CHAPTER 32: Light: Reflection and Refraction

Responses to Questions

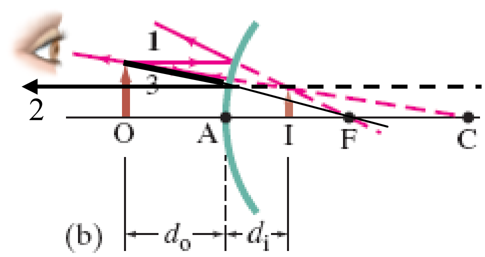
1. (*a*) The Moon would look just like it does now, since the surface is rough. Reflected sunlight is scattered by the surface of the Moon in many directions, making the surface appear white.
   (*b*) With a polished, mirror-like surface, the Moon would reflect an image of the Sun, the stars, and the Earth. The appearance of the Moon would be different as seen from different locations on the Earth.

2. Yes, it would have been possible, although certainly difficult. Several attempts have been made to reenact the event in order to test its feasibility. Two of the successful attempts include a 1975 experiment directed by Greek scientist Dr. Ioannis Sakkas and a 2005 experiment performed by a group of engineering students at MIT. (See www.mit.edu for links to both these and other similar experiments.) In both these cases, several individual mirrors operating together simulated a large spherical mirror and were used to ignite a wooden boat. If in fact the story is true, Archimedes would have needed good weather and an enemy fleet that cooperated by staying relatively still while the focused sunlight heated the wood.

3. The focal length of a plane mirror is infinite. The magnification of a plane mirror is 1.

4. The image is real and inverted, because the magnification is negative. The mirror is concave, because convex mirrors can only form virtual images. The image is on the same side of the mirror as the object; real images are formed by converging light rays and light rays cannot actually pass through a mirror.

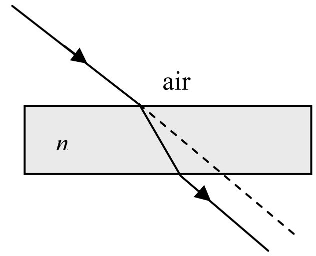
5. Ray 2 is directed as if it were going through the focal point and is reflected from the convex mirror parallel to the principal axis.



6. Yes. For a plane mirror, $d_o = -d_i$, since the object and image are equidistant from the mirror and the image is virtual, or behind the mirror. The focal length of a plane mirror is infinite, so the result of the mirror equation, Eq. 32-2, is $\frac{1}{d_o} + \frac{1}{d_i} = 0$, or $d_o = -d_i$, as expected.

7. Yes. When a concave mirror produces a real image of a real object, both d_o and d_i are positive. The magnification equation, $m = -\frac{d_i}{d_o}$, results in a negative magnification, which indicates that the image is inverted.

8. A light ray entering the solid rectangular object will exit the other side following a path that is parallel to its original path but displaced slightly from it. The angle of refraction in the glass can be determined geometrically from this displacement and the thickness of the object. The index of refraction can then be determined using Snell's Law with this angle of refraction and the original angle of incidence. The speed of light in the material follows from the definition of the index of refraction: $n = c/v$.

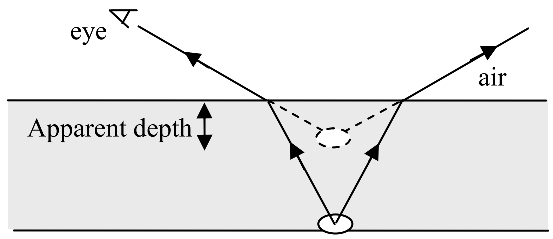


9. This effect is similar to diffuse reflection off of a rough surface. A ripply sea has multiple surfaces which are at an angle to reflect the image of the Moon into your eyes. This makes the image of the Moon appear elongated.

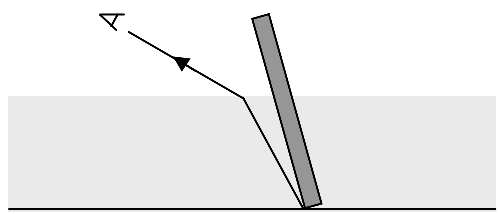
10. A negative object distance corresponds to a virtual object. This could occur if converging rays from another mirror or lens were intercepted by the mirror before actually forming an image. This image would be the object for the mirror.

11. The angle of refraction and the angle of incidence are both zero in this case.

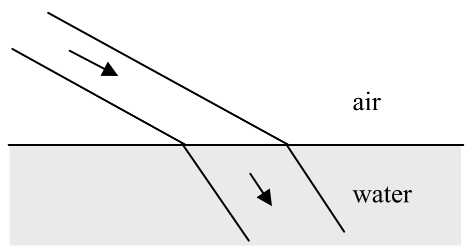
12. Underestimate. The light rays leaving the bottom of the pool bend away from the normal as they enter the air, so their source appears to be more shallow than it actually is. The greater the viewing angle, the more the bending of the light and therefore the less the apparent depth.



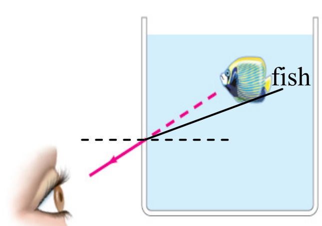
13. Your brain interprets the refracted rays as if the part of the stick that is under water is closer to the surface than it actually is, so the stick appears bent.



14. Because the broad beam hits the surface of the water at an angle, it illuminates an area of the surface that is wider than the beam width. Light from the beam bends towards the normal. The refracted beam is wider than the incident beam because one edge of the beam strikes the surface first, while the other edge travels farther in the air. (See the adjacent diagram.)



15. The light rays from the fish are bent away from the normal as they leave the tank. The fish will appear closer to the side of the tank than it really is.



16. The water drop acts like a lens, and refracts light as the light passes through it. Also, some of the light incident on the air/water boundary is reflected at the surface, so the drop can be seen in reflected light.

17. When the light ray passes from the blue material to the green material, the ray bends toward the normal. This indicates that the index of refraction of the blue material is less than that of the green material. When the light ray passes from the green material to the yellow material, the ray bends away from the normal, but not far enough to make the ray parallel to the initial ray, indicating that the index of refraction of the yellow material is less than that of the green material but larger than the index of refraction of the blue material. The ranking of the indices of refraction is, least to greatest, blue, yellow, and green.

18. No. Total internal reflection can only occur when light travels from a medium of higher index of refraction to a medium of lower index of refraction.

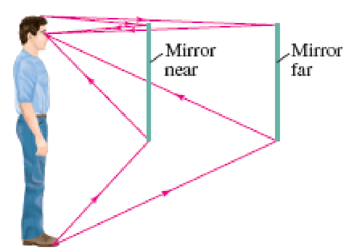
19. No. The refraction of light as it enters the pool will make the object look smaller. See Figure 32-32 and Conceptual Example 32-11.

20. The mirror is concave, and the person is standing inside the focal point so that a virtual, upright image is formed. (A convex mirror would also form a virtual, upright image but the image would be smaller than the object.) In addition, an image is also present at the far right edge of the mirror, which is only possible if the mirror is concave.

21. (*a*) Since the light is coming from a vacuum into the atmosphere, which has a larger index of refraction, the light rays should bend toward the normal (toward the vertical direction).
(*b*) The stars are closer to the horizon than they appear to be from the surface of the Earth.

Solutions to Problems

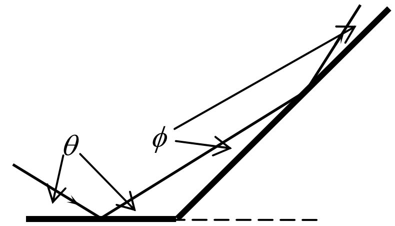
1. Because the angle of incidence must equal the angle of reflection, we see from the ray diagrams that the ray that reflects to your eye must be as far below the horizontal line to the reflection point on the mirror as the top is above the line, regardless of your position.



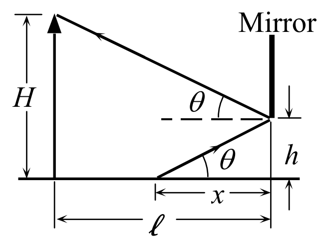
2. For a flat mirror the image is as far behind the mirror as the object is in front, so the distance from object to image is twice the distance from the object to the mirror, or $\boxed{5.6\,\text{m}}$.

3. The law of reflection can be applied twice. At the first reflection, the angle is θ, and at the second reflection, the angle is ϕ. Consider the triangle formed by the mirrors and the first reflected ray.

$$\theta + \alpha + \phi = 180° \;\rightarrow\; 38° + 135° + \phi = 180° \;\rightarrow\; \boxed{\phi = 7°}$$



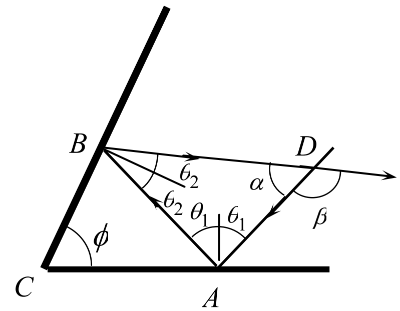
4. The angle of incidence is the angle of reflection. See the diagram for the appropriate lengths.



$$\tan\theta = \frac{(H-h)}{\ell} = \frac{h}{x} \rightarrow$$

$$\frac{(1.64\,\text{m} - 0.38\,\text{m})}{(2.30\,\text{m})} = \frac{(0.38\,\text{m})}{x} \rightarrow x = \boxed{0.69\,\text{m}}$$

5. The incoming ray is represented by line segment DA. For the first reflection at *A* the angles of incidence and reflection are θ_1. For the second reflection at *B* the angles of incidence and reflection are θ_2. We relate θ_1 and θ_2 to the angle at which the mirrors meet, ϕ, by using the sum of the angles of the triangle *ABC*.



$$\phi + (90° - \theta_1) + (90° - \theta_2) = 180° \rightarrow \phi = \theta_1 + \theta_2$$

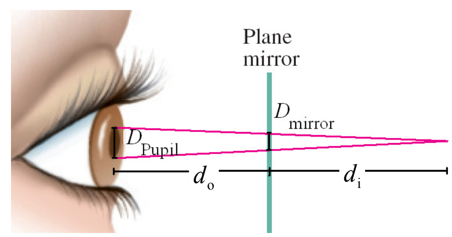
Do the same for triangle *ABD*.

$$\alpha + 2\theta_1 + 2\theta_2 = 180° \rightarrow \alpha = 180° - 2(\theta_1 + \theta_2) = 180° - 2\phi$$

At point *D* we see that the deflection is as follows.

$$\beta = 180° - \alpha = 180° - (180° - 2\phi) = \boxed{2\phi}$$

6. The rays entering your eye are diverging from the virtual image position behind the mirror. Thus the diameter of the area on the mirror and the diameter of your pupil must subtend the same angle from the image.

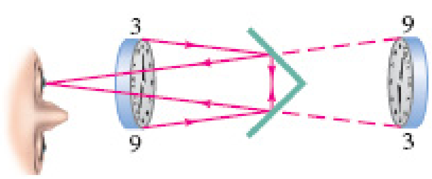


$$\frac{D_{\text{mirror}}}{d_i} = \frac{D_{\text{pupil}}}{(d_o + d_i)} \rightarrow D_{\text{mirror}} = D_{\text{pupil}} \frac{d_i}{(d_o + d_i)} = \tfrac{1}{2} D_{\text{pupil}}$$

$$A_{\text{mirror}} = \tfrac{1}{4}\pi D_{\text{mirror}}^2 = \tfrac{1}{4}\pi \tfrac{1}{4} D_{\text{pupil}}^2 = \frac{\pi}{16}(4.5\times10^{-3}\,\text{m})^2$$

$$= \boxed{4.0\times10^{-6}\,\text{m}^2}$$

[7.] See the "top view" ray diagram.



8. (*a*) The velocity of the incoming light wave is in the direction of the initial light wave. We can write this velocity in component form, where the three axes of our coordinate system are chosen to be perpendicular to the plane of each of the three mirrors. As the light reflects off any of the three mirrors, the component of the velocity perpendicular to that mirror reverses direction. The other two velocity components will remain unchanged. After the light has reflected off of each of the three mirrors, each of the three velocity components will be reversed and the light will be traveling directly back from where it came.

(*b*) If the mirrors are assumed to be large enough, the light can only reflect off two of the mirrors if the velocity component perpendicular to the third mirror is zero. Therefore, in this case the light is still reflected back directly to where it came.

9. The rays from the Sun will be parallel, so the image will be at the focal point, which is half the radius of curvature.

$$r = 2f = 2(18.8\,\text{cm}) = \boxed{37.6\,\text{cm}}$$

10. To produce an image at infinity, the object must be at the focal point, which is half the radius of curvature.

$$d_\text{o} = f = \tfrac{1}{2}r = \tfrac{1}{2}(24.0\,\text{cm}) = \boxed{12.0\,\text{cm}}$$

11. The image flips at the focal point, which is half the radius of curvature. Thus the radius is $\boxed{1.0\text{ m}}$.

12. (*a*) The focal length is half the radius of curvature, so $f = \tfrac{1}{2}r = \tfrac{1}{2}(24\,\text{cm}) = \boxed{12\,\text{cm}}$.

(*b*) Use Eq. 32-2.

$$\frac{1}{d_\text{o}} + \frac{1}{d_\text{i}} = \frac{1}{f} \quad \rightarrow \quad d_\text{i} = \frac{d_\text{o}f}{d_\text{o} - f} = \frac{(35\,\text{cm})(24\,\text{cm})}{35\,\text{cm} - 24\,\text{cm}} = \boxed{76\,\text{cm}}$$

(*c*) The image is $\boxed{\text{inverted}}$, since the magnification is negative.

