CHAPTER 28: Sources of Magnetic Field

Responses to Questions

$\boxed{1.}$ Alternating currents will have little effect on the compass needle, due to the rapid change of the direction of the current and of the magnetic field surrounding it. Direct currents will deflect a compass needle. The deflection depends on the magnitude and direction of the current and the distance from the current to the compass. The effect on the compass decreases with increasing distance from the wire.

2. The magnetic field due to a long straight current is proportional to the current strength. The electric field due to a long straight line of electric charge at rest is proportional to the charge per unit length. Both fields are inversely proportional to the distance from the wire or line of charge. The magnetic field lines form concentric circles around the wire; the electric field lines are directed radially outward if the line of charge is positive and radially inward if the line of charge is negative.

3. The magnetic forces exerted on one wire by the other try to align the wires. The net force on either wire is zero, but the net torque is not zero.

4. Yes. Assume the upper wire is fixed in position. Since the currents in the wires are in the same direction, the wires will attract each other. The lower wire will be held in equilibrium if this force of attraction (upward) is equal in magnitude to the weight of the wire (downward).

5. (*a*) The current in the lower wire is opposite in direction to the current in the upper wire.
   (*b*) The upper wire can be held in equilibrium due to the balance between the magnetic force from the lower wire and the gravitational force. The equilibrium will be stable for small vertical displacements, but not for horizontal displacements.

6. (*a*) Let $I_2 = I_1$. $\oint \vec{\mathbf{B}} \cdot d\vec{\ell} = \mu_0 I_{\text{encl}} = \mu_0 \left(I_1 + I_2 \right) = 2\mu_0 I_1$
   (*b*) Let $I_2 = -I_1$. $\oint \vec{\mathbf{B}} \cdot d\vec{\ell} = \mu_0 I_{\text{encl}} = \mu_0 \left(I_1 + I_2 \right) = 0$

$\boxed{7.}$ Inside the cavity $\vec{\mathbf{B}} = 0$ since the geometry is cylindrical and no current is enclosed.

8. Construct a closed path similar to that shown in part (*a*) of the figure, such that sides *ab* and *cd* are perpendicular to the field lines and sides *bc* and *da* lie along the field lines. Unlike part (*a*), the path will not form a rectangle; the sides *ab* and *cd* will flare outward so that side *bc* is longer than side *da*. Since the field is stronger in the region of *da* than it is in the region of *bc*, but *da* is shorter than *bc*, the contributions to the integral in Ampère's law may cancel. Thus, $\mu_0 I_{\text{encl}} = \oint \vec{\mathbf{B}} \cdot d\vec{\ell} = 0$ is possible and the field is consistent with Ampère's law. The lines could not curve upward instead of downward, because then *bc* would be shorter than *da* and it would not be possible for the contributions to sum to zero.

9. The equation for the magnetic field strength inside a solenoid is given by $B = \mu_0 n I$.
   (*a*) The magnetic field strength is not affected if the diameter of the loops doubles.
   (*b*) If the spacing between the loops doubles, the number of loops per unit length decreases by a factor of 2, and the magnetic field strength then also decreases by a factor of 2.

(*c*) If the solenoid's length is doubled along with the doubling of the total number of loops, then the number of loops per unit length remains the same, and the magnetic field strength is not affected.

10. The Biot-Savart law states that the net field at a point in space is the vector sum of the field contributions due to each infinitesimal current element. As shown in Example 28-12, the magnetic field along the axis of a current loop is parallel to the axis because the perpendicular field contributions cancel. However, for points off the axis, the perpendicular contributions will not cancel. The net field for a point off the axis will be dominated by the current elements closest to it. For example, in Figure 28-21, the field lines inside the loop but below the axis curve downward, because these points in space are closer to the lower segment of the loop (where the current goes into the page) than they are to the upper segment (where the current comes out of the page).

11. No. The magnetic field varies in strength and direction for points in the plane of the loop. The magnetic field is strongest at the center of the loop.

12. The lead-in wires to electrical devices have currents running in opposite directions. The magnetic fields due to these currents are therefore also opposite in direction. If the wires are twisted together, then the distance from a point outside the wires to each of the individual wires is about the same, and the field contributions from the two wires will cancel. If the wires were not twisted and were separate from each other, then a point outside the wires would be a different distance from one of the wires than from the other, and there would be a net field due to the currents in the wires.

13. The Biot-Savart law and Coulomb's law are both inverse-square in the radius and both contain a proportionality constant. Coulomb's law describes a central force; the Biot-Savart law involves a cross product of vectors and so cannot describe a central force.

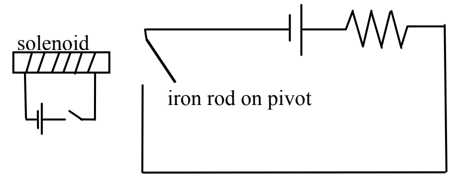
14. (*a*) The force between two identical electric charges is given by Coulomb's law: $F = \frac{kq^2}{r^2}$.

Magnetic pole strength of a bar magnet could be defined using an analogous expression for the magnetic force between the poles of two identical magnets: $F = \frac{\mu m^2}{4\pi r^2}$. Then, magnetic pole strength, *m*, would be given by $m = \sqrt{\frac{4\pi F r^2}{\mu}}$. To determine *m*, place two identical magnets with their poles facing each other a distance *r* apart and measure the force between them.

(*b*) The magnetic pole strength of a current loop could be defined the same way by using two identical current loops instead of two bar magnets.

15. Determine the magnetic field of the Earth at one of the magnetic poles (north or south), and use Equation 28-7b to calculate the magnetic moment. In this equation, *x* will be (approximately) the radius of the Earth.

16. To design a relay, place an iron rod inside a solenoid, with the solenoid oriented such that one end of it is facing a second iron rod on a pivot. The second iron rod functions as a switch for the large-current circuit and is normally held open by a spring. When current flows through the solenoid, the iron rod inside it becomes magnetized and attracts the second iron rod, closing the switch and allowing current to flow.



17. (*a*) The source of the kinetic energy is the attractive force produced by the magnetic field from the magnet acting on the magnetic moments of the atoms in the iron.
    (*b*) When the block strikes the magnet, some of the kinetic energy from the block is converted into kinetic energy in the iron atoms in the magnet, randomizing their magnetic moments and decreasing the overall field produced by the magnet. Some of the kinetic energy of the block as a whole is also converted into the kinetic energy of the individual atoms in the block, resulting in an increase in thermal energy.

18. No, a magnet with a steady field will only attract objects made of ferromagnetic materials. Aluminum is not ferromagnetic, so the magnetic field of the magnet will not cause the aluminum to become a temporary magnet and therefore there will be no attractive force. Iron is ferromagnetic, so in the presence of a magnet, the domains in a piece of iron will align such that it will be attracted to the magnet.

19. An unmagnetized nail has randomly oriented domains and will not generate an external magnetic field. Therefore, it will not attract an unmagnitized paper clip, which also has randomly oriented domains. When one end of the nail is in contact with a magnet, some of the domains in the nail align, producing an external magnetic field and turning the nail into a magnet. The magnetic nail will cause some of the domains in the paper clip to align, and it will be attracted to the nail.

20. Yes, an iron rod can attract a magnet and a magnet can attract an iron rod. Consider Newton's third law. If object A attracts object B then object B attracts object A.

21. Domains in ferromagnetic materials in molten form were aligned by the Earth's magnetic field and then fixed in place as the material cooled.

22. Yes. When a magnet is brought near an unmagnetized piece of iron, the magnet's field causes a temporary alignment of the domains of the iron. If the magnet's north pole is brought near the iron, then the domains align such that the temporary south pole of the iron is facing the magnet, and if the magnet's south pole is closest to the iron, then the alignment will be the opposite. In either case, the magnet and the iron will attract each other.

23. The two rods that have ends that repel each other will be the magnets. The unmagnetized rod will be attracted to both ends of the magnetized rods.

