CHAPTER 30: Inductance, Electromagnetic Oscillations, and AC Circuits

Responses to Questions

1. (*a*) For the maximum value of the mutual inductance, place the coils close together, face to face, on the same axis.
   (*b*) For the least possible mutual inductance, place the coils with their faces perpendicular to each other.

2. The magnetic field near the end of the first solenoid is less than it is in the center. Therefore the flux through the second coil would be less than that given by the formula, and the mutual inductance would be lower.

3. Yes. If two coils have mutual inductance, then they each have the capacity for self-inductance. Any coil that experiences a changing current will have a self-inductance.

4. The energy density is greater near the center of a solenoid, where the magnetic field is greater.

5. To create the greatest self-inductance, bend the wire into as many loops as possible. To create the least self-inductance, leave the wire as a straight piece of wire.

6. (*a*) No. The time needed for the *LR* circuit to reach a given fraction of its maximum possible current depends on the time constant, $\tau = L/R$, which is independent of the emf.
   (*b*) Yes. The emf determines the maximum value of the current ($I_{max} = V_0/R$,) and therefore will affect the time it takes to reach a particular value of current.

7. A circuit with a large inductive time constant is resistant to changes in the current. When a switch is opened, the inductor continues to force the current to flow. A large charge can build up on the switch, and may be able to ionize a path for itself across a small air gap, creating a spark.

8. Although the current is zero at the instant the battery is connected, the rate at which the current is changing is a maximum and therefore the rate of change of flux through the inductor is a maximum. Since, by Faraday's law, the induced emf depends on the rate of change of flux and not the flux itself, the emf in the inductor is a maximum at this instant.

9. When the capacitor has discharged completely, energy is stored in the magnetic field of the inductor. The inductor will resist a change in the current, so current will continue to flow and will charge the capacitor again, with the opposite polarity.

10. Yes. The instantaneous voltages across the different elements in the circuit will be different, but the current through each element in the series circuit is the same.

11. The energy comes from the generator. (A generator is a device that converts mechanical energy to electrical energy, so ultimately, the energy came from some mechanical source, such as falling water.) Some of the energy is dissipated in the resistor and some is stored in the fields of the capacitor and the inductor. An increase in *R* results in an increase in energy dissipated by the circuit. *L*, *C*, *R*, and the frequency determine the current flow in the circuit, which determines the power supplied by generator.

12. $X_L = X_C$ at the resonant frequency. If the circuit is predominantly inductive, such that $X_L > X_C$, then the frequency is greater than the resonant frequency and the voltage leads the current. If the circuit is predominantly capacitive, such that $X_C > X_L$, then the frequency is lower than the resonant frequency and the current leads the voltage. Values of L and C cannot be meaningfully compared, since they are in different units. Describing the circuit as "inductive" or "capacitive" relates to the values of X_L and X_C, which are both in ohms and which both depend on frequency.

13. Yes. When ω approaches zero, X_L approaches zero, and X_C becomes infinitely large. This is consistent with what happens in an ac circuit connected to a dc power supply. For the dc case, ω is zero and X_L will be zero because there is no changing current to cause an induced emf. X_C will be infinitely large, because steady direct current cannot flow across a capacitor once it is charged.

14. The impedance in an *LRC* circuit will be a minimum at resonance, when $X_L = X_C$. At resonance, the impedance equals the resistance, so the smallest R possible will give the smallest impedance.

15. Yes. The power output of the generator is $P = IV$. When either the instantaneous current or the instantaneous voltage in the circuit is negative, and the other variable is positive, the instantaneous power output can be negative. At this time either the inductor or the capacitor is discharging power back to the generator.

16. Yes, the power factor depends on frequency because X_L and X_C, and therefore the phase angle, depend on frequency. For example, at resonant frequency, $X_L = X_C$, the phase angle is 0º, and the power factor is one. The average power dissipated in an *LRC* circuit also depends on frequency, since it depends on the power factor: $P_{\text{avg}} = I_{\text{rms}} V_{\text{rms}} \cos\varphi$. Maximum power is dissipated at the resonant frequency. The value of the power factor decreases as the frequency gets farther from the resonant frequency.

17. (*a*) The impedance of a pure resistance is unaffected by the frequency of the source emf.
(*b*) The impedance of a pure capacitance decreases with increasing frequency.
(*c*) The impedance of a pure inductance increases with increasing frequency.
(*d*) In an *LRC* circuit near resonance, small changes in the frequency will cause large changes in the impedance.
(*e*) For frequencies far above the resonance frequency, the impedance of the *LRC* circuit is dominated by the inductive reactance and will increase with increasing frequency. For frequencies far below the resonance frequency, the impedance of the *LRC* circuit is dominated by the capacitive reactance and will decrease with increasing frequency.

18. In all three cases, the energy dissipated decreases as R approaches zero. Energy oscillates between being stored in the field of the capacitor and being stored in the field of the inductor.
(*a*) The energy stored in the fields (and oscillating between them) is a maximum at resonant frequency and approaches an infinite value as R approaches zero.
(*b*) When the frequency is near resonance, a large amount of energy is stored in the fields but the value is less than the maximum value.
(*c*) Far from resonance, a much lower amount of energy is stored in the fields.

19. In an *LRC* circuit, the current and the voltage in the circuit both oscillate. The energy stored in the circuit also oscillates and is alternately stored in the magnetic field of the inductor and the electric field of the capacitor.

20. In an *LRC* circuit, energy oscillates between being stored in the magnetic field of the inductor and being stored in the electric field of the capacitor. This is analogous to a mass on a spring, with energy alternating between kinetic energy of the mass and spring potential energy as the spring compresses and extends. The energy stored in the magnetic field is analogous to the kinetic energy of the moving mass, and *L* corresponds to the mass, *m*, on the spring. The energy stored in the electric field of the capacitor is analogous to the spring potential energy, and *C* corresponds to the reciprocal of the spring constant, 1/*k*.

Solutions to Problems

1. (*a*) The mutual inductance is found in Example 30-1.

$$M = \frac{\mu N_1 N_2 A}{\ell} = \frac{1850\left(4\pi \times 10^{-7}\,\text{T}\bullet\text{m/A}\right)(225)(115)\pi\left(0.0200\,\text{m}\right)^2}{2.44\,\text{m}} = \boxed{3.10 \times 10^{-2}\,\text{H}}$$

(b) The emf induced in the second coil can be found from Eq. 30-3b.

$$\mathscr{E}_2 = -M\frac{dI_1}{dt} = -M\frac{\Delta I_1}{\Delta t} = \left(-3.10 \times 10^{-2}\,\text{H}\right)\frac{(-12.0\,\text{A})}{0.0980\,\text{ms}} = \boxed{3.79\,\text{V}}$$

2. If we assume the outer solenoid is carrying current I_1, then the magnetic field inside the outer solenoid is $B = \mu_0 n_1 I_1$. The flux in each turn of the inner solenoid is $\Phi_{21} = B\pi r_2^2 = \mu_0 n_1 I_1 \pi r_2^2$. The mutual inductance is given by Eq. 30-1.

$$M = \frac{N_2 \Phi_{21}}{I_1} = \frac{n_2 \ell \mu_0 n_1 I_1 \pi r_2^2}{I_1} \quad\rightarrow\quad \frac{M}{\ell} = \boxed{\mu_0 n_1 n_2 \pi r_2^2}$$

3. We find the mutual inductance of the inner loop. If we assume the outer solenoid is carrying current I_1, then the magnetic field inside the outer solenoid is $B = \mu_0 \frac{N_1}{\ell} I_1$. The magnetic flux through each loop of the small coil is the magnetic field times the area perpendicular to the field. The mutual inductance is given by Eq. 30-1.

$$\Phi_{21} = BA_2 \sin\theta = \mu_0 \frac{N_1 I_1}{\ell} A_2 \sin\theta \;;\; M = \frac{N_2 \Phi_{21}}{I_1} = \frac{N_2 \mu_0 \frac{N_1 I_1}{\ell} A_2 \sin\theta}{I_1} = \boxed{\frac{\mu_0 N_1 N_2 A_2 \sin\theta}{\ell}}$$

4. We find the mutual inductance of the system using Eq. 30-1, with the flux equal to the integral of the magnetic field of the wire (Eq. 28-1) over the area of the loop.

$$M = \frac{\Phi_{12}}{I_1} = \frac{1}{I_1}\int_{\ell_1}^{\ell_2} \frac{\mu_0 I_1}{2\pi r} w\,dr = \boxed{\frac{\mu_0 w}{2\pi}\ln\left(\frac{\ell_2}{\ell_1}\right)}$$

5. Find the induced emf from Eq. 30-5.

$$\mathscr{E} = -L\frac{dI}{dt} = -L\frac{\Delta I}{\Delta t} = -(0.28\,\text{H})\frac{(10.0\,\text{A} - 25.0\,\text{A})}{0.36\,\text{s}} = \boxed{12\,\text{V}}$$

6. Use the relationship for the inductance of a solenoid, as given in Example 30-3.

$$L=\frac{\mu_0 N^2 A}{\ell} \quad\rightarrow\quad N=\sqrt{\frac{L\ell}{\mu_0 A}}=\sqrt{\frac{(0.13\,\text{H})(0.300\,\text{m})}{(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A})\pi(0.021\,\text{m})^2}}\approx\boxed{4700\,\text{turns}}$$

7. Because the current in increasing, the emf is negative. We find the self-inductance from Eq. 30-5.

$$\mathscr{E}=-L\frac{dI}{dt}=-L\frac{\Delta I}{\Delta t} \quad\rightarrow\quad L=-\mathscr{E}\frac{\Delta t}{\Delta I}=-(-2.50\,\text{V})\frac{0.0120\,\text{s}}{[0.0250\,\text{A}-(-0.0280\,\text{A})]}=\boxed{0.566\,\text{H}}$$

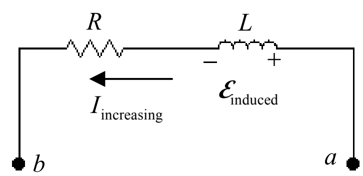
8. (*a*) The number of turns can be found from the inductance of a solenoid, which is derived in Example 30-3.

$$L=\frac{\mu_0 N^2 A}{\ell}=\frac{(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A})(2800)^2\pi(0.0125\,\text{m})^2}{(0.217\,\text{m})}=0.02229\,\text{H}\approx\boxed{0.022\,\text{H}}$$

(*b*) Apply the same equation again, solving for the number of turns.

$$L=\frac{\mu_0 N^2 A}{\ell} \quad\rightarrow\quad N=\sqrt{\frac{L\ell}{\mu_0 A}}=\sqrt{\frac{(0.02229\,\text{H})(0.217\,\text{m})}{(1200)(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A})\pi(0.0125\,\text{m})^2}}\approx\boxed{81\,\text{turns}}$$

9. We draw the coil as two elements in series, and pure resistance and a pure inductance. There is a voltage drop due to the resistance of the coil, given by Ohm's law, and an induced emf due to the inductance of the coil, given by Eq. 30-5. Since the current is increasing, the inductance will create a potential difference to oppose the increasing current, and so there is a drop in the potential due to the inductance. The potential difference across the coil is the sum of the two potential drops.



$$V_{ab}=IR+L\frac{dI}{dt}=(3.00\,\text{A})(3.25\,\Omega)+(0.44\,\text{H})(3.60\,\text{A/s})=\boxed{11.3\,\text{V}}$$

10. We use the result for inductance per unit length from Example 30-5.

$$\frac{L}{\ell}=\frac{\mu_0}{2\pi}\ln\frac{r_2}{r_1}\le 55\times10^{-9}\,\text{H/m} \quad\rightarrow\quad r_1\ge r_2 e^{-\frac{2\pi}{\mu_0}(55\times10^{-9}\,\text{H/m})}=(0.0030\,\text{m})e^{-\frac{2\pi(55\times10^{-9}\,\text{H/m})}{(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A})}}=0.00228\,\text{m}$$

$$\boxed{r_1\ge 0.0023\,\text{m}}$$

11. The self-inductance of an air-filled solenoid was determined in Example 30-3. We solve this equation for the length of the tube, using the diameter of the wire as the length per turn.

$$L=\frac{\mu_0 N^2 A}{\ell}=\mu_0 n^2 A\ell=\frac{\mu_0 A\ell}{d^2}$$

$$\ell=\frac{Ld^2}{\mu_0\pi r^2}=\frac{(1.0\text{H})(0.81\times10^{-3}\text{m})^2}{(4\pi\times10^{-7}\text{T}\bullet\text{m/A})\pi(0.060\,\text{m})^2}=46.16\,\text{m}\approx\boxed{46\,\text{m}}$$

The length of the wire is equal to the number of turns (the length of the solenoid divided by the diameter of the wire) multiplied by the circumference of the turn.

$$L=\frac{\ell}{d}\pi D=\frac{46.16\,\text{m}}{0.81\times10^{-3}\text{m}}\pi(0.12\,\text{m})=21{,}490\,\text{m}\approx\boxed{21\text{km}}$$

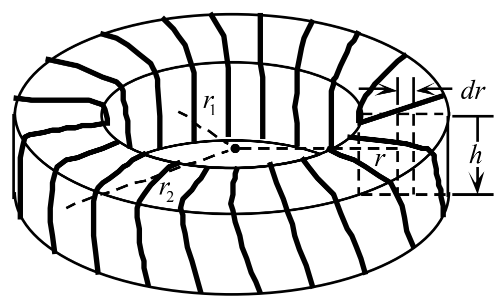
The resistance is calculated from the resistivity, area, and length of the wire.

$$R = \frac{\rho \ell}{A} = \frac{\left(1.68\times10^{-8}\,\Omega\bullet\text{m}\right)\left(21{,}490\,\text{m}\right)}{\pi\left(0.405\times10^{-3}\,\text{m}\right)^2} = \boxed{0.70\,\text{k}\Omega}$$

12. The inductance of the solenoid is given by $L = \dfrac{\mu_0 N^2 A}{\ell} = \dfrac{\mu_0 N^2}{\ell}\dfrac{\pi d^2}{4}$. The (constant) length of the wire is given by $\ell_{\text{wire}} = N\pi d_{\text{sol}}$, and so since $d_{\text{sol 2}} = 2.5\,d_{\text{sol 1}}$, we also know that $N_1 = 2.5N_2$. The fact that the wire is tightly wound gives $\ell_{\text{sol}} = Nd_{\text{wire}}$. Find the ratio of the two inductances.

$$\frac{L_2}{L_1} = \frac{\dfrac{\mu_0\pi}{4}\dfrac{N_2^2}{\ell_{\text{sol 2}}}d_{\text{sol 2}}^2}{\dfrac{\mu_0\pi}{4}\dfrac{N_1^2}{\ell_{\text{sol 1}}}d_{\text{sol 1}}^2} = \frac{\dfrac{N_2^2}{\ell_{\text{sol 2}}}d_{\text{sol 2}}^2}{\dfrac{N_1^2}{\ell_{\text{sol 1}}}d_{\text{sol 1}}^2} = \frac{\dfrac{\ell_{\text{wire}}^2/\pi^2}{\ell_{\text{sol 2}}}}{\dfrac{\ell_{\text{wire}}^2/\pi^2}{\ell_{\text{sol 1}}}} = \frac{\ell_{\text{sol 1}}}{\ell_{\text{sol 2}}} = \frac{N_1 d_{\text{wire}}}{N_2 d_{\text{wire}}} = \frac{N_1}{N_2} = \boxed{2.5}$$

13. We use Eq. 30-4 to calculate the self-inductance, where the flux is the integral of the magnetic field over a cross-section of the toroid. The magnetic field inside the toroid was calculated in Example 28-10.



$$L = \frac{N}{I}\Phi_B = \frac{N}{I}\int_{r_1}^{r_2}\frac{\mu_0 NI}{2\pi r}h\,dr = \boxed{\frac{\mu_0 N^2 h}{2\pi}\ln\left(\frac{r_2}{r_1}\right)}$$

14. (*a*) When connected in series the voltage drops across each inductor will add, while the currents in each inductor are the same.

$$\mathscr{E} = \mathscr{E}_1 + \mathscr{E}_2 = -L_1\frac{dI}{dt} - L_2\frac{dI}{dt} = -\left(L_1 + L_2\right)\frac{dI}{dt} = -L_{\text{eq}}\frac{dI}{dt} \;\rightarrow\; \boxed{L_{\text{eq}} = L_1 + L_2}$$

(*b*) When connected in parallel the currents in each inductor add to the equivalent current, while the voltage drop across each inductor is the same as the equivalent voltage drop.

$$\frac{dI}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt} \;\rightarrow\; \frac{\mathscr{E}}{L_{\text{eq}}} = \frac{\mathscr{E}}{L_1} + \frac{\mathscr{E}}{L_2} \;\rightarrow\; \boxed{\frac{1}{L_{\text{eq}}} = \frac{1}{L_1} + \frac{1}{L_2}}$$

Therefore, inductors in series and parallel add the same as resistors in series and parallel.