$\boxed{13.}$ The ball is a convex mirror with a focal length $f = \tfrac{1}{2}r = \tfrac{1}{2}(-4.6\,\text{cm}) = -2.3\,\text{cm}$. Use Eq. 32-3 to locate the image.

$$\frac{1}{d_\text{o}} + \frac{1}{d_\text{i}} = \frac{1}{f} \quad \rightarrow \quad d_\text{i} = \frac{d_\text{o}f}{d_\text{o} - f} = \frac{(25.0\,\text{cm})(-2.3\,\text{cm})}{25.0\,\text{cm} - (-2.3\,\text{cm})} = -2.106\,\text{cm} \approx -2.1\,\text{cm}$$

The image is $\boxed{\text{2.1 cm behind the surface of the ball, virtual, and upright}}$. Note that the magnification is $m = -\dfrac{d_\text{i}}{d_\text{o}} = \dfrac{-(-2.106\,\text{cm})}{(25.0\,\text{cm})} = +0.084$.

14. The image distance can be found from the object distance of 1.7 m and the magnification of +3. With the image distance and object distance, the focal length and radius of curvature can be found.

$$m = \frac{-d_\text{i}}{d_\text{o}} \quad \rightarrow \quad d_\text{i} = -md_\text{o}$$

$$\frac{1}{d_\text{o}} + \frac{1}{d_\text{i}} = \frac{1}{f} \quad \rightarrow \quad f = \frac{d_\text{o}d_\text{i}}{d_\text{o} + d_\text{i}} = \frac{d_\text{o}(-md_\text{o})}{d_\text{o} - md_\text{o}} = \frac{md_\text{o}}{m-1} = \frac{3(1.7\,\text{m})}{3-1} = 2.55\,\text{m}$$

$$r = 2f = 2(2.55\,\text{m}) = \boxed{5.1\,\text{m}}$$

15. The object distance of 2.00 cm and the magnification of +4.0 are used to find the image distance. The focal length and radius of curvature can then be found.

$$m = \frac{-d_\text{i}}{d_\text{o}} \quad \rightarrow \quad d_\text{i} = -md_\text{o}$$

$$\frac{1}{d_\text{o}} + \frac{1}{d_\text{i}} = \frac{1}{f} \quad \rightarrow \quad f = \frac{d_\text{o}d_\text{i}}{d_\text{o} + d_\text{i}} = \frac{d_\text{o}(-md_\text{o})}{d_\text{o} - md_\text{o}} = \frac{md_\text{o}}{m-1} = \frac{4(2.00\,\text{cm})}{4-1} = 2.677\,\text{cm}$$

$$r = 2f = 2(2.667\,\text{cm}) = \boxed{5.3\,\text{cm}}$$

Because the focal length is positive, the mirror is $\boxed{\text{concave}}$.

16. The mirror must be **convex**. Only convex mirrors produce images that are upright and smaller than the object. The object distance of 18.0 m and the magnification of +0.33 are used to find the image distance. The focal length and radius of curvature can then be found.

$$m = \frac{-d_i}{d_o} \rightarrow d_i = -md_o$$

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \rightarrow f = \frac{d_o d_i}{d_o + d_i} = \frac{d_o(-md_o)}{d_o - md_o} = \frac{md_o}{m-1} = \frac{0.33(18.0\,\text{m})}{0.33-1} = -8.866\,\text{m}$$

$$r = 2f = 2(-8.866\,\text{m}) = \boxed{-17.7\,\text{m}}$$

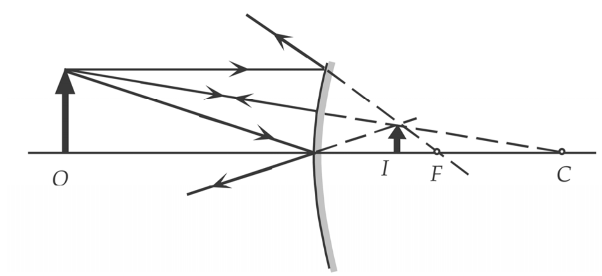
17. The object distance of 3.0 m and the magnification of +0.5 are used to find the image distance. The focal length and radius of curvature can then be found.

$$m = \frac{-d_i}{d_o} \rightarrow d_i = -md_o$$

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \rightarrow f = \frac{d_o d_i}{d_o + d_i} = \frac{d_o(-md_o)}{d_o - md_o} = \frac{md_o}{m-1} = \frac{0.5(3.0\,\text{m})}{0.5-1} = -3.0\,\text{m}$$

$$r = 2f = 2(-3.0\,\text{m}) = \boxed{-6.0\,\text{m}}$$

18. (*a*) From the ray diagram it is seen that the image is virtual. We estimate the image distance as $\boxed{-6\,\text{cm}.}$



(*b*) Use a focal length of $-9.0\,\text{cm}$ with the object distance of $18.0\,\text{cm}$.

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \rightarrow$$

$$d_i = \frac{d_o f}{d_o - f} = \frac{(18.0\,\text{cm})(-9.0\,\text{cm})}{18.0\,\text{cm} - (-9.0\,\text{cm})} = \boxed{-6.0\,\text{cm}}$$

(*c*) We find the image size from the magnification:

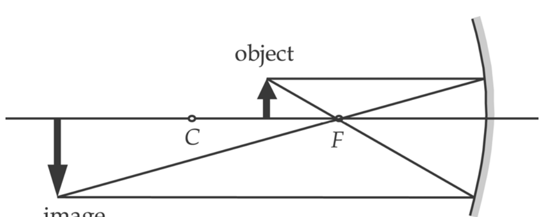
$$m = \frac{h_i}{h_o} = \frac{-d_i}{d_o} \rightarrow h_i = h_o\left(\frac{-d_i}{d_o}\right) = (3.0\,\text{mm})\left(\frac{6.0\,\text{cm}}{18.0\,\text{cm}}\right) = \boxed{1.0\,\text{mm}}$$

19. Take the object distance to be ∞, and use Eq. 32-3. Note that the image distance is negative since the image is behind the mirror.

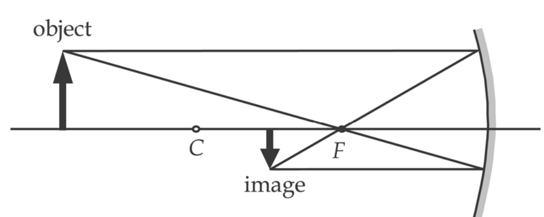
$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \rightarrow \frac{1}{\infty} + \frac{1}{d_i} = \frac{1}{f} \rightarrow f = d_i = -16.0\,\text{cm} \rightarrow r = 2f = \boxed{-32.0\,\text{cm}}$$

Because the focal length is negative, the mirror is **convex.**

20. (*a*)



$|h_i| > |h_o|$; $m > 1$; $d_o < r$



$|h_i| < |h_o|$; $m < 1$; $d_o > r$

(*b*) Apply Eq. 32-3 and Eq. 32-4.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}=\frac{2}{r} \rightarrow d_i=\frac{rd_o}{(2d_o-r)} \;;\; m=\frac{-d_i}{d_o}=\frac{-r}{(2d_o-r)}$$

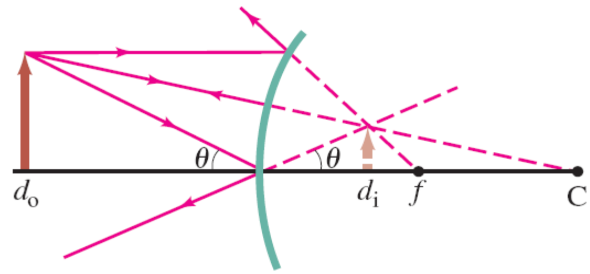
If $d_o>r$, then $(2d_o-r)>r$, so $|m|=\frac{r}{(2d_o-r)}=\frac{r}{(>r)}<1.$

If $d_o<r$, then $(2d_o-r)<r$, so $|m|=\frac{r}{(2d_o-r)}=\frac{r}{(<r)}>1.$

21. Consider the ray that reflects from the center of the mirror, and note that $d_i<0$.

$$\tan\theta=\frac{h_o}{d_o}=\frac{h_i}{-d_i} \rightarrow \frac{-d_i}{d_o}=\frac{h_i}{h_o}$$

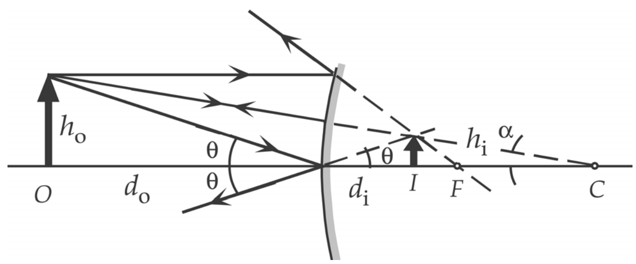
$$m=\frac{h_i}{h_o}=\boxed{-\frac{d_i}{d_o}}$$



22. From the ray diagram, we see that with a negative image distance, we have the following.

$$\tan\theta=\frac{h_o}{d_o}=\frac{h_i}{-d_i}$$

$$\tan\alpha=\frac{h_o}{(d_o+r)}=\frac{h_i}{(r+d_i)}$$



When we divide the two equations, we get

$$\frac{(d_o+r)}{d_o}=-\frac{(r+d_i)}{d_i} \rightarrow 1+\frac{r}{d_o}=-1-\frac{r}{d_i} \rightarrow \frac{r}{d_o}+\frac{r}{d_i}=-2 \rightarrow \frac{1}{d_o}+\frac{1}{d_i}=-\frac{2}{r}$$

If we define $f=\frac{r}{2}$ and consider the radius of curvature and focal length to be negative, then we have Eq. 32-2, $\boxed{\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}}$..

23. Use Eq. 32-2 and 32-3.

$$m=\frac{-d_i}{d_o} \rightarrow d_i=-md_o$$

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \rightarrow f=\frac{d_od_i}{d_o+d_i}=\frac{d_o(-md_o)}{d_o-md_o}=\frac{md_o}{m-1}=\frac{(0.55)(3.2\,\text{m})}{0.55-1}=\boxed{-3.9\,\text{m}}$$

24. (*a*) We are given that $d_i=d_o$. Use Eq. 32-3.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \rightarrow \frac{2}{d_o}=\frac{1}{f} \rightarrow \boxed{d_o=2f=r}$$

The object should be placed at the $\boxed{\text{center of curvature.}}$

(*b*) Because the image is in front of the mirror, $d_i>0$, it is $\boxed{\text{real.}}$

(*c*) The magnification is $m = \frac{-d_i}{d_o} = \frac{-d_o}{d_o} = -1$. Because the magnification is negative, the image is $\boxed{\text{inverted.}}$

(*d*) As found in part (*c*), $m = \boxed{-1.}$

$\boxed{25.}$ (*a*) To produce a smaller image located behind the surface of the mirror requires a $\boxed{\text{convex mirror.}}$

(*b*) Find the image distance from the magnification.

$$m = \frac{h_i}{h_o} = \frac{-d_i}{d_o} \rightarrow d_i = -\frac{d_o h_i}{h_o} = -\frac{(26\,\text{cm})(3.5\,\text{cm})}{(4.5\,\text{cm})} = -20.2\,\text{cm} \approx \boxed{-20\,\text{cm}} \text{ (2 sig. fig.)}$$

As expected, $d_i < 0$. The image is located $\boxed{\text{20 cm behind the surface.}}$

(*c*) Find the focal length from Eq. 32.3.

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \rightarrow f = \frac{d_o d_i}{d_o + d_i} = \frac{(26\,\text{cm})(-20.2\,\text{cm})}{(26\,\text{cm}) + (-20.2\,\text{cm})} = -90.55\,\text{cm} \approx \boxed{-91\,\text{cm}}$$

(*d*) The radius of curvature is twice the focal length.

$$r = 2f = 2(-90.55\,\text{cm}) = -181.1\,\text{cm} \approx \boxed{-180\,\text{cm}}$$

26. (*a*) To produce a larger upright image requires a $\boxed{\text{concave mirror.}}$

(*b*) The image will be $\boxed{\text{upright and virtual.}}$

(*c*) We find the image distance from the magnification:

$$m = \frac{h_i}{h_o} = \frac{-d_i}{d_o} \rightarrow d_i = -m d_o$$

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \rightarrow f = \frac{d_o d_i}{d_o + d_i} = \frac{d_o(-m d_o)}{d_o - m d_o} = \frac{m d_o}{m-1} \rightarrow$$

$$r = 2f = \frac{2 m d_o}{m-1} = \frac{2(1.35)(20.0\,\text{cm})}{1.35-1} = \boxed{154\,\text{cm}}$$

27. (*a*) We use the magnification equation, Eq. 32-3, to write the image distance in terms of the magnification and object distance. We then replace the image distance in the mirror equation, Eq. 32-2, and solve for the magnification in terms of the object distance and the focal length.

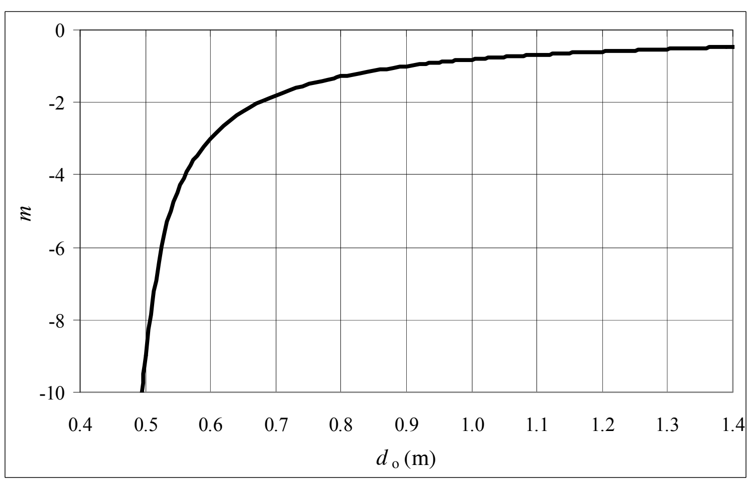
$$m = -d_i/d_o \rightarrow d_i = -m d_o$$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \rightarrow$$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{-m d_o} \rightarrow$$

$$\boxed{m = \frac{f}{f - d_o}}$$

(*b*) We set $f = 0.45$ m and draw a graph of the magnification as a function of the object distance. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH32.XLS," on tab "Problem 32.27b."