24. No. If they were both magnets, then they would repel one another when they were placed with like poles facing each other. However, if one is a magnet and the other isn't, they will attract each other no matter which ends are placed together. The magnet will cause an alignment of the domains of the non-magnet, causing an attraction.

25. (*a*) The magnetization curve for a paramagnetic substance is a straight line with slope slightly greater than 1. It passes through the origin; there is no hysteresis.
    (*b*) The magnetization curve for a diamagnetic substance is a straight line with slope slightly less than 1. It passes through the origin; there is no hysteresis.
    The magnetization curve for a ferromagnetic substance is a hysteresis curve (see Figure 28-29).

26. (*a*) Yes. Diamagnetism is present in all materials but in materials that are also paramagnetic or ferromagnetic, its effects will not be noticeable.
    (*b*) No. Paramagnetic materials are nonferromagnetic materials with a relative permeability greater than one.
    (*c*) No. Ferromagnetic materials are those that can be magnetized by alignment of their domains.

Solutions to Problems

$\boxed{1.}$ We assume the jumper cable is a long straight wire, and use Eq. 28-1.

$$B_{\text{cable}} = \frac{\mu_0 I}{2\pi r} = \frac{\left(4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A}\right)(65\text{A})}{2\pi(0.035\,\text{m})} = 3.714 \times 10^{-4}\,\text{T} \approx \boxed{3.7 \times 10^{-4}\,\text{T}}$$

Compare this to the Earth's field of $0.5 \times 10^{-4}\,\text{T}$.

$$B_{\text{cable}}/B_{\text{Earth}} = \frac{3.714 \times 10^{-4}\,\text{T}}{5.0 \times 10^{-5}\,\text{T}} = 7.43\text{, so } \boxed{\text{the field of the cable is over 7 times that of the Earth.}}$$

2. We assume that the wire is long and straight, and use Eq. 28-1.

$$B_{\text{wire}} = \frac{\mu_0 I}{2\pi r} \quad \to \quad I = \frac{2\pi r B_{\text{wire}}}{\mu_0} = \frac{2\pi(0.15\,\text{m})\left(0.50 \times 10^{-4}\,\text{T}\right)}{4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A}} = 37.5\,\text{A} \approx \boxed{38\,\text{A}}$$

3. Since the currents are parallel, the force on each wire will be attractive, toward the other wire. Use Eq. 28-2 to calculate the magnitude of the force.

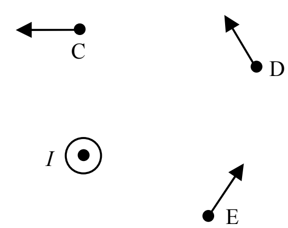
$$F_2 = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}\ell_2 = \frac{\left(4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A}\right)}{2\pi}\frac{(35\,\text{A})^2}{(0.040\,\text{m})}(25\,\text{m}) = \boxed{0.15\,\text{N, attractive}}$$

4. Since the force is attractive, the currents must be in the same direction, so the current in the second wire must also be upward. Use Eq. 28-2 to calculate the magnitude of the second current.

$$F_2 = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}\ell_2 \quad \to$$

$$I_2 = \frac{2\pi}{\mu_0}\frac{F_2}{\ell_2}\frac{d}{I_1} = \frac{2\pi}{4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A}}\left(7.8 \times 10^{-4}\,\text{N/m}\right)\frac{0.070\,\text{m}}{28\,\text{A}} = 9.75\,\text{A} \approx \boxed{9.8\,\text{A upward}}$$

5. To find the direction, draw a radius line from the wire to the field point. Then at the field point, draw a perpendicular to the radius line, directed so that the perpendicular line would be part of a counterclockwise circle.

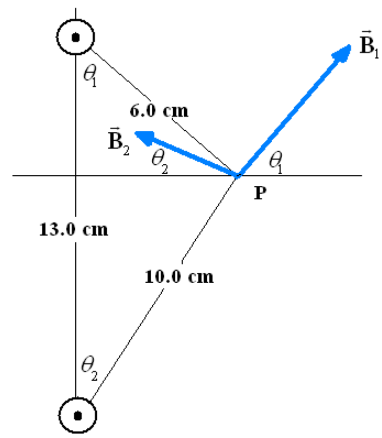


6. For the experiment to be accurate to $\pm 2.0\%$, the magnetic field due to the current in the cable must be less than or equal to 2.0% of the Earth's magnetic field. Use Eq. 28-1 to calculate the magnetic field due to the current in the cable.

$$B_{\text{cable}} = \frac{\mu_0 I}{2\pi r} \le 0.020 B_{\text{Earth}} \quad \to \quad I \le \frac{2\pi r\left(0.020 B_{\text{Earth}}\right)}{\mu_0} = \frac{2\pi(1.00\,\text{m})(0.020)\left(0.5 \times 10^{-4}\,\text{T}\right)}{4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A}} = 5.0\,\text{A}$$

Thus the maximum allowable current is $\boxed{5.0\,\text{A}}$.

7. Since the magnetic field from a current carrying wire circles the wire, the individual field at point P from each wire is perpendicular to the radial line from that wire to point P. We define $\vec{\mathbf{B}}_1$ as the field from the top wire, and $\vec{\mathbf{B}}_2$ as the field from the bottom wire. We use Eq. 28-1 to calculate the magnitude of each individual field.



$$B_1 = \frac{\mu_0 I}{2\pi r_1} = \frac{(4\pi\times10^{-7}\,\mathrm{T{\bullet}m/A})(35\,\mathrm{A})}{2\pi(0.060\,\mathrm{m})} = 1.17\times10^{-4}\,\mathrm{T}$$

$$B_2 = \frac{\mu_0 I}{2\pi r_2} = \frac{(4\pi\times10^{-7}\,\mathrm{T{\bullet}m/A})(35\,\mathrm{A})}{2\pi(0.100\,\mathrm{m})} = 7.00\times10^{-5}\,\mathrm{T}$$

We use the law of cosines to determine the angle that the radial line from each wire to point P makes with the vertical. Since the field is perpendicular to the radial line, this is the same angle that the magnetic fields make with the horizontal.

$$\theta_1 = \cos^{-1}\left(\frac{(0.060\text{ m})^2 + (0.130\text{ m})^2 - (0.100\text{ m})^2}{2(0.060\text{ m})(0.130\text{ m})}\right) = 47.7^\circ$$

$$\theta_2 = \cos^{-1}\left(\frac{(0.100\text{ m})^2 + (0.130\text{ m})^2 - (0.060\text{ m})^2}{2(0.100\text{ m})(0.130\text{ m})}\right) = 26.3^\circ$$

Using the magnitudes and angles of each magnetic field we calculate the horizontal and vertical components, add the vectors, and calculate the resultant magnetic field and angle.

$$B_{\text{net}\,x} = B_1\cos(\theta_1) - B_2\cos\theta_2 = (1.174\times10^{-4}\,\mathrm{T})\cos47.7^\circ - (7.00\times10^{-5}\,\mathrm{T})\cos26.3^\circ = 1.626\times10^{-5}\,\mathrm{T}$$

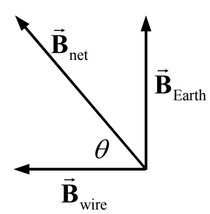
$$B_{\text{net}\,y} = B_1\sin(\theta_1) + B_2\sin\theta_1 = (1.17\times10^{-4}\,\mathrm{T})\sin47.7^\circ + (7.00\times10^{-5}\,\mathrm{T})\sin26.3^\circ = 1.18\times10^{-4}\,\mathrm{T}$$

$$B = \sqrt{B_{net,x}^2 + B_{net,y}^2} = \sqrt{(1.626\times10^{-5}\text{ T})^2 + (1.18\times10^{-4}\text{ T})^2} = 1.19\times10^{-4}\text{ T}$$

$$\theta = \tan^{-1}\frac{B_{net,y}}{B_{net,y}} = \tan^{-1}\frac{1.18\times10^{-4}\text{ T}}{1.626\times10^{-5}\text{ T}} = 82.2^\circ$$

$$\vec{\mathbf{B}} = 1.19\times10^{-4}\text{ T @ }82.2^\circ \approx \boxed{1.2\times10^{-4}\,\mathrm{T}\text{ @ }82^\circ}$$

