CHAPTER 29: Electromagnetic Induction and Faraday's Law

Responses to Questions

1. Using coils with many (N) turns increases the values of the quantities to be experimentally measured, because the induced emf and therefore the induced current are proportional to N.

2. Magnetic flux is a quantitative measure of the number of magnetic field lines passing through a given area. It depends not only on the field itself, but also on the area and on the angle between the field and the area.

3. Yes, the current is induced clockwise. No, there is no induced current if the magnet is steady, because there is no changing flux through the ring. Yes, the current is induced counterclockwise.

4. There is no induced current in the loop that is moving parallel to the wire because there is no change of magnetic flux through the loop. The induced current in the loop moving away from the wire is clockwise. The magnetic field through the loop due to the current is directed into the page, and the loop is moving such that its distance from the wire is increasing, resulting in a decrease in magnetic field strength and therefore a decrease in magnetic flux through the loop. By Lenz's law, a decreasing magnetic flux into the page results in a clockwise induced current.

5. Yes. The force is attractive. The induced clockwise current in the right loop will induce a counterclockwise current in the left loop which will slow the relative motion of the loops.

6. (*a*) Yes.
   (*b*) The current starts as soon as the battery is connected and current begins to flow in the first loop.
   (*c*) The induced current stops as soon as the current in the first loop has reached its steady value.
   (*d*) The induced current in the second loop will be counterclockwise, in order to oppose the change.
   (*e*) While there is an induced current, there will be a force between the two loops.
   (*f*) The force will be repulsive, since the currents are in opposite directions.

7. Yes, a current will be induced in the second coil. It will start when the battery is disconnected from the first coil and stop when the current falls to zero in the first coil. The current in the second loop will be clockwise.

8. Counterclockwise. If the area of the loop decreases, the flux through the loop (directed out of the page) decreases. By Lenz's law, the resulting induced current will be counterclockwise to oppose the change. Another way to approach this question is to use the right-hand rule. As the bar moves to the left, the negative electrons in the bar will experience a force down, which results in a counterclockwise current.

9. (*a*) The current through R_A will be to the right. The field due to the current in coil B will be to the left. As coil B is moved toward coil A, the flux through A will increase, so the induced field in coil A will be to the right, to oppose the change. This field corresponds to an induced current flowing from left to right in R_A.
   (*b*) The current through R_A will be to the left. When coil B is moved away from coil A, the flux through coil A will decrease, so the induced field will be to the left, to oppose the change. This field corresponds to an induced current flowing from right to left in R_A.
   (*c*) If R_B is increased, the current in the circuit will decrease, decreasing the flux through coil A, resulting in a current through R_A to the left.

10. The shielding prevents external fields from inducing a current which would cause a false signal in the inner signal wire.

11. The currents in the two wires will be 180º out of phase. If they are very close together, or wrapped around each other, then the magnetic fields created by the currents in the wires will very nearly cancel each other.

12. The straight wire will fall faster. Since the magnetic field is non-uniform, the flux through the loop will change as the loop falls, inducing a current which will oppose the change and therefore resist the downward motion. Eddy currents will also be induced in the straight wire, but they will be much smaller since the straight wire does not form a closed loop.

13. (*a*) Yes. If a rapidly changing magnetic field exists outside, then currents will be induced in the metal sheet. These currents will create magnetic fields which will partially cancel the external fields.
(*b*) Yes. Since the metal sheet is permeable, it will partially shield the interior from the exterior static magnetic field; some of the magnetic field lines will travel through the metal sheet.
(*c*) The superconducting sheet will shield the interior from magnetic fields.

14. Each of the devices mentioned has a different operating current and voltage, and each needs its own transformer with its own ratio of primary to secondary turns designed to convert normal household current and voltage into the required current and voltage. If the devices were designed to operate with the same current and voltage, they could all run on identical transformers.

15. You could hook the transformer up to a known ac voltage source. The ratio of the output voltage to the input voltage will give the ratio of turns on the two coils. If you pair up the leads incorrectly (one lead from each coil, rather than both leads from the same coil), there will be no output voltage. Alternatively, you could attach an ohmmeter to two of the leads. The resistance will be infinite if you have one lead from each pair, and nearly zero if you have both leads from the same pair.

16. Higher voltages are inherently more dangerous because of the increased risk of establishing large currents and large electromagnetic fields. The large potential differences between the wires and the ground could cause arcing and short circuits, leading to accidental electrocutions. In addition, higher-voltage power lines will have higher electromagnetic fields associated with them than lower-voltage power lines. Biological effects of exposure to high electromagnetic fields are not well understood, but there is evidence of increased health risks to people who live close to high voltage power lines.

17. When the transformer is connected to the 120-V dc source no back emf is generated, as would happen with an ac source. Therefore, the current in the transformer connected to the dc source will be very large. Because transformers generally are made with fine, low resistance wires, the large current could cause the wires to overheat, melt the insulation, and burn out.

18. A motor uses electric energy to create mechanical energy. When a large electric motor is running, the current in the motor's coil creates a back emf. When the motor is first turned on, the back emf is small, allowing the motor to draw maximum current. The back emf has a maximum value when the motor is running at full speed, reducing the amount of current required to run the motor. As the current flow in the motor's coil stabilizes, the motor will operate at its lower, normal current. The lights will dim briefly when the refrigerator motor starts due to the increased current load on the house circuit. Electric heaters operate by sending a large current through a large resistance, generating heat. When an electric heater is turned on, the current will increase quickly to its

maximum value (no coil, so no back emf) and will stay at its maximum value as long as the heater is on. Therefore, the lights will stay dim as long as the heater is on.

19. At the moment shown in Figure 29-15, the armature is rotating clockwise and so the current in length *b* of the wire loop on the armature is directed outward. (Use the right-hand rule: the field is north to south and the wire is moving with a component downward, therefore force on positive charge carriers is out.) This current is increasing, because as the wire moves down, the downward component of the velocity increases. As the current increases, the flux through the loop also increases, and therefore there is an induced emf to oppose this change. The induced emf opposes the current flowing in section *b* of the wire, and therefore creates a counter-torque.

20. Eddy currents exist in any conducting material, so eddy current brakes could work with wheels made of copper or aluminum.

21. The nonferrous materials are not magnetic but they are conducting. As they pass by the permanent magnets, eddy currents will be induced in them. The eddy currents provide a "braking" mechanism which will cause the metallic materials to slide more slowly down the incline than the nonmetallic materials. The nonmetallic materials will reach the bottom with larger speeds. The nonmetallic materials can therefore be separated from the metallic, nonferrous materials by placing bins at different distances from the bottom of the incline. The closest bin will catch the metallic materials, since their projectile velocities off the end of the incline will be small. The bin for the nonmetallic materials should be placed farther away to catch the higher-velocity projectiles.

22. The slots in the metal bar prevent the formation of large eddy currents, which would slow the bar's fall through the region of magnetic field.

23. As the aluminum sheet is moved through the magnetic field, eddy currents are created in the sheet. The magnetic force on these induced currents opposes the motion. Thus it requires some force to pull the sheet out. (See Figure 29-21.)

24. As the bar magnet falls, it sets up eddy currents in the metal tube which will interact with the magnet and slow its fall. The magnet will reach terminal velocity (due to the interactions with the magnetic dipoles set up by the eddy currents, not air resistance) when the weight of the magnet is balanced by the upward force from the eddy currents.

25. As the bar moves in the magnetic field, induced eddy currents are created in the bar. The magnetic field exerts a force on these currents that opposes the motion of the bar. (See Figure 29-21.)

26. Although in principle you could use a loudspeaker in reverse as a microphone, it would probably not work in actual practice. The membrane of the microphone is very lightweight and sensitive to the sound waves produced by your voice. The cardboard cone of a loudspeaker is much stiffer and would significantly dampen the vibrations so that the frequency of the impinging sound waves would not be translated into an induced emf with the same frequency.

Solutions to Problems

1. The average induced emf is given by Eq. 29-2b.

$$\mathscr{E} = -N\frac{d\Phi_B}{dt} = -N\frac{\Delta\Phi_B}{\Delta t} = -2\frac{38\,\text{Wb} - (-58\,\text{Wb})}{0.42\,\text{s}} = \boxed{-460\,\text{V}}$$

2. As the magnet is pushed into the coil, the magnetic flux increases to the right. To oppose this increase, flux produced by the induced current must be to the left, so the induced current in the resistor will be $\boxed{\text{from right to left}}$.

3. As the coil is pushed into the field, the magnetic flux through the coil increases into the page. To oppose this increase, the flux produced by the induced current must be out of the page, so the induced current is $\boxed{\text{counterclockwise}}$.

4. The flux changes because the loop rotates. The angle between the field and the normal to the loop changes from 0° to 90°. The average induced emf is given by the "difference" version of Eq. 29-2b.

$$\mathscr{E}_{\text{avg}} = -\frac{\Delta\Phi_B}{\Delta t} = -\frac{AB\Delta\cos\theta}{\Delta t} = -\frac{\pi(0.110\,\text{m})^2\,1.5\,\text{T}(\cos 90^\circ - \cos 0^\circ)}{0.20\,\text{s}}$$

$$= -\frac{\pi(0.110\,\text{m})^2\,1.5\,\text{T}(0-1)}{0.20\,\text{s}} = \boxed{0.29\,\text{V}}$$

5. Use Eq. 29-2a to calculate the emf. Setting the flux equal to the magnetic field multiplied by the area of the loop, $A = \pi r^2$, and the emf equal to zero, we can solve for the rate of change in the coil radius.

$$\mathscr{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left(B\pi r^2\right) = -\frac{dB}{dt}\pi r^2 - 2\pi Br\frac{dr}{dt} = 0$$

$$\frac{dr}{dt} = -\frac{dB}{dt}\frac{r}{2B} = -(-0.010\text{ T/s})\frac{0.12\text{ m}}{2(0.500\text{ T})} = 0.0012\text{ m/s} = \boxed{1.2\text{ mm/s}}$$

6. We choose up as the positive direction. The average induced emf is given by the "difference" version of Eq. 29-2a.

$$\mathscr{E} = -\frac{\Delta\Phi_B}{\Delta t} = -\frac{A\Delta B}{\Delta t} = -\frac{\pi(0.054\,\text{m})^2(-0.25\,\text{T} - 0.68\,\text{T})}{0.16\,\text{s}} = \boxed{5.3\times10^{-2}\,\text{V}}$$

$\boxed{7.}$ (*a*) When the plane of the loop is perpendicular to the field lines, the flux is given by the maximum of Eq. 29-1a.

$$\Phi_B = BA = B\pi r^2 = (0.50\,\text{T})\pi(0.080\,\text{m})^2 = \boxed{1.0\times10^{-2}\,\text{Wb}}$$

(*b*) The angle is $\theta = \boxed{55^\circ}$

(*c*) Use Eq. 29-1a.

$$\Phi_B = BA\cos\theta = B\pi r^2 = (0.50\,\text{T})\pi(0.080\,\text{m})^2\cos 55^\circ = \boxed{5.8\times10^{-3}\,\text{Wb}}$$

8. (*a*) As the resistance is increased, the current in the outer loop will decrease. Thus the flux through the inner loop, which is out of the page, will decrease. To oppose this decrease, the induced current in the inner loop will produce a flux out of the page, so the direction of the induced current will be $\boxed{\text{counterclockwise}}$.

