CHAPTER 31: Maxwell's Equations and Electromagnetic Waves

Responses to Questions

1. The magnetic field will be clockwise in both cases. In the first case, the electric field is away from you and is increasing. The direction of the displacement current (proportional to $\frac{d\Phi_E}{dt}$) is therefore away from you and the corresponding magnetic field is clockwise. In the second case, the electric field is directed towards you and is decreasing; the displacement current is still away from you, and the magnetic field is still clockwise.

2. The displacement current is to the right.

3. The displacement current is spread out over a larger area than the conduction current. Thus, the displacement current produces a less intense field at any given point.

4. One possible reason the term $\varepsilon_0 \frac{d\Phi_E}{dt}$ can be called a "current" is because it has units of amperes.

5. The magnetic field vector will oscillate up and down, perpendicular to the direction of propagation and to the electric field vector.

6. No. Sound is a longitudinal mechanical wave. It requires the presence of a medium; electromagnetic waves do not require a medium.

7. EM waves are self-propagating and can travel through a perfect vacuum. Sound waves are mechanical waves which require a medium, and therefore cannot travel through a perfect vacuum.

8. No. Electromagnetic waves travel at a very large but finite speed. When you flip on a light switch, it takes a very small amount of time for the electrical signal to travel along the wires.

9. The wavelengths of radio and television signals are longer than those of visible light.

10. The wavelength of the current is 5000 km; the house is only 200 km away. The phase of the current at the position of the house is $2\pi/25$ radians different from the phase at the source due to the position of the house.

11. The signals travel through the wires at close to the speed of light, so the length of the wires in a normal room will have an insignificant effect.

12. 10^3 km: radio wave; 1 km: radio wave; 1 m: microwave; 1 cm: microwave; 1 mm: microwave or infrared; 1 μm: infrared.

13. Yes, although the wavelengths for radio waves will be much longer than for sound waves, since the radio waves travel at the speed of light.

14. Both cordless phones and cellular phones are radio receivers and transmitters. When you speak, the phone converts the sound waves into electrical signals which are amplified, modulated, and transmitted. The receiver picks up the EM waves and converts them back into sound. Cordless phones and cell phones use different frequency ranges and different intensities.

15. Yes. If one signal is sent by amplitude modulation and the other signal is sent by frequency modulation, both could be carried over the same carrier frequency. There are other ways two signals can be sent on the same carrier frequency which are more complex.

16. The receiver's antenna should also be vertical for the best reception.

17. Diffraction is significant when the order of magnitude of the wavelength of the waves is the same as the size of the obstacles. AM waves have longer wavelengths than FM waves and will be more likely to diffract around hills and other landscape barriers.

18. It is amplitude modulated, or AM. The person flashing the light on and off is changing the amplitude of the light ("on" is maximum amplitude and "off" is zero). The frequency of the carrier wave is just the frequency of the visible light, approximately 10^{14} to 10^{15} Hz.

Solutions to Problems

1. The electric field between the plates is given by $E = \frac{V}{d}$, where *d* is the distance between the plates.

$$E = \frac{V}{d} \quad \rightarrow \quad \frac{dE}{dt} = \frac{1}{d}\frac{dV}{dt} = \left(\frac{1}{0.0011\,\text{m}}\right)(120\,\text{V/s}) = \boxed{1.1\times10^{5}\,\frac{\text{V/m}}{\text{s}}}$$

2. The displacement current is shown in section 31-1 to be $I_\text{D} = \varepsilon_0 A \frac{dE}{dt}$.

$$I_\text{D} = \varepsilon_0 A \frac{dE}{dt} = \left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)(0.058\,\text{m})^2\left(2.0\times10^{6}\,\frac{\text{V}}{\text{m}\bullet\text{s}}\right) = \boxed{6.0\times10^{-8}\,\text{A}}$$

3. The current in the wires must also be the displacement current in the capacitor. Use the displacement current to find the rate at which the electric field is changing.

$$I_D = \varepsilon_0 A \frac{dE}{dt} \quad \rightarrow \quad \frac{dE}{dt} = \frac{I_\text{D}}{\varepsilon_0 A} = \frac{(2.8\,\text{A})}{\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)(0.0160\,\text{m})^2} = \boxed{1.2\times10^{15}\,\frac{\text{V}}{\text{m}\bullet\text{s}}}$$

4. The current in the wires is the rate at which charge is accumulating on the plates and also is the displacement current in the capacitor. Because the location in question is outside the capacitor, use the expression for the magnetic field of a long wire.

$$B = \frac{\mu_0 I}{2\pi R} = \left(\frac{\mu_0}{4\pi}\right)\frac{2I}{R} = \frac{\left(10^{-7}\,\text{T}\bullet\text{m/A}\right)2\left(38.0\times10^{-3}\,\text{A}\right)}{(0.100\,\text{m})} = \boxed{7.60\times10^{-8}\,\text{T}}$$

After the capacitor is fully charged, all currents will be zero, so the magnetic field will be $\boxed{\text{zero.}}$

5. The electric field between the plates is given by $E = \frac{V}{d}$, where *d* is the distance between the plates.

The displacement current is shown in section 31-1 to be $I_D = \varepsilon_0 A \frac{dE}{dt}$.

$$I_D = \varepsilon_0 A \frac{dE}{dt} = \varepsilon_0 A \frac{1}{d}\frac{dV}{dt} = \frac{\varepsilon_0 A}{d}\frac{dV}{dt} = \boxed{C\frac{dV}{dt}}$$

6. (*a*) The footnote on page 816 indicates that Kirchhoff's junction rule is valid at a capacitor plate, and so the conduction current is the same value as the displacement current. Thus the maximum conduction current is $\boxed{35\mu\text{A}}$.

(*b*) The charge on the pages is given by $Q = CV = C\mathscr{E}_0 \cos\omega t$. The current is the derivative of this.

$$I = \frac{dQ}{dt} = -\omega C\mathscr{E}_0 \sin\omega t \;;\; I_{\max} = \omega C\mathscr{E}_0 \;\rightarrow$$

$$\mathscr{E}_0 = \frac{I_{\max}}{\omega \text{C}} = \frac{I_{\max} d}{2\pi f \varepsilon_0 A} = \frac{\left(35\times10^{-6}\,\text{A}\right)\left(1.6\times10^{-3}\,\text{m}\right)}{2\pi\left(76.0\,\text{Hz}\right)\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\pi\left(0.025\text{m}\right)^2}$$

$$= 6749\,\text{V} \approx \boxed{6700\,\text{V}}$$

(*c*) From Eq. 31-3, $I_D = \varepsilon_0 \frac{d\Phi_E}{dt}$.

$$I_D = \varepsilon_0 \frac{d\Phi_E}{dt} \;\rightarrow\; \left(\frac{d\Phi_E}{dt}\right)_{\max} = \frac{\left(I_D\right)_{\max}}{\varepsilon_0} = \frac{35\times10^{-6}\,\text{A}}{8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2} = \boxed{4.0\times10^6\,\text{V}\bullet\text{m/s}}$$

$\boxed{7.}$ (*a*) We follow the development and geometry given in Example 31-1, using R for the radial distance. The electric field between the plates is given by $E = \frac{V}{d}$, where *d* is the distance between the plates.

$$\oint \vec{\mathbf{B}}\bullet d\vec{\boldsymbol{\ell}} = \mu_0\varepsilon_0 \frac{d\Phi_E}{dt} \;\rightarrow\; B\left(2\pi R_{\text{path}}\right) = \mu_0\varepsilon_0 \frac{d\left(\pi R_{\text{flux}}^2 E\right)}{dt}$$

The subscripts are used on the radial variable because there might not be electric field flux through the entire area bounded by the amperian path. The electric field between the plates is given by $E = \frac{V}{d} = \frac{V_0 \sin\left(2\pi ft\right)}{d}$, where *d* is the distance between the plates.

$$B\left(2\pi R_{\text{path}}\right) = \mu_0\varepsilon_0 \frac{d\left(\pi R_{\text{flux}}^2 E\right)}{dt} \;\rightarrow$$

$$B = \frac{\mu_0\varepsilon_0\pi R_{\text{flux}}^2}{2\pi R_{\text{path}}}\frac{d\left(E\right)}{dt} = \frac{\mu_0\varepsilon_0\pi R_{\text{flux}}^2}{2\pi R_{\text{path}}}\frac{2\pi fV_0}{d}\cos\left(2\pi ft\right) = \frac{\mu_0\varepsilon_0 R_{\text{flux}}^2}{R_{\text{path}}}\frac{\pi fV_0}{d}\cos\left(2\pi ft\right)$$

We see that the functional form of the magnetic field is $\boxed{B = B_0\left(R\right)\cos\left(2\pi ft\right)}$.