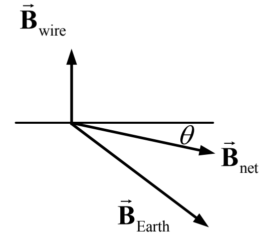
8. At the location of the compass, the magnetic field caused by the wire will point to the west, and the Earth's magnetic field points due North. The compass needle will point in the direction of the NET magnetic field.



$$B_{\text{wire}} = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi\times10^{-7}\,\mathrm{T{\bullet}m/A})(43\,\mathrm{A})}{2\pi(0.18\,\mathrm{m})} = 4.78\times10^{-5}\,\mathrm{T}$$

$$\theta = \tan^{-1}\frac{B_{\text{Earth}}}{B_{\text{wire}}} = \tan^{-1}\frac{4.5\times10^{-5}\,\mathrm{T}}{4.78\times10^{-5}\,\mathrm{T}} = \boxed{43^\circ\text{N of W}}$$

9. The magnetic field due to the long horizontal wire points straight up at the point in question, and its magnitude is given by Eq. 28-1. The two fields are oriented as shown in the diagram. The net field is the vector sum of the two fields.



$$B_{\text{wire}} = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi\times10^{-7}\,\mathrm{T{\bullet}m/A})(24.0\,\mathrm{A})}{2\pi(0.200\,\mathrm{m})} = 2.40\times10^{-5}\,\mathrm{T}$$

$$B_{\text{Earth}} = 5.0\times10^{-5}\,\text{T}$$

$$B_{\text{net}\,x} = B_{\text{Earth}}\cos 44^{\circ} = 3.60\times10^{-5}\,\text{T} \qquad B_{\text{net}\,y} = B_{\text{wire}} - B_{\text{Earth}}\sin 44^{\circ} = -1.07\times10^{-5}\,\text{T}$$

$$B_{\text{net}} = \sqrt{B_{\text{net}\,x}^2 + B_{\text{net}\,y}^2} = \sqrt{\left(3.60\times10^{-5}\,\text{T}\right)^2 + \left(-1.07\times10^{-5}\,\text{T}\right)^2} = \boxed{3.8\times10^{-5}\,\text{T}}$$

$$\theta = \tan^{-1}\frac{B_{\text{net}\,y}}{B_{\text{net}\,x}} = \tan^{-1}\frac{-1.07\times10^{-5}\,\text{T}}{3.60\times10^{-5}\,\text{T}} = \boxed{17^{\circ}\ \text{below the horizontal}}$$

10. The stream of protons constitutes a current, whose magnitude is found by multiplying the proton rate times the charge of a proton. Then use Eq. 28-1 to calculate the magnetic field.

$$B_{\text{stream}} = \frac{\mu_0 I}{2\pi r} = \frac{\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)\left(2.5\times10^{9}\,\text{protons/s}\right)\left(1.60\times10^{-19}\,\text{C/proton}\right)}{2\pi\left(2.0\,\text{m}\right)} = \boxed{4.0\times10^{-17}\,\text{T}}$$

11. (*a*) If the currents are in the same direction, the magnetic fields at the midpoint between the two currents will oppose each other, and so their magnitudes should be subtracted.

$$B_{\text{net}} = \frac{\mu_0 I_1}{2\pi r_1} - \frac{\mu_0 I_2}{2\pi r_2} = \frac{\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)}{2\pi\left(0.010\,\text{m}\right)}\left(I - 25\,\text{A}\right) = \boxed{\left(2.0\times10^{-5}\,\text{T/A}\right)\left(I - 25\,\text{A}\right)}$$

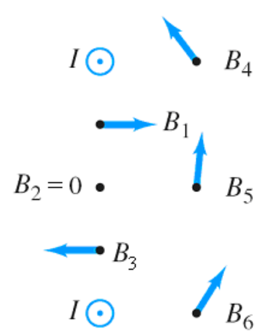
(*b*) If the currents are in the opposite direction, the magnetic fields at the midpoint between the two currents will reinforce each other, and so their magnitudes should be added.

$$B_{\text{net}} = \frac{\mu_0 I_1}{2\pi r_1} + \frac{\mu_0 I_2}{2\pi r_2} = \frac{\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)}{2\pi\left(0.010\,\text{m}\right)}\left(I + 25\,\text{A}\right) = \boxed{\left(2.0\times10^{-5}\,\text{T/A}\right)\left(I + 25\,\text{A}\right)}$$

12. Using the right-hand-rule we see that if the currents flow in the same direction, the magnetic fields will oppose each other between the wires, and therefore can equal zero at a given point. Set the sum of the magnetic fields from the two wires equal to zero at the point 2.2 cm from the first wire and use Eq. 28-1 to solve for the unknown current.

$$B_{net} = 0 = \frac{\mu_0 I_1}{2\pi r_1} - \frac{\mu_0 I_2}{2\pi r_2} \;\rightarrow\; I_2 = \left(\frac{r_2}{r_1}\right) I_1 = \left(\frac{6.0\,\text{cm} - 2.2\,\text{cm}}{2.2\,\text{cm}}\right)\left(2.0\,\text{A}\right) = \boxed{3.5\,\text{A}}$$

$\boxed{13.}$ Use the right hand rule to determine the direction of the magnetic field from each wire. Remembering that the magnetic field is inversely proportional to the distance from the wire, qualitatively add the magnetic field vectors. The magnetic field at point #2 is zero.



14. The fields created by the two wires will oppose each other, so the net field is the difference of the magnitudes of the two fields. The positive direction for the fields is taken to be into the paper, and so the closer wire creates a field in the positive direction, and the farther wire creates a field in the negative direction. Let *d* be the separation distance of the wires.

$$B_{\text{net}} = \frac{\mu_0 I}{2\pi r_{\text{closer}}} - \frac{\mu_0 I}{2\pi r_{\text{farther}}} = \frac{\mu_0 I}{2\pi}\left(\frac{1}{r_{\text{closer}}} - \frac{1}{r_{\text{farther}}}\right) = \frac{\mu_0 I}{2\pi}\left(\frac{1}{r - \frac{1}{2}d} - \frac{1}{r + \frac{1}{2}d}\right)$$

$$= \frac{\mu_0 I}{2\pi}\left(\frac{d}{\left(r-\frac{1}{2}d\right)\left(r+\frac{1}{2}d\right)}\right)$$

$$= \frac{\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)(28.0\,\text{A})}{2\pi}\left(\frac{0.0028\,\text{m}}{(0.10\,\text{m}-0.0014\,\text{m})(0.10\,\text{m}+0.0014\,\text{m})}\right)$$

$$= 1.568\times10^{-6}\,\text{T} \approx \boxed{1.6\times10^{-6}\,\text{T}}$$

Compare this to the Earth's field of $0.5\times10^{-4}\,\text{T}$.

$$B_{\text{net}}/B_{\text{Earth}} = \frac{1.568\times10^{-6}\,\text{T}}{0.5\times10^{-4}\,\text{T}} = 0.031$$

$\boxed{\text{The field of the wires is about 3\% that of the Earth.}}$

15. The center of the third wire is 5.6 mm from the left wire, and 2.8 mm from the right wire. The force on the near (right) wire will attract the near wire, since the currents are in the same direction. The force on the far (left) wire will repel the far wire, since the currents oppose each other. Use Eq. 28-2 to calculate the force per unit length.

