CHAPTER 35: Diffraction and Polarization

Responses to Questions

1. Radio waves have a much longer wavelength than visible light and will diffract around normal-sized objects (like hills). The wavelengths of visible light are very small and will not diffract around normal-sized objects.

2. You see a pattern of dark and bright lines parallel to your fingertips in the narrow opening between your fingers.

3. Light from all points of an extended source produces diffraction patterns, and these many different diffraction patterns overlap and wash out each other so that no distinct pattern can be easily seen. When using white light, the diffraction patterns of the different wavelengths will overlap because the locations of the fringes depend on wavelength. Monochromatic light will produce a more distinct diffraction pattern.

4. (*a*) If the slit width is increased, the diffraction pattern will become more compact.
   (*b*) If the wavelength of the light is increased, the diffraction pattern will spread out.

5. (*a*) A slit width of 50 nm would produce a central maximum so spread out that it would cover the entire width of the screen. No minimum (and therefore no diffraction pattern) will be seen. The different wavelengths will all overlap, so the light on the screen will be white. It will also be dim, compared to the source, because it is spread out.
   (*b*) For the 50,000 nm slit, the central maximum will be very narrow, about a degree in width for the blue end of the spectrum and about a degree and a half for the red. The diffraction pattern will not be distinct, because most of the intensity will be in the small central maximum and the fringes for the different wavelengths of white light will not coincide.

6. (*a*) If the apparatus is immersed in water, the wavelength of the light will decrease $\left(\lambda' = \frac{\lambda}{n}\right)$ and the diffraction pattern will become more compact.
   (*b*) If the apparatus is placed in a vacuum, the wavelength of the light will increase slightly, and the diffraction pattern will spread out very slightly.

7. The intensity pattern is actually a function of the form $\left(\frac{\sin x}{x}\right)^2$ (see equations 35-7 and 35-8). The maxima of this function do not coincide exactly with the maxima of $\sin^2 x$. You can think of the intensity pattern as the combination of a $\sin^2 x$ function and a $1/x^2$ function, which forces the intensity function to zero and shifts the maxima slightly.

8. Similarities: Both have a regular pattern of light and dark fringes. The angular separation of the fringes is proportional to the wavelength of the light, and inversely proportional to the slit size or slit separation. Differences: The single slit diffraction maxima decrease in brightness from the center. Maxima for the double slit interference pattern would be equally bright (ignoring single slit effects) and are equally spaced.

9. No. D represents the slit width and d the distance between the centers of the slits. It is possible for the distance between the slit centers to be greater than the width of the slits; it is not possible for the distance between the slit centers to be less than the width of the slits.

10. (*a*) Increasing the wavelength, λ, will spread out the diffraction pattern, since the locations of the minima are given by $\sin\theta = m\lambda/D$. The interference pattern will also spread out; the interference maxima are given by $\sin\theta = m\lambda/d$. The number of interference fringes in the central diffraction maximum will not change.
    (*b*) Increasing the slit separation, *d*, will decrease the spacing between the interference fringes without changing the diffraction, so more interference maxima will fit in the central maximum of the diffraction envelope.
    (*c*) Increasing the slit width, *D*, will decrease the angular width of the diffraction central maximum without changing the interference fringes, so fewer bright fringes will fit in the central maximum.

11. Yes. As stated in Section 35-5, "It is not possible to resolve detail of objects smaller than the wavelength of the radiation being used."

12. Yes. Diffraction effects will occur for both real and virtual images.

13. A large mirror has better resolution and gathers more light than a small mirror.

14. No. The resolving power of a lens is on the order of the wavelength of the light being used, so it is not possible to resolve details smaller than the wavelength of the light. Atoms have diameters of about 10^{-8} cm and the wavelength of visible light is on the order of 10^{-5} cm.

15. Violet light would give the best resolution in a microscope, because the wavelengths are shortest.

16. Yes. (See the introduction to Section 35-7.) The analysis for a diffraction grating of many slits is essentially the same as for Young's double slit interference. However, the bright maxima of a multiple-slit grating are much sharper and narrower than those in a double-slit pattern.

17. The answer depends on the slit spacing of the grating being used. If the spacing is small enough, only the first order will appear so there will not be any overlap. For wider slit spacing there can be overlap. If there is overlap, it will be the higher orders of the shorter wavelength light overlapping with lower orders of the longer wavelength light. See, for instance, Example 35-9, which shows the overlap of the third order blue light with the second order red light.

18. The bright lines will coincide, but those for the grating will be much narrower with wider dark spaces in between. The grating will produce a much sharper pattern.

19. (*a*) Violet light will be at the top of the rainbow created by the diffraction grating. Principal maxima for a diffraction grating are at positions given by $\sin\theta = \frac{m\lambda}{d}$. Violet light has a shorter wavelength than red light and so will appear at a smaller angle away from the direction of the horizontal incident beam.
    (*b*) Red light will appear at the top of the rainbow created by the prism. The index of refraction for violet light in a given medium is slightly greater than for red light in the same medium, and so the violet light will bend more and will appear farther from the direction of the horizontal incident beam.

20. The tiny peaks are produced when light from some but not all of the slits interferes constructively. The peaks are tiny because light from only some of the slits interferes constructively.

21. Polarization demonstrates the transverse wave nature of light, and cannot be explained if light is considered only as particles.

22. Take the sunglasses outside and look up at the sky through them. Rotate the sunglasses (about an axis perpendicular to the lens) through at least 180°. If the sky seems to lighten and darken as you rotate the sunglasses, then they are polarizing. You could also look at a liquid crystal display or reflections from the floor while rotating the glasses, or put one pair of glasses on top of the other and rotate them. If what you see through the glasses changes as you rotate them, then the glasses are polarizing.

23. Black. If there were no atmosphere, there would be no scattering of the sunlight coming to Earth.

Solutions to Problems

1. We use Eq. 35-1 to calculate the angular distance from the middle of the central peak to the first minimum. The width of the central peak is twice this angular distance.

$$\sin\theta_1 = \frac{\lambda}{D} \to \theta_1 = \sin^{-1}\left(\frac{\lambda}{D}\right) = \sin^{-1}\left(\frac{680\times10^{-9}\,\text{m}}{0.0365\times10^{-3}\,\text{m}}\right) = 1.067°$$

$$\Delta\theta = 2\theta_1 = 2(1.067°) = \boxed{2.13°}$$

2. The angle from the central maximum to the first dark fringe is equal to half the width of the central maximum. Using this angle and Eq. 35-1, we calculate the wavelength used.

$$\theta_1 = \tfrac{1}{2}\Delta\theta = \tfrac{1}{2}(32°) = 16°$$

$$\sin\theta_1 = \frac{\lambda}{D} \to \lambda = D\sin\theta_1 = \left(2.60\times10^{-3}\ \text{mm}\right)\ \sin(16°) = 7.17\times10^{-4}\ \text{mm} = \boxed{717\ \text{nm}}$$

3. The angle to the first maximum is about halfway between the angles to the first and second minima. We use Eq. 35-2 to calculate the angular distance to the first and second minima. Then we average these to values to determine the approximate location of the first maximum. Finally, using trigonometry, we set the linear distance equal to the distance to the screen multiplied by the tangent of the angle.

$$D\sin\theta_m = m\lambda \to \theta_m = \sin^{-1}\left(\frac{m\lambda}{D}\right)$$

$$\theta_1 = \sin^{-1}\left(\frac{1\times580\times10^{-9}\ \text{m}}{3.8\times10^{-6}\ \text{m}}\right) = 8.678° \qquad \theta_2 = \sin^{-1}\left(\frac{2\times580\times10^{-9}\ \text{m}}{3.8\times10^{-6}\ \text{m}}\right) = 17.774°$$

$$\theta = \frac{\theta_1+\theta_2}{2} = \frac{8.678°+17.774°}{2} = 13.23°$$

$$y = \ell\tan\theta_1 = (10.0\ \text{m})\tan(13.23°) = \boxed{2.35\ \text{m}}$$

4. (*a*) We use Eq. 35-2, using m=1,2,3,… to calculate the possible diffraction minima, when the wavelength is 0.50 cm.

$$D\sin\theta_m = m\lambda \to \theta_m = \sin^{-1}\left(\frac{m\lambda}{D}\right)$$

$$\theta_1 = \sin^{-1}\left(\frac{1\times0.50\ \text{cm}}{1.6\ \text{cm}}\right) = 18.2° \qquad \theta_2 = \sin^{-1}\left(\frac{2\times0.50\ \text{cm}}{1.6\ \text{cm}}\right) = 38.7°$$

$$\theta_3 = \sin^{-1}\left(\frac{3\times 0.50\text{ cm}}{1.6\text{ cm}}\right) = 69.6° \qquad \theta_4 = \sin^{-1}\left(\frac{4\times 0.50\text{ cm}}{1.6\text{ cm}}\right) \rightarrow \text{ no solution}$$

There are three diffraction minima: $\boxed{18°, 39°, \text{and } 70°.}$

(*b*) We repeat the process from part (*a*) using a wavelength of 1.0 cm.

$$\theta_1 = \sin^{-1}\left(\frac{1\times 1.0\text{ cm}}{1.6\text{ cm}}\right) = 38.7° \qquad \theta_2 = \sin^{-1}\left(\frac{2\times 1.0\text{ cm}}{1.6\text{ cm}}\right) = \text{no real solution}$$

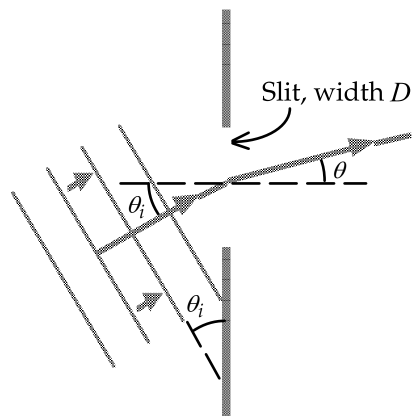
The only diffraction minima is at $\boxed{39°}$

(*c*) We repeat the process from part (*a*) using a wavelength of 3.0 cm.

$$\theta_1 = \sin^{-1}\left(\frac{1\times 3.0\text{ cm}}{1.6\text{ cm}}\right) = \text{no real solution}$$

There are $\boxed{\text{no diffraction minima}}$.