(*b*) If the small loop is placed to the left, the flux through the small loop will be into the page and will decrease. To oppose this decrease, the induced current in the inner loop will produce a flux into the page, so the direction of the induced current will be $\boxed{\text{clockwise}}$.

9. As the solenoid is pulled away from the loop, the magnetic flux to the right through the loop decreases. To oppose this decrease, the flux produced by the induced current must be to the right, so the induced current is $\boxed{\text{counterclockwise}}$ as viewed from the right end of the solenoid.

10. (*a*) The average induced emf is given by the "difference" version of Eq. 29-2b.

$$\mathscr{E} = -\frac{\Delta\Phi_B}{\Delta t} = -\frac{A\Delta B}{\Delta t} = -\frac{\pi(0.040\,\text{m})^2(-0.45\,\text{T} - 0.52\,\text{T})}{0.18\,\text{s}} = \boxed{2.7\times10^{-2}\,\text{V}}$$

(*b*) The positive result for the induced emf means the induced field is away from the observer, so the induced current is $\boxed{\text{clockwise}}$.

11. (*a*) The magnetic flux through the loop is into the paper and decreasing, because the area is decreasing. To oppose this decrease, the induced current in the loop will produce a flux into the paper, so the direction of the induced current will be $\boxed{\text{clockwise}}$.

(*b*) The average induced emf is given by the "difference" version of Eq. 29-2b.

$$\left|\mathscr{E}_{\text{avg}}\right| = \frac{\Delta\Phi_B}{\Delta t} = \frac{B|\Delta A|}{\Delta t} = \frac{(0.75\,\text{T})\pi\left[(0.100\,\text{m})^2 - (0.030\,\text{m})^2\right]}{0.50\,\text{s}}$$

$$= 4.288\times10^{-2}\,\text{V} \approx \boxed{4.3\times10^{-2}\,\text{V}}$$

(*c*) We find the average induced current from Ohm's law.

$$I = \frac{\mathscr{E}}{R} = \frac{4.288\times10^{-2}\,\text{V}}{2.5\,\Omega} = \boxed{1.7\times10^{-2}\,\text{A}}$$

12. As the loop is pulled from the field, the flux through the loop decreases, causing an induced EMF whose magnitude is given by Eq. 29-3, $\mathscr{E} = B\ell v$. Because the inward flux is decreasing, the induced flux will be into the page, so the induced current is clockwise, given by $I = \mathscr{E}/R$. Because this current in the left-hand side of the loop is in a downward magnetic field, there will be a magnetic force to the left. To keep the rod moving, there must be an equal external force to the right, given by $F = I\ell B$.

$$F = I\ell B = \frac{\mathscr{E}}{R}\ell B = \frac{B\ell v}{R}\ell B = \frac{B^2\ell^2 v}{R} = \frac{(0.650\,\text{T})^2(0.350\,\text{m})^2(3.40\,\text{m/s})}{0.280\,\Omega} = \boxed{0.628\,\text{N}}$$

$\boxed{13.}$ (*a*) Use Eq. 29-2a to calculate the emf induced in the ring, where the flux is the magnetic field multiplied by the area of the ring. Then using Eq. 25-7, calculate the average power dissipated in the ring as it is moved away. The thermal energy is the average power times the time.

$$\mathscr{E} = -\frac{\Delta\Phi_B}{\Delta t} = -\frac{\Delta BA}{\Delta t} = -\frac{\Delta B\left(\frac{1}{4}\pi d^2\right)}{\Delta t}$$

$$Q = P\Delta t = \left(\frac{\mathscr{E}^2}{R}\right)\Delta t = \left(\frac{\Delta B\left(\frac{1}{4}\pi d^2\right)}{\Delta t}\right)^2\left(\frac{\Delta t}{R}\right) = \frac{(\Delta B)^2\pi^2 d^4}{16R\Delta t}$$

$$= \frac{(0.80\,\text{T})^2\pi^2(0.015\,\text{m})^4}{16\left(55\times10^{-6}\,\Omega\right)\left(45\times10^{-3}\,\text{s}\right)} = 8.075\times10^{-3}\,\text{J} \approx \boxed{8.1\,\text{mJ}}$$

(*b*) The temperature change is calculated from the thermal energy using Eq. 19-2.

$$\Delta T = \frac{Q}{mc} = \frac{8.075\times10^{-3}\,\text{J}}{\left(15\times10^{-3}\,\text{kg}\right)\left(129\,\text{J/kg}\bullet{}^\circ\text{C}\right)} = \boxed{4.2\times10^{-3}\,\text{C}^\circ}$$

14. The average emf induced in the short coil is given by the "difference" version of Eq. 29-2b. N is the number of loops in the short coil, and the flux change is measured over the area of the short coil. The magnetic flux comes from the field created by the solenoid. The field in a solenoid is given by Eq. 28-4, $B = \mu_0 I N_{\text{solenoid}} / \ell_{\text{solenoid}}$, and the changing current in the solenoid causes the field to change.

$$|\mathscr{E}| = \frac{N_{\text{short}} A_{\text{short}} \Delta B}{\Delta t} = \frac{N_{\text{short}} A_{\text{short}} \Delta \left(\dfrac{\mu_0 I N_{\text{solenoid}}}{\ell_{\text{solenoid}}} \right)}{\Delta t} = \frac{\mu_0 N_{\text{short}} N_{\text{solenoid}} A_{\text{short}}}{\ell_{\text{solenoid}}} \frac{\Delta I}{\Delta t}$$

$$= \frac{\left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)(15)(420)\pi(0.0125\,\text{m})^2}{(0.25\,\text{m})} \frac{(5.0\,\text{A})}{(0.60\,\text{s})} = \boxed{1.3 \times 10^{-4}\,\text{V}}$$

15. (*a*) There is an emf induced in the coil since the flux through the coil changes. The current in the coil is the induced emf divided by the resistance of the coil. The resistance of the coil is found from Eq. 25-3.

$$|\mathscr{E}| = N A_{\text{coil}} \frac{dB}{dt} \qquad R = \frac{\rho \ell}{A_{\text{wire}}}$$

$$I = \frac{\mathscr{E}}{R} = \frac{N A_{\text{coil}} \dfrac{dB}{dt}}{\dfrac{\rho \ell}{A_{\text{wire}}}} = \frac{N A_{\text{coil}} A_{\text{wire}}}{\rho \ell} \frac{dB}{dt}$$

$$= \frac{28\left[\pi(0.110\,\text{m})^2\right]\left[\pi\left(1.3\times10^{-3}\,\text{m}\right)^2\right]\left(8.65\times10^{-3}\,\text{T/s}\right)}{\left(1.68\times10^{-8}\,\Omega\cdot\text{m}\right)28(2\pi)(0.110\,\text{m})} = 0.1504\,\text{A} \approx \boxed{0.15\,\text{A}}$$

(*b*) The rate at which thermal energy is produced in the wire is the power dissipated in the wire.

$$P = I^2 R = I^2 \frac{\rho \ell}{A_{\text{wire}}} = (0.1504\,\text{A})^2 \frac{\left(1.68\times10^{-8}\,\Omega\cdot\text{m}\right)28(2\pi)(0.11)}{\pi\left(1.3\times10^{-3}\,\text{m}\right)^2} = \boxed{1.4\times10^{-3}\,\text{W}}$$

16. The sinusoidal varying current in the power line creates a sinusoidal varying magnetic field encircling the power line, given by Eq. 28-1. Using Eq. 29-1b we integrate this field over the area of the rectangle to determine the flux through it. Differentiating the flux as in Eq. 29-2b gives the emf around the rectangle. Finally, by setting the maximum emf equal to 170 V we can solve for the necessary length of the rectangle.

$$B(t) = \frac{\mu_0 I_0}{2\pi r}\cos(2\pi f t) \ ;$$

$$\Phi_B(t) = \int B\,dA = \int_{5.0\,\text{m}}^{7.0\,\text{m}} \frac{\mu_0 I_0}{2\pi r}\cos(2\pi f t)\,\ell\,dr = \frac{\mu_0 I_0}{2\pi}\ell\cos(2\pi f t)\int_{5.0\,\text{m}}^{7.0\,\text{m}}\frac{dr}{r} = \frac{\mu_0 I_0}{2\pi}\ln(1.4)\,\ell\cos(2\pi f t)$$

$$\mathscr{E} = -N\frac{d\Phi_B}{dt} = -\frac{N\mu_0 I_0}{2\pi}\ln(1.4)\,\ell\left[\frac{d}{dt}\cos(2\pi f t)\right] = N\mu_0 I_0 f\ \ln(1.4)\,\ell\sin(2\pi f t) \ ;$$

$$\mathscr{E}_0 = N\mu_0 I_0 f\ \ln(1.4)\,\ell \ \rightarrow$$

$$\ell = \frac{\mathscr{E}_0}{N\mu_0 I_0 f\ \ln(1.4)} = \frac{170\,\text{V}}{10\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)(55{,}000\,\text{A})(60\,\text{Hz})\ln(1.4)} = \boxed{12\,\text{m}}$$

This is unethical because the current in the rectangle creates a back emf in the initial wire. This results in a power loss to the electric company, just as if the wire had been physically connected to the line.