(*b*) If $R \le R_0$, then there is electric flux throughout the area bounded by the amperian loop, and so $R_{\text{path}} = R_{\text{flux}} = R$.

$$B_0\left(R \le R_0\right) = \frac{\mu_0\varepsilon_0 R_{\text{flux}}^2}{R_{\text{path}}}\frac{\pi f V_0}{d} = \mu_0\varepsilon_0\frac{\pi f V_0}{d}R = \frac{\pi\left(60\,\text{Hz}\right)\left(150\,\text{V}\right)}{\left(3.00\times10^8\,\text{m/s}\right)^2\left(5.0\times10^{-3}\,\text{m}\right)}R$$

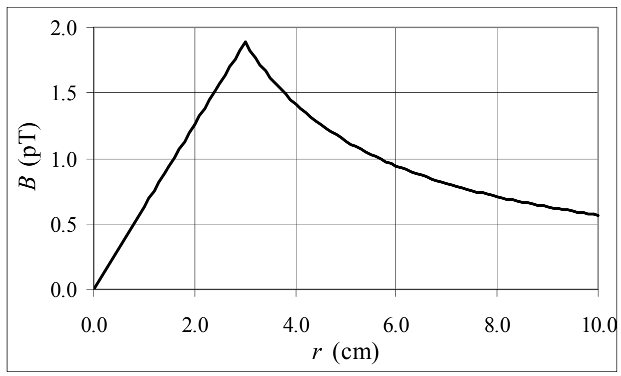
$$= \left(6.283\times10^{-11}\,\text{T/m}\right)R \approx \boxed{\left(6.3\times10^{-11}\,\text{T/m}\right)R}$$

If $R > R_0$, then there is electric flux only out a radial distance of R_0. Thus $R_{\text{path}} = R$ and $R_{\text{flux}} = R_0$.

$$B_0\left(R > R_0\right) = \frac{\mu_0\varepsilon_0 R_{\text{flux}}^2}{R_{\text{path}}}\frac{\pi f V_0}{d} = \mu_0\varepsilon_0\frac{\pi f V_0 R_0^2}{d}\frac{1}{R} = \frac{\pi\left(60\,\text{Hz}\right)\left(150\,\text{V}\right)\left(0.030\,\text{m}\right)^2}{\left(3.00\times10^8\,\text{m/s}\right)^2\left(5.0\times10^{-3}\,\text{m}\right)}\frac{1}{R}$$

$$= \left(5.655\times10^{-14}\,\text{T}\bullet\text{m}\right)\frac{1}{R} \approx \boxed{\left(5.7\times10^{-14}\,\text{T}\bullet\text{m}\right)\frac{1}{R}}$$

(*c*) See the adjacent graph. Note that the magnetic field is continuous at the transition from "inside" to "outside" the capacitor radius. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH31.XLS," on tab "Problem 31.7c."



8. Use Eq. 31-11 with $v = c$.

$$\frac{E_0}{B_0} = c \;\rightarrow\; B_0 = \frac{E_0}{c} = \frac{0.57\times10^{-4}\,\text{V/m}}{3.00\times10^8\,\text{m/s}} = \boxed{1.9\times10^{-13}\,\text{T}}$$

9. Use Eq. 31-11 with $v = c$.

$$\frac{E_0}{B_0} = c \;\rightarrow\; E_0 = B_0 c = \left(12.5\times10^{-9}\,\text{T}\right)\left(3.00\times10^8\,\text{m/s}\right) = \boxed{3.75\,\text{V/m}}$$

10. The frequency of the two fields must be the same: $\boxed{80.0\,\text{kHz}}$. The rms strength of the electric field can be found from Eq. 31-11 with $v = c$.

$$E_{\text{rms}} = cB_{\text{rms}} = \left(3.00\times10^8\,\text{m/s}\right)\left(7.75\times10^{-9}\,\text{T}\right) = \boxed{2.33\,\text{V/m}}$$

The electric field is perpendicular to both the direction of travel and the magnetic field, so the electric field oscillates along the $\boxed{\text{horizontal north-south line.}}$

11. (*a*) If we write the argument of the cosine function as $kz + \omega t = k(z + ct)$, we see that the wave is traveling in the $\boxed{-z\text{ direction}}$, or $\boxed{-\hat{\mathbf{k}}}$.

(*b*) $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ are perpendicular to each other and to the direction of propagation. At the origin, the electric field is pointing in the positive *x* direction. Since $\vec{\mathbf{E}}\times\vec{\mathbf{B}}$ must point in the negative *z* direction, $\vec{\mathbf{B}}$ must point in the the $\boxed{-y \text{ direction}}$, or $\boxed{-\hat{\mathbf{j}}}$. The magnitude of the magnetic field is found from Eq. 31-11 as $B_0 = \boxed{E_0/c}$.

12. The wave equation to be considered is $v^2 \frac{\partial^2 E}{\partial x^2} = \frac{\partial^2 E}{\partial t^2}$.

(*a*) Given $E(x,t) = Ae^{-\alpha(x-vt)^2}$.

$$\frac{\partial E}{\partial x} = Ae^{-\alpha(x-vt)^2}\left[-2\alpha(x-vt)\right]$$

$$\frac{\partial^2 E}{\partial x^2} = Ae^{-\alpha(x-vt)^2}(-2\alpha) + Ae^{-\alpha(x-vt)^2}\left[-2\alpha(x-vt)\right]^2 = -2\alpha Ae^{-\alpha(x-vt)^2}\left[1-2\alpha(x-vt)^2\right]$$

$$\frac{\partial E}{\partial t} = Ae^{-\alpha(x-vt)^2}\left[-2\alpha(x-vt)(-v)\right] = Ae^{-\alpha(x-vt)^2}\left[2\alpha v(x-vt)\right]$$

$$\frac{\partial^2 E}{\partial t^2} = Ae^{-\alpha(x-vt)^2}\left(-2\alpha v^2\right) + Ae^{-\alpha(x-vt)^2}\left[2\alpha v(x-vt)\right]^2 = -2\alpha v^2 Ae^{-\alpha(x-vt)^2}\left[1-2\alpha(x-vt)^2\right]$$

We see that $v^2 \frac{\partial^2 E}{\partial x^2} = \frac{\partial^2 E}{\partial t^2}$, and so the wave equation is satisfied.

(*b*) Given $E(x,t) = Ae^{-(\alpha x^2 - vt)}$.

$$\frac{\partial E}{\partial x} = Ae^{-(\alpha x^2-vt)}(-2\alpha x)$$

$$\frac{\partial^2 E}{\partial x^2} = Ae^{-(\alpha x^2-vt)}(-2\alpha) + Ae^{-(\alpha x^2-vt)}(-2\alpha x)^2 = -2\alpha Ae^{-(\alpha x^2-vt)}\left[1-2\alpha x^2\right]$$

$$\frac{\partial E}{\partial t} = Ave^{-(\alpha x^2-vt)} \quad ; \quad \frac{\partial^2 E}{\partial t^2} = Av^2 e^{-(\alpha x^2-vt)}$$

This does NOT satisfy $v^2 \frac{\partial^2 E}{\partial x^2} = \frac{\partial^2 E}{\partial t^2}$, since $-2\alpha v^2 Ae^{-(\alpha x^2-vt)}\left[1-2\alpha x^2\right] \neq Av^2 e^{-(\alpha x^2-vt)}$ in general.