5. The path-length difference between the top and bottom of the slit for the incident wave is $D \sin \theta_i$. The path-length difference between the top and bottom of the slit for the diffracted wave is $D \sin \theta$. When the net path-length difference is equal to a multiple of the wavelength, there will be an even number of segments of the wave having a path-length difference of $\lambda/2$. We set the path-length difference equal to m (an integer) times the wavelength and solve for the angle of the diffraction minimum.



$$D\sin\theta_i - D\sin\theta = m\lambda \quad \rightarrow$$

$$\boxed{\sin\theta = \sin\theta_i - \frac{m\lambda}{D}, \quad m = \pm 1, \pm 2, ...}$$

From this equation we see that when $\theta = 23.0°$, the minima will be symmetrically distributed around a central maximum at 23.0°

6. The angle from the central maximum to the first bright maximum is half the angle between the first bright maxima on either side of the central maximum. The angle to the first maximum is about halfway between the angles to the first and second minima. We use Eq. 35-2, setting $m = 3/2$, to calculate the slit width, D.

$$\theta_1 = \tfrac{1}{2}\Delta\theta = \tfrac{1}{2}(35°) = 17.5°$$

$$D\sin\theta_m = m\lambda \rightarrow D = \frac{m\lambda}{\sin\theta_1} = \frac{(3/2)(633\text{ nm})}{\sin 17.5°} = 3157.6\text{ nm} \approx \boxed{3.2\,\mu\text{m}}$$

$\boxed{7.}$ We use the distance to the screen and half the width of the diffraction maximum to calculate the angular distance to the first minimum. Then using this angle and Eq. 35-1 we calculate the slit width. Then using the slit width and the new wavelength we calculate the angle to the first minimum and the width of the diffraction maximum.

$$\tan\theta_1 = \frac{\left(\tfrac{1}{2}\Delta y_1\right)}{\ell} \quad \rightarrow \quad \theta_1 = \tan^{-1}\frac{\left(\tfrac{1}{2}\Delta y_1\right)}{\ell} = \tan^{-1}\frac{\left(\tfrac{1}{2}\times 0.06\,\text{m}\right)}{2.20\,\text{m}} = 0.781°$$

$$\sin\theta_1 = \frac{\lambda_1}{D} \quad \rightarrow \quad D = \frac{\lambda_1}{\sin\theta_1} = \frac{580\,\text{nm}}{\sin 0.781°} = 42{,}537\,\text{nm}$$

$$\sin\theta_2 = \frac{\lambda_2}{D} \rightarrow \theta_2 = \sin^{-1}\left(\frac{\lambda_2}{D}\right) = \sin^{-1}\left(\frac{460\,\text{nm}}{42{,}537\,\text{nm}}\right) = 0.620°$$

$$\Delta y_2 = 2\ell\tan\theta_2 = 2(2.20\,\text{m})\tan(0.620°) = 0.0476\,\text{m} \approx \boxed{4.8\,\text{cm}}$$

8. (*a*) There will be no diffraction minima if the angle for the first minimum is greater than 90°. We set the angle in Eq. 35-1 equal to 90° and solve for the slit width.

$$\sin\theta = \frac{\lambda}{D} \rightarrow D = \frac{\lambda}{\sin 90°} = \boxed{\lambda}$$

(*b*) For no visible light to exhibit a diffraction minimum, the slit width must be equal to the shortest visible wavelength.

$$D = \lambda_{\text{min}} = \boxed{400\text{ nm}}.$$

9. We set the angle to the first minimum equal to half of the separation angle between the dark bands. We insert this angle into Eq. 35-1 to solve for the slit width.

$$\theta = \tfrac{1}{2}\Delta\theta = \tfrac{1}{2}(55.0°) = 27.5°$$

$$\sin\theta = \frac{\lambda}{D} \rightarrow D = \frac{\lambda}{\sin\theta} = \frac{440\,\text{nm}}{\sin 27.5°} = \boxed{953\,\text{nm}}$$

10. We find the angle to the first minimum using Eq. 35-1. The distance on the screen from the central maximum is found using the distance to the screen and the tangent of the angle. The width of the central maximum is twice the distance from the central maximum to the first minimum.

$$\sin\theta_1 = \frac{\lambda}{D} \rightarrow \theta_1 = \sin^{-1}\left(\frac{\lambda}{D}\right) = \sin^{-1}\left(\frac{450\times10^{-9}\,\text{m}}{1.0\times10^{-3}\,\text{m}}\right) = \underline{\underline{0.02578°}}$$

$$y_1 = \ell\tan\theta_1 = (5.0\text{ m})\tan 0.02578° = 0.00225\text{ m}$$

$$\Delta y = 2y_1 = 2(0.00225\text{ m}) = 0.0045\text{ m} = \boxed{0.45\text{ cm}}$$

11. (*a*) For vertical diffraction we use the height of the slit (1.5 μm) as the slit width in Eq. 35-1 to calculate the angle between the central maximum to the first minimum. The angular separation of the first minima is equal to twice this angle.

$$\sin\theta_1 = \frac{\lambda}{D} \rightarrow \theta_1 = \sin^{-1}\frac{\lambda}{D} = \sin^{-1}\frac{780\times10^{-9}\,\text{m}}{1.5\times10^{-6}\,\text{m}} = 31.3°$$

$$\Delta\theta = 2\theta_1 = 2(31.3°) \approx \boxed{63°}$$

(*b*) To find the horizontal diffraction we use the width of the slit (3.0 μm) in Eq. 35-1.

$$\sin\theta_1 = \frac{\lambda}{D} \rightarrow \theta_1 = \sin^{-1}\frac{\lambda}{D} = \sin^{-1}\frac{780\times10^{-9}\,\text{m}}{3.0\times10^{-6}\,\text{m}} = 15.07°$$

$$\Delta\theta = 2\theta_1 = 2(15.07°) \approx \boxed{30°}$$

12. (*a*) If we consider the slit made up of *N* wavelets each of amplitude E_0, the total amplitude at the central maximum, where they are all in phase, is NE_0. Doubling the size of the slit doubles the number of wavelets and thus the total amplitude of the electric field. Because the intensity is proportional to the square of the electric field amplitude, the intensity at the central maximum is increased by a factor of 4.

$$I \propto E^2 = (2E_0)^2 = 4E_0^2 \propto \boxed{4I_0}$$

(*b*) From Eq. 35-1 we see that, for small angles, the width of the central maximum is inversely proportional to the slit width. Therefore doubling the slit width will cut the area of the central peak in half. Since the intensity is spread over only half the area, where the intensity is four times the initial intensity, the average intensity (or energy) over the central maximum has doubled. This is true for all fringes, so when the slit width is doubled, allowing twice the energy to pass through the slit, the average energy within each slit will also double, in accord with the conservation of energy.

13. We use Eq. 35-8 to calculate the intensity, where the angle θ is found from the displacement from the central maximum (15 cm) and the distance to the screen.

$$\tan\theta = \frac{y}{\ell} \rightarrow \theta = \tan^{-1}\left(\frac{15\,\text{cm}}{25\,\text{cm}}\right) = 31.0^\circ$$

$$\beta = \frac{2\pi}{\lambda}D\sin\theta = \frac{2\pi}{\left(750\times10^{-9}\,\text{m}\right)}\left(1.0\times10^{-6}\,\text{m}\right)\sin 31.0^\circ = 4.31\,\text{rad}$$

$$\frac{I}{I_0} = \left(\frac{\sin\beta/2}{\beta/2}\right)^2 = \left(\frac{\sin\left(4.31\,\text{rad}/2\right)}{4.31\,\text{rad}/2}\right)^2 = 0.1498 \approx \boxed{0.15}$$

So the light intensity at 15 cm is about 15% of the maximum intensity.

14. (*a*) The secondary maxima do not occur precisely where $\sin(\beta/2)$ is a maximum, that is at $\beta/2 = (m+\frac{1}{2})\pi$ where $m = 1,2,3,\ldots,$ because the diffraction intensity (Eq. 35-7) is the ratio of the sine function and $\beta/2$. Near the maximum of the sine function, the denominator of the intensity function causes the intensity to decrease more rapidly than the sine function causes it to increase. This results in the intensity reaching a maximum slightly before the sine function reaches its maximum.

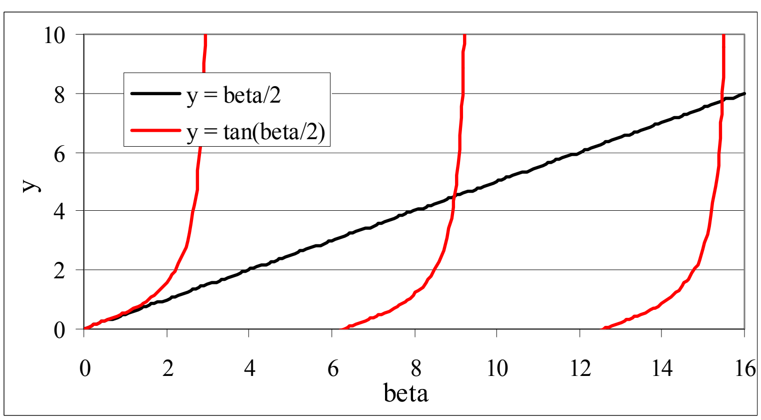
(*b*) We set the derivative of Eq. 35-7 with respect to β equal to zero to determine the intensity extrema.

$$0 = \frac{dI}{d\beta} = \frac{d}{d\beta}I_o\left[\frac{\sin(\beta/2)}{\beta/2}\right]^2 = 2I_o\left[\frac{\sin(\beta/2)}{\beta/2}\right]\left[\frac{\cos(\beta/2)}{\beta} - \frac{\sin(\beta/2)}{\beta^2/2}\right]$$

When the first term in brackets is zero, the intensity is a minimum, so the intensity is a maximum when the second term in brackets is zero.

$$0 = \frac{\cos(\beta/2)}{\beta} - \frac{\sin(\beta/2)}{\beta^2/2} \rightarrow \boxed{\beta/2 = \tan(\beta/2)}$$

(*c*) The first and secondary maxima are found where these two curves intersect, or $\boxed{\beta_1 = 8.987}$ and $\boxed{\beta_2 = 15.451.}$ We calculate the percent difference between these and the maxima of the sine curve, $\beta_1' = 3\pi$ and $\beta_2' = 5\pi$.



$$\left.\frac{\Delta\beta}{\beta}\right|_1 = \frac{\beta_1 - \beta_1'}{\beta_1'} = \frac{8.987 - 3\pi}{3\pi} = -0.0464 = \boxed{-4.64\%}$$

$$\left.\frac{\Delta\beta}{\beta}\right|_2 = \frac{15.451 - 5\pi}{5\pi} = -0.0164 = \boxed{-1.64\%}$$

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH35.XLS," on tab "Problem 35.14."

15. If the central diffraction peak contains nine fringes, there will be four fringes on each side of the central peak. Thus the fifth maximum of the double slit must coincide with the first minimum of the diffraction pattern. We use Eq. 34-2a with *m* = 5 to find the angle of the fifth interference maximum and set that angle equal to the first diffraction minimum, given by Eq. 35-1, to solve for the ratio of the slit separation to slit width.

$$d\sin\theta = m\lambda \;\rightarrow\; \sin\theta = \frac{5\lambda}{d} \;\; ; \;\; \sin\theta = \frac{\lambda}{D} = \frac{5\lambda}{d} \;\rightarrow\; \boxed{d = 5D}$$

16. (*a*) If the central diffraction peak is to contain seventeen fringes, there will be eight fringes on each side of the central peak. Thus, the ninth minimum of the double slit must coincide with the first minimum of the diffraction pattern. We use Eq. 34-2b with *m* = 8 to find the angle of the ninth interference minimum and set that angle equal to the first diffraction minimum, given by Eq. 35-1, to solve for the ratio of the slit separation to slit width.

$$d\sin\theta = \left(m + \tfrac{1}{2}\right)\lambda \;\rightarrow\; \sin\theta = \frac{\left(8 + \tfrac{1}{2}\right)\lambda}{d} = \frac{8.5\lambda}{d}$$

$$\sin\theta = \frac{\lambda}{D} = \frac{8.5\lambda}{d} \;\rightarrow\; \boxed{d = 8.5D}$$

Therefore, for the first diffraction minimum to be at the ninth interference minimum, the separation of slits should be 8.5 times the slit width.

(*b*) If the first diffraction minimum is to occur at the ninth interference maximum, we use Eq. 34-2a with *m* = 9 to find the angle of the ninth interference maximum and set that angle equal to the first diffraction minimum, given by Eq. 35-1, to solve for the ratio of the slit separation to slit width.

$$d\sin\theta = m\lambda \;\rightarrow\; \sin\theta = \frac{9\lambda}{d} = \frac{9\lambda}{d} \;\; ; \;\; \sin\theta = \frac{\lambda}{D} = \frac{9\lambda}{d} \;\rightarrow\; \boxed{d = 9D}$$

Therefore, for the first diffraction minimum to be at the ninth interference maximum, the separation of slits should be 9 times the slit width.