17. The charge that passes a given point is the current times the elapsed time, $Q = I\Delta t$. The current will be the emf divided by the resistance, $I = \frac{\mathcal{E}}{R}$. The resistance is given by Eq. 25-3, $R = \frac{\rho\ell}{A_{\text{wire}}}$, and the emf is given by the "difference" version of Eq. 29-2a. Combine these equations to find the charge during the operation.

$$|\mathcal{E}| = \frac{\Delta\Phi_B}{\Delta t} = \frac{A_{\text{loop}}|\Delta B|}{\Delta t} \;;\; R = \frac{\rho\ell}{A_{\text{wire}}} \;;\; I = \frac{\mathcal{E}}{R} = \frac{\frac{A_{\text{loop}}|\Delta B|}{\Delta t}}{\frac{\rho\ell}{A_{\text{wire}}}} = \frac{A_{\text{loop}}A_{\text{wire}}|\Delta B|}{\rho\ell\Delta t}$$

$$Q = I\Delta t = \frac{A_{\text{loop}}A_{\text{wire}}|\Delta B|}{\rho\ell} = \frac{\pi r_{\text{loop}}^2 \pi r_{\text{wire}}^2|\Delta B|}{\rho(2\pi)r_{\text{loop}}} = \frac{r_{\text{loop}}\pi r_{\text{wire}}^2|\Delta B|}{2\rho}$$

$$= \frac{(0.091\,\text{m})\pi\left(1.175\times10^{-3}\,\text{m}\right)^2(0.750\,\text{T})}{2\left(1.68\times10^{-8}\,\Omega\bullet\text{m}\right)} = \boxed{8.81\,\text{C}}$$

18. (*a*) Use Eq. 29-2b to calculate the emf.

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = (-75)\frac{d}{dt}\left[\left(8.8t - 0.51t^3\right)\times10^{-2}\,\text{T}\bullet\text{m}^2\right] = \left(-6.6 + 1.1475t^2\right)\text{V}$$

$$\approx \boxed{\left(-6.6 + 1.1t^2\right)\text{V}}$$

(*b*) Evaluate at the specific times.

$$\mathcal{E}(t = 1.0\,\text{s}) = \left(-6.6 + 1.1475(1.0)^2\right)\text{V} = \boxed{-5.5\,\text{V}}$$

$$\mathcal{E}(t = 4.0\,\text{s}) = \left(-6.6 + 1.1475(4.0)^2\right)\text{V} = \boxed{12\,\text{V}}$$

[19.] The energy dissipated in the process is the power dissipated by the resistor, times the elapsed time that the current flows. The average induced emf is given by the "difference" version of Eq. 29-2a.

$$\mathcal{E} = -\frac{\Delta\Phi_B}{\Delta t} \;;\; P = \frac{\mathcal{E}^2}{R} \;;$$

$$E = P\Delta t = \frac{\mathcal{E}^2}{R}\Delta t = \left(\frac{\Delta\Phi_B}{\Delta t}\right)^2\frac{\Delta t}{R} = \frac{A^2(\Delta B)^2}{R\Delta t} = \frac{\left[\pi(0.125\text{m})^2\right]^2(0.40\,\text{T})^2}{(150\,\Omega)(0.12\,\text{s})} = \boxed{2.1\times10^{-5}\,\text{J}}$$

20. The induced emf is given by Eq. 29-2a. Since the field is uniform and is perpendicular to the area, the flux is simply the field times the area.

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -B\frac{dA}{dt} = -(0.28\,\text{T})\left(-3.50\times10^{-2}\,\text{m}^2/\text{s}\right) = \boxed{9.8\,\text{mV}}$$

Since the area changes at a constant rate, and the area has not shrunk to 0 at $t = 2.00$ s, the emf is the same for both times.

21. The induced emf is given by Eq. 29-2a. Since the field is uniform and is perpendicular to the area, the flux is simply the field times the area of a circle. We calculate the initial radius from the initial area. To calculate the radius after one second we add the change in radius to the initial radius.

$$\mathscr{E}(t) = \frac{d\Phi_B}{dt} = B\frac{d\left(\pi r^2\right)}{dt} = 2\pi Br\frac{dr}{dt} \qquad A_0 = \pi r_0^2 \to r_0 = \sqrt{\frac{A_0}{\pi}}$$

$$\mathscr{E}(0) = 2\pi\left(0.28\,\text{T}\right)\sqrt{\frac{0.285\,\text{m}^2}{\pi}}\left(0.043\,\text{m/s}\right) = \boxed{23\,\text{mV}}$$

$$\mathscr{E}(1.00\,\text{s}) = 2\pi\left(0.28\,\text{T}\right)\left[\sqrt{\frac{0.285\,\text{m}^2}{\pi}} + \left(0.043\,\text{m/s}\right)\left(1.00\,\text{s}\right)\right]\left(0.043\,\text{m/s}\right) = \boxed{26\,\text{mV}}$$

22. The magnetic field inside the solenoid is given by Eq. 28-4, $B = \mu_0 nI$. Use Eq. 29-2a to calculate the induced emf. The flux causing the emf is the flux through the small loop.

$$\mathscr{E} = -\frac{d\Phi_B}{dt} = -A_1\frac{dB_{\text{solenoid}}}{dt} = -A_1\mu_0 n\frac{dI}{dt} = -A_1\mu_0 n\left(-\omega I_0 \sin\omega t\right) = \boxed{A_1\mu_0 n\omega I_0 \sin\omega t}$$

23. (*a*) If the magnetic field is parallel to the plane of the loop, no magnetic flux passes through the loop at any time. Therefore, the emf and the current in the loop are $\boxed{\text{zero}}$.

(*b*) When the magnetic field is perpendicular to the plane of the loop, we differentiate Eq. 29-1a with respect to time to obtain the emf in the loop. Then we divide the emf by the resistance to calculate the current in the loop.

$$I = \frac{\mathscr{E}}{R} = \frac{1}{R}\left(-\frac{d\Phi_B}{dt}\right) = -\frac{1}{R}\frac{d}{dt}\left[\left(\alpha t\right)\left(A_0 + \beta t\right)\right] = -\frac{\alpha}{R}\left[A_0 + 2\beta t\right]$$

$$= -\frac{\left(0.60\text{ T/s}\right)\left[\left(0.50\text{ m}^2\right) + 2\left(0.70\text{ m}^2\text{/s}\right)\left(2.0\text{ s}\right)\right]}{2.0\ \Omega} = -0.99\text{ A}$$

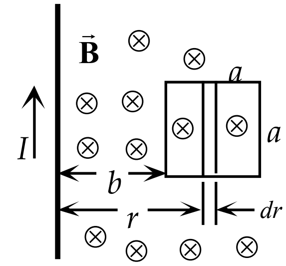
Since the magnetic field is pointing down into the page, the downward flux is increasing. The current then flows in a direction to create an upward flux. The resulting current is then $\boxed{\text{0.99 A in the counterclockwise direction}}$.

24. The magnetic field across the primary coil is constant and is that of a solenoid (Eq. 28-4). We multiply this magnetic field by the area of the secondary coil to calculate the flux through the secondary coil. Then using Eq. 29-2b we differentiate the flux to calculate the induced emf.

$$\Phi_B = BA = \mu_0 n_p I_0 \sin\left(2\pi ft\right)\left(\pi d^2/4\right)$$

$$\mathscr{E}_2 = N\frac{d\Phi_B}{dt} = N\mu_0 n_p I_0\left(\pi d^2/4\right)\left[\frac{d}{dt}\sin\left(2\pi ft\right)\right] = \boxed{-\tfrac{1}{2}\pi^2 d^2 fN\mu_0 n_p I_0 \cos\left(2\pi ft\right)}$$

$\boxed{25.}$ (*a*) The magnetic field a distance *r* from the wire is perpendicular to the wire and given by Eq. 28-1. Integrating this magnetic field over the area of the loop gives the flux through the loop.



$$\Phi_B = \int B\,dA = \int_b^{b+a}\frac{\mu_0 I}{2\pi r}a\,dr = \boxed{\frac{\mu_0 Ia}{2\pi}\ln\left(1+\frac{a}{b}\right)}$$

(*b*) Since the loop is being pulled away, $v = \dfrac{db}{dt}$. Differentiate the magnetic flux with respect to time to calculate the emf in the loop.

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left[\frac{\mu_0 Ia}{2\pi}\ln\left(1+\frac{a}{b}\right)\right] = -\frac{\mu_0 Ia}{2\pi}\frac{d}{db}\left[\ln\left(1+\frac{a}{b}\right)\right]\frac{db}{dt} = \boxed{\frac{\mu_0 Ia^2 v}{2\pi b(b+a)}}$$

Note that this is the emf at the instant the loop is a distance *b* from the wire. The value of *b* is changing with time.

(*c*) Since the magnetic field at the loop points into the page, and the flux is decreasing, the induced current will create a downward magnetic field inside the loop. The current in the loop then flows $\boxed{\text{clockwise}}$.

(*d*) The power dissipated in the loop as it is pulled away is related to the emf and resistance by Eq. 25-7b. This power is provided by the force pulling the loop away. We calculate this force from the power using Eq. 8-21. As in part (*b*), the value of *b* is changing with time.

$$F = \frac{P}{v} = \frac{\mathcal{E}^2}{Rv} = \boxed{\frac{\mu_0^2 I^2 a^4 v}{4\pi^2 R b^2 (b+a)^2}}$$

26. From Problem 25, the flux through the loop is given by $\Phi_B = \frac{\mu_0 Ia}{2\pi}\ln\left(1+\frac{a}{b}\right)$. The emf is found from Eq. 29-2a.

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left[\frac{\mu_0 Ia}{2\pi}\ln\left(1+\frac{a}{b}\right)\right] = -\frac{\mu_0 a}{2\pi}\ln\left(1+\frac{a}{b}\right)\frac{dI}{dt}$$

$$= -\frac{(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A})(0.120\,\text{m})}{2\pi}\ln\left(1+\frac{12.0}{15.0}\right)(15.0\,\text{A})(2500\,\text{rad/s})\cos(2500t)$$

$$= \boxed{(5.3\times10^{-4}\,\text{V})\cos(2500t)}$$

27. The velocity is found from Eq. 29-3.

$$\mathcal{E} = B\ell v \quad\rightarrow\quad v = \frac{\mathcal{E}}{B\ell} = \frac{0.12\,\text{V}}{(0.90\,\text{T})(0.132\,\text{m})} = \boxed{1.0\,\text{m/s}}$$

28. Because the velocity is perpendicular to the magnetic field and the rod, we find the induced emf from Eq. 29-3.

$$\mathcal{E} = B\ell v = (0.800\,\text{T})(0.120\,\text{m})(0.150\,\text{m/s}) = \boxed{1.44\times10^{-2}\,\text{V}}$$

29. (*a*) Because the velocity is perpendicular to the magnetic field and the rod, we find the induced emf from Eq. 29-3.

$$\mathcal{E} = B\ell v = (0.35\,\text{T})(0.250\,\text{m})(1.3\,\text{m/s}) = 0.1138\,\text{V} \approx \boxed{0.11\,\text{V}}$$

(*b*) Find the induced current from Ohm's law, using the **total** resistance.

$$I = \frac{\mathcal{E}}{R} = \frac{0.1138\,\text{V}}{25.0\,\Omega + 2.5\,\Omega} = 4.138\times10^{-3}\,\text{A} \approx \boxed{4.1\,\text{mA}}$$

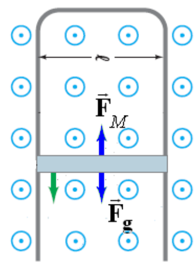
(*c*) The induced current in the rod will be down. Because this current is in an upward magnetic field, there will be a magnetic force to the left. To keep the rod moving, there must be an equal external force to the right, given by Eq. 27-1.

$$F = I\ell B = (4.138\times10^{-3}\,\text{A})(0.250\,\text{m})(0.35\,\text{T}) = 3.621\times10^{-4}\,\text{N} \approx \boxed{0.36\,\text{mN}}$$

30. The emf is given by Eq. 29-3 as $\mathcal{E} = B\ell v$. The resistance of the conductor is given by Eq. 25-3. The length in Eq. 25-3 is the length of resistive material. Since the movable rod starts at the bottom of the U at time $t = 0$, in a time t it will have moved a distance vt.

$$I = \frac{\mathcal{E}}{R} = \frac{B\ell v}{\frac{\rho L}{A}} = \frac{B\ell v}{\frac{\rho(2vt+\ell)}{A}} = \boxed{\frac{B\ell vA}{\rho(2vt+\ell)}}$$

31. The rod will descend at its terminal velocity when the magnitudes of the magnetic force (found in Example 29-8) and the gravitational force are equal. We set these two forces equal and solve for the terminal velocity.



