CHAPTER 37: Early Quantum Theory and Models of the Atom

Responses to Questions

1. A reddish star is the coolest, followed by a whitish-yellow star. Bluish stars have the highest temperatures. The temperature of the star is related to the frequency of the emitted light. Since red light has a lower frequency than blue light, red stars have a lower temperature than blue stars.

2. The energy radiated by an object may not be in the visible part of the electromagnetic spectrum. The spectrum of a blackbody with a temperature of 1000 K peaks in the IR and the object appears red, since it includes some radiation at the red end of the visible spectrum. Cooler objects will radiate less overall energy and peak at even longer wavelengths. Objects that are cool enough will not radiate any energy at visible wavelengths.

3. The lightbulb will not produce light as white as the Sun, since the peak of its emitted light is in the infrared. The lightbulb will appear more yellowish than the Sun, which has a spectrum that peaks in the visible range.

4. A bulb which appears red would emit very little radiant energy at higher visible frequencies and therefore would not expose black and white photographic paper. This strategy would not work in a darkroom for developing color photographs since the photographic paper would be sensitive to light at all visible frequencies, including red.

5. If the threshold wavelength increases for the second metal, then it has a smaller work function than the first metal. Longer wavelength corresponds to lower energy. It will take less energy for the electron to escape the surface of the second metal.

6. According to the wave theory, light of any frequency can cause electrons to be ejected as long as the light is intense enough. A higher intensity corresponds to a greater electric field magnitude and more energy. Therefore, there should be no frequency below which the photoelectric effect does not occur. According to the particle theory, however, each photon carries an amount of energy which depends upon its frequency. Increasing the intensity of the light increases the number of photons but does not increase the energy of the individual photons. The cutoff frequency is that frequency at which the energy of the photon equals the work function. If the frequency of the incoming light is below the cutoff, the electrons will not be ejected because no individual photon has enough energy to impart to an electron.

7. Individual photons of ultraviolet light are more energetic than photons of visible light and will deliver more energy to the skin, causing burns. UV photons also can penetrate farther into the skin, and, once at the deeper level, can deposit a large amount of energy that can cause damage to cells.

8. Cesium will give a higher maximum kinetic energy for the electrons. Cesium has a lower work function, so more energy is available for the kinetic energy of the electrons.

9. (*a*) No. The energy of a beam of photons depends not only on the energy of each individual photon but also on the total number of photons. If there are enough infrared photons, the infrared beam may have more energy than the ultraviolet beam.
   (*b*) Yes. The energy of a single photon depends on its frequency: $E = hf$. Since infrared light has a lower frequency than ultraviolet light, a single IR photon will always have less energy than a single UV photon.

10. Fewer electrons are emitted from the surface struck by the 400 nm photons. Each 400 nm photon has a higher energy than each 450 nm photon, so it will take fewer 400 nm photons to produce the same intensity (energy per unit area per unit time) as the 450 nm photon beam. The maximum kinetic energy of the electrons emitted from the surface struck by the 400 nm photons will be greater than the maximum kinetic energy of the electrons emitted from the surface struck by the 450 nm photons, again because each 400 nm photon has a higher energy.

11. (*a*) In a burglar alarm, when the light beam is interrupted (by an intruder, or a door or window opening), the current stops flowing in the circuit. An alarm could be set to go off when the current stops.
    (*b*) In a smoke detector, when the light beam is obscured by smoke, the current in the circuit would decrease or stop. An alarm could be set to go off when the current decreased below a certain level.
    (*c*) The amount of current in the circuit depends on the intensity of the light, as long as the frequency of the light is above the threshold frequency. The ammeter in the circuit could be calibrated to reflect the light intensity.

12. Yes, the wavelength increases. In the scattering process, some of the energy of the incident photon is transferred to the electron, so the scattered photon has less energy, and therefore a lower frequency and longer wavelength, than the incident photon. ($E = hf = hc/\lambda$.)

13. In the photoelectric effect the photon energy is completely absorbed by the electron. In the Compton effect, the photon is scattered from the electron and travels off at a lower energy.

14. According to both the wave theory and the particle theory the intensity of a point source of light decreases as the inverse square of the distance from the source. In the wave theory, the intensity of the waves obeys the inverse square law. In the particle theory, the surface area of a sphere increases with the square of the radius, and therefore the density of particles decreases with distance, obeying the inverse square law. The variation of intensity with distance cannot be used to help distinguish between the two theories.

15. The proton will have the shorter wavelength, since it has a larger mass than the electron and therefore a larger momentum $\left(\lambda = h/p\right)$.

16. Light demonstrates characteristics of both waves and particles. Diffraction and interference are wave characteristics, and are demonstrated, for example, in Young's double-slit experiment. The photoelectric effect and Compton scattering are examples of experiments in which light demonstrates particle characteristics. We can't say that light IS a wave or a particle, but it has properties of each.

17. Electrons demonstrate characteristics of both waves and particles. Electrons act like waves in electron diffraction and like particles in the Compton effect and other collisions.

18. Both a photon and an electron have properties of waves and properties of particles. They can both be associated with a wavelength and they can both undergo scattering. An electron has a negative charge and a rest mass, obeys the Pauli exclusion principle, and travels at less than the speed of light. A photon is not charged, has no rest mass, does not obey the Pauli exclusion principle, and travels at the speed of light.

19. Opposite charges attract, so the attractive Coulomb force between the positive nucleus and the negative electrons keeps the electrons from flying off into space.

20. Look at a solar absorption spectrum, measured above the Earth's atmosphere. If there are dark (absorption) lines at the wavelengths corresponding to oxygen transitions, then there is oxygen near the surface of the Sun.

21. At room temperature, nearly all the atoms in hydrogen gas will be in the ground state. When light passes through the gas, photons are absorbed, causing electrons to make transitions to higher states and creating absorption lines. These lines correspond to the Lyman series since that is the series of transitions involving the ground state or n = 1 level. Since there are virtually no atoms in higher energy states, photons corresponding to transitions from n $\geq$ 2 to higher states will not be absorbed.

22. The closeness of the spacing between energy levels near the top of Figure 37-26 indicates that the energy differences between these levels are small. Small energy differences correspond to small wavelength differences, leading to the closely spaced spectral lines in Figure 37-21.

23. There is no direct connection between the size of a particle and its de Broglie wavelength. It is possible for the wavelength to be smaller or larger than the particle.

24. On average the electrons of helium are closer to the nucleus than the electrons of hydrogen. The nucleus of helium contains two protons (positive charges), and so attracts each electron more strongly than the single proton in the nucleus of hydrogen. (There is some shielding of the nuclear charge by the second electron, but each electron still feels the attractive force of more than one proton's worth of charge.)

25. The lines in the spectrum of hydrogen correspond to all the possible transitions that the electron can make. The Balmer lines, for example, correspond to an electron moving from all higher energy levels to the n = 2 level. Although an individual hydrogen atom only contains one electron, a sample of hydrogen gas contains many atoms and all the different atoms will be undergoing different transitions.

26. The Balmer series spectral lines are in the visible light range and could be seen by early experimenters without special detection equipment.

27. The photon carries momentum, so according to conservation of momentum, the hydrogen atom will recoil as the photon is ejected. Some of the energy emitted in the transition of the atom to a lower energy state will be the kinetic energy of the recoiling atom, so the photon will have slightly less energy than predicted by the simple difference in energy levels.

28. No. At room temperature, virtually all the atoms in a sample of hydrogen gas will be in the ground state. Thus, the absorption spectrum will contain primarily just the Lyman lines, as photons corresponding to transitions from the n = 1 level to higher levels are absorbed. Hydrogen at very high temperatures will have atoms in excited states. The electrons in the higher energy levels will fall to all lower energy levels, not just the n = 1 level. Therefore, emission lines corresponding to transitions to levels higher than n = 1 will be present as well as the Lyman lines. In general, you would expect to see only Lyman lines in the absorption spectrum of room temperature hydrogen, but you would find Lyman, Balmer, Paschen, and other lines in the emission spectrum of high-temperature hydrogen.

Solutions to Problems

In several problems, the value of *hc* is needed. We often use the result of Problem 96, $hc = 1240\,\text{eV}\bullet\text{nm}$.

1. We use Wien's law, Eq. 37-1.

(*a*) $\lambda_{\text{P}} = \dfrac{(2.90\times10^{-3}\,\text{m}\bullet\text{K})}{T} = \dfrac{(2.90\times10^{-3}\,\text{m}\bullet\text{K})}{(273\,\text{K})} = 1.06\times10^{-5}\,\text{m} = \boxed{10.6\,\mu\text{m}}$

This wavelength is in the $\boxed{\text{far infrared.}}$

(*b*) $\lambda_{\text{P}} = \dfrac{(2.90\times10^{-3}\,\text{m}\bullet\text{K})}{T} = \dfrac{(2.90\times10^{-3}\,\text{m}\bullet\text{K})}{(3500\,\text{K})} = 8.29\times10^{-7}\,\text{m} = \boxed{829\,\text{nm}}$

This wavelength is in the $\boxed{\text{infrared.}}$

(*c*) $\lambda_{\text{P}} = \dfrac{(2.90\times10^{-3}\,\text{m}\bullet\text{K})}{T} = \dfrac{(2.90\times10^{-3}\,\text{m}\bullet\text{K})}{(4.2\,\text{K})} = 6.90\times10^{-4}\,\text{m} = \boxed{0.69\,\text{mm}}$

This wavelength is in the $\boxed{\text{microwave}}$ region.

(*d*) $\lambda_{\text{P}} = \dfrac{(2.90\times10^{-3}\,\text{m}\bullet\text{K})}{T} = \dfrac{(2.90\times10^{-3}\,\text{m}\bullet\text{K})}{(2.725\,\text{K})} = 1.06\times10^{-3}\,\text{m} = \boxed{1.06\,\text{mm}}$

This wavelength is in the $\boxed{\text{microwave}}$ region.

2. We use Wien's law to find the temperature for a peak wavelength of 460 nm.

$$T = \frac{(2.90\times10^{-3}\,\text{m}\bullet\text{K})}{\lambda_{\text{P}}} = \frac{(2.90\times10^{-3}\,\text{m}\bullet\text{K})}{(460\times10^{-9}\,\text{m})} = \boxed{6300\,\text{K}}$$

3. Because the energy is quantized according to Eq. 37-2, the difference in energy between adjacent levels is simply *E* = *nhf*.

$$\Delta E = hf = (6.63\times10^{-34}\,\text{J}\bullet\text{s})(8.1\times10^{13}\,\text{Hz}) = \boxed{5.4\times10^{-20}\,\text{J} = 0.34\,\text{eV}}$$

4. We use Eq. 37-1 with a temperature of $98^\circ\text{F} = 37^\circ\text{C} = 310\,\text{K}$.

$$\lambda_{\text{P}} = \frac{(2.90\times10^{-3}\,\text{m}\bullet\text{K})}{T} = \frac{(2.90\times10^{-3}\,\text{m}\bullet\text{K})}{(310\,\text{K})} = 9.4\times10^{-6}\,\text{m} = \boxed{9.4\,\mu\text{m}}$$

5. (*a*) Wien's displacement law says that $\lambda_{\text{P}}T =$ constant. We must find the wavelength at which $I(\lambda, T)$ is a maximum for a given temperature. This can be found by setting $\partial I/\partial\lambda = 0$.

$$\frac{\partial I}{\partial\lambda} = \frac{\partial}{\partial\lambda}\left(\frac{2\pi hc^2\lambda^{-5}}{e^{hc/\lambda kT}-1}\right) = 2\pi hc^2\frac{\partial}{\partial\lambda}\left(\frac{\lambda^{-5}}{e^{hc/\lambda kT}-1}\right)$$

$$= 2\pi hc^2\left[\frac{(e^{hc/\lambda kT}-1)(-5\lambda^{-6}) - \lambda^{-5}e^{hc/\lambda kT}\left(-\dfrac{hc}{kT\lambda^2}\right)}{(e^{hc/\lambda kT}-1)^2}\right]$$

$$= \frac{2\pi hc^2}{\lambda^6(e^{hc/\lambda kT}-1)^2}\left[5 + e^{hc/\lambda kT}\left(\frac{hc}{kT\lambda} - 5\right)\right] = 0 \;\rightarrow\; 5 = e^{hc/\lambda kT}\left(5 - \frac{hc}{kT\lambda}\right) \;\rightarrow$$

$$e^x(5-x)=5\ ;\ x=\frac{hc}{\lambda_{\text{P}}kT}$$

This transcendental equation will have some solution x = constant, and so $\frac{hc}{\lambda_{\text{P}}kT}$ = constant, and so $\boxed{\lambda_{\text{P}}T=\text{constant}}$. The constant could be evaluated from solving the transcendental equation,

(*b*) To find the value of the constant, we solve $e^x(5-x)=5$, or $5-x=5e^{-x}$. This can be done graphically, by graphing both $y=5-x$ and $y=5e^{-x}$ on the same set of axes and finding the intersection point. Or, the quantity $5-x-5e^{-x}$ could be calculated, and find for what value of x that expression is 0. The answer is x = 4.966. We use this value to solve for h. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH37.XLS," on tab "Problem 37.5."

$$\frac{hc}{\lambda_{\text{P}}kT}=4.966\ \rightarrow$$

$$h=4.966\frac{\lambda_{\text{P}}Tk}{c}=4.966\frac{(2.90\times10^{-3}\,\text{m}\bullet\text{K})(1.38\times10^{-23}\,\text{J/K})}{3.00\times10^{8}\,\text{m/s}}=\boxed{6.62\times10^{-34}\,\text{J}\bullet\text{s}}$$

(*c*) We integrate Planck's radiation formula over all wavelengths.

$$\int_0^\infty I(\lambda,T)\,d\lambda=\int_0^\infty\left(\frac{2\pi hc^2\lambda^{-5}}{e^{hc/\lambda kT}-1}\right)d\lambda\ ;\ \text{let}\ \frac{hc}{\lambda kT}=x\ ;\ \lambda=\frac{hc}{xkT}\ ;\ d\lambda=-\frac{hc}{x^2kT}dx$$

$$\int_0^\infty I(\lambda,T)\,d\lambda=\int_0^\infty\left(\frac{2\pi hc^2\lambda^{-5}}{e^{hc/\lambda kT}-1}\right)d\lambda=\int_\infty^0\left(\frac{2\pi hc^2\left(\frac{hc}{xkT}\right)^{-5}}{e^x-1}\right)\left(-\frac{hc}{x^2kT}dx\right)=\frac{2\pi k^4T^4}{h^3c^2}\int_0^\infty\left(\frac{x^3}{e^x-1}\right)dx$$

$$=\frac{2\pi k^4}{h^3c^2}\left[\int_0^\infty\left(\frac{x^3}{e^x-1}\right)dx\right]T^4\propto T^4$$

Thus the total radiated power per unit area is proportional to T^4. Everything else in the expression is constant with respect to temperature.