15. The magnetic energy in the field is derived from Eq. 30-7.

$$u = \frac{\text{Energy stored}}{\text{Volume}} = \tfrac{1}{2}\frac{B^2}{\mu_0} \;\rightarrow$$

$$\text{Energy} = \tfrac{1}{2}\frac{B^2}{\mu_0}\left(\text{Volume}\right) = \tfrac{1}{2}\frac{B^2}{\mu_0}\pi r^2\ell = \tfrac{1}{2}\frac{\left(0.600\,\text{T}\right)^2}{\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)}\pi\left(0.0105\,\text{m}\right)^2\left(0.380\,\text{m}\right) = \boxed{18.9\,\text{J}}$$

16. (*a*) We use Eq. 24-6 to calculate the energy density in an electric field and Eq. 30-7 to calculate the energy density in the magnetic field.

$$u_E = \tfrac{1}{2}\varepsilon_0 E^2 = \tfrac{1}{2}\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\bullet\text{m}^2\right)\left(1.0\times10^4\ \text{N/C}\right)^2 = \boxed{4.4\times10^{-4}\ \text{J/m}^3}$$

$$u_B = \frac{B^2}{2\mu_0} = \frac{(2.0\text{ T})^2}{2(4\pi \times 10^{-7}\text{ T}\bullet\text{m/A})} = 1.592 \times 10^6\text{ J/m}^3 \approx \boxed{1.6 \times 10^6\text{ J/m}^3}$$

(*b*) Use Eq. 24-6 to calculate the electric field from the energy density for the magnetic field given in part (*a*).

$$u_E = \tfrac{1}{2}\varepsilon_0 E^2 = u_B \rightarrow E = \sqrt{\frac{2u_B}{\varepsilon_0}} = \sqrt{\frac{2(1.592 \times 10^6\text{ J/m}^3)}{(8.85 \times 10^{-12}\text{ C}^2/\text{N}\bullet\text{m}^2)}} = \boxed{6.0 \times 10^8\text{ N/C}}$$

17. We use Eq. 30-7 to calculate the energy density with the magnetic field calculated in Example 28-12.

$$u_B = \frac{B^2}{2\mu_0} = \frac{1}{2\mu_0}\left(\frac{\mu_0 I}{2R}\right)^2 = \frac{\mu_0 I^2}{8R^2} = \frac{(4\pi \times 10^{-7}\text{ T}\bullet\text{m/A})(23.0\text{ A})^2}{8(0.280\text{ m})^2} = \boxed{1.06 \times 10^{-3}\text{ J/m}^3}$$

18. We use Eq. 30-7 to calculate the magnetic energy density, with the magnetic field calculated using Eq. 28-1.

$$u_B = \frac{B^2}{2\mu_0} = \frac{1}{2\mu_0}\left(\frac{\mu_0 I}{2\pi R}\right)^2 = \frac{\mu_0 I^2}{8\pi^2 R^2} = \frac{(4\pi \times 10^{-7}\text{ T}\bullet\text{m/A})(15\text{ A})^2}{8\pi^2(1.5 \times 10^{-3}\text{ m})^2} = \boxed{1.6\text{ J/m}^3}$$

To calculate the electric energy density with Eq. 24-6, we must first calculate the electric field at the surface of the wire. The electric field will equal the voltage difference along the wire divided by the length of the wire. We can calculate the voltage drop using Ohm's law and the resistance from the resistivity and diameter of the wire.

$$E = \frac{V}{\ell} = \frac{IR}{\ell} = \frac{I\rho\ell}{\ell\pi r^2} = \frac{I\rho}{\pi r^2}$$

$$u_E = \tfrac{1}{2}\varepsilon_0 E^2 = \tfrac{1}{2}\varepsilon_0\left(\frac{I\rho}{\pi r^2}\right)^2 = \tfrac{1}{2}(8.85 \times 10^{-12}\text{C}^2/\text{N}\bullet\text{m}^2)\left[\frac{(15\text{A})(1.68 \times 10^{-8}\Omega\bullet\text{m})}{\pi(1.5 \times 10^{-3}\text{m})^2}\right]^2$$

$$= \boxed{5.6 \times 10^{-15}\text{J/m}^3}$$

19. We use Eq. 30-7 to calculate the energy density in the toroid, with the magnetic field calculated in Example 28-10. We integrate the energy density over the volume of the toroid to obtain the total energy stored in the toroid. Since the energy density is a function of radius only, we treat the toroid as cylindrical shells each with differential volume $dV = 2\pi rhdr$.

$$u_B = \frac{B^2}{2\mu_0} = \frac{1}{2\mu_0}\left(\frac{\mu_0 NI}{2\pi r}\right)^2 = \boxed{\frac{\mu_0 N^2 I^2}{8\pi^2 r^2}}$$

$$U = \int u_B dV = \int_{r_1}^{r_2}\frac{\mu_0 N^2 I^2}{8\pi^2 r^2}2\pi rhdr = \frac{\mu_0 N^2 I^2 h}{4\pi}\int_{r_1}^{r_2}\frac{dr}{r} = \boxed{\frac{\mu_0 N^2 I^2 h}{4\pi}\ln\left(\frac{r_2}{r_1}\right)}$$

20. The magnetic field between the cables is given in Example 30-5. Since the magnetic field only depends on radius, we use Eq. 30-7 for the energy density in the differential volume $dV = 2\pi r\ell dr$ and integrate over the radius between the two cables.

$$\frac{U}{\ell} = \frac{1}{\ell}\int u_B dV = \int_{r_1}^{r_2}\frac{1}{2\mu_0}\left(\frac{\mu_0 I}{2\pi r}\right)^2 2\pi rdr = \frac{\mu_0 I^2}{4\pi}\int_{r_1}^{r_2}\frac{dr}{r} = \boxed{\frac{\mu_0 I^2}{4\pi}\ln\left(\frac{r_2}{r_1}\right)}$$

21. We create an Amperean loop of radius *r* to calculate the magnetic field within the wire using Eq. 28-3. Since the resulting magnetic field only depends on radius, we use Eq. 30-7 for the energy density in the differential volume $dV = 2\pi r\ell dr$ and integrate from zero to the radius of the wire.

$$\oint \vec{\mathbf{B}} \cdot d\vec{\ell} = \mu_0 I_{enc} \rightarrow B(2\pi r) = \mu_0\left(\frac{I}{\pi R^2}\right)(\pi r^2) \rightarrow B = \frac{\mu_0 I r}{2\pi R^2}$$

$$\frac{U}{\ell} = \frac{1}{\ell}\int u_B dV = \int_0^R \frac{1}{2\mu_0}\left(\frac{\mu_0 I r}{2\pi R^2}\right)^2 2\pi r dr = \frac{\mu_0 I^2}{4\pi R^4}\int_0^R r^3 dr = \boxed{\frac{\mu_0 I^2}{16\pi}}$$

22. For an *LR* circuit, we have $I = I_{\max}\left(1 - e^{-t/\tau}\right)$. Solve for t.

$$I = I_{\max}\left(1 - e^{-t/\tau}\right) \rightarrow e^{-t/\tau} = 1 - \frac{I}{I_{\max}} \rightarrow t = -\tau \ln\left(1 - \frac{I}{I_{\max}}\right)$$

(*a*) $I = 0.95 I_{\max} \rightarrow t = -\tau \ln\left(1 - \frac{I}{I_{\max}}\right) = -\tau \ln(1 - 0.95) = \boxed{3.0\,\tau}$

(*b*) $I = 0.990 I_{\max} \rightarrow t = -\tau \ln\left(1 - \frac{I}{I_{\max}}\right) = -\tau \ln(1 - 0.990) = \boxed{4.6\,\tau}$

(*c*) $I = 0.9990 I_{\max} \rightarrow t = -\tau \ln\left(1 - \frac{I}{I_{\max}}\right) = -\tau \ln(1 - 0.9990) = \boxed{6.9\,\tau}$

23. We set the current in Eq. 30-11 equal to $0.03I_0$ and solve for the time.

$$I = 0.03 I_0 = I_0 e^{-t/\tau} \rightarrow t = -\tau \ln(0.03) \approx \boxed{3.5\tau}$$

24. (*a*) We set *I* equal to 75% of the maximum value in Eq. 30-9 and solve for the time constant.

$$I = 0.75 I_0 = I_0\left(1 - e^{-t/\tau}\right) \rightarrow \tau = -\frac{t}{\ln(0.25)} = -\frac{(2.56 \text{ ms})}{\ln(0.25)} = 1.847 \text{ ms} \approx \boxed{1.85 \text{ ms}}$$

(*b*) The resistance can be calculated from the time constant using Eq. 30-10.

$$R = \frac{L}{\tau} = \frac{31.0 \text{ mH}}{1.847 \text{ ms}} = \boxed{16.8\ \Omega}$$

$\boxed{25.}$ (*a*) We use Eq. 30-6 to determine the energy stored in the inductor, with the current given by Eq. Eq 30-9.

$$U = \tfrac{1}{2} L I^2 = \boxed{\frac{L V_0^2}{2R^2}\left(1 - e^{-t/\tau}\right)^2}$$

(*b*) Set the energy from part (*a*) equal to 99.9% of its maximum value and solve for the time.

$$U = 0.999 \frac{V_0^2}{2R^2} = \frac{V_0^2}{2R^2}\left(1 - e^{-t/\tau}\right)^2 \rightarrow t = \tau \ln\left(1 - \sqrt{0.999}\right) \approx \boxed{7.6\tau}$$

26. (*a*) At the moment the switch is closed, no current will flow through the inductor. Therefore, the resistors R_1 and R_2 can be treated as in series.

$$\mathscr{E} = I(R_1 + R_2) \rightarrow \boxed{I_1 = I_2 = \frac{\mathscr{E}}{R_1 + R_2},\ I_3 = 0}$$

(*b*) A long time after the switch is closed, there is no voltage drop across the inductor so resistors R_2 and R_3 can be treated as parallel resistors in series with R_1.

$$I_1 = I_2 + I_3, \quad \mathscr{E} = I_1R_1 + I_2R_2, \quad I_2R_2 = I_3R_3$$

$$\frac{\mathscr{E} - I_2R_2}{R_1} = I_2 + \frac{I_2R_2}{R_3} \rightarrow \boxed{I_2 = \frac{\mathscr{E}R_3}{R_2R_3 + R_1R_3 + R_1R_2}}$$

$$I_3 = \frac{I_2R_2}{R_3} = \boxed{\frac{\mathscr{E}R_2}{R_2R_3 + R_1R_3 + R_1R_2}} \qquad I_1 = I_2 + I_3 = \boxed{\frac{\mathscr{E}(R_3 + R_2)}{R_2R_3 + R_1R_3 + R_1R_2}}$$

(*c*) Just after the switch is opened the current through the inductor continues with the same magnitude and direction. With the open switch, no current can flow through the branch with the switch. Therefore the current through R_2 must be equal to the current through R_3, but in the opposite direction.

$$I_3 = \boxed{\frac{\mathscr{E}R_2}{R_2R_3 + R_1R_3 + R_1R_2}}, \quad I_2 = \boxed{\frac{-\mathscr{E}R_2}{R_2R_3 + R_1R_3 + R_1R_2}}, \quad I_1 = \boxed{0}$$

(*d*) After a long time, with no voltage source, the energy in the inductor will dissipate and no current will flow through any of the branches.

$$I_1 = I_2 = I_3 = \boxed{0}$$

27. (*a*) We use Eq. 30-5 to determine the emf in the inductor as a function of time. Since the exponential term decreases in time, the maximum emf occurs when $t = 0$.

$$\mathscr{E} = -L\frac{dI}{dt} = -L\frac{d}{dt}\left[I_0e^{-tR/L}\right] = \frac{LI_0R}{L}e^{-t/\tau} = V_0e^{-t/\tau} \rightarrow \boxed{\mathscr{E}_{max} = V_0}.$$

(*b*) The current is the same just before and just after the switch moves from A to B. We use Ohm's law for a steady state current to determine I_0 before the switch is thrown. After the switch is thrown, the same current flows through the inductor, and therefore that current will flow through the resistor *R'*. Using Kirchhoff's loop rule we calculate the emf in the inductor. This will be a maximum at $t = 0$.

$$I_0 = \frac{V_0}{R}, \qquad \mathscr{E} - IR' = 0 \rightarrow \mathscr{E} = R'\frac{V_0}{R}e^{-t/\tau'} \rightarrow \mathscr{E}_{max} = \left(\frac{R'}{R}\right)V_0 = \left(\frac{55R}{R}\right)(120\,\text{V}) = \boxed{6.6\,\text{kV}}$$

28. The steady state current is the voltage divided by the resistance while the time constant is the inductance divided by the resistance, Eq. 30-10. To cut the time constant in half, we must double the resistance. If the resistance is doubled, we must double the voltage to keep the steady state current constant.

$$R' = 2R = 2(2200\ \Omega) = \boxed{4400\ \Omega} \qquad V_0' = 2V_0 = 2(240\ \text{V}) = \boxed{480\ \text{V}}$$

29. We use Kirchhoff's loop rule in the steady state (no voltage drop across the inductor) to determine the current in the circuit just before the battery is removed. This will be the maximum current after the battery is removed. Again using Kirchhoff's loop rule, with the current given by Eq. 30-11, we calculate the emf as a function of time.

$$V - I_0R = 0 \quad \rightarrow \quad I_0 = \frac{V}{R}$$

$$\mathscr{E} - IR = 0 \quad \rightarrow \quad \mathscr{E} = I_0Re^{-t/\tau} = V\,e^{-tR/L} = (12\,\text{V})e^{-t(2.2\,\text{k}\Omega)/(18\,\text{mH})} = \boxed{(12\,\text{V})e^{-\left(1.22\times10^5\,\text{s}^{-1}\right)t}}$$

The emf across the inductor is greatest at $\boxed{t = 0}$ with a value of $\boxed{\mathscr{E}_{max} = 12\,\text{V}}$.