(*c*) The image and object will have the same lateral size when the magnification is equal to negative one. Setting the magnification equal to negative one, we solve the equation found in part (a) for the object distance.

$$m = \frac{f}{f - d_o} = -1 \rightarrow d_o = 2f = \boxed{0.90\,\text{m}}$$

(*d*) From the graph we see that for the image to be much larger than the object, the object should be placed at a point $\boxed{\text{just beyond the focal point.}}$

28. We use the magnification equation, Eq. 32-3, to write the image distance in terms of the magnification and object distance. We then replace the image distance in the mirror equation, Eq. 32-2, and solve for the magnification in terms of the object distance and the focal length, with the focal length given as $f = -|f|$.

$$m = -\frac{d_i}{d_o} \rightarrow d_i = -md_o \rightarrow \frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \rightarrow \frac{1}{-|f|} = \frac{1}{d_o} + \frac{1}{-md_o} \rightarrow \boxed{m = \frac{|f|}{|f| + d_o}}$$

From this relation, the closer the object is to the mirror (i.e., smaller object distance) the greater the magnification. Since a person's nose is closer to the mirror than the rest of the face, its image appears larger.

29. (*a*) We use the magnification equation, Eq. 32-3, to write the image distance in terms of the magnification and object distance. We then replace the image distance in the mirror equation, Eq. 32-2, and solve for the object distance in terms of the magnification and the focal length.

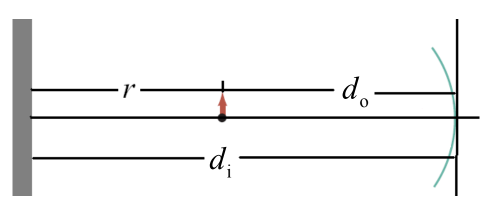
$$m = -\frac{d_i}{d_o} \rightarrow d_i = -md_o$$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \rightarrow \frac{1}{f} = \frac{1}{d_o} + \frac{1}{-md_o} = \frac{1}{d_o}\left(1 - \frac{1}{m}\right) \rightarrow \boxed{d_o = f\left(1 - \frac{1}{m}\right)}$$

(*b*) We set the object distance equal to the range of all positive numbers. Since the focal length of a convex lens is negative, the term in parentheses in the above equation must be the range of all negative numbers for the object distance to include the range of all positive numbers. We solve the resulting equation for all possible values of the magnification.

$$\left(1 - \frac{1}{m}\right) \le 0 \rightarrow 1 \le \frac{1}{m} \rightarrow \boxed{0 \le m \le 1}$$

30. The distance between the mirror and the wall is equal to the image distance, which we can calculate using Eq. 32-2. The object is located a distance *r* from the wall, so the object distance will be *r* less than the image distance. The focal length is given by Eq. 32-1. For the object distance to be real, the image distance must be greater than *r*.

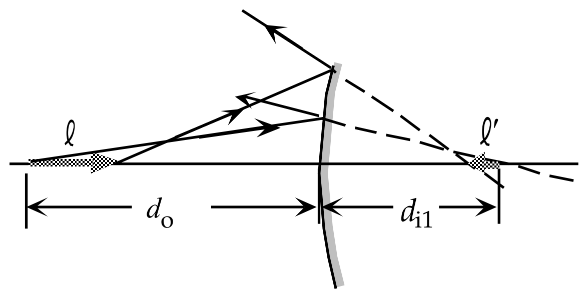


$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \rightarrow \frac{2}{r} = \frac{1}{d_i - r} + \frac{1}{d_i} \rightarrow 2d_i^2 - 4d_i r + r^2 = 0$$

$$d_i = \frac{4r \pm \sqrt{16r^2 - 8r^2}}{4} = r\left(1 \pm \frac{\sqrt{2}}{2}\right) \approx 0.292r \text{ or } \boxed{1.71r}$$

Use Eq. 32-3 to calculate the magnification: $m = -\frac{d_i}{d_o} = \frac{1.71r}{1.71r - r} = \boxed{-2.41}$

31. The lateral magnification of an image equals the height of the image divided by the height of the object. This can be written in terms of the image distance and focal length with Eqs. 32-2 and 32-3.



$$\frac{1}{f}=\frac{1}{d_i}+\frac{1}{d_o} \rightarrow d_i=\left(\frac{fd_o}{d_o-f}\right)$$

$$m=\frac{-d_i}{d_o}=-\frac{f}{d_o-f}$$

The longitudinal magnification will be the difference in image distances of the two ends of the object divided by the length of the image. Call the far tip of the wire object 1 with object distance d_o. The close end of the wire will be object 2 with object distance $d_o-\ell$. Using Eq. 32-2 we can find the image distances for both ends.

$$\frac{1}{f}=\frac{1}{d_o}+\frac{1}{d_{i1}} \rightarrow d_{i1}=\frac{d_o f}{d_o-f} \quad ; \quad \frac{1}{f}=\frac{1}{d_o-\ell}+\frac{1}{d_{i2}} \rightarrow d_{i2}=\frac{(d_o-\ell)f}{d_o-\ell-f}$$

Taking the difference in image distances and dividing by the object length gives the longitudinal magnification.

$$m_\ell=\frac{d_{i1}-d_{i2}}{\ell}=\frac{1}{\ell}\left(\frac{d_o f}{d_o-f}-\frac{(d_o-\ell)f}{d_o-\ell-f}\right)=\frac{d_o f(d_o-\ell-f)-(d_o-f)(d_o-\ell)f}{\ell(d_o-f)(d_o-\ell-f)}$$

$$=\frac{-f^2}{(d_o-f)(d_o-\ell-f)}$$

Set $\ell \ll d_o$, so that the ℓ drops out of the second factor of the denominator. Then rewrite the equation in terms of the lateral magnification, using the expression derived at the beginning of the problem.

$$m_\ell=\frac{-f^2}{(d_o-f)^2}=-\left[\frac{f}{(d_o-f)}\right]^2=\boxed{-m^2}$$

The negative sign indicates that the image is reversed front to back, as shown in the diagram.

32. We find the index of refraction from Eq. 32-1.

$$n=\frac{c}{v}=\frac{3.00\times10^8\ \text{m/s}}{2.29\times10^8\ \text{m/s}}=\boxed{1.31}$$

33. In each case, the speed is found from Eq. 32-1 and the index of refraction.

(*a*) Ethyl alcohol: $v=\frac{c}{n}=\frac{3.00\times10^8\ \text{m/s}}{1.36}=\boxed{2.21\times10^8\ \text{m/s}}$

(*b*) Lucite: $v=\frac{c}{n}=\frac{3.00\times10^8\ \text{m/s}}{1.51}=\boxed{1.99\times10^8\ \text{m/s}}$

(*c*) Crown glass: $v=\frac{c}{n}=\frac{3.00\times10^8\ \text{m/s}}{1.52}=\boxed{1.97\times10^8\ \text{m/s}}$

34. Find the distance traveled by light in 4.2 years.

$$d=c\Delta t=(3.00\times10^8\ \text{m/s})(4.2\ \text{yr})(3.16\times10^7\ \text{s/yr})=\boxed{4.0\times10^{16}\ \text{m}}$$

35. The time for light to travel from the Sun to the Earth is found from the distance between them and the speed of light.

$$\Delta t = \frac{d}{c} = \frac{1.50\times10^{11}\,\text{m}}{3.00\times10^{8}\,\text{m/s}} = 5.00\times10^{2}\,\text{s} = \boxed{8.33\,\text{min}}$$

36. We find the index of refraction from Eq. 32-1.

$$n = \frac{c}{v} = \frac{c}{0.88v_{\text{water}}} = \frac{c}{0.88\left(\dfrac{c}{n_{\text{water}}}\right)} = \frac{n_{\text{water}}}{0.88} = \frac{1.33}{0.88} = \boxed{1.51}$$

$\boxed{37.}$ The length in space of a burst is the speed of light times the elapsed time.

$$d = ct = \left(3.00\times10^{8}\,\text{m/s}\right)\left(10^{-8}\,\text{s}\right) = \boxed{3\,\text{m}}$$

38. Find the angle of refraction from Snell's law.

$$n_1\sin\theta_1 = n_2\sin\theta_2 \rightarrow \theta_2 = \sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_1\right) = \sin^{-1}\left(\frac{1.33}{1.00}\sin 38.5^\circ\right) = \boxed{55.9^\circ}$$

39. Find the angle of refraction from Snell's law.

$$n_1\sin\theta_1 = n_2\sin\theta_2 \rightarrow \theta_2 = \sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_1\right) = \sin^{-1}\left(\frac{1.00}{1.56}\sin 63^\circ\right) = \boxed{35^\circ}$$

40. We find the incident angle in the air (relative to the normal) from Snell's law.

$$n_1\sin\theta_1 = n_2\sin\theta_2 \rightarrow \theta_1 = \sin^{-1}\left(\frac{n_2}{n_1}\sin\theta_2\right) = \sin^{-1}\left(\frac{1.33}{1.00}\sin 33.0^\circ\right) = 46.4^\circ$$

Since this is the angle relative to the horizontal, the angle as measured from the horizon is $90.0^\circ - 46.4^\circ = \boxed{43.6^\circ}$.

41. We find the incident angle in the water from Snell's law.

$$n_1\sin\theta_1 = n_2\sin\theta_2 \rightarrow \theta_1 = \sin^{-1}\left(\frac{n_2}{n_1}\sin\theta_2\right) = \sin^{-1}\left(\frac{1.00}{1.33}\sin 56.0^\circ\right) = \boxed{38.6^\circ}$$

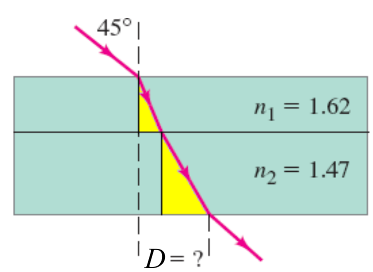
42. The angle of reflection is equal to the angle of incidence: $\theta_{\text{refl}} = \theta_1 = 2\theta_2$. Use Snell's law

$$n_{\text{air}}\sin\theta_1 = n_{\text{glass}}\sin\theta_2 \rightarrow (1.00)\sin 2\theta_2 = (1.56)\sin\theta_2$$

$$\sin 2\theta_2 = 2\sin\theta_2\cos\theta_2 = (1.56)\sin\theta_2 \rightarrow \cos\theta_2 = 0.780 \rightarrow \theta_2 = 38.74^\circ$$

$$\theta_1 = 2\theta_2 = \boxed{77.5^\circ}$$

$\boxed{43.}$ The beam forms the hypotenuse of two right triangles as it passes through the plastic and then the glass. The upper angle of the triangle is the angle of refraction in that medium. Note that the sum of the opposite sides is equal to the displacement *D*. First, we calculate the angles of refraction in each medium using Snell's Law (Eq. 32-5).



$$\sin 45 = n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\theta_1 = \sin^{-1}\left(\frac{\sin 45}{n_1}\right) = \sin^{-1}\left(\frac{\sin 45}{1.62}\right) = 25.88°$$

$$\theta_2 = \sin^{-1}\left(\frac{\sin 45}{n_2}\right) = \sin^{-1}\left(\frac{\sin 45}{1.47}\right) = 28.75°$$

We then use the trigonometric identity for tangent to calculate the two opposite sides, and sum to get the displacement.

$$D = D_1 + D_2 = h_1 \tan\theta_1 + h_1 \tan\theta_1 = (2.0\,\text{cm})\tan 25.88° + (3.0\,\text{cm})\tan 28.75° = \boxed{2.6\,\text{cm}}$$

44. (*a*) We use Eq. 32-5 to calculate the refracted angle as the light enters the glass (n=1.56) from the air (n=1.00).

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \rightarrow \theta_2 = \sin^{-1}\left[\frac{n_1}{n_2}\sin\theta_1\right] = \sin^{-1}\left[\frac{1.00}{1.56}\sin\ 43.5°\right] = 26.18° \approx \boxed{26.2°}$$

(*b*) We again use Eq. 32-5 using the refracted angle in the glass and the indices of refraction of the glass and water.

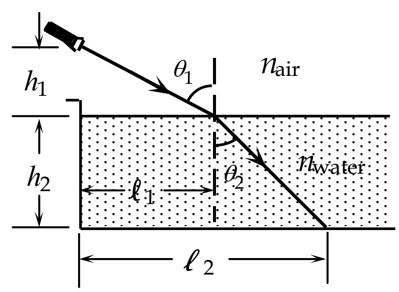
$$\theta_3 = \sin^{-1}\left[\frac{n_2}{n_3}\sin\theta_2\right] = \sin^{-1}\left[\frac{1.56}{1.33}\sin\ 26.18°\right] = 31.17° \approx \boxed{31.2°}$$

(*c*) We repeat the same calculation as in part (a), but using the index of refraction of water.

$$\theta_3 = \sin^{-1}\left[\frac{n_1}{n_3}\sin\theta_1\right] = \sin^{-1}\left[\frac{1.00}{1.33}\sin\ 43.5°\right] = 31.17° \approx \boxed{31.2°}$$

As expected the refracted angle in the water is the same whether the light beam first passes through the glass, or passes directly into the water.