17. Given light with $\lambda = 605\text{ nm}$ passing through double slits with separation $d = 0.120\text{ mm}$, we use Eq. 34-2a to find the highest integer m value for the interference fringe that occurs before the angle $\theta = 90^\circ$.

$$d\sin\theta = m\lambda \;\rightarrow\; m = \frac{\left(0.120\times10^{-3}\text{m}\right)\sin 90^{\circ\circ}}{605\times10^{-9}\text{m}} = 198$$

So, including the $m = 0$ fringe, and the symmetric pattern of interference fringes on each side of $\theta = 0$, there are potentially a total of $198 + 198 + 1 = 397$ fringes. However, since slits have width $a = 0.040\text{ mm}$, the potential interference fringes that coincide with the slits' diffraction minima will be absent. Let the diffraction minima be indexed by $m' = 1, 2, 3, \text{etc.}$ We then set the diffraction angles in Eq. 34-2a and Eq. 35-2 equal to solve for the *m* values of the absent fringes.

$$\sin\theta = \frac{m'\lambda}{D} = \frac{m\lambda}{d} \rightarrow \frac{m}{m'} = \frac{d}{D} = \frac{0.120\text{ mm}}{0.040\text{ mm}} = 3 \rightarrow m = 3m'$$

Using $m' = 1, 2, 3, \text{etc.}$, the 66 interference fringes on each side of $\theta = 0$ with $m = 3, 6, 9, \ldots, 198$ will be absent. Thus the number of fringes on the screen is $397 - 2(66) = \boxed{265}$.

18. In a double-slit experiment, if the central diffraction peak contains 13 interference fringes, there is the $m = 0$ fringe, along with fringes up to $m = 6$ on each side of $\theta = 0$. Then, at angle θ, the $m = 7$ interference fringe coincides with the first diffraction minima. We set this angle in Eq. 34-2a and 35-2 equal to solve for the relationship between the slit width and separation.

$$\sin\theta_1 = \frac{m'\lambda}{D} = \frac{m\lambda}{d} \rightarrow \frac{d}{D} = \frac{m}{m'} = \frac{7}{1} = 7 \rightarrow d = 7D$$

Now, we use these equations again to find the m value at the second diffraction minimum, $m' = 0$.

$$\sin\theta_2 = \frac{m'\lambda}{D} = \frac{m\lambda}{d} \rightarrow m = m'\frac{d}{D} = 2\frac{7D}{D} = 14$$

Thus, the $\boxed{\text{six fringes}}$ corresponding to $m = 8$ to $m = 13$ will occur within the first and second diffraction minima.

19. (*a*) The angle to each of the maxima of the double slit are given by Eq. 34-2a. The distance of a fringe on the screen from the center of the pattern is equal to the distance between the slit and screen multiplied by the tangent of the angle. For small angles, we can set the tangent equal to the sine of the angle. The slit spacing is found by subtracting the distance between two adjacent fringes.

$$\sin\theta_m = \frac{m\lambda}{d} \qquad y_m = \ell\tan\theta_m \approx \ell\sin\theta_m = \ell\frac{m\lambda}{d}$$

$$\Delta y = y_{m+1} - y_m = \ell\frac{(m+1)\lambda}{d} - \ell\frac{m\lambda}{d} = \frac{\ell\lambda}{d} = \frac{(1.0\text{ m})(580\times10^{-9}\text{ m})}{0.030\times10^{-3}\text{m}} = 0.019\text{ m} = \boxed{1.9\text{ cm}}$$

(*b*) We use Eq. 35-1 to determine the angle between the center and the first minimum. Then by multiplying the distance to the screen by the tangent of the angle we find the distance from the center to the first minima. The distance between the two first order diffraction minima is twice the distance from the center to one of the minima.

$$\sin\theta_1 = \frac{\lambda}{D} \rightarrow \theta_1 = \sin^{-1}\frac{\lambda}{D} = \sin^{-1}\frac{580\times10^{-9}\text{ m}}{0.010\times10^{-3}\text{m}} = 3.325^\circ$$

$$y_1 = \ell\tan\theta_1 = (1.0\text{ m})\tan 3.325^\circ = 0.0581\text{ m}$$

$$\Delta y = 2y_1 = 2(0.0581\text{ m}) = 0.116\text{ m} \approx \boxed{12\text{ cm}}$$

20. We set $d = D$ in Eqs. 34-4 and 35-6 to show $\beta = \delta$. Replacing δ with β in Eq. 35-9, and using the double angle formula we show that Eq. 35-9 reduces to Eq. 35-7, with $\beta' = 2\beta$. Finally using Eq. 35-6 again, we show that $\beta' = 2\beta$ implies that the new slit width D' is simply double the initial slit width.

$$\delta = \frac{2\pi}{\lambda}d\sin\theta = \frac{2\pi}{\lambda}D\sin\theta = \beta$$

$$I_\theta = I_0\left(\frac{\sin\beta/2}{\beta/2}\right)^2\cos^2(\delta/2) = I_0\frac{\sin^2(\beta/2)\cos^2(\beta/2)}{(\beta/2)^2} = I_0\frac{\frac{1}{4}\sin^2[2(\beta/2)]}{(\beta/2)^2}$$

$$= I_0 \frac{\sin^2 \beta}{\beta^2} = \boxed{I_0 \frac{\sin^2(\beta'/2)}{(\beta'/2)^2}}, \text{ where } \beta' = 2\beta.$$

$$\beta' = \frac{2\pi}{\lambda} D' \sin\theta = 2\left(\frac{2\pi}{\lambda} D \sin\theta\right) \rightarrow \boxed{D' = 2D}$$

21. Using Eq. 34-2a we determine the angle at which the third-order interference maximum occurs. Then we use Eq. 35-9 to determine the ratio of the intensity of the third-order maximum, where β is given by Eq. 35-6 and δ is given by Eq. 34-4.

$$d \sin\theta = m\lambda \Rightarrow \theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left(\frac{3\lambda}{40.0\lambda}\right) = 4.301°$$

$$\frac{\beta}{2} = \frac{2\pi}{2\lambda} D \sin\theta = \frac{\pi(40.0\lambda/5)}{\lambda} \sin(4.301°) = 1.885 \text{ rad}$$

$$\frac{\delta}{2} = \frac{2\pi d}{2\lambda} \sin\theta = \frac{\pi(40.0\lambda)}{\lambda} \sin(4.301°) = 9.424 \text{ rad}$$

$$I = I_o \left[\frac{\sin(\beta/2)}{\beta/2}\right]^2 \left[\cos\left(\frac{\delta}{2}\right)\right]^2 = I_o \left[\frac{\sin(1.885 \text{ rad})}{1.885 \text{ rad}}\right]^2 \left[\cos(9.424)\right]^2 = \boxed{0.255 I_o}$$

22. We use Eq. 34-2a to determine the order of the double slit maximum that corresponds to the same angle as the first order single slit minimum, from Eq. 35-1. Since this double slit maximum is darkened, inside the central diffraction peak, there will be the zeroth order fringe and on either side of the central peak a number of maximum equal to one less than the double slit order. Therefore, there will be $2(m-1)+1$, or $2m-1$ fringes.

$$d \sin\theta = m\lambda \rightarrow m = \frac{d \sin\theta}{\lambda} = \frac{d}{\lambda}\left(\frac{\lambda}{D}\right) = \frac{d}{D} \ ; \ N = 2m - 1 = 2\frac{d}{D} - 1$$

(*a*) We first set the slit separation equal to twice the slit width, $d = 2.00\,D$.

$$N = 2\frac{2.00D}{D} - 1 = \boxed{3}$$

(*b*) Next we set $d = 12.0\,D$.

$$N = 2\frac{12.00D}{D} - 1 = \boxed{23}$$

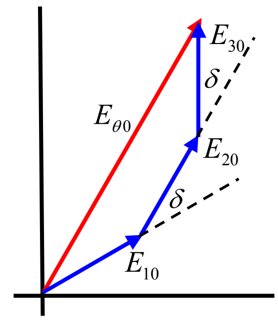
(*c*) For the previous two parts, the ratio of slits had been an integer value. This corresponded to the single slit minimum overlapping the double slit maximum. Now that $d = 4.50\,D$, the single slit minimum overlaps a double slit minimum. Therefore, the last order maximum, $m = 4$, is not darkened and $N = 2m + 1$.

$$N = 2m + 1 = 2(4) + 1 = \boxed{9}$$

(*d*) In this case the ratio of the slit separation to slit width is not an integer, nor a half-integer value. The first order single-slit minimum falls between the seventh order maximum and the seventh order minimum. Therefore, the seventh order maximum will partially be seen as a fringe.

$$N = 2m + 1 = 2(7) + 1 = \boxed{15}$$

23. (*a*) If $D \approx \lambda$, the central maximum of the diffraction pattern will be very wide. Thus we need consider only the interference between slits. We construct a phasor diagram for the interference, with $\delta = \frac{2\pi}{\lambda} d\sin\theta$ as the phase difference between adjacent slits. The magnitude of the electric fields of the slits will have the same magnitude, $E_{10} = E_{20} = E_{30} = E_0$. From the symmetry of the phasor diagram we see that $\phi = \delta$. Adding the three electric field vectors yields the net electric field.



$$E_{\theta 0} = E_{10}\cos\delta + E_{20} + E_{30}\cos\delta = E_0\left(1+2\cos\delta\right)$$

The central peak intensity occurs when $\delta = 0$. We set the intensity proportional to the square of the electric field and calculate the ratio of the intensities.

$$\frac{I_\theta}{I_0} = \frac{E_{\theta 0}^2}{E_{00}^2} = \frac{E_0^2\left(1+2\cos\delta\right)^2}{E_0^2\left(1+2\cos 0\right)^2} = \boxed{\frac{\left(1+2\cos\delta\right)^2}{9}}$$

(*b*) We find the locations of the maxima and minima by setting the first derivative of the intensity equal to zero.

$$\frac{dI_\theta}{d\delta} = \frac{d}{d\delta}\frac{I_0}{9}\left(1+2\cos\delta\right)^2 = \frac{2I_0}{9}\left(1+2\cos\delta\right)\left(-2\sin\delta\right) = 0$$

This equation is satisfied when either of the terms in parentheses is equal to zero. When $1+2\cos\delta = 0$, the intensity equals zero and is a minimum.

$$1+2\cos\delta = 0 \to \delta = \cos^{-1}\left(-\tfrac{1}{2}\right) = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}, \ldots$$

Maxima occur for $\sin\delta = 0$, which also says $\cos\delta = \pm 1$.

$$\sin\delta = 0 \to \delta = \sin^{-1}0 = 0, \pi, 2\pi, 3\pi, \ldots$$

When $\cos\delta = 1$, the intensity is a principal maximum. When $\cos\delta = -1$, the intensity is a secondary maximum.

$$I_\theta(0) = I_0\frac{\left(1+2\cos\delta\right)^2}{9} = I_0\frac{\left(1+2\cos 0\right)^2}{9} = I_0$$

$$I_\theta(\pi) = I_0\frac{\left(1+2\cos\pi\right)^2}{9} = I_0\frac{\left(1+2\left(-1\right)\right)^2}{9} = \frac{I_0}{9}$$

$$I_\theta(2\pi) = I_0\frac{\left(1+2\cos 2\pi\right)^2}{9} = I_0\frac{\left(1+2\right)^2}{9} = I_0$$

Thus we see that, since $\cos\delta$ alternates between +1 and –1, there is only a single secondary maximum between each principal maximum.

