R = \rho_\infty V_\infty \gamma \qquad c_l = \frac{\Gamma}{RV_\infty}$$

$$\Gamma = \int_a^b \gamma\,ds$$

$$C_p = 1 - \left(2\sin\theta + \frac{\Gamma}{2\pi R V_\infty}\right)^2 \qquad \text{where } R = \tfrac{1}{2}c$$

Symm. airfoil

$$\Gamma = \pi\alpha c V_\infty$$

$$L' = \rho_\infty V_\infty \Gamma = \pi\alpha c \rho_\infty V_\infty^2$$

$$c_l = \frac{L'}{\frac{1}{2}\rho_\infty V_\infty^2 c} = \frac{\pi\alpha c\rho_\infty V_\infty^2}{\frac{1}{2}\rho_\infty V_\infty^2 c} = 2\pi\alpha$$

$$c_{m,le} = -\frac{1}{4}c_l$$

Cambered airfoils

$$\frac{dz}{dx} = \alpha - A_0 + \sum_{n=1}^{\infty} A_n \cos n\theta_0$$

$$\alpha_{opt} = \frac{1}{\pi}\int_0^\pi \frac{dz}{dx}\theta_0$$

$$(c_l)_{design} = \pi A_1 = 2\int_0^\pi \frac{dz}{dx}\cos\theta_0\,d\theta_0$$

$$c_l = \pi(2A_0 + A_1), \qquad c_{m,c/4} = \frac{\pi}{4}(A_2 - A_1)$$

Biot–Savart

$$v = \frac{\Gamma}{4\pi r}[\cos A - \cos B]$$

Prandtl relation for normal shock wave

$$M_2^2 = \frac{2 + (\gamma - 1)M_1^2}{2\gamma M_1^2 - (\gamma - 1)}$$

If $M_1 > 1$ we have $M_2 < 1$

If $M_1 = 1$, then also $M_2 = 1$

Mach angle $\quad \sin\mu = \frac{at}{Vt} = \frac{a}{V} = \frac{1}{M}$

wave angle $\quad \beta > \mu$

Shock wave: pressure, density and temp. increase

Expansion wave: pressure, density and temp. decrease

Nozzle back pressure

- If $p_B > p_{e,6}$, **overexpanded** oblique shock waves
- If $p_B < p_{e,6}$, **underexpanded** expansion waves
- If $p_B = p_{e,6}$, without any waves

Prandtl-Glauert correction

$$C_p = \frac{C_{p,0}}{\sqrt{1 - M_\infty^2}}$$

Karman-Tsien

$$C_p = \frac{C_{p,0}}{\sqrt{1 - M_\infty^2} + \frac{M_\infty^2}{1 + \sqrt{1 - M_\infty^2}}\frac{C_{p,0}}{2}}$$

Lift and Drag Coefficients of a Flat Plate

$$C_{p,l} = \frac{2\alpha}{\sqrt{M_\infty^2 - 1}} \quad \text{and} \quad C_{p,u} = -\frac{2\alpha}{\sqrt{M_\infty^2 - 1}}$$

$$c_n = \frac{1}{c}\int_0^c (C_{p,l} - C_{p,u})\,dx = \frac{4\alpha}{\sqrt{M_\infty^2 - 1}}$$

$$c_l = c_n\cos\alpha \quad \text{and} \quad c_d = c_n \sin\alpha$$

$$c_l = \frac{4\alpha}{\sqrt{M_\infty^2 - 1}} \quad \text{and} \quad c_n = \frac{4\alpha^2}{\sqrt{M_\infty^2 - 1}}$$

Three-Dimensional Wings

Angles of attack $\quad \alpha = \alpha_{eff} + \alpha_i$

$$\alpha_i(y_0) = \frac{1}{4\pi V_\infty}\int_{-\frac{b}{2}}^{\frac{b}{2}} \frac{\left(\frac{d\Gamma}{dy}\right)dy}{(y_0 - y)}$$

$$\alpha_{eff}(y_0) = \frac{\Gamma(y_0)}{\pi V_\infty c(y_0)} + \alpha_{L=0}$$

Aspect ratio $\quad A = \frac{b^2}{S}$

Lift coefficient $\quad C_L = A_1 \pi A$

Induced drag $\quad D_i'(y) = L'\alpha_i = \rho_\infty V_\infty \Gamma \alpha_i$

$$C_{D,i} = \frac{C_L^2}{\pi A e}$$

$$e = A_1^2\left(\sum_1^N n A_n^2\right)^{-1}$$