6. We use Eq. 37-3.

$$E=hf=(6.626\times10^{-34}\,\text{J}\bullet\text{s})(104.1\times10^{6}\,\text{Hz})=\boxed{6.898\times10^{-26}\,\text{J}}$$

$\boxed{7.}$ We use Eq. 37-3 along with the fact that $f=c/\lambda$ for light. The longest wavelength will have the lowest energy.

$$E_1=hf_1=\frac{hc}{\lambda_1}=\frac{(6.63\times10^{-34}\,\text{J}\bullet\text{s})(3.00\times10^{8}\,\text{m/s})}{(410\times10^{-9}\,\text{m})}=4.85\times10^{-19}\,\text{J}\left(\frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}}\right)=3.03\,\text{eV}$$

$$E_2=hf_2=\frac{hc}{\lambda_2}=\frac{(6.63\times10^{-34}\,\text{J}\bullet\text{s})(3.00\times10^{8}\,\text{m/s})}{(750\times10^{-9}\,\text{m})}=2.65\times10^{-19}\,\text{J}\left(\frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}}\right)=1.66\,\text{eV}$$

Thus the range of energies is $\boxed{2.7\times10^{-19}\,\text{J}<E<4.9\times10^{-19}\,\text{J}}$ or $\boxed{1.7\,\text{eV}<E<3.0\,\text{eV}}$.

8. We use Eq. 37-3 with the fact that $f = c/\lambda$ for light.

$$\lambda = \frac{c}{f} = \frac{hc}{E} = \frac{(6.63\times10^{-34}\,\text{J•s})(3.00\times10^{8}\,\text{m/s})}{(1.60\times10^{-19}\,\text{J/eV})(380\times10^{3}\,\text{eV})} = 3.27\times10^{-12}\,\text{m} \approx \boxed{3.3\times10^{-3}\,\text{nm}}$$

Significant diffraction occurs when the opening is on the order of the wavelength. Thus there would be $\boxed{\text{insignificant diffraction}}$ through the doorway.

9. We use Eq. 37-3 with the fact that $f = c/\lambda$ for light.

$$E_{\text{min}} = hf_{\text{min}} \;\rightarrow\; f_{\text{min}} = \frac{E_{\text{min}}}{h} = \frac{(0.1\,\text{eV})(1.60\times10^{-19}\,\text{J/eV})}{(6.63\times10^{-34}\,\text{J•s})} = 2.41\times10^{13}\,\text{Hz} \approx \boxed{2\times10^{13}\,\text{Hz}}$$

$$\lambda_{\text{max}} = \frac{c}{f_{\text{min}}} = \frac{(3.00\times10^{8}\,\text{m/s})}{(2.41\times10^{13}\,\text{Hz})} = 1.24\times10^{-5}\,\text{m} \approx \boxed{1\times10^{-5}\,\text{m}}$$

10. We use Eq. 37-5.

$$p = \frac{h}{\lambda} = \frac{(6.63\times10^{-34}\,\text{J•s})}{(6.20\times10^{-7}\,\text{m})} = \boxed{1.07\times10^{-27}\,\text{kg•m/s}}$$

11. At the minimum frequency, the kinetic energy of the ejected electrons is 0. Use Eq. 37-4a.

$$K = hf_{\text{min}} - W_0 = 0 \;\rightarrow\; f_{\text{min}} = \frac{W_0}{h} = \frac{4.8\times10^{-19}\,\text{J}}{6.63\times10^{-34}\,\text{J•s}} = \boxed{7.2\times10^{14}\,\text{Hz}}$$

12. The longest wavelength corresponds to the minimum frequency. That occurs when the kinetic energy of the ejected electrons is 0. Use Eq. 37-4a.

$$K = hf_{\text{min}} - W_0 = 0 \;\rightarrow\; f_{\text{min}} = \frac{c}{\lambda_{\text{max}}} = \frac{W_0}{h} \;\rightarrow$$

$$\lambda_{\text{max}} = \frac{ch}{W_0} = \frac{(3.00\times10^{8}\,\text{m/s})(6.63\times10^{-34}\,\text{J•s})}{(3.70\,\text{eV})(1.60\times10^{-19}\,\text{J/eV})} = \boxed{3.36\times10^{-7}\,\text{m}} = 336\,\text{nm}$$

$\boxed{13.}$ The energy of the photon will equal the kinetic energy of the baseball. We use Eq. 37-3.

$$K = hf \;\rightarrow\; \tfrac{1}{2}mv^2 = h\frac{c}{\lambda} \;\rightarrow\; \lambda = \frac{2hc}{mv^2} = \frac{2(6.63\times10^{-34}\,\text{J•s})(3.00\times10^{8}\,\text{m/s})}{(0.145\,\text{kg})(30.0\,\text{m/s})^2} = \boxed{3.05\times10^{-27}\,\text{m}}$$

14. We divide the minimum energy by the photon energy at 550 nm to find the number of photons.

$$E = nhf = E_{\text{min}} \;\rightarrow\; n = \frac{E_{\text{min}}}{hf} = \frac{E_{\text{min}}\lambda}{hc} = \frac{(10^{-18}\,\text{J})(550\times10^{-9}\,\text{m})}{(6.63\times10^{-34}\,\text{J•s})(3.00\times10^{8}\,\text{m/s})} = 2.77 \approx \boxed{3\text{ photons}}$$

15. The photon of visible light with the maximum energy has the least wavelength. We use 410 nm as the lowest wavelength of visible light.

$$hf_{\text{max}} = \frac{hc}{\lambda_{\text{min}}} = \frac{(6.63\times10^{-34}\,\text{J•s})(3.00\times10^{8}\,\text{m/s})}{(1.60\times10^{-19}\,\text{J/eV})(410\times10^{-9}\,\text{m})} = 3.03\,\text{eV}$$

Electrons will not be emitted if this energy is less than the work function.
The metals with work functions greater than 3.03 eV are $\boxed{\text{copper and iron.}}$

16. (*a*) At the threshold wavelength, the kinetic energy of the photoelectrons is zero, so the work function is equal to the energy of the photon.

$$W_0 = hf - K_{max} = hf = \frac{hc}{\lambda} = \frac{1240\,\text{eV}\bullet\text{nm}}{520\,\text{nm}} = \boxed{2.4\,\text{eV}}$$

(*b*) The stopping voltage is the voltage that gives a potential energy change equal to the maximum kinetic energy. We use Eq. 37-4b to calculate the maximum kinetic energy.

$$K_{max} = hf - W_0 = \frac{hc}{\lambda} - W_0 = \frac{1240\,\text{eV}\bullet\text{nm}}{470\,\text{nm}} - 2.38\,\text{eV} = 0.25\,\text{eV}$$

$$V_0 = \frac{K_{max}}{e} = \frac{0.25\,\text{eV}}{e} = \boxed{0.25\,\text{V}}$$

17. The photon of visible light with the maximum energy has the minimum wavelength. We use Eq. 37-4b to calculate the maximum kinetic energy.

$$K_{max} = hf - W_0 = \frac{hc}{\lambda} - W_0 = \frac{1240\text{ eV}\bullet\text{nm}}{410\text{ nm}} - 2.48\text{ eV} = \boxed{0.54\text{ eV}}$$

18. We use Eq. 37-4b to calculate the maximum kinetic energy. Since the kinetic energy is much less than the rest energy, we use the classical definition of kinetic energy to calculate the speed.

$$K_{max} = hf - W_0 = \frac{hc}{\lambda} - W_0 = \frac{1240\,\text{eV}\bullet\text{nm}}{365\,\text{nm}} - 2.48\,\text{eV} = \boxed{0.92\,\text{eV}}$$

$$K_{max} = \tfrac{1}{2}mv^2 \quad \rightarrow \quad v = \sqrt{\frac{2K_{max}}{m}} = \sqrt{\frac{2(0.92\,\text{eV})(1.60\times10^{-19}\text{ J/eV})}{9.11\times10^{-31}\text{kg}}} = \boxed{5.7\times10^5\,\text{m/s}}$$

19. We use Eq. 37-4b to calculate the work function.

$$W_0 = hf - K_{max} = \frac{hc}{\lambda} - K_{max} = \frac{1240\text{ eV}\bullet\text{nm}}{285\text{ nm}} - 1.70\text{ eV} = \boxed{2.65\text{ eV}}$$

20. Electrons emitted from photons at the threshold wavelength have no kinetic energy. We use Eq. 37-4b with the threshold wavelength to determine the work function.

$$W_0 = \frac{hc}{\lambda} - K_{max} = \frac{hc}{\lambda_{max}} = \frac{1240\text{ eV}\bullet\text{nm}}{320\text{ nm}} = 3.88\text{ eV}.$$

(*a*) We now use Eq. 36-4b with the work function determined above to calculate the kinetic energy of the photoelectrons emitted by 280 nm light.

$$K_{max} = \frac{hc}{\lambda} - W_0 = \frac{1240\text{ eV}\bullet\text{nm}}{280\text{ nm}} - 3.88\text{ eV} = \boxed{0.55\text{ eV}}$$

(*b*) Because the wavelength is greater than the threshold wavelength, the photon energy is less than the work function, so there will be $\boxed{\text{no ejected electrons.}}$

21. The stopping voltage is the voltage that gives a potential energy change equal to the maximum kinetic energy of the photoelectrons. We use Eq. 37-4b to calculate the work function where the maximum kinetic energy is the product of the stopping voltage and electron charge.

$$W_0 = \frac{hc}{\lambda} - K_{max} = \frac{hc}{\lambda} - eV_0 = \frac{1240\text{ eV}\bullet\text{nm}}{230\text{ nm}} - (1.84\text{ V})e = \boxed{3.55\text{ eV}}$$

22. The energy required for the chemical reaction is provided by the photon. We use Eq. 37-3 for the energy of the photon, where $f = c/\lambda$.

$$E = hf = \frac{hc}{\lambda} = \frac{1240\ \text{eV}\bullet\text{nm}}{630\ \text{nm}} = \boxed{2.0\ \text{eV}}$$

Each reaction takes place in a molecule, so we use the appropriate conversions to convert eV/molecule to kcal/mol.

$$E = \left(\frac{2.0\ \text{eV}}{\text{molecule}}\right)\left(\frac{1.60\times10^{-19}\ \text{J}}{\text{eV}}\right)\left(\frac{6.02\times10^{23}\ \text{molecules}}{\text{mol}}\right)\left(\frac{\text{kcal}}{4186\ \text{J}}\right) = \boxed{45\ \text{kcal/mole}}$$

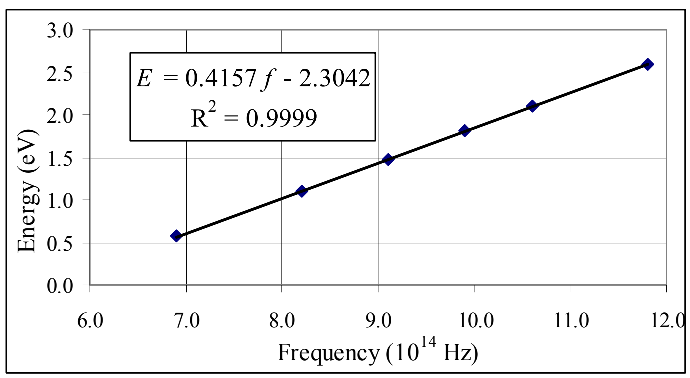
23. (*a*) Since $f = c/\lambda$, the photon energy given by Eq. 37-3 can be written in terms of the wavelength as $E = hc/\lambda$. This shows that the photon with the largest wavelength has the smallest energy. The 750-nm photon then delivers the minimum energy that will excite the retina.

$$E = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34}\ \text{J}\bullet\text{s})(3.00\times10^{8}\ \text{m/s})}{(750\times10^{-9}\ \text{m})}\left(\frac{1\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = \boxed{1.66\ \text{eV}}$$

(*b*) The eye cannot see light with wavelengths less than 410 nm. Obviously, these wavelength photons have more energy than the minimum required to initiate vision, so they must not arrive at the retina. That is, wavelength less than 410 nm are absorbed near the front portion of the eye. The threshold photon energy is that of a 410-nm photon.

$$E = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34}\ \text{J}\bullet\text{s})(3.00\times10^{8}\ \text{m/s})}{(410\times10^{-9}\ \text{m})}\left(\frac{1\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = \boxed{3.03\ \text{eV}}$$

24. We plot the maximum (kinetic) energy of the emitted electrons vs. the frequency of the incident radiation. Eq. 37-4b says $K_{\text{max}} = hf - W_0$. The best-fit straight line is determined by linear regression in Excel. The slope of the best-fit straight line to the data should give Planck's constant, the *x*-intercept is the cutoff frequency, and the *y*-intercept is the opposite of the work function. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH37.XLS," on tab "Problem 37.24."



