CHAPTER 34: The Wave Nature of Light; Interference

Responses to Questions

1. Yes, Huygens' principle applies to all waves, including sound and water waves.

2. Light from the Sun can be focused by a converging lens on a piece of paper and burn a hole in the paper. This provides evidence that light is energy. Also, you can feel the heat from the Sun beating down on you on a hot summer day. When you move into the shade you may still feel hot, but you don't feel the Sun's energy directly.

3. A ray shows the direction of propagation of a wave front. If this information is enough for the situation under discussion, then light can be discussed as rays. Sometimes, however, the wave nature of light is essential to the discussion. For instance, the double slit interference pattern depends on the interference of the waves, and could not be explained by examining light as only rays.

4. The bending of waves around corners or obstacles is called diffraction. Diffraction is most prominent when the size of the obstacle is on the order of the size of the wavelength. Sound waves have much longer wavelengths than do light waves. As a result, the diffraction of sound waves around a corner is noticeable and we can hear the sound in the "shadow region," but the diffraction of light waves around a corner is not noticeable.

5. The wavelength of light cannot be determined from reflection measurements alone, because the law of reflection is the same for all wavelengths. However, thin film interference, which involves interference of the rays reflecting from the front and back surfaces of the film, can be used to determine wavelength. Refraction can also be used to determine wavelength because the index of refraction for a given medium is different for different wavelengths.

6. For destructive interference, the path lengths must differ by an odd number of half wavelengths, such as $\lambda/2$, $3\lambda/2$, $5\lambda/2$, $7\lambda/2$, etc. In general, the path lengths must differ by $\lambda(m + ½)$, where m is an integer.

7. Blue light has a shorter wavelength than red light. The angles to each of the bright fringes for the blue light would be smaller than for the corresponding orders for the red light, so the bright fringes would be closer together for the blue light.

8. The fringes would be closer together because the wavelength of the light underwater is less than the wavelength in air.

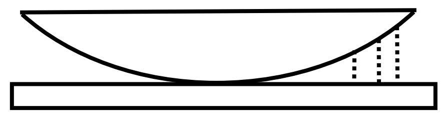
9. The two experiments are the same in principle. Each requires coherent sources and works best with a single frequency source. Each produces a pattern of alternating high and low intensity. Sound waves have much longer wavelengths than light waves, so the appropriate source separation for the sound experiment would be larger. Also, sound waves are mechanical waves which require a medium through which to travel, so the sound experiment could not be done in a vacuum and the light experiment could.

10. The red light and the blue light coming from the two different slits will have different wavelengths (and different frequencies) and will not have a constant phase relationship. In order for a double-slit pattern to be produced, the light coming from the slits must be coherent. No distinct double-slit interference pattern will appear. However, each slit will individually produce a "single-slit diffraction" pattern, as will be discussed in Chapter 35.

11. Light from the two headlights would not be coherent, so would not maintain a consistent phase relationship and therefore no stable interference pattern would be produced.

12. As the thickness of the film increases, the number of different wavelengths in the visible range that meet the constructive interference criteria increases. For a thick piece of glass, many different wavelengths will undergo constructive interference and these will all combine to produce white light.

13. Bright colored rings will occur when the path difference between the two interfering rays is λ/2, 3λ/2, 5λ/2, and so forth. A given ring, therefore, has a path difference that is exactly one wavelength longer than the path difference of its neighboring ring to the inside and one wavelength shorter than the path difference of its neighboring ring to the outside. Newton's rings are created by the thin film of air between a glass lens and the flat glass surface on which it is placed. Because the glass of the lens is curved, the thickness of this air film does not increase linearly. The farther a point is from the center, the less the horizontal distance that corresponds to an increase in vertical thickness of one wavelength. The horizontal distance between two neighboring rings therefore decreases with increasing distance from the center.

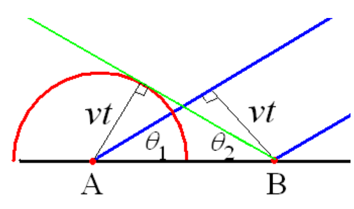


14. These lenses probably are designed to eliminate wavelengths at both the red and the blue ends of the spectrum. The thickness of the coating is designed to cause destructive interference for reflected red and blue light. The reflected light then appears yellow-green.

15. The index of refraction of the oil must be less than the index of refraction of the water. If the oil film appears bright at the edge, then the interference between the light reflected from the top of the oil film and from the bottom of the oil film at that point must be constructive. The light reflecting from the top surface (the air/oil interface) undergoes a 180º phase shift since the index of refraction of the oil is greater than that of air. The thickness of the oil film at the edge is negligible, so for there to be constructive interference, the light reflecting from the bottom of the oil film (the oil/water interface) must also undergo a 180º phase shift. This will occur only if the index of refraction of the oil is less than that of the water.

Solutions to Problems

1. Consider a wave front traveling at an angle θ_1 relative to a surface. At time $t = 0$, the wave front touches the surface at point A, as shown in the figure. After a time *t*, the wave front, moving at speed *v*, has moved forward such that the contact position has moved to point B. The distance between the two contact points is calculated using simple geometry: $\text{AB} = \dfrac{vt}{\sin\theta_1}$.



By Huygens' principle, at each point the wave front touches the surface, it creates a new wavelet. These wavelets expand out in all directions at speed *v*. The line passing through the surface of each of these wavelets is the reflected wave front. Using the radius of the wavelet created at $t = 0$, the center of the wavelet created at time *t*, and the distance between the two contact points (AB) we create a right triangle. Dividing the radius of the wavelet centered at AB (*vt*) by distance between the contact points gives the sine of the angle between the contact surface and the reflected wave, θ_2.

$$\sin\theta_2 = \frac{vt}{\text{AB}} = \frac{vt}{\dfrac{vt}{\sin\theta_1}} = \sin\theta_1 \quad \rightarrow \quad \boxed{\theta_2 = \theta_1}$$

Since these two angles are equal, their complementary angles (the incident and reflected angles) are also equal.

2. For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 34-2a. Apply this to the fifth order.

$$d\sin\theta = m\lambda \quad \rightarrow \quad \lambda = \frac{d\sin\theta}{m} = \frac{\left(1.8\times 10^{-5}\,\text{m}\right)\sin 9.8^\circ}{5} = \boxed{6.1\times 10^{-7}\,\text{m}}$$

3. For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 34-2a. Apply this to the third order.

$$d\sin\theta = m\lambda \quad \rightarrow \quad d = \frac{m\lambda}{\sin\theta} = \frac{3\left(610\times 10^{-9}\,\text{m}\right)}{\sin 28^\circ} = \boxed{3.9\times 10^{-6}\,\text{m}}$$

4. For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 34-2a. The location on the screen is given by $x = \ell\tan\theta$, as seen in Fig. 34-7(c). For small angles, we have $\sin\theta \approx \tan\theta \approx x/\ell$. Adjacent fringes will have $\Delta m = 1$.

$$d\sin\theta = m\lambda \quad \rightarrow \quad d\frac{x}{\ell} = m\lambda \quad \rightarrow \quad x = \frac{\lambda m\ell}{d}$$

$$x_1 = \frac{\lambda m_1\ell}{d} \;;\; x_2 = \frac{\lambda(m+1)\ell}{d} \quad \rightarrow \quad \Delta x = x_2 - x_1 = \frac{\lambda(m+1)\ell}{d} - \frac{\lambda m\ell}{d} = \frac{\lambda\ell}{d}$$

$$\lambda = \frac{d\Delta x}{\ell} = \frac{\left(4.8\times 10^{-5}\,\text{m}\right)\left(0.085\,\text{m}\right)}{6.00\,\text{m}} = \boxed{6.8\times 10^{-7}\,\text{m}} \;;\; f = \frac{c}{\lambda} = \frac{3.00\times 10^{8}\,\text{m/s}}{6.8\times 10^{-7}\,\text{m}} = \boxed{4.4\times 10^{14}\,\text{Hz}}$$

5. For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 34-2a. The location on the screen is given by $x = \ell\tan\theta$, as seen in Fig. 34-7(c). For small angles, we have $\sin\theta \approx \tan\theta \approx x/\ell$. Second order means $m = 2$.

$$d\sin\theta = m\lambda \quad \rightarrow \quad d\frac{x}{\ell} = m\lambda \quad \rightarrow \quad x = \frac{\lambda m\ell}{d} \;;\; x_1 = \frac{\lambda_1 m\ell}{d} \;;\; x_2 = \frac{\lambda_2 m\ell}{d} \quad \rightarrow$$

$$\Delta x = x_2 - x_1 = \frac{(\lambda_2 - \lambda_1)m\ell}{d} = \frac{\left[(720-660)\times 10^{-9}\,\text{m}\right](2)(1.0\,\text{m})}{\left(6.8\times 10^{-4}\,\text{m}\right)} = 1.76\times 10^{-4}\,\text{m} \approx \boxed{0.2\,\text{mm}}$$

This justifies using the small angle approximation, since $x \ll \ell$.

6. The slit spacing and the distance from the slits to the screen is the same in both cases. The distance between bright fringes can be taken as the position of the first bright fringe (m = 1) relative to the central fringe. We indicate the lab laser with subscript 1, and the laser pointer with subscript 2. For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 34-2a. The location on the screen is given by $x = \ell\tan\theta$, as seen in Fig. 34-7(c). For small angles, we have $\sin\theta \approx \tan\theta \approx x/\ell$.

$$d\sin\theta = m\lambda \quad \rightarrow \quad d\frac{x}{\ell} = m\lambda \quad \rightarrow \quad x = \frac{\lambda m\ell}{d} \;;\; x_1 = \frac{\lambda_1\ell}{d} \;;\; x_2 = \frac{\lambda_2\ell}{d} \quad \rightarrow$$

$$\lambda_2 = \frac{d}{\ell}x_2 = \frac{\lambda_1}{x_1}x_2 = \left(632.8\,\text{nm}\right)\frac{5.14\,\text{mm}}{5.00\,\text{mm}} = 650.52\,\text{nm} \approx \boxed{651\,\text{nm}}$$

7. Using a ruler on Fig. 35-9a, the distance from the $m=0$ fringe to the $m=10$ fringe is found to be about 13.5 mm. For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 34-2a. The location on the screen is given by $x=\ell\tan\theta$, as seen in Fig. 34-7(c). For small angles, we have $\sin\theta\approx\tan\theta\approx x/\ell$.

$$d\sin\theta=m\lambda \;\rightarrow\; d\frac{x}{\ell}=m\lambda \;\rightarrow\; \lambda=\frac{dx}{m\ell}=\frac{dx}{m\ell}=\frac{\left(1.7\times10^{-4}\,\text{m}\right)\left(0.0135\,\text{m}\right)}{\left(10\right)\left(0.35\,\text{m}\right)}=\boxed{6.6\times10^{-7}\,\text{m}}$$

8. For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 34-2a. The location on the screen is given by $x=\ell\tan\theta$, as seen in Fig. 34-7(c). For small angles, we have $\sin\theta\approx\tan\theta\approx x/\ell$.

