Probability & Observation Theory

23rd/06/'05

$\boxed{1}$ ①

X	1	2	3	4	5	6
1	X	8	8	8	8	8
2	8	X	8	8	8	8
3	8	8	X	8	8	8
4	8	8	8	X	8	8
5	8	8	8	8	X	8
6	8	8	8	8	8	8

$36 - 5 = 31$

dos $\frac{31}{36}$ (b)

② $P(H) = 0.1$ $\quad P(A|H) = 0.99$

$P(H^c) = 0.9$ $\quad P(A|H^c) = 0.02$

$$P(H|A) = \frac{P(A|H)(P(H))}{P(A)} = \frac{0.99 \times 0.1}{0.117} = 0.8462$$

$$P(A) = 0.1 \times 0.99 + 0.02 \times 0.9 = 0.117$$ (c)

③ $p(\text{not detected in 3 cycles}) < 0.001 = p_c^3$

$$\left.\begin{array}{l} \sqrt[3]{0.001} = p_c = 0.1 \\ p = 0.9 \end{array}\right\} 5\text{ km}$$ (a)

④ $$\left.\begin{array}{l} h_1 \sim N(7000, 150^2) \\ h_2 \sim N(8000, 150^2) \end{array}\right\} dh = h_2 - h_1 \sim N(1000, \underbrace{150^2 + 150^2}_{212,13})$$

$$p(dh < 212,13) = \phi\left(\frac{500 - 1000}{212,13}\right) = \phi(-2,36) = 0.0092$$ (c)

(5) $f_W(\omega) = \frac{1}{b}\left(\cancel{\frac{\omega}{b}}\right) e^{-\left(\frac{\omega - \omega_0}{b}\right)}$ for $\omega \geq \omega_0$

$$\bar{\omega} = \int_{\omega_0}^{\infty} \frac{\omega}{b} e^{\frac{\omega_0 - \omega}{b}} d\omega$$

$\Rightarrow \quad u = \frac{\omega}{b} \qquad v = -b e^{\frac{\omega_0 - \omega}{b}}$

$du = \frac{1}{b} \qquad dv = e^{\frac{\omega_0 - \omega}{b}}$

$$\left[-\frac{1}{b} e^{\frac{\omega_0 - \omega}{b}}\right] + \int e^{\frac{\omega_0 - \omega}{b}} d\omega$$

$$\left[-\frac{\omega}{\cancel{b}} e^{\frac{\omega_0 - \omega}{b}} - b e^{\frac{\omega_0 - \omega}{b}}\right]_{\omega_0}^{\infty}$$

$$= (0 - 0) - (-\omega_0 - b) = \omega_0 + b \qquad (c)$$

(6) $z \sim N\left(10 \cdot 10, \frac{0.5^2}{12} \cdot 10\right)$?

$\hookrightarrow$ st. dev $= \sqrt{0.20833} = 0.456$

$$P(z \leq 0.99) = \Phi\left(\frac{99 - 100}{0.456}\right) = \Phi(-2.19) = 0.0143$$

(7) positively correlated

(8) $f_X(x) = \lambda e^{-\lambda x} = 0.2 e^{-0.2x}$

$\bar{X}(x>2) = \int_2^{\infty} x e^{-0.2x} dx \quad , \quad u = x \qquad v = -5e^{-0.2x}$

$du = 1 \qquad dv = e^{-0.2x}$

$$X(x>2) = \left[-5x e^{-0.2x}\right]_2^{\infty} - \int_2^{\infty} -5e^{-0.2x} dx$$

$$= (0 - -10e^{-0.4}) - \left[25 e^{-0.2x}\right]_2^{\infty}$$

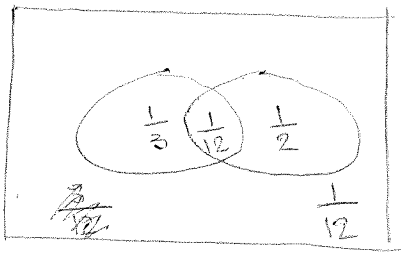
$=$

$\boxed{2}$

① $P(A) = \frac{1}{3} = \frac{4}{12}$

$P(B) = \frac{1}{2} = \frac{6}{12}$

$P(A \cup B) = \frac{3}{4} = \frac{9}{12}$



C

② $P(H) = 0.1$ $\quad P(A|H) = 0.99$ $\quad \cancel{P(A^c|H)}$ "A" switched on

$P(H^c)\, 0.9$ $\quad P(A|H^c) = 0.02$ $\quad$ "H" oil low

$$P(H|A^c) = \frac{P(A^c|H)\,P(H)}{P(A^c)}$$

$P(A^c|H) = 0.01$

$P(A^c|H^c) = 0.98$

$P(A^c) = \cancel{0.99 \times 0.1 +}\; 0.1 \times 0.01 + 0.98 \times 0.9 = 0.883$

$$P(H|A^c) = \frac{0.01 \times 0.1}{0.883} = 0.00113$$ C

③ $\bar{X}_1 = 3$ $\quad \sigma = \sqrt{2}$ $\quad E(3X_1^2 - 2X_2) =$

$\bar{X}_2 = 5$

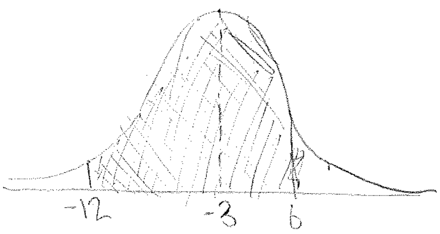
$\bar{Y} = g(\bar{x}) + \frac{1}{2} g''(\bar{x})\, \sigma_x^2$

$E(Z) = 3 \cdot 3^2 - 2 \cdot 5 + \frac{1}{2} \cdot 6 \cdot 2 = 23$ a

④ $X_1 \sim N(2,5)$ $\quad Z = 3X_1 - 2X_2 + 1$ $\quad Z \sim N(-3, 81)$

$X_2 \sim N(5,9)$ $\quad \sigma = 3^2 \cdot 5 + (-2)^2 \cdot 9 = 81$

$P(-12 \le Z \le 6) = \cancel{\Phi\left(\frac{12-3}{9}\right)}\; 2 \times \Phi(1) = 2 \times$



$= 1 - \cancel{0.8826}\; 2 \times 0.1587$

(5)

$$f_{X_1}(x_1) = \lambda e^{-\lambda x_1}$$
$$f_{X_2}(x_2) = \lambda e^{-\lambda x_2} \quad \Bigg\} \quad Y = \frac{X_1}{X_2}$$

$$f_Y(y) = \int_{-\infty}^{\infty} |x_2| \, f_{X_1 X_2}(yx_2, x_2) \, dx_2$$
$$\hookrightarrow = \lambda^2 e^{-\lambda y x_2 - \lambda x_2}$$

$$f_Y(y) = \int_0^{\infty} x_2 \lambda^2 e^{-\lambda y x_2 - \lambda x_2} \, dx_2$$

$$u = x_2 \qquad v = -\frac{1}{\lambda(y+1)} e^{-\lambda x_2 (y+1)}$$
$$du = 1 \qquad dv = e^{\cancel{-\lambda(y x_2)} - \lambda x_2 (y+1)}$$

$$f_Y(y) = \left[\frac{x_2 \lambda^2}{-\lambda(y+1)} e^{-\lambda x_2 (y+1)} \right]_0^{\infty} - \int \frac{\lambda^2}{-\cancel{y}\lambda(y+1)} e^{-\lambda x_2 (y+1)}$$

$$= \cancel{\left(0 - \left(\frac{x_2}{-\lambda} e^{-\lambda x_2}\right)\right) - \left[\frac{1}{\lambda^2 (y+1)^2} e^{-\lambda x_2 (y+1)} \right.}$$

$$= (0 - 0) - \left[\frac{\cancel{\lambda^2} e^{-\lambda x_2 (y+1)}}{\cancel{\lambda^2} (y+1)^2} \right]_0^{\infty}$$

$$= \frac{1}{(y+1)^2} \quad (d)$$

(6)

$$X_1(x_1) = 1 e^{-x_1}$$
$$X_2(x_2) = 3 e^{-3x_2}$$
$$f_{X_1 X_2} = 3 e^{(-x_1 - 3x_2)}$$

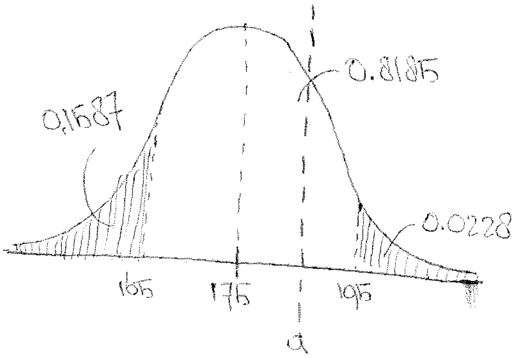
$$F_{X_1 X_2} = \int_0^{x_2} \int_0^{x_1} 3 e^{-x_1 - 3x_2} \, dx_1 \, dx_2$$

$$= \int_0^{x_2} \left[-3e^{-x_1 - x_2} \right]_0^{x_1} dx_2 = \int_0^{x_2} \left(-3e^{-x_1 - 3x_2} + 3e^{-3x_2} \right) dx_2$$

$$= \left[\cancel{3} e^{-x_1 - 3x_2} - e^{-3x_2} \right]_0^{x_2}$$

$$= \left(\left(e^{-x_1 - 3x_2} \right) - e^{-3x_2} \right) - \left(e^{-x_1} - 1 \right)$$

⑦ $X_1 \sim N(175, 10)$ $\qquad 165 \leq X_1 \leq 195$



$P(x > 195) = \phi\left(\frac{195-175}{10}\right) = \phi(2)$

$= 0.0228$

$P(x < 165) = 1 - P(x > 165)$

$\cancel{=1-}\phi\left(\frac{165-175}{10}\right) = \phi(-1)$

$= \cancel{1-0.1587} = 0.1587$

$P(x > a) = 0.8185/2 + 0.0228 = 0.43205 \Rightarrow 0.17$

$\cancel{\phi(}$

$175 + 0.17 \times 10 = 176.7$ $\qquad$ ⓑ

⑧ $f_Y(y) = f_X(g^{-1}(y)) \left| \frac{dg^{-1}(y)}{dy} \right|$ $\qquad g(x) = 3x^2 + 1$

$\frac{dg^{-1}(y)}{dy} = \left(\frac{1}{3}y - \frac{1}{3}\right)^{\frac{1}{2}}$ $\qquad g^{-1}(x) = \sqrt{\frac{y-1}{3}}$

$= \frac{1}{2}\left(\frac{1}{3}y - \frac{1}{3}\right)^{-\frac{1}{2}} \times \frac{1}{3} = \frac{1}{6\sqrt{\frac{y-1}{3}}}$

$\cancel{f_Y(y)} = f_X(g^{-1}(y)) =$

⑩ $f_X(x) = 0.4\, e^{-0.4x}$

$\bar{X} = \int_0^{\infty} x \cdot 0.4 e^{-0.4x}\, dx$

$u = x \qquad v = -e^{-0.4x}$

$du = 1 \qquad dv = 0.4 e^{-0.4x}$

$\bar{X} = \left[-xe^{-0.4x}\right]_0^{\infty} - \left[\frac{5}{2} e^{-\frac{2}{5}x}\right]_0^{\infty}$