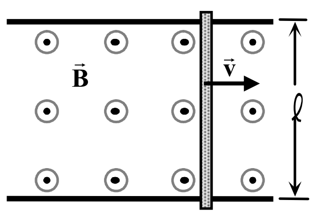
$$\frac{B^2\ell^2 v_t}{R} = mg \rightarrow$$

$$v_t = \frac{mgR}{B^2\ell^2} = \frac{(3.6\times10^{-3}\text{ kg})(9.80\text{ m/s}^2)(0.0013\ \Omega)}{(0.060\text{ T})^2(0.18\text{ m})^2} = \boxed{0.39\text{ m/s}}$$

32. Since the antenna is vertical, the maximum emf will occur when the car is traveling perpendicular to the horizontal component of the Earth's magnetic field. This occurs when the car is traveling in the east or west direction. We calculate the magnitude of the emf using Eq. 29-3, where B is the horizontal component of the Earth's magnetic field.

$$\mathcal{E} = B_x\ell v = (5.0\times10^{-5}\text{T}\cos45°)(0.750\text{m})(30.0\text{m/s}) = 8.0\times10^{-4}\text{V} = \boxed{0.80\text{mV}}$$

33. (*a*) As the rod moves through the magnetic field an emf will be built up across the rod, but no current can flow. Without the current, there is no force to oppose the motion of the rod, so yes, the rod travels at constant speed.



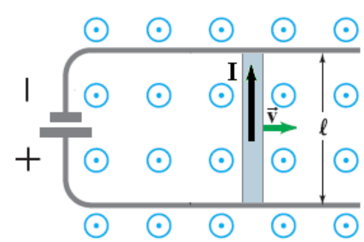
(*b*) We set the force on the moving rod, obtained in Example 29-8, equal to the mass times the acceleration of the rod. We then write the acceleration as the derivative of the velocity, and by separation of variables we integrate the velocity to obtain an equation for the velocity as a function of time.

$$F = ma = m\frac{dv}{dt} = -\frac{B^2\ell^2}{R}v \rightarrow \frac{dv}{v} = -\frac{B^2\ell^2}{mR}dt$$

$$\int_{v_0}^{v}\frac{dv'}{v'} = -\frac{B^2\ell^2}{mR}\int_0^t dt' \rightarrow \ln\frac{v}{v_0} = -\frac{B^2\ell^2}{mR}t \rightarrow \boxed{v(t) = v_0 e^{-\frac{B^2\ell^2}{mR}t}}$$

The magnetic force is proportional to the velocity of the rod and opposes the motion. This results in an exponentially decreasing velocity.

34. (*a*) For a constant current, of polarity shown in the figure, the magnetic force will be constant, given by Eq. 27-2. Using Newton's second law we can integrate the acceleration to calculate the velocity as a function of time.



$$F = m\frac{dv}{dt} = I\ell B \rightarrow \int_0^v dv = \frac{I\ell B}{m}\int_0^t dt \rightarrow \boxed{v(t) = \frac{I\ell B}{m}t}$$

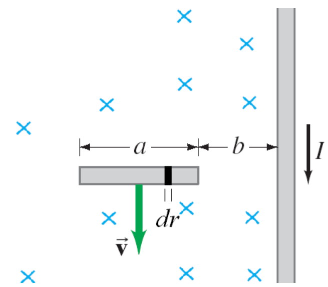
(*b*) For a constant emf, the current will vary with the speed of the rod, as motional emf opposes the motion of the rod. We again use Eq. 27-2 for the force on the rod, with the current given by Ohm's law, and the induced motional emf given by Eq 29-3. The current produced by the induced emf opposes the current produced by the battery.

$$F = m\frac{dv}{dt} = I\ell B = \left(\frac{\mathcal{E}_0 - B\ell v}{R}\right)\ell B \rightarrow \frac{dv}{\mathcal{E}_0 - B\ell v} = \frac{\ell B}{mR}dt \rightarrow \frac{dv}{v - \mathcal{E}_0/B\ell} = -\frac{B^2\ell^2}{mR}dt \rightarrow$$

$$\int_0^v \frac{dv}{v - \mathcal{E}_0/B\ell} = -\frac{B^2\ell^2}{mR}\int_0^t dt \rightarrow \ln\left(\frac{v - \mathcal{E}_0/B\ell}{-\mathcal{E}_0/B\ell}\right) = -\frac{B^2\ell^2}{mR}t \rightarrow \boxed{v(t) = \frac{\mathcal{E}_0}{B\ell}\left(1 - e^{-\frac{B^2\ell^2}{mR}t}\right)}$$

(*c*) With constant current, the acceleration is constant and so the velocity does not reach a terminal velocity. However, with constant emf, the increasing motional emf decreases the applied force. This results in a limiting, or terminal velocity of $\boxed{v_t = \mathcal{E}_0/B\ell}$.

35. (*a*) The magnetic field is perpendicular to the rod, with the magnetic field decreasing with distance from the rod, as in Eq. 28-1. The emf, $d\mathcal{E}$, across a short segment, *dr*, of the rod is given by the differential version of Eq. 29-3. Integrating this emf across the length of the wire gives the total emf.



$$d\mathcal{E} = Bvdr \rightarrow$$

$$\mathcal{E} = \int d\mathcal{E} = \int_b^{b+a} \frac{\mu_0 I}{2\pi r}vdr = \boxed{\frac{\mu_0 Iv}{2\pi}\ln\left(\frac{b+a}{b}\right)}$$

This emf points $\boxed{\text{toward the wire}}$, as positive charges are attracted toward the current.

(*b*) The only change is the direction of the current, so the magnitude of the emf remains the same, but points $\boxed{\text{away from the wire}}$, since positive charges are repelled from the current.

36. From Eq. 29-4, the induced voltage is proportional to the angular speed. Thus their quotient is a constant.

$$\frac{\mathcal{E}_1}{\omega_1} = \frac{\mathcal{E}_2}{\omega_2} \rightarrow \mathcal{E}_2 = \mathcal{E}_1\frac{\omega_2}{\omega_1} = (12.4\,\text{V})\frac{1550\,\text{rpm}}{875\,\text{rpm}} = \boxed{22.0\,\text{V}}$$

$\boxed{37.}$ We find the number of turns from Eq. 29-4. The factor multiplying the sine term is the peak output voltage.

$$\mathcal{E}_{\text{peak}} = NB\omega A \rightarrow N = \frac{\mathcal{E}_{\text{peak}}}{B\omega A} = \frac{24.0\,\text{V}}{(0.420\,\text{T})(2\pi\,\text{rad/rev})(60\,\text{rev/s})(0.0515\,\text{m})^2} = \boxed{57.2\text{ loops}}$$

38. From Eq. 29-4, the peak voltage is $\mathcal{E}_{\text{peak}} = NB\omega A$. Solve this for the rotation speed.

$$\mathcal{E}_{\text{peak}} = NB\omega A \rightarrow \omega = \frac{\mathcal{E}_{\text{peak}}}{NBA} = \frac{120\,\text{V}}{480(0.550\,\text{T})(0.220\,\text{m})^2} = 9.39\,\text{rad/s}$$

$$f = \frac{\omega}{2\pi} = \frac{9.39\,\text{rad/s}}{2\pi\,\text{rad/rev}} = \boxed{1.49\,\text{rev/s}}$$

39. From Eq. 29-4, the peak voltage is $\mathcal{E}_{\text{peak}} = NAB\omega$. The rms voltage is the peak voltage divided by $\sqrt{2}$, and so $V_{\text{rms}} = \mathcal{E}_{\text{peak}}/\sqrt{2} = NAB\omega/\sqrt{2}$.

40. Rms voltage is found from the peak induced emf. Peak induced emf is calculated from Eq. 29-4.

$$\mathcal{E}_{\text{peak}} = NB\omega A \rightarrow$$

$$V_{\text{rms}} = \frac{\mathcal{E}_{\text{peak}}}{\sqrt{2}} = \frac{NB\omega A}{\sqrt{2}} = \frac{(250)(0.45\,\text{T})(2\pi\,\text{rad/rev})(120\,\text{rev/s})\pi(0.050\,\text{m})^2}{\sqrt{2}}$$

$$= 471.1\,\text{V} \approx \boxed{470\,\text{V}}$$

To double the output voltage, you must $\boxed{\text{double the rotation frequency}}$ to 240 rev/s.