$$d\sin\theta=m\lambda \;\rightarrow\; d\frac{x}{\ell}=m\lambda \;\rightarrow\; d=\frac{\lambda m\ell}{x}=\frac{\left(680\times10^{-9}\,\text{m}\right)\left(3\right)\left(2.6\,\text{m}\right)}{38\times10^{-3}\,\text{m}}=\boxed{1.4\times10^{-4}\,\text{m}}$$

9. For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 34-2a. The location on the screen is given by $x=\ell\tan\theta$, as seen in Fig. 34-7(c). For small angles, we have $\sin\theta\approx\tan\theta\approx x/\ell$. For adjacent fringes, $\Delta m=1$.

$$d\sin\theta=m\lambda \;\rightarrow\; d\frac{x}{\ell}=m\lambda \;\rightarrow\; x=\frac{\lambda m\ell}{d} \;\rightarrow$$

$$\Delta x=\Delta m\frac{\lambda\ell}{d}=\left(1\right)\frac{\left(633\times10^{-9}\,\text{m}\right)\left(3.8\,\text{m}\right)}{\left(6.8\times10^{-5}\,\text{m}\right)}=0.035\,\text{m}=\boxed{3.5\,\text{cm}}$$

10. For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 34-2a. The location on the screen is given by $x=\ell\tan\theta$, as seen in Fig. 34-7(c). For small angles, we have $\sin\theta\approx\tan\theta\approx x/\ell$.

$$d\sin\theta=m\lambda \;\rightarrow\; d\frac{x}{\ell}=m\lambda \;\rightarrow\; d=\frac{\lambda m\ell}{x}=\frac{\left(633\times10^{-9}\,\text{m}\right)\left(1\right)\left(5.0\,\text{m}\right)}{\left(0.25\,\text{m}\right)}=\boxed{1.3\times10^{-5}\,\text{m}}$$

11. The 180° phase shift produced by the glass is equivalent to a path length of $\frac{1}{2}\lambda$. For constructive interference on the screen, the total path difference is a multiple of the wavelength:

$$\tfrac{1}{2}\lambda+d\sin\theta_{\text{max}}=m\lambda,\; m=0,1,2,\cdots \;\rightarrow\; d\sin\theta_{\text{max}}=\left(m-\tfrac{1}{2}\right)\lambda,\; m=1,2,\cdots$$

We could express the result as $d\sin\theta_{\text{max}}=\left(m+\tfrac{1}{2}\right)\lambda,\; m=0,1,2,\cdots$.

For destructive interference on the screen, the total path difference is

$$\tfrac{1}{2}\lambda+d\sin\theta_{\text{min}}=\left(m+\tfrac{1}{2}\right)\lambda,\; m=0,1,2,\cdots \;\rightarrow\; d\sin\theta_{\text{min}}=m\lambda,\; m=0,1,2,\cdots$$

Thus the pattern is just the **reverse of the usual double-slit pattern**. There will be a dark central line. Every place there was a bright fringe will now have a dark line, and vice versa.

12. We equate the expression from Eq. 34-2a for the second order blue light to Eq. 34-2b, since the slit separation and angle must be the same for the two conditions to be met at the same location.

$$d\sin\theta=m\lambda_b=\left(2\right)\left(480\,\text{nm}\right)=960\,\text{nm} \;;\; d\sin\theta=\left(m'+\tfrac{1}{2}\right)\lambda,\; m'=0,1,2,\cdots$$

$$\left(m'+\tfrac{1}{2}\right)\lambda=960\,\text{nm}\quad m'=0 \;\rightarrow\; \lambda=1920\,\text{nm} \;;\; m'=1 \;\rightarrow\; \lambda=640\,\text{nm}$$

$$m'=2 \;\rightarrow\; \lambda=384\,\text{nm}$$

The only one visible is **640 nm**. 384 nm is near the low-wavelength limit for visible light.

13. For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 34-2a. The location on the screen is given by $x = \ell \tan\theta$, as seen in Fig. 34-7(c). For small angles, we have $\sin\theta \approx \tan\theta \approx x/\ell$. For adjacent fringes, $\Delta m = 1$.

$$d\sin\theta = m\lambda \rightarrow d\frac{x}{\ell} = m\lambda \rightarrow x = \frac{\lambda m\ell}{d} \rightarrow$$

$$\Delta x = \Delta m\frac{\lambda\ell}{d} = (1)\frac{(544\times10^{-9}\,\text{m})(5.0\,\text{m})}{(1.0\times10^{-3}\,\text{m})} = \boxed{2.7\times10^{-3}\,\text{m}}$$

14. An expression is derived for the slit separation from the data for the 500 nm light. That expression is then used to find the location of the maxima for the 650 nm light. For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 34-2a. The location on the screen is given by $x = \ell \tan\theta$, as seen in Fig. 34-7(c). For small angles, we have $\sin\theta \approx \tan\theta \approx x/\ell$.

$$d\sin\theta = m\lambda \rightarrow d\frac{x}{\ell} = m\lambda \rightarrow d = \frac{\lambda m\ell}{x} = \frac{\lambda_1 m_1\ell}{x_1} \rightarrow x = \frac{\lambda m\ell}{d} \rightarrow$$

$$x_2 = \frac{\lambda_2 m_2\ell}{\frac{\lambda_1 m_1\ell}{x_1}} = x_1\frac{\lambda_2 m_2}{\lambda_1 m_1} = (12\,\text{mm})\frac{(650\,\text{nm})(2)}{(500\,\text{nm})(3)} = 10.4\,\text{mm} \approx \boxed{10\,\text{mm}}\ (2\text{ sig. fig.})$$

15. The presence of the water changes the wavelength according to Eq. 34-1, and so we must change λ to $\lambda_n = \lambda/n$. For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 34-2a. The location on the screen is given by $x = \ell\tan\theta$, as seen in Fig. 34-7(c). For small angles, we have $\sin\theta \approx \tan\theta \approx x/\ell$. Adjacent fringes will have $\Delta m = 1$.

$$d\sin\theta = m\lambda_n \rightarrow d\frac{x}{\ell} = m\lambda_n \rightarrow x = \frac{\lambda_n m\ell}{d}\ ;\ x_1 = \frac{\lambda m_1\ell}{d}\ ;\ x_2 = \frac{\lambda(m+1)\ell}{d} \rightarrow$$

$$\Delta x = x_2 - x_1 = \frac{\lambda_n(m+1)\ell}{d} - \frac{\lambda_n m\ell}{d} = \frac{\lambda_n\ell}{d} = \frac{\lambda\ell}{nd} = \frac{(470\times10^{-9}\,\text{m})(0.500\,\text{m})}{(1.33)(6.00\times10^{-5}\,\text{m})} = \boxed{2.94\times10^{-3}\,\text{m}}$$

16. To change the center point from constructive interference to destructive interference, the phase shift produced by the introduction of the plastic must be equivalent to half a wavelength. The wavelength of the light is shorter in the plastic than in the air, so the number of wavelengths in the plastic must be ½ greater than the number in the same thickness of air. The number of wavelengths in the distance equal to the thickness of the plate is the thickness of the plate divided by the appropriate wavelength.

$$N_{\text{plastic}} - N_{\text{air}} = \frac{t}{\lambda_{\text{plastic}}} - \frac{t}{\lambda} = \frac{tn_{\text{plastic}}}{\lambda} - \frac{t}{\lambda} = \frac{t}{\lambda}(n_{\text{plastic}} - 1) = \tfrac{1}{2} \rightarrow$$

$$t = \frac{\lambda}{2(n_{\text{plastic}} - 1)} = \frac{680\,\text{nm}}{2(1.60 - 1)} = \boxed{570\,\text{nm}}$$

17. The intensity is proportional to the square of the amplitude. Let the amplitude at the center due to one slit be E_0. The amplitude at the center with both slits uncovered is $2E_0$.

$$\frac{I_{1\text{ slit}}}{I_{2\text{ slits}}} = \left(\frac{E_0}{2E_0}\right)^2 = \boxed{\frac{1}{4}}$$

Thus the amplitude due to a single slit is one-fourth the amplitude when both slits are open.

18. The intensity as a function of angle from the central maximum is given by Eq. 34-6.

$$I_\theta = I_0 \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right) = \tfrac{1}{2} I_0 \;\rightarrow\; \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right) = \tfrac{1}{2} \;\rightarrow\; \cos\left(\frac{\pi d \sin\theta}{\lambda}\right) = \pm\frac{1}{\sqrt{2}}$$

$$\cos\left(\frac{\pi d \sin\theta}{\lambda}\right) = \pm\frac{1}{\sqrt{2}} \;\rightarrow\; \frac{\pi d \sin\theta}{\lambda} = \cos^{-1}\left(\pm\frac{1}{\sqrt{2}}\right) = 45^\circ \pm n(90^\circ) = \frac{\pi}{4} \pm n\frac{\pi}{2} \;\rightarrow$$

$$2d\sin\theta = \left(\tfrac{1}{2} \pm n\right)\lambda$$

To only consider $\theta \ge 0$, we take just the plus sign.

$$\boxed{2d\sin\theta = \left(n + \tfrac{1}{2}\right)\lambda,\ n = 0, 1, 2, \cdots}$$

[19.] The intensity of the pattern is given by Eq. 34-6. We find the angle where the intensity is half its maximum value.

$$I_\theta = I_0 \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right) = \tfrac{1}{2} I_0 \;\rightarrow\; \cos^2\left(\frac{\pi d \sin\theta_{1/2}}{\lambda}\right) = \tfrac{1}{2} \;\rightarrow\; \cos\left(\frac{\pi d \sin\theta_{1/2}}{\lambda}\right) = \frac{1}{\sqrt{2}} \;\rightarrow$$

$$\frac{\pi d \sin\theta_{1/2}}{\lambda} = \cos^{-1}\frac{1}{\sqrt{2}} = \frac{\pi}{4} \;\rightarrow\; \sin\theta_{1/2} = \frac{\lambda}{4d}$$

If $\lambda \ll d$, then $\sin\theta = \dfrac{\lambda}{4d} \ll 1$ and so $\sin\theta \approx \theta$. This is the angle from the central maximum to the location of half intensity. The angular displacement from the half-intensity position on one side of the central maximum to the half-intensity position on the other side would be twice this.

$$\Delta\theta = 2\theta_{1/2} = 2\frac{\lambda}{4d} = \boxed{\frac{\lambda}{2d}}$$

20. (*a*) The phase difference is given in Eq. 34-4. We are given the path length difference, $d\sin\theta$.

$$\frac{\delta}{2\pi} = \frac{d\sin\theta}{\lambda} \;\rightarrow\; \delta = 2\pi\frac{1.25\lambda}{\lambda} = \boxed{2.50\pi}$$

(*b*) The intensity is given by Eq. 34-6.

$$I = I_0\cos^2\left(\frac{\delta}{2}\right) = I_0\cos^2(1.25\pi) = \boxed{0.500 I_0}$$

21. A doubling of the intensity means that the electric field amplitude has increased by a factor of $\sqrt{2}$. We set the amplitude of the electric field of one slit equal to E_0 and of the other equal to $\sqrt{2}E_0$. We use Eq. 34-3 to write each of the electric fields, where the phase difference, δ, is given by Eq. 34-4. Summing these two electric fields gives the total electric field.