(*a*) $h = (0.4157\,\text{eV}/10^{14}\text{Hz})(1.60\times10^{-19}\ \text{J/eV}) = \boxed{6.7\times10^{-34}\ \text{J}\bullet\text{s}}$

(*b*) $hf_{\text{cutoff}} = W_0 \;\rightarrow\; f_{\text{cutoff}} = \dfrac{W_0}{h} = \dfrac{2.3042\,\text{eV}}{(0.4157\,\text{eV}/10^{14}\text{Hz})} = \boxed{5.5\times10^{14}\ \text{Hz}}$

(*c*) $W_0 = \boxed{2.3\,\text{eV}}$

$\boxed{25.}$ (*a*) Since $f = c/\lambda$, the photon energy is $E = hc/\lambda$ and the largest wavelength has the smallest energy. In order to eject electrons for all possible incident visible light, the metal's work function must be less than or equal to the energy of a 750-nm photon. Thus the maximum value for the metal's work function W_o is found by setting the work function equal to the energy of the 750-nm photon.

$$W_o = \frac{hc}{\lambda} = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}{\left(750\times10^{-9}\,\text{m}\right)}\left(\frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}}\right) = \boxed{1.66\,\text{eV}}$$

(*b*) If the photomultiplier is to function only for incident wavelengths less than 410-nm, then we set the work function equal to the energy of the 410-nm photon.

$$W_o = \frac{hc}{\lambda} = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}{\left(410\times10^{-9}\,\text{m}\right)}\left(\frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}}\right) = \boxed{3.03\,\text{eV}}$$

26. Since $f = c/\lambda$, the energy of each emitted photon is $E = hc/\lambda$. We multiply the energy of each photon by $1.0\times10^{6}/\text{s}$ to determine the average power output of each atom. At distance of $r = 25\,\text{cm}$, the light sensor measures an intensity of $I = 1.6\,\text{nW}/1.0\,\text{cm}^2$. Since light energy emitted from atoms radiates equally in all directions, the intensity varies with distance as a spherical wave. Thus, from Section 15–3 in the text, the average power emitted is $\bar{P} = 4\pi r^2 I$. Dividing the total average power by the power from each atom gives the number of trapped atoms.

$$N = \frac{\bar{P}}{\bar{P}_{\text{atom}}} = \frac{4\pi r^2 I}{nhc/\lambda} = \frac{4\pi\left(25\text{cm}\right)^2\left(1.6\times10^{-9}\,\text{W/cm}^2\right)}{\left(1.0\times10^{6}/\text{s}\right)\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(3.00\times10^{8}\text{m/s}\right)/\left(780\times10^{-9}\,\text{m}\right)}$$

$$= \boxed{4.9\times10^{7}\,\text{atoms}}$$

27. We set the kinetic energy in Eq. 37-4b equal to the stopping voltage, eV_0, and write the frequency of the incident light in terms of the wavelength, $f = c/\lambda$. We differentiate the resulting equation and solve for the fractional change in wavelength, and we take the absolute value of the final expression.

$$eV_0 = \frac{hc}{\lambda} - W_0 \;\rightarrow\; e\,dV_0 = -\frac{hc}{\lambda^2}d\lambda \;\rightarrow\; \frac{d\lambda}{\lambda} = -\frac{e\,dV_0\lambda}{hc} \;\approx\; \boxed{\frac{\Delta\lambda}{\lambda} = \frac{e\lambda}{hc}\Delta V_0}$$

$$\frac{\Delta\lambda}{\lambda} = \frac{\left(1.60\times10^{-19}\,\text{C}\right)\left(550\times10^{-9}\,\text{m}\right)}{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}\left(0.01\,\text{V}\right) = \boxed{0.004}$$

28. We use Eq. 37-6b. Note that the answer is correct to two significant figures.

$$\Delta\lambda = \frac{h}{m_e c}\left(1-\cos\phi\right) \;\rightarrow$$

$$\phi = \cos^{-1}\left(1-\frac{m_e c\Delta\lambda}{h}\right) = \cos^{-1}\left(1-\frac{\left(9.11\times10^{-31}\text{kg}\right)\left(3.00\times10^{8}\,\text{m/s}\right)\left(1.5\times10^{-13}\,\text{m}\right)}{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)}\right) = \boxed{20^\circ}$$

29. The Compton wavelength for a particle of mass *m* is h/mc.

(*a*) $$\frac{h}{m_e c} = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)}{\left(9.11\times10^{-31}\text{kg}\right)\left(3.00\times10^{8}\,\text{m/s}\right)} = \boxed{2.43\times10^{-12}\,\text{m}}$$

(*b*) $$\frac{h}{m_p c} = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)}{\left(1.67\times10^{-27}\text{kg}\right)\left(3.00\times10^{8}\,\text{m/s}\right)} = \boxed{1.32\times10^{-15}\,\text{m}}$$

(*c*) The energy of the photon is given by Eq. 37-3.

$$E_{\text{photon}} = hf = \frac{hc}{\lambda} = \frac{hc}{\left(h/mc\right)} = mc^2 = \text{rest energy}$$

30. We find the Compton wavelength shift for a photon scattered from an electron, using Eq. 37-6b. The Compton wavelength of a free electron is given in the text right after Eq. 37-6b.

$$\lambda' - \lambda = \left(\frac{h}{m_e c}\right)(1 - \cos\theta) = \lambda_C (1 - \cos\theta) = \left(2.43\times10^{-3}\text{nm}\right)(1 - \cos\theta)$$

(*a*) $\lambda'_a - \lambda = \left(2.43\times10^{-3}\text{nm}\right)\left(1-\cos 60°\right) = \boxed{1.22\times10^{-3}\text{nm}}$

(*b*) $\lambda'_b - \lambda = \left(2.43\times10^{-3}\text{nm}\right)\left(1-\cos 90°\right) = \boxed{2.43\times10^{-3}\text{nm}}$

(*c*) $\lambda'_c - \lambda = \left(2.43\times10^{-3}\text{nm}\right)\left(1-\cos 180°\right) = \boxed{4.86\times10^{-3}\text{nm}}$

31. (*a*) In the Compton effect, the maximum change in the photon's wavelength is when scattering angle $\phi = 180^{\circ}$. We use Eq. 37-6b to determine the maximum change in wavelength. Dividing the maximum change by the initial wavelength gives the maximum fractional change.

$$\Delta\lambda = \frac{h}{m_e c}(1-\cos\theta) \rightarrow$$

$$\frac{\Delta\lambda}{\lambda} = \frac{h}{m_e c\lambda}(1-\cos\theta) = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(1-\cos 180°\right)}{\left(9.11\times10^{-31}\text{kg}\right)\left(3.00\times10^{8}\text{ m/s}\right)\left(550\times10^{-9}\text{m}\right)} = \boxed{8.8\times10^{-6}}$$

(*b*) We replace the initial wavelength with $\lambda = 0.10$ nm.

$$\frac{\Delta\lambda}{\lambda} = \frac{h}{m_e c\lambda}(1-\cos\theta) = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(1-\cos 180°\right)}{\left(9.11\times10^{-31}\text{kg}\right)\left(3.00\times10^{8}\text{ m/s}\right)\left(0.10\times10^{-9}\text{m}\right)} = \boxed{0.049}$$

32. We find the change in wavelength for each scattering event using Eq. 37-6b, with a scattering angle of $\phi = 0.50^{\circ}$. To calculate the total change in wavelength, we subtract the initial wavelength, obtained from the initial energy, from the final wavelength. We divide the change in wavelength by the wavelength change from each event to determine the number of scattering events.

$$\Delta\lambda = \frac{h}{m_e c}\left(1-\cos 0.5^{\circ}\right) = \frac{\left(6.63\times10^{-34}\text{ J}\cdot\text{s}\right)\left(1-\cos 0.5°\right)}{\left(9.11\times10^{-31}\text{ kg}\right)\left(3.00\times10^{8}\text{ m/s}\right)} = 9.24\times10^{-17}\text{m} = 9.24\times10^{-8}\text{nm}$$

$$\lambda_0 = \frac{hc}{E_0} = \frac{\left(6.63\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m/s}\right)}{\left(1.0\times10^{6}\text{ eV}\right)\left(1.60\times10^{-19}\text{ J/eV}\right)} = 1.24\times10^{-12}\text{m} = 0.00124\,\text{nm}.$$

$$n = \frac{\lambda - \lambda_0}{\Delta\lambda} = \frac{(555\text{ nm})-(0.00124\text{ nm})}{9.24\times10^{-8}\text{ nm}} = \boxed{6\times10^{9}\text{events}}$$

33. (*a*) We use conservation of momentum to set the initial momentum of the photon equal to the sum of the final momentum of the photon and electron, where the momentum of the photon is given by Eq. 37-5 and the momentum of the electron is written in terms of the total energy (Eq. 36-13). We multiply this equation by the speed of light to simplify.

$$\frac{h}{\lambda} + 0 = -\left(\frac{h}{\lambda'}\right) + p_e \rightarrow \frac{hc}{\lambda} = -\left(\frac{hc}{\lambda'}\right) + \sqrt{E^2 - E_0^2}$$

Using conservation of energy we set the initial energy of the photon and rest energy of the electron equal to the sum of the final energy of the photon and the total energy of the electron.

$$\left(\frac{hc}{\lambda}\right) + E_0 = \left(\frac{hc}{\lambda'}\right) + E$$

By summing these two equations, we eliminate the final wavelength of the photon. We then solve the resulting equation for the kinetic energy of the electron, which is the total energy less the rest energy.

$$2\left(\frac{hc}{\lambda}\right)+E_0=\sqrt{E^2-E_0^{\,2}}+E \;\rightarrow\; \left[2\left(\frac{hc}{\lambda}\right)+E_0-E\right]^2=E^2-E_0^{\,2}$$

$$\left[2\left(\frac{hc}{\lambda}\right)+E_0\right]^2-2E\left[2\left(\frac{hc}{\lambda}\right)+E_0\right]+E^2=E^2-E_0^{\,2} \;\rightarrow\; E=\frac{\left[2\left(\frac{hc}{\lambda}\right)+E_0\right]^2+E_0^{\,2}}{2\left[2\left(\frac{hc}{\lambda}\right)+E_0\right]}$$

$$K=E-E_0=\frac{\left[2\left(\frac{hc}{\lambda}\right)+E_0\right]^2+E_0^{\,2}}{2\left[2\left(\frac{hc}{\lambda}\right)+E_0\right]}-\frac{2\left[2\left(\frac{hc}{\lambda}\right)+E_0\right]E_0}{2\left[2\left(\frac{hc}{\lambda}\right)+E_0\right]}=\frac{2\left(\frac{hc}{\lambda}\right)^2}{\left[2\left(\frac{hc}{\lambda}\right)+E_0\right]}$$

$$=\frac{2\left(\frac{1240\text{ eV}\bullet\text{nm}}{0.160\text{ nm}}\right)^2}{\left[2\left(\frac{1240\text{ eV}\bullet\text{nm}}{0.160\text{ nm}}\right)+5.11\times10^5\text{eV}\right]}=\boxed{228\,\text{eV}}$$

(*b*) We solve the energy equation for the final wavelength.

$$\left(\frac{hc}{\lambda}\right)+E_0=\left(\frac{hc}{\lambda'}\right)+E$$

$$\lambda'=\frac{hc}{\left(\frac{hc}{\lambda}\right)+E_0-E}=\left[\frac{1}{\lambda}-\frac{K}{hc}\right]^{-1}=\left[\frac{1}{0.160\,\text{nm}}-\frac{228\,\text{eV}}{1240\,\text{eV}\bullet\text{nm}}\right]^{-1}=\boxed{0.165\,\text{nm}}$$

34. First we use conservation of energy, where the energy of the photon is written in terms of the wavelength, to relate the initial and final energies. Solve this equation for the electron's final energy.

$$\left(\frac{hc}{\lambda}\right)+mc^2=\left(\frac{hc}{\lambda'}\right)+E \;\Rightarrow\; E=\left(\frac{hc}{\lambda}\right)-\left(\frac{hc}{\lambda'}\right)+mc^2$$

Next, we define the *x*-direction as the direction of the initial motion of the photon. We write equations for the conservation of momentum in the horizontal and vertical directions, where θ is the angle the photon makes with the initial direction of the photon and ϕ is the angle the electron makes.

$$p_x:\quad \frac{h}{\lambda}=p_e\cos\phi+\frac{h}{\lambda'}\cos\theta \qquad\qquad p_y:\quad 0=p_e\sin\phi+\frac{h}{\lambda'}\sin\theta$$

To eliminate the variable ϕ we solve the momentum equations for the electron's momentum, square the resulting equations and add the two equations together using the identity $\cos^2\theta+\sin^2\theta=1$.