$$F_{\text{near}} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d_{\text{near}}}\ell_{\text{near}} \quad \rightarrow$$

$$\frac{F_{\text{near}}}{\ell_{\text{near}}} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d_{\text{near}}} = \frac{4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}}{2\pi}\frac{(25.0\,\text{A})(28.0\,\text{A})}{2.8\times10^{-3}\,\text{m}} = \boxed{0.050\,\text{N/m, attractive}}$$

$$F_{\text{far}} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d_{\text{far}}}\ell_{\text{far}} \quad \rightarrow$$

$$\frac{F_{\text{far}}}{\ell_{\text{far}}} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d_{\text{far}}} = \frac{4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}}{2\pi}\frac{(25.0\,\text{A})(28.0\,\text{A})}{5.6\times10^{-3}\,\text{m}} = \boxed{0.025\,\text{N/m, repelling}}$$

16. (*a*) We assume that the power line is long and straight, and use Eq. 28-1.

$$B_{\text{line}} = \frac{\mu_0 I}{2\pi r} = \frac{\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)(95\,\text{A})}{2\pi(8.5\,\text{m})} = 2.235\times10^{-6}\,\text{T} \approx \boxed{2.2\times10^{-6}\,\text{T}}$$

The direction at the ground, from the right hand rule, is $\boxed{\text{south}}$. Compare this to the Earth's field of $0.5\times10^{-4}\,\text{T}$, which points approximately north.

$$B_{\text{line}}/B_{\text{Earth}} = \frac{2.235\times10^{-6}\,\text{T}}{0.5\times10^{-4}\,\text{T}} = 0.0447$$

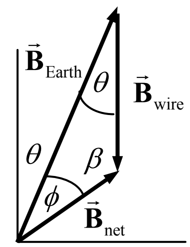
$\boxed{\text{The field of the cable is about 4\% that of the Earth.}}$

(*b*) We solve for the distance where $B_{\text{line}} = B_{\text{Earth}}$.

$$B_{\text{line}} = \frac{\mu_0 I}{2\pi r} = B_{\text{Earth}} \quad \rightarrow \quad r = \frac{\mu_0 I}{2\pi B_{\text{Earth}}} = \frac{\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)(95\,\text{A})}{2\pi\left(0.5\times10^{-4}\,\text{T}\right)} = 0.38\,\text{m} \approx \boxed{0.4\,\text{m}}$$

So about 0.4 m below the wire, the net B-field would be 0, assuming the Earth's field points straight north at this location.

17. The Earth's magnetic field is present at both locations in the problem, and we assume it is the same at both locations. The field east of a vertical wire must be pointing either due north or due south. The compass shows the direction of the net magnetic field, and it changes from 28° E of N to 55° E of N when taken inside. That is a "southerly" change (rather than a "northerly" change), and so the field due to the wire must be pointing due south. See the diagram. For the angles, $\theta = 28°$, $\theta + \phi = 55°$, and $\beta + \theta + \phi = 180°$ and so $\phi = 27°$ and $\beta = 125°$. Use the law of sines to find the magnitude of $\vec{\mathbf{B}}_{\text{wire}}$, and then use Eq. 28-1 to find the magnitude of the current.



$$\frac{B_{\text{wire}}}{\sin\phi} = \frac{B_{\text{Earth}}}{\sin\beta} \rightarrow B_{\text{wire}} = B_{\text{Earth}} \frac{\sin\phi}{\sin\beta} = \frac{\mu_0}{2\pi}\frac{I}{r} \rightarrow$$

$$I = B_{\text{Earth}} \frac{\sin\phi}{\sin\beta}\frac{2\pi}{\mu_0} r = \left(5.0\times10^{-5}\,\text{T}\right)\frac{\sin 27^\circ}{\sin 125^\circ}\frac{2\pi}{4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}}\left(0.120\,\text{m}\right) = \boxed{17\,\text{A}}$$

Since the field due to the wire is due south, the current in the wire must be $\boxed{\text{down}}$.

18. The magnetic field at the loop due to the long wire is into the page, and can be calculated by Eq. 28-1. The force on the segment of the loop closest to the wire is towards the wire, since the currents are in the same direction. The force on the segment of the loop farthest from the wire is away from the wire, since the currents are in the opposite direction.

Because the magnetic field varies with distance, it is more difficult to calculate the total force on the left and right segments of the loop. Using the right hand rule, the force on each small piece of the left segment of wire is to the left, and the force on each small piece of the right segment of wire is to the right. If left and right small pieces are chosen that are equidistant from the long wire, the net force on those two small pieces is zero. Thus the total force on the left and right segments of wire is zero, and so only the parallel segments need to be considered in the calculation. Use Eq. 28-2.

$$F_{\text{net}} = F_{\text{near}} - F_{\text{far}} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d_{\text{near}}}\ell_{\text{near}} - \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d_{\text{far}}}\ell_{\text{far}} = \frac{\mu_0}{2\pi} I_1 I_2 \ell\left(\frac{1}{d_{\text{near}}} - \frac{1}{d_{\text{far}}}\right)$$

$$= \frac{4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}}{2\pi}\left(3.5\,\text{A}\right)^2\left(0.100\,\text{m}\right)\left(\frac{1}{0.030\,\text{m}} - \frac{1}{0.080\,\text{m}}\right) = \boxed{5.1\times10^{-6}\,\text{N, towards wire}}$$

$\boxed{19.}$ The left wire will cause a field on the *x* axis that points in the *y* direction, and the right wire will cause a field on the *x* axis that points in the negative *y* direction. The distance from the left wire to a point on the *x* axis is *x*, and the distance from the right wire is $d - x$.

$$\vec{\mathbf{B}}_{\text{net}} = \frac{\mu_0 I}{2\pi x}\hat{\mathbf{j}} - \frac{\mu_0 I}{2\pi(d-x)}\hat{\mathbf{j}} = \boxed{\frac{\mu_0 I}{2\pi}\left(\frac{1}{x} - \frac{1}{d-x}\right)\hat{\mathbf{j}}} = \boxed{\frac{\mu_0 I}{2\pi}\left(\frac{d-2x}{x(d-x)}\right)\hat{\mathbf{j}}}$$

20. The left wire will cause a field on the *x* axis that points in the negative *y* direction, and the right wire will also cause a field on the *x* axis that points in the negative *y* direction. The distance from the left wire to a point on the *x* axis is *x*, and the distance from the right wire is $d - x$.

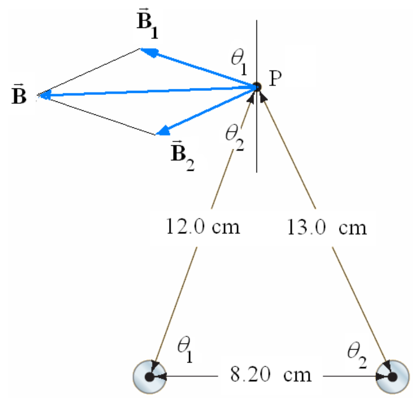
$$\vec{\mathbf{B}}_{\text{net}} = -\frac{\mu_0(2I)}{2\pi x}\hat{\mathbf{j}} - \frac{\mu_0 I}{2\pi(d-x)}\hat{\mathbf{j}} = \boxed{-\frac{\mu_0 I}{2\pi}\left(\frac{2}{x} + \frac{1}{d-x}\right)\hat{\mathbf{j}}}$$

21. The magnetic fields created by the individual currents will be at right angles to each other. The field due to the top wire will be to the right, and the field due to the bottom wire will be out of the page. Since they are at right angles, the net field is the hypotenuse of the two individual fields.

$$B_{\text{net}} = \sqrt{\left(\frac{\mu_0 I_{\text{top}}}{2\pi r_{\text{top}}}\right)^2 + \left(\frac{\mu_0 I_{\text{bottom}}}{2\pi r_{\text{bottom}}}\right)^2} = \frac{\mu_0}{2\pi r}\sqrt{I_{\text{top}}^2 + I_{\text{bottom}}^2} = \frac{4\pi\times 10^{-7}\,\text{T}\bullet\text{m/A}}{2\pi(0.100\,\text{m})}\sqrt{(20.0\,\text{A})^2 + (12.0\,\text{A})^2}$$

$$= \boxed{4.66\times 10^{-5}\,\text{T}}$$

22. The net magnetic field is the vector sum of the magnetic fields produced by each current carrying wire. Since the individual magnetic fields encircle the wire producing it, the field is perpendicular to the radial line from the wire to point P. We let $\vec{\mathbf{B}}_1$ be the field from the left wire, and $\vec{\mathbf{B}}_2$ designate the field from the right wire. The magnitude of the magnetic field vectors is calculated from Eq. 28-1.