$$E_\theta = E_0\sin\omega t + \sqrt{2}E_0\sin(\omega t + \delta) = E_0\sin\omega t + \sqrt{2}E_0\sin\omega t\cos\delta + \sqrt{2}E_0\cos\omega t\sin\delta$$

$$= E_0\left(1 + \sqrt{2}\cos\delta\right)\sin\omega t + \sqrt{2}E_0\cos\omega t\sin\delta$$

We square the total electric field intensity and integrate over the period to determine the average intensity.

$$\bar{E}_\theta^2 = \frac{1}{T}\int_0^T E_\theta^2\,dt = \frac{1}{T}\int_0^T\left[E_0\left(1+\sqrt{2}\cos\delta\right)\sin\omega t + \sqrt{2}E_0\cos\omega t\sin\delta\right]^2 dt$$

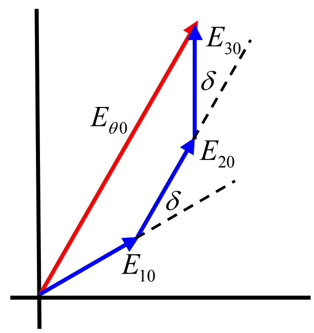
$$= \frac{E_0^2}{T}\int_0^T\left[\left(1+\sqrt{2}\cos\delta\right)^2\sin^2\omega t + 2\cos^2\omega t\sin^2\delta + 2\sqrt{2}\left(1+\sqrt{2}\cos\delta\right)\sin\delta\sin\omega t\cos\omega t\right]dt$$

$$= \frac{E_0^2}{2}\left[\left(1+\sqrt{2}\cos\delta\right)^2 + 2\sin^2\delta\right] = \frac{E_0^2}{2}\left[3+2\sqrt{2}\cos\delta\right]$$

Since the intensity is proportional to this average square of the electric field, and the intensity is maximum when $\delta = 0$, we obtain the relative intensity by dividing the square of the electric field by the maximum square of the electric field.

$$\frac{I_\theta}{I_0} = \frac{\bar{E}_\theta^2}{E_{\delta=0}^2} = \boxed{\frac{3+2\sqrt{2}\cos\delta}{3+2\sqrt{2}}, \text{ with } \delta = \frac{2\pi}{\lambda} d\sin\theta}$$

22. (*a*) If the sources have equal intensities, their electric fields will have the same magnitudes. We show a phasor diagram with each of the electric fields shifted by an angle δ. As shown in the sketch, the three electric fields and their sum form a symmetric trapezoid. Since E_{20} and $E_{\theta 0}$ are parallel, and E_{20} is rotated from E_{10} and E_{30} by the angle δ, the magnitude of $E_{\theta 0}$ is the sum of the components of E_{10}, E_{20}, and E_{30} that are parallel to E_{20}.



$$E_{\theta 0} = E_{10}\cos\delta + E_{20} + E_{30}\cos\delta = E_{10}\left(1+2\cos\delta\right)$$

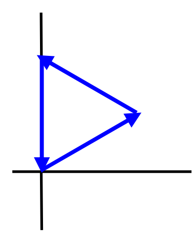
We set the intensity proportional to the square of the electric field magnitude and divide by the maximum intensity (at $\delta = 0$) to determine the relative intensity.

$$\frac{I_\theta}{I_0} = \frac{E_{\theta 0}^2}{E_{\delta=0}^2} = \frac{\left[E_{10}\left(1+2\cos\delta\right)\right]^2}{\left[E_{10}\left(1+2\cos 0\right)\right]^2} = \boxed{\frac{\left(1+2\cos\delta\right)^2}{9}, \ \delta = \frac{2\pi}{\lambda} d\sin\theta}$$

(*b*) The intensity will be at its maximum when $\cos\delta = 1$. In this case the three phasors are all in line.

$$\cos\delta_{\max} = 1 \ \rightarrow \ \delta_{\max} = 2m\pi = \frac{2\pi}{\lambda} d\sin\theta_{\max} \ \rightarrow \ \boxed{\sin\theta_{\max} = \frac{m\lambda}{d}, \quad m = 0, 1, 2, \cdots}$$

The intensity will be a minimum when $1+2\cos\delta = 0$. In this case the three phasors add to 0 and form an equilateral triangle as shown in the second diagram, for the case of $k = 1$, where k is defined below.



$$1 + 2\cos\delta_{\min} = 0 \ \rightarrow$$

$$\delta_{\min} = \cos^{-1}\left(-\tfrac{1}{2}\right) = \begin{cases} \tfrac{2}{3}\pi + 2m\pi = 2\pi\left(m+\tfrac{1}{3}\right) \\ \tfrac{4}{3}\pi + 2m\pi = 2\pi\left(m+\tfrac{2}{3}\right) \end{cases}, \ m = 0, 1, 2, \cdots$$

This can be written as one expression with two parameters.

$$\delta_{\min} = 2\pi\left(m + \tfrac{1}{3}k\right) = \frac{2\pi}{\lambda} d\sin\theta_{\min}, \ k = 1, 2; \ m = 0, 1, 2, \cdots \ \rightarrow$$

$$\boxed{\sin\theta_{\min} = \frac{\lambda}{d}\left(m + \tfrac{1}{3}k\right) = , \ k = 1, 2; \ m = 0, 1, 2, \cdots}$$

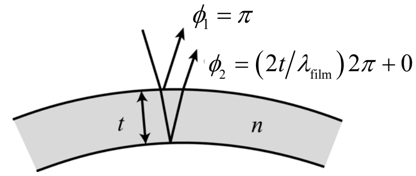
23. From Example 34-7, we see that the thickness is related to the bright color wavelength by $t = \lambda/4n$.

$$t = \lambda/4n \ \rightarrow \ \lambda = 4nt = 4(1.32)(120\,\text{nm}) = \boxed{634\,\text{nm}}$$

24. Between the 25 dark lines there are 24 intervals. When we add the half-interval at the wire end, we have 24.5 intervals over the length of the plates.

$$\frac{28.5\,\text{cm}}{24.5\,\text{intervals}} = \boxed{1.16\,\text{cm}}$$

25. (*a*) An incident wave that reflects from the outer surface of the bubble has a phase change of $\phi_1 = \pi$. An incident wave that reflects from the inner surface of the bubble has a phase change due to the additional path length, so $\phi_2 = \left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi$. For destructive interference with a minimum non-zero thickness of bubble, the net phase change must be π.



$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi\right] - \pi = \pi \quad \rightarrow \quad t = \tfrac{1}{2}\lambda_{\text{film}} = \frac{\lambda}{2n} = \frac{480\,\text{nm}}{2(1.33)} = \boxed{180\,\text{nm}}$$

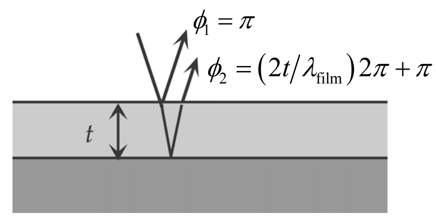
(*b*) For the next two larger thicknesses, the net phase change would be 3π and 5π.

$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi\right] - \pi = 3\pi \quad \rightarrow \quad t = \lambda_{\text{film}} = \frac{\lambda}{n} = \frac{480\,\text{nm}}{(1.33)} = \boxed{361\,\text{nm}}$$

$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi\right] - \pi = 5\pi \quad \rightarrow \quad t = \lambda_{\text{film}} = \frac{\lambda}{n} = \tfrac{3}{2}\frac{480\,\text{nm}}{(1.33)} = \boxed{541\,\text{nm}}$$

(*c*) If the thickness were much less than one wavelength, then there would be very little phase change introduced by additional path length, and so the two reflected waves would have a phase difference of about $\phi_1 = \pi$. This would produce destructive interference.

26. An incident wave that reflects from the top surface of the coating has a phase change of $\phi_1 = \pi$. An incident wave that reflects from the glass $(n \approx 1.5)$ at the bottom surface of the coating has a phase change due to both the additional path length and a phase change of π on reflection, so $\phi_2 = \left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi + \pi$. For constructive interference with a minimum non-zero thickness of coating, the net phase change must be 2π.



$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi + \pi\right] - \pi = 2\pi \quad \rightarrow \quad t = \tfrac{1}{2}\lambda_{\text{film}} = \tfrac{1}{2}\left(\frac{\lambda}{n_{\text{film}}}\right).$$

The lens reflects the most for $\lambda = 570\,\text{nm}$. The minimum non-zero thickness occurs for $m = 1$:

$$t_{\min} = \frac{\lambda}{2n_{\text{film}}} = \frac{(570\,\text{nm})}{2(1.25)} = \boxed{228\,\text{nm}}$$

Since the middle of the spectrum is being selectively reflected, the transmitted light will be stronger in the red and blue portions of the visible spectrum.

27. (*a*) When illuminated from above at A, a light ray reflected from the air-oil interface undergoes a phase shift of $\phi_1 = \pi$. A ray reflected at the oil-water interface undergoes no phase shift. If the oil thickness at A is negligible compared to the wavelength of the light, then there is no significant shift in phase due to a path distance traveled by a ray in the oil. Thus the light reflected from the two surfaces will destructively interfere for all visible wavelengths, and the oil will appear black when viewed from above.

(*b*) From the discussion in part (*a*), the ray reflected from the air-oil interface undergoes a phase shift of $\phi_1 = \pi$. A ray that reflects from the oil-water interface has no phase change due to reflection, but has a phase change due to the additional path length of $\phi_2 = \left(\frac{2t}{\lambda_{\text{oil}}}\right)2\pi.$ For constructive interference, the net phase change must be a multiple of $2\pi.$

$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda_{\text{oil}}}\right)2\pi\right] - \pi = m(2\pi) \rightarrow t = \tfrac{1}{2}\left(m+\tfrac{1}{2}\right)\lambda_{\text{oil}} = \tfrac{1}{2}\left(m+\tfrac{1}{2}\right)\frac{\lambda}{n_{\text{o}}}$$

From the diagram, we see that point B is the second thickness that yields constructive interference for 580 nm, and so we use $m = 1$. (The first location that yields constructive interference would be for $m = 0$.)

$$t = \tfrac{1}{2}\left(m+\tfrac{1}{2}\right)\frac{\lambda}{n_{\text{o}}} = \tfrac{1}{2}\left(1+\tfrac{1}{2}\right)\frac{580\,\text{nm}}{1.50} = \boxed{290\,\text{nm}}$$

28. When illuminated from above, the light ray reflected from the air-oil interface undergoes a phase shift of $\phi_1 = \pi.$ A ray reflected at the oil-water interface undergoes no phase shift due to reflection, but has a phase change due to the additional path length of $\phi_2 = \left(\frac{2t}{\lambda_{\text{oil}}}\right)2\pi.$ For constructive interference to occur, the net phase change must be a multiple of $2\pi.$

$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda_{\text{oil}}}\right)2\pi\right] - \pi = m(2\pi) \rightarrow t = \tfrac{1}{2}\left(m+\tfrac{1}{2}\right)\lambda_{\text{oil}} = \tfrac{1}{2}\left(m+\tfrac{1}{2}\right)\frac{\lambda}{n_{\text{o}}}$$

For $\lambda = 650\,\text{nm}$, the possible thicknesses are as follows.