$$\left(\frac{h}{\lambda}-\frac{h}{\lambda'}\cos\theta\right)^2=\left(p_e\cos\phi\right)^2 \qquad\qquad \left(\frac{h}{\lambda'}\sin\theta\right)^2=\left(p_e\sin\phi\right)^2$$

$$\left(p_e\cos\phi\right)^2+\left(p_e\sin\phi\right)^2=\left(\frac{h}{\lambda}-\frac{h}{\lambda'}\cos\theta\right)^2+\left(\frac{h}{\lambda'}\sin\theta\right)^2$$

$$p_e^2=\left(\frac{h}{\lambda}\right)^2-\frac{2h^2}{\lambda\lambda'}\cos\theta+\left(\frac{h}{\lambda'}\right)^2$$

We now apply the relativistic invariant equation, Eq. 36-13, to write the electron momentum in terms of the electron energy. Then using the electron energy obtained from the conservation of energy equation, we eliminate the electron energy and solve for the change in wavelength.

$$\left(\frac{h}{\lambda}\right)^2 - \frac{2h^2}{\lambda\lambda'}\cos\theta + \left(\frac{h}{\lambda'}\right)^2 = \frac{E^2 - m^2c^4}{c^2} = \left[\left(\frac{h}{\lambda}\right) - \left(\frac{h}{\lambda'}\right) + mc\right]^2 - m^2c^2$$

$$= \left(\frac{h}{\lambda}\right)^2 + \left(\frac{h}{\lambda'}\right)^2 + m^2c^2 + 2hmc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) - \frac{h^2}{\lambda\lambda'} - m^2c^2$$

$$-\frac{2h^2}{\lambda\lambda'}\cos\theta = 2hmc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) - \frac{h^2}{\lambda\lambda'}$$

$$-h\cos\theta = mc(\lambda' - \lambda) - h \quad \rightarrow \quad \boxed{\lambda' - \lambda = \frac{h}{mc}(1 - \cos\theta)}$$

35. The photon energy must be equal to the kinetic energy of the products plus the mass energy of the products. The mass of the positron is equal to the mass of the electron.

$$E_{\text{photon}} = K_{\text{products}} + m_{\text{products}}c^2 \quad \rightarrow$$

$$K_{\text{products}} = E_{\text{photon}} - m_{\text{products}}c^2 = E_{\text{photon}} - 2m_{\text{electron}}c^2 = 2.67\,\text{MeV} - 2(0.511\,\text{MeV}) = \boxed{1.65\,\text{MeV}}$$

36. The photon with the longest wavelength has the minimum energy in order to create the masses with no additional kinetic energy. Use Eq. 37-5.

$$\lambda_{\text{max}} = \frac{hc}{E_{\text{min}}} = \frac{hc}{2mc^2} = \frac{h}{2mc} = \frac{(6.63 \times 10^{-34}\,\text{J}\bullet\text{s})}{2(1.67 \times 10^{-27}\,\text{kg})(3.00 \times 10^8\,\text{m/s})} = \boxed{6.62 \times 10^{-16}\,\text{m}}$$

This must take place in the presence of some other object in order for momentum to be conserved.

37. The minimum energy necessary is equal to the rest energy of the two muons.

$$E_{\text{min}} = 2mc^2 = 2(207)(0.511\,\text{MeV}) = \boxed{212\,\text{MeV}}$$

The wavelength is given by Eq. 37-5.

$$\lambda = \frac{hc}{E} = \frac{(6.63\times10^{-34}\,\text{J}\bullet\text{s})(3.00\times10^8\,\text{m/s})}{(1.60\times10^{-19}\,\text{J/eV})(212\times10^6\,\text{eV})} = \boxed{5.86\times10^{-15}\,\text{m}}$$

38. Since $v < 0.001c$, the total energy of the particles is essentially equal to their rest energy. Both particles have the same rest energy of 0.511 MeV. Since the total momentum is 0, each photon must have half the available energy and equal momenta.

$$E_{\text{photon}} = m_{\text{electron}}c^2 = \boxed{0.511\,\text{MeV}} \quad ; \quad p_{\text{photon}} = \frac{E_{\text{photon}}}{c} = \boxed{0.511\,\text{MeV}/c}$$

39. The energy of the photon is equal to the total energy of the two particles produced. Both particles have the same kinetic energy and the same mass.

$$E_{\text{photon}} = 2(K + mc^2) = 2(0.375\,\text{MeV} + 0.511\,\text{MeV}) = \boxed{1.772\,\text{MeV}}$$

The wavelength is found from Eq. 37-5.

$$\lambda = \frac{hc}{E} = \frac{(6.63\times10^{-34}\,\text{J}\bullet\text{s})(3.00\times10^8\,\text{m/s})}{(1.60\times10^{-19}\,\text{J/eV})(1.772\times10^6\,\text{eV})} = \boxed{7.02\times10^{-13}\,\text{m}}$$

40. We find the wavelength from Eq. 37-7.

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)}{\left(0.23\,\text{kg}\right)\left(0.10\,\text{m/s}\right)} = \boxed{2.9\times10^{-32}\,\text{m}}$$

41. The neutron is not relativistic, so we can use $p = mv$. We also use Eq. 37-7.

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)}{\left(1.67\times10^{-27}\,\text{kg}\right)\left(8.5\times10^{4}\,\text{m/s}\right)} = \boxed{4.7\times10^{-12}\,\text{m}}$$

42. We assume the electron is non-relativistic, and check that with the final answer. We use Eq. 37-7.

$$\lambda = \frac{h}{p} = \frac{h}{mv} \quad\rightarrow\quad v = \frac{h}{m\lambda} = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)}{\left(9.11\times10^{-31}\,\text{kg}\right)\left(0.21\times10^{-9}\,\text{m}\right)} = 3.466\times10^{6}\,\text{m/s} = 0.01155c$$

Our use of classical expressions is justified. The kinetic energy is equal to the potential energy change.

$$eV = K = \tfrac{1}{2}mv^2 = \frac{\tfrac{1}{2}\left(9.11\times10^{-31}\,\text{kg}\right)\left(3.466\times10^{6}\,\text{m/s}\right)^2}{\left(1.60\times10^{-19}\,\text{J/eV}\right)} = 34.2\,\text{eV}$$

Thus the required potential difference is $\boxed{34\ \text{V}}$.

$\boxed{43.}$ The theoretical resolution limit is the wavelength of the electron. We find the wavelength from the momentum, and find the momentum from the kinetic energy and rest energy. We use the result from Problem 94. The kinetic energy of the electron is 85 keV.

$$\lambda = \frac{hc}{\sqrt{K^2 + 2mc^2K}} = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}{\left(1.60\times10^{-19}\,\text{J/eV}\right)\sqrt{\left(85\times10^{3}\text{eV}\right)^2 + 2\left(0.511\times10^{6}\text{eV}\right)\left(85\times10^{3}\text{eV}\right)}}$$

$$= \boxed{4.1\times10^{-12}\,\text{m}}$$

44. We use the relativistic expression for momentum, Eq. 36-8.

$$p = \frac{mv}{\sqrt{1 - v^2/c^2}} = \frac{mv}{\sqrt{1 - v^2/c^2}} = \frac{h}{\lambda} \quad\rightarrow$$

$$\lambda = \frac{h\sqrt{1 - v^2/c^2}}{mv} = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)\sqrt{1 - \left(0.98\right)^2}}{\left(9.11\times10^{-31}\,\text{kg}\right)\left(0.98\right)\left(3.00\times10^{8}\,\text{m/s}\right)} = \boxed{4.9\times10^{-13}\,\text{m}}$$

45. Since the particles are not relativistic, we may use $K = p^2/2m$. We then form the ratio of the kinetic energies, using Eq. 37-7.

$$K = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2} \quad ; \quad \frac{\lambda_e}{\lambda_p} = \frac{\dfrac{h^2}{2m_e\lambda^2}}{\dfrac{h^2}{2m_p\lambda^2}} = \frac{m_p}{m_e} = \frac{1.67\times10^{-27}\,\text{kg}}{9.11\times10^{-31}\,\text{kg}} = \boxed{1840}$$

46. We assume the neutron is not relativistic. If the resulting velocity is small, our assumption will be valid. We use Eq. 37-7.

$$\lambda = \frac{h}{p} = \frac{h}{mv} \quad \rightarrow \quad v = \frac{h}{m\lambda} = \frac{\left(6.63\times10^{-34}\,\text{J•s}\right)}{\left(1.67\times10^{-27}\,\text{kg}\right)\left(0.3\times10^{-9}\,\text{m}\right)} = 1300\,\text{m/s} \approx \boxed{1000\,\text{m/s}}$$

This is not relativistic, so our assumption was valid.

47. (*a*) We find the momentum from Eq. 37-7.

$$p = \frac{h}{\lambda} = \frac{6.63\times10^{-34}\,\text{J•s}}{6.0\times10^{-10}\,\text{m}} = \boxed{1.1\times10^{-24}\,\text{kg•m/s}}$$

(*b*) We assume the speed is non-relativistic.

$$\lambda = \frac{h}{p} = \frac{h}{mv} \quad \rightarrow \quad v = \frac{h}{m\lambda} == \frac{6.63\times10^{-34}\,\text{J•s}}{\left(9.11\times10^{-31}\,\text{kg}\right)\left(6.0\times10^{-10}\,\text{m}\right)} = \boxed{1.2\times10^{6}\,\text{m/s}}$$

Since $v/c = 4.04\times10^{-3}$, our assumption is valid.

(*c*) We calculate the kinetic energy classically.

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}\left(mc^2\right)\left(v/c\right)^2 = \tfrac{1}{2}\left(0.511\,\text{MeV}\right)\left(4.04\times10^{-3}\right)^2 = 4.17\times10^{-6}\,\text{MeV} = 4.17\,\text{eV}$$

This is the energy gained by an electron if accelerated through a potential difference of $\boxed{4.2\text{ V}}$.

48. Because all of the energies to be considered are much less than the rest energy of an electron, we can use non-relativistic relationships. We use Eq. 37-7 to calculate the wavelength.

$$K = \frac{p^2}{2m} \quad \rightarrow \quad p = \sqrt{2mK} \quad ; \quad \lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}}$$

(*a*) $$\lambda = \frac{h}{\sqrt{2mK}} = \frac{6.63\times10^{-34}\,\text{J•s}}{\sqrt{2\left(9.11\times10^{-31}\text{kg}\right)\left(20\,\text{eV}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)}} = 2.7\times10^{-10}\,\text{m} \approx \boxed{3\times10^{-10}\,\text{m}}$$

(*b*) $$\lambda = \frac{h}{\sqrt{2mK}} = \frac{6.63\times10^{-34}\,\text{J•s}}{\sqrt{2\left(9.11\times10^{-31}\text{kg}\right)\left(200\,\text{eV}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)}} = 8.7\times10^{-11}\,\text{m} \approx \boxed{9\times10^{-11}\,\text{m}}$$

(*c*) $$\lambda = \frac{h}{\sqrt{2mK}} = \frac{6.63\times10^{-34}\,\text{J•s}}{\sqrt{2\left(9.11\times10^{-31}\text{kg}\right)\left(2.0\times10^{3}\text{eV}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)}} = \boxed{2.7\times10^{-11}\,\text{m}}$$

$\boxed{49.}$ Since the particles are not relativistic, we may use $K = p^2/2m$. We then form the ratio of the wavelengths, using Eq. 37-7.

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} \quad ; \quad \frac{\lambda_p}{\lambda_e} = \frac{\dfrac{h}{\sqrt{2m_pK}}}{\dfrac{h}{\sqrt{2m_eK}}} = \sqrt{\frac{m_e}{m_p}} < 1$$

Thus we see the proton has the shorter wavelength, since $m_e < m_p$.

50. The final kinetic energy of the electron is equal to the negative change in potential energy of the electron as it passes through the potential difference. We compare this energy to the rest energy of the electron to determine if the electron is relativistic.

$$K = -q\Delta V = \left(1\text{e}\right)\left(33\times10^{3}\text{ V}\right) = 33\times10^{3}\text{eV}$$

Because this is greater than 1% of the electron rest energy, $\boxed{\text{the electron is relativistic}}$. We use Eq. 36-13 to determine the electron momentum and then Eq. 37-5 to determine the wavelength.

$$E^2 = \left[K + mc^2\right]^2 = p^2c^2 + m^2c^4 \;\Rightarrow p = \frac{\sqrt{K^2 + 2Kmc^2}}{c}$$

$$\lambda = \frac{h}{p} = \frac{hc}{\sqrt{K^2 + 2Kmc^2}} = \frac{1240\,\text{eV}\bullet\text{nm}}{\sqrt{\left(33\times10^3\text{eV}\right)^2 + 2\left(33\times10^3\text{eV}\right)\left(511\times10^3\text{eV}\right)}} = 0.0066\,\text{nm}$$

Because λ « 5 cm, $\boxed{\text{diffraction effects are negligible.}}$

51. We will assume that the electrons are non-relativistic, and then examine the result in light of that assumption. The wavelength of the electron can be found from Eq. 34-2a. The speed can then be found from Eq. 37-7.

$$d\sin\theta = m_{\text{order}}\lambda \;\rightarrow\; \lambda = \frac{d\sin\theta}{m_{\text{order}}} \;;\; \lambda = \frac{h}{p} = \frac{h}{m_e v} \;\rightarrow$$

$$v = \frac{hm_{\text{order}}}{m_e d\sin\theta} = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)(2)}{\left(9.11\times10^{-31}\text{kg}\right)\left(3.0\times10^{-6}\text{m}\right)\left(\sin 55^\circ\right)} = \boxed{590\,\text{m/s}}$$

This is far from being relativistic, so our original assumption was fine.