$$B_1 = \frac{\mu_0 I}{2\pi r_1} = \frac{(4\pi\times 10^{-7}\,\text{T}\bullet\text{m/A})(16.5\,\text{A})}{2\pi(0.12\,\text{m})} = 2.7500\times 10^{-5}\,\text{T}$$

$$B_2 = \frac{\mu_0 I}{2\pi r_2} = \frac{(4\pi\times 10^{-7}\,\text{T}\bullet\text{m/A})(16.5\,\text{A})}{2\pi(0.13\,\text{m})} = 2.5385\times 10^{-5}\,\text{T}$$

We use the law of cosines to determine the angle that the radial line from each wire to point P makes with the horizontal. Since the magnetic fields are perpendicular to the radial lines, these angles are the same as the angles the magnetic fields make with the vertical.

$$\theta_1 = \cos^{-1}\left(\frac{(0.12\text{ m})^2 + (0.082\text{ m})^2 - (0.13\text{ m})^2}{2(0.12\text{ m})(0.082\text{ m})}\right) = 77.606^\circ$$

$$\theta_2 = \cos^{-1}\left(\frac{(0.13\text{ m})^2 + (0.082\text{ m})^2 - (0.12\text{ m})^2}{2(0.13\text{ m})(0.082\text{ m})}\right) = 64.364^\circ$$

Using the magnitudes and angles of each magnetic field we calculate the horizontal and vertical components, add the vectors, and calculate the resultant magnetic field and angle.

$$B_{\text{net}\,x} = -B_1\sin(\theta_1) - B_2\sin\theta_2 = -(2.7500\times 10^{-5}\text{T})\sin 77.606^\circ - (2.5385\times 10^{-5}\text{T})\sin 64.364^\circ$$
$$= -49.75\times 10^{-6}\text{T}$$

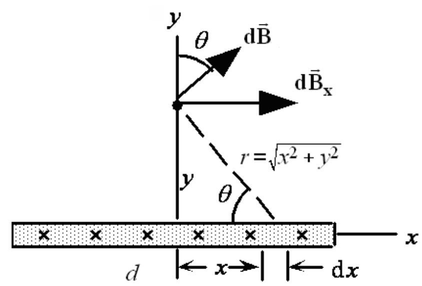
$$B_{\text{net}\,y} = B_1\cos(\theta_1) - B_2\cos\theta_1 = (2.7500\times 10^{-5}\text{T})\cos 77.606^\circ - (2.5385\times 10^{-5}\text{T})\cos 64.364^\circ$$
$$= -5.080\times 10^{-6}\text{T}$$

$$B = \sqrt{B_{net,x}^2 + B_{net,y}^2} = \sqrt{(-49.75\times 10^{-6}\text{ T})^2 + (-5.080\times 10^{-6}\text{ T})^2} = 5.00\times 10^{-5}\text{ T}$$

$$\theta = \tan^{-1}\frac{B_{net,y}}{B_{net,y}} = \tan^{-1}\frac{-5.08\times 10^{-6}\text{ T}}{-49.75\times 10^{-6}\text{ T}} = 5.83^\circ$$

$$\boxed{\vec{\mathbf{B}} = 5.00\times 10^{-5}\text{ T @ }5.83^\circ\text{ below the negative }x\text{-axis}}$$

23. (*a*) The net magnetic field a point y above the center of the strip can be found by dividing the strip into infinitely thin wires and integrating the field contribution from each wire. Since the point is directly above the center of the strip, we see that the vertical contributions to the magnetic field from symmetric points on either side of the center cancel out. Therefore, we only need to integrate the horizontal component of the magnetic field. We use Eq. 28-1 for the magnitude of the magnetic field, with the current given by



$$dI = \frac{I}{d}dx.$$

$$B_x = \int \frac{\mu_0 \sin\theta}{2\pi r} dI = \frac{\mu_0 I}{2\pi d}\int_{-\frac{1}{2}d}^{\frac{1}{2}d} \frac{dx}{\sqrt{x^2+y^2}}\left(\frac{y}{\sqrt{x^2+y^2}}\right) = \frac{\mu_0 I y}{2\pi d}\int_{-\frac{1}{2}d}^{\frac{1}{2}d}\frac{dx}{x^2+y^2}$$

$$= \frac{\mu_0 I y}{2\pi d}\frac{1}{y}\tan^{-1}\frac{x}{y}\bigg|_{-d/2}^{d/2} = \boxed{\frac{\mu_0 I}{\pi d}\tan^{-1}\left(\frac{d}{2y}\right)}$$

(*b*) In the limit of large y, $\tan^{-1} d/2y \approx d/2y$.

$$B_x = \frac{\mu_0 I}{\pi d}\tan^{-1}\left(\frac{d}{2y}\right) \approx \frac{\mu_0 I}{\pi d}\frac{d}{2y} = \boxed{\frac{\mu_0 I}{2\pi y}}$$

This is the same as the magnetic field for a long wire.

24. We break the current loop into the three branches of the triangle and add the forces from each of the three branches. The current in the parallel branch flows in the same direction as the long straight wire, so the force is attractive with magnitude given by Eq. 28-2.

$$F_1 = \frac{\mu_0 I I'}{2\pi d}a$$

By symmetry the magnetic force for the other two segments will be equal. These two wires can be broken down into infinitesimal segments, each with horizontal length dx. The net force is found by integrating Eq. 28-2 over the side of the triangle. We set x=0 at the left end of the left leg. The distance of a line segment to the wire is then given by $r = d + \sqrt{3}x$. Since the current in these segments flows opposite the direction of the current in the long wire, the force will be repulsive.

$$F_2 = \int_0^{a/2}\frac{\mu_0 I I'}{2\pi\left(d+\sqrt{3}x\right)}dx = \frac{\mu_0 I I'}{2\pi\sqrt{3}}\ln\left(d+\sqrt{3}x\right)\bigg|_0^{a/2} = \frac{\mu_0 I I'}{2\pi\sqrt{3}}\ln\left(1+\frac{\sqrt{3}a}{2d}\right)$$

We calculate the net force by summing the forces from the three segments.

$$F = F_1 - 2F_2 = \frac{\mu_0 I I'}{2\pi d}a - 2\frac{\mu_0 I I'}{2\pi\sqrt{3}}\ln\left(1+\frac{\sqrt{3}a}{2d}\right) = \boxed{\frac{\mu_0 I I'}{\pi}\left[\frac{a}{2d} - \frac{\sqrt{3}}{3}\ln\left(1+\frac{\sqrt{3}a}{2d}\right)\right]}$$

$\boxed{25.}$ Use Eq. 28-4 for the field inside a solenoid.

$$B = \frac{\mu_0 I N}{\ell} \rightarrow I = \frac{B\ell}{\mu_0 N} = \frac{\left(0.385\times10^{-3}\,\mathrm{T}\right)\left(0.400\,\mathrm{m}\right)}{\left(4\pi\times10^{-7}\,\mathrm{T\bullet m/A}\right)\left(765\right)} = \boxed{0.160\,\mathrm{A}}$$

26. The field inside a solenoid is given by Eq. 28-4.

$$B=\frac{\mu_0 IN}{\ell}\ \rightarrow\ N=\frac{B\ell}{\mu_0 I}=\frac{(0.30\,\text{T})(0.32\,\text{m})}{(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A})(4.5\,\text{A})}=\boxed{1.7\times10^4\,\text{turns}}$$

27. (*a*) We use Eq. 28-1, with *r* equal to the radius of the wire.