24. The angular resolution is given by Eq. 35-10.

$$\theta = 1.22\frac{\lambda}{D} = 1.22\frac{560\times10^{-9}\text{ m}}{254\times10^{-2}\text{ m}} = 2.69\times10^{-7}\text{ rad}\left(\frac{180^\circ}{\pi\text{ rad}}\right)\left(\frac{3600''}{1^\circ}\right) = \boxed{0.055''}$$

$\boxed{25.}$ The angular resolution is given by Eq. 35-10. The distance between the stars is the angular resolution times the distance to the stars from the Earth.

$$\theta = 1.22\frac{\lambda}{D}\ ;\ \ell = r\theta = 1.22\frac{r\lambda}{D} = 1.22\frac{\left(16\text{ly}\right)\left(\frac{9.46\times10^{15}\text{m}}{1\text{ly}}\right)\left(550\times10^{-9}\text{m}\right)}{\left(0.66\text{m}\right)} = \boxed{1.5\times10^{11}\text{m}}$$

26. We find the angle θ subtended by the planet by dividing the orbital radius by the distance of the star to the earth. Then using Eq. 35-10 we calculate the minimum diameter aperture needed to resolve this angle.

$$\theta = \frac{r}{d} = \frac{1.22\lambda}{D} \quad \rightarrow$$

$$D = \frac{1.22\lambda d}{r} = \frac{1.22\left(550\times10^{-9}\,\text{m}\right)\left(4\,\text{ly}\right)\left(9.461\times10^{15}\,\text{m/ly}\right)}{\left(1\,\text{AU}\right)\left(1.496\times10^{11}\,\text{m/AU}\right)} = 0.17\,\text{m} \approx \boxed{20\,\text{cm}}$$

27. We find the angular half-width of the flashlight beam using Eq. 35-10 with $D = 5$ cm and $\lambda = 550$ nm. We set the diameter of the beam equal to twice the radius, where the radius of the beam is equal to the angular half-width multiplied by the distance traveled, 3.84×10^{8} m.

$$\theta = \frac{1.22\lambda}{D} = \frac{1.22\left(550\times10^{-9}\,\text{m}\right)}{0.050\ \text{m}} = 1.3\times10^{-5}\,\text{rad}$$

$$d = 2\left(r\theta\right) = 2\left(3.84\times10^{8}\,\text{m}\right)\left(1.3\times10^{-5}\,\text{rad}\right) = \boxed{1.0\times10^{4}\,\text{m}}$$

28. To find the focal length of the eyepiece we use Eq. 33-7, where the objective focal length is 2.00 m, θ' is the ratio of the minimum resolved distance and 25 cm, and θ is the ratio of the object on the moon and the distance to the moon. We ignore the inversion of the image.

$$\frac{f_o}{f_e} = \frac{\theta'}{\theta} \rightarrow f_e = f_o\frac{\theta}{\theta'} = f_o\frac{\left(d_o/\ell\right)}{\left(d/N\right)} = \left(2.0\ \text{m}\right)\frac{\left(7.5\ \text{km}/384{,}000\ \text{km}\right)}{\left(0.10\ \text{mm}/250\ \text{mm}\right)} = 0.098\ \text{m} = \boxed{9.8\ \text{cm}}$$

We use Eq. 35-10 to determine the resolution limit.

$$\theta = 1.22\frac{\lambda}{D} = 1.22\frac{560\times10^{-9}\ \text{m}}{0.11\ \text{m}} = \boxed{6.2\times10^{-6}\ \text{rad}}$$

This corresponds to a minimum resolution distance, $r = \left(384{,}000\,\text{km}\right)\left(6.2\times10^{-6}\,\text{rad}\right) = \boxed{2.4\,\text{km}}$, which is smaller than the 7.5 km object we wish to observe.

29. We set the resolving power as the focal length of the lens multiplied by the angular resolution, as in Eq. 35-11. The resolution is the inverse of the resolving power.

$$\frac{1}{RP(f/2)} = \left[\frac{1.22\lambda f}{D}\right]^{-1} = \frac{D}{1.22\lambda f} = \frac{25\ \text{mm}}{1.22\left(560\times10^{-6}\,\text{mm}\right)\left(50.0\ \text{mm}\right)} = \boxed{730\ \text{lines/mm}}$$

$$\frac{1}{RP(f/16)} = \frac{3.0\ \text{mm}}{1.22\left(560\times10^{-6}\,\text{mm}\right)\left(50.0\ \text{mm}\right)} = \boxed{88\ \text{lines/mm}}$$

30. We use Eq. 35-13 to calculate the angle for the second order maximum.

$$d\sin\theta = m\lambda \rightarrow \theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left(\frac{2\left(480\times10^{-9}\ \text{m}\right)}{1.35\times10^{-5}\ \text{m}}\right) = \boxed{4.1^\circ}$$

31. We use Eq. 35-13 to calculate the wavelengths from the given angles. The slit separation, *d*, is the inverse of the number of lines per cm, *N*. We assume that 12,000 is good to 3 significant figures.

$$d\sin\theta = m\lambda \rightarrow \lambda = \frac{\sin\theta}{Nm}$$

$$\lambda_1 = \frac{\sin 28.8°}{12{,}000\ /\text{cm}} = 4.01\times10^{-5}\ \text{cm} = \boxed{401\ \text{nm}} \qquad \lambda_2 = \frac{\sin 36.7°}{12{,}000\ /\text{cm}} = 4.98\times10^{-5}\ \text{cm} = \boxed{498\ \text{nm}}$$

$$\lambda_3 = \frac{\sin 38.6°}{12{,}000\ /\text{cm}} = 5.201\times10^{-5}\ \text{cm} = \boxed{520\ \text{nm}} \qquad \lambda_4 = \frac{\sin 47.9°}{12{,}000\ /\text{cm}} = 6.18\times10^{-5}\ \text{cm} = \boxed{618\ \text{nm}}$$

32. We use Eq. 35-13 to find the wavelength, where the number of lines, *N,* is the inverse of the slit separation, or $d=1/N$.

$$d\sin\theta = m\lambda \to \lambda = \frac{\sin\theta}{mN} = \frac{\sin 26.0°}{3(3500\ /\text{cm})} = 4.17\times10^{-5}\text{cm} \approx \boxed{420\,\text{nm}}$$

33. Because the angle increases with wavelength, to have a complete order we use the largest wavelength. We set the maximum angle is 90° to determine the largest integer *m* in Eq. 35-13.

$$d\sin\theta = m\lambda \to m = \frac{\sin\theta}{\lambda N} = \frac{\sin 90°}{(700\times10^{-9}\ \text{m})(6800\ /\text{cm})(100\ \text{cm/m})} = 2.1$$

Thus, $\boxed{\text{two}}$ full spectral orders can be seen on each side of the central maximum, and a portion of the third order.

34. We find the slit separation from Eq. 35-13. Then set the number of lines per centimeter equal to the inverse of the slit separation, $N=1/d$.

$$d\sin\theta = m\lambda \to N = \frac{1}{d} = \frac{\sin\theta}{m\lambda} = \frac{\sin 15.0°}{3(650\times10^{-7}\text{cm})} = \boxed{1300\,\text{lines/cm}}$$

35. Since the same diffraction grating is being used for both wavelengths of light, the slit separation will be the same. We solve Eq. 35-13 for the slit separation for both wavelengths and set the two equations equal. The resulting equation is then solved for the unknown wavelength.

$$d\sin\theta = m\lambda \quad\Rightarrow\quad d = \frac{m_1\lambda_1}{\sin\theta_1} = \frac{m_2\lambda_2}{\sin\theta_2} \quad\Rightarrow\quad \lambda_2 = \frac{m_1}{m_2}\frac{\sin\theta_2}{\sin\theta_1}\lambda_1 = \frac{2}{1}\frac{\sin 20.6°}{\sin 53.2°}(632.8\,\text{nm}) = \boxed{556\,\text{nm}}$$

36. We find the first order angles for the maximum and minimum wavelengths using Eq. 35-13, where the slit separation distance is the inverse of the number of lines per centimeter. Then we set the distance from the central maximum of the maximum and minimum wavelength equal to the distance to the screen multiplied by the tangent of the first order angle. The width of the spectrum is the difference in these distances.

$$d\sin\theta = m\lambda \to \theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}(m\lambda N)$$

$$\theta_1 = \sin^{-1}\left[(410\times10^{-7}\text{cm})(7800\ \text{lines/cm})\right] = 18.65°$$

$$\theta_2 = \sin^{-1}\left[(750\times10^{-7}\text{cm})(7800\ \text{lines/cm})\right] = 35.80°$$

$$\Delta y = y_2 - y_1 = \ell(\tan\theta_2 - \tan\theta_1) = (2.80\ \text{m})(\tan 35.80° - \tan 18.65°) = \boxed{1.1\,\text{m}}$$

$\boxed{37.}$ We find the second order angles for the maximum and minimum wavelengths using Eq. 35-13, where the slit separation distance is the inverse of the number of lines per centimeter. Subtracting these two angles gives the angular width.

$$d\sin\theta = m\lambda \to \theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}(m\lambda N)$$

$$\theta_1 = \sin^{-1}\left[2\left(4.5\times10^{-7}\text{ m}\right)\left(6.0\times10^{5}\text{ /m}\right)\right] = 32.7^\circ$$
$$\theta_2 = \sin^{-1}\left[2\left(7.0\times10^{-7}\text{ m}\right)\left(6.0\times10^{5}\text{ /m}\right)\right] = 57.1^\circ$$
$$\Delta\theta = \theta_2 - \theta_1 = 57.1^\circ - 32.7^\circ = \boxed{24^\circ}$$

38. The $m = 1$ brightness maximum for the wavelength of 1200 nm occurs at angle θ. At this same angle $m = 2$, $m = 3$, etc. brightness maximum will form for other wavelengths. To find these wavelengths, we use Eq. 35-13, where the right hand side of the equation remains constant, and solve for the wavelengths of higher order.

$$d\sin\theta = m_1\lambda_1 = m\lambda_m \Rightarrow \lambda_m = \frac{m_1\lambda_1}{m} = \frac{\lambda_1}{m}$$
$$\lambda_2 = \frac{1200\text{ nm}}{2} = 600\text{ nm} \qquad \lambda_3 = \frac{1200\text{ nm}}{3} = 400\text{ nm} \qquad \lambda_4 = \frac{1200\text{ nm}}{4} = 300\text{ nm}$$

Higher order maxima will have shorter wavelengths. Therefore in the range 360 nm to 2000 nm, the only wavelengths that have a maxima at the angle θ are $\boxed{\text{600 nm and 400 nm}}$ besides the 1200 nm.

39. Because the angle increases with wavelength, we compare the maximum angle for the second order with the minimum angle for the third order, using Eq. 35-13, by calculating the ratio of the sines for each angle. Since this ratio is greater than one, the maximum angle for the second order is larger than the minimum angle for the first order and the spectra overlap.

$$d\sin\theta = m\lambda \quad\rightarrow\quad \sin\theta = \left(\frac{m\lambda}{d}\right) \;;\; \frac{\sin\theta_2}{\sin\theta_3} = \frac{2\lambda_2/d}{3\lambda_3/d} = \frac{2\lambda_2}{3\lambda_3} = \frac{2(700\text{ nm})}{3(400\text{ nm})} = 1.2$$

To determine which wavelengths overlap, we set this ratio of sines equal to one and solve for the second order wavelength that overlaps with the shortest wavelength of the third order. We then repeat this process to find the wavelength of the third order that overlaps with the longest wavelength of the second order.

$$\frac{\sin\theta_2}{\sin\theta_3} = 1 = \frac{2\lambda_2/d}{3\lambda_3/d} = \frac{2\lambda_2}{3\lambda_3} \quad\rightarrow\quad \lambda_3 = \frac{2}{3}\lambda_{2,\max} = \frac{2}{3}(700\text{ nm}) = 467\text{ nm}$$
$$\rightarrow\quad \lambda_2 = \frac{3}{2}\lambda_{3,\min} = \frac{3}{2}(400\text{ nm}) = 600\text{ nm}$$

Therefore, the wavelengths $\boxed{\text{600 nm – 700 nm of the second order}}$ overlap with the wavelengths $\boxed{\text{400 nm – 467 nm of the third order.}}$ Note that these wavelengths are independent of the slit spacing.