$= (0-0) - \left(0 - \frac{5}{2}\right) = 2.5$

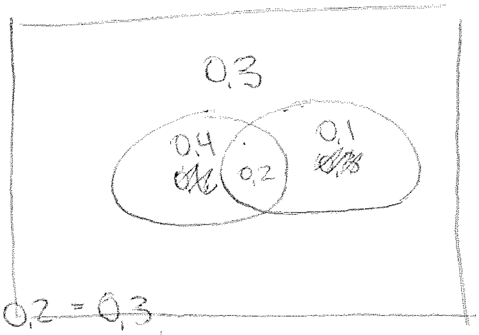
$5 \times 2.5 = 12.5$

$\boxed{3}$ ① $P(A) = 0.6$

$P(B) = 0.3$

$P(A \text{ and } B) = 0.2$

$P(A^c \text{ and } B^c) = 1 - 0.6 - 0.3 + 0.2 = 0.3$



ⓐ

② $P(T \mid A) = \frac{3}{5}$ $\qquad P(A) = \frac{1}{3}$

$P(T \mid B) = \frac{4}{5}$ $\qquad P(B) = \frac{2}{3}$

$$P(T) = P(T \mid A)P(A) + P(T \mid B)P(B)$$
$$= \frac{3}{5} \times \frac{1}{3} + \frac{4}{5} \times \frac{2}{3} = \frac{3}{15} + \frac{8}{15} = \frac{11}{15}$$

ⓒ

③ $P(D) = 0.05$

$P(X = 3) = 0.95 \times 0.95 \times 0.05 = 0.045$ ⓑ

④ $y = e^x$ $\qquad f_Y(y) = f_X\left(g^{-1}(y)\right) \left| \frac{dg^{-1}(y)}{dy} \right|$

$g^{-1}(y) = \ln y$ $\qquad \frac{dg^{-1}(y)}{dy} = \frac{1}{y}$

$\cancel{f_Y(y)} = f_X = \frac{1}{\sqrt{2\pi}\,\sigma_x y} \exp\left(-\frac{1}{2}\left(\frac{\ln y - \bar{x}}{\sigma_x}\right)^2\right) \cdot \frac{1}{y}$

⑤ $f_X(x) = \frac{3}{4}x(2-x)$ $\Rightarrow$ $y = \sqrt{x}$

$= \frac{3}{2}x - \frac{3}{4}x^2$

$\hookrightarrow g(x) = y = \sqrt{x} \Rightarrow g^{-1}(y) = y^2 = x$

$$\frac{dg^{-1}(y)}{dy} = 2y$$

$$f_Y(y) = \left(\frac{3}{2}y^2 - \frac{3}{4}y^4\right)2y = 3y^3 - \frac{3}{2}y^5$$ ⓒ

⑥ $f_{X_1X_2}(x_1, x_2) = \frac{1}{10}(3x_1^2 + 8x_1x_2)$ $\quad 0 < x_1 < 1$, $\quad 0 < x_2 < 2$

$$f_{X_1}(x_1) = \int_0^2 f_{X_1X_2}(x_1, x_2)\,dx_2$$

$$= \int_0^2 \frac{3}{10}x_1^2 + \frac{8}{10}x_1x_2 \, dx_2 = \left[\frac{3}{10}x_1^2x_2 + \frac{8}{20}x_1x_2^2\right]_0^2$$

$$= \frac{6}{10}x_1^2 + \frac{32}{20}x_1 = \frac{3}{5}x_1^2 + \frac{8}{5}x_1$$ ⓒ

⑦ $X \sim N(10, 0.01)$

$P(10.03 \le X \le 9.97) = 2P(X \geq 10.03)$

$$= 2\Phi\left(\frac{10.03 - 10.00}{0.01}\right) = 2\Phi(3) = 2 \times 0.0013$$

$= 0.0026$

$= 0.26\%$ ⓑ

(8) $\sigma^2 = \frac{k_B T}{m} \qquad 2E_k = \frac{1}{2} m v^2 \qquad \sigma_v = \sqrt{\frac{kT}{m}}$

(9) $v = \sqrt{\frac{2E_k}{m}} \qquad \frac{dv}{dE_k} = \frac{1}{2}\left(\frac{2}{m}E_k\right)^{-\frac{1}{2}} \times \frac{2}{m} = \frac{1}{m\sqrt{\frac{2}{m}E_k}}$

$$f_Y(y) = f_X\left(g^{-1}(y)\right)\left|\frac{d\,g^{-1}(y)}{dy}\right|$$

$$f_{E_k}(E_k) =$$
$$f_X = \frac{1}{\sqrt{2\pi}\,\sigma_x}\exp\left(-\frac{1}{2}\frac{\sqrt{\frac{2E_k}{m}}}{\sqrt{\frac{kT}{m}}}\right) \times$$

$$f_{E_k}(E_k) = \frac{1}{\sqrt{2\pi}\sqrt{\frac{kT}{m}}}\exp\left(-\frac{1}{2}\frac{2E_k}{2kT}\right) \times \frac{1}{\sqrt{\frac{2m}{\cancel{m}}E_k}}$$

$$\frac{1}{\sqrt{4\pi\,\cancel{\tfrac{1}{m}}\,kT\,\cancel{2m}\,E_k}} = \frac{1}{\sqrt{4\pi kT E_k}}$$

(10) $Y_1 = X_1 + 2X_2 = f_{X_1 Y_1} \qquad f_{Y_1 Y_2}(y_1, y_2) =$
$Y_2 = X_1 - X_2 = f_{X_2 Y_2}$

① $f_{Y_1}(y_1) = f_X\left(g^{-1}(y_1)\right)\left|\frac{dg(y_1)}{dy_1}\right)$

$$f_{X_1} = Y_1 = X_1 + 2X_2$$
$$X_1 = 2X_2 - Y_1$$
$$X_2 = \frac{1}{2}Y_1 - \frac{1}{2}X_1$$

$$Y = X_1 + X_2 \qquad f_Y(y) = \int_{-\infty}^{\infty} f_{X_1X_2}(y - x_2, x_2)\, dx_2$$

$$= \int_{-\infty}^{\infty} f_{X_1X_2}(x_1, y - x_1)\, dx_1$$

$$f_{Y_1}(y_1) = \int_{-\infty}^{\infty} 2y_1 - x_1 \; dx_1 = \left[2y_1 - 1\right]_{-\infty}^{\infty} = 2y_1 - 1$$

$$f_{X_1}(x_1, x_2) = x_1 + 2x_2$$
$$f_{X_1}(x_1, y_1 - x_1) = x_1 + 2(y_1 - x_1) = x_1 + 2y_1 - 2x_1 = 2y_1 - x_1$$

$$f_{Y_2}(y_2) = \int_{-\infty}^{\infty} 2x_1 - y_2 \; dx_1 = \left[2 - y_2\right]_{-\infty}^{\infty} = 2 - y_2$$

$$f_{X_2}(x_1, x_2) = x_1 - x_2$$
$$f_{X_2}(x_1, y_2 - x_1) = x_1 - (y_2 - x_1) = 2x_1 - y_2$$

$$f_{Y_1}(y_1) = 2y_1 - 1$$
$$f_{Y_2}(y_2) = 2 - y_2$$

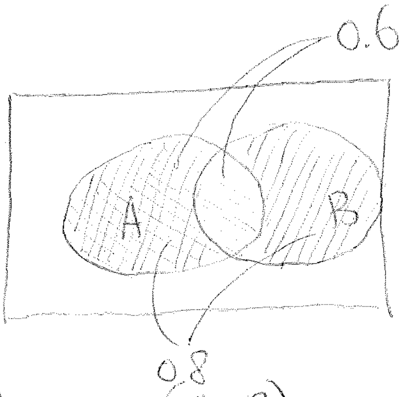
$$f_{Y_1} f_{Y_2} = (2y_1 - 1)(2 - y_2) = 4y_1 - 2y_1y_2 - 2 + y_2$$

$\boxed{4}$ ① $P(A) = 0.6$

$P(A \text{ of } B) = 0.8$

$P(A|B) = 0.5$

$P(B) = ?$



$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)} = \frac{P(A \cap B)}{P(B)}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(B)\,P(A|B)$$

$$P(B) = \frac{P(A \cap B)}{P(A|B)}$$

$$P(A \cup B) = P(A) + P(B) - P(B) \times P(A|B)$$

$$P(A \cup B) - P(A) = P(B)\left(1 - P(A|B)\right)$$

$$P(B) = \frac{P(A \cup B) - P(A)}{1 - P(A|B)} = \frac{0.8 - 0.6}{1 - 0.5} = 0.4$$

② $P(L|A) = \frac{3}{5}$ $\quad P(A) = \frac{1}{3}$ $\quad$ ⓐ

$P(L|B) = \frac{4}{5}$ $\quad P(B) = \frac{2}{3}$

$$P(A|L) = \frac{P(L|A)\,P(A)}{P(L)} = \frac{\frac{3}{5} \times \frac{1}{3}}{\frac{11}{15}} = \frac{3}{11} \quad ⓓ$$

$$P(L) = P(L|A)\,P(A) + P(L|B)\,P(B)$$

$$= \frac{3}{5} \times \frac{1}{3} + \frac{4}{5} \times \frac{2}{3} = \frac{11}{15}$$

(3) $P(X>4) = \cancel{P(X} 1 - P(X \leq 4)$

$= 1 -$

$\cancel{P(X} P_X(k) = e^{-\lambda}\left(\frac{\lambda^k}{k!}\right) = e^{-3}\left(\frac{3^4}{4!}\right)$

$P_X(k) = e^{-3}\left(\frac{3^0}{1} + \frac{3^1}{1} + \frac{3^2}{2} + \frac{3^3}{6} + \frac{3^4}{24}\right) = 0.8153$

$1 - 0.8153 = 0.1847$ (d)

(4) $y = \frac{9}{5}X + 32$

mean 10 $\Rightarrow \frac{9}{5} \times 10 + 32 = 50$

variance 4 $\Rightarrow \underline{(9/5)^2} \times 4 = 12.96 \Rightarrow$ st dev $= 13.6$ (d)

(5) $f_X(x) = -\frac{3}{4}X^2 + \frac{3}{2}X \qquad 0 < X < 2$

$F_X = \int_0^2 -\frac{3}{4}X^2 + \frac{3}{2}X \, dX = \cancel{[\frac{3}{3}\frac{3}{2}X}$

$= \left[-\frac{1}{4}X^3 + \cancel{\frac{2}{3}} \frac{3}{4}X^2\right]_0^2 = -\frac{8}{4} +$ (d)

(6) $f_{X_1X_2}(X_1 X_2) = \frac{1}{10}(3X_1^2 + 8X_1X_2)$

$= \int_0^2 \int_0^{\frac{1}{2}X_2} \frac{1}{10}(3X_1^2 + 8X_1X_2) \, dX_1 \, dX_2$

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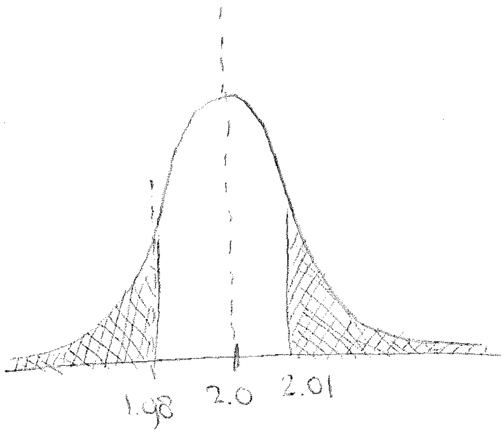
(7) $\bar{X} = 2$ cm

$\sigma_x = 0.01$ cm



$$P(X > 2.01) = \phi\left(\frac{2.01 - 2.0}{0.01}\right) = \phi(1) = 0.1587$$

$$P(X \le 1.98) = \phi\left(\frac{1.98 - 2.0}{0.01}\right) = \phi(-2) =$$

$$\phi(2) = 0.0228$$

$0.1587 + 0.0228 = 0.1815 = 18.15\%$ (b)