30. We use the inductance of a solenoid, as derived in Example 30-3: $L_{\text{sol}} = \dfrac{\mu_0 N^2 A}{\ell}$.

(*a*) Both solenoids have the same area and the same length. Because the wire in solenoid 1 is 1.5 times as thick as the wire in solenoid 2, solenoid 2 will have 1.5 times the number of turns as solenoid 1.

$$\frac{L_2}{L_1} = \frac{\dfrac{\mu_0 N_2^2 A}{\ell}}{\dfrac{\mu_0 N_1^2 A}{\ell}} = \frac{N_2^2}{N_1^2} = \left(\frac{N_2}{N_1}\right)^2 = 1.5^2 = 2.25 \rightarrow \boxed{\frac{L_2}{L_1} = 2.25}$$

(*b*) To find the ratio of the time constants, both the inductance and resistance ratios need to be known. Since solenoid 2 has 1.5 times the number of turns as solenoid 1, the length of wire used to make solenoid 2 is 1.5 times that used to make solenoid 1, or $\ell_{\text{wire 2}} = 1.5\ell_{\text{wire 1}}$, and the diameter of the wire in solenoid 1 is 1.5 times that in solenoid 2, or $d_{\text{wire 1}} = 1.5 d_{\text{wire 2}}$. Use this to find their relative resistances, and then the ratio of time constants.

$$\frac{R_1}{R_2} = \frac{\dfrac{\rho\ell_{\text{wire 1}}}{A_{\text{wire 1}}}}{\dfrac{\rho\ell_{\text{wire 2}}}{A_{\text{wire 2}}}} = \frac{\dfrac{\ell_{\text{wire 1}}}{\pi\left(d_{\text{wire 1}}/2\right)^2}}{\dfrac{\ell_{\text{wire 2}}}{\pi\left(d_{\text{wire 2}}/2\right)^2}} = \frac{\dfrac{\ell_{\text{wire 1}}}{d_{\text{wire 1}}^2}}{\dfrac{\ell_{\text{wire 2}}}{d_{\text{wire 2}}^2}} = \frac{\ell_{\text{wire 1}}}{\ell_{\text{wire 2}}}\left(\frac{d_{\text{wire 2}}}{d_{\text{wire 1}}}\right)^2 = \left(\frac{1}{1.5}\right)\left(\frac{1}{1.5}\right)^2 = \frac{1}{1.5^3} \rightarrow$$

$$\frac{R_1}{R_2} = \frac{1}{1.5^3} \; ; \; \frac{\tau_1}{\tau_2} = \frac{L_1/R_1}{L_2/R_2} = \frac{L_1}{L_2}\frac{R_2}{R_1} = \left(\frac{1}{2.25}\right)\left(1.5^3\right) = 1.5 \rightarrow \boxed{\frac{\tau_1}{\tau_2} = 1.5}$$

31. (*a*) The AM station received by the radio is the resonant frequency, given by Eq. 30-14. We divide the resonant frequencies to create an equation relating the frequencies and capacitances. We then solve this equation for the new capacitance.

$$\frac{f_1}{f_2} = \frac{\dfrac{1}{2\pi}\sqrt{\dfrac{1}{LC_1}}}{\dfrac{1}{2\pi}\sqrt{\dfrac{1}{LC_2}}} = \sqrt{\frac{C_2}{C_1}} \rightarrow C_2 = C_1\left(\frac{f_1}{f_2}\right)^2 = (1350\,\text{pF})\left(\frac{550\,\text{kHz}}{1600\,\text{kHz}}\right)^2 = \boxed{0.16\,\text{nF}}$$

(*b*) The inductance is obtained from Eq. 30-14.

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC_1}} \rightarrow L = \frac{1}{4\pi^2 f^2 C} = \frac{1}{4\pi^2\left(550\times10^3\,\text{Hz}\right)^2\left(1350\times10^{-12}\,\text{F}\right)} = \boxed{62\,\mu\text{H}}$$

32. (*a*) To have maximum current and no charge at the initial time, we set $t = 0$ in Eqs. 30-13 and 30-15 to solve for the necessary phase factor ϕ.

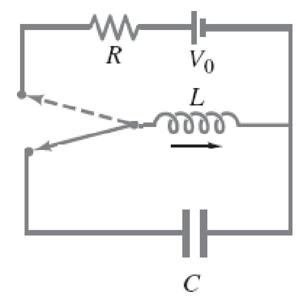
$$I_0 = I_0 \sin\phi \rightarrow \phi = \frac{\pi}{2} \rightarrow I(t) = I_0 \sin\left(\omega t + \frac{\pi}{2}\right) = I_0 \cos\omega t$$

$$Q(0) = Q_0 \cos\left(\frac{\pi}{2}\right) = 0 \rightarrow Q = Q_0 \cos\left(\omega t + \frac{\pi}{2}\right) = -Q_0 \sin(\omega t)$$

Differentiating the charge with respect to time gives the negative of the current. We use this to write the charge in terms of the known maximum current.

$$I = -\frac{dQ}{dt} = -Q_0\omega\cos(\omega t) = I_0\cos(\omega t) \rightarrow Q_0 = \frac{I_0}{\omega} \rightarrow \boxed{Q(t) = \frac{I_0}{\omega}\sin(\omega t)}$$

(*b*) As in the figure, attach the inductor to a battery and resistor for an extended period so that a steady state current flows through the inductor. Then at time $t = 0$, flip the switch connecting the inductor in series to the capacitor.



33. (*a*) We write the oscillation frequency in terms of the capacitance using Eq. 30-14, with the parallel plate capacitance given by Eq. 24-2. We then solve the resulting equation for the plate separation distance.

$$2\pi f = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{L(\varepsilon_0 A/x)}} \to x = \boxed{4\pi^2 A\varepsilon_0 f^2 L}$$

(*b*) For small variations we can differentiate *x* and divide the result by *x* to determine the fractional change.

$$dx = 4\pi^2 A\varepsilon_0 (2fdf)L \;\; ; \;\; \frac{dx}{x} = \frac{4\pi^2 A\varepsilon_0 (2fdf)L}{4\pi^2 A\varepsilon_0 f^2 L} = \frac{2df}{f} \to \boxed{\frac{\Delta x}{x} \approx \frac{2\Delta f}{f}}$$

(*c*) Inserting the given data, we can calculate the fractional variation on *x*.

$$\frac{\Delta x}{x} \approx \frac{2(1\text{ Hz})}{1\text{ MHz}} = 2\times 10^{-6} = \boxed{0.0002\%}$$

34. (*a*) We calculate the resonant frequency using Eq. 30-14.

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} = \frac{1}{2\pi}\sqrt{\frac{1}{(0.175\text{ H})(425\times 10^{-12}\text{ F})}} = 18{,}450\text{ Hz} \approx \boxed{18.5\text{ kHz}}$$

(*b*) As shown in Eq. 30-15, we set the peak current equal to the maximum charge (from Eq. 24-1) multiplied by the angular frequency.

$$I = Q_0\omega = CV(2\pi f) = (425\times 10^{-12}\text{ F})(135\text{ V})(2\pi)(18{,}450\text{ Hz})$$
$$= 6.653\times 10^{-3}\text{ A} \approx \boxed{6.65\text{ mA}}$$

(*c*) We use Eq. 30-6 to calculate the maximum energy stored in the inductor.

$$U = \tfrac{1}{2}LI^2 = \tfrac{1}{2}(0.175\text{ H})(6.653\times 10^{-3}\text{ A})^2 = \boxed{3.87\ \mu\text{J}}$$

35. (*a*) When the energy is equally shared between the capacitor and inductor, the energy stored in the capacitor will be one half of the initial energy in the capacitor. We use Eq. 24-5 to write the energy in terms of the charge on the capacitor and solve for the charge when the energy is equally shared.

$$\frac{Q^2}{2C} = \frac{1}{2}\frac{Q_0^2}{2C} \to Q = \boxed{\frac{\sqrt{2}}{2}Q_0}$$

(*b*) We insert the charge into Eq. 30-13 and solve for the time.

$$\frac{\sqrt{2}}{2}Q_0 = Q_0\cos\omega t \to t = \frac{1}{\omega}\cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{T}{2\pi}\left(\frac{\pi}{4}\right) = \boxed{\frac{T}{8}}$$

36. Since the circuit loses 3.5% of its energy per cycle, it is an underdamped oscillation. We use Eq. 24-5 for the energy with the charge as a function of time given by Eq. 30-19. Setting the change in energy equal to 3.5% and using Eq. 30-18 to determine the period, we solve for the resistance.

$$\frac{\Delta E}{E}=\frac{\dfrac{Q_0^2e^{-\frac{R}{L}T}\cos^2(2\pi)}{2C}-\dfrac{Q_0^2\cos^2(0)}{2C}}{\dfrac{Q_0^2\cos^2(0)}{2C}}=e^{-\frac{R}{L}T}-1=-0.035\rightarrow\frac{RT}{L}=\ln(1-0.035)=0.03563$$

$$0.03563=\frac{R}{L}\left(\frac{2\pi}{\omega'}\right)=\frac{R}{L}\frac{2\pi}{\sqrt{1/LC-R^2/4L^2}}\rightarrow R=\sqrt{\frac{4L(0.03563)^2}{C\left[16\pi^2+(0.03563)^2\right]}}$$

$$R=\sqrt{\frac{4(0.065\,\mathrm{H})(0.03563)^2}{(1.00\times10^{-6}\,\mathrm{F})\left[16\pi^2+(0.03563)^2\right]}}=1.4457\,\Omega\approx\boxed{1.4\,\Omega}$$

37. As in the derivation of 30-16, we set the total energy equal to the sum of the magnetic and electric energies, with the charge given by Eq. 30-19. We then solve for the time that the energy is 75% of the initial energy.

$$U=U_E+U_B=\frac{Q^2}{2C}+\frac{LI^2}{2}=\frac{Q_0^2}{2C}e^{-\frac{R}{L}t}\cos^2(\omega't+\phi)+\frac{Q_0^2}{2C}e^{-\frac{R}{L}t}\sin^2(\omega't+\phi)=\frac{Q_0^2}{2C}e^{-\frac{R}{L}t}$$

$$0.75\frac{Q_0^2}{2C}=\frac{Q_0^2}{2C}e^{-\frac{R}{L}t}\rightarrow t=-\frac{L}{R}\ln(0.75)=-\frac{L}{R}\ln(0.75)\approx\boxed{0.29\frac{L}{R}}$$

38. As shown by Eq. 30-18, adding resistance will $\boxed{\text{decrease}}$ the oscillation frequency. We use Eq. 30-14 for the pure LC circuit frequency and Eq. 30-18 for the frequency with added resistance to solve for the resistance.

$$\omega'=(1-.0025)\omega\quad\rightarrow\quad\sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}}=0.9975\sqrt{\frac{1}{LC}}\quad\rightarrow$$

$$R=\sqrt{\frac{4L}{C}\left(1-0.9975^2\right)}=\sqrt{\frac{4(0.350\,\mathrm{H})}{(1.800\times10^{-9}\,\mathrm{F})}\left(1-0.9975^2\right)}=\boxed{2.0\,\mathrm{k}\Omega}$$

39. We find the frequency from Eq. 30-23b for the reactance of an inductor.

$$X_L=2\pi fL\quad\rightarrow\quad f=\frac{X_L}{2\pi L}=\frac{660\,\Omega}{2\pi(0.0320\,\mathrm{H})}=3283\,\mathrm{Hz}\approx\boxed{3300\,\mathrm{Hz}}$$

40. The reactance of a capacitor is given by Eq. 30-25b, $X_C=\dfrac{1}{2\pi fC}$.

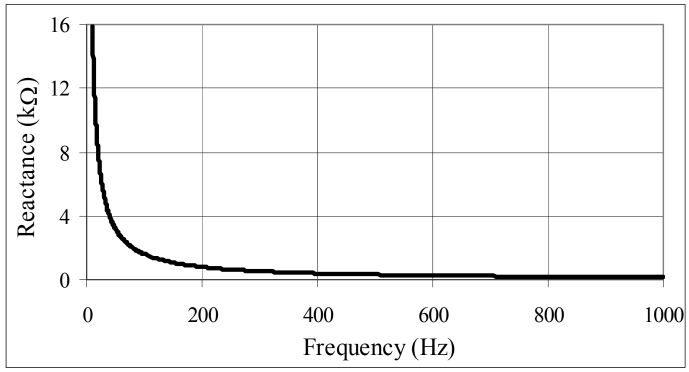
(*a*) $$X_C=\frac{1}{2\pi fC}=\frac{1}{2\pi(60.0\,\mathrm{Hz})(9.2\times10^{-6}\,\mathrm{F})}=\boxed{290\,\Omega}$$

(*b*) $$X_C=\frac{1}{2\pi fC}=\frac{1}{2\pi(1.00\times10^{6}\,\mathrm{Hz})(9.2\times10^{-6}\,\mathrm{F})}=\boxed{1.7\times10^{-2}\,\Omega}$$

41. The impedance is $X_C = \dfrac{1}{2\pi fC}$. The extreme values are as follows.

$$X_{\max} = \frac{1}{2\pi(10\,\text{Hz})(1.0\times10^{-6}\,\text{F})} = 16{,}000\,\Omega$$

$$X_{\min} = \frac{1}{2\pi(1000\,\text{Hz})(1.0\times10^{-6}\,\text{F})} = 160\,\Omega$$



The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH30.XLS," on tab "Problem 30.41."

42. We find the reactance from Eq. 30-23b, and the current from Ohm's law.

$$X_L = 2\pi fL = 2\pi(33.3\times10^{3}\,\text{Hz})(0.0360\,\text{H}) = 7532\,\Omega \approx \boxed{7530\,\Omega}$$

$$V = IX_L \quad\rightarrow\quad I = \frac{V}{X_L} = \frac{250\,\text{V}}{7532\,\Omega} = 0.03319\,\text{A} \approx \boxed{3.3\times10^{-2}\,\text{A}}$$

43. (*a*) At $\omega = 0$, the impedance of the capacitor is infinite. Therefore the parallel combination of the resistor *R* and capacitor *C* behaves as the resistor only, and so is *R*. Thus the impedance of the entire circuit is equal to the resistance of the two series resistors.

$$Z = \boxed{R + R'}$$

(*b*) At $\omega = \infty$,, the impedance of the capacitor is zero. Therefore the parallel combination of the resistor *R* and capacitor *C* is equal to zero. Thus the impedance of the entire circuit is equal to the resistance of the series resistor only.

$$Z = \boxed{R'}$$

44. We use Eq. 30-22a to solve for the impedance.

$$V_{\text{rms}} = I_{\text{rms}}\omega L \rightarrow L = \frac{V_{\text{rms}}}{I_{\text{rms}}\omega} = \frac{110\,\text{V}}{(3.1\,\text{A})2\pi(60\,\text{Hz})} = \boxed{94\,\text{mH}}$$

45. (*a*) We find the reactance from Eq. 30-25b.

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(660\,\text{Hz})(8.6\times10^{-8}\,\text{F})} = 2804\,\Omega \approx \boxed{2800\,\Omega}$$

(*b*) We find the peak value of the current from Ohm's law.

$$I_{\text{peak}} = \sqrt{2}I_{\text{rms}} = \sqrt{2}\frac{V_{\text{rms}}}{X_C} = \sqrt{2}\frac{22{,}000\,\text{V}}{2804\,\Omega} = \boxed{11\,\text{A at }660\,\text{Hz}}$$

46. (*a*) Since the resistor and capacitor are in parallel, they will have the same voltage drop across them. We use Ohm's law to determine the current through the resistor and Eq. 30-25 to determine the current across the capacitor. The total current is the sum of the currents across each element.

$$I_R = \frac{V}{R} \;\; ; \; I_C = \frac{V}{X_C} = V(2\pi fC)$$

$$\frac{I_C}{I_R + I_C} = \frac{V(2\pi fC)}{V(2\pi fC) + V/R} = \frac{R(2\pi fC)}{R(2\pi fC) + 1} = \frac{(490\ \Omega)2\pi(60\ \text{Hz})(0.35\times10^{-6}\ \text{F})}{(490\ \Omega)2\pi(60\ \text{Hz})(0.35\times10^{-6}\ \text{F}) + 1}$$

$$= 0.0607 \approx \boxed{6.1\%}$$

(*b*) We repeat part (*a*) with a frequency of 60,000 Hz.

$$\frac{I_C}{I_R + I_C} = \frac{(490\ \Omega)2\pi(60{,}000\ \text{Hz})(0.35\times10^{-6}\ \text{F})}{(490\ \Omega)2\pi(60{,}000\ \text{Hz})(0.35\times10^{-6}\ \text{F}) + 1} = 0.9847 \approx \boxed{98\%}$$

47. The power is only dissipated in the resistor, so we use the power dissipation equation obtained in section 25-7.

$$P_{\text{avg}} = \tfrac{1}{2} I_0^2 R = \tfrac{1}{2}(1.80\,\text{A})^2(1350\,\Omega) = 2187\,\text{W} \approx \boxed{2.19\,\text{kW}}$$

48. The impedance of the circuit is given by Eq. 30-28a without a capacitive reactance. The reactance of the inductor is given by Eq. 30-23b.