40. We set the diffraction angles as one half the difference between the angles on opposite sides of the center. Then we solve Eq. 35-13 for the wavelength, with *d* equal to the inverse of the number of lines per centimeter.

$$\theta_1 = \frac{\theta_r - \theta_\ell}{2} = \frac{26^\circ38' - (-26^\circ18')}{2} = 26^\circ28' = 26 + 28/60 = 26.47^\circ$$
$$\lambda_1 = d\sin\theta = \frac{\sin\theta}{N} = \frac{\sin 26.47^\circ}{9650\text{ line/cm}} = 4.618\times10^{-5}\text{ cm} = \boxed{462\text{ nm}}$$
$$\theta_2 = \frac{\theta_{2r} - \theta_{2\ell}}{2} = \frac{41^\circ02' - (-40^\circ27')}{2} = 40^\circ44.5' = 40 + 44.5/60 = 40.742^\circ$$
$$\lambda_2 = \frac{\sin 40.742^\circ}{9650\text{ line/cm}} = 6.763\times10^{-5}\text{ cm} = \boxed{676\text{ nm}}$$

41. If the spectrometer were immersed in water, the wavelengths calculated in Problem 40 would be wavelengths in water. To change those wavelengths into wavelengths in air, we must multiply by the index of refraction.

$$\lambda_{1\,\text{air}} = \left(4.618\times10^{-5}\,\text{cm}\right)\left(1.33\right) = \boxed{614\,\text{nm}} \;;\; \lambda_{2\,\text{air}} = \left(6.763\times10^{-5}\,\text{cm}\right)\left(1.33\right) = \boxed{899\,\text{nm}}$$

Note that the second wavelength is not in the visible range.

42. We solve Eq. 35-13 for the slit separation width, *d,* using the given information. Then setting *m*=3, we solve for the angle of the third order maximum.

$$\sin\theta = \frac{m\lambda}{d} \;\rightarrow\; d = \frac{m\lambda}{\sin\theta} = \frac{1\left(589\,\text{nm}\right)}{\sin 16.5^\circ} = 2074\,\text{nm} = \boxed{2.07\,\mu\text{m}}$$

$$\theta_3 = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left(\frac{3\times589\,\text{nm}}{2074\,\text{nm}}\right) = \boxed{58.4^\circ}$$

43. We find the angle for each "boundary" color from Eq. 35-13, and then use the fact that the displacement on the screen is given by $\tan\theta = \frac{y}{L}$, where *y* is the displacement on the screen from the central maximum, and *L* is the distance from the grating to the screen.

$$\sin\theta = \frac{m\lambda}{d} \;;\; d = \frac{1}{610\,\text{lines/mm}}\left(\frac{1\text{m}}{10^3\text{mm}}\right) = \left(1/6.1\times10^5\right)\text{m} \;;\; y = L\tan\theta = L\tan\left[\sin^{-1}\frac{m\lambda}{d}\right]$$

$$\ell_1 = L\tan\left[\sin^{-1}\frac{m\lambda_{\text{red}}}{d}\right] - L\tan\left[\sin^{-1}\frac{m\lambda_{\text{violet}}}{d}\right]$$

$$= \left(0.32\,\text{m}\right)\left\{\tan\left[\sin^{-1}\frac{(1)\left(700\times10^{-9}\,\text{m}\right)}{\left(1/6.1\times10^5\right)\text{m}}\right] - \tan\left[\sin^{-1}\frac{(1)\left(400\times10^{-9}\,\text{m}\right)}{\left(1/6.1\times10^5\right)\text{m}}\right]\right\}$$

$$= 0.0706\,\text{m} \approx \boxed{7\,\text{cm}}$$

$$\ell_2 = L\tan\left[\sin^{-1}\frac{m\lambda_{\text{red}}}{d}\right] - L\tan\left[\sin^{-1}\frac{m\lambda_{\text{violet}}}{d}\right]$$

$$= \left(0.32\,\text{m}\right)\left\{\tan\left[\sin^{-1}\frac{(2)\left(700\times10^{-9}\,\text{m}\right)}{\left(1/6.1\times10^5\right)\text{m}}\right] - \tan\left[\sin^{-1}\frac{(2)\left(400\times10^{-9}\,\text{m}\right)}{\left(1/6.1\times10^5\right)\text{m}}\right]\right\}$$

$$= 0.3464\,\text{m} \approx \boxed{35\,\text{cm}}$$

The second order rainbow is dispersed over a larger distance.

44. (*a*) Missing orders occur when the angle to the interference maxima (Eq. 34-2a) is equal to the angle of a diffraction minimum (Eq. 35-2). We set $d = 2D$ and show that the even interference orders are missing.

$$\sin\theta = \frac{m_1\lambda}{d} = \frac{m_2\lambda}{D} \;\rightarrow\; \frac{m_1}{m_2} = \frac{d}{D} = \frac{2D}{D} = 2 \;\rightarrow\; m_1 = 2m_2$$

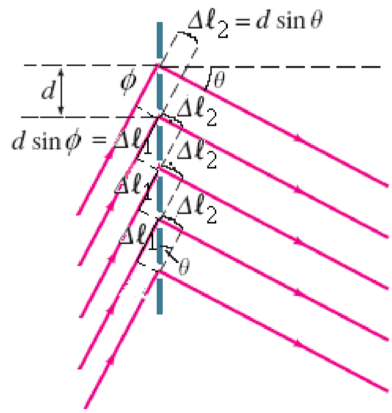
Since $m_2 = 1, 2, 3, 4, \ldots,$ all even orders of m_1 correspond to the diffraction minima and will be missing from the interference pattern.

(*b*) Setting the angle of interference maxima equal to the angle of diffraction minimum, with the orders equal to integers we determine the relationship between the slit size and separation that will produce missing orders.

$$\sin\theta = \frac{m_1\lambda}{d} = \frac{m_2\lambda}{D} \quad \rightarrow \quad \boxed{\frac{d}{D} = \frac{m_1}{m_2}}$$

(*c*) When $d = D$, all interference maxima will overlap with diffraction minima so that no fringes will exist. This is expected because if the slit width and separation distance are the same, the slits will merge into one single opening.

45. (*a*) Diffraction maxima occur at angles for which the incident light constructive interferes. That is, when the path length difference between two rays is equal to an integer number of wavelengths. Since the light is incident at an angle ϕ relative to the grating, each succeeding higher ray, as shown in the diagram, travels a distance $\Delta\ell_1 = d\sin\phi$ farther to reach the grating. After passing through the grating the higher rays travel a distance to the screen that is again longer by $\Delta\ell_2 = d\sin\theta$. By setting the total path length difference equal to an integer number of wavelengths, we are able to determine the location of the bright fringes.



$$\Delta\ell = \Delta\ell_1 + \Delta\ell_1 = d\left(\sin\phi + \sin\theta\right) = \pm m\lambda, \qquad m = 0,1,2,....$$

(*b*) The ± allows for the incident angle and the diffracted angle to have positive and negative values.

(*c*) We insert the given data, with *m*=1, to solve for the angles θ.

$$\theta = \sin^{-1}\left(-\sin\phi \pm \frac{m\lambda}{d}\right) = \sin^{-1}\left(-\sin 15° \pm \frac{550\times10^{-9}\,\text{m}}{0.01\,\text{m}/5000\,\text{lines}}\right) = \boxed{0.93° \text{ and } -32°}$$

46. Using Eq. 35-13 we calculate the maximum order possible for this diffraction grating, by setting the angle equal to 90°. Then we set the resolving power equal to the product of the number of grating lines and the order, where the resolving power is the wavelength divided by the minimum separation in wavelengths (Eq. 35-19) and solve for the separation.

$$\sin\theta = \frac{m\lambda}{d} \quad \rightarrow \quad m = \frac{d\sin\theta}{\lambda} = \frac{\left(0.01\,\text{m}/6500\,\text{lines}\right)\sin 90°}{624\times10^{-9}\text{m}} = 2.47 \approx 2$$

$$\frac{\lambda}{\Delta\lambda} = Nm \;\Rightarrow\; \Delta\lambda = \frac{\lambda}{Nm} = \frac{624\text{ nm}}{\left(6500\text{ lines/cm}\right)\left(3.18\text{ cm}\right)\left(2\right)} = \boxed{0.015\text{ nm}}$$

The resolution is best for the $\boxed{\text{second order}}$, since it is more spread out than the first order.

47. (*a*) The resolving power is given by Eq. 35-19.

$$\text{R} = Nm \quad \rightarrow \quad \text{R}_1 = \left(16{,}000\right)\left(1\right) = \boxed{16{,}000} \;;\; \text{R}_1 = \left(16{,}000\right)\left(2\right) = \boxed{32{,}000}$$

(*b*) The wavelength resolution is also given by Eq. 35-19.

$$\text{R} = \frac{\lambda}{\Delta\lambda} = Nm \quad \rightarrow \quad \Delta\lambda = \frac{\lambda}{Nm}$$

$$\Delta\lambda_1 = \frac{410\,\text{nm}}{\left(16{,}000\right)\left(1\right)} = 2.6\times10^{-2}\,\text{nm} = \boxed{26\,\text{pm}} \;;\; \Delta\lambda_1 = \frac{410\,\text{nm}}{\left(32{,}000\right)\left(1\right)} = 1.3\times10^{-2}\,\text{nm} = \boxed{13\,\text{pm}}$$

48. (*a*) We use Eq. 35-13, with the angle equal to 90° to determine the maximum order.

$$\sin\theta = \frac{m\lambda}{d} \rightarrow m = \frac{d\sin\theta}{\lambda} = \frac{(1050\,\text{nm})\sin 90°}{580\,\text{nm}} = 1.81$$

Since the order must be an integer number there will only be one principal maximum on either side of the central maximum. Counting the central maximum and the two other principal maxima there will be a total of three principal maxima.

(*b*) We use Eq. 35-17 to calculate the peak width, where the full peak width is double the half-peak width and the angle to the peak is given by Eq. 35-13.

$$\theta_0 = 0$$

$$\Delta\theta_0 = 2\frac{\lambda}{Nd\cos\theta_0} = \frac{2\lambda}{\ell\cos\theta_0} = \frac{2(580\,\text{nm})}{(1.80\times10^{-2}\,\text{m})\cos 0°} = 6.4\times10^{-5}\,\text{rad} = \boxed{0.0037°}$$

$$\theta_{\pm1} = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left(\frac{\pm1\times580\,\text{nm}}{1050\,\text{nm}}\right) = \pm33.5°$$

$$\Delta\theta_{\pm1} = \frac{2\lambda}{\ell\cos\theta_{\pm1}} = \frac{2(580\,\text{nm})}{(1.80\times10^{-2}\,\text{m})\cos(\pm33.5°)} = 7.7\times10^{-5}\,\text{rad} = \boxed{0.0044°}$$

49. We use Eq. 35-20, with $m = 1$.

$$m\lambda = 2d\sin\phi \rightarrow \phi = \sin^{-1}\frac{m\lambda}{2d} = \sin^{-1}\frac{(1)(0.138\,\text{nm})}{2(0.285\,\text{nm})} = \boxed{14.0°}$$

50. We use Eq. 35-20 for X-ray diffraction.

(*a*) Apply Eq. 35-20 to both orders of diffraction.

$$m\lambda = 2d\sin\phi \rightarrow \frac{m_1}{m_2} = \frac{\sin\phi_1}{\sin\phi_2} \rightarrow \phi_2 = \sin^{-1}\left(\frac{m_2}{m_1}\sin\phi_1\right) = \sin^{-1}\left(\frac{2}{1}\sin 26.8°\right) = \boxed{64.4°}$$

(*b*) Use the first order data.

$$m\lambda = 2d\sin\phi \rightarrow \lambda = \frac{2d\sin\phi}{m} = \frac{2(0.24\,\text{nm})\sin 26.8°}{1} = \boxed{0.22\,\text{nm}}$$

51. For each diffraction peak, we can measure the angle and count the order. Consider Eq. 35-20.

$$m\lambda = 2d\sin\phi \rightarrow \lambda = 2d\sin\phi_1 \;;\; 2\lambda = 2d\sin\phi_2 \;;\; 3\lambda = 2d\sin\phi_3$$

From each equation, all we can find is the ratio $\frac{\lambda}{d} = 2\sin\phi = \sin\phi_2 = \frac{2}{3}\sin\phi_3$. No, we cannot separately determine the wavelength or the spacing.