(8) $$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(5 < X < 6) = \int_{5}^{6} 0.2\, e^{-0.2x}\, dx = \left[-e^{-0.2x}\right]_5^6$$

$$= -e^{-1.2} + e^{-1}$$

$$P(3 < X < 6) = \int_{3}^{6} 0.2\, e^{-0.2x}\, dx = \left[-e^{-0.2x}\right]_3^6$$

$$= -e^{-1.2} + e^{-0.6}$$

$$P(A|B) = \frac{\cancel{-\frac{1}{e^{1.2}}}\ \frac{1}{e} - \frac{1}{e^{1.2}}}{\frac{1}{e^{0.6}} - \frac{1}{e^{1.2}}} = 0.269 \quad (a)$$

9) $\sigma^2 = \frac{kT}{m} \Rightarrow \sigma = \sqrt{\frac{kT}{m}}$

$$f_x = \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left(-\frac{1}{2}\left(\frac{X - \bar{X}}{\sigma_x}\right)^2\right)$$

$$f_V(V) = f_x\left(g^{-1}(V)\right) \left| \frac{dg^{-1}(V)}{dV} \right|$$

10) $f_x = 0{,}5\, e^{-0{,}5x}$

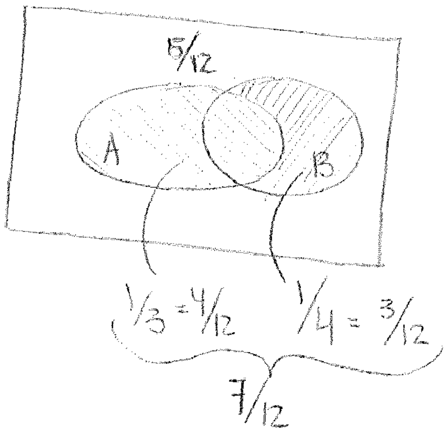
$$P(x > 15) = \int_{15}^{\infty} 0{,}5\, e^{-0{,}5x}\, dx = \left[-e^{-0{,}5x} \right]_{15}^{\infty}$$

$$= (0) - \left(-e^{-7{,}5}\right) = \frac{1}{e^{7{,}5}}$$

$\boxed{5}$ (1) $P(A) = \frac{1}{3}$

$P(B|A^c) = \frac{1}{4}$

$P(A \cup B) = ?$



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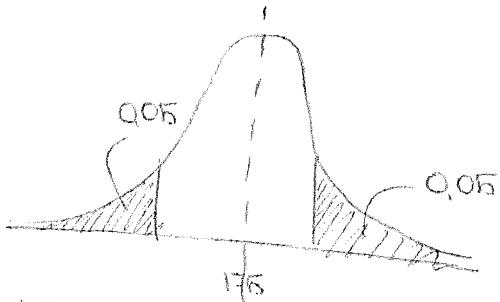
(2) $\frac{1}{4}\left(1 - \frac{3}{4}\right) = $ (b)

(3) b

(4) $P(X \geq 1) = 1 - P(X=0) = 1 - 0.1822 = 0.99996$ (b)

(5) $X \sim N(175, 8.5)$



$\frac{X - 175}{8.5} = \cancel{2.624}\ 1.645$

$X = 175 \pm 22.25$

$152.73 < X < 197.27$

(6) $X \sim N(0.79, 0.1^2)$

$Y \sim N(1.13, 0.14^2)$

$\Big\} \; Y - X \sim N(34, 0.1^2 + 0.14^2)$

0.172

(9) $P(D) = 0.005$ $\quad P(C|D) = 0.98$ $\quad P(NC|D) = 0.02$

$P(ND) = 0.995$ $\quad P(C|ND) = 0.8$ $\quad P(NC|ND) = 0.2$

A = diagnosis positive

B = decease

$P(B) = 0.005$

$P(A^c|B^c) = 0.8$

$P(A|B) = 0.98$

$$P(B^c|A) = \frac{P(A|B^c)\,P(B^c)}{P(A)}$$

$P(A) = 0.98 \times 0.005 + 0.2 \cdot 0.995 =$

(10) $y_1 = X_1 + X_2$

$y_2 = X_1 - X_2$

$$\begin{cases} f_{y_1} = \int (y - X_2) \cancel{+ X_2}\, dX_2 = \int y \, dX_2 = [y X_2 \\ f_{y_1} = \int X_1 + (y - X_1)\, dX_1 = \int y = [y X_1] \end{cases}$$

$f_{y_2} = \int X_1 - (y - X_1)\, dX_1 = \int 2X_1 - y = [X_1^2 - y X_1$

$f_{y_2} = \int (y - X_2) - X_2 \, dX_2 = \int y - 2X_2 \, dX_2 = [-X_2^2 + y X_2$

$f_{y_1} = y_1 X_2$

$= y_1 X_1$

(11) $(P_{X<5} | 3<X<6) = \dfrac{P(X<5 \cap \cancel{\&} 3<X<6)}{P(3<X<6)}$

(12) $f_X(x) = 1$ $\quad -y = \ln x$ $\quad x = e^{-y}$ $\quad \dfrac{dg^{-1}(y)}{dy} = -e^{-y}$

$f_Y(y) = 1(-e^{-y})$

$\boxed{1}$ $\quad f_X(x) = 0.2\, e^{-0.2x}$

$\bar{X} = \int_2^{\infty} x\; 0.2e^{-0.2x}$

$\boxed{2}$ ③ $E(g(x)) \simeq g(\bar{x}) + \frac{1}{2} g''(\bar{x}) \sigma_x^2$

$g(x) = 3X_1^2 - 2X_2$

$$\left.\begin{aligned} \bar{X}_1 &= 3 \\ \bar{X}_2 &= 5 \\ g''(x) &= 6 \end{aligned}\right\} E(g(x)) = 3(3)^2 - 2(5) + \tfrac{1}{2}(6)(2)$$

$$= 27 - 10 + 6 = 23$$

④ $X_1 \sim N(2,5)$ $\quad Z = 3X_1 - 2X_2 + 1$

$X_2 \sim N(5,9)$

$$\sigma^2 = \left(3^2 \cdot 5 - 2^2 \cdot 9\right)^2 = 81$$

$Z \sim N(-3, 81)$

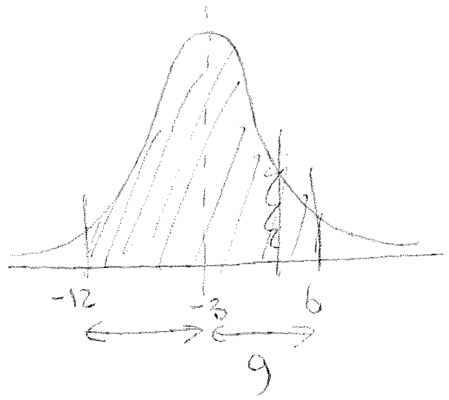
$$P(-12 \le X \le 6) = \cancel{P(X \ge 12)} +$$

$$1 - P(X > 12) - P(X > 6)$$

$$= 1 - \phi\left(\frac{\cancel{12+13}}{9}\right) - \phi\left(\frac{6 - -3}{9}\right) = 1 - \phi\left(\frac{15}{9}\right) - \phi(1)$$

$$= 1 - \phi(1.66) - \phi(1)$$

$= 1 \cancel{- 0.5} -$



$1 - 2 \cdot \phi(1) = 1 - 2 \cdot 0.1587 = 0.6826$

5

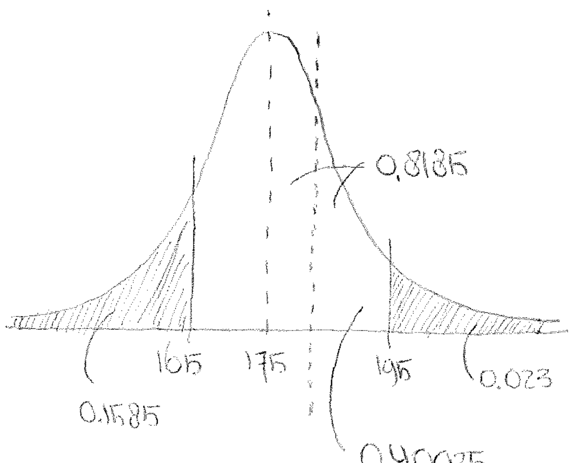
6 $X_1(x_1) = 1 \ \exp(-1\, x_1)$

$X_2(x_2) = 3 \ \exp(-3\, x_2)$

$$f_{x_1 x_2} = 3 \ \exp(-x_1 - 3x_2)$$

$$F_{x_1 x_2}(x_1, x_2) = \int_0^{x_2}\int_0^{x_1} 3 \ \exp(-x_1 - 3x_2)\, dx_1\, dx_2$$

7



0.40925 + 0.023 = 0, 43225

8 $X \sim U(0,2)$

$Y = 3X^2 + 1$

$X = \sqrt{\frac{Y-1}{3}} = \left(\frac{1}{3}Y - \frac{1}{3}\right)^{\frac{1}{2}}$

$\frac{1}{2}\frac{1}{3}\left(\frac{1}{3}Y - \frac{1}{3}\right)^{-\frac{1}{2}} = \frac{1}{6\sqrt{\frac{Y-1}{3}}}$

$X_2 = \frac{1}{3}Y_1 - \frac{1}{3}Y_2$

$X_1 = \frac{1}{3}Y_2 + \frac{2}{3}Y_1$

$X_2 = Y_1 - X_1$

10 $\sigma_1 = 2$ $\quad Y_1 = X_1 + X_2 \quad 3X_2 = Y_1 - Y_2$

$\sigma_2 = 1$ $\quad Y_2 = X_1 - 2X_2 \quad 3X_1 = Y_2 + 2Y_1$

$f_{Y_1Y_2}(y_1, y_2) = f_{X_1X_2}(x_1 x_2) \left| \det\left[\partial_y^T g^{-1}(y_1, y_2)\right] \right|$

$\partial_x^T g(x_1 x_2) = \begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix}$ $\quad \det = -2 - 1 = -3$

$\partial_x g(x_1 x_2)^{-1} = -\frac{1}{3}\begin{bmatrix} -2 & -1 \\ -1 & 1 \end{bmatrix}$ $\quad \det = -2 + 2 = -3$

$\det = \frac{1}{9}$

$\left(\frac{1}{3}Y_1 - \frac{1}{3}Y_2\right)\left(\frac{1}{3}Y_2 + \frac{2}{3}Y_1\right)$

$f_{X_1X_2} =$ $\quad -\frac{1}{9}Y_1Y_2 + \frac{2}{9}Y_1^2 - \frac{1}{9}Y_2^2 - \cancel{\frac{2}{9}Y_1Y_2}$

$f_X = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2}\left(\frac{x}{\sigma}\right)^2\right)$

$f_{X_1X_2} = \frac{1}{2\pi 2} \exp$ $\quad \frac{2}{9}Y_1^2 - \frac{1}{9}Y_1Y_2 + -\frac{1}{9}Y_2^2$

$= \frac{1}{2\pi 2} \cdot \frac{1}{9} \exp\left(-\frac{1}{2}\right.$

(4) $Y = \frac{9}{5} X + 32$

$X \sim N(10,4) \Rightarrow$ mean $= 18 + 32 = 50$

stand. dev. $= \left(\frac{9}{5}\right)^2 4 = \frac{81 \times 4}{5} \quad \frac{244}{5} = \frac{488}{10} = 48.8$

stand. dev $= \left(\frac{9}{5}\right) 4^2 = \frac{144}{5} = \frac{288}{10} = 2.88$

variance $= \left(\frac{9}{5}\right)^2 4 = \frac{81}{25} = 12.96$

stand. dev =

(3) Poisson $P_x(k) = \exp(-\lambda) \frac{\lambda^k}{k!}$

$P_x(X > 4) = 1 - P(X \leq 4)$

$1 + 3 + 4.5 + \frac{27}{6} + \frac{81}{24}$

$P(X \leq 4) = e^{-3} \left(\frac{3^0}{1} + \frac{3^1}{1} + \frac{3^2}{2} + \frac{3^3}{6} + \frac{3^4}{24} \right)$