(*a*) $Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + 4\pi^2 f^2 L^2} = \sqrt{(10.0\times10^3\Omega)^2 + 4\pi^2(55.0\,\text{Hz})^2(0.0260\,\text{H})^2}$

$= \boxed{1.00\times10^4\Omega}$

(*b*) $Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + 4\pi^2 f^2 L^2} = \sqrt{(10.0\times10^3\Omega)^2 + 4\pi^2(5.5\times10^4\,\text{Hz})^2(0.0260\,\text{H})^2}$

$= \boxed{1.34\times10^4\Omega}$

$\boxed{49.}$ The impedance of the circuit is given by Eq. 30-28a without an inductive reactance. The reactance of the capacitor is given by Eq. 30-25b.

(*a*) $Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \dfrac{1}{4\pi^2 f^2 C^2}} = \sqrt{(75\Omega)^2 + \dfrac{1}{4\pi^2(60\,\text{Hz})^2(6.8\times10^{-6}\text{F})^2}} = 397\,\Omega$

$\approx \boxed{400\,\Omega}$ (2 sig. fig.)

(*b*) $Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \dfrac{1}{4\pi^2 f^2 C^2}} = \sqrt{(75\Omega)^2 + \dfrac{1}{4\pi^2(60000\,\text{Hz})^2(6.8\times10^{-6}\text{F})^2}} = \boxed{75\,\Omega}$

50. We find the impedance from Eq. 30-27.

$$Z = \frac{V_{\text{rms}}}{I_{\text{rms}}} = \frac{120\,\text{V}}{70\times10^{-3}\,\text{A}} = \boxed{1700\,\Omega}$$

51. The impedance is given by Eq. 30-28a with no capacitive reactance.

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + (2\pi fL)^2}$$

$$Z_f = 2Z_{60} \;\rightarrow\; \sqrt{R^2 + 4\pi^2 f^2 L^2} = 2\sqrt{R^2 + 4\pi^2(60\,\text{Hz})^2 L^2} \;\rightarrow$$

$$R^2 + 4\pi^2 f^2 L^2 = 4\left[R^2 + 4\pi^2(60\,\text{Hz})^2 L^2\right] = 4R^2 + 16\pi^2(60\,\text{Hz})^2 L^2 \;\rightarrow$$

$$f=\sqrt{\frac{3R^2+16\pi^2\left(60\,\text{Hz}\right)^2L^2}{4\pi^2L^2}}=\sqrt{\frac{3R^2}{4\pi^2L^2}+4\left(60\,\text{Hz}\right)^2}=\sqrt{\frac{3\left(2500\,\Omega\right)^2}{4\pi^2\left(0.42\,\text{H}\right)^2}+4\left(60\,\text{Hz}\right)^2}$$

$$=1645\,\text{Hz}\approx\boxed{1.6\,\text{kHz}}$$

52. (*a*) The rms current is the rmv voltage divided by the impedance. The impedance is given by Eq. 30-28a with no inductive reactance,

$$Z=\sqrt{R^2+X_C^2}=\sqrt{R^2+\frac{1}{\left(2\pi fC\right)^2}}\ .$$

$$I_{\text{rms}}=\frac{V_{\text{rms}}}{Z}=\frac{V_{\text{rms}}}{\sqrt{R^2+\frac{1}{4\pi^2f^2L^2}}}=\frac{120\,\text{V}}{\sqrt{\left(3800\,\Omega\right)^2+\frac{1}{4\pi^2\left(60.0\,\text{Hz}\right)^2\left(0.80\times10^{-6}\,\text{F}\right)^2}}}$$

$$=\frac{120\,\text{V}}{5043\,\Omega}=2.379\times10^{-2}\,\text{A}\approx\boxed{2.4\times10^{-2}\,\text{A}}$$

(*b*) The phase angle is given by Eq. 30-29a with no inductive reactance.

$$\phi=\tan^{-1}\frac{-X_C}{R}=\tan^{-1}\frac{-\frac{1}{2\pi fC}}{R}=\tan^{-1}\frac{-\frac{1}{2\pi\left(60.0\,\text{Hz}\right)\left(0.80\times10^{-6}\,\text{F}\right)}}{3800\,\Omega}=\boxed{-41^\circ}$$

The current is leading the source voltage.

(*c*) The power dissipated is given by $P=I_{\text{rms}}^2R=\left(0.02379\text{A}\right)^2\left(6.0\times10^3\Omega\right)=\boxed{2.2\,\text{W}}$

(*d*) The rms voltage reading is the rms current times the resistance or reactance of the element.

$$V_{\underset{R}{\text{rms}}}=I_{\text{rms}}R=\left(2.379\times10^{-2}\,\text{A}\right)\left(3800\,\Omega\right)=90.4\,\text{V}\approx\boxed{90\,\text{V}}\quad\left(2\text{ sig. fig.}\right)$$

$$V_{\underset{C}{\text{rms}}}=I_{\text{rms}}X_C=\frac{I_{\text{rms}}}{2\pi fC}=\frac{\left(2.379\times10^{-2}\,\text{A}\right)}{2\pi\left(60.0\,\text{Hz}\right)\left(0.80\times10^{-6}\,\text{F}\right)}=78.88\,\text{V}\approx\boxed{79\,\text{V}}$$

Note that, because the maximum voltages occur at different times, the two readings do not add to the applied voltage of 120 V.

53. We use the rms voltage across the resistor to determine the rms current through the circuit. Then, using the rms current and the rms voltage across the capacitor in Eq. 30-25 we determine the frequency.

$$I_{\text{rms}}=\frac{V_{R,\text{rms}}}{R}\qquad V_{C,\text{rms}}=\frac{I_{\text{rms}}}{2\pi fC}$$

$$f=\frac{I_{\text{rms}}}{2\pi CV_{C,\text{rms}}}=\frac{V_{R,\text{rms}}}{2\pi CRV_{C,\text{rms}}}=\frac{\left(3.0\text{ V}\right)}{2\pi\left(1.0\times10^{-6}\text{ C}\right)\left(750\ \Omega\right)\left(2.7\text{ V}\right)}=\boxed{240\text{ Hz}}$$

Since the voltages in the resistor and capacitor are not in phase, the rms voltage across the power source will not be the sum of their rms voltages.

54. The total impedance is given by Eq. 30-28a.

$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2} = \sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}$$

$$= \sqrt{\left(8.70\times10^3\,\Omega\right)^2 + \left[2\pi\left(1.00\times10^4\,\text{Hz}\right)\left(3.20\times10^{-2}\,\text{H}\right) - \frac{1}{2\pi\left(1.00\times10^4\,\text{Hz}\right)\left(6.25\times10^{-9}\,\text{F}\right)}\right]^2}$$

$$= 8716.5\,\Omega \approx \boxed{8.72\,\text{k}\Omega}$$

The phase angle is given by Eq. 30-29a.

$$\phi = \tan^{-1}\frac{X_L - X_C}{R} = \tan^{-1}\frac{2\pi fL - \frac{1}{2\pi fC}}{R}$$

$$= \tan^{-1}\frac{2\pi\left(1.00\times10^4\,\text{Hz}\right)\left(3.20\times10^{-2}\,\text{H}\right) - \frac{1}{2\pi\left(1.00\times10^4\,\text{Hz}\right)\left(6.25\times10^{-9}\,\text{F}\right)}}{R}$$

$$= \tan^{-1}\frac{-535.9\,\Omega}{8.70\times10^3\,\Omega} = \boxed{-3.52^\circ}$$

The voltage is lagging the current, or the current is leading the voltage.
The rms current is given by Eq. 30-27.

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{725\,\text{V}}{8716.5\,\Omega} = \boxed{8.32\times10^{-2}\,\text{A}}$$

55. (*a*) The rms current is the rms voltage divided by the impedance. The impedance is given by Eq. 30-28a with no capacitive reactance.

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + \left(2\pi fL\right)^2}\ .$$

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{V_{\text{rms}}}{\sqrt{R^2 + 4\pi^2 f^2 L^2}} = \frac{120\,\text{V}}{\sqrt{\left(965\,\Omega\right)^2 + 4\pi^2\left(60.0\,\text{Hz}\right)^2\left(0.225\,\text{H}\right)^2}}$$

$$= \frac{120\,\text{V}}{968.7\,\Omega} = \boxed{0.124\,\text{A}}$$

(*b*) The phase angle is given by Eq. 30-29a with no capacitive reactance.

$$\phi = \tan^{-1}\frac{X_L}{R} = \tan^{-1}\frac{2\pi fL}{R} = \tan^{-1}\frac{2\pi\left(60.0\,\text{Hz}\right)\left(0.225\,\text{H}\right)}{965\,\Omega} = \boxed{5.02^\circ}$$

The current is lagging the source voltage.

(*c*) The power dissipated is given by $P = I_{\text{rms}}^2 R = \left(0.124\,\text{A}\right)^2\left(965\,\Omega\right) = \boxed{14.8\,\text{W}}$

(*d*) The rms voltage reading is the rms current times the resistance or reactance of the element.

$$V_{\text{rms}\atop R} = I_{\text{rms}}R = \left(0.124\,\text{A}\right)\left(965\,\Omega\right) = 119.7\,\text{V} \approx \boxed{120\,\text{V}}$$

$$V_{\text{rms}\atop L} = I_{\text{rms}}X_L = I_{\text{rms}}2\pi fL = \left(0.124\,\text{A}\right)2\pi\left(60.0\,\text{Hz}\right)\left(0.25\,\text{H}\right) = \boxed{10.5\,\text{V}}$$

Note that, because the maximum voltages occur at different times, the two readings do not add to the applied voltage of 120 V.

56. (*a*) The current is found from the voltage and impedance. The impedance is given by Eq. 30-28a.

$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2} = \sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}$$

$$= \sqrt{\left(2.0\,\Omega\right)^2 + \left[2\pi\left(60\,\text{Hz}\right)\left(0.035\,\text{H}\right) - \frac{1}{2\pi\left(60\,\text{Hz}\right)\left(26\times10^{-6}\,\text{F}\right)}\right]^2} = 88.85\,\Omega$$

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{45\,\text{V}}{88.85\,\Omega} = 0.5065\,\text{A} \approx \boxed{0.51\,\text{A}}$$

(*b*) Use Eq. 30-29a to find the phase angle.

$$\phi = \tan^{-1}\frac{X_L - X_C}{R} = \tan^{-1}\frac{2\pi fL - \frac{1}{2\pi fC}}{R}$$

$$= \tan^{-1}\frac{2\pi\left(60\,\text{Hz}\right)\left(0.035\,\text{H}\right) - \frac{1}{2\pi\left(60\,\text{Hz}\right)\left(26\times10^{-6}\,\text{F}\right)}}{2.0\,\Omega} = \tan^{-1}\frac{-88.83\,\Omega}{2.0\,\Omega} = \boxed{-88^\circ}$$

(*c*) The power dissipated is given by $P = I_{\text{rms}}^2 R = \left(0.5065\,\text{A}\right)^2\left(2.0\,\Omega\right) = \boxed{0.51\,\text{W}}$

57. For the current and voltage to be in phase, the reactances of the capacitor and inductor must be equal. Setting the two reactances equal enables us to solve for the capacitance.

$$X_L = 2\pi fL = X_C = \frac{1}{2\pi fC} \rightarrow C = \frac{1}{4\pi^2 f^2 L} = \frac{1}{4\pi^2\left(360\,\text{Hz}\right)^2\left(0.025\,\text{H}\right)} = \boxed{7.8\,\mu\text{F}}$$

58. The light bulb acts like a resistor in series with the inductor. Using the desired rms voltage across the resistor and the power dissipated by the light bulb we calculate the rms current in the circuit and the resistance. Then using this current and the rms voltage of the circuit we calculate the impedance of the circuit (Eq. 30-27) and the required inductance (Eq. 30-28b).

$$I_{\text{rms}} = \frac{P}{V_{R,\text{rms}}} = \frac{75\,\text{W}}{120\,\text{V}} = 0.625\,\text{A} \qquad R = \frac{V_{R,\text{rms}}}{I_{\text{rms}}} = \frac{120\,\text{V}}{0.625\,\text{A}} = 192\,\Omega$$

$$Z = \frac{V_{\text{rms}}}{I_{\text{rms}}} = \sqrt{R^2 + \left(2\pi fL\right)^2}$$

$$L = \frac{1}{2\pi f}\sqrt{\left(\frac{V_{\text{rms}}}{I_{\text{rms}}}\right)^2 - R^2} = \frac{1}{2\pi\left(60\text{ Hz}\right)}\sqrt{\left(\frac{240\text{ V}}{0.625\text{ A}}\right)^2 - \left(192\ \Omega\right)^2} = \boxed{0.88\text{ H}}$$

59. We multiply the instantaneous current by the instantaneous voltage to calculate the instantaneous power. Then using the trigonometric identity for the summation of sine arguments (inside back cover of text) we can simplify the result. We integrate the power over a full period and divide the result by the period to calculate the average power.

$$P = IV = \left(I_0 \sin\omega t\right)V_0\sin\left(\omega t + \phi\right) = I_0V_0\sin\omega t\left(\sin\omega t\cos\phi + \sin\phi\cos\omega t\right)$$

$$= I_0V_0\left(\sin^2\omega t\cos\phi + \sin\omega t\cos\omega t\sin\phi\right)$$

$$\overline{P} = \frac{1}{T}\int_0^T P dT = \frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}} I_0 V_0 \left(\sin^2 \omega t \cos\phi + \sin\omega t \cos\omega t \sin\phi\right) dt$$

$$= \frac{\omega}{2\pi} I_0 V_0 \cos\phi \int_0^{\frac{2\pi}{\omega}} \sin^2 \omega t \; dt + \frac{\omega}{2\pi} I_0 V_0 \sin\phi \int_0^{\frac{2\pi}{\omega}} \sin\omega t \cos\omega t \; dt$$

$$= \frac{\omega}{2\pi} I_0 V_0 \cos\phi \left(\frac{1}{2}\frac{2\pi}{\omega}\right) + \frac{\omega}{2\pi} I_0 V_0 \sin\phi \left(\frac{1}{\omega}\sin^2 \omega t \Big|_0^{\frac{2\pi}{\omega}}\right) = \boxed{\tfrac{1}{2} I_0 V_0 \cos\phi}$$

60. Given the resistance, inductance, capacitance, and frequency, we calculate the impedance of the circuit using Eq. 30-28b.

$$X_L = 2\pi f L = 2\pi (660 \text{ Hz})(0.025 \text{ H}) = 103.67 \ \Omega$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi (660 \text{ Hz})(2.0\times10^{-6} \text{ F})} = 120.57 \ \Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(150 \ \Omega)^2 + (103.67 \ \Omega - 120.57 \ \Omega)^2} = 150.95 \ \Omega$$

(*a*) From the impedance and the peak voltage we calculate the peak current, using Eq. 30-27.

$$I_0 = \frac{V_0}{Z} = \frac{340 \text{ V}}{150.95 \ \Omega} = 2.252 \text{ A} \approx \boxed{2.3 \text{ A}}$$

(*b*) We calculate the phase angle of the current from the source voltage using Eq. 30-29a.

$$\phi = \tan^{-1}\frac{X_L - X_C}{R} = \tan^{-1}\frac{103.67 \ \Omega - 120.57 \ \Omega}{150 \ \Omega} = \boxed{-6.4^\circ}$$

(*c*) We multiply the peak current times the resistance to obtain the peak voltage across the resistor. The voltage across the resistor is in phase with the current, so the phase angle is the same as in part (*b*).