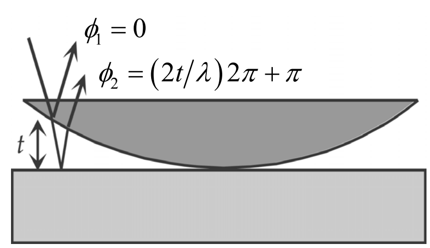
$$t_{650} = \tfrac{1}{2}\left(m+\tfrac{1}{2}\right)\frac{650\,\text{nm}}{1.50} = 108\,\text{nm},\ 325\,\text{nm},\ 542\,\text{nm},\ \cdots$$

For $\lambda = 390\,\text{nm}$, the possible thicknesses are as follows.

$$t_{390} = \tfrac{1}{2}\left(m+\tfrac{1}{2}\right)\frac{390\,\text{nm}}{1.50} = 65\,\text{nm},\ 195\,\text{nm},\ 325\,\text{nm},\ 455\,\text{nm},\ \cdots$$

The minimum thickness of the oil slick must be $\boxed{325\,\text{nm}}$.

29. An incident wave that reflects from the convex surface of the lens has no phase change, so $\phi_1 = 0.$ An incident wave that reflects from the glass underneath the lens has a phase change due to both the additional path length and a phase change of π on reflection, so $\phi_2 = \left(\frac{2t}{\lambda}\right)2\pi + \pi.$ For destructive interference (dark rings), the net phase change must be an odd-integer multiple of π, so $\phi_{\text{net}} = \phi_2 - \phi_1 = (2m+1)\pi,\ m = 0, 1, 2, \cdots.$ Because $m = 0$ corresponds to the dark center, m represents the number of the ring.

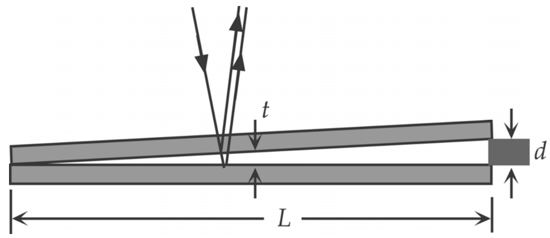


$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda}\right)2\pi + \pi\right] - 0 = (2m+1)\pi,\ m = 0, 1, 2, \cdots \rightarrow$$

$$t = \tfrac{1}{2}m\lambda_{\text{air}} = \tfrac{1}{2}(31)(560\,\text{nm}) = 8680\,\text{nm} = \boxed{8.68\,\mu\text{m}}$$

The thickness of the lens is the thickness of the air at the edge of the lens:

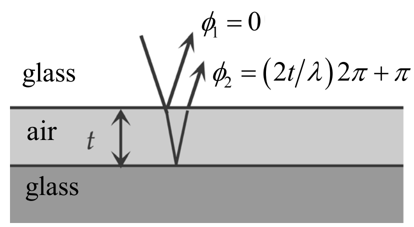
30. An incident wave that reflects from the second surface of the upper piece of glass has no phase change, so $\phi_1 = 0$. An incident wave that reflects from the first surface of the second piece of glass has a phase change due to both the additional path length and a phase change of π on reflection, so $\phi_2 = \left(\frac{2t}{\lambda}\right)2\pi + \pi$. For destructive interference (dark lines), the net phase change must be an odd-integer multiple of π, so $\phi_{net} = \phi_2 - \phi_1 = (2m+1)\pi,\ m = 0, 1, 2, \cdots$. Because $m = 0$ corresponds to the left edge of the diagram, the 28th dark line corresponds to $m = 27$. The 28th dark line also has a gap thickness of d.



$$\phi_{net} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda}\right)2\pi + \pi\right] - 0 = (2m+1)\pi \ \rightarrow\ t = \tfrac{1}{2}m\lambda \ \rightarrow$$

$$d = \tfrac{1}{2}(27)(670\,\text{nm}) = 9045\,\text{nm} \approx \boxed{9.0\,\mu\text{m}}$$

31. With respect to the incident wave, the wave that reflects from the air at the top surface of the air layer has a phase change of $\phi_1 = 0$. With respect to the incident wave, the wave that reflects from the glass at the bottom surface of the air layer has a phase change due to both the additional path length and reflection, so $\phi_2 = \left(\frac{2t}{\lambda}\right)2\pi + \pi$. For constructive interference, the net phase change must be an even non-zero integer multiple of π.



$$\phi_{net} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda}\right)2\pi + \pi\right] - 0 = 2m\pi \ \rightarrow\ t = \tfrac{1}{2}\left(m - \tfrac{1}{2}\right)\lambda,\ m = 1, 2, \cdots.$$

The minimum thickness is with $m = 1$.

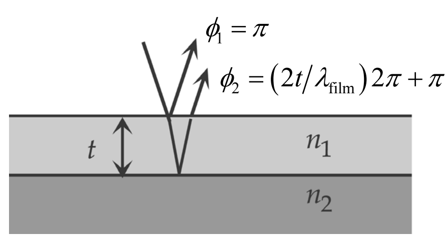
$$t_{min} = \tfrac{1}{2}(450\,\text{nm})\left(1 - \tfrac{1}{2}\right) = \boxed{113\,\text{nm}}$$

For destructive interference, the net phase change must be an odd-integer multiple of π.

$$\phi_{net} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda}\right)2\pi + \pi\right] - 0 = (2m+1)\pi \ \rightarrow\ t = \tfrac{1}{2}m\lambda,\ = 0, 1, 2, \cdots$$

The minimum non-zero thickness is $t_{min} = \tfrac{1}{2}(450\,\text{nm})(1) = \boxed{225\,\text{nm}}$.

32. With respect to the incident wave, the wave that reflects from the top surface of the alcohol has a phase change of $\phi_1 = \pi$. With respect to the incident wave, the wave that reflects from the glass at the bottom surface of the alcohol has a phase change due to both the additional path length and a phase change of π on reflection, so $\phi_2 = \left(\frac{2t}{\lambda_{film}}\right)2\pi + \pi$. For constructive interference, the net phase change must be an even non-zero integer multiple of π.



$$\phi_{net} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda_{1film}}\right)2\pi + \pi\right] - \pi = m_1 2\pi \ \rightarrow\ t = \tfrac{1}{2}\lambda_{1film}m_1 = \tfrac{1}{2}\frac{\lambda_1}{n_{film}}m_1,\ m_1 = 1, 2, 3, \ldots.$$

For destructive interference, the net phase change must be an odd-integer multiple π.

$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda_{2\,\text{film}}}\right)2\pi + \pi\right] - \pi = (2m_2+1)\pi \quad \rightarrow \quad t = \tfrac{1}{4}\frac{\lambda_2}{n_{\text{film}}}(2m_2+1),\ m_2 = 0, 1, 2, \ldots .$$

Set the two expressions for the thickness equal to each other.

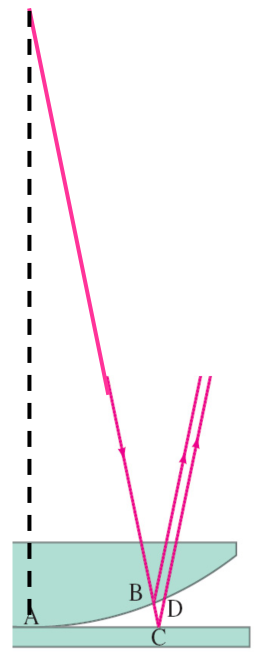
$$\tfrac{1}{2}\frac{\lambda_1}{n_{\text{film}}}m_1 = \tfrac{1}{4}\frac{\lambda_2}{n_{\text{film}}}(2m_2+1) \quad \rightarrow \quad \frac{2m_2+1}{2m_1} = \frac{\lambda_1}{\lambda_2} = \frac{(635\,\text{nm})}{(512\,\text{nm})} = 1.24 \approx 1.25 = \frac{5}{4}$$

Thus we see that $m_1 = m_2 = 2,$ and the thickness of the film is

$$t = \tfrac{1}{2}\frac{\lambda_1}{n_{\text{film}}}m_1 = \tfrac{1}{2}\left(\frac{635\,\text{nm}}{1.36}\right)(2) = \boxed{467\,\text{nm}} \text{ or } t = \tfrac{1}{4}\frac{\lambda_2}{n_{\text{film}}}(2m_2+1) = \tfrac{1}{4}\left(\frac{512\,\text{nm}}{1.36}\right)(5) = \boxed{471\,\text{nm}}$$

With 2 sig.fig., the thickness is 470 nm. The range of answers is due to rounding λ_1/λ_2.

33. With respect to the incident wave, the wave that reflects from point B in the first diagram will not undergo a phase change, and so $\phi_B = 0$. With respect to the incident wave, the wave that reflects from point C in the first diagram has a phase change due to both the additional path length in air, and a phase change of π on reflection, and so we say that $\phi_D = \frac{2y}{\lambda}(2\pi) + \pi$, where y is the thickness of the air gap from B to C (or C to D). For dark rings, the net phase difference of the waves that recombine as they leave the glass moving upwards must be an odd-integer multiple of π.

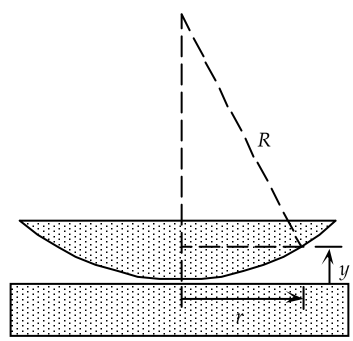


$$\phi_{\text{net}} = \phi_D - \phi_B = \frac{2y}{\lambda}(2\pi) + \pi = (2m+1)\pi \quad \rightarrow$$

$$y_{\text{dark}} = \tfrac{1}{2}m\lambda,\ m = 0, 1, 2, \cdots$$

Because $m = 0$ corresponds to the dark center, m represents the number of the dark ring.