45. We find the angle of incidence from the distances.



$$\tan\theta_1 = \frac{\ell_1}{h_1} = \frac{(2.5\,\text{m})}{(1.3\,\text{m})} = 1.9231 \quad\rightarrow\quad \theta_1 = 62.526°$$

For the refraction from air into water, we have

$$n_{\text{air}} \sin\theta_1 = n_{\text{water}} \sin\theta_2;$$

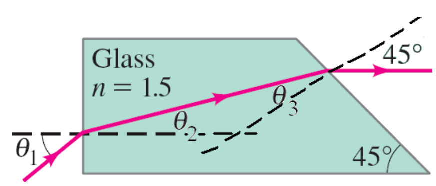
$$(1.00)\sin 62.526° = (1.33)\sin\theta_2 \quad\rightarrow\quad \theta_2 = 41.842°$$

We find the horizontal distance from the edge of the pool from

$$\ell = \ell_1 + \ell_2 = \ell_1 + h_2 \tan\theta_2$$

$$= 2.5\,\text{m} + (2.1\,\text{m})\tan 41.842° = 4.38\,\text{m} \approx \boxed{4.4\,\text{m}}$$

46. Since the light ray travels parallel to the base when it exits the glass, and the back edge of the glass makes a 45° angle to the horizontal, the exiting angle of refraction is 45°. We use Snell's law, Eq. 32-5, to calculate the incident angle at the back pane.



$$\theta_3 = \sin^{-1}\left[\frac{n_4}{n_3}\sin\theta_4\right] = \sin^{-1}\left[\frac{1.0}{1.5}\sin\ 45°\right] = 28.13°$$

We calculate the refracted angle at the front edge of the glass by noting that the angles θ_2 and θ_3 in the figure form two angles of a triangle. The third angle, as determined by the perpendiculars to the surface, is 135°.

$$\theta_2 + \theta_3 + 135° = 180° \to \theta_2 = 45° - \theta_3 = 45° - 28.13° = 16.87°$$

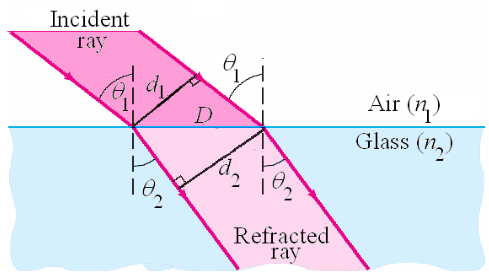
Finally, we use Snell's law at the front face of the glass to calculate the incident angle.

$$\theta_1 = \sin^{-1}\left[\frac{n_2}{n_1}\sin\theta_2\right] = \sin^{-1}\left[\frac{1.5}{1.0}\sin\ 16.87°\right] = 25.81° \approx \boxed{26°}$$

47. As the light ray passes from air into glass with an angle of incidence of 25°, the beam will refract. Determine the angle of refraction by applying Snell's law.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \quad \to$$

$$\theta_2 = \sin^{-1}\left[\frac{n_1}{n_2}\sin\theta_1\right] = \sin^{-1}\left[\frac{1.00}{1.5}\sin\ 25°\right] = 16.36°$$



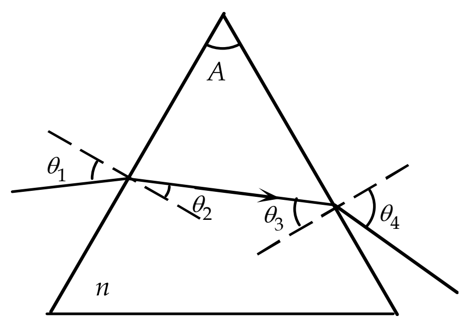
We now consider the two right triangles created by the diameters of the incident and refracted beams with the air–glass interface, as shown in the figure. The diameters form right angles with the ray direction and using complementary angles we see that the angle between the diameter and the interface is equal to the incident and refracted angles. Since the air–glass interface creates the hypotenuse for both triangles we use the definition of the cosine to solve for this length in each triangle and set the lengths equal. The resulting equation is solved for the diameter of the refracted ray.

$$D = \frac{d_1}{\cos\theta_1} = \frac{d_2}{\cos\theta_2} \quad \to \quad d_2 = d_1\frac{\cos\theta_2}{\cos\theta_1} = (3.0\,\text{mm})\frac{\cos 16.36°}{\cos 25°} = \boxed{3.2\,\text{mm}}$$

48. Find the angle θ_2 for the refraction at the first surface.

$$n_{\text{air}} \sin\theta_1 = n \sin\theta_2$$

$$(1.00)\sin 45.0° = (1.54)\sin\theta_2 \quad \to \quad \theta_2 = 27.33°$$



Find the angle of incidence at the second surface from the triangle formed by the two sides of the prism and the light path.

$$(90° - \theta_2) + (90° - \theta_3) + A = 180° \quad \to$$

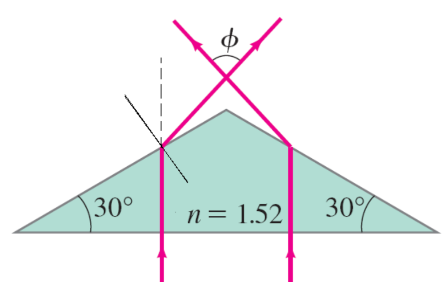
$$\theta_3 = A - \theta_2 = 60° - 27.33° = 32.67°$$

Use refraction at the second surface to find θ_4.

$$n \sin\theta_3 = n_{\text{air}} \sin\theta_4 \quad \to \quad (1.54)\sin 32.67° = (1.00)\sin\theta_4 \quad \to \quad \theta_4 = \boxed{56.2° \text{ from the normal}}$$

$\boxed{49.}$ Since the angle of incidence at the base of the prism is $0°$, the rays are undeflected there. The angle of incidence at the upper face of the prism is $30°$. Use Snell's law to calculate the angle of refraction as the light exits the prism.

$$n_1 \sin\theta_1 = \sin\theta_r \to \theta_r = \sin^{-1}(1.52 \sin 30°) = 49.46°$$



From the diagram, note that a normal to either top surface makes a 30° angle from the vertical. Subtracting 30° from the refracted angle will give the angle of the beam with respect to the vertical. By symmetry, the angle ϕ is twice the angle of the refracted beam from the vertical.

$$\phi = 2(\theta_r - 30°) = 2(49.46° - 30°) = \boxed{38.9°}$$

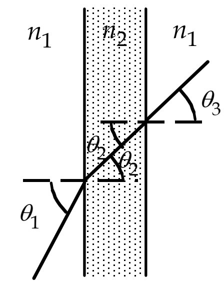
50. Because the surfaces are parallel, the angle of refraction from the first surface is the angle of incidence at the second. Thus for the two refractions, we have the following.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \;\; ; \;\; n_2 \sin\theta_2 = n_1 \sin\theta_3$$

Substitute the second equation into the first.

$$n_1 \sin\theta_1 = n_1 \sin\theta_3 \;\; \rightarrow \;\; \boxed{\theta_3 = \theta_1}$$

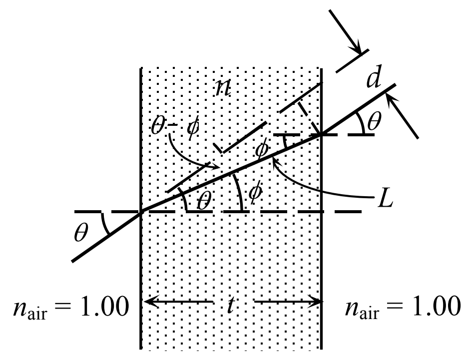
Because the ray emerges in the same index of refraction, it is undeviated.



51. Because the glass surfaces are parallel, the exit beam will be traveling in the same direction as the original beam. Find the angle inside the glass from Snell's law, $n_{air} \sin\theta = n \sin\phi$. Since the angles are small, $\cos\phi \approx 1$ and $\sin\phi \approx \phi$, where ϕ is in radians.

$$(1.00)\,\theta = n\phi \;\; \rightarrow \;\; \phi = \frac{\theta}{n}$$

Find the distance along the ray in the glass from $L = \dfrac{t}{\cos\phi} \approx t$, and then find the perpendicular displacement from the original direction.



$$d = L\sin(\theta - \phi) \approx t(\theta - \phi) = t\left[\theta - \left(\frac{\theta}{n}\right)\right] = \boxed{\frac{t\theta(n-1)}{n}}$$

52. We find the speed of light from the speed of light in a vacuum divided by the index of refraction. Examining the graph we estimate that the index of refraction of 450 nm light in silicate flint glass is 1.643 and of 680 nm light is 1.613. There will be some variation in the answers due to estimation from the graph.

$$\frac{v_{red} - v_{blue}}{v_{red}} = \frac{c/n_{680} - c/n_{450}}{c/n_{680}} = \frac{1/1.613 - 1/1.643}{1/1.613} = 0.01826 \approx \boxed{1.8\%}$$

53. We find the angles of refraction in the glass from Snell's law, Eq. 32-5.

$$(1.00)\sin 60.00° = (1.4831)\sin\theta_{2,blue} \;\; \rightarrow \;\; \theta_{2,blue} = 35.727°$$

$$(1.00)\sin 60.00° = (1.4754)\sin\theta_{2,red} \;\; \rightarrow \;\; \theta_{2,red} = 35.943° \text{ which gives } \theta_{2,700} = 35.943°.$$

Thus the angle between the refracted beams is

$$\theta_{2,red} - \theta_{2,blue} = 35.943° - 35.727° = 0.216° \approx \boxed{0.22°.}$$

54. The indices of refraction are estimated from Figure 32-28 as 1.642 for 465 nm and 1.619 for 652 nm. Consider the refraction at the first surface.

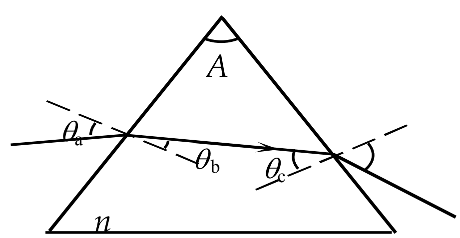
$$n_{air} \sin\theta_a = n \sin\theta_b \;\; \rightarrow$$

$$(1.00)\sin 45° = (1.642)\sin\theta_{b1} \;\; \rightarrow \;\; \theta_{b1} = 25.51°$$

$$(1.00)\sin 45° = (1.619)\sin\theta_{b2} \;\; \rightarrow \;\; \theta_{b2} = 25.90°$$

We find the angle of incidence at the second surface from the upper triangle.

$$(90° - \theta_b) + (90° - \theta_c) + A = 180° \;\; \rightarrow$$



$\theta_{c1} = A - \theta_{b1} = 60.00° - 25.51° = 34.49°$; $\theta_{c2} = A - \theta_{b2} = 60.00° - 25.90° = 34.10°$

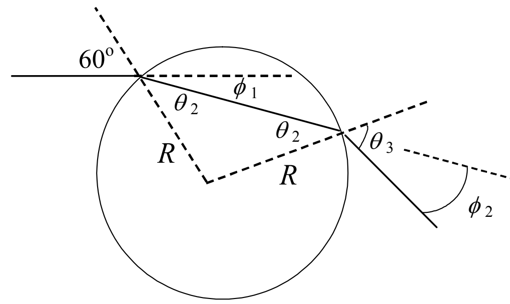
Apply Snell's law at the second surface.

$$n \sin\theta_c = n_{air} \sin\theta_d$$

$$(1.642)\sin 34.49° = (1.00)\sin\theta_{d1} \rightarrow \boxed{\theta_{d1} = 68.4° \text{ from the normal}}$$

$$(1.619)\sin 34.10° = (1.00)\sin\theta_{d2} \rightarrow \boxed{\theta_{d2} = 65.2° \text{ from the normal}}$$

55. At the first surface, the angle of incidence $\theta_1 = 60^\circ$ from air $(n_1 = 1.000)$ and the angle of refraction θ_2 into water $(n_2 = n)$ is found using Snell's law.



$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \rightarrow$$

$$(1.000)\sin 60° = (n)\sin\theta_2 \rightarrow$$

$$\theta_2 = \sin^{-1}\left(\frac{\sin 60°}{n}\right)$$

Note that at this surface the ray has been deflected from its initial direction by angle $\phi_1 = 60° - \theta_2$. From the figure we see that the triangle that is interior to the drop is an isosceles triangle, so the angle of incidence from water $(n_2 = n)$ at the second surface is θ_2 and angle of refraction is θ_3 into air $(n_3 = 1.000)$. This relationship is identical to the relationship at the first surface, showing that the refracted angle as the light exits the drop is again 60°.

$$n_2 \sin\theta_2 = n_3 \sin\theta_3 \rightarrow (n)\sin\theta_2 = (1.000)\sin\theta_3 \rightarrow \sin\theta_3 = n\sin\theta_2 \rightarrow$$

$$\sin\theta_3 = n\left(\frac{\sin 60^\circ}{n}\right) = \sin 60^\circ \rightarrow \theta_3 = 60^\circ$$

Note that at this surface the ray has been deflected from its initial direction by the angle $\phi_2 = \theta_3 - \theta_2 = 60^\circ - \theta_2$. The total deflection of the ray is equal to the sum of the deflections at each surface.

$$\phi = \phi_1 + \phi_2 = \left(60^\circ - \theta_2\right) + \left(60^\circ - \theta_2\right) = 120^\circ - 2\theta_2 = 120^\circ - 2\sin^{-1}\left(\frac{\sin 60^\circ}{n}\right)$$

Inserting the indices of refraction for the two colors and subtracting the angles gives the difference in total deflection.

$$\Delta\phi = \phi_{violet} - \phi_{red} = \left\{120^\circ - 2\sin^{-1}\left[\frac{\sin 60^\circ}{n_{violet}}\right]\right\} - \left\{120^\circ - 2\sin^{-1}\left[\frac{\sin 60^\circ}{n_{red}}\right]\right\}$$

$$= 2\left\{\sin^{-1}\left[\frac{\sin 60^\circ}{n_{red}}\right] - \sin^{-1}\left[\frac{\sin 60^\circ}{n_{violet}}\right]\right\} = 2\left\{\sin^{-1}\left[\frac{\sin 60^\circ}{1.330}\right] - \sin^{-1}\left[\frac{\sin 60^\circ}{1.341}\right]\right\} = \boxed{0.80^\circ}$$

56. (*a*) We solve Snell's law for the refracted angle. Then, since the index varies by only about 1%, we differentiate the angle with respect to the index of refraction to determine the spread in angle.

$$\sin\theta_1 = n\sin\theta_2 \rightarrow \theta_2 = \sin^{-1}\left(\frac{\sin\theta_1}{n}\right) \rightarrow$$

$$\frac{\Delta\theta_2}{\Delta n} \approx \frac{d\theta_2}{dn} = \frac{\sin\theta_1}{n^2\sqrt{1 - \frac{\sin^2\theta_1}{n^2}}} \rightarrow \boxed{\Delta\theta_2 = \frac{\Delta n}{n}\frac{\sin\theta_1}{\sqrt{n^2 - \sin^2\theta_1}}}$$

(*b*) We set $n = 1.5$ and $\theta_1 = 0° = 0\,\text{rad}$ and solve for the spread in refracted angle.

$$\Delta\theta_2 = \frac{\Delta n}{n}\frac{\sin\theta_1}{\sqrt{n^2 - \sin^2\theta_1}} = (0.01)\frac{\sin\ 0}{\sqrt{1.5^2 - \sin^2 0}} = \boxed{0}$$

(*c*) We set $n = 1.5$ and $\theta_1 = 90°$ and solve for the spread in refracted angle. We must convert the spread from radians back to degrees.

$$\Delta\theta_2 = (0.01)\frac{\sin\ 90°}{\sqrt{1.5^2 - \sin^2 90°}} = 0.0089\ \text{rad}\left(\frac{180°}{\pi\ \text{rad}}\right) = \boxed{0.5°}$$

57. When the light in the material with a higher index is incident at the critical angle, the refracted angle is 90°. Use Snell's law.