$$V_{0,R} = I_0 R = (2.252 \text{ A})(150 \ \Omega) = \boxed{340 \text{ V}} \ ; \ \boxed{\phi = -6.4^\circ}$$

(*d*) We multiply the peak current times the inductive reactance to calculate the peak voltage across the inductor. The voltage in the inductor is 90º ahead of the current. Subtracting the phase difference between the current and source from the 90º between the current and inductor peak voltage gives the phase angle between the source voltage and the inductive peak voltage.

$$V_{0,L} = I_0 X_L = (2.252 \text{ A})(103.67 \ \Omega) = \boxed{230 \text{ V}}$$

$$\phi_L = 90.0^\circ - \phi = 90.0^\circ - (-6.4)^\circ = \boxed{96.4^\circ}$$

(*e*) We multiply the peak current times the capacitive reactance to calculate the peak voltage across the capacitor. Subtracting the phase difference between the current and source from the -90º between the current and capacitor peak voltage gives the phase angle between the source voltage and the capacitor peak voltage.

$$V_{0,C} = I_0 X_C = (2.252 \text{ A})(120.57 \ \Omega) = \boxed{270 \text{ V}}$$

$$\phi_C = -90.0^\circ - \phi = -90.0^\circ - (-6.4^\circ) = \boxed{-83.6^\circ}$$

61. Using Eq. 30-23b we calculate the impedance of the inductor. Then we set the phase shift in Eq. 30-29a equal to 25º and solve for the resistance. We calculate the output voltage by multiplying the current through the circuit, from Eq. 30-27, by the inductive reactance (Eq. 30-23b).

$$X_L = 2\pi f L = 2\pi (175 \text{ Hz})(0.055 \text{ H}) = 60.48 \ \Omega$$

$$\tan\phi = \frac{X_L}{R} \Longrightarrow R = \frac{X_L}{\tan\phi} = \frac{60.48 \ \Omega}{\tan \ 25^\circ} = 129.7 \Omega \approx \boxed{130 \ \Omega}$$

$$\frac{V_{\text{output}}}{V_0} = \frac{V_R}{V_0} = \frac{IR}{IZ} = \frac{R}{Z} = \frac{129.70\Omega}{\sqrt{(129.70\Omega)^2 + (60.48\Omega)^2}} = \boxed{0.91}$$

62. The resonant frequency is found from Eq. 30-32. The resistance does not influence the resonant frequency.

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} = \frac{1}{2\pi}\sqrt{\frac{1}{(26.0\times10^{-6}\,\text{H})(3800\times10^{-12}\,\text{F})}} = \boxed{5.1\times10^5\,\text{Hz}}$$

63. We calculate the resonant frequency using Eq. 30-32 with the inductance and capacitance given in the example. We use Eq. 30-30 to calculate the power dissipation, with the impedance equal to the resistance.

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(0.0300\text{ H})(12.0\times10^{-6}\text{ F})}} = \boxed{265\text{ Hz}}$$

$$\bar{P} = I_{\text{rms}}V_{\text{rms}}\cos\phi = \left(\frac{V_{\text{rms}}}{R}\right)V_{\text{rms}}\left(\frac{R}{R}\right) = \frac{V^2_{\text{rms}}}{R} = \frac{(90.0\,\text{V})^2}{25.0\Omega} = \boxed{324\,\text{W}}$$

64. (*a*) We find the capacitance from the resonant frequency, Eq. 30-32.

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} \rightarrow C = \frac{1}{4\pi^2 L f_0^2} = \frac{1}{4\pi^2(4.15\times10^{-3}\,\text{H})(33.0\times10^3\,\text{Hz})^2} = \boxed{5.60\times10^{-9}\,\text{F}}$$

(*b*) At resonance the impedance is the resistance, so the current is given by Ohm's law.

$$I_{\text{peak}} = \frac{V_{\text{peak}}}{R} = \frac{136\,\text{V}}{3800\Omega} = \boxed{35.8\,\text{mA}}$$

65. (*a*) The peak voltage across the capacitor is the peak current multiplied by the capacitive reactance. We calculate the current in the circuit by dividing the source voltage by the impedance, where at resonance the impedance is equal to the resistance.

$$V_{C0} = X_C I_0 = \frac{1}{2\pi f_0 C}\frac{V_0}{R} = \frac{V_0}{2\pi(RC)}\frac{1}{f_0} = \boxed{\frac{V_0}{2\pi\tau}T_0}$$

(*b*) We set the amplification equal to 125 and solve for the resistance.

$$\beta = \frac{T_0}{2\pi\tau} = \frac{1}{2\pi f_0 RC} \rightarrow R = \frac{1}{2\pi f_0 \beta C} = \frac{1}{2\pi(5000\,\text{Hz})(125)(2.0\times10^{-9}\,\text{F})} = \boxed{130\Omega}$$

66. (*a*) We calculate the resonance frequency from the inductance and capacitance using Eq.30-32.

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(0.055\text{ H})(1.0\times10^{-9}\text{ F})}} = 21460\text{ Hz} \approx \boxed{21\text{ kHz}}$$

(*b*) We use the result of Problem 65 to calculate the voltage across the capacitor.

$$V_{C0} = \frac{V_0}{2\pi(RC)}\frac{1}{f_0} = \frac{2.0\text{ V}}{2\pi(35\ \Omega)(1.0\times10^{-9}\text{ F})(21460\text{ Hz})} = \boxed{420\text{ V}}$$

(*c*) We divide the voltage across the capacitor by the voltage source.

$$\frac{V_{C0}}{V_0} = \frac{420\text{ V}}{2.0\text{ V}} = \boxed{210}$$

67. (*a*) We write the average power using Eq. 30-30, with the current in terms of the impedance (Eq. 30-27) and the power factor in terms of the resistance and impedance (Eq. 30-29b). Finally we write the impedance using Eq. 30-28b.

$$\bar{P} = I_{\text{rms}}V_{\text{rms}}\cos\phi = \frac{V_{\text{rms}}}{Z}V_{\text{rms}}\frac{R}{Z} = \frac{V_{\text{rms}}^2 R}{Z^2} = \boxed{\frac{V_0^2 R}{2\left[R^2 + \left(\omega L - 1/\omega C\right)^2\right]}}$$

(*b*) The power dissipation will be a maximum when the inductive reactance is equal to the capacitive reactance, which is the resonant frequency.

$$f = \boxed{\frac{1}{2\pi\sqrt{LC}}}$$

(*c*) We set the power dissipation equal to ½ of the maximum power dissipation and solve for the angular frequencies.

$$\bar{P} = \frac{1}{2}\bar{P}_{\text{max}} = \frac{V_0^2 R}{2\left[R^2 + \left(\omega L - 1/\omega C\right)^2\right]} = \frac{1}{2}\left(\frac{V_0^2 R}{2R^2}\right) \rightarrow \left(\omega L - 1/\omega C\right) = \pm R$$

$$\rightarrow\ 0 = \omega^2 LC \pm RC\omega - 1 \ \rightarrow\ \omega = \frac{\pm RC \pm \sqrt{R^2C^2 + 4LC}}{2LC}$$

We require the angular frequencies to be positive and for a sharp peak, $R^2C^2 \ll 4LC$. The angular width will then be the difference between the two positive frequencies.

$$\omega = \frac{2\sqrt{LC} \pm RC}{2LC} = \frac{1}{\sqrt{LC}} \pm \frac{R}{2L} \ \rightarrow\ \Delta\omega = \left(\frac{1}{\sqrt{LC}} + \frac{R}{2L}\right) - \left(\frac{1}{\sqrt{LC}} - \frac{R}{2L}\right) = \boxed{\frac{R}{L}}$$

68. (*a*) We write the charge on the capacitor using Eq. 24-1, where the voltage drop across the capacitor is the inductive capacitance multiplied by the circuit current (Eq. 30-25a) and the circuit current is found using the source voltage and circuit impedance (Eqs. 30-27 and 30-28b).

$$Q_0 = CV_{C0} = CI_0X_C = C\left(\frac{V_0}{Z}\right)X_C = \frac{CV_0}{\omega C\sqrt{R^2 + \left(\omega L - 1/\omega C\right)^2}} = \boxed{\frac{V_0}{\sqrt{\omega^2R^2 + \left(\omega^2 L - 1/C\right)^2}}}$$

(*b*) We set the derivative of the charge with respect to the frequency equal to zero to calculate the frequency at which the charge is a maximum.

$$\frac{dQ_0}{d\omega} = \frac{d}{d\omega}\frac{V_0}{\sqrt{\omega'^2R^2 + \left(\omega'^2L - 1/C\right)^2}} = \frac{-V_0\left(2\omega' R^2 + 4\omega'^3L^2 - 4\omega' L/C\right)}{\left[\omega'^2R^2 + \left(\omega'^2L - 1/C\right)^2\right]^{\frac{3}{2}}} = 0$$

$$\rightarrow \omega' = \boxed{\sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}}$$

(*c*) The amplitude in a forced damped harmonic oscillation is given by Eq. 14-23. This is equivalent to the *LRC* circuit with $F_0 \leftrightarrow V_0,\ k \leftrightarrow 1/C,\ \ m \leftrightarrow L,$ and $b \leftrightarrow R.$

69. Since the circuit is in resonance, we use Eq. 30-32 for the resonant frequency to determine the necessary inductance. We set this inductance equal to the solenoid inductance calculated in Example 30-3, with the area equal to the area of a circle of radius *r*, the number of turns equal to the length of the wire divided by the circumference of a turn, and the length of the solenoid equal to the diameter of the wire multiplied by the number of turns. We solve the resulting equation for the number of turns.

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \rightarrow L = \frac{1}{4\pi^2 f_o^2 C} = \frac{\mu_0 N^2 A}{\ell} = \frac{\mu_0 \left(\frac{\ell_{wire}}{2\pi r}\right)^2 \pi r^2}{Nd} \rightarrow$$

$$N = \frac{\pi f_o^2 C \mu_0 \ell^2{}_{wire}}{d} = \frac{\pi \left(18.0\times10^3\,\text{Hz}\right)^2 \left(2.20\times10^{-7}\,\text{F}\right)\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)\left(12.0\,\text{m}\right)^2}{1.1\times10^{-3}\,\text{m}} = \boxed{37 \text{ loops}}$$

70. The power on each side of the transformer must be equal. We replace the currents in the power equation with the number of turns in the two coils using Eq. 29-6. Then we solve for the turn ratio.

$$P_p = I_p^2 Z_p = P_s = I_s^2 Z_s \rightarrow \frac{Z_p}{Z_s} = \left(\frac{I_s}{I_p}\right)^2 = \left(\frac{N_p}{N_s}\right)^2$$

$$\rightarrow \frac{N_p}{N_s} = \sqrt{\frac{Z_p}{Z_s}} = \sqrt{\frac{45\times10^3\ \Omega}{8.0\ \Omega}} = \boxed{75}$$

71. (*a*) We calculate the inductance from the resonance frequency.

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \rightarrow$$

$$L = \frac{1}{4\pi^2 f_o^2 C} = \frac{1}{4\pi^2\left(17\times10^3\,\text{Hz}\right)^2\left(2.2\times10^{-9}\,\text{F}\right)} = 0.03982\,\text{H} \approx \boxed{0.040\,\text{H}}$$

(*b*) We set the initial energy in the electric field, using Eq. 24-5, equal to the maximum energy in the magnetic field, Eq. 30-6, and solve for the maximum current.

$$\tfrac{1}{2}CV_0^2 = \tfrac{1}{2}LI_{\max}^2 \rightarrow I_{\max} = \sqrt{\frac{CV_0^2}{L}} = \sqrt{\frac{\left(2.2\times10^{-9}\,\text{F}\right)\left(120\,\text{V}\right)^2}{\left(0.03984\,\text{H}\right)}} = \boxed{0.028\,\text{A}}$$

(*c*) The maximum energy in the inductor is equal to the initial energy in the capacitor.

$$U_{L,\max} = \tfrac{1}{2}CV_0^2 = \tfrac{1}{2}\left(2.2\times10^{-9}\,\text{F}\right)\left(120\,\text{V}\right)^2 = \boxed{16\,\mu\text{J}}$$

72. We use Eq. 30-6 to calculate the initial energy stored in the inductor.

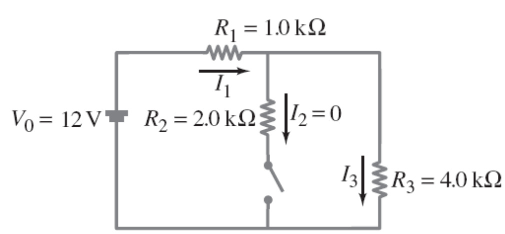
$$U_0 = \tfrac{1}{2}LI_0^2 = \tfrac{1}{2}\left(0.0600\,\text{H}\right)\left(0.0500\,\text{A}\right)^2 = \boxed{7.50\times10^{-5}\,\text{J}}$$

We set the energy in the inductor equal to five times the initial energy and solve for the current. We set the current equal to the initial current plus the rate of increase multiplied by time and solve for the time.

$$U = \tfrac{1}{2}LI^2 \rightarrow I = \sqrt{\frac{2U}{L}} = \sqrt{\frac{2\left(5.0\times7.50\times10^{-5}\,\text{J}\right)}{0.0600\,\text{H}}} = 111.8\,\text{mA}$$

$$I = I_0 + \beta t \rightarrow t = \frac{I - I_0}{\beta} = \frac{111.8\,\text{mA} - 50.0\,\text{mA}}{78.0\,\text{mA/s}} = \boxed{0.79\,\text{s}}$$

73. When the currents have acquired their steady-state values, the capacitor will be fully charged, and so no current will flow through the capacitor. At this time, the voltage drop across the inductor will be zero, as the current flowing through the inductor is constant. Therefore, the current through R_1 is zero, and the resistors R_2 and R_3 can be treated as in series.



$$I_1 = I_3 = \frac{V_0}{R_1 + R_3} = \frac{12\,\text{V}}{5.0\,\text{k}\Omega} = \boxed{2.4\,\text{mA}}\ ;\ I_2 = \boxed{0}$$

74. (*a*) The self inductance is written in terms of the magnetic flux in the toroid using Eq. 30-4. We set the flux equal to the magnetic field of a toroid, from Example 28-10. The field is dependent upon the radius of the solenoid, but if the diameter of the solenoid loops is small compared with the radius of the solenoid, it can be treated as approximately constant.

$$L = \frac{N\Phi_B}{I} = \frac{N\left(\pi d^2/4\right)\left(\mu_0 NI/2\pi r_0\right)}{I} = \boxed{\frac{\mu_0 N^2 d^2}{8r_0}}$$

This is consistent with the inductance of a solenoid for which the length is $\ell = 2\pi r_0$.