13. Use Eq. 31-14 to find the frequency of the microwave.

$$c = \lambda f \quad\rightarrow\quad f = \frac{c}{\lambda} = \frac{\left(3.00\times10^8\,\text{m/s}\right)}{\left(1.50\times10^{-2}\,\text{m}\right)} = \boxed{2.00\times10^{10}\,\text{Hz}}$$

14. Use Eq. 31-14 to find the wavelength and frequency.

(*a*) $$c = \lambda f \quad\rightarrow\quad \lambda = \frac{c}{f} = \frac{\left(3.000\times10^8\,\text{m/s}\right)}{\left(25.75\times10^9\,\text{Hz}\right)} = \boxed{1.165\times10^{-2}\,\text{m}}$$

(*b*) $$c = \lambda f \quad\rightarrow\quad f = \frac{c}{\lambda} = \frac{\left(3.00\times10^8\,\text{m/s}\right)}{\left(0.12\times10^{-9}\,\text{m}\right)} = \boxed{2.5\times10^{18}\,\text{Hz}}$$

15. Use the relationship that $d = vt$ to find the time.

$$d = vt \;\rightarrow\; t = \frac{d}{v} = \frac{\left(1.50 \times 10^{11}\,\text{m}\right)}{\left(3.00 \times 10^{8}\,\text{m/s}\right)} = 5.00 \times 10^{2}\,\text{s} = \boxed{8.33\,\text{min}}$$

16. Use Eq. 31-14 to find the wavelength.

$$c = \lambda f \;\rightarrow\; \lambda = \frac{c}{f} = \frac{\left(3.00 \times 10^{8}\,\text{m/s}\right)}{\left(8.56 \times 10^{14}\,\text{Hz}\right)} = \boxed{3.50 \times 10^{-7}\,\text{m}} = 311\,\text{nm}$$

This wavelength is just outside the violet end of the visible region, so it is $\boxed{\text{ultraviolet.}}$

17. (*a*) Use Eq. 31-14 to find the wavelength.

$$c = \lambda f \;\rightarrow\; \lambda = \frac{c}{f} = \frac{\left(3.00 \times 10^{8}\,\text{m/s}\right)}{\left(1.00 \times 10^{3}\,\text{Hz}\right)} = \boxed{3.00 \times 10^{5}\,\text{m}}$$

(*b*) Again use Eq. 31-14, with the speed of sound in place of the speed of light.

$$v = \lambda f \;\rightarrow\; \lambda = \frac{v}{f} = \frac{\left(341\,\text{m/s}\right)}{\left(1.00 \times 10^{3}\,\text{Hz}\right)} = \boxed{0.341\,\text{m}}$$

(*c*) $\boxed{\text{No}}$, you cannot hear a 1000-Hz EM wave. It takes a pressure wave to excite the auditory system. However, if you applied the 1000-Hz EM wave to a speaker, you could hear the 1000-Hz pressure wave.

18. The length of the pulse is $\Delta d = c\Delta t$. Use this to find the number of wavelengths in a pulse.

$$N = \frac{\left(c\Delta t\right)}{\lambda} = \frac{\left(3.00 \times 10^{8}\,\text{m/s}\right)\left(38 \times 10^{-12}\,\text{s}\right)}{\left(1062 \times 10^{-9}\,\text{m}\right)} = 10734 \approx \boxed{11{,}000\text{ wavelengths}}$$

If the pulse is to be only one wavelength long, then its time duration is the period of the wave, which is the reciprocal of the wavelength.

$$T = \frac{1}{f} = \frac{\lambda}{c} = \frac{\left(1062 \times 10^{-9}\,\text{m}\right)}{\left(3.00 \times 10^{8}\,\text{m/s}\right)} = \boxed{3.54 \times 10^{-15}\,\text{s}}$$

$\boxed{19.}$ (*a*) The radio waves travel at the speed of light, and so $\Delta d = v\Delta t$. The distance is found from the radii of the orbits. For Mars when nearest the Earth, the radii should be subtracted.

$$\Delta t = \frac{\Delta d}{c} = \frac{\left(227.9 \times 10^{9}\,\text{m} - 149.6 \times 10^{9}\,\text{m}\right)}{\left(3.000 \times 10^{8}\,\text{m/s}\right)} = \boxed{261\,\text{s}}$$

(*b*) For Mars when farthest from Earth, the radii should be subtracted.

$$\Delta t = \frac{\Delta d}{c} = \frac{\left(227.9 \times 10^{9}\,\text{m} + 149.6 \times 10^{9}\,\text{m}\right)}{\left(3.000 \times 10^{8}\,\text{m/s}\right)} = \boxed{1260\,\text{s}}$$

20. (*a*) The general form of a plane wave is given in Eq. 31-7. For this wave, $E_x = E_0 \sin\left(kz - \omega t\right)$.

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{0.077\,\text{m}^{-1}} = 81.60\,\text{m} \approx \boxed{82\,\text{m}}$$

$$f = \frac{\omega}{2\pi} = \frac{2.3\times10^{7}\,\text{rad/s}}{2\pi} = 3.661\times10^{6}\,\text{Hz} \approx \boxed{3.7\,\text{MHz}}$$

Note that $\lambda f = \left(81.60\,\text{m}\right)\left(3.661\times10^{6}\,\text{Hz}\right) = 2.987\times10^{8}\,\text{m/s} \approx c$.

(*b*) The magnitude of the magnetic field is given by $B_0 = E_0/c$. The wave is traveling in the $\hat{\mathbf{k}}$ direction, and so the magnetic field must be in the $\hat{\mathbf{j}}$ direction, since the direction of travel is given by the direction of $\vec{\mathbf{E}} \times \vec{\mathbf{B}}$.

$$B_0 = \frac{E_0}{c} = \frac{225\,\text{V/m}}{3.00\times10^8\,\text{m/s}} = 7.50\times10^{-7}\,\text{T} \quad \rightarrow$$

$$\vec{\mathbf{B}} = \boxed{\hat{\mathbf{j}}\left(7.50\times10^{-7}\,\text{T}\right)\sin\left[\left(0.077\text{m}^{-1}\right)z - \left(2.3\times10^7\,\text{rad/s}\right)t\right]}$$

21. The eight-sided mirror would have to rotate 1/8 of a revolution for the succeeding mirror to be in position to reflect the light in the proper direction. During this time the light must travel to the opposite mirror and back.

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{\frac{1}{8}(2\pi\,\text{rad})}{(2\Delta x/c)} = \frac{(\pi\,\text{rad})c}{8\Delta x} = \frac{(\pi\,\text{rad})\left(3.00\times10^8\,\text{m/s}\right)}{8\left(35\times10^3\text{m}\right)} = \boxed{3400\,\text{rad/s}}\ \left(3.2\times10^4\,\text{rev/min}\right)$$

22. The average energy transferred across unit area per unit time is the average magnitude of the Poynting vector, and is given by Eq. 31-19a.

$$\overline{S} = \tfrac{1}{2}\varepsilon_0 c E_0^2 = \tfrac{1}{2}\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\left(3.00\times10^8\,\text{m/s}\right)\left(0.0265\,\text{V/m}\right) = \boxed{9.32\times10^{-7}\,\text{W/m}^2}$$

23. The energy per unit area per unit time is given by the magnitude of the Poynting vector. Let ΔU represent the energy that crosses area A in a time ΔT.

$$S = \frac{cB_{\text{rms}}^2}{\mu_0} = \frac{\Delta U}{A\Delta t} \quad \rightarrow$$

$$\Delta t = \frac{\mu_0 \Delta U}{AcB_{\text{rms}}^2} = \frac{\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)(335\,\text{J})}{\left(1.00\times10^{-4}\,\text{m}^2\right)\left(3.00\times10^8\,\text{m/s}\right)\left(22.5\times10^{-9}\,\text{T}\right)^2} = 0.194\,\text{W/m}^2$$

$$= \boxed{2.77\times10^7\,\text{s}} \approx 321\ \text{days}$$

24. The energy per unit area per unit time is given by the magnitude of the Poynting vector. Let ΔU represent the energy that crosses area A in a time Δt.

$$S = c\varepsilon_0 E_{\text{rms}}^2 = \frac{\Delta U}{A\Delta t} \quad \rightarrow$$

$$\frac{\Delta U}{\Delta t} = c\varepsilon_0 E_{\text{rms}}^2 A$$

$$= \left(3.00\times10^8\,\text{m/s}\right)\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\left(0.0328\,\text{V/m}\right)^2\left(1.00\times10^{-4}\text{m}^2\right)\left(3600\text{s/h}\right)$$

$$= \boxed{1.03\times10^{-6}\,\text{J/h}}$$

25. The intensity is the power per unit area, and also is the time averaged value of the Poynting vector. The area is the surface area of a sphere, since the wave is spreading spherically.