52. We relate the kinetic energy to the momentum with a classical relationship, since the electrons are non-relativistic. We also use Eq. 37-7. We then assume that the kinetic energy was acquired by electrostatic potential energy.

$$K = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2} = eV \quad\rightarrow$$

$$V = \frac{h^2}{2me\lambda^2} = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)^2}{2\left(9.11\times10^{-31}\,\text{kg}\right)\left(1.60\times10^{-19}\text{C}\right)\left(0.28\times10^{-9}\text{m}\right)^2} = \boxed{19\,\text{V}}$$

53. The kinetic energy is 3450 eV. That is small enough compared to the rest energy of the electron for the electron to be non-relativistic. We use Eq. 37-7.

$$\lambda = \frac{h}{p} = \frac{h}{(2mK)^{1/2}} = \frac{hc}{\left(2mc^2K\right)^{1/2}} = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(3.00\times10^8\,\text{m/s}\right)}{\left(1.60\times10^{-19}\,\text{J/eV}\right)\left[2\left(0.511\times10^6\,\text{eV}\right)\left(3450\text{ eV}\right)\right]^{1/2}}$$

$$= 2.09\times10^{-11}\,\text{m} = \boxed{20.9\,\text{pm}}$$

54. The energy of a level is $E_n = -\dfrac{(13.6\text{ eV})}{n^2}$.

(*a*) The transition from $n = 1$ to $n' = 3$ is an $\boxed{\text{absorption,}}$ because the $\boxed{\text{final state,}}$ $n' = 3$, has a higher energy. The photon energy is the difference between the energies of the two states.

$$hf = E_{n'} - E_n = -(13.6\text{ eV})\left[\left(\frac{1}{3^2}\right) - \left(\frac{1}{1^2}\right)\right] = 12.1\text{ eV}$$

(*b*) The transition from $n = 6$ to $n' = 2$ is an $\boxed{\text{emission,}}$ because the $\boxed{\text{initial state,}}$ $n' = 2$, has a higher energy. The photon energy is the difference between the energies of the two states.

$$hf = -\left(E_{n'} - E_n\right) = (13.6\text{ eV})\left[\left(\frac{1}{2^2}\right) - \left(\frac{1}{6^2}\right)\right] = 3.0\text{ eV}$$

(*c*) The transition from $n = 4$ to $n' = 5$ is an $\boxed{\text{absorption,}}$ because the $\boxed{\text{final state,}}$ $n' = 5$, has a higher energy. The photon energy is the difference between the energies of the two states.

$$hf = E_{n'} - E_n = -(13.6\text{ eV})\left[\left(\frac{1}{5^2}\right) - \left(\frac{1}{4^2}\right)\right] = 0.31\text{ eV}$$

The photon for the transition from $\boxed{n = 1 \text{ to } n' = 3}$ has the largest energy.

55. To ionize the atom means removing the electron, or raising it to zero energy.

$$E_{\text{ionization}} = 0 - E_n = 0 - \frac{(-13.6\,\text{eV})}{n^2} = \frac{(13.6\,\text{eV})}{3^2} = \boxed{1.51\,\text{eV}}$$

56. We use the equation that appears above Eq. 37-15 in the text.

(*a*) The second Balmer line is the transition from $n = 4$ to $n = 2$.

$$\lambda = \frac{hc}{(E_4 - E_2)} = \frac{1240\,\text{eV}\bullet\text{nm}}{[-0.85\,\text{eV} - (-3.4\,\text{eV})]} = \boxed{490\,\text{nm}}$$

(*b*) The third Lyman line is the transition from $n = 4$ to $n = 1$.

$$\lambda = \frac{hc}{(E_4 - E_1)} = \frac{1240\,\text{eV}\bullet\text{nm}}{[-0.85\text{ eV} - (-13.6\text{ eV})]} = \boxed{97.3\,\text{nm}}$$

(*c*) The first Balmer line is the transition from $n = 3$ to $n = 2$.
For the jump from $n = 5$ to $n = 2$, we have

$$\lambda = \frac{hc}{(E_3 - E_2)} = \frac{1240\,\text{eV}\bullet\text{nm}}{[-1.5\text{ eV} - (-3.4\text{ eV})]} = \boxed{650\,\text{nm}}$$

57. Doubly ionized lithium is similar to hydrogen, except that there are three positive charges ($Z = 3$) in the nucleus. The square of the product of the positive and negative charges appears in the energy term for the energy levels. We can use the results for hydrogen, if we replace e^2 by Ze^2:

$$E_n = -\frac{Z^2(13.6\,\text{eV})}{n^2} = -\frac{3^2(13.6\,\text{eV})}{n^2} = -\frac{(122\,\text{eV})}{n^2}$$

$$E_{\text{ionization}} = 0 - E_1 = 0 - \left[-\frac{(122\,\text{eV})}{(1)^2}\right] = \boxed{122\,\text{eV}}$$

58. We evaluate the Rydberg constant using Eq. 37-8 and 37-15. We use hydrogen so $Z = 1$.

$$\frac{1}{\lambda} = R\left(\frac{1}{(n')^2} - \frac{1}{(n)^2}\right) = \frac{Z^2e^4m}{8\varepsilon_0^2h^3c}\left(\frac{1}{(n')^2} - \frac{1}{(n)^2}\right) \rightarrow$$

$$R = \frac{Z^2e^4m}{8\varepsilon_0^2h^3c} = \frac{(1)^2(1.602176\times10^{-19}\,\text{C})^4(9.109382\times10^{-31}\,\text{kg})}{8(8.854188\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2)^2(6.626069\times10^{-34}\,\text{J}\bullet\text{s})^3(2.997925\times10^8\,\text{m/s})}$$

$$= 1.0974\times10^7\,\frac{\text{C}^4\bullet\text{kg}}{\frac{\text{C}^4}{\text{N}^2\bullet\text{m}^4}\text{J}^3\text{s}^3\,\text{m/s}} = \boxed{1.0974\times10^7\,\text{m}^{-1}}$$

59. The longest wavelength corresponds to the minimum energy, which is the ionization energy:

$$\lambda = \frac{hc}{E_{\text{ion}}} = \frac{(6.63\times10^{-34}\,\text{J}\bullet\text{s})(3.00\times10^8\,\text{m/s})}{(1.60\times10^{-19}\text{ J/eV})(13.6\,\text{eV})} = 9.14\times10^{-8}\,\text{m} = \boxed{91.4\,\text{nm}}$$

60. Singly ionized helium is like hydrogen, except that there are two positive charges ($Z = 2$) in the nucleus. The square of the product of the positive and negative charges appears in the energy term for the energy levels. We can use the results for hydrogen, if we replace e^2 by Ze^2.

$$E_n = -\frac{Z^2(13.6\text{ eV})}{n^2} = -\frac{2^2(13.6\text{ eV})}{n^2} = -\frac{(54.4\text{ eV})}{n^2}$$

We find the energy of the photon from the $n = 5$ to $n = 2$ transition in singly-ionized helium.

$$\Delta E = E_5 - E_2 = -(54.4\text{ eV})\left[\left(\frac{1}{5^2}\right) - \left(\frac{1}{2^2}\right)\right] = 11.4\text{ eV}$$

Because this is NOT the energy difference between any two specific energy levels for hydrogen, the photon CANNOT be absorbed by hydrogen.

61. The energy of the photon is the sum of the ionization energy of 13.6 eV and the kinetic energy of 20.0eV. The wavelength is found from Eq. 37-3.

$$hf = \frac{hc}{\lambda} = E_{\text{total}} \rightarrow \lambda = \frac{hc}{E_{\text{total}}} = \frac{(6.63\times10^{-34}\,\text{J}\bullet\text{s})(3.00\times10^{8}\,\text{m/s})}{(1.60\times10^{-19}\,\text{J/eV})(33.6\text{ eV})} = 3.70\times10^{-8}\,\text{m} = \boxed{37.0\,\text{nm}}$$

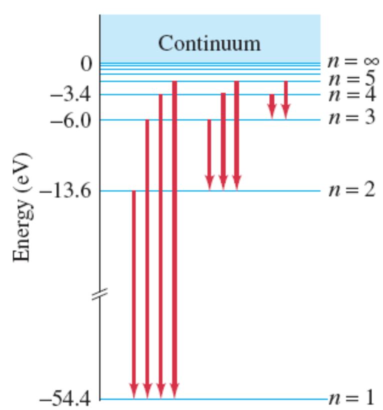
62. A collision is elastic if the kinetic energy before the collision is equal to the kinetic energy after the collision. If the hydrogen atom is in the ground state, then the smallest amount of energy it can absorb is the difference in the $n = 1$ and $n = 2$ levels. So as long as the kinetic energy of the incoming electron is less than that difference, the collision must be elastic.

$$K < E_2 - E_1 = \left(-\frac{13.6\text{eV}}{4}\right) - (-13.6\text{eV}) = \boxed{10.2\,\text{eV}}$$

63. Singly ionized helium is like hydrogen, except that there are two positive charges ($Z = 2$) in the nucleus. The square of the product of the positive and negative charges appears in the energy term for the energy levels. We can use the results for hydrogen, if we replace e^2 by Ze^2:

$$E_n = -\frac{Z^2(13.6\text{eV})}{n^2} = -\frac{2^2(13.6\text{eV})}{n^2} = -\frac{(54.4\text{eV})}{n^2}$$

$$E_1 = -54.5\text{eV},\ E_2 = -13.6\text{eV},\ E_3 = -6.0\text{eV},\ E_4 = -3.4\text{eV}$$

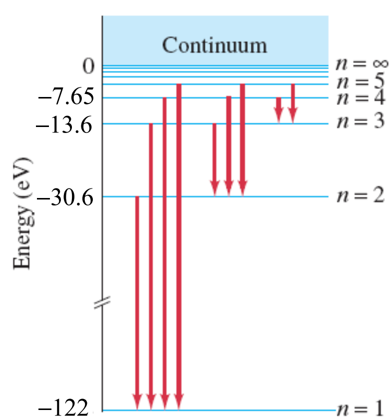


64. Doubly ionized lithium is like hydrogen, except that there are three positive charges ($Z = 3$) in the nucleus. The square of the product of the positive and negative charges appears in the energy term for the energy levels. We can use the results for hydrogen, if we replace e^2 by Ze^2:

$$E_n = -\frac{Z^2(13.6\text{eV})}{n^2} = -\frac{3^2(13.6\text{eV})}{n^2} = -\frac{(122.4\text{eV})}{n^2}$$

$$E_1 = -122\text{eV},\ E_2 = -30.6\text{eV},\ E_3 = -13.6\text{eV},$$

$$E_4 = -7.65\text{eV}$$



65. The potential energy for the ground state is given by the charge of the electron times the electric potential caused by the proton.

$$U=(-e)V_{\text{proton}}=(-e)\frac{1}{4\pi\varepsilon_0}\frac{e}{r_1}=-\frac{\left(9.00\times10^9\,\text{N}\bullet\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\,\text{C}\right)^2\left(1\,\text{eV}/1.60\times10^{-19}\,\text{J}\right)}{\left(0.529\times10^{-10}\,\text{m}\right)}$$

$$=\boxed{-27.2\,\text{eV}}$$

The kinetic energy is the total energy minus the potential energy.

$$K=E_1-U=-13.6\,\text{eV}-(-27.2\,\text{eV})=\boxed{+13.6\,\text{eV}}$$

66. The value of *n* is found from $r_n=n^2r_1$, and then find the energy from Eq. 37-14b.

$$r_n=n^2r_1\ \rightarrow\ n=\sqrt{\frac{r_n}{r_1}}=\sqrt{\frac{\frac{1}{2}\left(0.10\times10^{-3}\,\text{m}\right)}{0.529\times10^{-10}\,\text{m}}}=\boxed{972}$$

$$E=-\frac{(13.6\text{ eV})}{n^2}=-\frac{(13.6\text{ eV})}{972^2}=-\frac{(13.6\text{ eV})}{1375^2}=\boxed{-1.4\times10^{-5}\,\text{eV}}$$

67. The velocity is found from Eq. 37-10 evaluated for *n* = 1.

$$mvr_n=\frac{nh}{2\pi}\ \rightarrow$$

$$v=\frac{h}{2\pi r_1m_e}=\frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)}{2\pi\left(0.529\times10^{-10}\,\text{m}\right)\left(9.11\times10^{-31}\,\text{kg}\right)}=2.190\times10^6\,\text{m/s}=\boxed{7.30\times10^{-3}c}$$

We see that $v\ll c$, and so $\boxed{\text{yes}}$, non-relativistic formulas are justified.
The relativistic factor is as follows.

$$\left[1-\left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}}\approx1-\tfrac{1}{2}\left(\frac{v}{c}\right)^2=1-\tfrac{1}{2}\left(\frac{2.190\times10^6\,\text{m/s}}{3.00\times10^8\,\text{m/s}}\right)^2=\boxed{1-2.66\times10^{-5}}\approx0.99997$$

We see that $\sqrt{1-v^2/c^2}$ is essentially 1, and so again the answer is $\boxed{\text{yes}}$, non-relativistic formulas are justified.