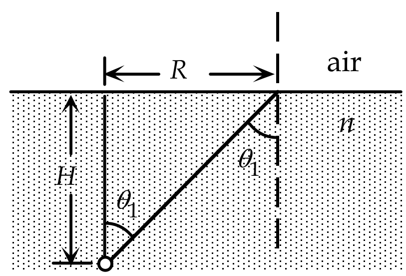
$$n_{\text{diamond}}\sin\theta_1 = n_{\text{water}}\sin\theta_2 \quad\rightarrow\quad \theta_1 = \sin^{-1}\left(\frac{n_{\text{water}}}{n_{\text{diamond}}}\right) = \sin^{-1}\frac{1.33}{2.42} = \boxed{33.3°}$$

Because diamond has the higher index, the light must start in $\boxed{\text{diamond.}}$

58. When the light in the liquid is incident at the critical angle, the refracted angle is 90°. Use Snell's law.

$$n_{\text{liquid}}\sin\theta_1 = n_{\text{air}}\sin\theta_2 \quad\rightarrow\quad n_{\text{liquid}} = n_{\text{air}}\frac{\sin\theta_2}{\sin\theta_1} = (1.00)\frac{1}{\sin 49.6°} = \boxed{1.31}$$

59. We find the critical angle for light leaving the water:



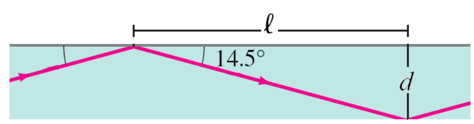
$$n_{\text{water}}\sin\theta_1 = n_{\text{air}}\sin\theta_2 \quad\rightarrow$$

$$\theta_1 = \sin^{-1}\left(\frac{n_{\text{air}}}{n_{\text{water}}}\right) = \sin^{-1}\frac{1.00}{1.33} = 48.75°$$

If the light is incident at a greater angle than this, it will totally reflect. Find *R* from the diagram.

$$R > H\tan\theta_1 = (72.0\,\text{cm})\tan 48.75° = \boxed{82.1\,\text{cm}}$$

60. The ray reflects at the same angle, so each segment makes a 14.5° angle with the side. We find the distance ℓ between reflections from the definition of the tangent function.



$$\tan\theta = \frac{d}{\ell} \rightarrow \ell = \frac{d}{\tan\theta} = \frac{1.40\times10^{-4}\ \text{m}}{\tan\ 14.5°} = \boxed{5.41\times10^{-4}\ \text{m}}$$

$\boxed{61.}$ We find the angle of incidence from the distances.

$$\tan\theta_1 = \frac{\ell}{h} = \frac{(7.6\,\text{cm})}{(8.0\,\text{cm})} = 0.95 \quad\rightarrow\quad \theta_1 = 43.53°$$

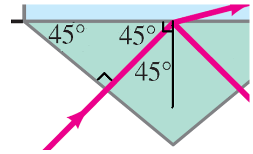
The relationship for the maximum incident angle for refraction from liquid into air gives this.

$$n_{\text{liquid}}\sin\theta_1 = n_{\text{air}}\sin\theta_2 \quad\rightarrow\quad n_{\text{liquid}}\sin\theta_{1\,\text{max}} = (1.00)\sin 90° \quad\rightarrow\quad \sin\theta_{1\,\text{max}} = \frac{1}{n_{\text{liquid}}}$$

Thus we have the following.

$$\sin\theta_1 \geq \sin\theta_{1\max} = \frac{1}{n_{\text{liquid}}} \rightarrow \sin 43.53° = 0.6887 \geq \frac{1}{n_{\text{liquid}}} \rightarrow \boxed{n_{\text{liquid}} \geq 1.5}$$

62. For the device to work properly, the light should experience total internal reflection at the top surface of the prism when it is a prism to air interface, but not total internal reflection when the top surface is a prism to water interface. Since the incident ray is perpendicular to the lower surface of the prism, light does not experience refraction at that surface. As shown in the diagram, the incident angle for the upper surface will be 45°. We then use Eq. 32-7 to determine the minimum index of refraction for total internal reflection with an air interface, and the maximum index of refraction for a water interface. The usable indices of refraction will lie between these two values.



$$\frac{n_2}{n_1} = \sin\theta_c \rightarrow n_{1,\min} = \frac{n_{\text{air}}}{\sin\theta_c} = \frac{1.00}{\sin 45°} = 1.41 \rightarrow n_{1,\max} = \frac{n_{\text{water}}}{\sin\theta_c} = \frac{1.33}{\sin 45°} = 1.88$$

The index of refraction must fall within the range $1.41 < n < 1.88$. $\boxed{\text{A Lucite prism will work.}}$

63. (*a*) We calculate the critical angle using Eq. 32-7. We calculate the time for each ray to pass through the fiber by dividing the length the ray travels by the speed of the ray in the fiber. The length for ray A is the horizontal length of the fiber. The length for ray B is equal to the length of the fiber divided by the critical angle, since ray B is always traveling along a diagonal line at the critical angle relative to the horizontal. The speed of light in the fiber is the speed of light in a vacuum divided by the index of refraction in the fiber.

$$\sin\theta_c = \frac{n_2}{n_1} \;;\; \Delta t = t_B - t_A = \frac{\ell_B}{v} - \frac{\ell_A}{v} = \frac{\ell_A}{v\sin\theta_c} - \frac{\ell_A}{v} = \frac{\ell_A}{c/n_1}\left(\frac{n_1}{n_2} - 1\right)$$

$$= \frac{(1.0\,\text{km})(1.465)}{(3.00\times10^5\,\text{km/s})}\left(\frac{1.465}{1.000} - 1\right) = \boxed{2.3\times10^{-6}\,\text{s}}$$

(*b*) We now replace the index of refraction of air (n = 1.000) with the index of refraction of the glass "cladding" (n = 1.460).

$$\Delta t = \frac{\ell_A n_1}{c}\left(\frac{n_1}{n_2} - 1\right) = \frac{(1.0\,\text{km})(1.465)}{3.00\times10^5\,\text{km/s}}\left(\frac{1.465}{1.460} - 1\right) = \boxed{1.7\times10^{-8}\,\text{s}}$$

64. (*a*) The ray enters normal to the first surface, so there is no deviation there. The angle of incidence is 45° at the second surface. When there is air outside the surface, we have the following.

$$n_1\sin\theta_1 = n_2\sin\theta_2 \rightarrow n_1\sin 45° = (1.00)\sin\theta_2$$

For total internal reflection to occur, $\sin\theta_2 \geq 1$, and so $n_1 \geq \frac{1}{\sin 45°} = \boxed{1.41}$.

(*b*) When there is water outside the surface, we have the following.

$$n_1\sin\theta_1 = n_2\sin\theta_2 \rightarrow (1.58)\sin 45° = (1.33)\sin\theta_2 \rightarrow \sin\theta_2 = 0.84$$

Because $\sin\theta_2 < 1$, $\boxed{\text{the prism will not be totally reflecting.}}$

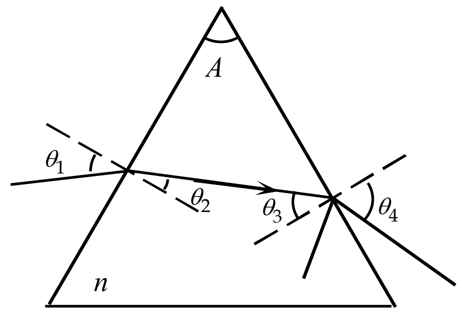
(*c*) For total reflection when there is water outside the surface, we have the following.

$$n_1\sin\theta_1 = n_2\sin\theta_2 \rightarrow n_1\sin 45° = (1.33)\sin\theta_2$$

$$n_1\sin 45° = (1.33)\sin\theta_2.$$

For total internal reflection to occur, $\sin\theta_2 \geq 1$.

$$n_1 \geq \frac{1.33}{\sin 45°} = \boxed{1.88}$$



65. For the refraction at the first surface, we have the following.

$$n_{\text{air}}\sin\theta_1 = n\sin\theta_2 \rightarrow (1.00)\sin\theta_1 = n\sin\theta_2 \rightarrow$$

$$\sin\theta_2 = \sin\frac{\theta_1}{n}.$$

Find the angle of incidence at the second surface.

$$(90^\circ - \theta_2) + (90^\circ - \theta_3) + A = 180^\circ \rightarrow$$

$$\theta_3 = A - \theta_2 = 60.0^\circ - \theta_2$$

For the refraction at the second surface, we have this.

$$n\sin\theta_3 = n_{\text{air}}\sin\theta_4 = (1.00)\sin\theta_4$$

The maximum value of θ_4 before internal reflection takes place at the second surface is 90°. For internal reflection to occur, we have the following.

$$n\sin\theta_3 = n\sin(A - \theta_2) \geq 1 \rightarrow n(\sin A\cos\theta_2 - \cos A\sin\theta_2) \geq 1$$

Use the result from the first surface to eliminate n.

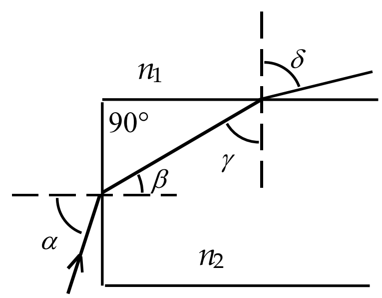
$$\frac{\sin\theta_1(\sin A\cos\theta_2 - \cos A\sin\theta_2)}{(\sin\theta_2)} = \sin\theta_1\left(\frac{\sin A}{\tan\theta_2 - \cos A}\right) \geq 1 \rightarrow$$

$$\frac{1}{\tan\theta_2} \geq \frac{\left[\left(\frac{1}{\sin\theta_1}\right) + \cos A\right]}{\sin A} = \frac{\left[\left(\frac{1}{\sin 45.0^\circ}\right) + \cos 60.0^\circ\right]}{\sin 60.0^\circ} = 2.210 \rightarrow \text{or}$$

$$\tan\theta_2 \leq 0.452 \rightarrow \theta_2 \leq 24.3^\circ$$

Use the result from the first surface.

$$n_{\min} = \frac{\sin\theta_1}{\sin\theta_{2\max}} = \frac{\sin 45.0^\circ}{\sin 24.3^\circ} = 1.715 \rightarrow \boxed{n \geq 1.72}$$



66. For the refraction at the side of the rod, we have $n_2\sin\gamma = n_1\sin\delta$.

The minimum angle for total reflection $\gamma_{\min}$ occurs when $\delta = 90^\circ$.

$$n_2\sin\gamma_{\min} = (1.00)(1) = 1 \rightarrow \sin\gamma_{\min} = \frac{1}{n_2}$$

Find the maximum angle of refraction at the end of the rod.

$$\beta_{\max} = 90^\circ - \gamma_{\min}$$

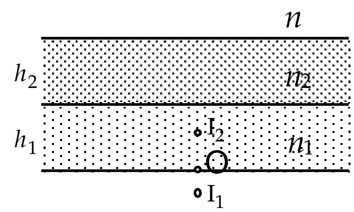
Because the sine function increases with angle, for the refraction at the end of the rod, we have the following.

$$n_1\sin\alpha_{\max} = n_2\sin\beta_{\max} \rightarrow (1.00)\sin\alpha_{\max} = n_2\sin(90^\circ - \gamma_{\min}) = n_2\cos\gamma_{\min}$$

If we want total internal reflection to occur for any incident angle at the end of the fiber, the maximum value of α is 90°, so $n_2\cos\gamma_{\min} = 1$. When we divide this by the result for the refraction at the side, we get $\tan\gamma_{\min} = 1 \rightarrow \gamma_{\min} = 45^\circ$. Thus we have the following.

$$\boxed{n_2 \geq \frac{1}{\sin\gamma_{\min}} = \frac{1}{\sin 45^\circ} = 1.414}$$

67. We find the location of the image of a point on the bottom from the refraction from water to glass, using Eq. 32-8, with $R = \infty$.



$$\frac{n_1}{d_o} + \frac{n_2}{d_i} = \frac{n_2 - n_1}{R} = 0 \quad \rightarrow$$

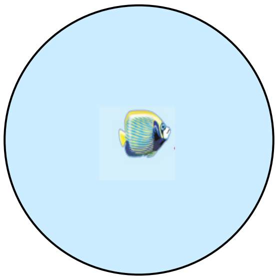
$$d_i = -\frac{n_2 d_o}{n_1} = -\frac{1.58(12.0\,\text{cm})}{1.33} = -14.26\,\text{cm}$$

Using this image distance from the top surface as the object for the refraction from glass to air gives the final image location, which is the apparent depth of the water.

$$\frac{n_2}{d_{o2}} + \frac{n_3}{d_{i2}} = \frac{n_3 - n_2}{R} = 0 \quad \rightarrow \quad d_{i2} = -\frac{n_3 d_{o2}}{n_2} = -\frac{1.00(13.0\,\text{cm} + 14.26\,\text{cm})}{1.58} = -17.25\,\text{cm}$$

Thus the bottom appears to be **17.3 cm below the surface of the glass**. In reality it is 25 cm.