(*b*) We calculate the value of the inductance from the given data, with r_0 equal to half of the diameter.

$$L = \frac{\mu_0 N^2 d^2}{8r_0} = \frac{\left(4\pi\times10^{-7}\ \text{T}\bullet\text{m/A}\right)(550)^2(0.020\ \text{m})^2}{8(0.33\ \text{m})} = \boxed{58\ \mu\text{H}}$$

75. We use Eq. 30-4 to calculate the self inductance between the two wires. We calculate the flux by integrating the magnetic field from the two wires, using Eq. 28-1, over the region between the two wires. Dividing the inductance by the length of the wire gives the inductance per unit length.

$$L = \frac{\Phi_B}{I} = \frac{1}{I}\int_r^{\ell-r}\left[\frac{\mu_0 I}{2\pi r'} + \frac{\mu_0 I}{2\pi(\ell - r')}\right]h\,dr' = \frac{\mu_0 h}{2\pi}\int_r^{\ell-r}\left[\frac{1}{r'} + \frac{1}{(\ell - r')}\right]dr'$$

$$\frac{L}{h} = \frac{\mu_0}{2\pi}\left[\ln(r') - \ln(\ell - r')\right]_r^{\ell-r} = \frac{\mu_0}{2\pi}\left[\ln\left(\frac{\ell - r}{r}\right) - \ln\left(\frac{r}{\ell - r}\right)\right] = \boxed{\frac{\mu_0}{\pi}\ln\left(\frac{\ell - r}{r}\right)}$$

76. The magnetic energy is the energy density (Eq. 30-7) multiplied by the volume of the spherical shell enveloping the earth.

$$U = u_B V = \frac{B^2}{2\mu_0}\left(4\pi r^2 h\right) = \frac{\left(0.50\times10^{-4}\text{T}\right)^2}{2\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)}\left[4\pi\left(6.38\times10^6\text{m}\right)^2\left(5.0\times10^3\text{m}\right)\right] = \boxed{2.5\times10^{15}\text{J}}$$

77. (*a*) For underdamped oscillation, the charge on the capacitor is given by Eq. 30-19, with $\phi = 0$. Differentiating the current with respect to time gives the current in the circuit.

$$Q(t) = Q_0 e^{-\frac{R}{2L}t}\cos\omega' t\ ;\ I(t) = \frac{dQ}{dt} = -Q_0 e^{-\frac{R}{2L}t}\left(\frac{R}{2L}\cos\omega' t + \omega'\sin\omega' t\right)$$

The total energy is the sum of the energies stored in the capacitor (Eq. 24-5) and the energy stored in the inductor (Eq. 30-6). Since the oscillation is underdamped ($\omega' \gg R/2L$), the cosine term in the current is much smaller than the sine term and can be ignored. The frequency of oscillation is approximately equal to the undamped frequency of Eq. 30-14.

$$U = U_C + U_L = \frac{Q^2}{2C} + \frac{LI^2}{2} = \frac{\left(Q_0 e^{-\frac{R}{2L}t}\cos\omega' t\right)^2}{2C} + \frac{L\left(Q_0 e^{-\frac{R}{2L}t}\right)^2\left(\omega'\sin\omega' t\right)^2}{2}$$

$$= \frac{Q_0^2 e^{-\frac{R}{L}t}}{2C}\left(\cos^2\omega' t + \omega'^2 LC\sin^2\omega' t\right) \approx \boxed{\frac{Q_0^2 e^{-\frac{R}{L}t}}{2C}}$$

(*b*) We differentiate the energy with respect to time to show the average power dissipation. We then set the power loss per cycle equal to the resistance multiplied by the square of the current. For a lightly damped oscillation, the exponential term does not change much in one cycle, while the sine squared term averages to ½ .

$$\frac{dU}{dt}=\frac{d}{dt}\left(\frac{Q_0^2 e^{-\frac{R}{L}t}}{2C}\right)=-\frac{RQ_0^2 e^{-\frac{R}{L}t}}{2LC}$$

$$P=-I^2R=-Q_0^2e^{-\frac{R}{L}t}\left(\omega'^2\sin^2\omega' t\right)\approx -Q_0^2e^{-\frac{R}{L}t}\left(\frac{1}{LC}\right)\left(\frac{1}{2}\right)=\boxed{-\frac{RQ_0^2e^{-\frac{R}{L}t}}{2LC}}$$

The change in power in the circuit is equal to the power dissipated by the resistor.

78. Putting an inductor in series with the device will protect it from sudden surges in current. The growth of current in an *LR* circuit is given is Eq. 30-9.

$$I=\frac{V}{R}\left(1-e^{-tR/L}\right)=I_{\max}\left(1-e^{-tR/L}\right)$$

The maximum current is 33 mA, and the current is to have a value of 7.5 mA after a time of 75 microseconds. Use this data to solve for the inductance.

$$I=I_{\max}\left(1-e^{-tR/L}\right)\ \rightarrow\ e^{-tR/L}=1-\frac{I}{I_{\max}}\ \rightarrow$$

$$L=-\frac{tR}{\ln\left(1-\frac{I}{I_{\max}}\right)}=-\frac{\left(75\times10^{-6}\text{sec}\right)\left(150\,\Omega\right)}{\ln\left(1-\frac{7.5\,\text{mA}}{33\,\text{mA}}\right)}=4.4\times10^{-2}\,\text{H}$$

$\boxed{\text{Put an inductor of value } 4.4\times10^{-2}\,\text{H in series with the device.}}$

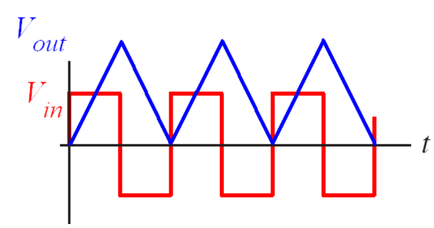
79. We use Kirchhoff's loop rule to equate the input voltage to the voltage drops across the inductor and resistor. We then multiply both sides of the equation by the integrating factor $e^{\frac{Rt}{L}}$ and integrate the right-hand side of the equation using a *u* substitution with $u=IRe^{\frac{Rt}{L}}$ and $du=dIRe^{\frac{Rt}{L}}+Ie^{\frac{Rt}{L}}dt/L$

$$V_{in}=L\frac{dI}{dt}+IR\ \rightarrow$$

$$\int V_{in}e^{\frac{Rt}{L}}dt=\int\left(L\frac{dI}{dt}+IR\right)e^{\frac{Rt}{L}}dt=\frac{L}{R}\int du=IR\frac{L}{R}e^{\frac{Rt}{L}}=V_{out}\frac{L}{R}e^{\frac{Rt}{L}}$$

For $L/R<<t$, $e^{\frac{Rt}{L}}\approx1$. Setting the exponential term equal to unity on both sides of the equation gives the desired results.

$$\int V_{in}dt=V_{out}\frac{L}{R}$$



80. (*a*) Since the capacitor and resistor are in series, the impedance of the circuit is given by Eq. 30-28a. Divide the source voltage by the impedance to determine the current in the circuit. Finally, multiply the current by the resistance to determine the voltage drop across the resistor.

$$V_R=IR=\frac{V_{in}}{Z}R=\frac{V_{in}R}{\sqrt{R^2+1/\left(2\pi fC\right)^2}}$$

$$= \frac{(130\text{ mV})(550\ \Omega)}{\sqrt{(550\ \Omega)^2 + 1\Big/\left[2\pi(60\text{ Hz})(1.2\times10^{-6}\text{ F})\right]^2}} = \boxed{31\text{ mV}}$$

(*b*) Repeat the calculation with a frequency of 6.0 kHz.

$$V_R = \frac{(130\text{ mV})(550\ \Omega)}{\sqrt{(550\ \Omega)^2 + 1\Big/\left[2\pi(6000\text{ Hz})(1.2\times10^{-6}\text{ F})\right]^2}} = \boxed{130\text{ mV}}$$

Thus the capacitor allows the higher frequency to pass, but attenuates the lower frequency.

81. (*a*) We integrate the power directly from the current and voltage over one cycle.

$$\overline{P} = \frac{1}{T}\int_0^T IV dt = \frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}} I_0 \sin(\omega t) V_0 \sin(\omega t + 90°)dt = \frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}} I_0 \sin(\omega t) V_0 \cos(\omega t)dt$$

$$= \frac{\omega}{2\pi} I_0 V_0 \frac{\sin^2(\omega t)}{2\omega}\Bigg|_0^{\frac{2\pi}{\omega}} = \frac{I_0 V_0}{4\pi}\left[\sin^2\left(\cancel{\omega}\frac{2\pi}{\cancel{\omega}}\right) - \sin^2(0)\right] = \boxed{0}$$

(*b*) We apply Eq. 30-30, with $\phi = 90°$.

$$\overline{P} = I_{\text{rms}} V_{\text{rms}} \cos 90° = \boxed{0}$$

As expected the average power is the same for both methods of calculation.

82. Since the current lags the voltage one of the circuit elements must be an $\boxed{\text{inductor}}$. Since the angle is less than 90º, the other element must be a $\boxed{\text{resistor}}$. We use 30-29a to write the resistance in terms of the impedance. Then using Eq. 30-27 to determine the impedance from the voltage and current and Eq. 30-28b, we solve for the unknown inductance and resistance.

$$\tan\phi = \frac{2\pi fL}{R} \rightarrow R = 2\pi fL\cot\phi$$

$$Z = \frac{V_{\text{rms}}}{I_{\text{rms}}} = \sqrt{R^2 + (2\pi fL)^2} = \sqrt{(2\pi fL\cot\phi)^2 + (2\pi fL)^2} = 2\pi fL\sqrt{1+\cot^2\phi}$$

$$L = \frac{V_{\text{rms}}}{2\pi f\, I_{\text{rms}}\sqrt{1+\cot^2\phi}} = \frac{120\text{V}}{2\pi(60\text{Hz})(5.6\text{A})\sqrt{1+\cot^2 65°}} = 51.5\text{mH} \approx \boxed{52\text{mH}}$$

$$R = 2\pi f\, L\cot\phi = 2\pi(60\text{Hz})(51.5\text{mH})\cot 65° = \boxed{9.1\Omega}$$

83. We use Eq. 30-28b to calculate the impedance at 60 Hz. Then we double that result and solve for the required frequency.

$$Z_0 = \sqrt{R^2 + (2\pi f_0 L)^2} = \sqrt{(3500\Omega)^2 + \left[2\pi(60\text{Hz})(0.44\text{H})\right]^2} = 3504\Omega$$

$$2Z_0 = \sqrt{R^2 + (2\pi fL)^2} \rightarrow f = \frac{\sqrt{4Z_0^2 - R^2}}{2\pi L} = \frac{\sqrt{4(3504\Omega)^2 - (3500\Omega)^2}}{2\pi(0.44\text{H})} = \boxed{2.2\text{kHz}}$$

84. (*a*) We calculate capacitive reactance using Eq. 30-25b. Then using the resistance and capacitive reactance we calculate the impedance. Finally, we use Eq. 30-27 to calculate the rms current.

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(60.0\text{Hz})(1.80\times10^{-6}\text{F})} = 1474\Omega$$

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{(5700\Omega)^2 + (1474\Omega)^2} = 5887\Omega$$

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{120\,\text{V}}{5887\,\Omega} = 20.38\,\text{mA} \approx \boxed{20.4\,\text{mA}}$$

(*b*) We calculate the phase angle using Eq. 30-29a.

$$\phi = \tan^{-1}\frac{-X_C}{R} = \tan^{-1}\frac{-1474\,\Omega}{5700\,\Omega} = \boxed{-14.5^\circ}$$

(*c*) The average power is calculated using Eq. 30-30.

$$\bar{P} = I_{\text{rms}}V_{\text{rms}}\cos\phi = (0.0204\,\text{A})(120\,\text{V})\cos(-14.5^\circ) = \boxed{2.37\,\text{W}}$$

(*d*) The voltmeter will read the rms voltage across each element. We calculate the rms voltage by multiplying the rms current through the element by the resistance or capacitive reactance.

$$V_R = I_{\text{rms}}R = (20.38\,\text{mA})(5.70\,\text{k}\Omega) = \boxed{116\,\text{V}}$$

$$V_C = I_{\text{rms}}X_C = (20.38\,\text{mA})(1474\,\Omega) = \boxed{30.0\,\text{V}}$$

Note that since the voltages are out of phase they do not sum to the applied voltage. However, since they are 90º out of phase their squares sum to the square of the input voltage.

85. We find the resistance using Ohm's law with the dc voltage and current. When then calculate the impedance from the ac voltage and current, and using Eq. 30-28b.

$$R = \frac{V}{I} = \frac{45\,\text{V}}{2.5\,\text{A}} = \boxed{18\,\Omega}\ ;\ Z = \frac{V_{\text{rms}}}{I_{\text{rms}}} = \frac{120\,\text{V}}{3.8\,\text{A}} = 31.58\,\Omega$$

$$\sqrt{R^2 + (2\pi fL)^2} \to L = \frac{\sqrt{Z^2 - R^2}}{2\pi f} = \frac{\sqrt{(31.58\,\Omega)^2 - (18\,\Omega)^2}}{2\pi(60\,\text{Hz})} = \boxed{69\,\text{mH}}$$

86. (*a*) From the text of the problem, the Q factor is the ratio of the voltage across the capacitor or inductor to the voltage across the resistor, at resonance. The resonant frequency is given by Eq. 30-32.

$$Q = \frac{V_L}{V_R} = \frac{I_{\text{res}}X_L}{I_{\text{res}}R} = \frac{2\pi f_0 L}{R} = \frac{2\pi\frac{1}{2\pi}\sqrt{\frac{1}{LC}}L}{R} = \boxed{\frac{1}{R}\sqrt{\frac{L}{C}}}$$

(*b*) Find the inductance from the resonant frequency, and the resistance from the Q factor.

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} \quad \to$$

$$L = \frac{1}{4\pi^2 C f_0^2} = \frac{1}{4\pi^2(1.0\times10^{-8}\,\text{F})(1.0\times10^{6}\,\text{Hz})^2} = 2.533\times10^{-6}\,\text{H} \approx \boxed{2.5\times10^{-6}\,\text{H}}$$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} \quad \to \quad R = \frac{1}{Q}\sqrt{\frac{L}{C}} = \frac{1}{350}\sqrt{\frac{2.533\times10^{-6}\,\text{H}}{1.0\times10^{-8}\,\text{F}}} = \boxed{4.5\times10^{-2}\,\Omega}$$

87. We calculate the period of oscillation as 2π divided by the angular frequency. Then set the total energy of the system at the beginning of each cycle equal to the charge on the capacitor as given by Eq. 24-5, with the charge given by Eq. 30-19, with $\cos(\omega' t + \phi) = \cos[\omega'(t+T) + \phi] = 1$. We take the difference in energies at the beginning and end of a cycle, divided by the initial energy. For small damping, the argument of the resulting exponential term is small and we replace it with the first two terms of the Taylor series expansion.

$$T = \frac{2\pi}{\omega'} \approx \frac{2\pi}{\omega} \qquad U_{\max} = \frac{Q_0^2 e^{-\frac{R}{L}t}\cos^2(\omega' t + \phi)}{2C} = \frac{Q_0^2 e^{-\frac{R}{L}t}}{2C}$$

$$\frac{\Delta U}{U} = \frac{Q_0^2 e^{-\frac{R}{L}t} - Q_0^2 e^{-\frac{R}{L}\left(t+\frac{2\pi}{\omega}\right)}}{Q_0^2 e^{-\frac{R}{L}t}} = 1 - e^{-\frac{2\pi R}{\omega L}} \approx 1 - \left(1 - \frac{2\pi R}{\omega L}\right) = \frac{2\pi R}{\omega L} = \boxed{\frac{2\pi}{Q}}$$

88. We set the power factor equal to the resistance divided by the impedance (Eq. 30-28a) with the impedance written in terms of the angular frequency (Eq. 30-28b). We rearrange the resulting equation to form a quadratic equation in terms of the angular frequency. We divide the positive angular frequencies by 2π to determine the desired frequencies.

$$\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \quad \rightarrow \quad \omega^2 LC \pm \omega C\sqrt{R^2\left(\frac{1}{\cos^2\phi} - 1\right)} - 1 = 0$$

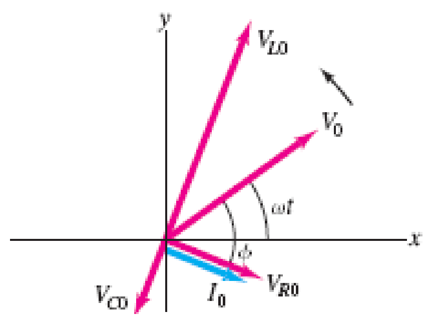
$$\omega^2(0.033\,\text{H})(55\times10^{-9}\,\text{F}) \pm \omega(55\times10^{-9}\,\text{F})\sqrt{(1500\,\Omega)^2\left(\frac{1}{0.17^2} - 1\right)} - 1 = 0$$

$$(1.815\times10^{-9}\,\text{F}\bullet\text{H})\omega^2 \pm (4.782\times10^{-4}\,\Omega\bullet\text{F})\omega - 1 = 0$$

$$\omega = \frac{\pm 4.78225\times10^{-4}\,\Omega\bullet\text{F} \pm 4.85756\times10^{-4}\,\Omega\bullet\text{F}}{3.63\times10^{-9}\,\text{F}\bullet\text{H}} = \pm 2.65\times10^5\,\text{rad/s},\ \pm 2.07\times10^3\,\text{rad/s}$$

$$f = \frac{\omega}{2\pi} = \frac{2.65\times10^5\,\text{rad/s}}{2\pi} = \boxed{42\,\text{kHz}} \quad \text{and} \quad \frac{2.07\times10^3\,\text{rad/s}}{2\pi} = \boxed{330\,\text{Hz}}$$

89. (*a*) We set $V = V_0 \sin\omega t$ and assume the inductive reactance is greater than the capacitive reactance. The current will lag the voltage by an angle ϕ. The voltage across the resistor is in phase with the current and the voltage across the inductor is 90º ahead of the current. The voltage across the capacitor is smaller than the voltage in the inductor, and antiparallel to it.