$$\overline{S} = \frac{P}{A} = \frac{(1500\,\text{W})}{\left[4\pi(5.0\,\text{m})^2\right]} = 4.775\,\text{W/m}^2 \approx \boxed{4.8\,\text{W/m}^2}$$

$$\overline{S} = c\varepsilon_0 E_{\text{rms}}^2 \quad \rightarrow \quad E_{\text{rms}} = \sqrt{\frac{\overline{S}}{c\varepsilon_0}} = \sqrt{\frac{4.775\,\text{W/m}^2}{\left(3.00\times10^8\,\text{m/s}\right)\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)}} = \boxed{42\,\text{V/m}}$$

26. (*a*) We find E using Eq. 31-11 with $v = c$.

$$E = cB = \left(3.00 \times 10^{8}\,\text{m/s}\right)\left(2.5 \times 10^{-7}\,\text{T}\right) = \boxed{75\,\text{V/m}}$$

(*b*) The average power per unit area is given by the Poynting vector, from Eq. 31-19a.

$$\overline{I} = \frac{E_0 B_0}{\left(2\mu_0\right)} = \frac{\left(75\,\text{V/m}\right)\left(2.5 \times 10^{-7}\,\text{T}\right)}{\left[2\left(4\pi \times 10^{-7}\,\text{N}\bullet\text{s}^2/\text{C}^2\right)\right]} = \boxed{7.5\,\text{W/m}^2}$$

27. From Eq. 31-16b, the instantaneous energy density is $u = \varepsilon_0 E^2$. From Eq. 31-17, we see that this instantaneous energy density is also given by S/c. The time-averaged value is therefore $\overline{S}/c$. Multiply that times the volume to get the energy.

$$U = uV = \frac{\overline{S}}{c}V = \frac{1350\,\text{W/m}^2}{3.00\times10^{8}\,\text{m/s}}\left(1.00\,\text{m}^3\right) = \boxed{4.50\times10^{-6}\,\text{J}}$$

28. The power output per unit area is the intensity, and also is the magnitude of the Poynting vector. Use Eq. 31-19a with rms values.

$$S = \frac{P}{A} = c\varepsilon_0 E_{\text{rms}}^2 \quad \rightarrow$$

$$E_{\text{rms}} = \sqrt{\frac{P}{Ac\varepsilon_0}} = \sqrt{\frac{0.0158\,\text{W}}{\pi\left(1.00\times10^{-3}\,\text{m}\right)^2\left(3.00 \times 10^{8}\,\text{m/s}\right)\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)}}$$

$$= 1376.3\,\text{V/m} \approx \boxed{1380\,\text{V/m}}$$

$$B_{\text{rms}} = \frac{E_{\text{rms}}}{c} = \frac{1376.3\,\text{V/m}}{3.00 \times 10^{8}\,\text{m/s}} = \boxed{4.59\times10^{-6}\,\text{T}}$$

29. The radiation from the Sun has the same intensity in all directions, so the rate at which it reaches the Earth is the rate at which it passes through a sphere centered at the Sun with a radius equal to the Earth's orbit radius. The $1350\,\text{W/m}^2$ is the intensity, or the magnitude of the Poynting vector.

$$S = \frac{P}{A} \quad \rightarrow \quad P = SA = 4\pi R^2 S = 4\pi\left(1.496 \times 10^{11}\,\text{m}\right)^2\left(1350\,\text{W/m}^2\right) = \boxed{3.80 \times 10^{26}\,\text{W}}$$

30. (*a*) The energy emitted in each pulse is the power output of the laser times the time duration of the pulse.

$$P = \frac{\Delta W}{\Delta t} \quad \rightarrow \quad \Delta W = P\Delta t = \left(1.8 \times 10^{11}\,\text{W}\right)\left(1.0 \times 10^{-9}\,\text{s}\right) = \boxed{180\,\text{J}}$$

(*b*) We find the rms electric field from the intensity, which is the power per unit area. That is also the magnitude of the Poynting vector. Use Eq. 31-19a with rms values.

$$S = \frac{P}{A} = c\varepsilon_0 E_{\text{rms}}^2 \quad \rightarrow$$

$$E_{\text{rms}} = \sqrt{\frac{P}{Ac\varepsilon_0}} = \sqrt{\frac{\left(1.8 \times 10^{11}\,\text{W}\right)}{\pi\left(2.2\times10^{-3}\,\text{m}\right)^2\left(3.00 \times 10^{8}\,\text{m/s}\right)\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)}}$$

$$= \boxed{2.1\times10^{9}\,\text{V/m}}$$

31. In each case, the required area is the power requirement of the device divided by 10% of the intensity of the sunlight.

(*a*) $A = \dfrac{P}{I} = \dfrac{50\times10^{-3}\,\text{W}}{100\,\text{W/m}^2} = 5\times10^{-4}\,\text{m}^2 = \boxed{5\,\text{cm}^2}$

A typical calculator is about 17 cm x 8 cm, which is about $140\,\text{cm}^2$. So yes, the solar panel can be mounted directly on the calculator.

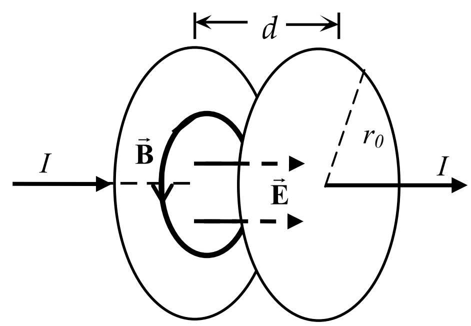
(*b*) $A = \dfrac{P}{I} = \dfrac{1500\,\text{W}}{100\,\text{W/m}^2} = 15\,\text{m}^2 \approx \boxed{20\,\text{m}^2}$ (to one sig. fig.)

A house of floor area $1000\,\text{ft}^2$ would have on the order of $100\,\text{m}^2$ of roof area. So yes, a solar panel on the roof should be able to power the hair dryer.

(*c*) $A = \dfrac{P}{I} = \dfrac{20\,\text{hp}(746\,\text{W/hp})}{100\,\text{W/m}^2} = 149\,\text{m}^2 \approx \boxed{100\,\text{m}^2}$ (to one sig. fig.)

This would require a square panel of side length about 12 m. So no, this panel could not be mounted on a car and used for real-time power.

32. (*a*) Example 31-1 refers back to Example 21-13 and Figure 21-31. In that figure, and the figure included here, the electric field between the plates is to the right. The magnetic field is shown as counterclockwise circles. Take any point between the capacitor plates, and find the direction of $\vec{\mathbf{E}}\times\vec{\mathbf{B}}$. For instance, at the top of the circle shown in Figure 31-4, $\vec{\mathbf{E}}$ is toward the viewer, and $\vec{\mathbf{B}}$ is to the left. The cross product $\vec{\mathbf{E}}\times\vec{\mathbf{B}}$ points down, directly to the line connecting the center of the plates. Or take the rightmost point on the circle.



$\vec{\mathbf{E}}$ is again toward the viewer, and $\vec{\mathbf{B}}$ is upwards. The cross product $\vec{\mathbf{E}}\times\vec{\mathbf{B}}$ points to the left, again directly to the line connecting the center of the plates. In cylindrical coordinates, $\vec{\mathbf{E}} = E\,\hat{\mathbf{k}}$ and $\vec{\mathbf{B}} = B\hat{\boldsymbol{\varphi}}$. The cross product $\hat{\mathbf{k}}\times\hat{\boldsymbol{\varphi}} = -\hat{\mathbf{r}}$.

(*b*) We evaluate the Poynting vector, and then integrate it over the curved cylindrical surface between the capacitor plates. The magnetic field (from Example 31-1) is $B = \tfrac{1}{2}\mu_0\varepsilon_0 r_0 \dfrac{dE}{dt}$, evaluated at $r = r_0$. $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ are perpendicular to each other, so $\vec{\mathbf{S}} = \dfrac{1}{\mu_0}\vec{\mathbf{E}}\times\vec{\mathbf{B}} = \tfrac{1}{2}\varepsilon_0 r_0 E\dfrac{dE}{dt}$, inward. In calculating $\iint \vec{\mathbf{S}}\cdot d\vec{\mathbf{A}}$ for energy flow into the capacitor volume, note that both $\vec{\mathbf{S}}$ and $d\vec{\mathbf{A}}$ point inward, and that S is constant over the curved surface of the cylindrical volume.

$$\iint \vec{\mathbf{S}}\cdot d\vec{\mathbf{A}} = \iint S\,dA = S\iint dA = SA = S2\pi r_0 d = \tfrac{1}{2}\varepsilon_0 r_0 E\frac{dE}{dt}2\pi r_0 d = \varepsilon_0 d\pi r_0^2 E\frac{dE}{dt}$$

The amount of energy stored in the capacitor is the energy density times the volume of the capacitor. The energy density is given by Eq. 24-6, $u = \tfrac{1}{2}\varepsilon_0 E^2$, and the energy stored is the energy density times the volume of the capacitor. Take the derivative of the energy stored with respect to time.

$$U = u(\text{Volume}) = \tfrac{1}{2}\varepsilon_0 E^2\pi r_0^2 d \quad\to\quad \frac{dU}{dt} = \varepsilon_0 E\pi r_0^2 d\frac{dE}{dt}$$

We see that $\iint \vec{\mathbf{S}}\cdot d\vec{\mathbf{A}} = \dfrac{dU}{dt}$.