$= e^{-3} (16.375) = 0.8153$

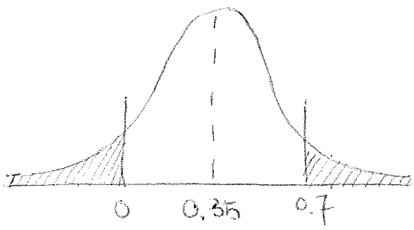
$1 - 0.8153 = 0.1847$ (d)

6 $X \sim N(0.78, 0.10)$ $\quad 0.10^2 + 0.14^2 =$

$Y \sim N(1.13, 0.14)$ $\quad 0.172$

$Z \sim N(0.35, 0.172)$

$P(Z < 0) = \phi\left(\frac{0 - 0.35}{0.172}\right) = \phi(\cancel{0.7}) =$



$\phi(4.07) = 0.024$

$= \cancel{0.242}$

7 $Z = \pi r^2 h$

$\cancel{\sigma_z^2 = \sigma_r^2 (460\pi)^2} +$

$$\tau_T = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T \quad , \tau_T = \frac{1}{\rho}\left(\frac{\partial \rho}{\partial p}\right)_T$$

$$\rho = \frac{p}{RT} \longrightarrow \frac{\partial \rho}{\partial p} = \frac{1}{RT} = \frac{1}{\rho RT}$$

$$Q_{conv} = h \cdot A \cdot \Delta T$$

$$Q_{total} = Q_{conv} + Q_{rad.} = h \cdot A \cdot \Delta T + A \cdot \sigma \cdot \varepsilon (T_1^4 - T_2$$

deviation from ideal gas behaviour ; 1 :

P & O

$$Y = (A) x \qquad A = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad N = \begin{bmatrix} W_{11} & 0 \\ 0 & W_{22} \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

① $$\hat{x} = \left(\begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} W_{11} & 0 \\ 0 & W_{22} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right)^{-1} \times \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} W_{11} & 0 \\ 0 & W_{22} \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix}$$

②

Probability and Observation Theory ; Exams.

23-06-2005

1) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

	1	2	3	4	5	6
1	2	2	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$$\left.\begin{array}{l} P(A) = 5/6 \\ P(B) = 3/36 \\ P(A \cap B) = 2/36 \end{array}\right\} P(A \cup B) = 31/36 \rightarrow B$$

2) Bayes' Role : $P(B_j | A) = \dfrac{P(A|B_j) \cdot P(B_j)}{P(A)}$

A ; warning light on

B ; oil pressure too low

$P(B) = 0{,}1 \rightarrow P(B^c) = 0{,}9$

$P(A|B) = 0{,}99$

$P(A|B^c) = 0{,}02$

$$\hookrightarrow P(H|A) = \frac{0{,}99 \cdot 0{,}1}{0{,}1 \cdot 0{,}99 + 0{,}02 \cdot 0{,}9} = \frac{0{,}099}{\underbrace{0{,}099 + 0{,}018}_{0{,}117}} = 0{,}846$$

$\downarrow$

C

3) ~~$p^3 = 0{,}999 \rightarrow p = \sqrt[3]{0{,}999}$~~

~~$p = 1 - p$~~ $\quad p^c$ = no detection $\rightarrow (p^c)^3 < 0{,}001$

$p^c < 0{,}1$

$p > 0{,}9$

$\hookrightarrow$ A

4) $\bar{h}_1 = 7000 \text{ m} \qquad \sigma_{h_1} = 150 \text{ m}$

$\bar{h}_2 = 8000 \text{ m} \qquad \sigma_{h_2} = 150 \text{ m}$

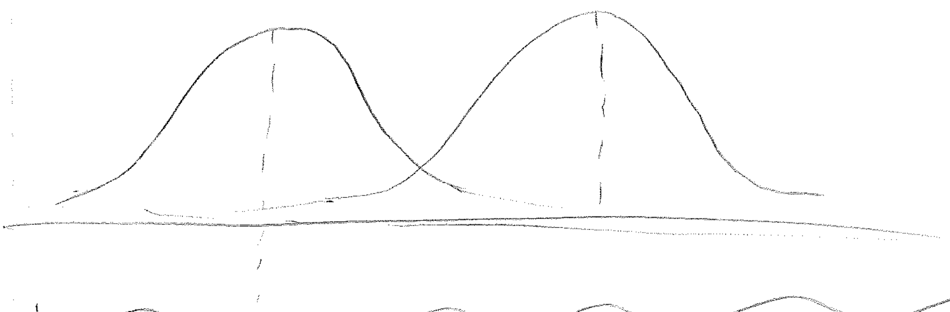
$\left.\right\} (h_2 - h_1) < 500 \text{ m}$

$y = h_2 - h_1 \rightarrow \bar{y} = 1000 \quad , \sigma_y = 150$

~~$\hookrightarrow P(z_1 < z_2) = \Phi(z_2)$~~

$P(y < 500) = \Phi(z_2) = \Phi\left(\frac{500 - 1000}{150}\right) = -3{,}33$

$\hookrightarrow = 1 - \alpha_{3{,}33} = 1 - \rightarrow$ ~~$0{,}00041$~~ $0{,}0092$



5) $f_w(\omega) = \frac{1}{b} \cdot \exp\left(-\left(\frac{\omega - \omega_0}{b}\right)\right)$

$\hookrightarrow E(x) = \bar{x} = \int_{-\infty}^{\infty} x \cdot f_x(x)\,dx = \frac{1}{b} \cdot \int_{-\infty}^{\infty} \omega \exp\left(-\left(\frac{\omega - \omega_0}{b}\right)\right) d\omega$

$u = \omega \qquad v = \frac{\exp\left(-\frac{\omega}{b} + \frac{\omega_0}{b}\right)}{-1/b} = -b \cdot \exp\left(-\frac{\omega}{b} + \frac{\omega_0}{b}\right)$

$du = 1 \qquad dv = \exp\left(-\frac{\omega}{b} + \frac{\omega_0}{b}\right)$

$\frac{1}{b} \cdot \left(\left[-\omega b \cdot \exp\left(-\frac{\omega}{b} + \frac{\omega_0}{b}\right) \right]_{-\infty}^{\infty} + \int_{-\infty}^{\infty} \exp\left(-\frac{\omega}{b} + \frac{\omega_0}{b}\right) d\omega \right.$

$$\left((x_1 + x_2) - \left(\frac{\bar{x}_1 + \bar{x}_2}{2}\right)\right)^2$$

variance $= \sigma^2$, standard dev. $= \sigma$

$$= \frac{1}{b}\left(\left[-\omega b \cdot \exp\left(-\frac{\omega}{b} + \frac{\omega_0}{b}\right)\right]_{\omega_0}^{\infty} + \left[-b \cdot \exp\left(-\frac{\omega}{b} + \frac{\omega_0}{b}\right)\right]_{\omega_0}^{\infty}\right)$$

$$= \frac{1}{b}\left(\left(\overset{0 - -\omega_0 b}{\cancel{0+0}}\right) + \left(\overset{0 - -b}{\cancel{0 + b \cdot \exp\left(\frac{\omega_0}{b}\right)}}\right)\right) \quad ??$$

$$= \frac{1}{b}\left(\omega_0 \cdot b + b\right)$$

~~b) $\underline{y} = x_1 + x_2$~~

~~$\sigma_y^2 = D(y) = E((y - \bar{y})^2) = E\left(\left((x_1 + x_2) - \left(\frac{\bar{x}_1 + \bar{x}_2}{2}\right)\right)^2\right)$~~

~~c) $Z = 10 \cdot 10 = 100$~~

~~$\sigma_z = \left(\frac{(10{,}25 - 9{,}75)^2}{12}\right) \cdot \sqrt{10}$~~

~~$\sigma_z = 0{,}008$~~

$\hookrightarrow$

6) $\sigma_x = \sqrt{\frac{(0{,}50)^2}{12}} = 0{,}5 \cdot \sqrt{1/12}$

$\sigma_z = 0{,}5 \cdot \sqrt{1/12} \cdot \sqrt{10} = 0{,}5 \cdot \sqrt{5/6}$

$\bar{z} = 100$

$\hookrightarrow p(z < 99) = \Phi\left(\frac{99 - 100}{0{,}5 \cdot \sqrt{5/6}}\right) = \Phi(-2{,}19) = 0{,}0143$

A

8) $E(x) = \bar{x} = \int_{-\infty}^{\infty} x \cdot f_x(x)\, dx$
$\hookrightarrow = 0{,}2 \cdot e^{-0{,}2 \cdot x}$

$\bar{x} = \int_{2}^{\infty} x \cdot 0{,}2 \cdot \exp(-0{,}2 \cdot x)\, dx$ $\longrightarrow u = x$ $\quad v = -\exp(-0{,}2\, x)$
$du = 1 \quad dv = 0{,}2 \cdot \exp(-0{,}2x$

$\bar{x} = \left[-x \cdot \exp(-0{,}2 \cdot x)\right]_2^{\infty} + \int_2^{\infty} (\exp(-0{,}2\, x))\, dx$

$= (0 - -2 \cdot \exp(-0{,}4) + \left[\frac{\exp(-0{,}2 \cdot x)}{-0{,}2}\right]_2^{\infty}$

$= 2 \cdot \exp(-0{,}4) + (0 + 5 \cdot \exp(-0{,}4)) = 7 \cdot \exp(-0{,}4)$
$= 6{,}69 \rightarrow 7$ maanden
$\hookrightarrow$ D

9) PDF of quotient of st. normal distributions ;
$\rightarrow$ page 55 ; $y = x_1/x_2 \rightarrow f_{\underline{y}}(y) = \int_{-\infty}^{\infty} |x_2| \cdot f_{\underline{x}_1 \underline{x}_2}(yx_2, x_2)\, d$

$\hookrightarrow f_{\underline{x}_1 \underline{x}_2} = \frac{1}{2\pi} \cdot \exp\left(-\frac{1}{2} x_1^2 - \frac{1}{2} x_2^2\right) = \frac{1}{2\pi} \cdot \exp\left(-\frac{1}{2}\left(x_1^2 + x_2^2\right)\right)$

$f_{\underline{y}}(y) = \int_{-\infty}^{\infty} |x_2| \cdot \frac{1}{2\pi} \cdot \exp\left(-\frac{1}{2}\left(y^2 x_2^2 + x_2^2\right)\right) dx_2$

$x_1 \sim N(0,1)$, $x_2 \sim N(0,2)$

10) $Z_1 = \underline{x}_1 + \underline{x}_2$

$$f_{x_1 x_2}(x_1, x_2) = \frac{1}{8\pi \cdot 2} \cdot \exp\left(-\tfrac{1}{2}\left(x_1 + \frac{x_2}{2}\right)\right)$$

$$F_y(y) = P(\underline{x}_1 + \underline{x}_2 \leq y) = \frac{1}{8\pi} \cdot \iint\limits_{(x_1+x_2) \leq y} \exp\left(-\tfrac{1}{2}\left(x_1 + \frac{x_2}{2}\right)\right) dx_1\, dx_2$$

??