41. From Eq. 29-4, the induced voltage (back emf) is proportional to the angular speed. Thus their quotient is a constant.

$$\frac{\mathcal{E}_1}{\omega_1} = \frac{\mathcal{E}_2}{\omega_2} \rightarrow \mathcal{E}_2 = \mathcal{E}_1\frac{\omega_2}{\omega_1} = (72\,\text{V})\frac{2500\,\text{rpm}}{1200\,\text{rpm}} = \boxed{150\,\text{V}}$$

42. When the motor is running at full speed, the back emf opposes the applied emf, to give the net across the motor.

$$\mathcal{E}_{\text{applied}} - \mathcal{E}_{\text{back}} = IR \rightarrow \mathcal{E}_{\text{back}} = \mathcal{E}_{\text{applied}} - IR = 120\,\text{V} - (7.20\,\text{A})(3.05\,\Omega) = \boxed{98\,\text{V}}$$

$\boxed{43.}$ The back emf is proportional to the rotation speed (Eq. 29-4). Thus if the motor is running at half speed, the back emf is half the original value, or 54 V. Find the new current from writing a loop equation for the motor circuit, from Figure 29-20.

$$\mathcal{E} - \mathcal{E}_{\text{back}} - IR = 0 \rightarrow I = \frac{\mathcal{E} - \mathcal{E}_{\text{back}}}{R} = \frac{120\,\text{V} - 54\,\text{V}}{5.0\,\Omega} = \boxed{13\,\text{A}}$$

44. The magnitude of the back emf is proportional to both the rotation speed and the magnetic field, from Eq. 29-4. Thus $\frac{\mathcal{E}}{B\omega}$ is constant.

$$\frac{\mathcal{E}_1}{B_1\omega_1} = \frac{\mathcal{E}_2}{B_2\omega_2} \rightarrow B_2 = \frac{\mathcal{E}_2}{\omega_2}\frac{B_1\omega_1}{\mathcal{E}_1} = \frac{(75\,\text{V})}{(2300\,\text{rpm})}\frac{B_1(1100\,\text{rpm})}{(85\,\text{V})} = 0.42B_1$$

So $\boxed{\text{reduce the magnetic field to 42\% of its original value}}$.

45. (*a*) The generator voltage rating is the generator emf less the back emf. The ratio of the generator voltage rating to the generator emf is equal to the ratio of the effective resistance to the armature resistance. We solve this ratio for the generator emf, which is the same as the "no load" voltage.

$$V_{\text{nl}} = \mathcal{E} = V_{\text{load}}\frac{R_{\text{load}}}{R_{\text{nl}}} = V_{\text{load}}\frac{V_{\text{load}}/I_{\text{load}}}{R_{\text{nl}}} = 250\,\text{V}\left(\frac{250\,\text{V}/64\,\text{A}}{0.40\,\Omega}\right) = 2441\,\text{V} \approx \boxed{2.4\,\text{kV}}$$

(*b*) The generator voltage is proportional to the rotation frequency. From this proportionality we solve for the new generator voltage.

$$\frac{V_2}{V_1} = \frac{\omega_2}{\omega_1} \rightarrow V_2 = V_1\frac{\omega_2}{\omega_1} = (250\,\text{V})\frac{750\,\text{rpm}}{1000\,\text{rpm}} = \boxed{190\,\text{V}}$$

46. Because $N_S < N_P$, this is a $\boxed{\text{step-down}}$ transformer. Use Eq. 29-5 to find the voltage ratio, and Eq. 29-6 to find the current ratio.

$$\frac{V_S}{V_P} = \frac{N_S}{N_P} = \frac{85\,\text{turns}}{620\,\text{turns}} = \boxed{0.14} \qquad \frac{I_S}{I_P} = \frac{N_P}{N_S} = \frac{620\,\text{turns}}{85\,\text{turns}} = \boxed{7.3}$$

47. We find the ratio of the number of turns from Eq. 21-6.

$$\frac{N_S}{N_P} = \frac{V_S}{V_P} = \frac{12000\,\text{V}}{240\,\text{V}} = \boxed{50}$$

If the transformer is connected backward, the role of the turns will be reversed:

$$\frac{N_S}{N_P} = \frac{V_S}{V_P} \rightarrow \frac{1}{50} = \frac{V_S}{240\,\text{V}} \rightarrow V_S = \frac{1}{50}(240\,\text{V}) = \boxed{4.8\,\text{V}}$$

48. (*a*) Use Eqs. 29-5 and 29-6 to relate the voltage and current ratios.

$$\frac{V_S}{V_P} = \frac{N_S}{N_P}\ ;\ \frac{I_S}{I_P} = \frac{N_P}{N_S} \rightarrow \frac{V_S}{V_P} = \frac{I_P}{I_S} \rightarrow V_S = V_P\frac{I_P}{I_S} = (120\,\text{V})\frac{0.35\,\text{A}}{7.5\,\text{A}} = \boxed{5.6\,\text{V}}$$

(*b*) Because $V_S < V_P$, this is a $\boxed{\text{step-down}}$ transformer.

$\boxed{49.}$ (*a*) We assume 100% efficiency, and find the input voltage from $P = IV$.

$$P = I_P V_P \rightarrow V_P = \frac{P}{I_P} = \frac{75\,\text{W}}{22\,\text{A}} = 3.409\,\text{V}$$

Since $V_P < V_S$, this is a $\boxed{\text{step-up}}$ transformer.

(*b*) $$\frac{V_S}{V_P} = \frac{12\,\text{V}}{3.409\,\text{V}} = \boxed{3.5}$$

50. (*a*) The current in the transmission lines can be found from Eq. 25-10a, and then the emf at the end of the lines can be calculated from Kirchhoff's loop rule.

$$P_{\text{town}} = V_{\text{rms}} I_{\text{rms}} \rightarrow I_{\text{rms}} = \frac{P_{\text{town}}}{V_{\text{rms}}} = \frac{65\times10^6\,\text{W}}{45\times10^3\,\text{V}} = 1444\,\text{A}$$

$$\mathcal{E} - IR - V_{\text{output}} = 0 \rightarrow$$

$$\mathcal{E} = IR + V_{\text{output}} = \frac{P_{\text{town}}}{V_{\text{rms}}}R + V_{\text{rms}} = \frac{65\times10^6\,\text{W}}{45\times10^3\,\text{V}}(3.0\,\Omega) + 45\times10^3\,\text{V} = 49333\,\text{V} = \boxed{49\,\text{kV}(\text{rms})}$$

(*b*) The power loss in the lines is given by $P_{\text{loss}} = I_{\text{rms}}^2 R$.

$$\text{Fraction wasted} = \frac{P_{\text{loss}}}{P_{\text{total}}} = \frac{P_{\text{loss}}}{P_{\text{town}} + P_{\text{loss}}} = \frac{I_{\text{rms}}^2 R}{P_{\text{town}} + I_{\text{rms}}^2 R} = \frac{(1444\,\text{A})^2(3.0\,\Omega)}{65\times10^6\,\text{W} + (1444\,\text{A})^2(3.0\,\Omega)}$$

$$= \boxed{0.088} = 8.8\%$$

51. (*a*) If the resistor *R* is connected between the terminals, then it has a voltage V_0 across it and current I_0 passing through it. Then by Ohm's law the equivalent resistance is equal to the resistance of the resistor.

$$R_{\text{eq}} = \frac{V_0}{I_0} = \boxed{R}$$

(*b*) We use Eqs. 29-5 and 29-6 to write the voltage drop and current through the resistor in terms of the source voltage and current to calculate the effective resistance.

$$R = \frac{V_s}{I_s} = \frac{\frac{N_S}{N_P}V_0}{\frac{N_P}{N_S}I_0} \rightarrow R_{\text{eq}} = \frac{V_0}{I_0} = \boxed{\left(\frac{N_P}{N_S}\right)^2 R}$$

52. We set the power loss equal to 2% of the total power. Then using Eq. 25-7a we write the power loss in terms of the current (equal to the power divided by the voltage drop) and the resistance. Then, using Eq. 25-3, we calculate the cross-sectional area of each wire and the minimum wire diameter. We assume there are two lines to have a complete circuit.

$$P_{\text{loss}} = 0.020P = I^2R = \left(\frac{P}{V}\right)^2\left(\frac{\rho\ell}{A}\right) \rightarrow A = \frac{P\rho\ell}{0.020V^2} = \frac{\pi d^2}{4} \rightarrow$$

$$d = \sqrt{\frac{4}{0.020\pi}\frac{P\rho\ell}{V^2}} = \sqrt{\frac{4\left(225\times10^6\,\text{W}\right)\left(2.65\times10^{-8}\,\Omega\bullet\text{m}\right)2\left(185\times10^3\ \text{m}\right)}{0.020\pi\left(660\times10^3\,\text{V}\right)^2}} = 0.01796\,\text{m} \approx \boxed{1.8\,\text{cm}}$$

The transmission lines must have a diameter greater than or equal to 1.8 cm.

53. Without the transformers, we find the delivered current, which is the current in the transmission lines, from the delivered power, and the power lost in the transmission lines.

$$P_{\text{out}} = V_{\text{out}}I_{\text{line}} \rightarrow I_{\text{line}} = \frac{P_{\text{out}}}{V_{\text{out}}} = \frac{85000\,\text{W}}{120\,\text{V}} = 708.33\,\text{A}$$

$$P_{\text{lost}} = I_{\text{line}}^2R_{\text{line}} = \left(708.33\,\text{A}\right)^2 2\left(0.100\,\Omega\right) = 100346\,\text{W}$$

Thus there must be $85000\,\text{W} + 100346\,\text{W} = 185346\text{W} \approx 185\,\text{kW}$ of power generated at the start of the process.

With the transformers, to deliver the same power at 120 V, the delivered current from the step-down transformer must still be 708.33 A. Using the step-down transformer efficiency, we calculate the current in the transmission lines, and the loss in the transmission lines.

$$P_{\text{out}} = 0.99P_{\substack{\text{line}\\ \text{end}}} \rightarrow V_{\text{out}}I_{\text{out}} = 0.99V_{\text{line}}I_{\text{line}} \rightarrow I_{\text{line}} = \frac{V_{\text{out}}I_{\text{out}}}{0.99V_{\text{line}}} = \frac{\left(120\,\text{V}\right)\left(708.33\,\text{A}\right)}{\left(0.99\right)\left(1200\,\text{V}\right)} = 71.548\,\text{A}$$

$$P_{\text{lost}} = I_{\text{line}}^2R_{\text{line}} = \left(71.548\,\text{A}\right)^2 2\left(0.100\,\Omega\right) = 1024\,\text{W}$$

The power to be delivered is 85000 W. The power that must be delivered to the step-down transformer is $\frac{85000\,\text{W}}{0.99} = 85859\,\text{W}$. The power that must be present at the start of the transmission must be $85859\,\text{W} + 1024\,\text{W} = 86883\,\text{W}$ to compensate for the transmission line loss. The power that must enter the transmission lines from the 99% efficient step-up transformer is $\frac{86883\,\text{W}}{0.99} = 87761 \approx 88\,\text{kW}$. So the power saved is $185346\,\text{W} - 87761\,\text{W} = 97585\,\text{W} \approx \boxed{98\,\text{kW}}$.