68. The angular momentum can be used to find the quantum number for the orbit, and then the energy can be found from the quantum number. Use Eqs. 37-10 and 37-14b.

$$L=n\frac{h}{2\pi}\ \rightarrow\ n=\frac{2\pi L}{h}=\frac{2\pi\left(5.273\times10^{-34}\,\text{kg}\bullet\text{m}^2/\text{s}\right)}{\left(6.626\times10^{-34}\,\text{J}\bullet\text{s}\right)}=5.000\approx5$$

$$E_n=-(13.6\text{eV})\frac{Z^2}{n^2}=-\frac{13.6\text{eV}}{25}=\boxed{0.544\,\text{eV}}$$

69. Hydrogen atoms start in the $n=1$ orbit ("ground state"). Using Eq. 37-9 and Eq. 37-14b, we determine the orbit to which the atom is excited when it absorbs a photon of 12.75 Ev via collision with an electron. Then, using Eq. 37-15, we calculate all possible wavelengths that can be emitted as the electron cascades back to the ground state.

$$\Delta E=E_U-E_L\ \rightarrow\ E_U=-\frac{13.6\text{ eV}}{n^2}=E_L+\Delta E\ \rightarrow$$

$$n=\sqrt{\frac{-13.6\text{ eV}}{E_L+\Delta E}}=\sqrt{\frac{-13.6\text{ eV}}{-13.6\text{ eV}+12.75\text{ eV}}}=4$$

Starting with the electron in the $n=4$ orbit, the following transitions are possible: $n=4$ to $n=3$; $n=4$ to $n=2$; $n=4$ to $n=1$; $n=3$ to $n=2$; $n=3$ to $n=1$; $n=2$ to $n=1$.

$$\frac{1}{\lambda}=\left(1.097\times10^{7}\text{ m}^{-1}\right)\left(\frac{1}{3^2}-\frac{1}{4^2}\right)=5.333\times10^{5}\text{ m}^{-1}\Rightarrow\lambda=\boxed{1875\text{ nm}}$$

$$\frac{1}{\lambda}=\left(1.097\times10^{7}\text{ m}^{-1}\right)\left(\frac{1}{2^2}-\frac{1}{4^2}\right)=2.057\times10^{6}\text{ m}^{-1}\Rightarrow\lambda=\boxed{486.2\text{ nm}}$$

$$\frac{1}{\lambda}=\left(1.097\times10^{7}\text{ m}^{-1}\right)\left(\frac{1}{1^2}-\frac{1}{4^2}\right)=1.028\times10^{7}\text{ m}^{-1}\Rightarrow\lambda=\boxed{97.23\text{ nm}}$$

$$\frac{1}{\lambda}=\left(1.097\times10^{7}\text{ m}^{-1}\right)\left(\frac{1}{2^2}-\frac{1}{3^2}\right)=1.524\times10^{6}\text{ m}^{-1}\Rightarrow\lambda=\boxed{656.3\text{ nm}}$$

$$\frac{1}{\lambda}=\left(1.097\times10^{7}\text{ m}^{-1}\right)\left(\frac{1}{1^2}-\frac{1}{3^2}\right)=9.751\times10^{6}\text{ m}^{-1}\Rightarrow\lambda=\boxed{102.6\text{ nm}}$$

$$\frac{1}{\lambda}=\left(1.097\times10^{7}\text{ m}^{-1}\right)\left(\frac{1}{1^2}-\frac{1}{2^2}\right)=8.228\times10^{6}\text{ m}^{-1}\Rightarrow\lambda=\boxed{121.5\text{ nm}}$$

70. When we compare the gravitational and electric forces we see that we can use the same expression for the Bohr orbits, Eq. 37-11 and 37-14a, if we replace $Ze^2/4\pi\varepsilon_0$ with Gm_em_p.

$$r_1=\frac{h^2\varepsilon_0}{\pi m_e Ze^2}=\frac{h^2}{4\pi^2 m_e}\frac{4\pi\varepsilon_0}{Ze^2}\quad\rightarrow$$

$$r_1=\frac{h^2}{4\pi^2 Gm^2{}_e m_p}=\frac{\left(6.626\times10^{-34}\text{J}\bullet\text{s}\right)^2}{4\pi^2\left(6.67\times10^{-11}\text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(9.11\times10^{-31}\text{kg}\right)^2\left(1.67\times10^{-27}\text{kg}\right)}$$

$$=\boxed{1.20\times10^{29}\text{m}}$$

$$E_1=-\frac{Z^2e^4m_e}{8\varepsilon_0^2h^2}=-\left(\frac{Ze^2}{4\pi\varepsilon_0}\right)^2\frac{2\pi^2m_e}{h^2}\quad\rightarrow\quad E_1=-\frac{2\pi^2G^2m_e{}^3m_p{}^2}{h^2}$$

$$=-\frac{2\pi^2\left(6.67\times10^{-11}\text{N}\bullet\text{m}^2/\text{kg}^2\right)^2\left(9.11\times10^{-31}\text{kg}\right)^3\left(1.67\times10^{-27}\text{kg}\right)^2}{\left(6.626\times10^{-34}\text{J}\bullet\text{s}\right)^2}=\boxed{-4.22\times10^{-97}\text{J}}$$

71. We know that the radii of the orbits are given by $r_n=n^2r_1$. Find the difference in radius for adjacent orbits.

$$\Delta r=r_n-r_{n-1}=n^2r_1-\left(n-1\right)^2r_1=n^2r_1-\left(n^2-2n+1\right)r_1=\left(2n-1\right)r_1$$

If $n\gg1$, we have $\Delta r\approx 2nr_1=2n\dfrac{r_n}{n^2}=\dfrac{2r_n}{n}$.

In the classical limit, the separation of radii (and energies) should be very small. We see that letting $n\rightarrow\infty$ accomplishes this. If we substitute the expression for r_1 from Eq. 37-11, we have this.

$$\Delta r\approx 2nr_1=\frac{2nh^2\varepsilon_0}{\pi me^2}$$

We see that $\Delta r\propto h^2$, and so letting $h\rightarrow0$ is equivalent to considering $n\rightarrow\infty$.

72. We calculate the energy from the light bulb that enters the eye by calculating the intensity of the light at a distance of 250 m by dividing the power in the visible spectrum by the area of a sphere of radius 250 m. We multiply the intensity of the light by the area of the pupil to determine the energy entering the eye per second. We divide this energy by the energy of a photon (Eq. 37-3) to calculate the number of photons entering the eye per second.

$$I = \frac{P}{4\pi\ell^2} \qquad P_e = I\left(\pi D^2/4\right) = \frac{P}{16}\left(\frac{D}{\ell}\right)^2$$

$$n = \frac{P_e}{hc/\lambda} = \frac{P\lambda}{16hc}\left(\frac{D}{\ell}\right)^2 = \frac{0.030(75\,\text{W})(550\times10^{-9}\,\text{m})}{16(6.626\times10^{-34}\,\text{J}\cdot\text{s})(3.00\times10^{8}\,\text{m/s})}\left(\frac{4.0\times10^{-3}\,\text{m}}{250\,\text{m}}\right)^2$$

$$= \boxed{1.0\times10^{8}\,\text{photons/sec}}$$

73. To produce a photoelectron, the hydrogen atom must be ionized, so the minimum energy of the photon is 13.6 eV. We find the minimum frequency of the photon from Eq. 37-3.

$$E = hf \;\rightarrow\; f = \frac{E}{h} \;\rightarrow\; f_{\text{min}} = \frac{E_{\text{min}}}{h} = \frac{(13.6\text{ eV})(1.60\times10^{-19}\text{ J/eV})}{(6.63\times10^{-34}\,\text{J}\cdot\text{s})} = \boxed{3.28\times10^{15}\,\text{Hz}}$$

74. From Section 35-10, the spacing between planes, d, for the first-order peaks is given by Eq. 35-20, $\lambda = 2d\sin\theta$. The wavelength of the electrons can be found from their kinetic energy. The electrons are not relativistic at the energy given.

$$K = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2} \;\rightarrow\; \lambda = \frac{h}{\sqrt{2mK}} = 2d\sin\theta \;\rightarrow$$

$$d = \frac{h}{2\sin\theta\sqrt{2mK}} = \frac{(6.63\times10^{-34}\,\text{J}\cdot\text{s})}{2(\sin 38^\circ)\sqrt{2(9.11\times10^{-31}\,\text{kg})(125\,\text{eV})(1.60\times10^{-19}\,\text{J/eV})}} = \boxed{8.9\times10^{-11}\,\text{m}}$$

75. The power rating is the amount of energy produced per second. If this is divided by the energy per photon, then the result is the number of photons produced per second.

$$E_{\text{photon}} = hf = \frac{hc}{\lambda} \;;\; \frac{P}{E_{\text{photon}}} = \frac{P\lambda}{hc} = \frac{(860\,\text{W})(12.2\times10^{-2}\,\text{m})}{(6.63\times10^{-34}\,\text{J}\cdot\text{s})(3.00\times10^{8}\,\text{m/s})} = \boxed{5.3\times10^{26}\text{ photons/s}}$$

76. The intensity is the amount of energy per second per unit area reaching the Earth. If that intensity is divided by the energy per photon, the result will be the photons per second per unit area reaching the Earth. We use Eq. 37-3.

$$E_{\text{photon}} = hf = \frac{hc}{\lambda}$$

$$I_{\text{photons}} = \frac{I_{\text{sunlight}}}{E_{\text{photon}}} = \frac{I_{\text{sunlight}}\lambda}{hc} = \frac{(1350\,\text{W/m}^2)(550\times10^{-9}\,\text{m})}{(6.63\times10^{-34}\,\text{J}\cdot\text{s})(3.00\times10^{8}\,\text{m/s})} = \boxed{3.7\times10^{21}\text{ photons/s}\cdot\text{m}^2}$$

77. The impulse on the wall is due to the change in momentum of the photons. Each photon is absorbed, and so its entire momentum is transferred to the wall.

$$F_{\text{on wall}}\Delta t = \Delta p_{\text{wall}} = -\Delta p_{\text{photons}} = -\left(0 - np_{\text{photon}}\right) = np_{\text{photon}} = \frac{nh}{\lambda} \rightarrow$$

$$\frac{n}{\Delta t} = \frac{F\lambda}{h} = \frac{\left(6.5\times10^{-9}\,\text{N}\right)\left(633\times10^{-9}\,\text{m}\right)}{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)} = \boxed{6.2\times10^{18}\text{ photons/s}}$$

78. We find the peak wavelength from Wien's law, Eq. 37-1.

$$\lambda_{\text{P}} = \frac{\left(2.90\times10^{-3}\,\text{m}\bullet\text{K}\right)}{T} = \frac{\left(2.90\times10^{-3}\,\text{m}\bullet\text{K}\right)}{\left(2.7\,\text{K}\right)} = 1.1\times10^{-3}\,\text{m} = \boxed{1.1\,\text{mm}}$$

79. The total energy of the two photons must equal the total energy (kinetic energy plus mass energy) of the two particles. The total momentum of the photons is 0, so the momentum of the particles must have been equal and opposite. Since both particles have the same mass and the same momentum, they each have the same kinetic energy.

$$E_{\text{photons}} = E_{\text{particles}} = 2\left(m_{\text{e}}c^2 + K\right) \rightarrow$$

$$K = \tfrac{1}{2}E_{\text{photons}} - m_{\text{e}}c^2 = 0.755\,\text{MeV} - 0.511\,\text{MeV} = \boxed{0.244\,\text{MeV}}$$

80. We calculate the required momentum from de Broglie's relation, Eq. 37-7.

$$p = \frac{h}{\lambda} = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)}{\left(6.0\times10^{-12}\,\text{m}\right)} = 1.11\times10^{22}\ \text{kg}\bullet\text{m/s}$$

(*a*) For the proton, we use the classical definition of momentum to determine the speed of the electron, and then the kinetic energy. We divide the kinetic energy by the charge of the proton to determine the required potential difference.

$$v = \frac{p}{m} = \frac{1.11\times10^{-22}\,\text{kg}\bullet\text{m/s}}{1.67\times10^{-27}\,\text{kg}} = 6.65\times10^{4}\,\text{m/s} \ll c$$

$$V = \frac{K}{e} = \frac{mv^2}{2e} = \frac{\left(1.67\times10^{-27}\,\text{kg}\right)\left(6.65\times10^{4}\,\text{m/s}\right)^2}{2\left(1.60\times10^{-19}\,\text{C}\right)} = \boxed{23\,\text{V}}$$

(*b*) For the electron, if we divide the momentum by the electron mass we obtain a speed greater than 10% of the speed of light. Therefore, we must use the relativistic invariant equation to determine the energy of the electron. We then subtract the rest energy from the total energy to determine the kinetic energy of the electron. Finally, we divide the kinetic energy by the electron charge to calculate the potential difference.

$$E = \left[\left(pc\right)^2 + \left(m_0c^2\right)^2\right]^{\frac{1}{2}}$$

$$= \left[\left(1.11\times10^{-22}\,\text{kg}\bullet\text{m/s}\right)^2\left(3.00\times10^{8}\,\text{m/s}\right)^2 + \left(9.11\times10^{-31}\,\text{kg}\right)^2\left(3.00\times10^{8}\,\text{m/s}\right)^4\right]^{\frac{1}{2}}$$

$$= 8.85\times10^{-14}\,\text{J}$$

$$K = E - m_0c^2 = 8.85\times10^{-14}\,\text{J} - \left(9.11\times10^{-31}\,\text{kg}\right)\left(3.00\times10^{8}\,\text{m/s}\right)^2 = 6.50\times10^{-15}\,\text{J}$$

$$V = \frac{K}{e} = \frac{6.50\times10^{-15}\,\text{J}}{1.60\times10^{-19}\,\text{C}} = \boxed{41\text{ kV}}$$