33. (*a*) The intensity from a point source is inversely proportional to the distance from the source.

$$\frac{I_{\text{Sun}}}{I_{\text{Star}}}=\frac{r^2_{\text{Star-Earth}}}{r^2_{\text{Sun-Earth}}} \rightarrow r_{\text{Star-Earth}}=r_{\text{Sun-Earth}}\sqrt{\frac{I_{\text{Sun}}}{I_{\text{Star}}}}=\left(1.496\times10^{11}\,\text{m}\right)\sqrt{\frac{1350\,\text{W/m}^2}{1\times10^{-23}\,\text{W/m}^2}}\left(\frac{1\,\text{ly}}{9.46\times10^{15}\,\text{m}}\right)$$

$$=1.84\times10^{8}\,\text{ly}\approx\boxed{2\times10^{8}\,\text{ly}}$$

(*b*) Compare this distance to the galactic size.

$$\frac{r_{\text{Star-Earth}}}{\text{galactic size}}=\frac{1.84\times10^{8}\,\text{ly}}{1\times10^{5}\,\text{ly}}=1840\approx\boxed{2000}$$

The distance to the star is about 2000 times the size of our galaxy.

34. We assume the light energy is all absorbed, and so use Eq. 31-21a.

$$P=\frac{\bar{S}}{c}=\frac{\dfrac{75\,\text{W}}{4\pi\left(8.0\times10^{-2}\,\text{m}\right)^2}}{\left(3.00\times10^{8}\,\text{m/s}\right)}=3.108\times10^{-6}\,\text{N/m}^2\approx\boxed{3.1\times10^{-6}\,\text{N/m}^2.}$$

The force is pressure times area. We approximate the area of a fingertip to be 1.0 cm^2.

$$F=PA=\left(3.108\times10^{-6}\,\text{N/m}^2\right)\left(1.0\times10^{-4}\,\text{m}^2\right)=\boxed{3.1\times10^{-10}\,\text{N}}$$

35. The acceleration of the cylindrical particle will be the force on it (due to radiation pressure) divided by its mass. The light is delivering electromagnetic energy to an area A at a rate of $\frac{dU}{dt}=1.0\,\text{W}$. That power is related to the average magnitude of the Poynting vector by $\bar{S}=\frac{dU/dt}{A}$. From Eq. 31-21a, that causes a pressure on the particle of $P=\frac{\bar{S}}{c}$, and the force due to that pressure is $F_{\text{laser}}=PA$. Combine these relationships with Newton's second law to calculate the acceleration. The mass of the particle is its volume times the density of water.

$$F_{\text{laser}}=PA=\frac{\bar{S}}{c}A=\frac{1}{c}\frac{dU}{dt}=ma=\rho_{H_2O}\pi r^2 ra \rightarrow$$

$$a=\frac{dU/dt}{c\rho_{H_2O}\pi r^3}=\frac{(1.0\,\text{W})}{\left(3.00\times10^{8}\,\text{m/s}\right)\left(1000\,\text{kg/m}^2\right)\pi\left(5\times10^{-7}\,\text{m}\right)^3}=\boxed{8\times10^{6}\,\text{m/s}^2}$$

36. (*a*) The light is delivering electromagnetic energy to an area A of the suit at a rate of $\frac{dU}{dt}=3.0\,\text{W}$. That power is related to the average magnitude of the Poynting vector by $\bar{S}=\frac{dU/dt}{A}$. From Eq. 31-21b, that causes a pressure on the suit of $P=\frac{2\bar{S}}{c}$, and the force due to that pressure is $F_{\text{laser}}=PA$. Combine these relationships to calculate the force.

$$F_{\text{laser}}=PA=\frac{2\bar{S}}{c}A=\frac{2}{c}\frac{dU}{dt}=\frac{2(3.0\,\text{W})}{3.00\times10^{8}\,\text{m/s}}=\boxed{2.0\times10^{-8}\,\text{N}}$$

(*b*) Use Newton's law of universal gravitation, Eq. 6-1, to estimate the gravitational force. We take the 20 m distance as having 2 significant figures.

$$F_{\text{grav}} = G\frac{m_{\text{shuttle}}m_{\text{astronaut}}}{r^2} = \left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\frac{\left(1.03\times10^5\,\text{kg}\right)\left(120\,\text{kg}\right)}{\left(20\,\text{m}\right)^2}$$

$$= 2.061\times10^{-6}\,\text{N} \approx \boxed{2.1\times10^{-6}\,\text{N}}$$

(*c*) The gravity force is larger, by a factor of approximately 100.

37. The intensity from a point source is inversely proportional to the distance from the source.

$$\frac{I_{\text{Earth}}}{I_{\text{Jupiter}}} = \frac{r^2_{\text{Sun-Jupiter}}}{r^2_{\text{Sun-Earth}}} = \frac{\left(7.78\times10^{11}\,\text{m}\right)^2}{\left(1.496\times10^{11}\,\text{m}\right)^2} = 27.0$$

So it would take an area of $\boxed{27\text{m}^2}$ at Jupiter to collect the same radiation as a 1.0-m^2 solar panel at the Earth.

38. Use Eq. 31-14. Note that the higher frequencies have the shorter wavelengths.

(*a*) For FM radio we have the following.

$$\lambda = \frac{c}{f} = \frac{\left(3.00\times10^8\,\text{m/s}\right)}{\left(1.08\times10^8\,\text{Hz}\right)} = \boxed{2.78\,\text{m}} \text{ to } \lambda = \frac{c}{f} = \frac{\left(3.00\times10^8\,\text{m/s}\right)}{\left(8.8\times10^7\,\text{Hz}\right)} = \boxed{3.41\,\text{m}}$$

(*b*) For AM radio we have the following.

$$\lambda = \frac{c}{f} = \frac{\left(3.00\times10^8\,\text{m/s}\right)}{\left(1.7\times10^6\,\text{Hz}\right)} = \boxed{180\,\text{m}} \text{ to } \lambda = \frac{c}{f} = \frac{\left(3.00\times10^8\,\text{m/s}\right)}{\left(5.35\times10^5\,\text{Hz}\right)} = \boxed{561\,\text{m}}$$

39. Use Eq. 31-14.

$$\lambda = \frac{c}{f} = \frac{\left(3.00\times10^8\,\text{m/s}\right)}{\left(1.9\times10^9\,\text{Hz}\right)} = \boxed{0.16\,\text{m}}$$

40. The resonant frequency of an *LC* circuit is given by $f = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{LC}}$. We assume the inductance is constant, and form the ratio of the two frequencies.

$$\frac{f_1}{f_2} = \frac{\dfrac{2\pi}{\sqrt{LC_1}}}{\dfrac{2\pi}{\sqrt{LC_2}}} = \sqrt{\frac{C_2}{C_1}} \rightarrow C_2 = \left(\frac{f_1}{f_2}\right)^2 C_1 = \left(\frac{550\,\text{kHz}}{1610\,\text{kHz}}\right)^2\left(2200\,\text{pF}\right) = \boxed{260\,\text{pF}}$$

41. The resonant frequency of an *LC* circuit is given by $f = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$. Solve for the inductance.

$$f = \frac{1}{2\pi\sqrt{LC}} \rightarrow L = \frac{1}{4\pi^2 f^2 C}$$

$$L_1 = \frac{1}{4\pi^2 f_1^2 C} = \frac{1}{4\pi^2\left(88\times10^6\,\text{Hz}\right)^2\left(620\times10^{-12}\,\text{F}\right)} = 5.3\times10^{-9}\,\text{H}$$

$$L_2 = \frac{1}{4\pi^2 f_2^2 C} = \frac{1}{4\pi^2\left(108\times10^6\,\text{Hz}\right)^2\left(620\times10^{-12}\,\text{F}\right)} = 3.5\times10^{-9}\,\text{H}$$