52. Use Eq. 35-21. Since the initial light is unpolarized, the intensity after the first polarizer will be half the initial intensity. Let the initial intensity be I_0.

$$I_1 = \tfrac{1}{2}I_0 \;;\; I_2 = I_1\cos^2\theta = \tfrac{1}{2}I_0\cos^2\theta \rightarrow \frac{I_2}{I_0} = \frac{\cos^2 65°}{2} = \boxed{0.089}$$

53. If I_0 is the intensity passed by the first Polaroid, the intensity passed by the second will be I_0 when the two axes are parallel. To calculate a reduction to half intensity, we use Eq. 35-21.

$$I = I_0\cos^2\theta = \tfrac{1}{2}I_0 \rightarrow \cos^2\theta = \tfrac{1}{2} \rightarrow \theta = \boxed{45°}$$

54. We assume that the light is coming from air to glass, and use Eq. 35-22b.

$$\tan\theta_{\text{p}} = n_{\text{glass}} = 1.58 \quad\rightarrow\quad \theta_{\text{p}} = \tan^{-1}1.58 = \boxed{57.7^\circ}$$

$\boxed{55.}$ The light is traveling from water to diamond. We use Eq. 35-22a.

$$\tan\theta_{\text{p}} = \frac{n_{\text{diamond}}}{n_{\text{water}}} = \frac{2.42}{1.33} = 1.82 \quad\rightarrow\quad \theta_{\text{p}} = \tan^{-1}1.82 = \boxed{61.2^\circ}$$

56. The critical angle exists when light passes from a material with a higher index of refraction (n_1) into a material with a lower index of refraction (n_2). Use Eq. 32-7.

$$\frac{n_2}{n_1} = \sin\theta_{\text{C}} = \sin 55^\circ$$

To find the Brewster angle, use Eq. 35-22a. If light is passing from high index to low index, we have the following.

$$\frac{n_2}{n_1} = \tan\theta_{\text{p}} = \sin 55^\circ \quad\rightarrow\quad \theta_{\text{p}} = \tan^{-1}\left(\sin 55^\circ\right) = \boxed{39^\circ}$$

If light is passing from low index to high index, we have the following.

$$\frac{n_1}{n_2} = \tan\theta_{\text{p}} = \frac{1}{\sin 55^\circ} \quad\rightarrow\quad \theta_{\text{p}} = \tan^{-1}\left(\frac{1}{\sin 55^\circ}\right) = \boxed{51^\circ}$$

57. Let the initial intensity of the unpolarized light be I_0. The intensity after passing through the first Polaroid will be $I_1 = \frac{1}{2}I_0$. Then use Eq. 35-21.

$$I_2 = I_1\cos^2\theta = \tfrac{1}{2}I_0\cos^2\theta \quad\rightarrow\quad \theta = \cos^{-1}\sqrt{\frac{2I_2}{I_0}}$$

(*a*) $\theta = \cos^{-1}\sqrt{\dfrac{2I_2}{I_0}} = \cos^{-1}\sqrt{\dfrac{2}{3}} = \boxed{35.3^\circ}$

(*b*) $\theta = \cos^{-1}\sqrt{\dfrac{2I_2}{I_0}} = \cos^{-1}\sqrt{\dfrac{2}{10}} = \boxed{63.4^\circ}$

58. For the first transmission, the angle between the light and the polarizer is 18.0°. For the second transmission, the angle between the light and the polarizer is 36.0°. Use Eq. 35-21 twice.

$$I_1 = I_0\cos^2 18.0^\circ \;;\; I_2 = I_1\cos^2 36.0^\circ = I_0\cos^2 18.0^\circ\cos^2 36.0^\circ = 0.592 I_0$$

Thus the transmitted intensity is $\boxed{59.2\%}$ of the incoming intensity.

59. First case: the light is coming from water to air. Use Eq. 35-22a.

$$\tan\theta_{\text{p}} = \frac{n_{\text{air}}}{n_{\text{water}}} \quad\rightarrow\quad \theta_{\text{p}} = \tan^{-1}\frac{n_{\text{air}}}{n_{\text{water}}} = \tan^{-1}\frac{1.00}{1.33} = \boxed{36.9^\circ}$$

Second case: for total internal reflection, the light must also be coming from water into air. Use Eq. 32-7.

$$\sin\theta_{\text{C}} = \frac{n_{\text{air}}}{n_{\text{water}}} \quad\rightarrow\quad \theta_{\text{p}} = \sin^{-1}\frac{n_{\text{air}}}{n_{\text{water}}} = \sin^{-1}\frac{1.00}{1.33} = \boxed{48.8^\circ}$$

Third case: the light is coming from air to water. Use Eq. 35-22b.

$$\tan\theta_{\text{p}} = n_{\text{water}} \quad\rightarrow\quad \theta_{\text{p}} = \tan^{-1}n_{\text{water}} = \tan^{-1}1.33 = \boxed{53.1^\circ}$$

Note that the two Brewster's angles add to give 90.0°.

60. When plane-polarized light passes through a sheet oriented at an angle θ, the intensity decreases according to Eq. 35-21, $I = I_0 \cos^2\theta$. For $\theta = 45°$, $\cos^2\theta = \frac{1}{2}$. Thus sheets 2 through 6 will each reduce the intensity by a factor of $\frac{1}{2}$. The first sheet reduces the intensity of the unpolarized incident light by $\frac{1}{2}$ as well. Thus we have the following.

$$I = I_0\left(\tfrac{1}{2}\right)^6 = \boxed{0.016 I_0}$$

61. We assume vertically polarized light of intensity I_0 is incident upon the first polarizer. The angle between the polarization direction and the polarizer is θ. After the light passes that first polarizer, the angle between that light and the next polarizer will be $90° - \theta$. Apply Eq. 35-21.

$$I_1 = I_0 \cos^2\theta \;;\; I = I_1 \cos^2(90° - \theta) = I_0 \cos^2\theta \cos^2(90° - \theta) = \boxed{I_0 \cos^2\theta \sin^2\theta}$$

We can also use the trigonometric identity $\sin\theta\cos\theta = \frac{1}{2}\sin 2\theta$ to write the final intensity as

$$I = I_0 \cos^2\theta \sin^2\theta = \boxed{\tfrac{1}{4} I_0 \sin^2 2\theta}.$$

$$\frac{dI}{d\theta} = \frac{d}{d\theta}\left(\tfrac{1}{4} I_0 \sin^2 2\theta\right) = \tfrac{1}{4} I_0 (2\sin 2\theta)(\cos 2\theta)2 = I_0 \sin 2\theta \cos 2\theta = \boxed{\tfrac{1}{2} I_0 \sin 4\theta}$$

$$\tfrac{1}{2} I_0 \sin 4\theta = 0 \;\rightarrow\; 4\theta = 0, 180°, 360° \;\rightarrow\; \theta = 0, 45°, 90°$$

Substituting the three angles back into the intensity equation, we see that the angles $0°$ and $90°$ both give minimum intensity. The angle $\boxed{45° \text{ gives the maximum intensity}}$ of $\frac{1}{4} I_0$.

62. We set the intensity of the beam as the sum of the maximum and minimum intensities. Using Eq. 35-21, we determine the intensity of the beam after it has passed through the polarizer. Since I_{min} is polarized perpendicular to I_{max} and the polarizer is rotated at an angle ϕ from the polarization of I_{max}, the polarizer is oriented at an angle of $(90° - \phi)$ from I_{min}.

$$I_0 = I_{max} + I_{min}$$

$$I = I_0 \cos^2\phi = I_{max}\cos^2\phi + I_{min}\cos^2(90 - \phi) = I_{max}\cos^2\phi + I_{min}\sin^2\phi$$

We solve the percent polarization equation for I_{min} and insert the result into our intensity equation.

$$p = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \;\rightarrow\; I_{min} = \frac{1-p}{1+p} I_{max}$$

$$I = I_{max}\cos^2\phi + \left(\frac{1-p}{1+p} I_{max}\right)\sin^2\phi = I_{max}\left[\frac{(1+p)\cos^2\phi + (1-p)\sin^2\phi}{1+p}\right]$$

$$= I_{max}\left[\frac{(\cos^2\phi + \sin^2\phi) + p(\cos^2\phi - \sin^2\phi)}{1+p}\right] = \boxed{I_{max}\left[\frac{1 + p\cos 2\phi}{1+p}\right]}$$

63. Because the width of the pattern is much smaller than the distance to the screen, the angles from the diffraction pattern for this first order will be small. Thus we may make the approximation that $\sin\theta = \tan\theta$. We find the angle to the first minimum from the distances, using half the width of the full first order pattern. Then we use Eq. 35-2 to find the slit width.

$$\tan\theta_{1\,min} = \tfrac{1}{2}\frac{(8.20\,\text{cm})}{(285\,\text{cm})} = 0.01439 = \sin\theta_{1\,min}$$

$$D\sin\theta = m\lambda \;\rightarrow\; D = \frac{m\lambda}{\sin\theta} = \frac{(1)(415\,\text{nm})}{0.01439} = 2.88\times10^4\,\text{nm} = \boxed{2.88\times10^{-5}\,\text{m}}$$

64. If the original intensity is I_0, the first polarizers will reduce the intensity to one half the initial intensity, or $I_1 = \frac{1}{2}I_0$. Each subsequent polarizer oriented at an angle θ to the preceding one will reduce the intensity by $\cos^2\theta$, as given by Eq. 35-21. We set the final intensity equal to one quarter of the initial intensity, with $\theta = 10°$ for each polarizer and solve for the minimum number of polarizers.

$$I = \tfrac{1}{2}I_0\left(\cos^2\theta\right)^{n-1} \;\Rightarrow\; n = 1 + \frac{\ln\left(2I/I_0\right)}{\ln\left(\cos^2\theta\right)} = 1 + \frac{\ln\left(2\times 0.25\right)}{\ln\left(\cos^2 10°\right)} = 23.6 \approx \boxed{24 \text{ polarizers}}$$

We round the number of lenses up to the integer number of polarizers, so that the intensity will be less than 25% of the initial intensity.