---

11) $t_1 = 1 \longrightarrow x_1 = 10$
$t_2 = 2 \longrightarrow x_2 = 504$
$t_3 = 3 \longrightarrow x_3 = 996$

$\boxed{\text{WLSE}}$

10
504
996

$\hat{x}(0)$
$\hat{v}$

$[y] = [A][x]$

$W = 5 \cdot I_3$

$\hat{x}(t) = \hat{x}(0) + \hat{v} \cdot t$

$\hookrightarrow A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}$

$\hat{x} = (A^T \cdot W \cdot A)^{-1} \cdot A^T \cdot W \cdot y$

$$= \left(\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \right)^{-1}$$

$$= \begin{bmatrix} 5 & 5 & 5 \\ 5 & 10 & 15 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 15 & 30 \\ 30 & 70 \end{bmatrix}$$

$$\left(\begin{bmatrix} 15 & 30 \\ 30 & 70 \end{bmatrix} \right)^{-1} = \frac{1}{150} \begin{bmatrix} 70 & -30 \\ -30 & 15 \end{bmatrix}$$

$$\hat{x} = \frac{1}{150}\begin{bmatrix} 70 & -30 \\ -30 & 15 \end{bmatrix}\begin{bmatrix} 5 & 5 & 5 \\ 5 & 10 & 15 \end{bmatrix}\begin{bmatrix} 10 \\ 504 \\ 996 \end{bmatrix}$$

$$\begin{bmatrix} 50 + 2520 + 4980 \\ 50 + 5040 + 14940 \end{bmatrix} = \begin{bmatrix} 7550 \\ 20030 \end{bmatrix}$$

$$\frac{1}{150}\begin{bmatrix} -72400 \\ 73950 \end{bmatrix} = \begin{bmatrix} -482,667 \\ 495 \end{bmatrix} = \begin{bmatrix} \hat{x}(0) \\ \hat{v} \end{bmatrix}$$

$\hookrightarrow$ B

12) ??? (Variance of a BLUE)

$\underline{h}_i = h_i + \underline{e}_i \quad ; i = 1,2,3$

$E(\underline{e}_i) = 0$

$D(\underline{e}_i) = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} cm^2$

$$D\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} cm^2$$

Variance prop. law,

$\underline{y} = A\underline{x} + b \rightarrow Q_{yy} = A \cdot Q_{xx} \cdot A^T$

$\hookrightarrow = D(\underline{y}) \quad \hookrightarrow = D(\underline{x})$

$Q_{\hat{x}\hat{x}} = (A^T$

Standard deviation ;

$$\begin{array}{llr} 90.2 & \cancel{16.15} & 8136.04 \\ 59.1 & \cancel{-14.95} & 3492.81 \\ 87.5 & \cancel{13.25} & 7621.29 \\ 59.6 & \cancel{-14.45} & 5552.16 \\ \hline 74.05 & & 5700.575 \end{array}$$

-5483.4025

$$219.1 \left\{ \begin{array}{lll} 59.1 & \rightarrow & 3492.81 \\ 559.7 & \rightarrow & \\ 267.7 & \rightarrow & \\ 209.9 & \rightarrow & \end{array} \right\} 58652.55$$

$$1/2^2 + 5/6^2 + 2/3^2 =$$

$$\sigma_z^2 = E(z^2) - \bar{z}^2$$

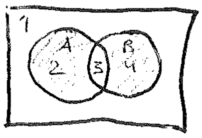
$$\underline{X}_1 = 1/2 \cdot \underline{y}_1 + 1/6 \cdot \underline{y}_2 + 1/3 \cdot \underline{y}_3$$
$$\underline{X}^2 = (1/2 \cdot x + 1/6 \cdot x + 1/3 x)^2 = \bar{x}^2$$

$$\frac{1/2 \cdot y_1 + 1/6 \cdot \underline{y}_2 + 1/3 \cdot y_3}{3} = 1/6 =$$

31-08-2005 ;

$$P(A^c \cup B^c) = P(A^c) + P(B^c) - P(A^c \cap B^c)$$

$$P(A^c) = 2/3$$
$$P(B^c) = 1/2$$



$2 + 3 = 1/3$
$3 + 4 = 1/2$
$2 + 4 = 3/4$

1	2	3	4	
1	$\cancel{1}0$	1	$\cancel{1}0$	$\cancel{4}\ 1/4$
0	$\cancel{1}0$	$\cancel{1}2$	$\cancel{0}\ -1$	$1/3 - 3/4 + 1/2$
0	0	1	1	$1/2$
0	$1\cancel{2}$	0	$1\cancel{0}$	$3/4$

$$2 \cdot 3 = 4/12 - 9/12 + 6/12 = 1/12$$

$$\cancel{P(A \cap B) = P(A) + P(B) - P(A \cup B)}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\begin{aligned} P(A \cap B) &= P(A) + P(B) - P(A \cup B) \\ &= 1/3 + 1/2 - 3/4 \\ &= 4/12 + 6/12 - 9/12 = 1/12 \end{aligned}$$

$$1 = P(A) + P(B) - 1/12 = 10/12 - 1/12 = 9/12$$

$$P(A^c \cup B^c) = 1 - 3 - 1 = 1 - 2/12 = 10/12 \rightarrow a$$

2) Bayes' Rule ; $P(H|A^c) = \dfrac{P(A^c|H) \cdot P(H)}{P(A_c)}$

$$A = \begin{bmatrix}1 & 1 & 1\end{bmatrix} \rightarrow A^T = \begin{bmatrix}1\\1\\1\end{bmatrix}$$

$$Q_{44} = D(e) = \begin{bmatrix}2 & 0 & 0\\0 & 2 & 0\\0 & 0 & 2\end{bmatrix} \rightarrow Q_{44}^{-1} = \begin{bmatrix}1/2 & 0 & 0\\0 & 1/2 & 0\\0 & 0 & 1/2\end{bmatrix}$$

$$\cancel{W = \begin{bmatrix}1/3 & 0 & 0\\0 & 1/3 & 0\\0 & 0 & 1/3\end{bmatrix}}$$

$$\left(A^T Q_{44}^{-1} A\right)^{-1} = \left(\begin{bmatrix}1\\1\\1\end{bmatrix} \begin{bmatrix}1/2 & 0 & 0\\0 & 1/2 & 0\\0 & 0 & 1/2\end{bmatrix} \begin{bmatrix}1 & 1 & 1\end{bmatrix} \right)^{-1}$$

$=$

13

$$y = (A)\,x$$

$$y \rightarrow \begin{pmatrix}y_1\\ \vdots \\ y_n\end{pmatrix} \qquad A \rightarrow \begin{pmatrix}1\\1\\ \vdots \\ n\end{pmatrix}$$

$$\begin{bmatrix}1 & 1 \ldots 1 & m\end{bmatrix} \begin{bmatrix}5 & & \\ 0 & \ddots & \\ 0 & & \\ 0 & & m\end{bmatrix} \begin{bmatrix}1\\ \vdots \\ m\end{bmatrix} = \cancel{\begin{bmatrix}5\\5\\5\\5\end{bmatrix}} \begin{bmatrix}5 & 5 & 5 & 5\end{bmatrix} \begin{bmatrix}1\\ \vdots \\ m\end{bmatrix}$$

$$= 5 \cdot m \rightarrow (\ )^{-1} = \cancel{\tfrac{1}{5}m}\ \frac{1}{5m}$$

$$A^T W = \begin{bmatrix}1 & 1 & 1 \ldots & m\end{bmatrix} \begin{bmatrix}5 & & \\ & \ddots & 5\end{bmatrix} = \begin{bmatrix}5 & 5 & 5 & 5\end{bmatrix}$$

$\cancel{2(1/2 \cdot 3/4) + 1/4 \cdot 1/4 =}$ $\quad 3/4 + 1/16 = 10/16$

$\cancel{3/4 + 1/4 = 1}$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} x \quad , \; W = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix} , \; Q_{yy} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

$$A^T W A = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= 8$$

$$()^{-1} = 1/8$$

$$A^T W Q_{yy} W A = \begin{bmatrix} 2 & 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 6 & 8 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 2 \\ 2 \end{bmatrix}$$

$$= 4 + 8 + 12 + 16$$

$$= 40$$

$$Q_{xx} = 1/8 \cdot 40 \cdot 1/8 = \frac{40}{64} = 5/8$$

(c)

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} x \; , \; Q_{yy} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \rightarrow Q_{yy}^{-1} = \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/2 \end{bmatrix}$$

↳ A

$$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 3/2$$

$$\frac{2}{3}$$

A : light is on $\rightarrow A^c$: light is off.
H : oil pressure low

$P(H) = 0,1$
$P(A|H) = 0,99$
$P(A|H^c) = 0,02$
$P(A^c|H^c) = 0,98$

$$P(H|A^c) = \frac{0,01 \cdot 0,1}{0,01 \cdot 0,1 + 0,9 \cdot 0,98} = 0,00115 \longrightarrow c$$

false

3) $\bar{X}_1 = 3$ $\quad \sigma_{x_1} = \sqrt{2}$
$\bar{X}_2 = 5$

$$\bar{Y} = g(\bar{x}) + \tfrac{1}{2} \cdot g''(\bar{x}) \cdot \sigma_x^2$$
$$= (3 \cdot 3^2 - 2 \cdot 5) + \tfrac{1}{2}(6) \cdot 2$$
$$= 27 - 10 + 6 = 23 \longrightarrow a$$

4) X

5) $\underline{Y} = \underline{X_1}/\underline{X_2} \rightarrow f_{\underline{Y}}(y) = \int_0^\infty |x_2| \, f_{\underline{X}_1, \underline{X}_2}(y x_2, x_2) \, dx_2$

$f_{\underline{X}_1} \cdot f_{\underline{X}_2} = f_{\underline{X}_1, \underline{X}_2}$

$\hookrightarrow f_{\underline{X}_1 \underline{X}_2} = \lambda^2 \cdot \exp(-\lambda(x_1 + x_2))$

$y = g(x_1, x_2) = \frac{x_1}{x_2} \rightarrow g^{-1}(y, x_2) = x_1 = y \cdot x_2$
$\frac{dg^{-1}}{dy} = x_2$

$$f_{\underline{Y}}(y) = \lambda^2 \int_0^\infty \exp(-\lambda(y \cdot x_2 + x_2)) \cdot x_2 \, dx_2$$
$$\exp(-\lambda y \cdot x_2 - \lambda x_2) \cdot x_2$$

11) $D(\underline{e}) = D(\underline{y}) = Q_{yy} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

$\cancel{x_1 + x_2 + x_3 = c}$

$\cancel{\begin{bmatrix} \\ \\ \\ \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}$

$\underbrace{[c]}_{\hookrightarrow y} = \underbrace{[1 \quad 1 \quad 1]}_{\hookrightarrow A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{\hookrightarrow x}$

$P = (A^T A)^{-1} \cdot A^T = \left[\frac{1}{3}\right] \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \cancel{\begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}} \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$

$Q_{yy} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

$A (A^T \cdot A)^{-1} = [1 \quad 1 \quad 1] = [3 \quad 3 \quad 3]$

$Q_{\hat{x}\hat{x}} = \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} [3 \quad 3 \quad 3]$

$= \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} [3 \quad 3 \quad 3]$

$$f_{\underline{x}_i}(x_i) = \lambda \cdot \exp(-\lambda x_i)$$
$$= 0{,}4 \cdot \exp(-0{,}4 \cdot x_i)$$

$$E(\underline{x}_i) = \int_0^\infty x \cdot 0{,}4 \cdot \exp(-0{,}4 \cdot x)\, dx$$

$\hookrightarrow$ $u = x$ $\quad$ $v = -\exp(-0{,}4\, x)$

$du = 1$ $\quad$ $dv = 0{,}4 \cdot \exp(-0{,}4 \cdot x)$

$$E(\underline{x}) = \left[-x \cdot \exp(-0{,}4\, x)\right]_0^\infty + \int^\infty \exp(-0{,}4\, x)\, dx$$
$$= (0-0) + \left[-\frac{\exp(-0{,}4\, x)}{0{,}4}\right]_0^\infty$$
$$= \left[-2{,}5 \cdot \exp(-0{,}4 \cdot x)\right]_0^\infty = (-0 - -2{,}5) = 2{,}5$$

$\hookrightarrow$ $5 * 2{,}5 = 12{,}5$ maanden $\longrightarrow$ a

---

9) a)