54. We choose a circular path centered at the origin with radius 10 cm. By symmetry the electric field is uniform along this path and is parallel to the path. We then use Eq. 29-8 to calculate the electric field at each point on this path. From the electric field we calculate the force on the charged particle.

$$\oint \vec{E}\bullet d\vec{\ell} = E\left(2\pi r\right) = -\frac{d\Phi_B}{dt} = -\left(\pi r^2\right)\frac{dB}{dt}$$

$$F = QE = -Q\frac{r}{2}\frac{dB}{dt} = -\left(1.0\times10^{-6}\ \text{C}\right)\frac{0.10\ \text{m}}{2}\left(-0.10\ \text{T/s}\right) = \boxed{5.0\ \text{nN}}$$

Since the magnetic field points into the page and is decreasing, Lenz's law tells us that an induced circular current centered at the origin would flow in the clockwise direction. Therefore, the force on a positive charge along the positive x-axis would be down, or $\boxed{\text{in the } -\hat{\mathbf{j}} \text{ direction.}}$

55. (*a*) The increasing downward magnetic field creates a circular electric field along the electron path. This field applies an electric force to the electron causing it to accelerate.

(*b*) For the electrons to move in a circle, the magnetic force must provide a centripetal acceleration. With the magnetic field pointing downward, the right-hand-rule requires the electrons travel in a clockwise direction for the force to point inward.

(*c*) For the electrons to accelerate, the electric field must point in the counterclockwise direction. A current in this field would create an upward magnetic flux. So by Lenz's law, the downward magnetic field must be increasing.

(*d*) For the electrons to move in a circle and accelerate, the field must be pointing downward and increasing in magnitude. For a sinusoidal wave, the field is downward half of the time and upward the other half. For the half that it is downward its magnitude is decreasing half of the time and increasing the other half. Therefore, the magnetic field is pointing downward and increasing for only one fourth of every cycle.

56. In Example 29-14 we found the electric field along the electron's path from Faraday's law. Multiplying this field by the electron charge gives the force on the electron, and from the force, we calculate the change in tangential velocity.

$$\frac{dv}{dt}=\frac{F}{m}=E\frac{q}{m}=\frac{q}{m}\frac{r}{2}\frac{dB_{\text{avg}}}{dt}$$

We set the centripetal force on the electron equal to the magnetic force (using Eq. 27-5b) and solve for the velocity. Differentiating the velocity with respect to time (keeping the radius constant) yields a relation for the acceleration in terms of the changing magnetic field.

$$qvB=m\frac{v^2}{r}\quad\rightarrow\quad v=\frac{qBr}{m}\quad\rightarrow\quad\frac{dv}{dt}=\frac{q}{m}r\frac{dB_0}{dt}$$

Equating these two equations for the electron acceleration, we see that the change in magnetic field at the electron must equal $\frac{1}{2}$ of the average change in magnetic field. This relation is satisfied if at all times $B_0=\frac{1}{2}B_{\text{avg}}$.

57. (*a*) The electric field is the change in potential across the rod (obtained from Ohm's law) divided by the length of the rod.

$$E=\frac{\Delta V}{\ell}=\boxed{\frac{IR}{\ell}}$$

(*b*) Again the electric field is the change in potential across the rod divided by the length of the rod. The electric potential is the supplied potential less the motional emf found using Eq. 29-3 and the results of Problem 34(b).

$$E=\frac{\Delta V}{\ell}=\frac{\mathcal{E}_0-B\ell v}{\ell}=\frac{\mathcal{E}_0-B\ell\,\mathcal{E}_0/B\ell\left(1-e^{-\frac{B^2\ell^2}{mR}t}\right)}{\ell}=\boxed{\frac{\mathcal{E}_0}{\ell}e^{-\frac{B^2\ell^2}{mR}t}}$$

58. (*a*) The clockwise current in the left-hand loop produces a magnetic field which is into the page within the loop and out of the page outside the loop. Thus the right-hand loop is in a magnetic field that is directed out of the page. Before the current in the left-hand loop reaches its steady state, there will be an induced current in the right-hand loop that will produce a magnetic field into the page to oppose the increase of the field from the left-hand loop. Thus the induced current will be clockwise.

(*b*) After a long time, the current in the left-hand loop is constant, so there will be no induced current in the right-hand coil.

(*c*) If the second loop is pulled to the right, the magnetic field out of the page from the left-hand loop through the second loop will decrease. During the motion, there will be an induced current in the right-hand loop that will produce a magnetic field out of the page to oppose the decrease of the field from the left-hand loop. Thus the induced current will be $\boxed{\text{counterclockwise}}$.

59. The electrical energy is dissipated because there is current flowing in a resistor. The power dissipation by a resistor is given by $P = I^2R$, and so the energy dissipated is $E = P\Delta t = I^2R\Delta t$. The current is created by the induced emf caused by the changing B-field. The average induced emf is given by the "difference" version of Eq. 29-2b.

$$\mathcal{E} = -\frac{\Delta\Phi_B}{\Delta t} = -\frac{A\Delta B}{\Delta t} \qquad I = \frac{\mathcal{E}}{R} = -\frac{A\Delta B}{R\Delta t}$$

$$E = P\Delta t = I^2R\Delta t = \frac{A^2(\Delta B)^2}{R^2(\Delta t)^2}R\Delta t = \frac{A^2(\Delta B)^2}{R(\Delta t)} = \frac{\left[(0.270\,\text{m})^2\right]^2\left[(0-0.755\,\text{T})\right]^2}{(7.50\,\Omega)(0.0400\,\text{s})}$$

$$= \boxed{1.01\times10^{-2}\,\text{J}}$$

60. Because there are perfect transformers, the power loss is due to resistive heating in the transmission lines. Since the town requires 65 MW, the power at the generating plant must be $\frac{65\,\text{MW}}{0.985} = 65.99$ MW. Thus the power lost in the transmission is 0.99 MW. This can be used to determine the current in the transmission lines.

$$P = I^2R \;\rightarrow\; I = \sqrt{\frac{P}{R}} = \sqrt{\frac{0.99\times10^6\,\text{W}}{2(85\,\text{km})0.10\,\Omega/\text{km}}} = 241.3\,\text{A}$$

To produce 65.99 MW of power at 241.3 A requires the following voltage.

$$V = \frac{P}{I} = \frac{65.99\times10^6\,\text{W}}{241.3\,\text{A}} = 2.73\times10^5\,\text{V} \approx \boxed{270\,\text{kV}}$$

$\boxed{61.}$ The charge on the capacitor can be written in terms of the voltage across the battery and the capacitance using Eq. 24-1. When fully charged the voltage across the capacitor will equal the emf of the loop, which we calculate using Eq. 29-2b.

$$Q = CV = C\frac{d\Phi_B}{dt} = CA\frac{dB}{dt} = (5.0\times10^{-12}\text{ F})(12\text{ m}^2)(10\text{ T/s}) = \boxed{0.60\text{ nC}}$$

62. (*a*) From the efficiency of the transformer, we have $P_S = 0.85P_P$. Use this to calculate the current in the primary.

$$P_S = 0.85P_P = 0.85I_PV_P \;\rightarrow\; I_P = \frac{P_S}{0.85V_P} = \frac{75\,\text{W}}{0.85(110\,\text{V})} = 0.8021\,\text{A} \approx \boxed{0.80\,\text{A}}$$

(*b*) The voltage in both the primary and secondary is proportional to the number of turns in the respective coil. The secondary voltage is calculated from the secondary power and resistance since $P = V^2/R$.

$$\frac{N_P}{N_S} = \frac{V_P}{V_S} = \frac{V_P}{\sqrt{P_SR_S}} = \frac{110\,\text{V}}{\sqrt{(75\,\text{W})(2.4\,\Omega)}} = \boxed{8.2}$$

63. (*a*) The voltage drop across the lines is due to the resistance.

$$V_{\text{out}} = V_{\text{in}} - IR = 42000\,\text{V} - (740\,\text{A})(2)(0.80\,\Omega) = 40816\,\text{V} \approx \boxed{41\,\text{kV}}$$

(*b*) The power input is given by $P_{\text{in}} = IV_{\text{in}}$.

$$P_{\text{in}} = IV_{\text{in}} = (740\,\text{A})(42000\,\text{V}) = 3.108\times10^7\,\text{W} \approx \boxed{3.1\times10^7\,\text{W}}$$

(*c*) The power loss in the lines is due to the current in the resistive wires.

$$P_{\text{loss}} = I^2R = (740\,\text{A})^2(1.60\,\Omega) = 8.76\times10^5\,\text{W} \approx \boxed{8.8\times10^5\,\text{W}}$$

(*d*) The power output is given by $P_{\text{out}} = IV_{\text{out}}$.

$$P_{\text{out}} = IV_{\text{out}} = (740\,\text{A})(40816\,\text{V}) = 3.020\times10^7\,\text{W} \approx \boxed{3.0\times10^7\,\text{W}}.$$

This could also be found by subtracting the power lost from the input power.

$$P_{\text{out}} = P_{\text{in}} - P_{\text{loss}} = 3.108\times10^7\,\text{W} - 8.76\times10^5\,\text{W} = 3.020\times10^7\,\text{W} \approx \boxed{3.0\times10^7\,\text{W}}$$

64. We find the current in the transmission lines from the power transmitted to the user, and then find the power loss in the lines.

$$P_{\text{T}} = I_{\text{L}}V \rightarrow I_{\text{L}} = \frac{P_{\text{T}}}{V} \qquad P_{\text{L}} = I_{\text{L}}^2R_{\text{L}} = \left(\frac{P_{\text{T}}}{V}\right)^2 R_{\text{L}} = \boxed{\frac{P_{\text{T}}^2R_{\text{L}}}{V^2}}$$

65. (*a*) Because $V_{\text{S}} < V_{\text{P}}$, this is a $\boxed{\text{step-down}}$ transformer.

(*b*) Assuming 100% efficiency, the power in both the primary and secondary is 35 W. Find the current in the secondary from the relationship $P = IV$.