Let the air gap of y be located a horizontal distance r from the center of the lens, as seen in the second diagram. Consider the dashed right triangle in the second diagram.



$$R^2 = r^2 + (R-y)^2 \quad \rightarrow$$

$$R^2 = r^2 + R^2 - 2Ry + y^2 \quad \rightarrow$$

$$r^2 = 2Ry - y^2$$

If we assume that $y \ll R$, then $r^2 \approx 2Ry$.

$$r^2 = 2Ry \quad \rightarrow \quad r_{\text{dark}}^2 = 2Ry_{\text{dark}} = 2R\left(\tfrac{1}{2}m\lambda\right) \quad \rightarrow \quad \boxed{r_{\text{dark}} = \sqrt{m\lambda R},\ m = 0, 1, 2, \cdots}$$

34. From Problem 33, we have $r = \sqrt{m\lambda R} = (m\lambda R)^{1/2}$. To find the distance between adjacent rings, we assume $m >> 1 \ \rightarrow \ \Delta m = 1 \ll m$. Since $\Delta m \ll m,\ \Delta r \approx \frac{dr}{dm}\Delta m.$

$$r = (m\lambda R)^{1/2} \quad ; \quad \frac{dr}{dm} = \tfrac{1}{2}(m\lambda R)^{-1/2}\lambda R$$

$$\Delta r \approx \frac{dr}{dm}\Delta m = \left[\tfrac{1}{2}(m\lambda R)^{-1/2}\lambda R\right](1) = \left[\frac{\lambda^2 R^2}{4m\lambda R}\right]^{1/2} = \boxed{\sqrt{\frac{\lambda R}{4m}}}$$

35. The radius of the *m*-th ring in terms of the wavelength of light and the radius of curvature is derived in Problem 33 as $r = \sqrt{m\lambda R}$. Using this equation, with the wavelength of light in the liquid given by Eq. 34-1, we divide the two radii and solve for the index of refraction.

$$\frac{r_{\text{air}}}{r_{\text{liquid}}} = \frac{\sqrt{m\lambda R}}{\sqrt{m(\lambda/n)R}} = \sqrt{n} \rightarrow n = \left(\frac{r_{\text{air}}}{r_{\text{liquid}}}\right)^2 = \left(\frac{2.92\text{ cm}}{2.54\text{ cm}}\right)^2 = \boxed{1.32}$$

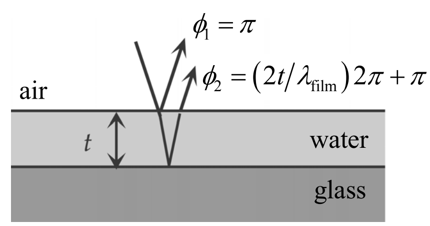
36. We use the equation derived in Problem 33, where *r* is the radius of the lens (1.7 cm) to solve for the radius of curvature. Since the outer edge is the 44th bright ring, which would be halfway between the 44th and 45th dark fringes, we set *m*=44.5

$$r = \sqrt{m\lambda R} \rightarrow R = \frac{r^2}{m\lambda} = \frac{(0.017\,\text{m})^2}{(44.5)(580\times10^{-9}\,\text{m})} = 11.20\,\text{m} \approx \boxed{11\,\text{m}}$$

We calculate the focal length of the lens using Eq. 33-4 (the lensmaker's equation) with the index of refraction of lucite taken from Table 32-1.

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right) = (1.51-1)\left(\frac{1}{11.2\,\text{m}}+\frac{1}{\infty}\right) = 0.0455\,\text{m}^{-1} \rightarrow f = \frac{1}{0.0455\,\text{m}^{-1}} = \boxed{22\,\text{m}}$$

37. (*a*) Assume the indices of refraction for air, water, and glass are 1.00, 1.33, and 1.50, respectively. When illuminated from above, a ray reflected from the air-water interface undergoes a phase shift of $\phi_1 = \pi$, and a ray reflected at the water-glass interface also undergoes a phase shift of π. Thus, the two rays are unshifted in phase relative to each other due to reflection. For constructive interference, the path difference $2t$ must equal an integer number of wavelengths in water.



$$2t = m\lambda_{\text{water}} = m\frac{\lambda}{n_{\text{water}}},\ m = 0, 1, 2, \cdots \rightarrow \boxed{\lambda = \frac{2n_{\text{water}}t}{m}}$$

(*b*) The above relation can be solved for the *m*-value associated with the reflected color. If this *m*-value is an integer the wavelength undergoes constructive interference upon reflection.

$$\lambda = \frac{2n_{\text{water}}t}{m} \rightarrow m = \frac{2n_{\text{water}}t}{\lambda}$$

For a thickness $t = 200\,\mu\text{m} = 2\times10^5\,\text{nm}$ the *m*-values for the two wavelengths are calculated.

$$m_{700\,\text{nm}} = \frac{2n_{\text{water}}t}{\lambda} = \frac{2(1.33)(2\times10^5\,\text{nm})}{700\,\text{nm}} = 760$$

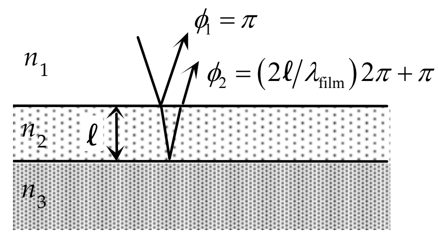
$$m_{400\,\text{nm}} = \frac{2n_{\text{water}}t}{\lambda} = \frac{2(1.33)(2\times10^5\,\text{nm})}{400\,\text{nm}} = 1330$$

Since both wavelengths yield integers for m, they are both reflected.

(*c*) All *m*-values between *m* = 760 and *m* = 1330 will produce reflected visible colors. There are 1330 – (760 – 1) = $\boxed{571}$ such values.

(*d*) This mix of a large number of wavelengths from throughout the visible spectrum will give the thick layer a white or grey appearance.

38. We assume $n_1 < n_2 < n_3$ and that most of the incident light is transmitted. If the amplitude of an incident ray is taken to be E_0, then the amplitude of a reflected ray is rE_0, with $r << 1$. The light reflected from the top surface of the film therefore has an amplitude of rE_0 and is phase shifted by $\phi_1 = \pi$ from the incident wave, due to the higher index of refraction. The light transmitted at that top surface has an amplitude of $(1-r)E_0$. That light is then reflected off the bottom surface of the film, and we assume that it has the same reflection coefficient. Thus the amplitude of that second reflected ray is $r(1-r)E_0 = (r-r^2)E_0 \approx rE_0$, the same amplitude as the first reflected ray. Due to traveling through the film and reflecting from the glass, the second ray has a phase shift of $\phi_2 = \pi + 2\pi(2\ell/\lambda_{\text{film}}) = \pi + 4\pi\ell n_2/\lambda$, where ℓ is the thickness of the film. Summing the two reflected rays gives the net reflected wave.



$$E = rE_0\cos(\omega t + \pi) + rE_0\cos(\omega t + \pi + 4\pi\ell/\lambda_n)$$
$$= rE_0\left[(1+\cos 4\pi\ell n_2/\lambda)\cos(\omega t+\pi) - \sin(4\pi\ell n_2/\lambda)\sin(\omega t+\pi)\right]$$

As with the double slit experiment, we set the intensity proportional to the square of the wave amplitude and integrate over one period to calculate the average intensity.

$$I \propto \frac{1}{T}\int_0^T E^2 dt = \frac{1}{T}\int_0^T \left[rE_0\left[(1+\cos 4\pi\ell n_2/\lambda)\cos(\omega t+\pi) - \sin(4\pi\ell n_2/\lambda)\sin(\omega t+\pi)\right]\right]^2 dt$$
$$= \frac{r^2E_0^2}{T}\int_0^T \begin{bmatrix}(1+\cos 4\pi\ell n_2/\lambda)^2\cos^2(\omega t+\pi) + \sin^2(4\pi\ell n_2/\lambda)\sin^2(\omega t+\pi) \\ -2(1+\cos 4\pi\ell n_2/\lambda)\sin(4\pi\ell n_2/\lambda)\cos(\omega t+\pi)\sin(\omega t+\pi)\end{bmatrix} dt$$
$$= \frac{r^2E_0^2}{2}\left[(1+\cos 4\pi\ell n_2/\lambda)^2 + \sin^2(4\pi\ell n_2/\lambda)\right] = r^2E_0^2(1+\cos 4\pi\ell n_2/\lambda)$$

The reflected intensity without the film is proportional to the square of the intensity of the single reflected electric field.

$$I_0 \propto \frac{1}{T}\int_0^T E_{\text{no film}}^2 dt = \frac{1}{T}\int_0^T \left[rE_0\cos(\omega t+\pi)\right]^2 dt = \frac{r^2E_0^2}{T}\int_0^T \left[\cos^2(\omega t+\pi)\right]dt = \frac{r^2E_0^2}{2}$$

Dividing the intensity with the film to that without the film gives the factor by which the intensity is reduced.

$$\frac{I}{I_0} = \frac{r^2E_0^2(1+\cos 4\pi\ell n_2/\lambda)}{\frac{1}{2}r^2E_0^2} = 2(1+\cos 4\pi\ell n_2/\lambda)$$

To determine the thickness of the film, the phase difference between the two reflected waves with $\lambda = 550\,\text{nm}$ must be an odd integer multiple of π so that there is destructive interference. The minimum thickness will be for *m* = 0.

$$\phi_2 - \phi_1 = [\pi + 4\pi\ell n_2/\lambda] - \pi = (2m+1)\pi \;\rightarrow\; \ell = \frac{\lambda_n}{4} = \frac{\lambda}{4n} = \frac{550\,\text{nm}}{4n}$$

It is interesting to see that the same result is obtained if we set the reflected intensity equal to zero for a wavelength of 550 nm.

$$\frac{I}{I_0} = 2(1+\cos 4\pi\ell n_2/\lambda) = 0 \;\rightarrow\; \cos 4\pi\ell n_2/\lambda = -1 \;\rightarrow\; 4\pi\ell n_2/\lambda = \pi \;\rightarrow\; \ell = \frac{550\,\text{nm}}{4n}$$

Finally, we insert the two given wavelengths (430 nm and 670 nm) into the intensity equation to determine the reduction in intensities.

For $\lambda = 430$ nm, $\frac{I}{I_0} = 2\left(1+\cos 4\pi \frac{550\text{ nm}/4n}{430\text{ nm}/n}\right) = 2\left(1+\cos 4\pi \frac{550\text{ nm}}{4(430\text{ nm})}\right) = 0.721 \approx \boxed{72\%}$

For $\lambda = 670$ nm, $\frac{I}{I_0} = 2\left(1+\cos 4\pi \frac{550\text{ nm}}{4(670\text{ nm})}\right) = 0.308 \approx \boxed{31\%}$

39. From the discussion in section 34-6, we see that the path length change is twice the distance that the mirror moves. One fringe shift corresponds to a change in path length of λ, and so corresponds to a mirror motion of $\frac{1}{2}\lambda$. Let N be the number of fringe shifts produced by a mirror movement of Δx.

$$N = \frac{\Delta x}{\frac{1}{2}\lambda} \rightarrow \Delta x = \tfrac{1}{2}N\lambda = \tfrac{1}{2}(650)(589\times10^{-9}\text{ m}) = \boxed{1.91\times10^{-4}\text{ m}}$$

40. From the discussion in section 34-6, we see that the path length change is twice the distance that the mirror moves. One fringe shift corresponds to a change in path length of λ, and so corresponds to a mirror motion of $\frac{1}{2}\lambda$. Let N be the number of fringe shifts produced by a mirror movement of Δx.

$$N = \frac{\Delta x}{\frac{1}{2}\lambda} \rightarrow \lambda = \frac{2\Delta x}{N} = \frac{2(1.25\times10^{-4}\text{ m})}{384} = 6.51\times10^{-7}\text{ m} = \boxed{651\text{ nm}}$$

41. From the discussion in section 34-6, we see that the path length change is twice the distance that the mirror moves. One fringe shift corresponds to a change in path length of λ, and so corresponds to a mirror motion of $\frac{1}{2}\lambda$. Let N be the number of fringe shifts produced by a mirror movement of Δx. The thickness of the foil is the distance that the mirror moves during the 272 fringe shifts.

$$N = \frac{\Delta x}{\frac{1}{2}\lambda} \rightarrow \Delta x = \tfrac{1}{2}N\lambda = \tfrac{1}{2}(272)(589\times10^{-9}\text{ m}) = \boxed{8.01\times10^{-5}\text{ m}}$$

42. One fringe shift corresponds to an effective change in path length of λ. The actual distance has not changed, but the number of wavelengths in the depth of the cavity has. If the cavity has a length d, the number of wavelengths in vacuum is $\frac{d}{\lambda}$, and the (greater) number with the gas present is $\frac{d}{\lambda_{\text{gas}}} = \frac{n_{\text{gas}}d}{\lambda}$. Because the light passes through the cavity twice, the number of fringe shifts is twice the difference in the number of wavelengths in the two media.

$$N = 2\left(\frac{n_{\text{gas}}d}{\lambda} - \frac{d}{\lambda}\right) = 2\frac{d}{\lambda}(n_{\text{gas}}-1) \rightarrow n_{\text{gas}} = \frac{N\lambda}{2d} + 1 = \frac{(176)(632.8\times10^{-9}\text{ m})}{2(1.155\times10^{-2}\text{ m})} + 1 = \boxed{1.00482}$$

$\boxed{43.}$ There are two interference patterns formed, one by each of the two wavelengths. The fringe patterns overlap but do not interfere with each other. Accordingly, when the bright fringes of one pattern occurs at the same locations as the dark fringes of the other patterns, there will be no fringes seen, since there will be no dark bands to distinguish one fringe from the adjacent fringes.