$$B=\frac{\mu_0 I}{2\pi r}=\frac{(4\pi\times10^{-7}\ \text{T}\bullet\text{m/A})(33\ \text{A})}{2\pi(1.25\times10^{-3}\ \text{m})}=\boxed{5.3\ \text{mT}}$$

(*b*) We use the results of Example 28-6, for points inside the wire. Note that $r=(1.25-0.50)\,\text{mm}$ $=0.75\,\text{mm}$.

$$B=\frac{\mu_0 Ir}{2\pi R^2}=\frac{(4\pi\times10^{-7}\text{T}\bullet\text{m/A})(33\,\text{A})(0.75\times10^{-3}\,\text{m})}{2\pi(1.25\times10^{-3}\,\text{m})^2}=\boxed{3.2\,\text{mT}}$$

(*c*) We use Eq. 28-1, with *r* equal to the distance from the center of the wire.

$$B=\frac{\mu_0 I}{2\pi r}=\frac{(4\pi\times10^{-7}\ \text{T}\bullet\text{m/A})(33\ \text{A})}{2\pi(1.25\times10^{-3}\ \text{m}+2.5\times10^{-3}\ \text{m})}=\boxed{1.8\ \text{mT}}$$

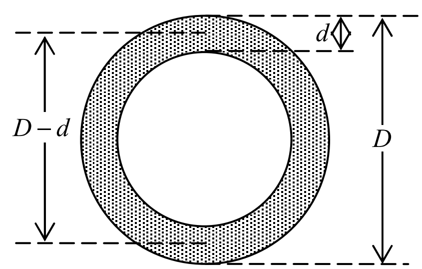
28. We use the results of Example 28-10 to find the maximum and minimum fields.

$$B_{\text{min}}=\frac{\mu_0 NI}{2\pi r_{\text{max}}}=\frac{(4\pi\times10^{-7}\ \text{T}\bullet\text{m/A})(687)(25.0\ \text{A})}{2\pi(0.270\ \text{m})}=12.7\ \text{mT}$$

$$B_{\text{max}}=\frac{\mu_0 NI}{2\pi r_{\text{min}}}=\frac{(4\pi\times10^{-7}\ \text{T}\bullet\text{m/A})(687)(25.0\ \text{A})}{2\pi(0.250\ \text{m})}=13.7\ \text{mT}$$

$$\boxed{12.7\ \text{mT}<B<13.7\ \text{mT}}$$

29. (*a*) The copper wire is being wound about an average diameter that is approximately equal to the outside diameter of the solenoid minus the diameter of the wire, or $D-d$. See the (not to scale) end-view diagram. The length of each wrapping is $\pi(D-d)$. We divide the length of the wire L by the length of a single winding to determine the number of loops. The length of the solenoid is the number of loops multiplied by the outer diameter of the wire, *d*.



$$\ell=d\frac{L}{\pi(D-d)}=(2.00\times10^{-3}\text{m})\frac{20.0\,\text{m}}{\pi\left[2.50\times10^{-2}\text{m}-(2.00\times10^{-3}\text{m})\right]}=\boxed{0.554\,\text{m}}$$

(*b*) The field inside the solenoid is found using Eq. 28-4. Since the coils are wound closely together, the number of turns per unit length is equal to the reciprocal of the wire diameter.

$$n=\frac{\#\ \text{turns}}{\ell}=\frac{\dfrac{L}{\pi(D-d)}}{\ell}=\frac{\ell/d}{\ell}=\frac{1}{d}$$

$$B=\mu_0 nI=\frac{\mu_0 I}{d}=\frac{(4\pi\times10^{-7}\text{T}\bullet\text{m/A})(16.7\,\text{A})}{2.00\times10^{-3}\text{m}}=\boxed{10.5\,\text{mT}}$$

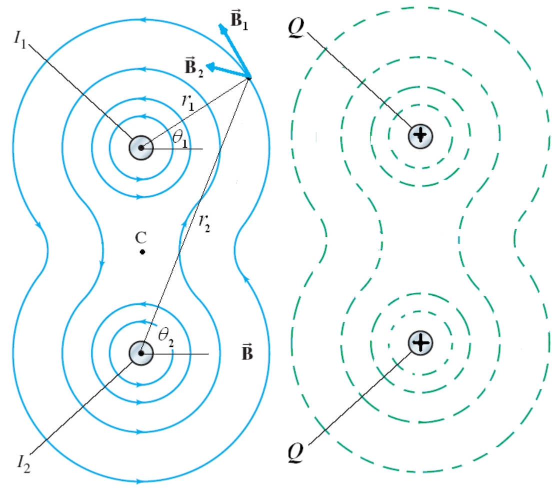
30. (*a*) The magnitude of the magnetic field from each wire is found using Eq. 28-1. The direction of the magnetic field is perpendicular to the radial vector from the current to the point of interest. Since the currents are both coming out of the page, the magnetic fields will point counterclockwise from the radial line. The total magnetic field is the vector sum of the individual fields.

$$\vec{\mathbf{B}} = \vec{\mathbf{B}}_1 + \vec{\mathbf{B}}_2 = \frac{\mu_0 I}{2\pi r_1}\left(-\sin\theta_1\hat{\mathbf{i}} + \cos\theta_1\hat{\mathbf{j}}\right) + \frac{\mu_0 I}{2\pi r_2}\left(-\sin\theta_2\hat{\mathbf{i}} + \cos\theta_2\hat{\mathbf{j}}\right)$$
$$= \frac{\mu_0 I}{2\pi}\left[\left(-\frac{\sin\theta_1}{r_1} - \frac{\sin\theta_1}{r_2}\right)\hat{\mathbf{i}} + \left(\frac{\cos\theta_1}{r_1} + \frac{\cos\theta_2}{r_2}\right)\hat{\mathbf{j}}\right]$$

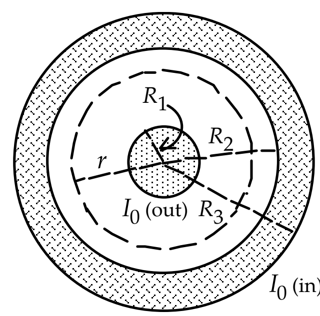
This equation for the magnetic field shows that the x-component of the magnetic field is symmetric and the y-component is anti-symmetric about $\theta = 90°$.

(*b*) See sketch.

(*c*) The two diagrams are similar in shape, as both form loops around the central axes. However, the magnetic field lines form a vector field, showing the direction, not necessarily the magnitude of the magnetic field. The equipotential lines are from a scalar field showing the points of constant magnitude. The equipotential lines do not have an associated direction.



31. Because of the cylindrical symmetry, the magnetic fields will be circular. In each case, we can determine the magnetic field using Ampere's law with concentric loops. The current densities in the wires are given by the total current divided by the cross-sectional area.



$$J_{\text{inner}} = \frac{I_0}{\pi R_1^2} \qquad J_{\text{outer}} = -\frac{I_0}{\pi\left(R_3^2 - R_2^2\right)}$$

(*a*) Inside the inner wire the enclosed current is determined by the current density of the inner wire.

$$\oint \vec{\mathbf{B}} \cdot d\vec{s} = \mu_0 I_{\text{encl}} = \mu_0\left(J_{\text{inner}}\pi R^2\right)$$
$$B(2\pi R) = \mu_0 \frac{I_0 \pi R^2}{\pi R_1^2} \rightarrow \boxed{B = \frac{\mu_0 I_0 R}{2\pi R_1^2}}$$

(*b*) Between the wires the current enclosed is the current on the inner wire.

$$\oint \vec{\mathbf{B}} \cdot d\vec{s} = \mu_0 I_{\text{encl}} \quad\rightarrow\quad B(2\pi R) = \mu_0 I_0 \quad\rightarrow\quad \boxed{B = \frac{\mu_0 I_0}{2\pi R}}$$