The range of inductances is $\boxed{3.5\times10^{-9}\,\text{H} \le L \le 5.3\times10^{-9}\,\text{H}}$

42. The rms electric field strength of the beam can be found from the Poynting vector.

$$S = \frac{P}{A} = c\varepsilon_0 E_{\text{rms}}^2 \quad \rightarrow$$

$$E_{\text{rms}} = \sqrt{\frac{P}{Ac\varepsilon_0}} = \sqrt{\frac{1.2\times10^4\,\text{W}}{\pi\left(750\,\text{m}\right)^2\left(3.00\times10^8\,\text{m/s}\right)\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)}} = \boxed{1.6\,\text{V/m}}$$

$\boxed{43.}$ The electric field is found from the desired voltage and the length of the antenna. Then use that electric field to calculate the magnitude of the Poynting vector.

$$E_{\text{rms}} = \frac{V_{\text{rms}}}{d} = \frac{1.00\times10^{-3}\,\text{V}}{1.60\,\text{m}} = \boxed{6.25\times10^{-4}\,\text{V/m}}$$

$$S = c\varepsilon_0 E_{\text{rms}}^2 = c\varepsilon_0\frac{V_{\text{rms}}^2}{d^2} = \left(3.00\times10^8\,\text{m/s}\right)\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)\frac{\left(1.00\times10^{-3}\,\text{V}\right)^2}{\left(1.60\,\text{m}\right)^2}$$

$$= \boxed{1.04\times10^{-9}\,\text{W/m}^2}$$

44. We ignore the time for the sound to travel to the microphone. Find the difference between the time for sound to travel to the balcony and for a radio wave to travel 3000 km.

$$\Delta t = t_{\text{radio}} - t_{\text{sound}} = \left(\frac{d_{\text{radio}}}{c}\right) - \left(\frac{d_{\text{sound}}}{v_{\text{sound}}}\right) = \left(\frac{3\times10^6\,\text{m}}{3.00\times10^8\,\text{m/s}}\right) - \left(\frac{50\,\text{m}}{343\,\text{m/s}}\right) = -0.14\,\text{s},$$

so the $\boxed{\text{person at the radio hears the voice 0.14 s sooner.}}$

45. The length is found from the speed of light and the duration of the burst.

$$d = ct = \left(3.00\times10^8\,\text{m/s}\right)\left(10^{-8}\,\text{s}\right) = \boxed{3\,\text{m}}$$

46. The time travel delay is the distance divided by the speed of radio waves (which is the speed of light).

$$t = \frac{d}{c} = \frac{3\times10^6\,\text{m}}{3.00\times10^8\,\text{m/s}} = \boxed{0.01\,\text{s}}$$

47. The time consists of the time for the radio signal to travel to Earth and the time for the sound to travel from the loudspeaker. We use 343 m/s for the speed of sound.

$$t = t_{\text{radio}} + t_{\text{sound}} = \left(\frac{d_{\text{radio}}}{c}\right) + \left(\frac{d_{\text{sound}}}{v_{\text{sound}}}\right) = \left(\frac{3.84\times10^8\,\text{m}}{3.00\times10^8\,\text{m/s}}\right) + \left(\frac{25\,\text{m}}{343\,\text{m/s}}\right) = \boxed{1.35\,\text{s}}$$

Note that about 5% of the time is for the sound wave.

48. (*a*) The rms value of the associated electric field is found from Eq. 24-6.

$$u = \tfrac{1}{2}\varepsilon_0 E^2 = \varepsilon_0 E_{\text{rms}}^2 \quad \rightarrow \quad E_{\text{rms}} = \sqrt{\frac{u}{\varepsilon_0}} = \sqrt{\frac{4\times10^{-14}\,\text{J/m}^3}{8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2}} = 0.0672\,\text{V/m} \approx \boxed{0.07\,\text{V/m}}$$

(*b*) A comparable value can be found using the magnitude of the Poynting vector.

$$\overline{S} = \varepsilon_0 c E_{\text{rms}}^2 = \frac{P}{4\pi r^2} \rightarrow$$

$$r = \frac{1}{E_{\text{rms}}}\sqrt{\frac{P}{4\pi\varepsilon_0 c}} = \frac{1}{0.0672\,\text{V/m}}\sqrt{\frac{7500\,\text{W}}{4\pi\left(8.85\times10^{-12}\,\text{C}^2/\text{n}\bullet\text{m}^2\right)\left(3.00\times10^8\,\text{m/s}\right)}}$$

$$= 7055\,\text{m} \approx \boxed{7\,\text{km}}$$

49. The light has the same intensity in all directions, so use a spherical geometry centered on the source to find the value of the Poynting vector. Then use Eq. 31-19a to find the magnitude of the electric field, and Eq. 31-11 with $v = c$ to find the magnitude of the magnetic field.

$$S = \frac{P_0}{A} = \frac{P_0}{4\pi r^2} = \tfrac{1}{2}c\varepsilon_0 E_0^2 \rightarrow$$

$$E_0 = \sqrt{\frac{P_0}{2\pi r^2 c\varepsilon_0}} = \sqrt{\frac{(75\,\text{W})}{2\pi(2.00\,\text{m})^2\left(3.00\times10^8\,\text{m/s}\right)\left(8.85\times10^{-12}\,\text{C}^2/\text{n}\bullet\text{m}^2\right)}} = 33.53\,\text{V/m}$$

$$\approx \boxed{34\,\text{V/m}}$$

$$B_0 = \frac{E_0}{c} = \frac{(33.53\,\text{V/m})}{\left(3.00\times10^8\,\text{m/s}\right)} = \boxed{1.1\times10^{-7}\,\text{T}}$$

50. The radiation from the Sun has the same intensity in all directions, so the rate at which energy passes through a sphere centered at the Sun is $P = S4\pi R^2$. This rate must be the same at any distance from the Sun. Use this fact to calculate the magnitude of the Poynting vector at Mars, and then use the Poynting vector to calculate the rms magnitude of the electric field at Mars.

$$S_{\text{Mars}}\left(4\pi R_{\text{Mars}}^2\right) = S_{\text{Earth}}\left(4\pi R_{\text{Earth}}^2\right) \rightarrow S_{\text{Mars}} = S_{\text{Earth}}\left(\frac{R_{\text{Earth}}^2}{R_{\text{Mars}}^2}\right) = c\varepsilon_0 E_{\text{rms Mars}}^2 \rightarrow$$

$$E_{\text{rms Mars}} = \sqrt{\frac{S_{\text{Earth}}}{c\varepsilon_0}}\left(\frac{R_{\text{Earth}}}{R_{\text{Mars}}}\right) = \sqrt{\frac{1350\,\text{W/m}^2}{\left(3.00\times10^8\,\text{m/s}\right)\left(8.85\times10^{-12}\,\text{C}^2/\text{n}\bullet\text{m}^2\right)}}\left(\frac{1}{1.52}\right) = \boxed{469\,\text{V/m}}$$

51. The direction of the wave velocity is the direction of the cross product $\vec{\mathbf{E}}\times\vec{\mathbf{B}}$. "South" crossed into "west" gives the direction $\boxed{\text{downward}}$. The electric field is found from the Poynting vector, Eq. 31-19a, and then the magnetic field is found from Eq. 31-11 with $v = c$.

$$S = \tfrac{1}{2}c\varepsilon_0 E_0^{\,2} \rightarrow$$

$$E_0 = \sqrt{\frac{2S}{c\varepsilon_0}} = \sqrt{\frac{2\left(560\,\text{W/m}^2\right)}{\left(3.00\times10^8\,\text{m/s}\right)\left(8.85\times10^{-12}\,\text{C}^2/\text{n}\bullet\text{m}^2\right)}} = 649\,\text{V/m} \approx \boxed{650\,\text{V/m}}$$

$$B_0 = \frac{E_0}{c} = \frac{(649\,\text{V/m})}{\left(3.00\times10^8\,\text{m/s}\right)} = \boxed{2.2\times10^{-6}\,\text{T}}$$

52. From the hint, we use Eq. 29-4, which says $\mathscr{E} = \mathscr{E}_0 \sin\omega t = NBA\omega\sin\omega t$. The intensity is given, and this can be used to find the magnitude of the magnetic field.