---

8) $f_{\underline{y}}(y) = f_{\underline{x}}(g^{-1}(y)) \cdot \left|\frac{dg^{-1}(y)}{dy}\right|$

$\hookrightarrow$ $f_{\underline{x}}(x) = 1/2$

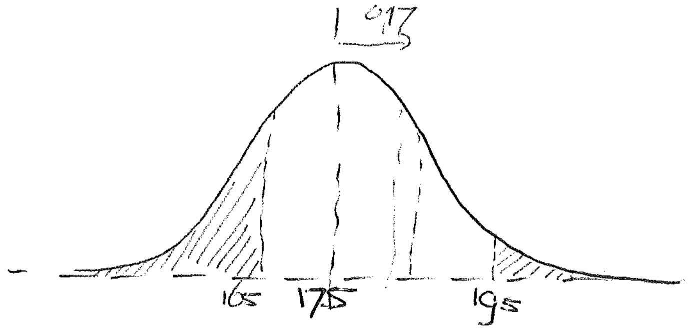
$y = g(x) = 3x^2 + 1 \longrightarrow x^2 = \frac{y-1}{3} \longrightarrow x = \sqrt{\frac{y-1}{3}}$

$$\frac{dx}{dy} = \left(\frac{y-1}{3}\right)^{1/2} = 1/2 \cdot 1/3 \cdot \left(\frac{y-1}{3}\right)^{-1/2} = \frac{1}{6 \cdot \left(\frac{y-1}{3}\right)^{1/2}} = \frac{1}{6\sqrt{\frac{y-1}{3}}}$$

$$\frac{3}{6 \cdot \sqrt{3(y-1)}} = \frac{1}{2 \cdot \sqrt{3(y-1)}} \longrightarrow d$$

a

7) $\bar{x} = 175$ cm
$\sigma_x = 10$ cm



$$E(\underline{x}) = \bar{x} = \int_{-165}^{195} x \cdot \frac{1}{\sqrt{2\pi}\cdot 10} \cdot \exp\left(-\tfrac{1}{2}\left(\frac{x-\cancel{\bar{x}}}{10}\right)^2\right) dx$$

$x = 0$ $\qquad v =$
$dx = 1$ $\qquad dv = \exp(-\tfrac{1}{2}($

$P(\underline{x} < 165) = 0{,}1585$ } $1 - \ldots = 0{,}8185$
$P(\underline{x} > 195) = 0{,}023$

$P(\underline{x} > a) = 0{,}40925 + 0{,}023 = 0{,}4323$

$\hookrightarrow \Phi(0{,}17) = 0{,}4323$

$\hookrightarrow 0{,}17 = \frac{a-175}{10} = 1{,}7 = 176{,}7 \rightarrow b$

a

~~6) JDF : $f_{\underline{x}_1} = e^{-x_1}$, $f_{\underline{x}_2} = 3e^{-3x_2}$~~

~~$F_{x_1} = -e^{-3x}$, $F_{x_2} = -e^{-x}$ } $F_{x_1} \cdot F_{x_2}$~~

~~$\hookrightarrow f_{\underline{x}_1,\underline{x}_2}(x_1, x_2) = 3e^{(-3x_2 - x_1)}$~~

~~$$F_{\underline{x}_1\underline{x}_2}(x_1, x_2) = \int_{-\infty}^{x_1}\int_{-\infty}^{x_2} 3 \cdot e^{(-3x_2 - x_1)} dx_2\, dx_1$$~~

~~$$= \int_{-\infty}^{x_1} \left[-e^{(-3x_2 - x_1)} \right]_{-\infty}^{x_2} dx_1 = \int_{-\infty}^{x_1} (-e^{(-3x_2 - x_1)}$$~~

6) $f_{\underline{X}_1} = e^{-x_1}$, $f_{\underline{X}_2} = 3 \cdot e^{-3x_2}$

$$f_{\underline{X}_1,\underline{X}_2} = 3e^{-x_1-3x_2}$$

$$F_{\underline{X}_1,\underline{X}_2} = \int_{0}^{x_1}\int_{0}^{x_2} 3\cdot \exp(-x_1-3\cdot x_2)\,dx_2\,dx_1$$

$$= \int_{0}^{x_1} \left[-3\cdot \exp(-x_1-3x_2)\cdot \frac{1}{3}\right]_{0}^{x_2} dx_1$$

$$= \int_{0}^{x_1} \left(-\exp(-x_1-3x_2)+\exp(-x_1)\right)$$

$$= \left[\exp(-x_1-3x_2)-\exp(-x_1)\right]_{0}^{x_1}$$

$$= \exp(-x_1-3x_2)-\exp(-x_1)-\exp(-3x_2)+1$$

↳ (b)

---

5) $\underline{Y} = \underline{X}_1 / \underline{X}_2$ → $f_{\underline{X}_1} = \lambda\cdot\exp(-\lambda x_1)$

$f_{\underline{X}_2} = \lambda\cdot\exp(-\lambda x_2)$

$$f_{\underline{Y}}(y) = \int_{-\infty}^{\infty} |x_2| \cdot f_{\underline{X}_1,\underline{X}_2}(yx_2, x_2)\,dx_2$$

↳ $f_{\underline{X}_1,\underline{X}_2} = \lambda^2\cdot\exp(-\lambda x_1-\lambda x_2)$ → $x_1 = y\cdot x_2$

$x_2 = x_2$

$$f_{\underline{X}_1,\underline{X}_2}(y\cdot x_2, x_2) = \lambda^2\exp(-\lambda y\cdot x_2-\lambda x_2)$$

$$f_{\underline{Y}}(y) = \left[|x_2|\cdot \frac{\lambda^2\cdot\exp(-\lambda y x_2-\lambda x_2)}{-\lambda y-\lambda}\right]_{0}^{\infty}$$

$$f_{\underline{Y}}(y) = \left(\frac{|\infty|\cdot\lambda^2\cdot 1}{-\lambda y-\lambda}\right.$$

$$f_{\underline{X}_1,\underline{X}_2}(x_1,x_2) = \lambda^2 \cdot \exp(-\lambda x_1 - \lambda x_2)$$

$$f_{\underline{Y}}(y) = \int_0^\infty f_{\underline{X}_1,\underline{X}_2}\left(g^{-1}(y,x_2), x_2\right) \cdot \left| \frac{dg^{-1}}{dy}(y,x_2) \right| dx_2$$

$$g(x_1, x_2) = \frac{x_1}{x_2} \rightarrow g^{-1}(y, x_2) = x_1 = y \cdot x_2$$

$$\left| \frac{dg^{-1}}{dy} \right| = |x_2|$$

$$f_{\underline{Y}}(y) = \int_0^\infty x_2 \cdot \lambda^2 \exp(-\lambda y x_2 - \lambda x_2)\, dx_2$$

$\hookrightarrow u = x_2$ $\quad v = \dfrac{\lambda^2 \exp(-\lambda y x_2 - \lambda x_2)}{-\cancel{\lambda} y - \cancel{\lambda} 1}$

$du = 1$ $\quad dv = \lambda^2 \exp(-\lambda y x_2 - \lambda x_2)$

$$\hookrightarrow f_{\underline{Y}}(y) = \left[x_2 \frac{\lambda \cdot \exp(-\lambda y x_2 - \lambda x_2)}{-y-1} \right]_0^\infty + \int_0^\infty \frac{\lambda \cdot \exp(-\lambda y x_2 - \lambda x_2)}{y+1} dx_2$$

$$= \left(0 + \frac{\lambda}{y+1} \right) + \left[\frac{\lambda \exp(-\lambda y x_2 - \lambda x_2)}{(-\lambda y - \lambda)(y+1)} \right]$$

$$\hookrightarrow = -\frac{\exp(-\lambda y x_2 - \lambda x_2)}{(y+1)^2}$$

$$\hookrightarrow \frac{\lambda}{y+1} + \left(0 + \frac{1}{(y+1)^2} \right)$$

$$= \cancel{\frac{(y+1)\lambda}{(y+1)^2}} + \frac{1}{(y+1)^2} = \frac{\lambda(y+1)+1}{(y+1)^2}$$

$\hookrightarrow$ d

4) $\underline{x}_1 \sim N(2{,}5) \quad , \underline{x}_2 \sim N(5{,}9)$

$\underline{z} = 3 \cdot \underline{x}_1 - 2 \cdot \underline{x}_2 + 1 \quad \longrightarrow \underline{z} \sim N(-3,$

$\cancel{\hookrightarrow \underline{z} \sim N(-3,}$

$$f_{\underline{y}}(y) = \int_{-\infty}^{\infty} f_{\underline{x}_1,\underline{x}_2}\left(g^{-1}(z, x_2), x_2\right) \left| \frac{dg^{-1}}{dy}(z, x_2) \right| dx_2$$

$$g(x_1, x_2) = z = 3x_1 - 2x_2 + 1$$

$$\hookrightarrow 3x_1 - 2x_2 + 1 = z$$

$$\cancel{-2x_2 = z + 1 \ldots}$$

$$\cancel{3x_1 = z + 1 - 2}$$

$$3x_1 = 2x_2 + z - 1$$

$$x_1 = \tfrac{2}{3}x_2 + \tfrac{1}{3}z - \tfrac{1}{3}$$

$$f_{\underline{x}_1\underline{x}_2} = \frac{1}{2\pi \cdot 45} \cdot \exp\left(-\tfrac{1}{2}\left(\frac{x_1-2}{5}\right)^2 - \tfrac{1}{2}\left(\frac{x_2-5}{9}\right)^2\right)$$

$$\frac{dg^{-1}}{dz} = \tfrac{1}{3}$$

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4) $h_1 \sim N(7000, 150)$, $h^2 \sim N(8000, 150)$ $\Big\}$ $dh = h_2 - h_1 \sim N(\overset{1000}{\cancel{8000-7000}}, 212{,}13)$

$$\hookrightarrow P(dh \leq 500) = \Phi\left(\frac{500-1000}{212{,}13}\right) = \Phi(2{,}357)$$

$$= 0{,}0091 \rightarrow \text{C}$$

4) $\underline{x}_1 \sim N(2,5)$, $\underline{x}_2 \sim N(5,9)$ $\Big\}$ $z = 3\underline{x}_1 - 2\underline{x}_2 + 1 \sim N(-3;\ 23{,}43)$

$$P(-12 < \underline{z} < 6) = \Phi\left(\frac{6--3}{23{,}43}\right) - \Phi\left(\frac{-12--3}{23{,}43}\right) = \Phi(0{,}38) - \Phi(-0{,}38)$$

$$= 1{,}00$$

$$\sqrt{150^2 + 150^2}$$

$$\left.\begin{array}{l} N_1 \sim (7000, 150) \\ N_2 \sim (8000, 150) \end{array}\right\} dh = N_2 - N_1 \sim N(1000,\ 212,13)$$

$$\left.\begin{array}{l} N_1 \sim (2, 5) \\ N_2 \sim (5, 9) \end{array}\right\} z = 3 N_1 - 2 N_2 + 1 \sim N(-3,\ 9)$$

$$\sqrt{3^2 \cdot 5 + 2^2 \cdot 9}$$

$$\left.\begin{array}{l} N_1 \sim N(7000, 150^2) \\ N_2 \sim N(8000, 150^2) \end{array}\right\} dh = N_2 - N_1 \sim N\left(1000, \sqrt{150^2 + 150^2}\right)$$