To shift from one "no fringes" occurrence to the next, the mirror motion must produce an integer number of fringe shifts for each wavelength, and the number of shifts for the shorter wavelength must be one more than the number for the longer wavelength. From the discussion in section 34-6, we see that the path length change is twice the distance that the mirror moves. One fringe shift corresponds to a change in path length of λ, and so corresponds to a mirror motion of $\frac{1}{2}\lambda$. Let *N* be the number of fringe shifts produced by a mirror movement of Δx.

$$N_1 = 2\frac{\Delta x}{\lambda_1} \;;\; N_2 = 2\frac{\Delta x}{\lambda_2} \;;\; N_2 = N_1 + 1 \;\rightarrow\; 2\frac{\Delta x}{\lambda_2} = 2\frac{\Delta x}{\lambda_1} + 1 \;\rightarrow$$

$$\Delta x = \frac{\lambda_1 \lambda_2}{2(\lambda_1 - \lambda_2)} = \frac{(589.6\,\text{nm})(589.0\,\text{nm})}{2(0.6\,\text{nm})} = 2.89 \times 10^5\,\text{nm} \approx \boxed{2.9 \times 10^{-4}\,\text{m}}$$

44. We assume the luminous flux is uniform, and so is the same in all directions.

$$F_\ell = E_\ell A = E_\ell 4\pi r^2 = \left(10^5\,\text{lm/m}^2\right)4\pi\left(1.496 \times 10^{11}\,\text{m}\right)^2 = 2.81 \times 10^{28}\,\text{lm} \approx \boxed{3 \times 10^{28}\,\text{lm}}$$

$$I_\ell = \frac{F_\ell}{4\pi\,\text{sr}} = \frac{2.81 \times 10^{28}\,\text{lm}}{4\pi\,\text{sr}} = 2.24 \times 10^{27}\,\text{cd} \approx \boxed{2 \times 10^{27}\,\text{cd}}$$

45. (*a*) The wattage of the bulb is the electric power input to the bulb.

$$\text{luminous efficiency} = \frac{F_\ell}{P} = \frac{1700\,\text{lm}}{100\,\text{W}} = \boxed{17\,\text{lm/W}}$$

(*b*) The illuminance is the luminous flux incident on a surface, divided by the area of the surface. Let *N* represent the number of lamps, each contributing an identical amount of luminous flux.

$$E_\ell = \frac{F_\ell}{A} = \frac{N\left[\frac{1}{2}(\text{luminous efficiency})P\right]}{A} \;\rightarrow$$

$$N = \frac{2E_\ell A}{(\text{luminous efficiency})P} = \frac{2\left(250\,\text{lm/m}^2\right)(25\,\text{m})(30\,\text{m})}{(60\,\text{lm/W})(40\,\text{W})} = 156\,\text{lamps} \approx \boxed{160\,\text{lamps}}$$

46. (*a*) For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 34-2a. The location on the screen is given by $x = \ell \tan\theta$, as seen in Fig. 34-7(c). For small angles, we have $\sin\theta \approx \tan\theta \approx x/\ell$. For adjacent fringes, $\Delta m = 1$.

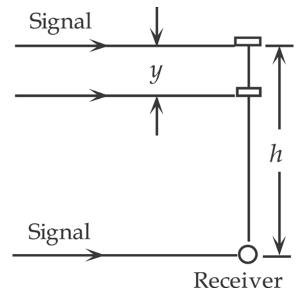
$$d\sin\theta = m\lambda \;\rightarrow\; d\frac{x}{\ell} = m\lambda \;\rightarrow\; x = \frac{\lambda m \ell}{d} \;\rightarrow\; \Delta x = \Delta m \frac{\lambda \ell}{d} \;\rightarrow$$

$$d = \frac{\lambda \ell \Delta m}{\Delta x} = \frac{\left(5.0 \times 10^{-7}\,\text{m}\right)(4.0\,\text{m})(1)}{\left(2.0 \times 10^{-2}\,\text{m}\right)} = \boxed{1.0 \times 10^{-4}\,\text{m}}$$

(*b*) For minima, we use Eq. 34-2b. The fourth-order minimum corresponds to *m* = 3, and the fifth-order minimum corresponds to *m* = 4. The slit separation, screen distance, and location on the screen are the same for the two wavelengths.

$$d\sin\theta = \left(m + \tfrac{1}{2}\right)\lambda \;\rightarrow\; d\frac{x}{\ell} = \left(m + \tfrac{1}{2}\right)\lambda \;\rightarrow\; \left(m_A + \tfrac{1}{2}\right)\lambda_A = \left(m_B + \tfrac{1}{2}\right)\lambda_B \;\rightarrow$$

$$\lambda_B = \lambda_A \frac{\left(m_A + \frac{1}{2}\right)}{\left(m_B + \frac{1}{2}\right)} = \left(5.0 \times 10^{-7}\,\text{m}\right)\frac{3.5}{4.5} = \boxed{3.9 \times 10^{-7}\,\text{m}}$$



47. The wavelength of the signal is $\lambda = \dfrac{v}{f} = \dfrac{\left(3.00 \times 10^{8}\ \text{m/s}\right)}{\left(75 \times 10^{6}\ \text{Hz}\right)} = 4.00\,\text{m}.$

(*a*) There is a phase difference between the direct and reflected signals from both the path difference, $\left(\dfrac{h}{\lambda}\right)2\pi$, and the reflection, π. The total phase difference is the sum of the two.

$$\phi = \left(\frac{h}{\lambda}\right)2\pi + \pi = \frac{(122\,\text{m})}{(4.00\,\text{m})}2\pi + \pi = 62\pi = 31(2\pi)$$

Since the phase difference is an integer multiple of 2π, the interference is **constructive.**

(*b*) When the plane is 22 m closer to the receiver, the phase difference is as follows.

$$\phi = \left[\frac{(h-y)}{\lambda}\right]2\pi + \pi = \left[\frac{(122\,\text{m} - 22\,\text{m})}{(4.00\,\text{m})}\right]2\pi + \pi = 51\pi = \frac{51}{2}(2\pi)$$

Since the phase difference is an odd-half-integer multiple of 2π, the interference is **destructive.**

48. Because the measurements are made far from the antennas, we can use the analysis for the double slit. Use Eq. 34-2a for constructive interference, and 34-2b for destructive interference. The wavelength of the signal is $\lambda = \dfrac{v}{f} = \dfrac{\left(3.00\times10^{8}\ \text{m/s}\right)}{\left(88.5\times10^{6}\,\text{Hz}\right)} = 3.39\,\text{m}.$

For constructive interference, the path difference is a multiple of the wavelength:

$$d\sin\theta = m\lambda,\ m = 0, 1, 2, 3, \ldots;\quad \to\quad \theta = \sin^{-1}\frac{m\lambda}{d}$$

$$\theta_{1\,\text{max}} = \sin^{-1}\frac{(1)(3.39\,\text{m})}{9.0\,\text{m}} = \boxed{22^\circ}\ ;\ \theta_{2\,\text{max}} = \sin^{-1}\frac{(2)(3.39\,\text{m})}{9.0\,\text{m}} = \boxed{49^\circ}\ ;$$

$$\theta_{3\,\text{max}} = \sin^{-1}\frac{(3)(3.39\,\text{m})}{9.0\,\text{m}} = \text{impossible}$$

For destructive interference, the path difference is an odd multiple of half a wavelength:

$$d\sin\theta = \left(m+\tfrac{1}{2}\right)\lambda,\ m = 0, 1, 2, 3, \ldots;\quad \to\quad \theta = \sin^{-1}\frac{\left(m+\frac{1}{2}\right)\lambda}{d}$$

$$\theta_{0\,\text{max}} = \sin^{-1}\frac{\left(\frac{1}{2}\right)(3.39\,\text{m})}{9.0\,\text{m}} = \boxed{11^\circ}\ ;\ \theta_{1\,\text{max}} = \sin^{-1}\frac{\left(\frac{3}{2}\right)(3.39\,\text{m})}{9.0\,\text{m}} = \boxed{34^\circ}\ ;$$

$$\theta_{2\,\text{max}} = \sin^{-1}\frac{\left(\frac{5}{2}\right)(3.39\,\text{m})}{9.0\,\text{m}} = \boxed{70^\circ}\ ;\ \theta_{3\,\text{max}} = \sin^{-1}\frac{\left(\frac{7}{2}\right)(3.39\,\text{m})}{9.0\,\text{m}} = \text{impossible}$$

These angles are applicable both above and below the midline, and both to the left and the right of the antennas.

49. For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 34-2a. The location on the screen is given by $x = \ell\tan\theta$, as seen in Fig. 34-7(c). For small angles, we have $\sin\theta \approx \tan\theta \approx x/\ell$. Second order means $m = 2$.

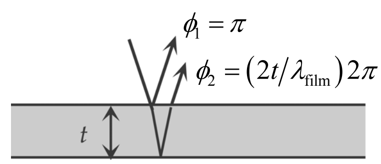
$$d\sin\theta = m\lambda \quad\to\quad d\frac{x}{\ell} = m\lambda \quad\to\quad x = \frac{\lambda m\ell}{d}\ ;\ x_1 = \frac{\lambda_1 m\ell}{d}\ ;\ x_2 = \frac{\lambda_2 m\ell}{d} \quad\to$$

$$\Delta x = x_1 - x_2 = \frac{\lambda_1 m\ell}{d} - \frac{\lambda_2 m\ell}{d} \rightarrow$$

$$\lambda_2 = \lambda_1 - \frac{d\Delta x}{m\ell} = 690\times10^{-9}\,\text{m} - \frac{(6.6\times10^{-4}\,\text{m})(1.23\times10^{-3}\,\text{m})}{2(1.60\,\text{m})} = 4.36\times10^{-7}\,\text{m} \approx \boxed{440\,\text{nm}}$$

50. PLEASE NOTE: In early versions of the textbook, in which the third line of this problem states that "… light is a minimum only for …," the resulting answer does not work out properly. It yields values of $m = 6$ and $m = 4$ for the integers in the interference relationship. Accordingly, the problem was changed to read "… light is a maximum only for … ." The solution here reflects that change.