$$\overline{S} = \frac{E_{\text{rms}}B_{\text{rms}}}{\mu_0} = \frac{cB_{\text{rms}}^2}{\mu_0} \rightarrow B_{\text{rms}} = \sqrt{\frac{\mu_0\overline{S}}{c}} \;;\; \mathscr{E} = \mathscr{E}_0\sin\omega t = NBA\omega\sin\omega t \rightarrow$$

$$\mathcal{E}_{\text{rms}} = NA\omega B_{\text{rms}} = NA\omega\sqrt{\frac{\mu_0 \overline{S}}{c}}$$

$$= (320)\pi(0.011\,\text{m})^2\, 2\pi(810\times10^3\,\text{Hz})\sqrt{\frac{(4\pi\times10^{-7}\,\text{N}\bullet\text{s}^2/\text{C}^2)(1.0\times10^{-4}\,\text{W/m}^2)}{3.00\times10^8\,\text{m/s}}}$$

$$= \boxed{4.0\times10^{-4}\,\text{V}}$$

53. (*a*) Since intensity is energy per unit time per unit area, the energy is found by multiplying the intensity times the area of the antenna times the elapsed time.

$$U = IAt = (1.0\times10^{-13}\,\text{W/m}^2)\pi\left(\frac{0.33\,\text{m}}{2}\right)^2(6.0\,\text{h})(3600\,\text{s/h}) = \boxed{1.8\times10^{-10}\,\text{J}}$$

(*b*) The electric field amplitude can be found from the intensity, which is the magnitude of the Poynting vector. The magnitude of the magnetic field is then found from Eq. 31-11 with $v = c$.

$$\overline{I} = \tfrac{1}{2}\varepsilon_0 c E_0^2 \quad \rightarrow$$

$$E_0 = \sqrt{\frac{2\overline{I}}{\varepsilon_0 c}} = \sqrt{\frac{2(1.0\times10^{-13}\,\text{W/m}^2)}{(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2)(3.00\times10^8\,\text{m/s})}} = 8.679\times10^{-6}\,\text{V/m}$$

$$\approx \boxed{8.7\times10^{-6}\,\text{V/m}}$$

$$B_0 = \frac{E_0}{c} = \frac{8.679\times10^{-6}\,\text{V/m}}{3.00\times10^8\,\text{m/s}} = \boxed{2.9\times10^{-14}\,\text{T}}$$

54. Use the relationship between average intensity (the magnitude of the Poynting vector) and electric field strength, as given by Eq. 31-19a. Also use the fact that intensity is power per unit area. We assume that the power is spherically symmetric about source.

$$\overline{S} = \tfrac{1}{2}\varepsilon_0 c E_0^{\,2} = \frac{P}{A} = \frac{P}{4\pi r^2} \quad \rightarrow$$

$$r = \sqrt{\frac{P}{2\pi\varepsilon_0 c E_0^{\,2}}} = \sqrt{\frac{25{,}000\,\text{W}}{2\pi(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2)(3.00\times10^8\,\text{m/s})(0.020\,\text{V/m})^2}} = 61{,}200\,\text{m}$$

$$\approx \boxed{61\,\text{km}}$$

Thus, to receive the transmission one should be within 61 km of the station.

55. The light has the same intensity in all directions. Use a spherical geometry centered at the source with the definition of the Poynting vector.

$$S = \frac{P_0}{A} = \frac{P_0}{4\pi r^2} = \tfrac{1}{2}c\varepsilon_0 E_0^{\,2} = \tfrac{1}{2}c\left(\frac{1}{c^2\mu_0}\right)E_0 \;\rightarrow\; \tfrac{1}{2}c\left(\frac{1}{c^2\mu_0}\right)E_0 = \frac{P_0}{4\pi r^2} \;\rightarrow\; \boxed{E_0 = \sqrt{\frac{\mu_0 c P_0}{2\pi r^2}}}$$

56. (*a*) The radio waves have the same intensity in all directions. The power crossing a given area is the intensity times the area. The intensity is the total power through the area of a sphere centered at the source.

$$P = IA = \frac{P_0}{A_{\text{total}}}A = \frac{35{,}000\,\text{W}}{4\pi(1.0\times10^3\,\text{m})^2}(1.0\,\text{m}^2) = 2.785\times10^{-3}\,\text{W} \approx \boxed{2.8\,\text{mW}}$$

(*b*) We find the rms value of the electric field from the intensity, which is the magnitude of the Poynting vector.

$$S = c\varepsilon_0 E_{\text{rms}}^2 = \frac{P_0}{4\pi r^2} \quad \rightarrow$$

$$E_{\text{rms}} = \sqrt{\frac{P_0}{4\pi r^2 c\varepsilon_0}} = \sqrt{\frac{35{,}000\,\text{W}}{4\pi\left(1.0\times 10^3\,\text{m}\right)^2\left(3.00\times 10^8\,\text{m/s}\right)\left(8.85\times 10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)}}$$

$$= 1.024\,\text{V/m} \approx \boxed{1.0\,\text{V/m}}$$

(*c*) The voltage over the length of the antenna is the electric field times the length of the antenna.

$$V_{\text{rms}} = E_{\text{rms}} d = \left(1.024\,\text{V/m}\right)\left(1.0\,\text{m}\right) = \boxed{1.0\,\text{V}}$$

(*d*) We calculate the electric field at the new distance, and then calculate the voltage.

$$E_{\text{rms}} = \sqrt{\frac{P_0}{4\pi r^2 c\varepsilon_0}} = \sqrt{\frac{35{,}000\,\text{W}}{4\pi\left(5.0\times 10^4\,\text{m}\right)^2\left(3.00\times 10^8\,\text{m/s}\right)\left(8.85\times 10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)}}$$

$$= 2.048\times 10^{-2}\,\text{V/m} \;;\; V_{\text{rms}} = E_{\text{rms}} d = \left(2.048\times 10^{-2}\,\text{V/m}\right)\left(1.0\,\text{m}\right) = \boxed{2.0\times 10^{-2}\,\text{V}}$$

57. The power output of the antenna would be the intensity at a given distance from the antenna, times the area of a sphere surrounding the antenna. The intensity is the magnitude of the Poynting vector.

$$S = \tfrac{1}{2} c\varepsilon_0 E_0^2$$

$$P = 4\pi r^2 S = 2\pi r^2 c\varepsilon_0 E_0^2 = 2\pi\left(0.50\,\text{m}\right)^2\left(3.00\times 10^8\,\text{m/s}\right)\left(8.85\times 10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)\left(3\times 10^6\,\text{V/m}\right)^2$$

$$\approx \boxed{4\times 10^{10}\,\text{W}}$$

This is many orders of magnitude higher than the power output of commercial radio stations, which are no higher than the 10's of kilowatts.

58. We calculate the speed of light in water according to the relationship given.

$$v_{\text{water}} = \frac{1}{\sqrt{K\varepsilon_0\mu_0}} = \frac{1}{\sqrt{K}}\frac{1}{\sqrt{\varepsilon_0\mu_0}} = \frac{1}{\sqrt{K}}c = \frac{1}{\sqrt{1.77}}\left(3.00\times 10^8\,\text{m/s}\right) = \boxed{2.25\times 10^8\,\text{m/s}}$$

$$\frac{v_{\text{water}}}{c} = \frac{\frac{1}{\sqrt{K}}c}{c} = \frac{1}{\sqrt{K}} = 0.752 = \boxed{75.2\%}$$

59. A standing wave has a node every half-wavelength, including the endpoints. For this wave, the nodes would occur at the spacing calculated here.

$$\tfrac{1}{2}\lambda = \tfrac{1}{2}\frac{c}{f} = \tfrac{1}{2}\frac{3.00\times 10^8\,\text{m/s}}{2.45\times 10^9\,\text{Hz}} = \boxed{0.0612\,\text{m}}$$

Thus there would be nodes at the following distances from a wall:

0, 6.12 cm, 12.2 cm, 18.4 cm, 24.5 cm, 30.6 cm, and 36.7 cm (approximately the other wall).

So there are $\boxed{\text{5 nodes}}$, not counting the ones at (or near) the walls.