↳ st. dev. $= \sqrt{150^2 + 150^2}$

$$\left.\begin{array}{l} N_1 \sim (2, 5^2) \\ N_2 \sim (5, 9^2) \end{array}\right\} z = 3 \cdot N_1 - 2 \cdot N_2 + 1$$

$$\sim N\left(-3, (3^2 \cdot 5 + 2^2 \cdot 9)\right)$$

$$f(x \mid a \le \underline{X} \le b) \quad \frac{f(x)}{F(b) - F(a)}$$

$$\hookrightarrow f_{\underline{X}}(x) = \frac{1}{0{,}8185} \cdot \frac{1}{\sqrt{2\pi} \cdot 10} \cdot \exp\left(-\frac{1}{2}\left(\frac{x - 175}{10}\right)^2\right) \longrightarrow$$

$$E(\underline{Y} \mid x) = \int_{-\infty}^{\infty} y \cdot f(y \mid x)\, dy$$

$$= \int_{165}^{195}$$

$$E(\underline{x}) = \frac{1}{\sqrt{2\pi}\,\sigma_x} \int_{-\infty}^{\infty} x \cdot \exp\left(-\tfrac{1}{2}\left(\frac{x-\bar{x}}{\sigma_x}\right)^2\right) dx = \bar{x}$$

$$\hookrightarrow \frac{x-\bar{x}}{\sigma_x} = z \rightarrow \cancel{x}\ \bar{x} =$$

$$\frac{x}{\cancel{\sigma_x}} - \frac{\cancel{\bar{x}}}{\cancel{\sigma_x}} = z \cdot \sigma_x + \cancel{\bar{x}}$$

$$dx = \sigma_x \cdot dz$$

$$E(\underline{x}) = \frac{1}{\sqrt{2\pi}} \cdot \int_{105}^{195} z \cdot \sigma_x \cdot \exp\left(-\tfrac{1}{2} z^2\right) dz + \frac{\bar{x}}{\sqrt{2\pi}} \int_{105}^{195} 1 \cdot \exp\left(-\tfrac{1}{2}(\cancel{\ \ }) z^2\right) dz$$

$$E(\underline{x}) = \frac{1}{\sqrt{2\pi}}$$

---

2)

$$A = \begin{bmatrix} 1 & -3 \\ 2 & 1 \\ 3 & -1 \\ 4 & 1 \end{bmatrix}, \quad \sigma_{44} = \begin{bmatrix} \sigma^2 & 0 & 0 & 0 \\ 0 & \sigma^2 & 0 & 0 \\ 0 & 0 & \sigma^2 & 0 \\ 0 & 0 & 0 & \sigma^2 \end{bmatrix} \rightarrow \sigma_{44}^{-1} = \begin{bmatrix} 1/\sigma^2 & 0 & 0 & 0 \\ 0 & 1/\sigma^2 & 0 & 0 \\ 0 & 0 & 1/\sigma^2 & 0 \\ 0 & 0 & 0 & 1/\sigma^2 \end{bmatrix}$$

$$\hookrightarrow \begin{bmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{bmatrix}^{-1} = \frac{1}{\sigma^4} \begin{bmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{bmatrix}$$

$$\hookrightarrow \begin{bmatrix} 1 & 2 & 3 & 4 \\ -3 & 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1/\sigma^2 & 0 & 0 & 0 \\ 0 & 1/\sigma^2 & 0 & 0 \\ 0 & 0 & 1/\sigma^2 & 0 \\ 0 & 0 & 0 & 1/\sigma^2 \end{bmatrix} = \begin{bmatrix} 1/\sigma^2 & 2/\sigma^2 & 3/\sigma^2 & 4/\sigma^2 \\ -3/\sigma^2 & 1/\sigma^2 & -1/\sigma^2 & 1/\sigma^2 \end{bmatrix}$$

$$\begin{bmatrix} 1/\sigma^2 & 2/\sigma^2 & 3/\sigma^2 & 4/\sigma^2 \\ -3/\sigma^2 & 1/\sigma^2 & -1/\sigma^2 & 1/\sigma^2 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ 2 & 1 \\ 3 & -1 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 30/\sigma^2 & 0 \\ 0 & 12/\sigma^2 \end{bmatrix}$$

$$\sigma_{x_1 x_1} = \frac{30}{\sigma^2} < 1 \rightarrow \sigma_{xx}^2 > 30 \rightarrow \sigma_{xx} > 5.47$$

$$\begin{bmatrix} 30/\sigma^2 & 0 \\ 0 & 12/\sigma^2 \end{bmatrix}^{-1} = \frac{1}{\frac{360}{\sigma^4}} \begin{bmatrix} 12/\sigma^2 & 0 \\ 0 & 30/\sigma^2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{0{,}0833}{\sigma^2} & 0 \\ 0 & \frac{0{,}0833}{\sigma^2} \end{bmatrix}$$

$$\cancel{\frac{12/\sigma^2}{360/\sigma^4} = \left(\frac{12}{360}\right)\sigma^2 = 1 \quad \hookrightarrow \sigma^2 = \frac{360}{12} \rightarrow \sigma = \sqrt{360/12}}$$

$$\cancel{\sigma_{x_2} = \left(\frac{12}{360}\right)\cdot\sigma^2 = 1}$$

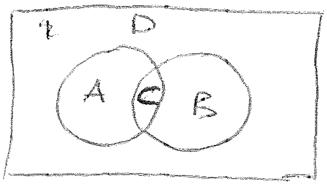
$$\frac{30/\sigma^2}{360/\sigma^4} = \left(\frac{30}{360}\right)\sigma^2$$

$$\hookrightarrow \sigma^2_{\hat{x}_2 \hat{x}_2} = 1 = \frac{3}{36}\cdot\sigma^2$$

$$\frac{36}{3} = 12 = \sigma^2 \longrightarrow \sigma = \sqrt{12} \leq 3{,}46$$

$$< 3{,}46 \rightarrow B$$

$P(A) = 0{,}6$, $P(A \cup B) = 0{,}8$, $P(A|B) = 0{,}5$



$A + C = 0{,}6$
$A = 0{,}8 - B$
$C = 0{,}5 \cdot B$

$0{,}8 - B + 0{,}5 \cdot B = 0{,}6$
$0{,}2 = B - 0{,}5 \cdot B = 0{,}5 \cdot B$
$\hookrightarrow B = 0{,}4 \longrightarrow A$

2) $\cancel{\frac{3}{5}\cdot\frac{1}{3} + \frac{2}{3}\cdot\frac{4}{5} = \frac{3}{15} + \frac{8}{15} = \frac{11}{15}}$

~~B lasts more than 3 months~~
A > 3 months
B ; type A
C ; type B

$$P(B|A) = \frac{P(A|B)\cdot P(B)}{P(A)}$$

$P(A|B)$ = kans more 3 monts, given : type A $^{3/5}$
$P(B) = 1/3$
$P(A) = 11/15$

$\hookrightarrow P(B|A) = \frac{3/5 \cdot 1/3}{11/15} = \frac{3/15 \cdot 15}{11} = \frac{3}{11} \rightarrow$ D

3) $P_{\underline{x}}(k) = \exp(-\lambda)\cdot \frac{\lambda^k}{k!} = \exp(-2)\cdot\frac{\lambda^2}{2}$

$\exp(-2)\left(\frac{2^0}{1} + \frac{2^1}{1} + \frac{2^2}{2} + \frac{2^3}{6}\right) =$

$(1 + 2 + 2 + 8/6) =$

$P_{\underline{x}}(k > 4) = 1 - \exp(-3)\cdot\left(\frac{3^0}{1} + \frac{3^1}{1} + \frac{3^2}{2} + \frac{3^3}{6} + \frac{3^4}{24}\right)$

$\hookrightarrow (1 + 3 + 4{,}5 + 4{,}5 + \frac{81}{24}) = \frac{131}{8}$

$\hookrightarrow 1 - e^{-3}\cdot\frac{131}{8} = 0{,}1847 \rightarrow$ D

4) $\underline{x} \sim N(10, 4) \rightarrow \underline{y} = 9/5 \cdot \underline{x} + 32$

$\underline{y} \sim N(50, 3{,}6)$

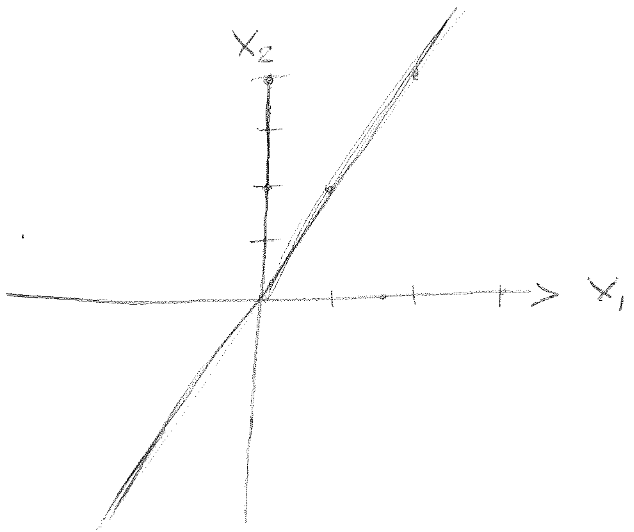
$\sigma_y^2 = a^2\sigma_x^2$

$\frac{81}{25}\cdot 4$

$\hookrightarrow \sigma_y = \sqrt{\frac{81}{25}\cdot 4} = a\cdot\sqrt{4}$

5) $F_{\underline{x}}(x) = -\frac{3}{4}x^2 + \frac{3}{2}x \rightarrow F_{\underline{x}}(x) = \left[-\frac{1}{4}x^3 + \frac{3}{4}x^2\right]_0^{x_1}$

$\hookrightarrow$ D



$\cancel{\tfrac{1}{2}}\, x_1 < \tfrac{1}{2} x_2$

$x = \tfrac{1}{2} y$

$y = 2x$

$x_1 = \tfrac{1}{2} \cdot x_2$

$0 \leq x_1 \leq \tfrac{1}{2} \cdot x_2$

$0 \leq x_2 \leq 2$

$$\int_0^2 \int_0^{\frac{1}{2} \cdot x_2} \left(\frac{3}{10} \cdot x_1^2 + \frac{4}{5} \cdot x_1 \cdot x_2 \right) dx_1\, dx_2$$

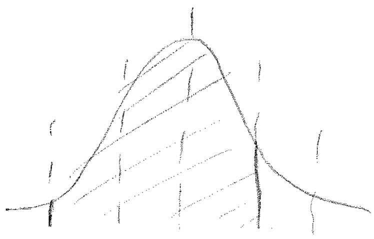
$$= \int_0^2 \left[\frac{1}{10} \cdot x_1^3 + \frac{2}{5} \cdot x_1^2 \cdot x_2 \right]_0^{\frac{1}{2} x_2} dx_2$$

$$= \int_0^2 \left(\tfrac{1}{8} \cdot \tfrac{1}{10} \cdot x_2^3 + \tfrac{2}{5} \cdot \tfrac{1}{4} \cdot x_2^3 \right) dx_2$$

$$= \left[\tfrac{1}{80} \cdot \tfrac{1}{4} \cdot x_2^4 + \cancel{\tfrac{2}{20}}\, \tfrac{1}{10} \cdot \tfrac{1}{4} \cdot x_2^4 \right]_0^2$$

$$= \tfrac{1}{320} \cdot 16 + \tfrac{1}{40} \cdot 16 = \tfrac{9}{20} \rightarrow C$$

7) $x \sim N(2{,}0, (0{,}01)^2)$



$$= 1 - \left(0{,}683 + \left(\frac{0{,}954 - 0{,}683}{2} \right) \right)$$

$$= 0{,}1815 \rightarrow B$$

8) exp. $f_x = 0{,}2 \cdot e^{-0{,}2 \cdot x}$, $x \geq 0$

$$P(\underline{x} > 5 \mid 3 < \underline{x} < 6) = \frac{P(5 < \underline{x} < 6)}{P(3 < \underline{x} < 6)}$$

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1)
2)
3)
4)
5)
6)*
7)
8)
9)
10)
11)*
12)
13)
14)

1)
2)
3)
4)
5)
6)
7)
8)
9)
10)
11)
12)
13)
14)