With respect to the incident wave, the wave that reflects at the top surface of the film has a phase change of $\phi_1 = \pi$. With respect to the incident wave, the wave that reflects from the bottom surface of the film has a phase change due to the additional path length and no phase change due to reflection, so



$\phi_2 = \left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi + 0$. For constructive interference, the net phase change must be an integer multiple of 2π.

$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi - \pi = 2\pi m \rightarrow t = \tfrac{1}{2}\left(m+\tfrac{1}{2}\right)\lambda_{\text{film}} = \tfrac{1}{2}\left(m+\tfrac{1}{2}\right)\frac{\lambda}{n_{\text{film}}},\ m = 0, 1, 2, \ldots.$$

Evaluate the thickness for the two wavelengths.

$$t = \tfrac{1}{2}\left(m_1+\tfrac{1}{2}\right)\frac{\lambda_1}{n_{\text{film}}} = \tfrac{1}{2}\left(m_2+\tfrac{1}{2}\right)\frac{\lambda_2}{n_{\text{film}}} \rightarrow \frac{(m_2+\frac{1}{2})}{(m_1+\frac{1}{2})} = \frac{\lambda_1}{\lambda_2} = \frac{688.0\,\text{nm}}{491.4\,\text{nm}} = 1.40 = \frac{7}{5}$$

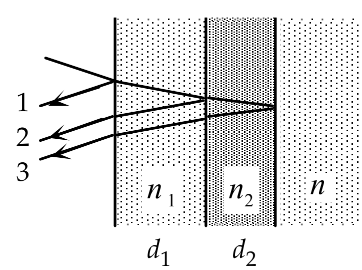
Thus $m_2 = 3$ and $m_1 = 2$. Evaluate the thickness with either value and the corresponding wavelength.

$$t = \tfrac{1}{2}\left(m_1+\tfrac{1}{2}\right)\frac{\lambda_1}{n_{\text{film}}} = \tfrac{1}{2}\left(\tfrac{5}{2}\right)\frac{688.0\,\text{nm}}{1.58} = \boxed{544\,\text{nm}}\ ;\ t = \tfrac{1}{2}\left(m_2+\tfrac{1}{2}\right)\frac{\lambda_2}{n_{\text{film}}} = \tfrac{1}{2}\left(\tfrac{7}{2}\right)\frac{491.4\,\text{nm}}{1.58} = \boxed{544\,\text{nm}}$$

51. From the discussion in section 34-6, we see that the path length change is twice the distance that the mirror moves. The phase shift is 2π for every wavelength of path length change. The intensity as a function of phase shift is given by Eq. 34-6.

$$\frac{\delta}{2\pi} = \frac{\text{path change}}{\lambda} = \frac{2x}{\lambda} \rightarrow \delta = \frac{4\pi x}{\lambda}\ ;\ I = I_0\cos^2\frac{\delta}{2} = \boxed{I_0\cos^2\left(\frac{2\pi x}{\lambda}\right)}$$

52. To maximize reflection, the three rays shown in the figure should be in phase. We first compare rays 2 and 3. Ray 2 reflects from $n_2 > n_1$, and so has a phase shift of $\phi_2 = \pi$. Ray 3 will have a phase change due to the additional path length in material 2, and a phase shift of π because of reflecting from $n > n_2$. Thus



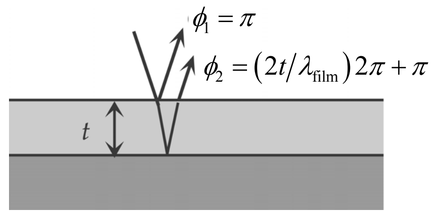
$\phi_3 = \left(\frac{2d_2}{\lambda_2}\right)2\pi + \pi$. For constructive interference the net phase change for rays 2 and 3 must be a non-zero integer multiple of 2π.

$$\Delta\phi_{2-3} = \phi_3 - \phi_2 = \left[\left(\frac{2d_2}{\lambda_2}\right)2\pi + \pi\right] - \pi = 2m\pi \rightarrow d_2 = \tfrac{1}{2}m\lambda_2,\ m = 1, 2, 3\cdots$$

The minimum thickness is for $m = 1$, and so $d_2 = \frac{1}{2} m\lambda_2 = \boxed{\dfrac{\lambda}{2n_2}}$.

Now consider rays 1 and 2. The exact same analysis applies, because the same relationship exists between the indices of refraction: $n_1 > n$ and $n_2 > n_1$. Thus $d_1 = \boxed{\dfrac{\lambda}{2n_1}}$.

53. With respect to the incident wave, the wave that reflects from the top surface of the coating has a phase change of $\phi_1 = \pi$. With respect to the incident wave, the wave that reflects from the glass $(n \approx 1.5)$ at the bottom surface of the coating has a phase change due to both the additional path length and reflection, so $\phi_2 = \left(\dfrac{2t}{\lambda_{\text{film}}}\right)2\pi + \pi$. For destructive interference, the net phase change must be an odd-integer multiple of π.



$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi + \pi\right] - \pi = (2m+1)\pi \quad \rightarrow$$

$$t = \tfrac{1}{4}(2m+1)\lambda_{\text{film}} = \tfrac{1}{4}(2m+1)\frac{\lambda}{n_{\text{film}}},\ m = 0, 1, 2, \ldots$$

The minimum thickness has $m = 0$, and so $t_{\min} = \frac{1}{4}\dfrac{\lambda}{n_{\text{film}}}$.

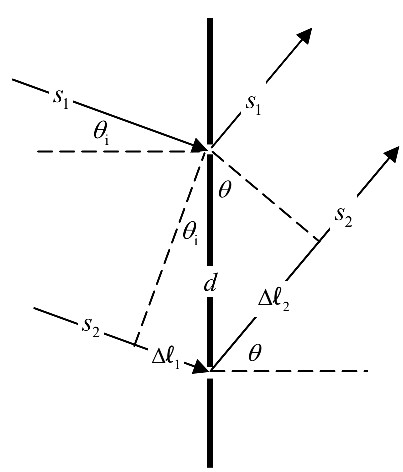
(*a*) For the blue light: $t_{\min} = \frac{1}{4}\dfrac{(450\,\text{nm})}{(1.38)} = 81.52\,\text{nm} \approx \boxed{82\,\text{nm}}$.

(*b*) For the red light: $t_{\min} = \frac{1}{4}\dfrac{(700\,\text{nm})}{(1.38)} = 126.8\,\text{nm} \approx \boxed{130\,\text{nm}}$.

54. The phase difference caused by the path difference back and forth through the coating must correspond to half a wavelength in order to produce destructive interference.

$$2t = \tfrac{1}{2}\lambda \quad \rightarrow \quad t = \tfrac{1}{4}\lambda = \tfrac{1}{4}(2\,\text{cm}) = \boxed{0.5\,\text{cm}}$$

55. We consider a figure similar to Figure 34-12, but with the incoming rays at an angle of θ_i to the normal. Ray s_2 will travel an extra distance $\Delta\ell_1 = d\sin\theta_i$ before reaching the slits, and an extra distance $\Delta\ell_2 = d\sin\theta$ after leaving the slits. There will be a phase difference between the waves due to the path difference $\Delta\ell_1 + \Delta\ell_2$. When this total path difference is a multiple of the wavelength, constructive interference will occur.



$$\Delta\ell_1 + \Delta\ell_2 = d\sin\theta_i + d\sin\theta = m\lambda \quad \rightarrow$$

$$\sin\theta = \frac{m\lambda}{d} - \sin\theta_i,\quad m = 0, 1, 2, \cdots$$

Since the rays leave the slits at all angles in the forward direction, we could have drawn the leaving rays with a downward tilt instead of an upward tilt. This would make the ray s_2 traveling a longer distance from the slits to the screen. In this case the path difference would be $\Delta\ell_2 - \Delta\ell_1$, and would result in the following expression.

$$\Delta\ell_2 - \Delta\ell_1 = d\sin\theta - d\sin\theta_i = m\lambda \;\rightarrow\; \sin\theta = \frac{m\lambda}{d} + \sin\theta_i,\ \ m = 0, 1, 2, \cdots$$

$$\Delta\ell_1 - \Delta\ell_2 = d\sin\theta_i - d\sin\theta = m\lambda \;\rightarrow\; \sin\theta = -\frac{m\lambda}{d} + \sin\theta_i,\ \ m = 0, 1, 2, \cdots$$

We combine the statements as follows.

$$\boxed{\sin\theta = \frac{m\lambda}{d} \pm \sin\theta_i,\ \ m = 0, 1, 2, \cdots}$$

Because of an arbitrary choice of taking $\Delta\ell_2 - \Delta\ell_1$, we could also have formulated the problem so that the result would be expressed as $\boxed{\sin\theta = \sin\theta_i \pm \frac{m\lambda}{d},\ \ m = 0, 1, 2, \cdots}$.

56. The signals will be out of phase when the path difference equals an odd number of half-wavelengths. Let the 175-m distance be represented by *d*.

$$\sqrt{y^2 + d^2} - y = \left(m + \tfrac{1}{2}\right)\lambda,\ m = 0, 1, 2, 3, \ldots \;\rightarrow\; \sqrt{y^2 + d^2} = y + \left(m + \tfrac{1}{2}\right)\lambda \;\rightarrow$$

$$y^2 + d^2 = y^2 + 2y\left(m + \tfrac{1}{2}\right)\lambda + \left(m + \tfrac{1}{2}\right)^2\lambda^2 \;\rightarrow\; y = \frac{d^2 - \left(m + \tfrac{1}{2}\right)^2\lambda^2}{2\left(m + \tfrac{1}{2}\right)\lambda}$$

We evaluate this for the first three values of *m*. The wavelength is $\lambda = \frac{c}{f} = \frac{3.00 \times 10^8\ \text{m/s}}{6.0 \times 10^6\ \text{Hz}} = 50\ \text{m}$.

$$y = \frac{d^2 - \left(m + \tfrac{1}{2}\right)^2\lambda^2}{2\left(m + \tfrac{1}{2}\right)\lambda} = \frac{(175\ \text{m})^2 - \left(m + \tfrac{1}{2}\right)^2(50\ \text{m})^2}{2\left(m + \tfrac{1}{2}\right)(50\ \text{m})} = 600\ \text{m},\ 167\ \text{m},\ 60\ \text{m},\ 0\ \text{m}$$

The first three points on the *y* axis where the signals are out of phase are at $y = \boxed{0, 60\ \text{m, and } 167\ \text{m}}$.