81. If we ignore the recoil motion, at the closest approach the kinetic energy of both particles is zero. The potential energy of the two charges must equal the initial kinetic energy of the α particle:

$$K_\alpha = U = \frac{1}{4\pi\varepsilon_0}\frac{(Z_\alpha e)(Z_{\text{Ag}}e)}{r_{\min}} \rightarrow$$

$$r_{\min} = \frac{1}{4\pi\varepsilon_0}\frac{(Z_\alpha e)(Z_{\text{Ag}}e)}{K_\alpha} = \frac{(9.00\times10^{9}\,\text{N}\bullet\text{m}^2/\text{C}^2)(2)(47)(1.60\times10^{-19}\,\text{C})^2}{(4.8\,\text{MeV})(1.60\times10^{-13}\,\text{J/MeV})} = \boxed{2.8\times10^{-14}\,\text{m}}$$

82. The electrostatic potential energy is given by Eq. 23-5. The kinetic energy is given by the total energy, Eq. 37-14a, minus the potential energy. The Bohr radius is given by Eq. 37-11.

$$U = -eV = -\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r_n} = -\frac{1}{4\pi\varepsilon_0}\frac{Ze^2\pi mZe^2}{n^2h^2\varepsilon_0} = -\frac{Z^2e^4m}{4n^2h^2\varepsilon_0^2}$$

$$K = E - U = -\frac{Z^2e^4m}{8\varepsilon_0^2h^2n^2} - \left(-\frac{Z^2e^4m}{4n^2h^2\varepsilon_0^2}\right) = \frac{Z^2e^4m}{8n^2h^2\varepsilon_0^2} \;;\; \frac{|U|}{K} = \frac{\dfrac{Z^2e^4m}{4n^2h^2\varepsilon_0^2}}{\dfrac{Z^2e^4m}{8n^2h^2\varepsilon_0^2}} = \frac{Z^2e^4m}{4n^2h^2\varepsilon_0^2}\frac{8n^2h^2\varepsilon_0^2}{Z^2e^4m} = \boxed{2}$$

83. We calculate the ratio of the forces.

$$\frac{F_{\text{gravitational}}}{F_{\text{electric}}} = \frac{\left(\dfrac{Gm_em_p}{r^2}\right)}{\left(\dfrac{ke^2}{r^2}\right)} = \frac{Gm_em_p}{ke^2} = \frac{(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2)(9.11\times10^{-31}\,\text{kg})^2(1.67\times10^{-27}\,\text{kg})}{(9.00\times10^{9}\,\text{N}\bullet\text{m}^2/\text{C}^2)(1.60\times10^{-19}\,\text{C})^2}$$

$$= \boxed{4.4\times10^{-40}}$$

$\boxed{\text{Yes,}}$ the gravitational force may be safely ignored.

84. The potential difference gives the electrons a kinetic energy of 12.3 eV, so it is possible to provide this much energy to the hydrogen atom through collisions. From the ground state, the maximum energy of the atom is $-13.6\text{ eV} + 12.3\text{ eV} = -1.3\text{ eV}$. From the energy level diagram, Figure 37-26, we see that this means the atom could be excited to the $n = 3$ state, so the possible transitions when the atom returns to the ground state are $n = 3$ to $n = 2$, $n = 3$ to $n = 1$, and $n = 2$ to $n = 1$. We calculate the wavelengths from the equation above Eq. 37-15.

$$\lambda_{3\to2} = \frac{hc}{(E_3 - E_2)} = \frac{1240\,\text{eV}\bullet\text{nm}}{[-1.5\text{ eV} - (-3.4\text{ eV})]} = \boxed{650\,\text{nm}}$$

$$\lambda_{3\to1} = \frac{hc}{(E_3 - E_1)} = \frac{1240\,\text{eV}\bullet\text{nm}}{[-1.5\text{ eV} - (-13.6\text{ eV})]} = \boxed{102\,\text{nm}}$$

$$\lambda_{2\to1} = \frac{hc}{(E_2 - E_1)} = \frac{1240\,\text{eV}\bullet\text{nm}}{[-3.4\text{ eV} - (-13.6\text{ eV})]} = \boxed{122\,\text{nm}}$$

$\boxed{85.}$ The stopping potential is the voltage that gives a potential energy change equal to the maximum kinetic energy. We use Eq. 37-4b to first find the work function, and then find the stopping potential for the higher wavelength.

$$K_{\max} = eV_0 = \frac{hc}{\lambda} - W_0 \;\rightarrow\; W_0 = \frac{hc}{\lambda_0} - eV_0$$

$$eV_1 = \frac{hc}{\lambda_1} - W_0 = \frac{hc}{\lambda_1} - \left(\frac{hc}{\lambda_0} - eV_0\right) = hc\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_0}\right) + eV_0$$

$$= \frac{(6.63 \times 10^{-34}\,\text{J•s})(3.00 \times 10^8\,\text{m/s})}{(1.60 \times 10^{-19}\,\text{J/eV})}\left(\frac{1}{440 \times 10^{-9}\,\text{m}} - \frac{1}{380 \times 10^{-9}\,\text{m}}\right) + 2.70\,\text{eV} = 2.25\,\text{eV}$$

The potential difference needed to cancel an electron kinetic energy of 2.25 eV is $\boxed{2.25\text{ V}}$.

86. (*a*) The electron has a charge *e*, so the potential difference produces a kinetic energy of eV. The shortest wavelength photon is produced when all the kinetic energy is lost and a photon is emitted.

$$hf_{\text{max}} = \frac{hc}{\lambda_0} = eV \rightarrow \lambda_0 = \boxed{\frac{hc}{eV}} \text{ which gives } \lambda_0 = \frac{hc}{eV}.$$

(*b*) $\lambda_0 = \frac{hc}{eV} = \frac{1240\,\text{eV•nm}}{33 \times 10^3\,\text{eV}} = \boxed{0.038\,\text{nm}}$

87. The average force on the sail is equal to the impulse on the sail divided by the time (Eq. 9-2). Since the photons bounce off the mirror the impulse is equal to twice the incident momentum. We use Eq. 37-5 to write the momentum of the photon in terms of the photon energy. The total photon energy is the intensity of the sunlight multiplied by the area of the sail

$$\bar{F} = \frac{\Delta p}{\Delta t} = \frac{2(E/c)}{\Delta t} = \frac{2(E/\Delta t)}{c} = \frac{2IA}{c} = \frac{2(1350\text{ W/m}^2)(1000\text{ m})^2}{3.00\times10^8\text{ m/s}} = \boxed{9.0\text{ N}}$$

88. We first find the work function from the given data. A photon energy of 9.0 eV corresponds with a stopping potential of 4.0 V.

$$eV_0 = hf - W_0 \rightarrow W_0 = hf - eV_0 = 9.0\,\text{eV} - 4.0\,\text{eV} = 5.0\,\text{eV}$$

If the photons' wavelength is doubled, the energy is halved, from 9.0 eV to 4.5 eV. This is smaller than the work function, and so no current flows. Thus the maximum kinetic energy is $\boxed{0}$. Likewise, if the photon's wavelength is tripled, the energy is only 3.0 eV, which is still less than the work function, and so $\boxed{\text{no current flows}}$.

89. The electrons will be non-relativistic at that low energy. The maximum kinetic energy of the photoelectrons is given by Eq. 37-4b. The kinetic energy determines the momentum, and the momentum determines the wavelength of the emitted electrons. The shortest electron wavelength corresponds to the maximum kinetic energy.

$$K_{\text{electron}} = \frac{hc}{\lambda} - W_0 = \frac{p^2}{2m} = \frac{h^2}{2m\lambda_{\text{electron}}^2} \rightarrow \lambda_{\text{electron}} = \frac{h}{\sqrt{2m\left(\frac{hc}{\lambda} - W_0\right)}}$$

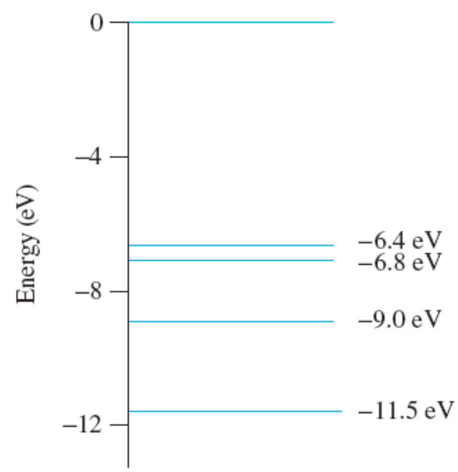
$$= \frac{(6.63 \times 10^{-34}\,\text{J•s})}{\sqrt{2(9.11 \times 10^{-31}\,\text{kg})\left(\frac{1240\,\text{eV•nm}}{360\,\text{nm}} - 2.4\,\text{eV}\right)(1.60 \times 10^{-19}\,\text{J/eV})}} = \boxed{1.2\times10^{-9}\,\text{m}}$$

90. The wavelength is found from Eq. 35-13. The velocity of electrons with the same wavelength (and thus the same diffraction pattern) is found from their momentum, assuming they are not relativistic. We use Eq. 37-7 to relate the wavelength and momentum.

$$d\sin\theta = n\lambda \quad\rightarrow\quad \lambda = \frac{d\sin\theta}{n} = \frac{h}{p} = \frac{h}{mv} \quad\rightarrow$$

$$v = \frac{hn}{md\sin\theta} = \frac{(6.63\times10^{-34}\,\text{J}\bullet\text{s})(1)}{(9.11\times10^{-31}\,\text{kg})(0.012\times10^{-3}\,\text{m})(\sin 3.5^\circ)} = \boxed{990\,\text{m/s}}$$

91. (*a*) See the adjacent figure.



(*b*) Absorption of a 5.1 eV photon represents a transition from the ground state to the state 5.1 eV above that, the third excited state. Possible photon emission energies are found by considering all the possible downward transitions that might occur as the electron makes its way back to the ground state.

$$-6.4\,\text{eV} - (-6.8\,\text{eV}) = \boxed{0.4\,\text{eV}}$$

$$-6.4\,\text{eV} - (-9.0\,\text{eV}) = \boxed{2.6\,\text{eV}}$$

$$-6.4\,\text{eV} - (-11.5\,\text{eV}) = \boxed{5.1\,\text{eV}}$$

$$-6.8\,\text{eV} - (-9.0\,\text{eV}) = \boxed{2.2\,\text{eV}}$$

$$-6.8\,\text{eV} - (-11.5\,\text{eV}) = \boxed{4.7\,\text{eV}}$$

$$-9.0\,\text{eV} - (-11.5\,\text{eV}) = \boxed{2.5\,\text{eV}}$$

92. (*a*) We use Eq. 37-4b to calculate the maximum kinetic energy of the electron and set this equal to the product of the stopping voltage and the electron charge.

$$K_{\text{max}} = hf - W_0 = eV_0 \quad\rightarrow\quad V_0 = \frac{hf - W_0}{e} = \frac{hc/\lambda - W_0}{e}$$

$$V_0 = \frac{(1240\,\text{eV}\bullet\text{nm})/(424\,\text{nm}) - 2.28\,\text{eV}}{\text{e}} = \boxed{0.65\,\text{V}}$$

(*b*) We calculate the speed from the non-relativistic kinetic energy equation and the maximum kinetic energy found in part (*a*).

$$K_{\text{max}} = \tfrac{1}{2}mv_{\text{max}}^2 \quad\rightarrow\quad v_{\text{max}} = \sqrt{\frac{2K_{\text{max}}}{m}} = \sqrt{\frac{2(0.65\,\text{eV})(1.60\times10^{-19}\,\text{J/eV})}{9.11\times10^{-31}\,\text{kg}}} = \boxed{4.8\times10^5\,\text{m/s}}$$

(*c*) We use Eq. 37-7 to calculate the de Broglie wavelength.