68. (*a*) We use Eq. 32-8 to calculate the location of the image of the fish. We assume that the observer is outside the circle in the diagram, to the right of the diagram. The fish is located at the center of the sphere so the object distance is 28.0 cm. Since the glass is thin we use the index of refraction of the water and of the air. Index 1 refers to the water, and index 2 refers to the air. The radius of curvature of the right side of the bowl is negative.



$$\frac{n_1}{d_o} + \frac{n_2}{d_i} = \frac{n_2 - n_1}{R} \quad \rightarrow$$

$$d_i = n_2\left[\frac{n_2 - n_1}{R} - \frac{n_1}{d_o}\right]^{-1} = 1.00\left[\frac{1.00 - 1.33}{-28.0\,\text{cm}} - \frac{1.33}{28.0\,\text{cm}}\right]^{-1} = \boxed{-28.0\,\text{cm}}$$

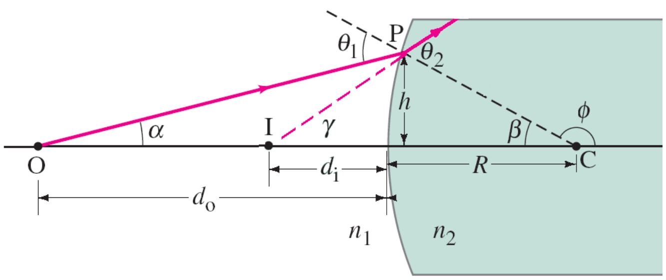
The image is also at the center of the bowl. When the fish is at the center of the bowl, all small-angle light rays traveling outward from the fish are approximately perpendicular to the surface of the bowl, and therefore do not refract at the surface. This causes the image of the fish to also be located at the center of the bowl.

(*b*) We repeat the same calculation as above with the object distance 20.0 from the right side of the bowl, so $d_o = 20.0\,\text{cm}$.

$$d_i = n_2\left[\frac{n_2 - n_1}{R} - \frac{n_1}{d_o}\right]^{-1} = 1.00\left[\frac{1.00 - 1.33}{-28.0\,\text{cm}} - \frac{1.33}{20.0\,\text{cm}}\right]^{-1} = \boxed{-18.3\,\text{cm}}$$

The fish appears closer to the center of the bowl than it actually is.

69. (*a*) The accompanying figure shows a light ray originating at point O and entering the convex spherical surface at point P. In this case $n_2 < n_1$. The ray bends away from the normal and creates a virtual image at point I. From the image and supplementary angles we obtain the relationships between the angles.



$$\theta_1 = \alpha + \beta \qquad \theta_2 = \gamma + \beta$$

We then use Snell's law to relate the incident and refracted angle. For this derivation we assume these are small angles.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \quad \rightarrow \quad n_1\theta_1 = n_2\theta_2$$

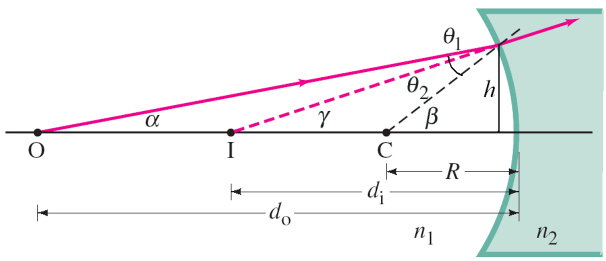
From the diagram we can create three right triangles, each with height *h* and lengths d_o, d_i, and *R*. Again, using the small angle approximation we obtain a relationship between the angles and lengths. Combining these definitions to eliminate the angles we obtain Eq. 32-8, noting that by our definition d_i is a negative value.

$$\alpha = \frac{h}{d_o} \;;\; \gamma = \frac{h}{R} \;;\; \beta = \frac{h}{(-d_i)}$$

$$n_1\theta_1 = n_2\theta_2 \quad \rightarrow \quad n_1(\alpha+\beta) = n_2(\gamma+\beta) = n_1\alpha + n_1\beta = n_2\gamma + n_2\beta \quad \rightarrow$$

$$n_1\frac{h}{d_o} + n_1\frac{h}{R} = n_2\frac{h}{(-d_i)} + n_2\frac{h}{R} \quad \rightarrow \quad \boxed{\frac{n_1}{d_o} + \frac{n_2}{d_i} = \frac{n_2 - n_1}{R}}$$

(*b*) This image shows a concave surface with $n_2 > n_1$. Again, we use the approximation of small angles and sign convention that $R < 0$ and $d_i < 0$. We write relationships between the angles using supplementary angles, Snell's law, and right triangles. Combining these equations to eliminate the angles we arrive at Eq. 32-8.

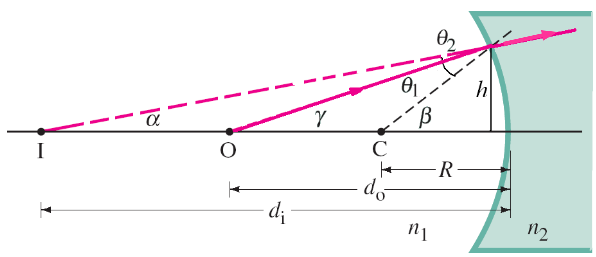


$$\theta_1 = \beta - \alpha \;;\; \theta_2 = \beta - \gamma \;;\; \alpha = \frac{h}{d_o} \;;\; \gamma = \frac{h}{(-d_i)} \;;\; \beta = \frac{h}{(-R)}$$

$$n_1\theta_1 = n_2\theta_2 \quad \rightarrow \quad n_1(\beta-\alpha) = n_2(\beta-\gamma) = n_1\beta - n_1\alpha = n_2\beta - n_2\gamma \quad \rightarrow$$

$$n_1\frac{h}{(-R)} - n_1\frac{h}{d_o} = n_2\frac{h}{(-R)} - n_2\frac{h}{(-d_i)} \quad \rightarrow \quad \boxed{\frac{n_1}{d_0} + \frac{n_2}{d_i} = \frac{n_2 - n_1}{R}}$$

(*c*) This image shows a concave surface with $n_2 < n_1$. Again, we use the approximation of small angles and sign convention that $R < 0$ and $d_i < 0$. We write relationships between the angles using supplementary angles, Snell's law, and right triangles. Combining these equations to eliminate the angles we arrive at Eq. 32-8.

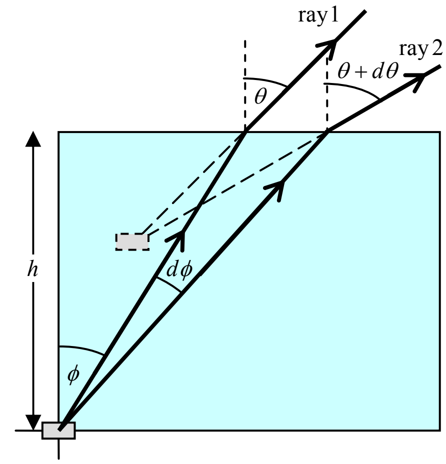


$$\theta_1 = \beta - \gamma \;;\; \theta_2 = \beta - \alpha \;;\; \alpha = \frac{h}{(-d_i)} \;;\; \gamma = \frac{h}{d_o} \;;\; \beta = \frac{h}{(-R)}$$

$$n_1\theta_1 = n_2\theta_2 \quad \rightarrow \quad n_1(\beta-\gamma) = n_2(\beta-\alpha) = n_1\beta - n_1\gamma = n_2\beta - n_2\alpha \quad \rightarrow$$

$$n_1\frac{h}{(-R)} - n_1\frac{h}{d_o} = n_2\frac{h}{(-R)} - n_2\frac{h}{(-d_i)} \quad \rightarrow \quad \boxed{\frac{n_1}{d_o} + \frac{n_2}{d_i} = \frac{n_2 - n_1}{R}}$$

70. We consider two rays leaving the coin. These rays refract upon leaving the surface and reach the observer's eye with angles of refraction all very near $\theta = 45°$. Let the origin of coordinates be at the actual location of the coin. We will write straight-line equations for each of two refracted rays, one with a refraction angle of θ and the other with a refraction angle of $\theta + d\theta$, and extrapolate them back to where they intersect to find the location of the image. We utilize the relationship $f(x+dx) = f(x) + \left(\frac{df}{dx}\right)dx.$



First, apply Snell's law to both rays.

Ray # 1, leaving the coin at angle ϕ.

$$n\sin\phi = \sin\theta$$

Ray # 2, leaving the coin at angle $\phi + d\phi$.

$$n\sin(\phi + d\phi) = \sin(\theta + d\theta)$$

Note the following relationship involving the differential angles.

$$\sin(\phi + d\phi) = \sin\phi + \frac{d(\sin\phi)}{d\phi}d\phi = \sin\phi + \cos\phi d\phi \;\; ; \;\; \sin(\theta + d\theta) = \sin\theta + \cos\theta d\theta$$

So for Ray # 2, we would have the following Snell's law relationship.

$$n[\sin\phi + \cos\phi d\phi] = [\sin\theta + \cos\theta d\theta] \;\; \rightarrow \;\; n\sin\phi + n\cos\phi d\phi = \sin\theta + \cos\theta d\theta \;\; \rightarrow$$

$$n\cos\phi d\phi = \cos\theta d\theta \;\; \rightarrow \;\; d\phi = \frac{\cos\theta}{n\cos\phi}d\theta$$

This relationship between $d\phi$ and $d\theta$ will be useful later in the solution.

Ray # 1 leaves the water at coordinates $x_1 = h\tan\phi,\ y_1 = h$ and has a slope after it leaves the water of $m_1 = \tan(90° - \theta) = \cot\theta.$ Thus a straight-line equation describing ray # 1 after it leaves the water is as follows.

$$y - y_1 = (x - x_1)m_1 \;\; \rightarrow \;\; y = h + (x - h\tan\phi)\cot\theta$$

Ray # 2 leaves the water at the following coordinates.

$$x_2 = h\tan(\phi + d\phi) = h\left[\tan\phi + \frac{d(\tan\phi)}{d\phi}d\phi\right] = h\left[\tan\phi + \sec^2\phi d\phi\right],\ y_2 = h$$

Ray # 2 has the following slope after it leaves the water.

$$m_2 = \tan\left[90° - (\theta + d\theta)\right] = \cot(\theta + d\theta) = \cot\theta + \frac{d(\cot\theta)}{d\theta}d\theta = \cot\theta - \csc^2\theta d\theta$$

Thus a straight-line equation describing ray # 2 after it leaves the water is as follows.

$$y - y_2 = (x - x_2)m_2 \;\; \rightarrow \;\; y = h + \left(x - h\left[\tan\phi + \sec^2\phi d\phi\right]\right)\left[\cot\theta - \csc^2\theta d\theta\right]$$

To find where these rays intersect, which is the image location, set the two expressions for y equal to each other.

$$h + (x - h\tan\phi)\cot\theta = h + \left(x - h\left[\tan\phi + \sec^2\phi d\phi\right]\right)\left[\cot\theta - \csc^2\theta d\theta\right] \;\; \rightarrow$$

Expanding the terms and subtracting common terms gives us the following.

$$x\csc^2\theta d\theta = h\tan\phi\csc^2\theta d\theta - h\sec^2\phi d\phi\cot\theta + h\sec^2\phi d\phi\csc^2\theta d\theta$$

The first three terms each have a differential factor, but the last term has two differential factors.

That means the last term is much smaller than the other terms, and so can be ignored. So we delete the last term, and use the relationship between the differentials derived earlier.

$$x\csc^2\theta d\theta = h\tan\phi\csc^2\theta d\theta - h\sec^2\phi d\phi\cot\theta \quad ; \quad d\phi = \frac{\cos\theta}{n\cos\phi}d\theta \quad \rightarrow$$

$$x\csc^2\theta d\theta = h\tan\phi\csc^2\theta d\theta - h\sec^2\phi\frac{\cos\theta}{n\cos\phi}d\theta\cot\theta$$

$$x = h\left[\tan\phi - \sec^2\phi\frac{\cos\theta}{n\cos\phi}\frac{\cot\theta}{\csc^2\theta}\right] = h\left[\tan\phi - \frac{\cos^2\theta\sin\theta}{n\cos^3\phi}\right]$$

Now we may substitute in values. We know that $\theta = 45°$ and $h = 0.75\,\text{m}$. We use the original relationship for ray # 1 to solve for ϕ. And once we solve for *x*, we use the straight-line equation for ray # 1 to solve for *y*.

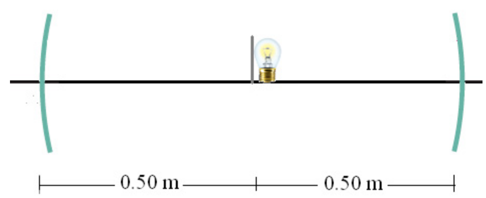
$$n\sin\phi = \sin\theta \quad \rightarrow \quad \phi = \sin^{-1}\frac{\theta}{n} = \sin^{-1}\frac{\sin 45°}{1.33} = 32.12°$$

$$x = h\left[\tan\phi - \frac{\cos^2\theta\sin\theta}{n\cos^3\phi}\right] = 0.75\left[\tan 32.12 - \frac{\cos^2 45\sin 45}{1.33\cos^3 32.12}\right] = 0.1427\,\text{m}$$

$$y = h + (x - h\tan\phi)\cot\theta = 0.75 + (0.1427 - 0.75\tan 32.12)\cot 45 = 0.4264\,\text{m}$$

The image of the coin is located 0.14 m toward the viewer and 0.43 m above the actual coin.

71. Use Eq. 32-2 to determine the location of the image from the right mirror, in terms of the focal length. Since this distance is measured from the right mirror, we subtract that distance from the separation distance between the two mirrors to obtain the object distance for the left mirror. We then insert this object distance back into Eq. 32-2, with the known image distance and combine terms to write a quadratic equation for the focal length.



$$\frac{1}{f} = \frac{1}{d_{o1}} + \frac{1}{d_{i1}} \quad \rightarrow \quad d_{i1} = \left(\frac{1}{f} - \frac{1}{d_{o1}}\right)^{-1} = \frac{fd_{o1}}{d_{o1} - f}$$

$$d_{o2} = D - d_{i1} = D - \frac{fd_{o1}}{d_{o1} - f} = \frac{Dd_{o1} - fD - fd_{o1}}{d_{o1} - f}$$

$$\frac{1}{f} = \frac{1}{d_{o2}} + \frac{1}{d_{i2}} = \frac{d_{o1} - f}{Dd_{o1} - fD - fd_{o1}} + \frac{1}{d_{i2}} = \frac{d_{i2}d_{o1} - fd_{i2} + Dd_{o1} - fD - fd_{o1}}{d_{i2}\left(Dd_{o1} - fD - fd_{o1}\right)}$$

$$d_{i2}\left(Dd_{o1} - fD - fd_{o1}\right) = d_{i2}d_{o1}f - f^2 d_{i2} + fDd_{o1} - f^2 D - f^2 d_{o1}$$

$$f^2\left[d_{i2} + D + d_{o1}\right] - f\left[2d_{i2}d_{o1} + Dd_{o1} + Dd_{i2}\right] + Dd_{o1}d_{i2} = 0$$

We insert the values for the initial object distance, final image distance, and mirror separation distance and then solve the quadratic equation.