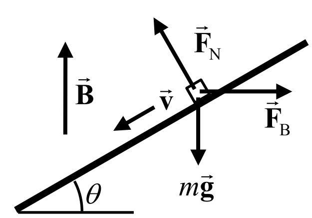
$$P_{\text{S}} = I_{\text{S}}V_{\text{S}} \rightarrow I_{\text{S}} = \frac{P_{\text{S}}}{V_{\text{S}}} = \frac{35\,\text{W}}{12\,\text{V}} = \boxed{2.9\,\text{A}}$$

(*c*) $P_{\text{P}} = I_{\text{P}}V_{\text{P}} \rightarrow I_{\text{P}} = \dfrac{P_{\text{P}}}{V_{\text{P}}} = \dfrac{35\,\text{W}}{120\,\text{V}} = \boxed{0.29\,\text{A}}$

(*d*) Find the resistance of the bulb from Ohm's law. The bulb is in the secondary circuit.

$$V_{\text{S}} = I_{\text{S}}R \rightarrow R = \frac{V_{\text{S}}}{I_{\text{S}}} = \frac{12\,\text{V}}{2.9\,\text{A}} = \boxed{4.1\,\Omega}$$

66. A side view of the rail and bar is shown in the figure. From Section 21-3, the emf in the bar is produced by the components of the magnetic field, the length of the bar, and the velocity of the bar, which are all mutually perpendicular. The magnetic field and the length of the bar are already perpendicular. The component of the velocity of the bar that is perpendicular to the magnetic field is $v\cos\theta$, and so the induced emf is given by the following.



$$\mathcal{E} = B\ell v\cos\theta$$

This produces a current in the wire, which can be found by Ohm's law. That current is pointing into the page on the diagram.

$$I = \frac{\mathcal{E}}{R} = \frac{B\ell v\cos\theta}{R}$$

Because the current is perpendicular to the magnetic field, the force on the wire from the magnetic field can be calculated from Eq. 27-2, and will be horizontal, as shown in the diagram.

$$F_B = I\ell B = \frac{B\ell v\cos\theta}{R}\ell B = \frac{B^2\ell^2 v\cos\theta}{R}$$

For the wire to slide down at a steady speed, the net force along the rail must be zero. Write Newton's second law for forces along the rail, with up the rail being positive.

$$F_{\text{net}} = F_B\cos\theta - mg\sin\theta = 0 \quad\rightarrow\quad \frac{B^2\ell^2 v\cos^2\theta}{R} = mg\sin\theta \quad\rightarrow$$

$$v = \frac{Rmg\sin\theta}{B^2\ell^2\cos^2\theta} = \frac{(0.60\,\Omega)(0.040\,\text{kg})(9.80\,\text{m/s}^2)\sin 6.0^\circ}{(0.55\,\text{T})^2(0.32\,\text{m})^2\cos^2 6.0^\circ} = \boxed{0.80\,\text{m/s}}$$

67. The induced current in the coil is the induced emf divided by the resistance. The induced emf is found from the changing flux by Eq. 29-2a. The magnetic field of the solenoid, which causes the flux, is given by Eq. 28-4. For the area used in Eq. 29-2a, the cross-sectional area of the solenoid (not the coil) must be used, because all of the magnetic flux is inside the solenoid.

$$I = \frac{\mathcal{E}_{\text{ind}}}{R} \qquad |\mathcal{E}_{\text{ind}}| = N_{\text{coil}}\frac{d\Phi}{dt} = N_{\text{coil}}A_{\text{sol}}\frac{dB_{\text{sol}}}{dt} \qquad B_{\text{sol}} = \mu_0\frac{N_{\text{sol}}I_{\text{sol}}}{\ell_{\text{sol}}}$$

$$I = \frac{N_{\text{coil}}A_{\text{sol}}\mu_0\dfrac{N_{\text{sol}}}{\ell_{\text{sol}}}\dfrac{dI_{\text{sol}}}{dt}}{R} = \frac{N_{\text{coil}}A_{\text{sol}}\mu_0}{R}\frac{N_{\text{sol}}}{\ell_{\text{sol}}}\frac{dI_{\text{sol}}}{dt}$$

$$= \frac{(150\,\text{turns})\pi(0.045\,\text{m})^2(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A})}{12\,\Omega}\frac{(230\,\text{turns})}{(0.01\,\text{m})}\frac{2.0\,\text{A}}{0.10\,\text{s}} = \boxed{4.6\times10^{-2}\,\text{A}}$$

As the current in the solenoid increases, a magnetic field from right to left is created in the solenoid and the loop. The induced current will flow in such a direction as to oppose that field, and so must flow from $\boxed{\text{left to right}}$ through the resistor.

68. The average induced emf is given by the "difference" version of Eq. 29-2b. Because the coil orientation changes by 180°, the change in flux is the opposite of twice the initial flux. The average current is the induced emf divided by the resistance, and the charge that flows in a given time is the current times the elapsed time.

$$\mathcal{E}_{\text{avg}} = -N\frac{\Delta\Phi_B}{\Delta t} = -NA\frac{\Delta B}{\Delta t} = -NA\frac{[(-B)-(+B)]}{\Delta t} = \frac{2NAB}{\Delta t}$$

$$Q = I\Delta t = \frac{\mathcal{E}_{\text{avg}}}{R}\Delta t = \frac{\left(\dfrac{2NAB}{\Delta t}\right)}{R}\Delta t = \frac{2NAB}{R} \quad\rightarrow\quad \boxed{B = \frac{RQ}{2NA}}$$

69. Calculate the current in the ring from the magnitude of the emf (from Eq. 29-2a) divided by the resistance. Setting the current equal to the derivative of the charge, we integrate the charge and flux over the 90° rotation, with the flux given by Eq. 29-1a. This results in the total charge flowing past a given point in the ring. Note that the initial orientation of the ring area relative to the magnetic field is not given.

$$\mathscr{E} = \frac{d\Phi_B}{dt} \;;\; I = \frac{\mathscr{E}}{R} = \frac{dQ}{dt} \;\rightarrow\; dQ = \frac{\mathscr{E}}{R}dt \;\rightarrow$$

$$Q = \int dQ = \int \frac{\mathscr{E}}{R}dt = \frac{1}{R}\int \frac{d\Phi}{dt}dt = \frac{1}{R}\int d\Phi = \frac{1}{R}\int_{BA\cos(\theta)}^{BA\cos(\theta+90°)} d\Phi = \frac{BA\left[\cos(\theta+90°)-\cos\theta\right]}{R}$$

$$= \frac{(0.23\,\text{T})\pi(0.030\,\text{m})^2}{0.025\,\Omega}\left[\cos(\theta+90°)-\cos\theta\right] = 0.02601\,\text{C}\left[\cos(\theta+90°)-\cos\theta\right]$$

To find the maximum charge, we set the derivative of the charge with respect to the starting angle, θ, equal to zero to find the extremes. Inserting the maximum angle into our equation, we find the maximum charge passing through the ring. Finally, we divide the maximum charge by the charge of a single electron to obtain the number of electrons passing the point in the ring.

$$\frac{dQ}{d\theta} = 0.02601\text{ C}\left[-\sin(\theta+90°)+\sin\theta\right] = 0.02601\text{ C}\left[-\cos\theta+\sin\theta\right] = 0 \;\rightarrow\; \tan\theta = 1 \;\rightarrow$$

$$\theta = 45° \text{ or } 225°$$

$$Q_{\text{max}} = 0.02601\text{ C}\left[\cos(225°+90°)-\cos 225°\right] = 0.03678\text{ C}$$

$$N_{\text{max}} = \frac{Q_{\text{max}}}{q} = \frac{0.03678\text{ C}}{1.60\times10^{-19}\text{ C/e}} = \boxed{2.3\times10^{17}\,\text{electrons}}$$

70. The coil should have a diameter about equal to the diameter of a standard flashlight D-cell so that it will be simple to hold and use. This would give the coil a radius of about 1.5 cm. As the magnet passes through the coil the field changes direction, so the change in flux for each pass is twice the maximum flux. Let us assume that the magnet is shaken with a frequency of about two shakes per second, so the magnet passes through the coil four times per second. We obtain the number of turns in the coil using Eq. 29-2b.

$$N = \frac{\mathscr{E}}{\Delta\Phi/\Delta t} = \frac{\mathscr{E}\Delta t}{\Delta\Phi} = \frac{\mathscr{E}\Delta t}{2B_0A} = \frac{(3.0\,\text{V})(0.25\,\text{s})}{2(0.050\,\text{T})\pi(0.015\,\text{m})^2} \approx \boxed{11{,}000\text{ turns}}$$

71. (*a*) Since the coils are directly connected to the wheels, the torque provided by the motor (Eq. 27-9) balances the torque caused by the frictional force.

$$NIAB = Fr \rightarrow I = \frac{Fr}{NAB} = \frac{(250\text{ N})(0.29\text{ m})}{270(0.12\text{ m})(0.15\text{ m})(0.60\text{ T})} = 24.86\text{ A} \approx \boxed{25\text{ A}}$$

(*b*) To maintain this speed the power loss due to the friction (Eq. 8-21) must equal the net power provided by the coils. The power provided by the coils is the current through the coils multiplied by the back emf.

$$P = Fv = I\mathscr{E}_{\text{back}} \rightarrow \mathscr{E}_{\text{back}} = \frac{Fv}{I} = \frac{(250\,\text{N})(35\,\text{km/h})}{24.86\,\text{A}}\left(\frac{1000\,\text{m/km}}{3600\text{ s/h}}\right) = 97.76\,\text{V} \approx \boxed{98\,\text{V}}$$

(*c*) The power dissipated in the coils is the difference between the power produced by the coils and the net power provided to the wheels.

$$P_{\text{loss}} = P - P_{\text{net}} = I\mathscr{E} - I\mathscr{E}_{\text{back}} = (24.86\,\text{A})(120\,\text{V} - 97.76\,\text{V}) = 553\,\text{W} \approx \boxed{600\,\text{W}}$$

(*d*) We divide the net power by the total power to determine the percent used to drive the car.

$$\frac{P_{\text{net}}}{P} = \frac{I\mathcal{E}_{\text{back}}}{I\mathcal{E}} = \frac{97.76\,\text{V}}{120\,\text{V}} = 0.8147 \approx \boxed{81\%}$$

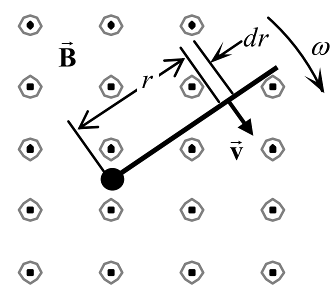
72. The energy is dissipated by the resistance. The power dissipated by the resistor is given by Eq. 25-7b, and the energy is the integral of the power over time. The induced emf is given by Eq. 29-2a.