57. As explained in Example 34-6 the $\frac{1}{2}$-cycle phase change at the lower surface means that destructive interference occurs when the thickness *t* is such that $2t = m\lambda,\ m = 0, 1, 2, \ldots$. Set $m = 1$ to find the smallest nonzero value of *t*.

$$t = \tfrac{1}{2}\lambda = \tfrac{1}{2}(680\ \text{nm}) = \boxed{340\ \text{nm}}$$

As also explained in Example 34-6, constructive interference will occur when $2t = \left(m + \tfrac{1}{2}\right)\lambda$, $m = 0, 1, 2, \ldots$. We set $m = 0$ to find the smallest value of *t*:

$$t = \tfrac{1}{4}\lambda = \tfrac{1}{4}(680\ \text{nm}) = \boxed{170\ \text{nm}}$$

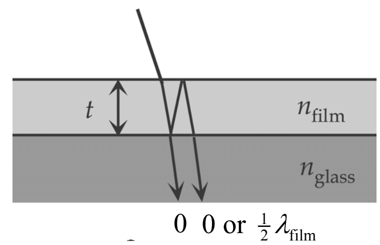
58. The reflected wave appears to be coming from the virtual image, so this corresponds to a double slit, with the separation being *d* = 2S. The reflection from the mirror produces a π phase shift, however, so the maxima and minima are interchanged, as described in Problem 11.

$$\sin\theta_{\max} = \left(m + \tfrac{1}{2}\right)\frac{\lambda}{2\text{S}},\ m = 0, 1, 2, \cdots\ ;\ \sin\theta_{\min} = m\frac{\lambda}{2\text{S}},\ m = 0, 1, 2, \cdots$$

59. Since the two sources are 180° out of phase, destructive interference will occur when the path length difference between the two sources and the receiver is 0, or an integer number of wavelengths. Since the antennae are separated by a distance of $d = \lambda/2$, the path length difference can never be greater than $\lambda/2$, so the only points of destructive interference occur when the receiver is equidistant from each antenna, that is, at $\boxed{\theta_{\text{destructive}} = 0° \text{ and } 180°}$. Constructive interference occurs when the path difference is a half integer wavelength. Again, since the separation distance between the two

antennas is $d = \lambda/2$, the maximum path length difference is $\lambda/2$, which occurs along the line through the antennae, therefore the constructive interference only occurs at $\boxed{\theta_{\text{constructive}} = 90° \text{ and } 270°}$. As expected, these angles are reversed from those in phase, found in Example 34-5c.

60. If we consider the two rays shown in the diagram, we see that the first ray passes through with no reflection, while the second ray has reflected twice. If $n_{\text{film}} < n_{\text{glass}}$, the first reflection from the glass produces a phase shift equivalent to $\frac{1}{2}\lambda_{\text{film}}$, while the second reflection from the air produces no shift. When we compare the two rays at the film-glass surface, we see that the second ray has a total shift in phase, due to its longer path length (2t) and reflection $\left(\frac{1}{2}\lambda_{\text{film}}\right)$. We set this path difference equal to an integer number of wavelengths for maximum intensity and equal to a half-integer number of wavelengths for minimum intensity.



$$\text{max:}\quad 2t + \tfrac{1}{2}\lambda_{\text{film}} = m\lambda_{\text{film}},\ m = 1, 2, 3, \ldots \quad\rightarrow\quad \boxed{t = \frac{\frac{1}{2}\left(m - \frac{1}{2}\right)\lambda}{n_{\text{film}}},\ m = 1, 2, 3, \ldots}$$

$$\text{min:}\quad 2t + \tfrac{1}{2}\lambda_{\text{film}} = \left(m + \tfrac{1}{2}\right)\lambda_{\text{film}},\ m = 0, 1, 2, 3, \ldots \quad\rightarrow\quad \boxed{t = \frac{\frac{1}{2}m\lambda}{n_{\text{film}}},\ m = 0, 1, 2, 3, \ldots}$$

At $t = 0$, or in the limit $t \ll \lambda/n_{\text{film}}$, the transmitted beam will be at a minimum. Each time the thickness increases by a quarter wavelength the intensity switches between a maximum and a minimum.

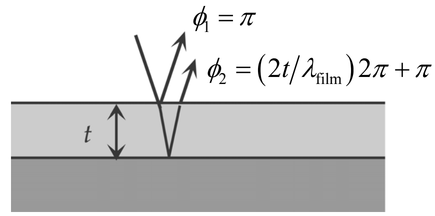
If $n_{\text{film}} > n_{\text{glass}}$, the first reflection from the glass produces no shift, while the second reflection from the air also produces no shift. When we compare the two rays at the film-glass surface, we see that the second ray has a total shift due solely to the difference in path lengths, 2t.
For maxima, we have

$$\text{max:}\quad 2t = m\lambda_{\text{film}},\ m = 0, 1, 2, 3, \ldots \quad\rightarrow\quad \boxed{t = \frac{\frac{1}{2}m\lambda}{n_{\text{film}}},\ m = 0, 1, 2, 3, \ldots}$$

$$\text{min:}\quad 2t = \left(m - \tfrac{1}{2}\right)\lambda_{\text{film}},\ m = 1, 2, 3, \ldots \quad\rightarrow\quad \boxed{t = \frac{\frac{1}{2}\left(m - \frac{1}{2}\right)\lambda}{n_{\text{film}}},\ m = 1, 2, 3, \ldots}$$

At $t = 0$, or in the limit $t \ll \lambda/n_{\text{film}}$, the transmitted beam will be at a maximum. Each time the thickness increases by a quarter wavelength the intensity switches between a maximum and a minimum.

61. With respect to the incident wave, the wave that reflects from the top surface of the film has a phase change of $\phi_1 = \pi$. With respect to the incident wave, the wave that reflects from the glass $(n = 1.52)$ at the bottom surface of the film has a phase change due to both the additional path length and reflection, so $\phi_2 = \left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi + \pi$. For constructive interference, the net phase change must be an even non-zero integer multiple of π.



$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi + \pi\right] - \pi = m2\pi \quad \rightarrow \quad t = \tfrac{1}{2}m\lambda_{\text{film}} = \tfrac{1}{2}m\frac{\lambda}{n_{\text{film}}},\ m = 1, 2, 3, \ldots .$$

The minimum non-zero thickness occurs for $m = 1$.

$$t_{\min} = \frac{\lambda}{2n_{\text{film}}} = \frac{643\,\text{nm}}{2(1.34)} = \boxed{240\,\text{nm}}$$

62. The path difference to a point on the x axis from the two sources is $\Delta d = d_2 - d_1 = \sqrt{x^2 + d^2} - x$. For the two signals to be out of phase, this path difference must be an odd number of half-wavelengths, so $\Delta d = \left(m + \tfrac{1}{2}\right)\lambda,\ m = 0, 1, 2, \cdots$. Also, the maximum path difference is $d = 3\lambda$. Thus the path difference must be $\tfrac{1}{2}\lambda,\ \tfrac{3}{2}\lambda$, or $\tfrac{5}{2}\lambda$ for the signals to be out of phase (m = 0, 1, or 2). We solve for x for the three path differences.

$$\Delta d = \sqrt{x^2 + d^2} - x = \left(m + \tfrac{1}{2}\right)\lambda \quad \rightarrow \quad \sqrt{x^2 + d^2} = x + \left(m + \tfrac{1}{2}\right)\lambda \quad \rightarrow$$

$$x^2 + d^2 = x^2 + 2x\left(m + \tfrac{1}{2}\right)\lambda + \left(m + \tfrac{1}{2}\right)^2\lambda^2 \quad \rightarrow$$

$$x = \frac{d^2 - \left(m + \tfrac{1}{2}\right)^2\lambda^2}{2\left(m + \tfrac{1}{2}\right)\lambda} = \frac{9\lambda^2 - \left(m + \tfrac{1}{2}\right)^2\lambda^2}{2\left(m + \tfrac{1}{2}\right)\lambda} = \frac{9 - \left(m + \tfrac{1}{2}\right)^2}{2\left(m + \tfrac{1}{2}\right)}\lambda$$

$$m = 0: x = \frac{9 - \left(0 + \tfrac{1}{2}\right)^2}{2\left(0 + \tfrac{1}{2}\right)}\lambda = \boxed{8.75\lambda} \ ; \ m = 1: x = \frac{9 - \left(1 + \tfrac{1}{2}\right)^2}{2\left(1 + \tfrac{1}{2}\right)}\lambda = \boxed{2.25\lambda}$$

$$m = 2: x = \frac{9 - \left(2 + \tfrac{1}{2}\right)^2}{2\left(2 + \tfrac{1}{2}\right)}\lambda = \boxed{0.55\lambda}$$

63. For both configurations, we have $d\sin\theta = m\lambda$. The angles and the orders are to be the same. The slit separations and wavelengths will be different. Use the fact that frequency and wavelength are related by $v = f\lambda$. The speed of sound in room-temperature air is given in Chapter 16 as 343 m/s.

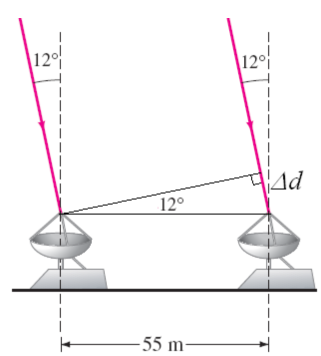
$$d\sin\theta = m\lambda \quad \rightarrow \quad \frac{\sin\theta}{m} = \frac{\lambda}{d} = \frac{\lambda_{\text{L}}}{d_{\text{L}}} = \frac{\lambda_{\text{S}}}{d_{\text{S}}} \quad \rightarrow$$

$$d_{\text{S}} = d_{\text{L}}\frac{\lambda_{\text{S}}}{\lambda_{\text{L}}} = d_{\text{L}}\frac{\dfrac{v_{\text{S}}}{f_{\text{S}}}}{\dfrac{v_{\text{L}}}{f_{\text{L}}}} = d_{\text{L}}\frac{v_{\text{S}}f_{\text{L}}}{v_{\text{L}}f_{\text{S}}} = \left(1.0\times10^{-4}\,\text{m}\right)\left(\frac{343\,\text{m/s}}{3.00\times10^{8}\,\text{m/s}}\right)\left(\frac{4.6\times10^{14}\,\text{Hz}}{262\,\text{Hz}}\right) = \boxed{200\,\text{m}}$$

The answer has 2 significant figures.

64. Light traveling from a region 12° from the vertical would have to travel a slightly longer distance to reach the far antenna. Using trigonometry we calculate this distance, as was done in Young's double slit experiment. Dividing this additional distance by the speed of light gives us the necessary time shift.

$$\Delta t = \frac{\Delta d}{c} = \frac{\ell\sin\theta}{c} = \frac{(55\text{ m})\sin12^\circ}{3.00\times10^{8}\text{ m/s}} = 3.81\times10^{-8}\text{ s} = \boxed{38.1\text{ ns}}$$



65. In order for the two reflected halves of the beam to be $180°$ out of phase with each other, the minimum path difference (2*t*) should be $\frac{1}{2}\lambda$ in the plastic. Notice that there is no net phase difference between the two halves of the beam due to reflection, because both halves reflect from the same material.

$$2t = \tfrac{1}{2}\frac{\lambda}{n} \quad \rightarrow \quad t = \frac{\lambda}{4n} = \frac{780\,\text{nm}}{4(1.55)} = \boxed{126\,\text{nm}}$$

66. We determine *n* for each angle using a spreadsheet. The results are shown below.

N	25	50	75	100	125	150
θ(degree)	5.5	6.9	8.6	10.0	11.3	12.5
n	1.75	2.19	2.10	2.07	2.02	1.98

The average value is $n_{\text{avg}} = \boxed{2.02}$. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH34.XLS," on tab "Problem 34.66."