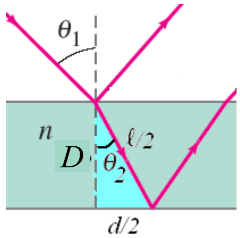
$$f^2\left[0.50\,\text{m} + 1.00\,\text{m} + 0.50\,\text{m}\right] - f\left[2(0.50\,\text{m})^2 + 2(1.00\,\text{m})(0.50\,\text{m})\right] + (1.00\,\text{m})(0.50\,\text{m})^2 = 0$$

$$(2.00\,\text{m})f^2 - (1.50\,\text{m}^2)f + 0.25\,\text{m}^3 = 0$$

$$f = \frac{1.50\,\text{m}^2 \pm \sqrt{(1.50\,\text{m}^2)^2 - 4(2.00\,\text{m})(0.25\,\text{m}^3)}}{2(2.00\,\text{m})} = \boxed{0.25\,\text{m} \text{ or } 0.50\,\text{m}}$$

If the focal length is 0.25 m, the right mirror creates an image at the location of the object. With the paper in place, this image would be blocked out. With a focal length of 0.50 m, the light from the right mirror comes out as parallel light. No image is formed from the right mirror. When this parallel light enters the second mirror it is imaged at the focal point (0.50 m) of the second mirror.

72. (*a*) We use Snell's law to calculate the refracted angle within the medium. Then using the right triangle formed by the ray within the medium, we can use the trigonometric identities to write equations for the horizontal displacement and path length.



$$\sin\theta_1 = n\sin\theta_2 \quad\rightarrow\quad \sin\theta_2 = \frac{\sin\theta_1}{n}$$

$$\cos\theta_2 = \frac{D}{\ell/2} \quad\rightarrow\quad \ell = \frac{2D}{\cos\theta_2} = \frac{2D}{\sqrt{1-\sin^2\theta_2}} = \boxed{\frac{2nD}{\sqrt{n^2-\sin^2\theta_1}}}$$

$$\sin\theta_2 = \frac{d/2}{\ell/2} \quad\rightarrow\quad d = \ell\sin\theta_2 = \frac{2nD}{\sqrt{n^2-\sin^2\theta_1}}\,\frac{\sin\theta_1}{n} = \boxed{\frac{2D\sin\theta_1}{\sqrt{n^2-\sin^2\theta_1}}}$$

(*b*) Evaluate the above expressions for $\theta_1 = 0°$.

$$\ell = \frac{2nD}{\sqrt{n^2-\sin^2\theta_1}} = \frac{2nD}{\sqrt{n^2}} = 2D \;;\; \sin\theta_2 = \frac{d/2}{\ell/2} \quad\rightarrow\quad d = \frac{2D\sin\theta_1}{\sqrt{n^2-\sin^2\theta_1}} = 0$$

These are the expected values.

73. (*a*) The first image seen will be due to a single reflection off the front glass. This image will be equally far behind the mirror as you are in front of the mirror.

$$D_1 = 2\times 1.5\text{ m} = \boxed{3.0\text{ m}}$$

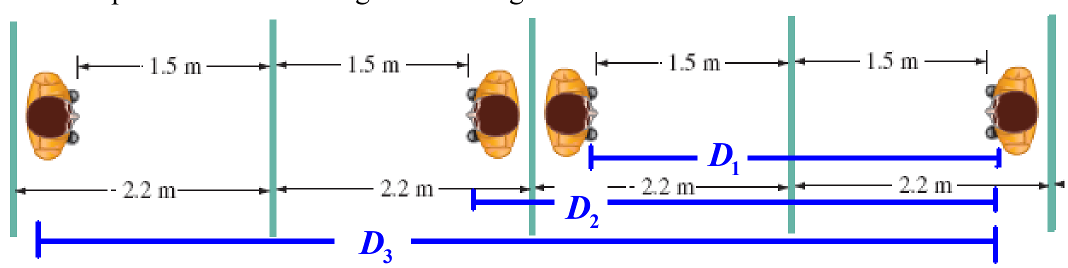
The second image seen will be the image reflected once off the front mirror and once off the back mirror. As seen in the diagram, this image will appear to be twice the distance between the mirrors.

$$D_2 = 1.5\text{ m} + 2.2\text{ m} + (2.2\text{ m} - 1.5\text{ m}) = 2\times 2.2\text{ m} = \boxed{4.4\text{ m}}$$

The third image seen will be the image reflected off the front mirror, the back mirror, and off the front mirror again. As seen in the diagram this image distance will be the sum of twice your distance to the mirror and twice the distance between the mirrors.

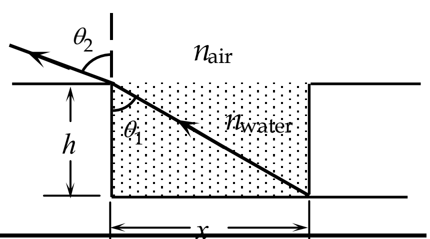
$$D_3 = 1.5\text{ m} + 2.2\text{ m} + 2.2\text{ m} + 1.5\text{ m} = 2\times 1.5\text{ m} + 2\times 2.2\text{ m} = \boxed{7.4\text{ m}}$$

The actual person is to the far right in the diagram.



(*b*) We see from the diagram that the first image is facing $\boxed{\text{toward you}}$; the second image is facing $\boxed{\text{away from you}}$; and the third image is facing $\boxed{\text{toward you.}}$

74. Find the angle of incidence for refraction from water into air.



$$n_{\text{water}}\sin\theta_1 = n_{\text{air}}\sin\theta_2 \quad\rightarrow$$

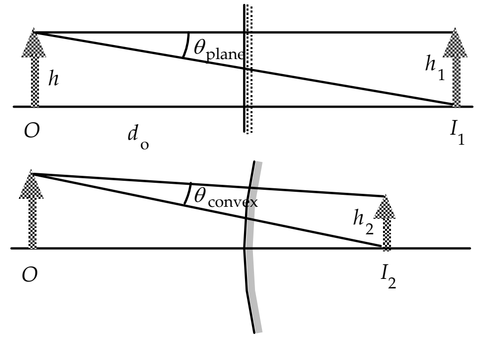
$$(1.33)\sin\theta_1 = (1.00)\sin(90.0° - 13.0°) \quad\rightarrow\quad \theta_1 = 47.11°$$

$$(1.33)\sin\theta_1 = (1.00)\sin(90.0° - 13.0°),$$

We find the depth of the pool from $\tan\theta_1 = x/h$.

$$\tan 47.11° = (5.50\,\text{m})/h \quad \rightarrow \quad h = \boxed{5.11\,\text{m}}$$

75. The apparent height of the image is related to the angle subtended by the image. For small angles, this angle is the height of the image divided by the distance between the image and viewer. Since both images are virtual, which gives a negative image distance, the image to viewer (object) distance will be the object distance minus the image distance. For the plane mirror the object and image heights are the same, and the image distance is the negative of the object distance.



$$h_i = h_o \;\; ; \;\; d_i = -d_o \;\; ; \;\; \theta_{\text{plane}} = \frac{h_i}{d_o - d_i} = \frac{h_o}{2d_o}$$

We use Eq. 32-2 and 32-3 to write the angle of the image in the convex mirror in terms of the object size and distance.

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \quad \rightarrow \quad d_i = \frac{d_o f}{d_o - f} \quad \rightarrow \quad d_o - d_i = \frac{d_o^2 - 2d_o f}{d_o - f}$$

$$\frac{h_i}{h_o} = -\frac{d_i}{d_o} \quad \rightarrow \quad h_i = -\frac{h_o d_i}{d_o} = -\frac{h_o f}{d_o - f}$$

$$\theta_{\text{convex}} = \frac{h_i}{d_o - d_i} = -\left(\frac{h_o f}{d_o - f}\right)\left(\frac{d_o - f}{d_o^2 - 2d_o f}\right) = \frac{-h_o f}{d_o^2 - 2d_o f}$$

We now set the angle in the convex mirror equal to ½ of the angle in the plane mirror and solve for the focal length.

$$\theta_{\text{convex}} = \tfrac{1}{2}\theta_{\text{plane}} \quad \rightarrow \quad \frac{-h_o f}{d_o^2 - 2d_o f} = \frac{h_o}{4d_o} \quad \rightarrow \quad -4d_o f = d_o^2 - 2d_o f \quad \rightarrow \quad f = -\tfrac{1}{2}d_o$$

We use Eq. 32-1 to calculate the radius of the mirror.

$$r = 2f = 2\left(-\tfrac{1}{2}d_o\right) = -d_o = \boxed{-3.80\,\text{m}}$$

76. For the critical angle, the refracted angle is 90°. For the refraction from plastic to air, we have the following.

$$n_{\text{plastic}} \sin\theta_{\text{plastic}} = n_{\text{air}} \sin\theta_{\text{air}} \quad \rightarrow \quad n_{\text{plastic}} \sin 39.3° = (1.00)\sin 90° \quad \rightarrow \quad n_{\text{plastic}} = 1.5788$$

For the refraction from plastic to water, we have the following.

$$n_{\text{plastic}} \sin\theta'_{\text{plastic}} = n_{\text{water}} \sin\theta_{\text{water}} \quad \rightarrow \quad (1.5788)\sin\theta'_{\text{plastic}} = (1.33)\sin 90° \quad \rightarrow \quad \theta'_{\text{plastic}} = \boxed{57.4°}$$

77. The two students [chose different signs for the magnification,] i.e., one upright and one inverted. The focal length of the concave mirror is $f = \tfrac{1}{2}R = \tfrac{1}{2}(46\,\text{cm}) = 23\,\text{cm}$. We relate the object and image distances from the magnification.

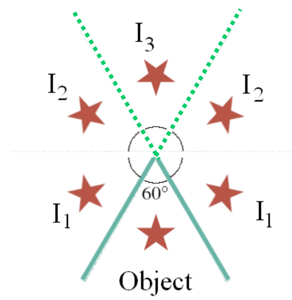
$$m = -\frac{d_i}{d_o} \quad \rightarrow \quad \pm 3 = -\frac{d_i}{d_o} \quad \rightarrow \quad d_i = \mp 3d_o$$

Use this result in the mirror equation.

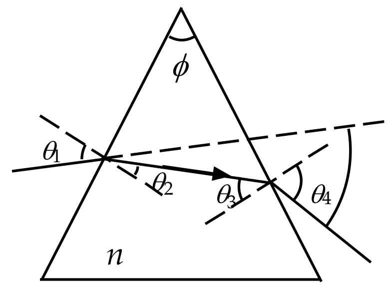
$$\left(\frac{1}{d_o}\right) + \left(\frac{1}{d_i}\right) = \frac{1}{f} \quad \rightarrow \quad \left(\frac{1}{d_o}\right) + \left[\frac{1}{(\mp 3d_o)}\right] = \frac{1}{f} \quad \rightarrow \quad d_o = \frac{2f}{3}, \frac{4f}{3} = 15.3\,\text{cm}, 30.7\,\text{cm}$$

So the object distances are [15 cm (produces virtual image)], and [+31 cm (produces real image)].

78. The object "creates" the I_1 images as reflections from the actual mirrors. The I_2 images can be considered as images of the I_1 "objects," formed by the original mirrors. A specific I_2 image is the image of the I_1 "object" that is diametrically opposite it. Then the I_3 image can be considered as an image of the I_2 "objects." Each I_2 "object" would make the I_3 "image" at the same location. We can consider the extension of the actual mirrors, shown as dashed lines, to help understand the image formation.



79. The total deviation of the beam is the sum of the deviations at each surface. The deviation at the first surface is the refracted angle θ_2 subtracted from the incident angle θ_1. The deviation at the second surface is the incident angle θ_3 subtracted from the refracted angle θ_4. This gives the total deviation.



$$\delta = \delta_1 + \delta_2 = \theta_1 - \theta_2 + \theta_4 - \theta_3$$

We will express all of the angles in terms of θ_2. To minimize the deviation, we will take the derivative of the deviation with respect to θ_2, and then set that derivative equal to zero. Use Snell's law at the first surface to write the incident angle in terms of the refracted angle.

$$\sin\theta_1 = n\sin\theta_2 \quad\rightarrow\quad \theta_1 = \sin^{-1}(n\sin\theta_2)$$

The angle of incidence at the second surface is found using complementary angles, such that the sum of the refracted angle from the first surface and the incident angle at the second surface must equal the apex angle.

$$\phi = \theta_2 + \theta_3 \quad\rightarrow\quad \theta_3 = \phi - \theta_2$$

The refracted angle from the second surface is again found using Snell's law with the deviation in angle equal to the difference between the incident and refracted angles at the second surface.

$$n\sin\theta_3 = \sin\theta_4 \quad\rightarrow\quad \theta_4 = \sin^{-1}(n\sin\theta_3) = \sin^{-1}\left(n\sin(\phi-\theta_2)\right)$$

Inserting each of the angles into the deviation and setting the derivative equal to zero allows us to solve for the angle at which the deviation is a minimum.

$$\delta = \sin^{-1}(n\sin\theta_2) - \theta_2 + \sin^{-1}\left(n\sin(\phi-\theta_2)\right) - (\phi-\theta_2)$$

$$= \sin^{-1}(n\sin\theta_2) + \sin^{-1}\left(n\sin(\phi-\theta_2)\right) - \phi$$

$$\frac{d\delta}{d\theta_2} = \frac{n\cos\theta_2}{\sqrt{1-n^2\sin^2\theta_2}} - \frac{n\cos(\phi-\theta_2)}{\sqrt{1-n^2\sin^2(\phi-\theta_2)}} = 0 \quad\rightarrow\quad \theta_2 = \phi-\theta_2 \quad\rightarrow\quad \theta_2 = \theta_3 = \tfrac{1}{2}\phi$$

In order for $\theta_2 = \theta_3$, the ray must pass through the prism horizontally, which is perpendicular to the bisector of the apex angle ϕ. Set $\theta_2 = \frac{1}{2}\phi$ in the deviation equation (for the minimum deviation, δ_m) and solve for the index of refraction.