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63\times10^{-34}\,\text{J}\bullet\text{s}}{(9.11\times10^{-31}\,\text{kg})(4.8\times10^5\,\text{m/s})} = 1.52\times10^{-9}\,\text{m} = \boxed{1.5\,\text{nm}}$$

93. (*a*) We use Bohr's analysis of the hydrogen atom, where we replace the proton mass with Earth's mass, the electron mass with the Moon's mass, and the electrostatic force $F_e = \frac{ke^2}{r^2}$ with the gravitational force $F_g = \frac{Gm_E m_M}{r^2}$. To account for the change in force, we replace ke^2 with $Gm_E m_M$. With these replacements, we write expressions similar to Eq. 37-11 and Eq. 37-14a for the Bohr radius and energy.

$$r_n = \frac{h^2n^2}{4\pi^2 mke^2} \quad \rightarrow$$

$$r_n = \frac{h^2n^2}{4\pi^2 Gm_M^2 m_E} = \frac{\left(6.626\times10^{-34}\,\text{J}\bullet\text{s}\right)^2}{4\pi^2\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(7.35\times10^{22}\,\text{kg}\right)^2\left(5.98\times10^{24}\,\text{kg}\right)}n^2$$

$$= \boxed{n^2\left(5.16\times10^{-129}\,\text{m}\right)}$$

$$E_n = -\frac{2\pi^2 e^4 mk^2}{n^2h^2} \quad \rightarrow$$

$$E_n = -\frac{2\pi^2 G^2 m_E^2 m_M^3}{n^2h^2} = -\frac{2\pi^2\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)^2\left(5.98\times10^{24}\,\text{kg}\right)^2\left(7.35\times10^{22}\,\text{kg}\right)^3}{n^2\left(6.626\times10^{-34}\,\text{J}\bullet\text{s}\right)^2}$$

$$= \boxed{-\frac{2.84\times10^{165}\,\text{J}}{n^2}}$$

(*b*) We insert the known masses and Earth–Moon distance into the Bohr radius equation to determine the Bohr state.

$$n = \sqrt{\frac{4\pi^2 Gm_M^2 m_E r_n}{h^2}}$$

$$= \sqrt{\frac{4\pi^2\left(6.67\times10^{-11}\ \text{Nm}^2/\text{kg}^2\right)\left(7.35\times10^{22}\ \text{kg}\right)^2\left(5.98\times10^{24}\ \text{kg}\right)\left(3.84\times10^{8}\ \text{m}\right)}{\left(6.626\times10^{-34}\ \text{Js}\right)^2}}$$

$$= 2.73\times10^{68}$$

Since $n \approx 10^{68}$, a value of $\Delta n = 1$ is negligible compared to *n*. Hence the quantization of energy and radius is $\boxed{\text{not apparent.}}$

94. We use Eqs. 36-13, 36-11, and 37-7 to derive the expression.

$$p^2c^2 + m^2c^4 = E^2 \ ;\ E = K + mc^2 \ \rightarrow\ p^2c^2 + m^2c^4 = \left(K + mc^2\right)^2 = K^2 + 2mc^2K + m^2c^4 \ \rightarrow$$

$$K^2 + 2mc^2K = p^2c^2 = \frac{h^2c^2}{\lambda^2} \ \rightarrow\ \lambda^2 = \frac{h^2c^2}{\left(K^2 + 2mc^2K\right)} \ \rightarrow\ \boxed{\lambda = \frac{hc}{\sqrt{K^2 + 2mc^2K}}}$$

95. As light leaves the flashlight it gains momentum. This change in momentum is given by Eq. 31-20. Dividing the change in momentum by the elapsed time gives the force the flashlight must apply to the light to produce this momentum. This is equal to the reaction force that light applies to the flashlight.

$$\frac{\Delta p}{\Delta t} = \frac{\Delta U}{c\Delta t} = \frac{P}{c} = \frac{3.0\,\text{W}}{3.00\times10^8\,\text{m/s}} = \boxed{1.0\times10^{-8}\,\text{N}}$$

96. (*a*) Since $f = c/\lambda$, the energy of each emitted photon is $E = hc/\lambda$. We insert the values for *h* and *c* and convert the resulting units to $\text{eV}\bullet\text{nm}$.

$$E = \frac{hc}{\lambda} = \frac{\left(6.626\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(2.998\times10^8\,\text{m/s}\right)}{\lambda}\frac{\left(1\,\text{eV}/1.602\times10^{-19}\,\text{J}\right)}{\left(10^{-9}\text{m}/1\,\text{nm}\right)} = \boxed{\frac{1240\ \text{eV}\bullet\text{nm}}{\lambda\left(\text{in nm}\right)}}$$

(*b*) Insert 650 nm into the above equation.

$$E = \frac{1240\ \text{eV}\bullet\text{nm}}{650\ \text{nm}} = \boxed{1.9\ \text{eV}}$$

97. (*a*) We write the Planck time as $t_P = G^{\alpha}h^{\beta}c^{\gamma}$, and the units of t_P must be $[T]$.

$$t_P = G^{\alpha}h^{\beta}c^{\gamma} \rightarrow [T] = \left[\frac{L^3}{MT^2}\right]^{\alpha}\left[\frac{ML^2}{T}\right]^{\beta}\left[\frac{L}{T}\right]^{\gamma} = [L]^{3\alpha+2\beta+\gamma}[M]^{\beta-\alpha}[T]^{-2\alpha-\beta-\gamma}$$

There are no mass units in $[T]$, and so $\beta = \alpha$, and $[T] = [L]^{5\alpha+\gamma}[T]^{-3\alpha-\gamma}$. There are no length units in $[T]$, and so $\gamma = -5\alpha$ and $[T] = [T]^{-3\alpha+5\alpha} = [T]^{2\alpha}$. Thus $\alpha = \frac{1}{2} = \beta$ and $\gamma = -\frac{5}{2}$.

$$\boxed{t_P = G^{1/2}h^{1/2}c^{-5/2} = \sqrt{\frac{Gh}{c^5}}}$$

(*b*) $$t_P = \sqrt{\frac{Gh}{c^5}} = \sqrt{\frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(6.63\times10^{-34}\ \text{J}\bullet\text{s})}{(3.00\times10^8\ \text{m/s})^5}} = \boxed{1.35\times10^{-43}\text{s}}$$

(*c*) We write the Planck length as $\lambda_P = G^{\alpha}h^{\beta}c^{\gamma}$, and the units of λ_P must be $[L]$.

$$\lambda_P = G^{\alpha}h^{\beta}c^{\gamma} \rightarrow [L] = \left[\frac{L^3}{MT^2}\right]^{\alpha}\left[\frac{ML^2}{T}\right]^{\beta}\left[\frac{L}{T}\right]^{\gamma} = [L]^{3\alpha+2\beta+\gamma}[M]^{\beta-\alpha}[T]^{-2\alpha-\beta-\gamma}$$

There are no mass units in $[L]$, and so $\beta = \alpha$, and $[L] = [L]^{5\alpha+\gamma}[T]^{-3\alpha-\gamma}$. There are no time units in $[L]$, and so $\gamma = -3\alpha$ and $[L] = [L]^{5\alpha-3\alpha} = [L]^{2\alpha}$. Thus $\alpha = \frac{1}{2} = \beta$ and $\gamma = -\frac{3}{2}$.

$$\boxed{t_P = G^{1/2}h^{1/2}c^{-3/2} = \sqrt{\frac{Gh}{c^3}}}$$

(*d*) $$\lambda_P = \sqrt{\frac{Gh}{c^3}} = \sqrt{\frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(6.63\times10^{-34}\ \text{J}\bullet\text{s})}{(3.00\times10^8\ \text{m/s})^5}} = \boxed{4.05\times10^{-35}\text{m}}$$

98. For standing matter waves, there are nodes at the two walls. For the ground state (first harmonic), the wavelength is twice the distance between the walls, or $\ell = \frac{1}{2}\lambda$ (see Figure 15-26b). We use Eq. 37-7 to find the velocity and then the kinetic energy.

$$\ell = \tfrac{1}{2}\lambda \rightarrow \lambda = 2\ell \ ; \ p = \frac{h}{\lambda} = \frac{h}{2\ell} \ ; \ K = \frac{p^2}{2m} = \frac{1}{2m}\left(\frac{h}{2\ell}\right)^2 = \boxed{\frac{h^2}{8m\ell^2}}$$

For the second harmonic, the distance between the walls is a full wavelength, and so $\ell = \lambda$.

$$\ell = \lambda \rightarrow p = \frac{h}{\lambda} = \frac{h}{\ell} \ ; \ K = \frac{p^2}{2m} = \frac{1}{2m}\left(\frac{h}{\ell}\right)^2 = \boxed{\frac{h^2}{2m\ell^2}}$$

99. (*a*) Apply conservation of momentum before and after the emission of the photon to determine the recoil speed of the atom, where the momentum of the photon is given by Eq. 37-7.

$$0=\frac{h}{\lambda}-mv \quad\to\quad v=\frac{h}{m\lambda}=\frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)}{85\left(1.66\times10^{-27}\,\text{kg}\right)\left(780\times10^{-9}\,\text{m}\right)}=\boxed{6.0\times10^{-3}\,\text{m/s}}$$

(*b*) We solve Eq. 18-5 for the lowest achievable temperature, where the recoil speed is the rms speed of the rubidium gas.

$$v=\sqrt{\frac{3kT}{m}} \quad\to\quad T=\frac{mv^2}{3k}=\frac{85\left(1.66\times10^{-27}\,\text{kg}\right)\left(6.0\times10^{-3}\,\text{m/s}\right)^2}{3\left(1.38\times10^{-23}\,\text{J/K}\right)}=1.2\times10^{-7}\,\text{K}=\boxed{0.12\,\mu\text{K}}$$

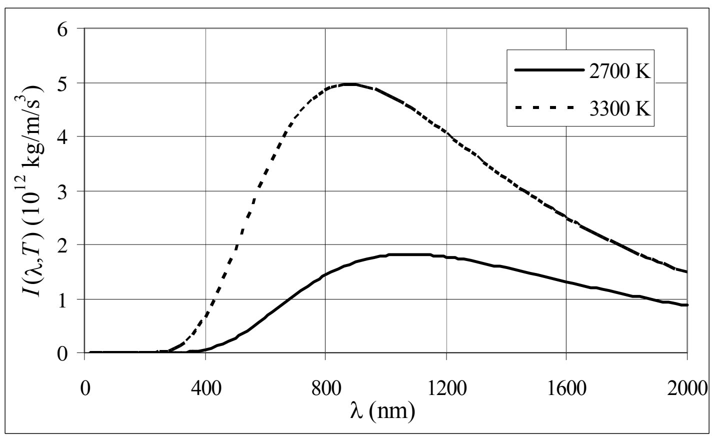
100. Each time the rubidium atom absorbs a photon its momentum decreases by the momentum of the photon. Dividing the initial momentum of the rubidium atom by the momentum of the photon, Eq. 37-7, gives the number of collisions necessary to stop the atom. Multiplying the number of collisions by the absorption time, 25 ns per absorption, provides the time to completely stop the atom.

$$n=\frac{mv}{h/\lambda}=\frac{mv\lambda}{h}=\frac{\left(8\,\text{u}\right)\left(1.66\times10^{-27}\,\text{kg/u}\right)\left(290\,\text{m/s}\right)\left(780\times10^{-9}\,\text{m}\right)}{6.63\times10^{-34}\,\text{J}\bullet\text{s}}=48{,}140$$

$$T=48{,}140\left(25\,\text{ns}\right)=\boxed{1.2\,\text{ms}}$$

101. (*a*) See the adjacent graphs.

(*b*) To compare the intensities, the two graphs are numerically integrated from 400 nm to 760 nm, which is approximately the range of wavelengths for visible light. The result of those integrations is that the higher temperature bulb is about $\boxed{\text{4.8 times more intense}}$ than the lower temperature bulb.



The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH37.XLS," on tab "Problem 37.101."

102. Planck's radiation formula $I(\lambda,T)$ was calculated for a temperature of 6000 K, for wavelengths from 20 nm to 2000 nm. A plot of those calculations is in the spreadsheet for this problem. To estimate the % of emitted sunlight that is in the visible, this ratio was calculated by numeric integration. The details are in the spreadsheet.

$$\%\,\text{visible}=\frac{\displaystyle\int_{400\,\text{nm}}^{700\,\text{nm}} I(\lambda,T)\,d\lambda}{\displaystyle\int_{20\,\text{nm}}^{2000\,\text{nm}} I(\lambda,T)\,d\lambda}=\boxed{0.42}$$

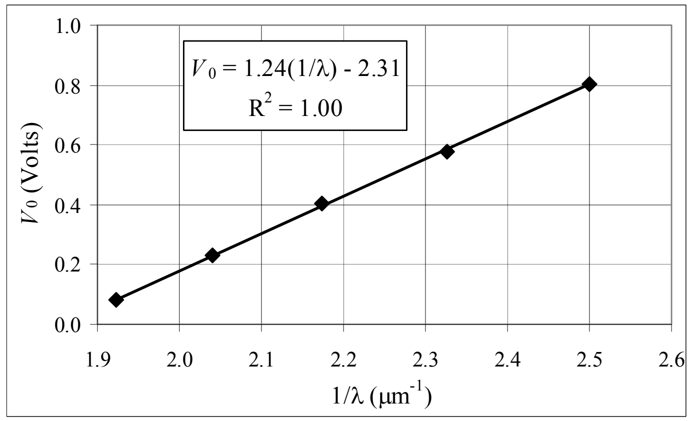
So our estimate is that 42% of emitted sunlight is in the visible wavelengths. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH37.XLS," on tab "Problem 37.102."

103. (*a*) For the photoelectric effect experiment, Eq. 37-4b can be expressed as $K_{max} = hf - W_0$. The maximum kinetic energy is equal to the potential energy associated with the stopping voltage, so $K_{max} = eV_0$. We also have $f = c/\lambda$. Combine those relationships as follows.

$$K_{max} = hf - W_0 \quad \rightarrow \quad eV_0 = \frac{hc}{\lambda} - W_0 \quad \rightarrow \quad V_0 = \frac{hc}{e}\frac{1}{\lambda} - \frac{W_0}{e}$$

A plot of V_0 vs. $\frac{1}{\lambda}$ should yield a straight line with a slope of $\boxed{\frac{hc}{e}}$ and a y-intercept of $\boxed{-\frac{W_0}{e}}$.

(*b*) The graph is shown, with a linear regression fit as given by Excel.



(*c*) The slope is $a = \frac{hc}{e} = 1.24\,\text{V}\bullet\mu\text{m}$, and the y-intercept is $b = -2.31\,\text{V}$.

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH37.XLS," on tab "Problem 37.103."

(*d*) $b = -\frac{W_0}{e} = -2.31\,\text{V} \quad \rightarrow \quad W_0 = \boxed{2.31\,\text{eV}}$

(*e*) $h = \frac{ea}{c} = \frac{(1.60\times10^{-19}\,\text{C})(1.24\times10^{-6}\,\text{V}\bullet\text{m})}{3.00\times10^{8}\,\text{m/s}} = \boxed{6.61\times10^{-34}\,\text{J}\bullet\text{s}}$