(*b*) From the diagram, the current is the projection of the maximum current onto the *y* axis, with the current lagging the voltage by the angle ϕ. This is the same angle obtained in Eq. 30-29a. The magnitude of the maximum current is the voltage divided by the impedance, Eq. 30-28b.

$$I(t) = I_0 \sin(\omega t - \phi) = \boxed{\frac{V_0}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}\sin(\omega t - \phi)} \ ; \ \phi = \tan^{-1}\frac{\omega L - 1/\omega C}{R}$$

90. (*a*) We use Eq. 30-28b to calculate the impedance and Eq. 30-29a to calculate the phase angle.

$$X_L = \omega L = (754\,\text{rad/s})(0.0220\,\text{H}) = 16.59\,\Omega$$

$$X_C = 1/\omega C = 1\big/(754\,\text{rad/s})(0.42\times10^{-6}\,\text{F}) = 3158\,\Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(23.2\times10^3\,\Omega)^2 + [16.59\,\Omega - 3158\,\Omega]^2} = \boxed{23.4\,\text{k}\Omega}$$

$$\phi = \tan^{-1}\frac{X_L - X_C}{R} = \tan^{-1}\frac{16.59\,\Omega - 3158\,\Omega}{23.2\times10^3\,\Omega} = \boxed{-7.71^\circ}$$

(*b*) We use Eq. 30-30 to obtain the average power. We obtain the rms voltage by dividing the maximum voltage by $\sqrt{2}$. The rms current is the rms voltage divided by the impedance.

$$\bar{P} = I_{\text{rms}}V_{\text{rms}}\cos\phi = \frac{V_{\text{rms}}^2}{Z}\cos\phi = \frac{V_0^2}{2Z}\cos\phi = \frac{(0.95\,\text{V})^2}{2(23.4\times10^3\,\Omega)}\cos(-7.71°) = \boxed{19\,\mu\text{W}}$$

(*c*) The rms current is the peak voltage, divided by $\sqrt{2}$, and then divided by the impedance.

$$I_{\text{rms}} = \frac{V_0/\sqrt{2}}{Z} = \frac{0.95\,\text{V}/\sqrt{2}}{23.4\times10^3\,\Omega} = 2.871\times10^{-5}\,\text{A} \approx \boxed{29\,\mu\text{A}}$$

The rms voltage across each element is the rms current times the resistance or reactance of the element.

$$V_R = I_{\text{rms}}R = (2.871\times10^{-5}\,\text{A})(23.2\times10^3\,\Omega) = \boxed{0.67\,\text{V}}$$

$$V_C = I_{\text{rms}}X_C = (2.871\times10^{-5}\,\text{A})(3158\,\Omega) = \boxed{0.091\,\text{V}}$$

$$V_L = I_{\text{rms}}X_L = (2.871\times10^{-5}\,\text{A})(16.59\,\Omega) = \boxed{4.8\times10^{-4}\,\text{V}}$$

91. (*a*) The impedance of the circuit is given by Eq. 30-28b with $X_L > X_C$ and $R = 0$. We divide the magnitude of the ac voltage by the impedance to get the magnitude of the ac current in the circuit. Since $X_L > X_C$, the voltage will lead the current by $\phi = \pi/2$. No dc current will flow through the capacitor.

$$Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2} = \omega L - 1/\omega C \qquad I_0 = \frac{V_{20}}{Z} = \frac{V_{20}}{\omega L - 1/\omega C}$$

$$I(t) = \boxed{\frac{V_{20}}{\omega L - 1/\omega C}\sin(\omega t - \pi/2)}$$

(*b*) The voltage across the capacitor at any instant is equal to the charge on the capacitor divided by the capacitance. This voltage is the sum of the ac voltage and dc voltage. There is no dc voltage drop across the inductor so the dc voltage drop across the capacitor is equal to the input dc voltage.

$$V_{\text{out,ac}} = V_{\text{out}} - V_1 = \frac{Q}{C} - V_1$$

We treat the emf as a superposition of the ac and dc components. At any instant of time the sum of the voltage across the inductor and capacitor will equal the input voltage. We use Eq. 30-5 to calculate the voltage drop across the inductor. Subtracting the voltage drop across the inductor from the input voltage gives the output voltage. Finally, we subtract off the dc voltage to obtain the ac output voltage.

$$V_L = L\frac{dI}{dt} = L\frac{d}{dt}\left[\frac{V_{20}}{\omega L - 1/\omega C}\sin(\omega t - \pi/2)\right] = \frac{V_{20}L\omega}{\omega L - 1/\omega C}\cos(\omega t - \pi/2)$$

$$= \frac{V_{20}L\omega}{\omega L - 1/\omega C}\sin(\omega t)$$

$$V_{\text{out}} = V_{\text{in}} - V_L = V_1 + V_{20}\sin\omega t - \left(\frac{V_{20}L\omega}{\omega L - 1/\omega C}\sin(\omega t)\right)$$

$$= V_1 + V_{20}\left(1 - \frac{L\omega}{\omega L - 1/\omega C}\right)\sin(\omega t) = V_1 - V_{20}\left(\frac{1/\omega C}{\omega L - 1/\omega C}\right)\sin(\omega t)$$

$$V_{\text{out,ac}} = V_{\text{out}} - V_1 = -V_{20}\left(\frac{1/\omega C}{\omega L - 1/\omega C}\right)\sin(\omega t) = \boxed{\left(\frac{V_{20}}{\omega^2 LC - 1}\right)\sin(\omega t - \pi)}$$

(*c*) The attenuation of the ac voltage is greatest when the denominator is large.

$$\omega^2 LC \gg 1 \to \omega L \gg \frac{1}{\omega C} \to X_L \gg X_C$$

We divide the output ac voltage by the input ac voltage to obtain the attenuation.

$$\frac{V_{2,\text{out}}}{V_{2,\text{in}}} = \frac{\dfrac{V_{20}}{\omega^2 LC - 1}}{V_{20}} = \frac{1}{\omega^2 LC - 1} \approx \boxed{\frac{1}{\omega^2 LC}}$$

(*d*) The dc output is equal to the dc input, since there is no dc voltage drop across the inductor.

$$\boxed{V_{1,\text{out}} = V_1}$$

92. Since no dc current flows through the capacitor, there will be no dc current through the resistor. Therefore the dc voltage passes through the circuit with little attenuation. The ac current in the circuit is found by dividing the input ac voltage by the impedance (Eq. 30-28b) We obtain the output ac voltage by multiplying the ac current by the capacitive reactance. Dividing the result by the input ac voltage gives the attenuation.

$$V_{2,\text{out}} = IX_C = \frac{V_{20}X_C}{\sqrt{R^2 + X_C^2}} \quad \to \quad \frac{V_{2,\text{out}}}{V_{20}} = \frac{1}{\sqrt{R^2\omega^2C^2 + 1}} \approx \boxed{\frac{1}{R\omega C}}$$

93. (*a*) Since the three elements are connected in parallel, at any given instant in time they will all three have the same voltage drop across them. That is the voltages across each element will be in phase with the source. The current in the resistor is in phase with the voltage source with magnitude given by Ohm's law.

$$I_R(t) = \boxed{\frac{V_0}{R}\sin\omega t}$$

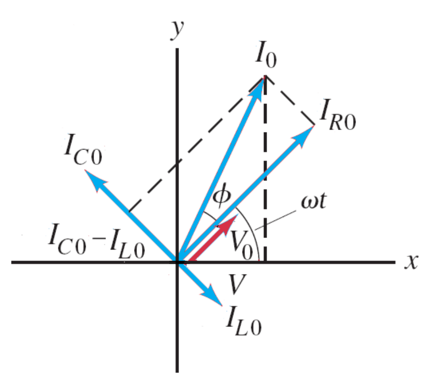
(*b*) The current through the inductor will lag behind the voltage by π/2, with magnitude equal to the voltage source divided by the inductive reactance.

$$I_L(t) = \boxed{\frac{V_0}{X_L}\sin\left(\omega t - \frac{\pi}{2}\right)}$$

(*c*) The current through the capacitor leads the voltage by π/2, with magnitude equal to the voltage source divided by the capacitive reactance.

$$I_C(t) = \boxed{\frac{V_0}{X_C}\sin\left(\omega t + \frac{\pi}{2}\right)}$$

(*d*) The total current is the sum of the currents through each element. We use a phasor diagram to add the currents, as was used in Section 30-8 to add the voltages with different phases. The net current is found by subtracting the current through the inductor from the current through the capacitor. Then using the Pythagorean theorem to add the current through the resistor. We use the tangent function to find the phase angle between the current and voltage source.



$$I_0 = \sqrt{I_{R0}^2 + (I_{C0} - I_{L0})^2} = \sqrt{\left(\frac{V_0}{R}\right)^2 + \left(\frac{V_0}{X_C} - \frac{V_0}{X_L}\right)^2} = \frac{V_0}{R}\sqrt{1 + \left(R\omega C - \frac{1}{R\omega L}\right)^2}$$

$$I(t) = \boxed{\frac{V_0}{R}\sqrt{1+\left(R\omega C - \frac{R}{\omega L}\right)^2}\sin(\omega t + \phi)}$$

$$\tan\phi = \frac{\dfrac{V_0}{X_C} - \dfrac{V_0}{X_L}}{\dfrac{V_0}{R}} \to \phi = \tan^{-1}\left(\frac{R}{X_C} - \frac{R}{X_L}\right) = \boxed{\tan^{-1}\left(R\omega C - \frac{R}{\omega L}\right)}$$

(*e*) We divide the magnitude of the voltage source by the magnitude of the current to find the impedance.

$$Z = \frac{V_0}{I_0} = \frac{V_0}{\dfrac{V_0}{R}\sqrt{1+\left(R\omega C - \dfrac{R}{\omega L}\right)^2}} = \boxed{\frac{R}{\sqrt{1+\left(R\omega C - \dfrac{R}{\omega L}\right)^2}}}$$

(*f*) The power factor is the ratio of the power dissipated in the circuit divided by the product of the rms voltage and current.

$$\frac{I_{R,\text{rms}}^2 R}{V_{\text{rms}} I_{\text{rms}}} = \frac{I_R^2 R}{V_0 I_0} = \frac{\left(\dfrac{V_0}{R}\right)^2 R}{V_0 \dfrac{V_0}{R}\sqrt{1+\left(R\omega C - \dfrac{R}{\omega L}\right)^2}} = \boxed{\frac{1}{\sqrt{1+\left(R\omega C - \dfrac{R}{\omega L}\right)^2}}}$$

94. We find the equivalent values for each type of element in series. From the equivalent values we calculate the impedance using Eq. 30-28b.

$$R_{\text{eq}} = R_1 + R_2 \qquad \frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} \qquad L_{\text{eq}} = L_1 + L_2$$

$$Z = \sqrt{R_{\text{eq}}^2 + \left(\omega L_{\text{eq}} - \frac{1}{\omega C_{\text{eq}}}\right)^2} = \boxed{\sqrt{(R_1 + R_2)^2 + \left(\omega L_1 + \omega L_2 - \frac{1}{\omega C_1} - \frac{1}{\omega C_2}\right)^2}}$$

95. If there is no current in the secondary, there will be no induced emf from the mutual inductance. Therefore, we set the ratio of the voltage to current equal to the inductive reactance and solve for the inductance.

$$\frac{V_{\text{rms}}}{I_{\text{rms}}} = X_L = 2\pi f L \to L = \frac{V_{\text{rms}}}{2\pi f I_{\text{rms}}} = \frac{220\text{ V}}{2\pi(60\text{ Hz})(4.3\text{ A})} = \boxed{0.14\text{ H}}$$

96. (*a*) We use Eq. 24-2 to calculate the capacitance, assuming a parallel plate capacitor.

$$C = \frac{K\varepsilon_0 A}{d} = \frac{(5.0)\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)\left(1.0\times10^{-4}\,\text{m}^2\right)}{2.0\times10^{-3}\,\text{m}} = 2.213\times10^{-12}\,\text{F} \approx \boxed{2.2\,\text{pF}}$$

(*b*) We use Eq. 30-25b to calculate the capacitive reactance.

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(12000\,\text{Hz})\left(2.2\times10^{-12}\,\text{F}\right)} = 5.995\times10^6\,\Omega \approx \boxed{6.0\,\text{M}\Omega}$$

(*c*) Assuming that the resistance in the plasma and in the person is negligible compared with the capacitive reactance, calculate the current by dividing the voltage by the capacitive reactance.

$$I_\text{o} \approx \frac{V_\text{o}}{X_C} = \frac{2500\text{ V}}{5.995\times10^6\,\Omega} = 4.17\times10^{-4}\,\text{A} \approx \boxed{0.42\,\text{mA}}$$

This is not a dangerous current level.

(*d*) We replace the frequency with 1.0 MHz and recalculate the current.

$$I_0 \approx \frac{V_0}{X_C} = 2\pi fCV_0 = 2\pi\left(1.0\times10^6\,\text{Hz}\right)\left(2.2\times10^{-12}\,\text{F}\right)\left(2500\,\text{V}\right) = \boxed{35\,\text{mA}}$$

This current level is dangerous.

97. We calculate the resistance from the power dissipated and the current. Then setting the ratio of the voltage to current equal to the impedance, we solve for the inductance.

$$\overline{P} = I_{\text{rms}}^2 R \rightarrow R = \frac{\overline{P}}{I_{\text{rms}}^2} = \frac{350\,\text{W}}{\left(4.0\,\text{A}\right)^2} = 21.88\,\Omega \approx \boxed{22\,\Omega}$$

$$Z = \frac{V_{\text{rms}}}{I_{\text{rms}}} = \sqrt{R^2 + \left(2\pi fL\right)^2} \quad \rightarrow$$

$$L = \frac{\sqrt{\left(V_{\text{rms}}/I_{\text{rms}}\right)^2 - R^2}}{2\pi f} = \frac{\sqrt{\left(120\,\text{V}/4.0\,\text{A}\right)^2 - \left(21.88\,\Omega\right)^2}}{2\pi\left(60\,\text{Hz}\right)} = \boxed{54\,\text{mH}}$$

98. We insert the proposed current into the differential equation and solve for the unknown peak current and phase.

$$\begin{aligned}
V_0 \sin\omega t &= L\frac{d}{dt}\left[I_0 \sin\left(\omega t - \phi\right)\right] + RI_0 \sin\left(\omega t - \phi\right) \\
&= L\omega I_0 \cos\left(\omega t - \phi\right) + RI_0 \sin\left(\omega t - \phi\right) \\
&= L\omega I_0 \left(\cos\omega t\cos\phi + \sin\omega t\sin\phi\right) + RI_0\left(\sin\omega t\cos\phi - \cos\omega t\sin\phi\right) \\
&= \left(L\omega I_0\cos\phi - RI_0\sin\phi\right)\cos\omega t + \left(L\omega I_0\sin\phi + RI_0\cos\phi\right)\sin\omega t
\end{aligned}$$

For the given equation to be a solution for all time, the coefficients of the sine and cosine terms must independently be equal.