65. The lines act like a grating. We assume that we see the first diffractive order, so $m = 1$. Use Eq. 35-13.

$$d\sin\theta = m\lambda \;\rightarrow\; d = \frac{m\lambda}{\sin\theta} = \frac{(1)(480\,\text{nm})}{\sin 56°} = \boxed{580\,\text{nm}}$$

66. We assume the sound is diffracted when it passes through the doorway, and find the angles of the minima from Eq. 35-2.

$$\lambda = \frac{v}{f}\;;\; D\sin\theta = m\lambda = \frac{mv}{f} \;\rightarrow\; \theta = \sin^{-1}\frac{mv}{Df},\; m = 1, 2, 3, ...$$

$$m = 1:\;\theta = \sin^{-1}\frac{mv}{Df} = \sin^{-1}\frac{(1)(340\,\text{m/s})}{(0.88\,\text{m})(850\,\text{Hz})} = \boxed{27°}$$

$$m = 2:\;\theta = \sin^{-1}\frac{mv}{Df} = \sin^{-1}\frac{(2)(340\,\text{m/s})}{(0.88\,\text{m})(850\,\text{Hz})} = \boxed{65°}$$

$$m = 3:\;\theta = \sin^{-1}\frac{mv}{Df} = \sin^{-1}\frac{(3)(340\,\text{m/s})}{(0.88\,\text{m})(850\,\text{Hz})} = \sin^{-1}1.36 = \text{impossible}$$

Thus the whistle would not be heard clearly at angles of $\boxed{27° \text{ and } 65° \text{ on either side of the normal.}}$

$\boxed{67.}$ We find the angles for the first order from Eq. 35-13.

$$\theta_1 = \sin^{-1}\frac{m\lambda}{d} = \sin^{-1}\frac{(1)\left(4.4\times10^{-7}\text{ m}\right)}{0.01\text{ m}/7600} = 19.5°$$

$$\theta_2 = \sin^{-1}\frac{(1)\left(6.8\times10^{-7}\text{ m}\right)}{0.01\text{ m}/7600} = 31.1°$$

The distances from the central white line on the screen are found using the tangent of the angle and the distance to the screen.

$$y_1 = L\tan\theta_1 = (2.5\,\text{m})\tan 19.5° = 0.89\,\text{m}$$

$$y_2 = L\tan\theta_2 = (2.5\,\text{m})\tan 31.1° = 1.51\,\text{m}$$

Subtracting these two distances gives the linear separation of the two lines.

$$y_2 - y_1 = 1.51\,\text{m} - 0.89\,\text{m} = \boxed{0.6\,\text{m}}$$

68. Because the angle increases with wavelength, to miss a complete order we use the smallest visible wavelength, 400 nm. The maximum angle is 90°. With these parameters we use Eq. 35-13 to find the slit separation, *d*. The inverse of the slit separation gives the number of lines per unit length.

$$d\sin\theta = m\lambda \quad\rightarrow\quad d = \frac{m\lambda}{\sin\theta} = \frac{2(400\,\text{nm})}{\sin 90^\circ} = \boxed{800\,\text{nm}}$$

$$\frac{1}{d} = \frac{1}{800\times10^{-7}\,\text{cm}} = \boxed{12{,}500\,\text{lines/cm}}$$

69. We find the angles for the two first-order peaks from the distance to the screen and the distances along the screen to the maxima from the central peak.

$$\tan\theta_1 = \frac{y_1}{\ell} \quad\rightarrow\quad \theta_1 = \tan^{-1}\frac{y_1}{\ell} = \tan^{-1}\frac{(3.32\,\text{cm})}{(66.0\,\text{cm})} = 2.88^\circ$$

$$\tan\theta_2 = \frac{y_2}{\ell} \quad\rightarrow\quad \theta_2 = \tan^{-1}\frac{y_2}{\ell} = \tan^{-1}\frac{(3.71\,\text{cm})}{(66.0\,\text{cm})} = 3.22^\circ$$

Inserting the wavelength of yellow sodium light and the first order angle into Eq. 35-13, we calculate the separation of lines. Then, using the separation of lines and the second angle, we calculate the wavelength of the second source. Finally, we take the inverse of the line separation to determine the number of lines per centimeter on the grating.

$$d\sin\theta_1 = m\lambda_1 \quad\rightarrow\quad d = \frac{m\lambda_1}{\sin\theta_1} = \frac{1(589\text{ nm})}{\sin 2.88^\circ} = 11{,}720\,\text{nm}$$

$$\lambda_2 = \frac{d\sin\theta_1}{m} = (11{,}720\,\text{nm})\sin 3.22^\circ = \boxed{658\,\text{nm}}$$

$$\frac{1}{d} = \frac{1\text{ line}}{11{,}720\times10^{-7}\,\text{cm}} = \boxed{853\,\text{lines/cm}}$$

70. We find the angles for the first order from Eq. 35-13, with $m = 1$. The slit spacing is the inverse of the lines/cm of the grating.

$$d = \frac{1}{8100\,\text{lines/cm}}\times\frac{1\,\text{m}}{100\,\text{cm}} = \frac{1}{8.1\times10^5}\,\text{m} \;;\; d\sin\theta = m\lambda \quad\rightarrow\quad \theta = \sin^{-1}\frac{m\lambda}{d} \quad\rightarrow$$

$$\Delta\theta = \sin^{-1}\frac{\lambda_1}{d} - \sin^{-1}\frac{\lambda_2}{d} = \sin^{-1}\frac{656\times10^{-9}\,\text{m}}{\left(\frac{1}{8.1\times10^5}\,\text{m}\right)} - \sin^{-1}\frac{410\times10^{-9}\,\text{m}}{\left(\frac{1}{8.1\times10^5}\,\text{m}\right)} = \boxed{13^\circ}$$

71. (*a*) This is very similar to Example 35-6. We use the same notation as in that Example, and solve for the distance ℓ.

$$s = \ell\theta = \ell\frac{1.22\lambda}{D} \quad\rightarrow\quad \ell = \frac{Ds}{1.22\lambda} = \frac{(6.0\times10^{-3}\,\text{m})(2.0\,\text{m})}{1.22(560\times10^{-9}\,\text{m})} = \boxed{1.8\times10^4\,\text{m}} = 18\,\text{km}$$

(*b*) We use the same data for the eye and the wavelength.

$$\theta = \frac{1.22\lambda}{D} = \frac{1.22(560\times10^{-9}\,\text{m})}{(6.0\times10^{-3}\,\text{m})} = 1.139\times10^{-4}\,\text{rad}\left(\frac{180^\circ}{\pi\,\text{rad}}\right)\left(\frac{3600''}{1^\circ}\right) = \boxed{23''}$$

Our answer is less than the real resolution, because of atmospheric effects and aberrations in the eye.

72. We first find the angular half-width for the first order, using Eq. 35-1, $\sin\theta = \frac{\lambda}{D}$. Since this angle is small, we may use the approximation that $\sin\theta \approx \tan\theta$. The width from the central maximum to the first minimum is given by $y = L\tan\theta$. That width is then doubled to find the width of the beam, from the first diffraction minimum on one side to the first diffraction minimum on the other side.

$$y = L\tan\theta = L\sin\theta$$

$$\Delta y = 2y = 2L\sin\theta = 2L\frac{\lambda}{D} = \frac{2\left(3.8\times10^{8}\,\text{m}\right)\left(633\times10^{-9}\,\text{m}\right)}{0.010\,\text{m}} = \boxed{4.8\times10^{4}\,\text{m}}$$

73. The distance between lines on the diffraction grating is found by solving Eq. 35-13 for *d*, the grating spacing. The number of lines per meter is the reciprocal of *d*.

$$d = \frac{m\lambda}{\sin\theta} \quad\rightarrow\quad \frac{1}{d} = \frac{\sin\theta}{m\lambda} = \frac{\sin 21.5^\circ}{(1)6.328\times10^{-7}\,\text{m}} = \boxed{5.79\times10^{5}\ \text{lines/m}}$$

74. (*a*) We calculate the wavelength of the mother's sound by dividing the speed of sound by the frequency of her voice. We use Eq. 34-2b to determine the double slit interference minima with $d = 3.0$ m.

$$\lambda = v/f = (340\ \text{m/s})/(400\ \text{Hz}) = 0.85\ \text{m}$$

$$\theta = \sin^{-1}\left[\frac{\left(m+\frac{1}{2}\right)\lambda}{d}\right] = \sin^{-1}\left[\frac{\left(m+\frac{1}{2}\right)(0.85\ \text{m})}{(3.0\ \text{m})}\right] = \sin^{-1}\left[0.2833\left(m+\tfrac{1}{2}\right)\right],\ m = 0,1,2,...$$

$$= \boxed{8.1^\circ,\ 25^\circ,\ 45^\circ,\ \text{and}\ 83^\circ}$$

We use Eq. 35-2 to determine the angles for destructive interference from single slit diffraction, with $D = 1.0$ m.

$$\theta = \sin^{-1}\left[\frac{m\lambda}{D}\right] = \sin^{-1}\left[\frac{m(0.85\ \text{m})}{(1.0\ \text{m})}\right] = \sin^{-1}\left[0.85m\right],\ m = 1,2,...$$

$$\theta = \boxed{58^\circ}$$

(*b*) We use the depth and length of the room to determine the angle the sound would need to travel to reach the son.

$$\theta = \tan^{-1}\left(\frac{8.0\ \text{m}}{5.0\ \text{m}}\right) = 58^\circ$$

This angle is close to the single slit diffraction minimum, so the son has a good explanation for not hearing her.

75. We use the Brewster angle, Eq. 35-22b, for light coming from air to water.

$$\tan\theta_{\text{p}} = n \quad\rightarrow\quad \theta_{\text{p}} = \tan^{-1} n = \tan^{-1} 1.33 = 53.1^\circ$$

This is the angle from the normal, as seen in Fig. 35-41, so the angle above the horizontal is the complement of $90.0^\circ - 53.1^\circ = \boxed{36.9^\circ}$.

76. (*a*) Let the initial unpolarized intensity be I_0. The intensity of the polarized light after passing the first polarizer is $I_1 = \frac{1}{2}I_0$. Apply Eq. 35-21 to find the final intensity.

$$I_2 = I_1\cos^2\theta = I_1\cos^2 90^\circ = \boxed{0}.$$

(*b*) Now the third polarizer is inserted. The angle between the first and second polarizers is 66°, so the angle between the second and third polarizers is 24°. It is still true that $I_1 = \frac{1}{2}I_0$.

$$I_2 = I_1\cos^2 66° = \tfrac{1}{2}I_0\cos^2 66° \ ; \ I_3 = I_2\cos^2 24° = \tfrac{1}{2}I_0\cos^2 66°\cos^2 24° = 0.069 \ \rightarrow$$

$$\frac{I_3}{I_1} = \boxed{0.069}$$

(*c*) The two crossed polarizers, which are now numbers 2 and 3, will still not allow any light to pass through them if they are consecutive to each other. Thus $\frac{I_3}{I_1} = \boxed{0}$.

77. The reduction being investigated is that which occurs when the polarized light passes through the second Polaroid. Let I_1 be the intensity of the light that emerges from the first Polaroid, and I_2 be the intensity of the light after it emerges from the second Polaroid. Use Eq. 35-21.

(*a*) $I_2 = I_1\cos^2\theta = 0.25I_1 \ \rightarrow \ \theta = \cos^{-1}\sqrt{0.25} = \boxed{60°}$

(*b*) $I_2 = I_1\cos^2\theta = 0.10I_1 \ \rightarrow \ \theta = \cos^{-1}\sqrt{0.10} = \boxed{72°}$

(*c*) $I_2 = I_1\cos^2\theta = 0.010I_1 \ \rightarrow \ \theta = \cos^{-1}\sqrt{0.010} = \boxed{84°}$

78. (*a*) We apply Eq. 35-21 through the successive polarizers. The initial light is unpolarized. Each polarizer is then rotated $30°$ from the previous one.

$$I_1 = \tfrac{1}{2}I_0 \ ; \ I_2 = I_1\cos^2\theta_2 = \tfrac{1}{2}I_0\cos^2\theta_2 \ ; \ I_3 = I_2\cos^2\theta_3 = \tfrac{1}{2}I_0\cos^2\theta_2\cos^2\theta_3 \ ;$$

$$I_4 = I_3\cos^2\theta_4 = \tfrac{1}{2}I_0\cos^2\theta_2\cos^2\theta_3\cos^2\theta_4 = \tfrac{1}{2}I_0\cos^2 30°\cos^2 30°\cos^2 30° = \boxed{0.21I_0}$$

(*b*) If we remove the second polarizer, then the angle between polarizers # 1 and # 3 is now $60°$.

$$I_1 = \tfrac{1}{2}I_0 \ ; \ I_3 = I_1\cos^2\theta_3 = \tfrac{1}{2}I_0\cos^2\theta_3 \ ;$$

$$I_4 = I_3\cos^2\theta_4 = \tfrac{1}{2}I_0\cos^2\theta_3\cos^2\theta_4 = \tfrac{1}{2}I_0\cos^2 60°\cos^2 30° = 0.094\,I_0$$

The same value would result by removing the third polarizer, because then the angle between polarizers # 2 and # 4 would be $60°$. Thus we can decrease the intensity by removing either the $\boxed{\text{second or third polarizer.}}$

(*c*) If we remove both the $\boxed{\text{second and third polarizers,}}$ we will have two polarizers with their axes perpendicular, so no light will be transmitted.