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -NA\frac{dB}{dt} = \frac{NAB_0}{\tau}e^{-t/\tau} \quad ; \quad P = I^2R = \frac{\mathcal{E}^2}{R} = \left(\frac{N^2A^2B_0^2}{R\tau^2}\right)e^{-2t/\tau}$$

$$E = \int P\,dt = \int_0^t \left(\frac{N^2A^2B_0^2}{R\tau^2}\right)e^{-2t/\tau}dt = \left(\frac{N^2A^2B_0^2}{R\tau^2}\right)\left(-\frac{\tau}{2}e^{-2t/\tau}\right)_0^t = \frac{\left(NAB_0\right)^2}{2R\tau}\left(1-e^{-2t/\tau}\right)$$

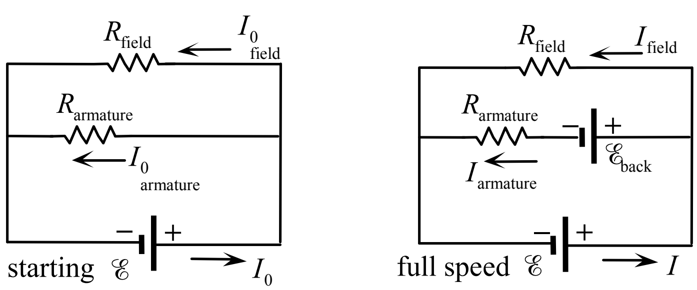
$$= \frac{\left[18\pi\left(0.100\,\text{m}\right)^2\left(0.50\,\text{T}\right)\right]^2}{2\left(2.0\,\Omega\right)\left(0.10\,\text{s}\right)}\left(1-e^{-2t/(0.10\,\text{s})}\right) = \boxed{\left(0.20\,\text{J}\right)\left(1-e^{-20t}\right)}$$

73. The total emf across the rod is the integral of the differential emf across each small segment of the rod. For each differential segment, *dr*, the differential emf is given by the differential version of Eq. 29-3. The velocity is the angular speed multiplied by the radius. The figure is a top view of the spinning rod.



$$d\mathcal{E} = Bv\,d\ell = B\omega r\,dr \quad \rightarrow \quad \mathcal{E} = \int d\mathcal{E} = \int_0^\ell B\omega r\,dr = \boxed{\tfrac{1}{2}B\omega\ell^2}$$

74. (*a*)



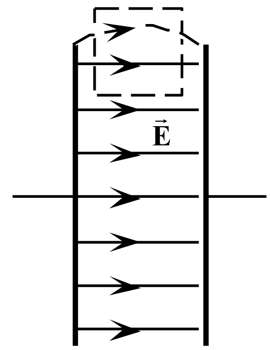
(*b*) At startup there is no back emf. We therefore treat the circuit as two parallel resistors, each with the same voltage drop. The current through the battery is the sum of the currents through each resistor.

$$I = I_{0\,\text{field}} + I_{0\,\text{armature}} = \frac{\mathcal{E}}{R_{\text{field}}} + \frac{\mathcal{E}}{R_{\text{armature}}} = \frac{115\text{ V}}{36.0\ \Omega} + \frac{115\text{ V}}{3.00\ \Omega} = \boxed{41.5\text{ A}}$$

(*c*) At full speed the back emf decreases the voltage drop across the armature resistor.

$$I = I_{\text{field}} + I_{\text{armature}} = \frac{\mathcal{E}}{R_{\text{field}}} + \frac{\mathcal{E}-\mathcal{E}_{\text{back}}}{R_{\text{armature}}} = \frac{115\,\text{V}}{36.0\,\Omega} + \frac{115\,\text{V}-105\,\text{V}}{3.00\,\Omega} = \boxed{6.53\,\text{A}}$$

75. Assume that the electric field does not fringe, but only has a horizontal component between the plates and zero field outside the plates. Apply Faraday's law (Eq. 29-8) to this situation for a rectangular loop with one horizontal leg inside the plates and the second horizontal leg outside the plates. We integrate around this path in the counterclockwise direction. Since the field only has a horizontal component between the plates, only the horizontal leg will contribute to the electric field integral. Since the field is constant in this region, the integral is the electric field times the length of the leg.

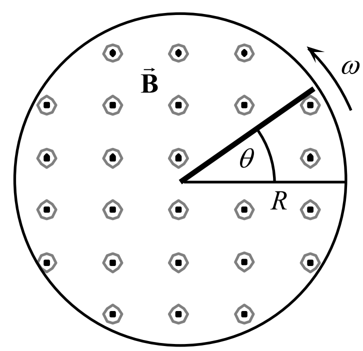


$$\oint \vec{\mathbf{E}}\cdot d\vec{\ell} = \int_0^{\ell} E\,d\ell = E\ell$$

For a static electric field, the magnetic flux is unchanging. Therefore $\frac{d\Phi_B}{dt} = 0.$

Using Faraday's law, we have $\oint \vec{\mathbf{E}}\cdot d\vec{\ell} = -\frac{d\Phi_B}{dt} \;\rightarrow\; E\ell = 0,$ which is not possible. Thus one of the initial assumptions must be false. We conclude that the field must have some fringing at the edges.

76. The total emf across the disk is the integral of the differential emf across each small segment of the radial line passing from the center of the disk to the edge. For each differential segment, dr, the emf is given by the differential version of Eq. 29-3. The velocity is the angular speed multiplied by the radius. Since the disk is rotating in the counterclockwise direction, and the field is out of the page, the emf is increasing with increasing radius. Therefore the rim is at the higher potential.



$$d\mathscr{E} = Bv\,d\ell = B\omega r\,dr \;\rightarrow$$

$$\mathscr{E} = \int d\mathscr{E} = \int_0^R B\omega r\,dr = \boxed{\tfrac{1}{2}B\omega R^2}$$

77. We set the electric field equal to the negative gradient of the electric potential (Eq. 23-8), with the differential potential given by Eq. 29-3, as in Problem 76.

$$\vec{\mathbf{E}} = -\frac{d\mathscr{E}}{dr}\hat{\mathbf{r}} = -\frac{Br\omega\,dr}{dr}\hat{\mathbf{r}} = \boxed{-Br\omega\hat{\mathbf{r}}}$$

The electric field has magnitude $Br\omega$ and points radially inwards, toward the center of the disk.

78. The emf around the loop is equal to the time derivative of the flux, as in Eq. 29-2a. Since the area of the coil is constant, the time derivative of the flux is equal to the derivative of the magnetic field multiplied by the area of the loop. To calculate the emf in the loop we add the voltage drop across the capacitor to the voltage drop across the resistor. The current in the loop is the derivative of the charge on the capacitor (Eq. 24-1).

$$I = \frac{dQ}{dt} = \frac{dCV}{dt} = \frac{d}{dt}\left[CV_0\left(1-e^{-t/\tau}\right)\right] = \frac{CV_0}{\tau}e^{-t/\tau} = \frac{V_0}{R}e^{-t/\tau}$$

$$\mathscr{E} = IR + V_C = \left(\frac{V_0}{R}e^{-t/\tau}\right)R + V_0\left(1-e^{-t/\tau}\right) = V_0 = \frac{d\Phi_B}{dt} = A\frac{dB}{dt} = \pi r^2\frac{dB}{dt} \;\rightarrow\; \boxed{\frac{dB}{dt} = \frac{V_0}{\pi r^2}}$$

Since the charge is building up on the top plate of the capacitor, the induced current is flowing clockwise. By Lenz's law this produces a downward flux, so the external downward magnetic field must be $\boxed{\text{decreasing}}$.

79. (*a*) As the loop falls out of the magnetic field, the flux through the loop decreases with time creating an induced emf in the loop. The current in the loop is equal to the emf divided by the resistance, which can be written in terms of the resistivity using Eq. 25-3.

$$I = \frac{\mathcal{E}}{R} = \left(\frac{\pi d^2/4}{\rho 4\ell}\right)\frac{d\Phi_B}{dt} = \left(\frac{\pi d^2}{16\rho\ell}\right)B\frac{dA}{dt} = \frac{\pi d^2}{16\rho\ell}B\ell v$$

This current induces a force on the three sides of the loop in the magnetic field. The forces on the two vertical sides are equal and opposite and therefore cancel.

$$F = I\ell B = \frac{\pi d^2}{16\rho\ell}B\ell v\ell B = \boxed{\frac{\pi d^2 B^2 \ell v}{16\rho}}$$

By Lenz's law this force is upward to slow the decrease in flux.

(*b*) Terminal speed will occur when the gravitational force is equal to the magnetic force.

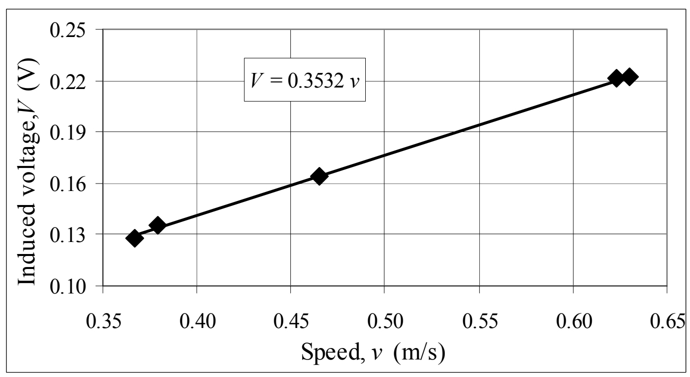
$$F_g = \rho_m\left(4\pi\ell\frac{d^2}{4}\right)g = \frac{\pi d^2 B^2 \ell v_T}{16\rho} \rightarrow v_T = \boxed{\frac{16\rho\rho_m g}{B^2}}$$

(*c*) We calculate the terminal velocity using the given magnetic field, the density of copper from Table 13-1, and the resistivity of copper from Table 25-1

$$v_T = \frac{16\left(8.9\times10^3\ \text{kg/m}^3\right)\left(1.68\times10^{-8}\ \Omega\text{m}\right)\left(9.80\ \text{m/s}^2\right)}{\left(0.80\ \text{T}\right)^2} = \boxed{3.7\ \text{cm/s}}$$

80. (*a*) See the graph, with best fit linear trend line (with the *y* intercept forced to be 0).

(*b*) The theoretical slope is the induced voltage divided by the velocity. Take the difference between the experimental value found in part (*a*) and the theoretical value and divide the result by the theoretical value to obtain the percent difference.



$$\%\ \text{diff} = \left(\frac{m_{\text{exp}} - m_{\text{theory}}}{m_{\text{theory}}}\right)100 = \left(\frac{m_{\text{exp}}}{BN\ell} - 1\right)100 = \left(\frac{0.3532\ \text{V}\bullet\text{s/m}}{(0.126\text{T})(50)(0.0561\text{m})} - 1\right)100$$

$$= \boxed{-0.065\%}$$

(*c*) Use the theoretical equation to calculate the voltage at each experimental speed. Then calculate the percent difference at each speed.

Speed (m/s)	Induced Voltage (V)	Theoretical Induced Voltage (V)	% diff.
0.367	0.128	0.130	– 1.32%
0.379	0.135	0.134	0.78%
0.465	0.164	0.164	– 0.21%
0.623	0.221	0.220	0.37%
0.630	0.222	0.223	– 0.30%

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH29.XLS," on tab "Problem 29.80."