60. (*a*) Assume that the wire is of length ℓ and cross-sectional area A. There must be a voltage across the ends of the wire to make the current flow $(V = IR)$, and there must be an electric field associated with that voltage $(E = V/\ell)$. Use these relationships with the definition of displacement current, Eq. 31-3.

$$I_D = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 \frac{d(EA)}{dt} = \varepsilon_0 A \frac{dE}{dt} = \varepsilon_0 A \frac{d(V/\ell)}{dt} = \varepsilon_0 \frac{A}{\ell}\frac{dV}{dt} = \varepsilon_0 \rho \frac{A}{\rho\ell}\frac{d(IR)}{dt}$$

$$= \varepsilon_0 \rho \frac{1}{R} R \frac{dI}{dt} = \boxed{\varepsilon_0 \rho \frac{dI}{dt}}$$

(*b*) Calculate the displacement current found in part (*a*).

$$I_D = \varepsilon_0 \rho \frac{dI}{dt} = \left(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)\left(1.68\times10^{-8}\,\Omega\bullet\text{m}\right)\left(\frac{1.0\,\text{A}}{1.0\times10^{-3}\,\text{s}}\right)$$

$$= 1.4868\times10^{-16}\,\text{A} \approx \boxed{1.5\times10^{-16}\,\text{A}}$$

(*c*) From example 28-6, Ampere's law gives the magnetic field created by a cylinder of current as $B = \frac{\mu_0 I}{2\pi r}$ at a distance of *r* from the axis of the cylindrical wire. This is true whether the current is displacement current or steady current.

$$B_D = \frac{\mu_0 I_D}{2\pi r} = \frac{\left(4\pi\times10^{-7}\,\text{N}\bullet\text{s}^2/\text{C}^2\right)\left(1.486\times10^{-16}\,\text{A}\right)}{2\pi\left(1.0\times10^{-3}\,\text{m}\right)} = 2.97\times10^{-20}\,\text{T} \approx \boxed{3.0\times10^{-20}\,\text{T}}$$

$$\frac{B_D}{B_{\text{steady}}} = \frac{\dfrac{\mu_0 I_D}{2\pi r}}{\dfrac{\mu_0 I_{\text{steady}}}{2\pi r}} = \frac{I_D}{I_{\text{steady}}} = \frac{1.486\times10^{-16}\,\text{A}}{1.0\,\text{A}} = 1.486\times10^{-16} \approx \boxed{1.5\times10^{-16}}$$

61. (*a*) We note that $-\alpha x^2 - \beta^2 t^2 + 2\alpha\beta xt = -(\alpha x - \beta t)^2$ and so $E_y = E_0 e^{-(\alpha x-\beta t)^2} = E_0 e^{-\alpha^2\left(x-\frac{\beta}{\alpha}t\right)^2}$. Since the wave is of the form $f(x - vt)$, with $v = \beta/\alpha$, the wave is moving in the $\boxed{+x \text{ direction}}$.

(*b*) The speed of the wave is $v = \beta/\alpha = c$, and so $\boxed{\beta = \alpha c}$.

(*c*) The electric field is in the *y* direction, and the wave is moving in the *x* direction. Since $\vec{\mathbf{E}}\times\vec{\mathbf{B}}$ must be in the direction of motion, the magnetic field must be in the *z* direction. The magnitudes are related by $|\vec{\mathbf{B}}| = |\vec{\mathbf{E}}|/c$.

$$\boxed{B_z = \frac{E_0}{c} e^{-(\alpha x-\beta t)^2}}$$

62. (*a*) Use the $\sin A \pm \sin B = 2\sin\left(\frac{A\pm B}{2}\right)\cos\left(\frac{A\mp B}{2}\right)$ from page A-4 in Appendix A.

$$E_y = E_0\left[\sin(kx-\omega t) + \sin(kx+\omega t)\right]$$

$$= 2E_0 \sin\left(\frac{(kx-\omega t)+(kx+\omega t)}{2}\right)\cos\left(\frac{(kx-\omega t)-(kx+\omega t)}{2}\right) = 2E_0\sin(kx)\cos(-\omega t)$$

$$= \boxed{2E_0\sin(kx)\cos(\omega t)}$$

$$B_z = B_0\left[\sin(kx-\omega t) - \sin(kx+\omega t)\right]$$

$$= 2B_0 \sin\left(\frac{(kx-\omega t)-(kx+\omega t)}{2}\right)\cos\left(\frac{(kx-\omega t)+(kx+\omega t)}{2}\right) = 2B_0\sin(-\omega t)\cos(kx)$$

$$= \boxed{-2B_0\cos(kx)\sin(\omega t)}$$

(*b*) The Poynting vector is given by $\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}}\times\vec{\mathbf{B}}.$

$$\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}}\times\vec{\mathbf{B}} = \frac{1}{\mu_0}\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 0 & 2E_0\sin(kx)\cos(\omega t) & 0 \\ 0 & 0 & -2B_0\cos(kx)\sin(\omega t)\end{vmatrix}$$

$$= \frac{1}{\mu_0}\hat{\mathbf{i}}\left[-4E_0B_0\sin(kx)\cos(kx)\sin(\omega t)\cos(\omega t)\right] = \boxed{-\frac{1}{\mu_0}\hat{\mathbf{i}}\,E_0B_0\sin(2kx)\sin(2\omega t)}$$

This is 0 for all times at positions where $\sin(2kx)=0.$

$$\sin(2kx)=0 \;\rightarrow\; 2kx=n\pi \;\rightarrow\; \boxed{x=\frac{n\pi}{2k},\; n=0,\pm1,\pm2,\ldots}$$

63. (*a*) To show that $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ are perpendicular, calculate their dot product.

$$\vec{\mathbf{E}}\bullet\vec{\mathbf{B}} = \left[E_0\sin(kx-\omega t)\hat{\mathbf{j}} + E_0\cos(kx-\omega t)\hat{\mathbf{k}}\right]\bullet\left[B_0\cos(kx-\omega t)\hat{\mathbf{j}} - B_0\sin(kx-\omega t)\hat{\mathbf{k}}\right]$$

$$= E_0\sin(kx-\omega t)B_0\cos(kx-\omega t) - E_0\cos(kx-\omega t)B_0\sin(kx-\omega t) = 0$$

Since $\vec{\mathbf{E}}\bullet\vec{\mathbf{B}}\equiv 0,$ $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ are perpendicular to each other at all times.

(*b*) The wave moves in the direction of the Poynting vector, which is given by $\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}}\times\vec{\mathbf{B}}.$

$$\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}}\times\vec{\mathbf{B}} = \frac{1}{\mu_0}\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 0 & E_0\sin(kx-\omega t) & E_0\cos(kx-\omega t) \\ 0 & B_0\cos(kx-\omega t) & -B_0\sin(kx-\omega t)\end{vmatrix}$$

$$= \frac{1}{\mu_0}\hat{\mathbf{i}}\left[-E_0B_0\sin^2(kx-\omega t) - E_0B_0\cos^2(kx-\omega t)\right] + \hat{\mathbf{j}}(0) + \hat{\mathbf{k}}(0) = -\frac{1}{\mu_0}E_0B_0\hat{\mathbf{i}}$$

We see that the Poynting vector points in the negative $\underline{x}$ direction, and so the wave moves in the negative x direction, which is perpendicular to both $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$.

(*c*) We find the magnitude of the electric field vector and the magnetic field vector.

$$\left|\vec{\mathbf{E}}\right| = E = \left(\left[E_0\sin(kx-\omega t)\right]^2 + \left[E_0\cos(kx-\omega t)\right]^2\right)^{1/2}$$

$$= \left[E_0^2\sin^2(kx-\omega t) + E_0^2\cos^2(kx-\omega t)\right]^{1/2} = E_0$$

$$\left|\vec{\mathbf{B}}\right| = B = \left(\left[B_0\cos(kx-\omega t)\right]^2 + \left[B_0\sin(kx-\omega t)\right]^2\right)^{1/2}$$

$$= \left[B_0^2\cos^2(kx-\omega t) + B_0^2\sin^2(kx-\omega t)\right]^{1/2} = B_0$$

(*d*) At $x=0$ and $t=0,$ $\vec{\mathbf{E}} = E_0\hat{\mathbf{k}}$ and $\vec{\mathbf{B}} = B_0\hat{\mathbf{j}}.$ See the figure. The x axis is coming out of the page toward the reader. As time increases, the component of the electric field in the z direction electric field begins to get smaller and the component in the negative y direction begins to get larger. At the same time, the component of the magnetic field in the y direction begins to get smaller, and the component in the z direction begins to get larger. The net effect is that both vectors rotate counterclockwise.