$y = \begin{bmatrix} 0.2 & 0.8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$Q_{xx} = \begin{bmatrix} 0.5^2 & \\ & 2.9^2 \end{bmatrix}$

~~31-08-2005~~
~~23-06-2005~~
~~03-07-2006~~

$1^2 \cdot 4 + \cancel{5}(-2)^2 \cdot 5 \cdot 1 = 4 +$ [illegible]

40 $\underline{x}_1 \sim N(0, 4)$ $\underline{z}_1 = \underline{x}_1 + \underline{x}_2 \; ; \sim N(0, 5)$

$\underline{x}_2 \sim N(0, 1)$ $\underline{z}_2 = \underline{x}_1 - 2 \cdot \underline{x}_2 \; ; \sim N(0, 8)$

$$f_{z_1} = \frac{1}{\sqrt{2\pi} \cdot \sqrt{5}} \cdot \exp\left(-\tfrac{1}{2}\left(\frac{z_1}{\sqrt{5}}\right)^2\right)$$

$$f_{z_2} = \frac{1}{\sqrt{2\pi} \cdot \sqrt{8}} \cdot \exp\left(-\tfrac{1}{2}\left(\frac{z_2}{\sqrt{8}}\right)^2\right)$$

$$Q_{yy} = A \cdot Q_{xx} \cdot A^T$$

$$f_\omega(\omega) = \frac{1}{b} \cdot \exp\left(-\left(\frac{\omega-\omega_0}{b}\right)\right)$$

$$\bar{\omega} = \int_{\omega_0}^{\infty} \frac{\omega}{b} \cdot \exp\left(-\left(\frac{\omega-\omega_0}{b}\right)\right) d\omega$$

$$\hookrightarrow u = \frac{\omega}{b} \qquad v = \frac{\exp\left(\frac{\omega_0-\omega}{b}\right)}{-1/b} = -b \cdot \exp\left(\frac{\omega_0-\omega}{b}\right)$$

$$du = \frac{1}{b} \qquad dv = \exp\left(\frac{\omega_0-\omega}{b}\right)$$

$$\bar{\omega} = \left[-\frac{\omega}{\cancel{b}} \cdot \cancel{b} \exp\left(\frac{\omega_0-\omega}{b}\right)\right]_{\omega_0}^{\infty} + \int_{\omega_0}^{\infty} \frac{1}{\cancel{b}} \cdot \cancel{b} \cdot \exp\left(\frac{\omega_0-\omega}{b}\right) d\omega$$

$$= (0 - -\omega_0) + \left[-b \cdot \exp\left(\frac{\omega_0-\omega}{b}\right)\right]_{\omega_0}^{\infty}$$

$$= \omega_0 + (0 - -b) = \omega_0 + b$$

mean of Z : $n \cdot \bar{x} = 10 \cdot 10 = 100$

st dev. : $\sigma\sqrt{n} = \sqrt{\frac{0{,}5^2}{12}} \cdot \sqrt{10} \rightarrow 0{,}456$

$$P(Z < 99) = \frac{99-100}{0{,}456} = -2{,}19$$

$$f_x(x) = 0{,}2 \cdot \exp(-0{,}2 \cdot x)$$

$$\int_{2}^{\infty} x \cdot 0{,}2 \cdot \exp(-0{,}2 \cdot x)\, dx$$

$$\hookrightarrow u = x \qquad v = -\exp(-0{,}2 \cdot x)$$

$$du = 1 \qquad dv = 0{,}2 \cdot \exp(-0{,}2 \cdot x)$$

$$\bar{x}_{(x>2)} = \left[-x \cdot \exp(-0{,}2 \cdot x)\right]_2^{\infty} + \int_2^{\infty} \exp(-0{,}2\, x)\, dx$$

$$= (0 - - 2 \cdot \exp(-0{,}4)) + \left[\frac{\exp(0{,}2\, x)}{-0{,}2}\right]_2^{\infty}$$

$$= 2 \cdot \exp(-0{,}4) + \left(0 + 5\, \exp(-0{,}4)\right)$$

$$= 7 \cdot \exp(-0{,}4) = 4{,}69 \text{ months}$$

$$P(5 < \underline{x} < 6) = \int_5^6 0{,}2 \cdot e^{-0{,}2 \cdot x}\, dx = \left[-e^{-0{,}2x}\right]_5^6$$

$$= (-0{,}3012 - -0{,}3679)$$

$$= \cancel{0{,}0669} \; 0{,}0669$$

$$P(3 < \underline{x} < 6) = \left[-e^{-0{,}2x}\right]_3^6 = -0{,}3012 - -0{,}548812$$

$$= \cancel{0{,}185} \; 0{,}2476$$

$$\rightarrow \cancel{\frac{0{,}0669}{0{,}35} = 0{,}1871} = \frac{0{,}0669}{0{,}2476} = 0{,}27 \rightarrow A$$

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10) $\exp : \lambda \cdot e^{-\lambda x} = 0{,}5 \cdot e^{-0{,}5 \cdot x}$

$$P(x > 15) = \int_{15}^{\infty} 0{,}5 \cdot e^{-0{,}5x}\, dx = \left[-e^{-0{,}5}\right]_{15}^{\infty}$$

$$= 0 + 5{,}53 \cdot 10^{-4}$$

$$P(x < 15) = 1 - P(x > 15) = 1 - 5{,}53 \cdot 10^{-4}$$

$$(P(x < 15))^{365} = 0{,}817 = P(\text{geen overstroming})$$

$$P(\text{overstoming}) = 1 - 0{,}817$$

$$= 0{,}183$$

$$\rightarrow A$$

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12)

$$Q_{yy}^{-1} = \begin{bmatrix} 1/\sigma^2 & 0 & 0 & 0 \\ 0 & 1/\sigma^2 & 0 & 0 \\ 0 & 0 & 1/\sigma^2 & 0 \\ 0 & 0 & 0 & 1/\sigma^2 \end{bmatrix} \quad A^T \begin{bmatrix} 1 & 2 & 3 & 4 \\ -3 & 1 & -1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1/\sigma^2 & 2/\sigma^2 & 3/\sigma^2 & 4/\sigma^2 \\ -3/\sigma^2 & 1/\sigma^2 & -1/\sigma^2 & 1/\sigma^2 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ 2 & 1 \\ 3 & -1 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 30/\sigma^2 & 0 \\ 0 & 12/\sigma^2 \end{bmatrix}$$

$$\begin{bmatrix} 30/\sigma^2 & 0 \\ 0 & 12/\sigma^2 \end{bmatrix}^{-1} = \frac{1}{\frac{360}{\sigma^4}} \begin{bmatrix} 12/\sigma^2 & 0 \\ 0 & 30/\sigma^2 \end{bmatrix}$$

$$\rightarrow = \frac{30\sigma^2}{360/\cancel{\sigma^4}} = \left(\frac{30}{360}\right)\sigma^2 = \sigma_{\hat{x}\hat{x}} = 1$$

$$\sigma^2 = \frac{36}{3} = 12$$

$$\rightarrow \sigma = \sqrt{12} \leq 3{,}46 \rightarrow B$$

$$\begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \end{bmatrix}$$

$$\sigma_{44} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}$$

$(\& \sigma_1$

$$\sqrt{\sigma_1^2 \cdot \bar{x}_2^2 + \sigma_2^2 \cdot \bar{x}_1^2 + \sigma_1^2 \cdot \sigma_2^2}$$

$\hookrightarrow 1 \quad \hookrightarrow 500 \quad \hookrightarrow 1 \quad 1000 \quad 1 \cdot 1$

---

i)

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$\hookrightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\hookrightarrow 5/6 + 3/36 - 2/36$

$= 30/36 + 3/36 - 2/36 = 31/36$

A : light on

H : pressure low

$P(H) = 0{,}1$

$P(A|H) = 0{,}99$

$P(A|H^c) = 0{,}02$

$$P(H|A) = \frac{P(A|H) \cdot P(H)}{P(A)} = \frac{0{,}99 \cdot 0{,}1}{0{,}1 \cdot 0{,}99 + 0{,}9 \cdot 0{,}02} = 0{,}846$$

$\downarrow$

C

10 $p(x) = 0{,}5 \exp(-0{,}5\, x)$

$p(x < 15) = \int_0^{15} 0{,}5 e(-0{,}5 x)\, dx$

$= \left[-\exp(-0{,}5 x) \right]_0^{15} = \left(-\exp(-7{,}5) + \cancel{(-\exp(0))}\right)$ 1

$= 0{,}999447$

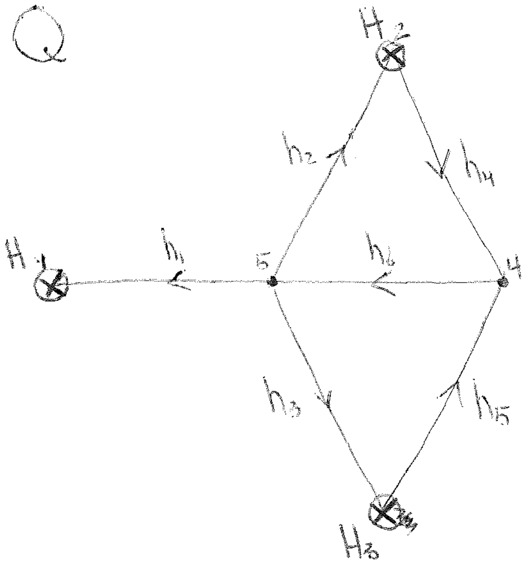
$\Rightarrow (0{,}999)^{365} = 0{,}0172 \quad \Rightarrow \quad p = 0{,}1828$

11 $E(y) = Ax$

$D(y) = \sigma^2 I_4$

Q

14



$H_1\ H_2\ H_3$ are known

$H_4 =$

$H_5 =$

$H_1 = H_5 + h_1$

$H_2 = H_5 + h_2$

$H_3 = H_5 + h_3$

$H_4 = H_3 + h_5$

$H_5 = H_4 + h_6$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$W = y_1 + y_2 \longrightarrow f = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

Q.

$E(y) = Ax$

$$\begin{vmatrix} h_{12} \\ h_{23} \\ h_{34} \\ h_{41} \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \end{vmatrix} \begin{vmatrix} h_2 \\ h_3 \\ h_4 \end{vmatrix}$$

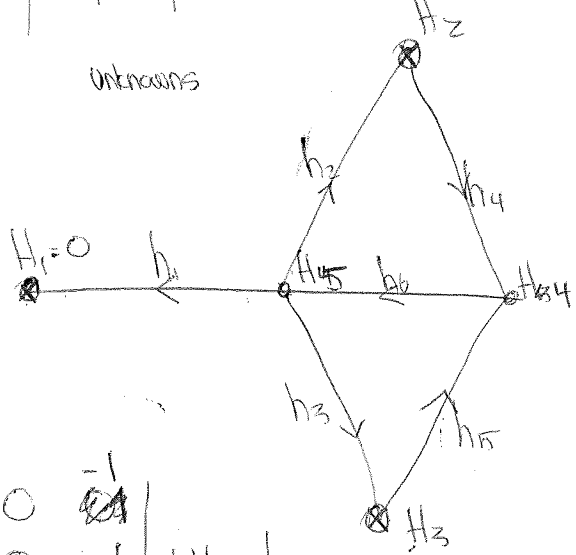
measurements unknowns

$h_{12} = h_2$

$h_{23} = h_3 - h_2$

$h_{34} = h_4 - h_3$

$h_{41} = -h_4$



$$\begin{vmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \end{vmatrix} = \begin{vmatrix} 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{vmatrix} \begin{vmatrix} H_2 \\ H_3 \\ H_4 \\ H_5 \end{vmatrix}$$

$y \qquad A \qquad x$

$$\begin{vmatrix} 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{vmatrix}$$