$\boxed{79.}$ For the minimum aperture the angle subtended at the lens by the smallest feature is the angular resolution, given by Eq. 35-10. We let ℓ represent the spatial separation, and *r* represent the altitude of the camera above the ground.

$$\theta = \frac{1.22\lambda}{D} = \frac{\ell}{r} \ \rightarrow \ D = \frac{1.22\lambda r}{\ell} = \frac{1.22(580\times10^{-9}\,\text{m})(25000\,\text{m})}{(0.05\,\text{m})} = 0.3538\,\text{m} \approx \boxed{0.4\,\text{m}}$$

80. Let I_0 be the initial intensity. Use Eq. 35-21 for both transmissions of the light.

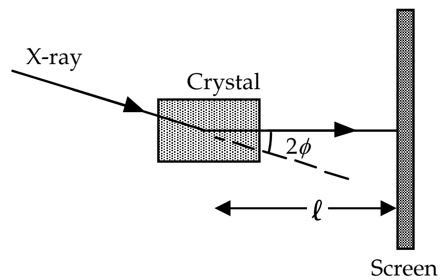
$$I_1 = I_0\cos^2\theta_1 \ ; \ I_2 = I_1\cos^2\theta_2 = I_0\cos^2\theta_1\cos^2\theta_2 = 0.25I_0 \ \rightarrow$$

$$\theta_1 = \cos^{-1}\left(\frac{\sqrt{0.25}}{\cos\theta_2}\right) = \cos^{-1}\left(\frac{\sqrt{0.25}}{\cos 48°}\right) = \boxed{42°}$$

81. We find the spacing from Eq. 35-20.

$$m\lambda = 2d\sin\phi \ \rightarrow \ d = \frac{m\lambda}{2\sin\phi} = \frac{(2)(9.73\times10^{-11}\,\text{m})}{2\sin 23.4°} = \boxed{2.45\times10^{-10}\,\text{m}}$$

82. The angles for Bragg scattering are found from Eq. 35-20, for $m = 1$ and $m = 2$. If the distance from the crystal to the screen is ℓ, the radius of the diffraction ring is given by $r = \ell \tan 2\phi$.



$$2d \sin\phi = m\lambda \ ; \ r = \ell \tan 2\phi = \ell \tan\left[2\sin^{-1}\left(\frac{m\lambda}{2d}\right)\right]$$

$$r_1 = \ell \tan\left[2\sin^{-1}\left(\frac{m\lambda}{2d}\right)\right]$$

$$= (0.12\,\text{m})\tan\left[2\sin^{-1}\left(\frac{(1)(0.10\times10^{-9}\,\text{m})}{2(0.22\times10^{-9}\,\text{m})}\right)\right] = \boxed{0.059\,\text{m}}$$

$$r_2 = \ell \tan\left[2\sin^{-1}\left(\frac{m\lambda}{2d}\right)\right] = (0.12\,\text{m})\tan\left[2\sin^{-1}\left(\frac{(2)(0.10\times10^{-9}\,\text{m})}{2(0.22\times10^{-9}\,\text{m})}\right)\right] = \boxed{0.17\,\text{m}}$$

83. From Eq. 35-10 we calculate the minimum resolvable separation angle. We then multiply this angle by the distance between the Earth and Moon to obtain the minimum distance between two objects on the Moon that the Hubble can resolve.

$$\theta = \frac{1.22\lambda}{D} = \frac{1.22(550\times10^{-9}\,\text{m})}{2.4\,\text{m}} = 2.796\times10^{-7}\,\text{rad}$$

$$\ell = s\theta = (3.84\times10^{8}\,\text{m})(2.796\times10^{-7}\,\text{rad}) = \boxed{110\,\text{m}}$$

84. From Eq. 35-10 we calculate the minimum resolvable separation angle. We then multiply this angle by the distance between Mars and Earth to obtain the minimum distance between two objects that can be resolved by a person on Mars

$$\theta = \frac{1.22\lambda}{D} = \frac{1.22(550\times10^{-9}\ \text{m})}{0.005\ \text{m}} = 1.34\times10^{-4}\ \text{rad}$$

$$\ell = s\theta = (8\times10^{10}\ \text{m})(1.34\times10^{-4}\ \text{rad}) = \boxed{1.07\times10^{7}\ \text{m}}$$

Since the minimum resolvable distance is much less than the Earth-Moon distance, a person standing on Mars could resolve the Earth and Moon as two separate objects without a telescope.

85. The distance x is twice the distance to the first minima. We can write x in terms of the slit width D using Eq. 35-2, with $m = 1$. The ratio $\frac{\lambda}{D}$ is small, so we may approximate $\sin\theta \approx \tan\theta \approx \theta$.

$$\sin\theta = \frac{\lambda}{D} \approx \theta \ ; \ x = 2y = 2\ell\tan\theta = 2\ell\theta = 2\ell\frac{\lambda}{D}$$

When the plate is heated up the slit width increases due to thermal expansion. Eq. 17-1b is used to determine the new slit width, with the coefficient of thermal expansion, α, given in Table 17-1. Each slit width is used to determine a value for x. Subtracting the two values for x gives the change Δx. We use the binomial expansion to simplify the evaluation.

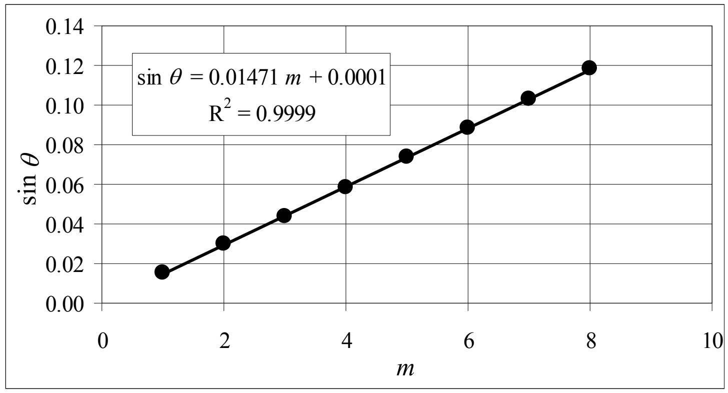
$$\Delta x = x - x_0 = 2\ell\left(\frac{\lambda}{D_0(1+\alpha\Delta T)}\right) - 2\ell\left(\frac{\lambda}{D_0}\right) = \frac{2\ell\lambda}{D_0}\left(\frac{1}{(1+\alpha\Delta T)} - 1\right) = \frac{2\ell\lambda}{D_0}\left((1+\alpha\Delta T)^{-1} - 1\right)$$

$$= \frac{2\ell\lambda}{D_0}(1 - \alpha\Delta T - 1) = -\frac{2\ell\lambda}{D_0}\alpha\Delta T = -\frac{2(2.0\,\text{m})(650\times10^{-9}\,\text{m})}{(22\times10^{-6}\,\text{m})}\left[25\times10^{-6}(\text{C}^\circ)^{-1}\right](55\text{C}^\circ)$$

$$= \boxed{-1.7\times10^{-4}\,\text{m}}$$

86. The tangent of the angle for each order is the distance in the table divided by the distance to the screen. If we call the distance in the table y and the distance to the screen ℓ, then we have this relationship.

$$\tan\theta = \frac{y}{\ell} \quad \rightarrow \quad \theta = \tan^{-1}\frac{y}{\ell}$$



The relationship between the angle and the wavelength is given by Eq. 35-2, $D\sin\theta = m\lambda$, which can be written as $\sin\theta = \frac{\lambda}{D}m$. A plot of $\sin\theta$ vs. m should have a slope of $\frac{\lambda}{D}$, and so the wavelength can be determined from the slope and the slit width. The graph is shown, and the slope used to calculate the wavelength.

$$\frac{\lambda}{D} = \text{slope} \quad \rightarrow \quad \lambda = (\text{slope})D = (0.01471)(4.000\times10^{-5}\,\text{m}) = \boxed{588.4\,\text{nm}}$$

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH35.XLS," on tab "Problem 35.86."

87. We have N polarizers providing a rotation of 90°. Thus, each polarizer must rotate the light by an angle of $\theta_N = (90/N)°$. As the light passes through each polarizer, the intensity will be reduced by a factor of $\cos^2\theta_N$. Let the original intensity be I_0.

$$I_1 = I_0\cos^2\theta_N \ ; \ I_2 = I_1\cos^2\theta_N = I_0\cos^4\theta_N \ ; \ I_3 = I_2\cos^2\theta_N = I_0\cos^6\theta_N$$

$$I_N = I_0(\cos\theta_N)^{2N} = 0.90I_0 \quad \rightarrow \quad \left[\cos(90°/N)\right]^{2N} = 0.90$$

We evaluate $\left[\cos(90°/N)\right]^{2N}$ for various values of N. A table for a few values of N is shown here. We see that N = 24 satisfies the criteria, and so $\theta_N = (90/24N)° = (90/24N)° = 3.75°$. So we need to $\boxed{\text{put 24 polarizers in the path of the original polarized light, each rotated } 3.75° \text{ from the previous one.}}$ The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH35.XLS," on tab "Problem 35.87."

N	$[\cos(90/N)]^{2N}$
21	0.8890
22	0.8938
23	0.8982
24	0.9022
25	0.9060

88. (*a*) The intensity of the diffraction pattern is given by Eqs. 35-6 and 35-7. We want to find the angle where $I = \frac{1}{2}I_0$. Doubling this angle will give the desired $\Delta\theta$.

$$I_\theta = I_0\left(\frac{\sin\beta/2}{\beta/2}\right)^2 = \tfrac{1}{2}I_0 \quad \rightarrow \quad \sin\beta/2 = \frac{\beta/2}{\sqrt{2}} \text{ or } \sin\alpha = \frac{\alpha}{\sqrt{2}}, \text{ with } \alpha = \tfrac{1}{2}\beta$$

This equation must be solved numerically. A spreadsheet was developed to find the non-zero values of α that satisfy $\sin\alpha - \frac{\alpha}{\sqrt{2}} = 0$. It is apparent from this expression that there will be no solutions for $\alpha > \sqrt{2}$. The only non-zero value is $\alpha = 1.392$. Now use Eq. 35-6 to find θ.

$$\beta = \frac{2\pi}{\lambda}D\sin\theta \quad \rightarrow \quad \theta = \sin^{-1}\frac{\lambda\beta}{2\pi D} = \sin^{-1}\frac{2\lambda\alpha}{2\pi D} = \sin^{-1}\frac{\lambda(1.392)}{\pi D} \ ;$$

$$\Delta\theta = 2\theta = \boxed{2\sin^{-1}\frac{\lambda(1.392)}{\pi D}}$$

(*b*) For $D = \lambda$: $\Delta\theta = 2\sin^{-1}\frac{\lambda(1.392)}{\pi D} = 2\sin^{-1}\frac{(1.392)}{\pi} = \boxed{52.6^\circ}$

For $D = 100\lambda$: $\Delta\theta = 2\sin^{-1}\frac{\lambda(1.392)}{\pi D} = 2\sin^{-1}\frac{(1.392)}{100\pi} = \boxed{0.508^\circ}$

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH35.XLS," on tab "Problem 35.88."