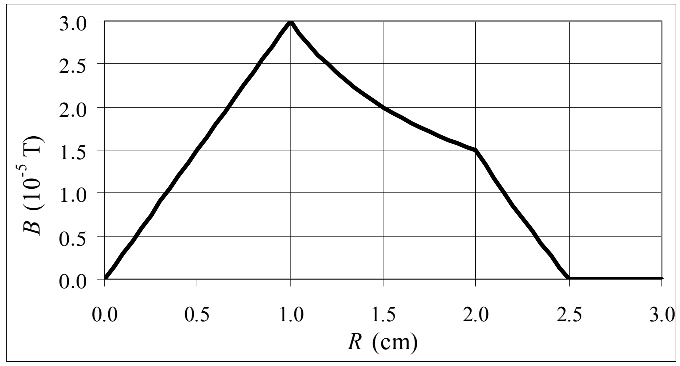
(*c*) Inside the outer wire the current enclosed is the current from the inner wire and a portion of the current from the outer wire.

$$\oint \vec{\mathbf{B}} \cdot d\vec{s} = \mu_0 I_{\text{encl}} = \mu_0\left[I_0 + J_{\text{outer}}\pi\left(R^2 - R_2^2\right)\right]$$
$$B(2\pi r) = \mu_0\left[I_0 - I_0\frac{\pi\left(R^2 - R_2^2\right)}{\pi\left(R_3^2 - R_2^2\right)}\right] \rightarrow \boxed{B = \frac{\mu_0 I_0}{2\pi R}\frac{\left(R_3^2 - R^2\right)}{\left(R_3^2 - R_2^2\right)}}$$

(*d*) Outside the outer wire the net current enclosed is zero.

$$\oint \vec{\mathbf{B}} \cdot d\vec{s} = \mu_0 I_{\text{encl}} = 0 \rightarrow B(2\pi R) = 0 \rightarrow \boxed{B = 0}$$

(*e*) See the adjacent graph. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH28.XLS," on tab "Problem 28.31e."



32. We first find the constants C_1 and C_2 by integrating the currents over each cylinder and setting the integral equal to the total current.

$$I_0 = \int_0^{R_1} C_1 R 2\pi R dR = 2\pi C_1 \int_0^{R_1} R^2 dR = \frac{2}{3}\pi R_1^3 C_1 \rightarrow C_1 = \frac{3I_0}{2\pi R_1^3}$$

$$-I_0 = 2\pi C_2 \int_{R_2}^{R_3} R^2 dR = \frac{2}{3}\pi\left(R_3^3 - R_2^3\right)C_2 \rightarrow C_2 = \frac{-3I_0}{2\pi\left(R_3^3 - R_2^3\right)}$$

(*a*) Inside the inner wire the enclosed current is determined by integrating the current density inside the radius R.

$$\oint \vec{\mathbf{B}} \cdot d\vec{s} = \mu_0 I_{\text{encl}} = \mu_0 \int_0^R (C_1 R') 2\pi R' dR' = \frac{2}{3}\mu_0 \pi C_1 R^3 = \frac{2}{3}\mu_0 \pi \left(\frac{3I_0}{2\pi R_1^3}\right) R^3$$

$$B(2\pi R) = \mu_0 \frac{I_0 \pi R^3}{\pi R_1^3} \rightarrow \boxed{B = \frac{\mu_0 I_0 R^2}{2\pi R_1^3}}$$

(*b*) Between the wires the current enclosed is the current on the inner wire.

$$\oint \vec{\mathbf{B}} \cdot d\vec{s} = \mu_0 I_{\text{encl}} \rightarrow B(2\pi R) = \mu_0 I_0 \rightarrow \boxed{B = \frac{\mu_0 I_0}{2\pi R}}$$

(*c*) Inside the outer wire the current enclosed is the current from the inner wire and a portion of the current from the outer wire.

$$\oint \vec{\mathbf{B}} \cdot d\vec{s} = \mu_0 I_{\text{encl}} = \mu_0 \left[I_0 + \int_{R_2}^{R} (C_2 R) 2\pi R dR \right] = \mu_0 \left[I_0 + \int_{R_2}^{R} (C_2 R) 2\pi R dR \right]$$

$$= \mu_0 I_0 \left[1 - \frac{2}{3}\pi C_2 \left(R^3 - R_2^3\right) \right] = \mu_0 \left[I_0 - \frac{\left(R^3 - R_2^3\right)}{\left(R_3^3 - R_2^3\right)} \right]$$

$$B(2\pi r) = \mu_0 I_0 \left[\frac{\left(R_3^3 - R_2^3\right)}{\left(R_3^3 - R_2^3\right)} - \frac{\left(R^3 - R_2^3\right)}{\left(R_3^3 - R_2^3\right)} \right] \rightarrow \boxed{B = \frac{\mu_0 I_0}{2\pi R} \frac{\left(R_3^3 - R^3\right)}{\left(R_3^3 - R_2^3\right)}}$$

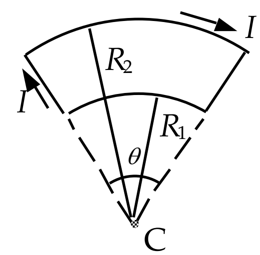
(*d*) Outside the outer wire the net current enclosed is zero.

$$\oint \vec{\mathbf{B}} \cdot d\vec{s} = \mu_0 I_{\text{encl}} = 0 \rightarrow B(2\pi R) = 0 \rightarrow \boxed{B = 0}$$

33. Use Eq. 28-7b to write a ratio of the magnetic fields at the surface of the earth and 13,000 km above the surface. Use the resulting ratio to determine the magnetic field above the surface.

$$\frac{B_2}{B_1}=\frac{\frac{\mu_0}{2\pi}\frac{\mu}{x_2^3}}{\frac{\mu_0}{2\pi}\frac{\mu}{x_1^3}}=\frac{x_1^3}{x_2^3}\quad\rightarrow\quad B_2=B_1\frac{x_1^3}{x_2^3}=\left(1.0\times10^{-4}\text{T}\right)\left(\frac{6.38\times10^3\text{km}}{19.38\times10^3\text{km}}\right)^3=\boxed{3.6\times10^{-6}\text{T}}$$

34. Since the point C is along the line of the two straight segments of the current, these segments do not contribute to the magnetic field at C. We calculate the magnetic field by integrating Eq. 28-5 along the two curved segments. Along each integration the line segment is perpendicular to the radial vector and the radial distance is constant.



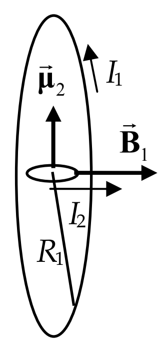
$$\vec{\mathbf{B}}=\frac{\mu_0 I}{4\pi}\int_0^{R_1\theta}\frac{d\vec{\ell}\times\hat{r}}{R_1^2}+\frac{\mu_0 I}{4\pi}\int_{R_2\theta}^{0}\frac{d\vec{\ell}\times\hat{r}}{R_2^2}=\frac{\mu_0 I}{4\pi R_1^2}\hat{\mathbf{k}}\int_0^{R_1\theta}ds+\frac{\mu_0 I}{4\pi R_2^2}\hat{\mathbf{k}}\int_{R_2\theta}^{0}ds$$

$$=\frac{\mu_0 I\theta}{4\pi R_1}\hat{\mathbf{k}}-\frac{\mu_0 I\theta}{4\pi R_2}\hat{\mathbf{k}}=\boxed{\frac{\mu_0 I\theta}{4\pi}\left(\frac{R_2-R_1}{R_1R_2}\right)\hat{\mathbf{k}}}$$

35. Since the current in the two straight segments flows radially toward and away from the center of the loop, they do not contribute to the magnetic field at the center. We calculate the magnetic field by integrating Eq. 28-5 along the two curved segments. Along each integration segment, the current is perpendicular to the radial vector and the radial distance is constant. By the right-hand-rule the magnetic field from the upper portion will point into the page and the magnetic field from the lower portion will point out of the page.

$$\vec{\mathbf{B}}=\frac{\mu_0 I_1}{4\pi}\int_{\text{upper}}\frac{ds}{R^2}\hat{\mathbf{k}}+\frac{\mu_0 I_2}{4\pi}\int_{\text{lower}}\frac{ds}{R^2}\left(-\hat{\mathbf{k}}\right)=\frac{\mu_0\left(\pi R\right)}{4\pi R^2}\hat{\mathbf{k}}\left(I_1-I_2\right)=\frac{\mu_0}{4R}\hat{\mathbf{k}}\left(0.35I-0.65I\right)=\boxed{-\frac{3\mu_0 I}{40R}}$$

36. We assume that the inner loop is sufficiently small that the magnetic field from the larger loop can be considered to be constant across the surface of the smaller loop. The field at the center of the larger loop is illustrated in Example 28-12. Use Eq. 27-10 to calculate the magnetic moment of the small loop, and Eq. 27-11 to calculate the torque.