For the $\cos\omega t$ term:

$$0 = L\omega I_0\cos\phi - RI_0\sin\phi \rightarrow \tan\phi = \frac{\omega L}{R} \rightarrow \boxed{\phi = \tan^{-1}\frac{\omega L}{R}}$$

For the $\sin\omega t$ term:

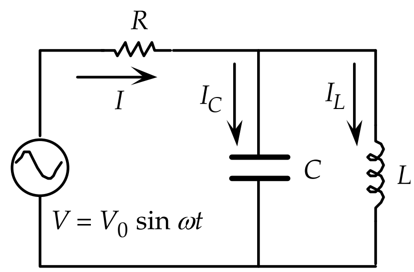
$$V_0 = L\omega I_0\sin\phi + RI_0\cos\phi$$

$$I_0 = \frac{V_0}{L\omega\sin\phi + R\cos\phi} = \frac{V_0}{L\omega\frac{\omega L}{\sqrt{R^2+\omega^2L^2}} + R\frac{R}{\sqrt{R^2+\omega^2L^2}}} = \boxed{\frac{V_0}{\sqrt{R^2+\omega^2L^2}}}$$

99. The peak voltage across either element is the current through the element multiplied by the reactance. We set the voltage across the inductor equal to six times the voltage across the capacitor and solve for the frequency in terms of the resonant frequency, Eq. 30-14.

$$V_L = I_0 2\pi fC = 6V_C = \frac{6I_0}{2\pi fC} \quad \rightarrow \quad \rightarrow f = \frac{1}{2\pi}\sqrt{\frac{6}{LC}} = \boxed{\sqrt{6}f_0}$$

100. We use Kirchhoff's junction rule to write an equation relating the currents in each branch, and the loop rule to write two equations relating the voltage drops around each loop. We write the voltage drops across the capacitor and inductor in terms of the charge and derivative of the current.



$$I_R = I_L + I_C$$

$$V_0 \sin \omega t - I_R R - \frac{Q_C}{C} = 0 \;\; ; \;\; V_0 \sin \omega t - I_R R - L\frac{dI_L}{dt} = 0$$

We combine these equations to eliminate the charge in the capacitor and the current in the inductor to write a single differential equation in terms of the current through the resistor.

$$\frac{dI_L}{dt} = \frac{V_0}{L}\sin \omega t - \frac{I_R R}{L}$$

$$\frac{dI_C}{dt} = \frac{d^2 Q_C}{dt^2} = \frac{d^2}{dt^2}\left(CV_0 \sin \omega t - I_R RC\right) = -CV_0\omega^2 \sin \omega t - RC\frac{dI_R{}^2}{dt^2}$$

$$\frac{dI_R}{dt} = \frac{dI_L}{dt} + \frac{dI_C}{dt} = -\frac{V_0}{L}\sin \omega t + \frac{I_R R}{L} - CV_0\omega^2 \sin \omega t - RC\frac{dI_R{}^2}{dt^2}$$

We set the current in the resistor, $I_R = I_0 \sin\left(\omega t + \phi\right) = I_0\left(\sin \omega t \cos \phi + \cos \omega t \sin \phi\right)$, equal to the current provided by the voltage source and take the necessary derivatives.

$$I_0\frac{d}{dt}\left(\sin \omega t \cos \phi + \cos \omega t \sin \phi\right) = \frac{V_0}{L}\sin \omega t - \left(\sin \omega t \cos \phi + \cos \omega t \sin \phi\right)\frac{I_0 R}{L} - CV_0\omega^2 \sin \omega t$$
$$- RCI_0\frac{d^2}{dt^2}\left(\sin \omega t \cos \phi + \cos \omega t \sin \phi\right)$$

$$I_0\omega \cos \omega t \cos \phi - I_0\omega \sin \omega t \sin \phi = \frac{V_0}{L}\sin \omega t - \frac{I_0 R}{L}\sin \omega t \cos \phi + \frac{I_0 R}{L}\cos \omega t \sin \phi - CV_0\omega^2 \sin \omega t$$
$$+ RCI_0\omega^2 \sin \omega t \cos \phi + RCI_0\omega^2 \cos \omega t \sin \phi$$

Setting the coefficients of the time dependent sine and cosine terms separately equal to zero enables us to solve for the magnitude and phase of the current through the voltage source. We also use Eq. 30-23b and Eq. 30-25b to write the inductance and capacitance in terms of their respective reactances.

From the $\cos(\omega t)$ term:

$$I_0\omega \cos \phi = \frac{I_0\omega R}{X_L}\sin \phi - \frac{I_0 R\omega}{X_C}\sin \phi \to \tan \phi = \frac{X_L X_C}{R\left(X_L - X_C\right)} \to \phi = \tan^{-1}\left[\frac{X_L X_C}{R\left(X_L - X_C\right)}\right]$$

From the $\sin(\omega t)$ term:

$$-I_0\omega \sin \phi = \frac{V_0\omega}{X_L} - \frac{I_0\omega R}{X_L}\cos \phi - \frac{V_0\omega}{X_C} + \frac{RI_0\omega}{X_C}\cos \phi$$

$$I_0 = \frac{V_0\left(X_C - X_L\right)}{X_C X_L \sin \phi + R\left(X_C - X_L\right)\cos \phi}$$

$$= \frac{V_0\left(X_C - X_L\right)}{X_C X_L\dfrac{X_C X_L}{\sqrt{\left(X_C X_L\right)^2 + R^2\left(X_C - X_L\right)^2}} + R\left(X_C - X_L\right)\dfrac{R\left(X_C - X_L\right)}{\sqrt{\left(X_C X_L\right)^2 + R^2\left(X_C - X_L\right)^2}}}$$

$$= \frac{V_0\left(X_C - X_L\right)}{\sqrt{\left(X_C X_L\right)^2 + R^2\left(X_C - X_L\right)^2}}$$

This gives us the current through the power source and resistor. We insert these values back into the junction and loop equations to determine the current in each element as a function of time. We calculate the impedance of the circuit by dividing the peak voltage by the peak current through the voltage source.

$$Z = \frac{V_0}{I_0} = \boxed{\frac{\sqrt{\left(X_C X_L\right)^2 + R^2\left(X_C - X_L\right)^2}}{\left(X_C - X_L\right)}} \;;\; \phi = \boxed{\tan^{-1}\left[\frac{X_L X_C}{R\left(X_L - X_C\right)}\right]} \;;\; I_R = \boxed{\frac{V_0}{Z}\sin\left(\omega t + \phi\right)}$$

$$I_C = \frac{dQ_C}{dt} = \frac{d}{dt}\left(CV_0 \sin\omega t - I_R RC\right) = CV_0\omega\cos\omega t - RC\frac{dI_R}{dt}$$

$$= \boxed{\frac{V_0}{X_C}\left[\cos\omega t - \frac{R}{Z}\cos\left(\omega t + \phi\right)\right]}$$

$$I_L = I_R - I_C = \frac{V_0}{Z}\sin\left(\omega t + \phi\right) - \frac{V_0}{X_C}\left[\cos\omega t - \frac{R}{Z}\cos\left(\omega t + \phi\right)\right]$$

$$= \boxed{\frac{V_0}{Z}\left[\sin\left(\omega t + \phi\right) + \frac{R}{X_C}\cos\left(\omega t + \phi\right)\right] - \frac{V_0}{X_C}\cos\omega t}$$

101. (*a*) The resonant frequency is given by Eq. 30-32. At resonance, the impedance is equal to the resistance, so the rms voltage of the circuit is equal to the rms voltage across the resistor.

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{\left(0.0050\,\text{H}\right)\left(0.10\times10^{-6}\,\text{F}\right)}} = 7118\,\text{Hz} \approx \boxed{7.1\,\text{kHz}}$$

$$\left(V_R\right)_{\text{rms}} = \boxed{V_{\text{rms}}}$$

(*b*) We set the inductance equal to 90% of the initial inductance and use Eq. 30-28b to calculate the new impedance. Dividing the rms voltage by the impedance gives the rms current. We multiply the rms current by the resistance to determine the voltage drop across the resistor.

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi\left(7118\,\text{Hz}\right)\left(0.10\times10^{-6}\,\text{F}\right)} = 223.6\,\Omega$$

$$X_L = 2\pi fL = 2\pi\left(7118\,\text{Hz}\right)\left(0.90\right)\left(0.0050\,\text{H}\right) = 201.3\,\Omega$$

$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2} = \sqrt{\left(45\,\Omega\right)^2 + \left(201.3\,\Omega - 223.6\,\Omega\right)^2} = 50.24\,\Omega$$

$$\left(V_R\right)_{\text{rms}} = \left(\frac{R}{Z}\right)V_{\text{rms}} = \left(\frac{45\,\Omega}{50.24\,\Omega}\right)V_{\text{rms}} = \boxed{0.90V_{\text{rms}}}$$

102. With the given applied voltage, calculate the rms current through each branch as the rms voltage divided by the impedance in that branch.

$$I_{C,\text{rms}} = \frac{V_{\text{rms}}}{\sqrt{R_1^2 + X_C^2}} \qquad I_{L,\text{rms}} = \frac{V_{\text{rms}}}{\sqrt{R_2^2 + X_L^2}}$$

Calculate the potential difference between points a and b in two ways. First pass through the capacitor and then through R_2. Then pass through R_1 and the inductor.

$$V_{\text{ab}} = I_C X_C - I_L R_2 = \frac{V_{\text{rms}} X_C}{\sqrt{R_1^2 + X_C^2}} - \frac{V_{\text{rms}} R_2}{\sqrt{R_2^2 + X_L^2}}$$

$$V_{ab} = -I_C R_1 + X_L I_L = -\frac{V_{rms} R_1}{\sqrt{R_1^2 + X_C^2}} + \frac{V_{rms} X_L}{\sqrt{R_2^2 + X_L^2}}$$

Set these voltage differences equal to zero, and rearrange the equations.

$$\frac{V_{rms} X_C}{\sqrt{R_1^2 + X_C^2}} - \frac{V_{rms} R_2}{\sqrt{R_2^2 + X_L^2}} = 0 \to X_C\sqrt{R_2^2 + X_L^2} = R_2\sqrt{R_1^2 + X_C^2}$$

$$-\frac{V_{rms} R_1}{\sqrt{R_1^2 + X_C^2}} + \frac{V_{rms} X_L}{\sqrt{R_2^2 + X_L^2}} = 0 \to R_1\sqrt{R_2^2 + X_L^2} = X_L\sqrt{R_1^2 + X_C^2}$$

Divide the resulting equations and solve for the product of the resistances. Write the reactances in terms of the capacitance and inductance to show that the result is frequency independent.

$$\frac{X_C\sqrt{R_2^2 + X_L^2}}{R_1\sqrt{R_2^2 + X_L^2}} = \frac{R_2\sqrt{R_1^2 + X_C^2}}{X_L\sqrt{R_1^2 + X_C^2}} \to R_1 R_2 = X_L X_C = \frac{\omega L}{\omega C} \to \boxed{R_1 R_2 = \frac{L}{C}}$$

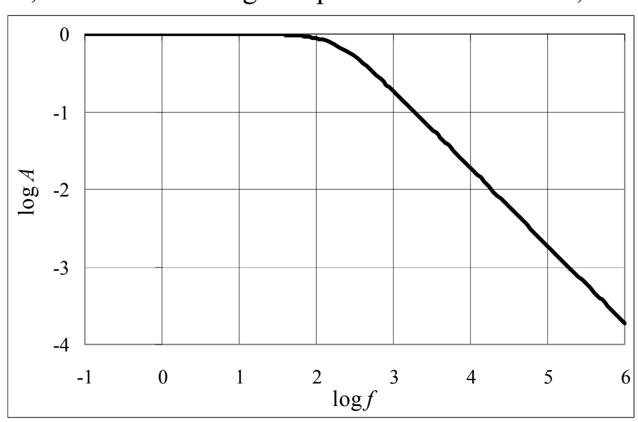
103. (*a*) The output voltage is the voltage across the capacitor, which is the current through the circuit multiplied by the capacitive reactance. We calculate the current by dividing the input voltage by the impedance. Finally, we divide the output voltage by the input voltage to calculate the gain.

$$V_{out} = IX_C = \frac{V_{in} X_C}{\sqrt{R^2 + X_C^2}} = \frac{V_{in}}{\sqrt{(R/X_C)^2 + 1}} = \frac{V_{in}}{\sqrt{(2\pi fCR)^2 + 1}}$$

$$A = \frac{V_{out}}{V_{in}} = \boxed{\frac{1}{\sqrt{4\pi^2 f^2 C^2 R^2 + 1}}}$$

(*b*) As the frequency goes to zero, the gain becomes one. In this instance the capacitor becomes fully charged, so no current flows across the resistor. Therefore the output voltage is equal to the input voltage. As the frequency becomes very large, the capacitive reactance becomes very small, allowing a large current. In this case, most of the voltage drop is across the resistor, and the gain goes to zero.

(*c*) See the graph of the log of the gain as a function of the log of the frequency. Note that for frequencies less than about 100 Hz the gain is ~ 1. For higher frequencies the gain drops off proportionately to the frequency. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH30.XLS," on tab "Problem 30.103c."



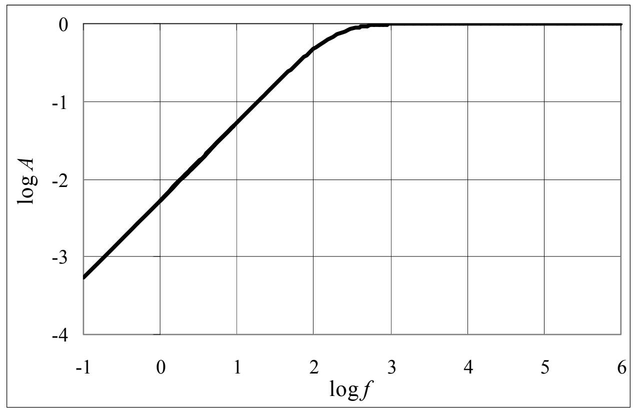
104. (*a*) The output voltage is the voltage across the resistor, which is the current through the circuit multiplied by the resistance. We calculate the current by dividing the input voltage by the impedance. Finally, we divide the output voltage by the input voltage to calculate the gain.

$$V_{out} = IR = \frac{V_{in} R}{\sqrt{R^2 + X_C^2}} = \frac{V_{in} R}{\sqrt{R^2 + \left(\frac{1}{2\pi fC}\right)^2}} = \frac{2\pi fCRV_{in}}{\sqrt{(2\pi fCR)^2 + 1}}$$

$$A = \frac{V_{\text{out}}}{V_{\text{in}}} = \boxed{\frac{2\pi fCR}{\sqrt{4\pi^2 f^2 C^2 R^2 + 1}}}$$

(*b*) As the frequency goes to zero, the gain drops to zero. In this instance the capacitor becomes fully charged, so no current flows across the resistor. Therefore the output voltage drops to zero. As the frequency becomes very large, the capacitive reactance becomes very small, allowing a large current. In this case, most of the voltage drop is across the resistor, and the gain is equal to unity.

(*c*) See the graph of the log of the gain as a function of the log of the frequency. Note that for frequencies greater than about 1000 Hz the gain is ~ 1. For lower frequencies the gain drops off proportionately to the inverse of the frequency. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH30.XLS," on tab "Problem 30.104c."



105. We calculate the resonant frequency using Eq. 30-32.

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\left(50\times10^{-6}\,\text{H}\right)\left(50\times10^{-6}\,\text{F}\right)}} = 20{,}000\,\text{rad/s}$$

Using a spreadsheet, we calculate the impedance as a function of frequency using Eq. 30-28b. We divide the rms voltage by the impedance to plot the rms current as a function of frequency. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH30.XLS," on tab "Problem 30.105."