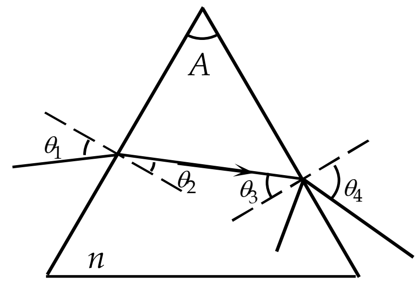
$$\delta_m = \sin^{-1}(n\sin\theta_2) + \sin^{-1}\left(n\sin(\phi-\theta_2)\right) - \phi$$

$$= \sin^{-1}\left(n\sin\tfrac{1}{2}\phi\right) + \sin^{-1}\left(n\sin\tfrac{1}{2}\phi\right) - \phi = 2\sin^{-1}\left(n\sin\tfrac{1}{2}\phi\right) - \phi$$

$$\rightarrow \boxed{n = \frac{\sin\left(\frac{1}{2}(\delta_m+\phi)\right)}{\sin\frac{1}{2}\phi}}$$

80. For the refraction at the second surface, we have this.

$$n\sin\theta_3 = n_{\text{air}}\sin\theta_4 \rightarrow (1.58)\sin\theta_3 = (1.00)\sin\theta_4$$



The maximum value of θ_4 before internal reflection takes place at the second surface is 90°. Thus for internal reflection not to occur, we have

$$(1.58)\sin\theta_3 \le 1.00 \rightarrow \sin\theta_3 \le 0.6329 \rightarrow \theta_3 \le 39.27°$$

We find the refraction angle at the second surface.

$$(90° - \theta_2) + (90° - \theta_3) + A = 180° \rightarrow$$

$$\theta_2 = A - \theta_3 = 72° - \theta_3$$

Thus $\theta_2 \ge 72° - 39.27° = 32.73°$.

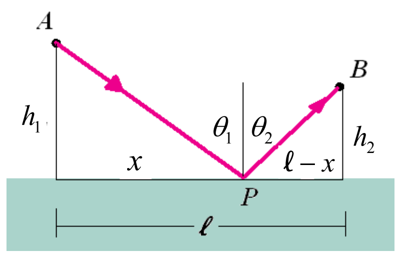
For the refraction at the first surface, we have the following.

$$n_{\text{air}}\sin\theta_1 = n\sin\theta_2 \rightarrow (1.00)\sin\theta_1 = (1.58)\sin\theta_2 \rightarrow \sin\theta_1 = (1.50)\sin\theta_2$$

Now apply the limiting condition.

$$\sin\theta_1 \ge (1.58)\sin 32.73° = 0.754 \rightarrow \boxed{\theta_1 \ge 58.69°}$$

81. (*a*) Consider the light ray shown in the figure. A ray of light starting at point A reflects off the surface at point P before arriving at point B, a horizontal distance ℓ from point A. We calculate the length of each path and divide the length by the speed of light to determine the time required for the light to travel between the two points.



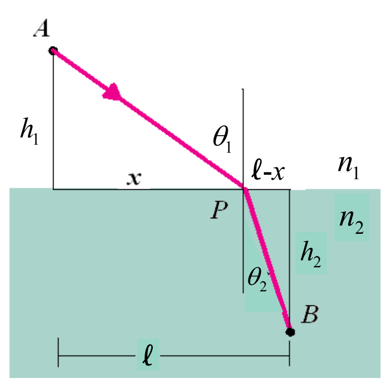
$$t = \frac{\sqrt{x^2 + h_1^2}}{c} + \frac{\sqrt{(\ell - x)^2 + h_2^2}}{c}$$

To minimize the time we set the derivative of the time with respect to *x* equal to zero. We also use the definition of the sine as opposite side over hypotenuse to relate the lengths to the angles of incidence and reflection.

$$0 = \frac{dt}{dx} = \frac{x}{c\sqrt{x^2 + h_1^2}} + \frac{-(\ell - x)}{c\sqrt{(\ell - x)^2 + h_2^2}} \rightarrow$$

$$\frac{x}{\sqrt{x^2 + h_1^2}} = \frac{(\ell - x)}{\sqrt{(\ell - x)^2 + h_2^2}} \rightarrow \sin\theta_1 = \sin\theta_2 \rightarrow \boxed{\theta_1 = \theta_2}$$

(*b*) Now we consider a light ray traveling from point A to point B in media with different indices of refraction, as shown in the figure. The time to travel between the two points is the distance in each medium divided by the speed of light in that medium.



$$t = \frac{\sqrt{x^2 + h_1^2}}{c/n_1} + \frac{\sqrt{(\ell - x)^2 + h_2^2}}{c/n_2}$$

To minimize the time we set the derivative of the time with respect to *x* equal to zero. We also use the definition of the sine as opposite side over hypotenuse to relate the lengths to the angles of incidence and reflection.

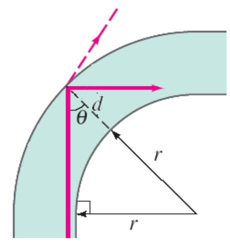
$$0=\frac{dt}{dx}=\frac{n_1x}{c\sqrt{x^2+h_1^2}}+\frac{-n_2(\ell-x)}{c\sqrt{(\ell-x)^2+h_2^2}}\rightarrow\frac{n_1x}{\sqrt{x^2+h_1^2}}=\frac{n_2(\ell-x)}{\sqrt{(\ell-x)^2+h_2^2}}\rightarrow\boxed{n_1\sin\theta_1=n_2\sin\theta_2}$$

82. We use Eq. 32-8 to calculate the location of the image and Eq. 32-3 to calculate the height of the image.

$$\frac{n_1}{d_o}+\frac{n_2}{d_i}=\frac{n_2-n_1}{R}\rightarrow d_i=n_2\left[\frac{n_2-n_1}{R}-\frac{n_1}{d_o}\right]^{-1}=1.53\left[\frac{1.53-1.33}{2.00\text{cm}}-\frac{1.33}{23\text{cm}}\right]^{-1}=\boxed{36.3\text{cm}}$$

$$\frac{h_i}{h_o}=-\frac{d_i}{d_o}\rightarrow h_i=-h_o\frac{d_i}{d_o}=-(2.0\text{mm})\frac{36.3\text{cm}}{23\text{cm}}=\boxed{3.2\text{mm}}$$

83. A ray of light initially on the inside of the beam will strike the far surface at the smallest angle, as seen in the associated figure. The angle is found using the triangle shown in the figure, with side *r* and hypotenuse *r*+*d*. We set this angle equal to the critical angle, using Eq. 32-7, and solve for the minimum radius of curvature.



$$\sin\theta_C=\frac{r}{r+d}=\frac{n_2}{n_1}=\frac{1}{n}\rightarrow\boxed{r=\frac{d}{n-1}}$$

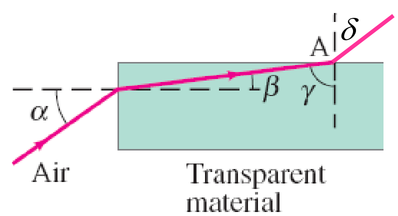
84. A relationship between the image and object distances can be obtained from the given information.

$$m=-\frac{1}{2}=-\frac{d_i}{d_o}\rightarrow d_i=\tfrac{1}{2}d_o=\boxed{7.5\text{cm}}$$

Now we find the focal length and the radius of curvature.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}\rightarrow\frac{1}{15\text{cm}}+\frac{1}{7.5\text{cm}}=\frac{1}{f}\rightarrow f=5.0\text{cm}\rightarrow\boxed{r=10\text{cm}}$$

85. If total internal reflection fails at all, it fails for $\alpha\approx90^\circ$. Assume $\alpha=90^\circ$ and use Snell's law to determine the maximum β.



$$n_2\sin\beta=n_1\sin\alpha=n_1\sin90^\circ=n_1\rightarrow\sin\beta=\frac{n_1}{n_2}$$

Snell's law can again be used to determine the angle δ for which light (if not totally internally reflected) would exit the top surface, using the relationship $\beta+\gamma=90^\circ$ since they form two angles of a right triangle.

$$n_1\sin\delta=n_2\sin\gamma=n_2\sin(90^\circ-\beta)=n_2\cos\beta\rightarrow\sin\delta=\frac{n_2}{n_1}\cos\beta$$

Using the trigonometric relationship $\cos\beta=\sqrt{1-\sin^2\beta}$ we can solve for the exiting angle in terms of the indices of refraction.

$$\sin\delta=\frac{n_2}{n_1}\sqrt{1-\sin^2\beta}=\frac{n_2}{n_1}\sqrt{1-\left(\frac{n_1}{n_2}\right)^2}$$

Insert the values for the indices ($n_1=1.00$ and $n_2=1.51$) to determine the sine of the exit angle.

$$\sin\delta=\frac{1.51}{1.00}\sqrt{1-\left(\frac{1.00}{1.51}\right)^2}=1.13$$

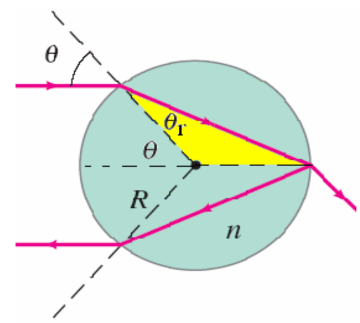
Since the sine function has a maximum value of 1, the light totally internally reflects at the glass–air interface for any incident angle of light.

If the glass is immersed in water, then $n_1 = 1.33$ and $n_2 = 1.51$.

$$\sin\delta = \frac{1.51}{1.33}\sqrt{1-\left(\frac{1.33}{1.51}\right)^2} = 0.538 \quad\rightarrow\quad \delta = \sin^{-1}0.538 = 32.5^\circ$$

Light entering the glass from water at 90° can escape out the top at 32.5°, therefore total internal reflection only occurs for incident angles $\boxed{\le 32.5^\circ}$.

86. The path of the ray in the sphere forms an isosceles triangle with two radii. The two identical angles of the triangle are equal to the refracted angle. Since the incoming ray is horizontal, the third angle is the supplementary angle of the incident angle. We set the sum of these angles equal to 180° and solve for the ratio of the incident and refracted angles. Finally we use Snell's law in the small angle approximation to calculate the index of refraction.

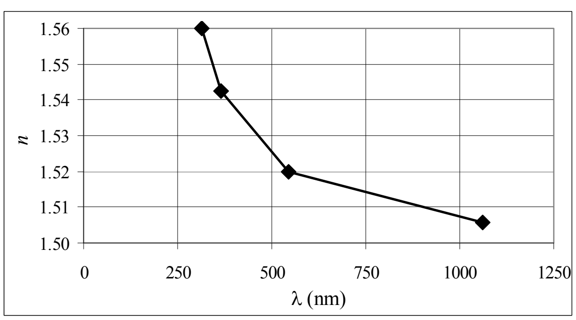


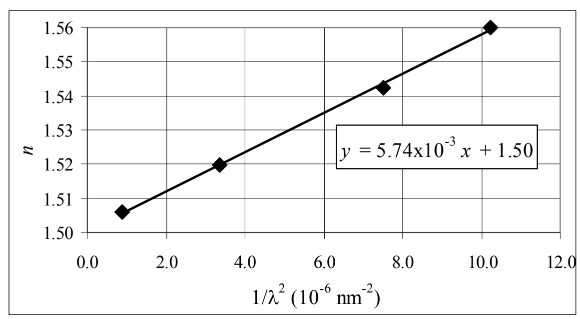
$$2\theta_r + (180^\circ - \theta) = 180^\circ \quad\rightarrow\quad \theta = 2\theta_r$$

$$n_1\sin\theta = n_2\sin\theta_r \quad\rightarrow\quad \theta = n\theta_r = 2\theta_r \quad\rightarrow\quad \boxed{n = 2}$$

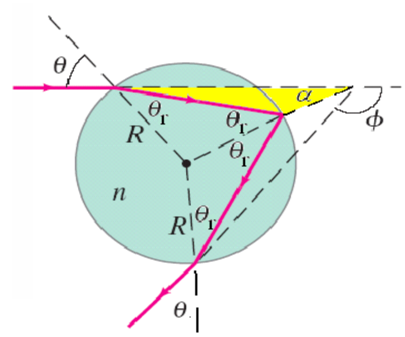
87. The first graph is a graph of n vs. λ. The second graph is a graph n vs. of $1/\lambda^2$. By fitting a line of the form $n = A + B/\lambda^2$, we have $A = 1.50$ and $B = (5.74\times10^{-3})/10^{-6}\,\text{nm}^{-2} = \boxed{5740\,\text{nm}^2}$.

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH32.XLS," on tab "Problem 32.87."





88. (*a*) As the light ray enters the water drop, its path changes by the difference between the incident and refracted angles. We use Snell's law to calculate the refracted angle. The light ray then reflects off the back surface of the droplet. At this surface its path changes by $180^\circ - 2\theta_r$, as seen in the diagram. As the light exits the droplet it refracts again, changing its path by the difference between the incident and refracted angles. Summing these three angles gives the total path change.



$$\sin\theta = n\sin\theta_r \rightarrow \theta_r = \sin^{-1}\left(\frac{\sin\theta}{n}\right)$$

$$\phi = (\theta-\theta_r) + (180^\circ - 2\theta_r) + (\theta - \theta_r) = 180^\circ + 2\theta - 4\theta_r = \boxed{180^\circ + 2\theta - 4\sin^{-1}\left(\frac{\sin\theta}{n}\right)}$$

(*b*) Here is the graph of ϕ vs θ. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH32.XLS," on tab "Problem 32.88."

(*c*) On the spreadsheet, the incident angles that give scattering angles from 138° to 140° are approximately $48.5° \le \theta \le 54.5°$ and $64.5° \le \theta \le 69.5°$. This is 11/90 of the possible incident angles, or about $\boxed{12\%}$.