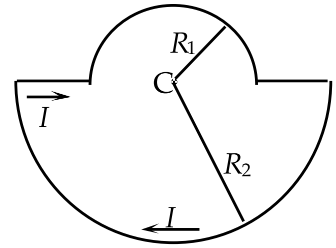
$$\vec{\mathbf{B}}=\frac{\mu_0 I}{2R}\hat{\mathbf{i}}\qquad\qquad \vec{\boldsymbol{\mu}}=I\vec{\mathbf{A}}=I\pi R_2^2\hat{\mathbf{j}}$$

$$\vec{\boldsymbol{\tau}}=\vec{\boldsymbol{\mu}}\times\vec{\mathbf{B}}=I\pi R_2^2\hat{\mathbf{j}}\times\frac{\mu_0 I}{2R}\hat{\mathbf{i}}=-\frac{\mu_0\pi I^2R_2^2}{2R}\hat{\mathbf{k}}$$

$$=-\frac{\left(4\pi\times10^{-7}\ \text{T}\bullet\text{m/A}\right)\pi\left(7.0\ \text{A}\right)^2\left(0.018\ \text{m}\right)^2}{2\left(0.25\ \text{m}\right)}\hat{\mathbf{k}}=\boxed{-1.3\times10^{-7}\hat{\mathbf{k}}\ \text{m}\bullet\text{N}}$$

This torque would cause the inner loop to rotate into the same plane as the outer loop with the currents flowing in the same direction.

$\boxed{37.}$ (*a*) The magnetic field at point C can be obtained using the Biot-Savart law (Eq. 28-5, integrated over the current). First break the loop into four sections: 1) the upper semi-circle, 2) the lower semi-circle, 3) the right straight segment, and 4) the left straight segment. The two straight segments do not contribute to the magnetic field as the point C is in the same direction that the



current is flowing. Therefore, along these segments $\hat{r}$ and $d\hat{\ell}$ are parallel and $d\hat{\ell}\times\hat{r}=0$. For the upper segment, each infinitesimal line segment is perpendicular to the constant magnitude radial vector, so the magnetic field points downward with constant magnitude.

$$\vec{\mathbf{B}}_{\text{upper}}=\int\frac{\mu_0 I}{4\pi}\frac{d\hat{\ell}\times\hat{r}}{r^2}=\frac{\mu_0 I}{4\pi}\frac{-\hat{k}}{R_1^{\,2}}\left(\pi R_1\right)=-\frac{\mu_0 I}{4R_1}\hat{k}\,.$$

Along the lower segment, each infinitesimal line segment is also perpendicular to the constant radial vector.

$$\vec{\mathbf{B}}_{\text{lower}}=\int\frac{\mu_0 I}{4\pi}\frac{d\hat{\ell}\times\hat{r}}{r^2}=\frac{\mu_0 I}{4\pi}\frac{-\hat{k}}{R_2^{\,2}}\left(\pi R_2\right)=-\frac{\mu_0 I}{4R_2}\hat{k}$$

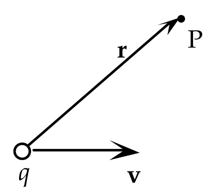
Adding the two contributions yields the total magnetic field.

$$\vec{\mathbf{B}}=\vec{\mathbf{B}}_{\text{upper}}+\vec{\mathbf{B}}_{\text{lower}}=-\frac{\mu_0 I}{4R_1}\hat{k}-\frac{\mu_0 I}{4R_2}\hat{k}=\boxed{-\frac{\mu_0 I}{4}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)\hat{k}}$$

(*b*) The magnetic moment is the product of the area and the current. The area is the sum of the two half circles. By the right-hand-rule, curling your fingers in the direction of the current, the thumb points into the page, so the magnetic moment is in the $-\hat{k}$ direction.

$$\vec{\boldsymbol{\mu}}=-\left(\frac{\pi R_1^2}{2}+\frac{\pi R_2^2}{2}\right)I\hat{k}=\boxed{\frac{-\pi I}{2}\left(R_1^2+R_2^2\right)\hat{k}}$$

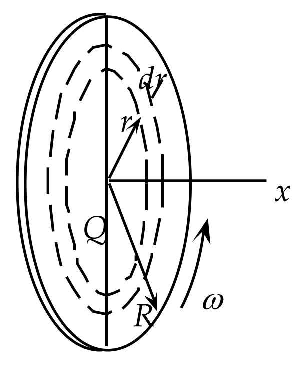
38. Treat the moving point charge as a small current segment. We can write the product of the charge and velocity as the product of a current and current segment. Inserting these into the Biot-Savart law gives us the magnetic field at point P.



$$q\vec{\mathbf{v}}=q\frac{d\vec{\boldsymbol{\ell}}}{dt}=\frac{dq}{dt}d\vec{\boldsymbol{\ell}}=Id\vec{\boldsymbol{\ell}}$$

$$\vec{\mathbf{B}}=\frac{\mu_0 I}{4\pi}\frac{d\vec{\boldsymbol{\ell}}\times\hat{r}}{r^2}=\frac{\mu_0}{4\pi}\frac{q\vec{\mathbf{v}}\times\hat{r}}{r^2}=\boxed{\frac{\mu_0}{4\pi}\frac{q\vec{\mathbf{v}}\times\vec{\mathbf{r}}}{r^3}}$$

39. (*a*) The disk can be broken down into a series of infinitesimal thick rings. As the charge in each of these rings rotates it produces a current of magnitude $dI=\left(\omega/2\pi\right)dq$, where dq is the surface charge density multiplied by the area of the ring. We use Eq. 27-10 to calculate the magnetic dipole moment of each current loop and integrate the dipole moments to obtain the total magnetic dipole moment.



$$d\vec{\boldsymbol{\mu}}=dI\vec{\mathbf{A}}=\left(\frac{Q}{\pi R^2}2\pi r dr\frac{\omega}{2\pi}\right)\left(\pi r^2\right)=\frac{Q\omega}{R^2}r^3dr\hat{\mathbf{i}}$$

$$\vec{\boldsymbol{\mu}}=\int_0^R\frac{Q\omega}{R^2}r^3dr\,\hat{\mathbf{i}}=\boxed{\frac{Q\omega R^2}{4}\hat{\mathbf{i}}}$$

(*b*) To find the magnetic field a distance *x* along the axis of the disk, we again consider the disk as a series of concentric currents. We use the results of Example 28-12 to determine the magnetic field from each current loop in the disk, and then integrate to obtain the total magnetic field.

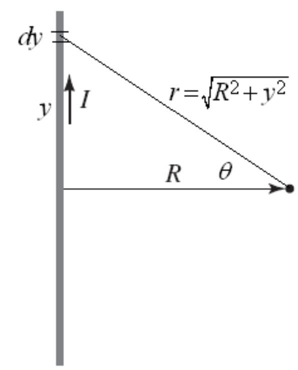
$$d\vec{\mathbf{B}}=\frac{\mu_0 r^2}{2\left(r^2+x^2\right)^{\frac{3}{2}}}dI=\frac{\mu_0 r^2}{2\left(r^2+x^2\right)^{\frac{3}{2}}}\frac{Q\omega}{\pi R^2}rdr$$

$$\vec{\mathbf{B}} = \frac{\mu_0 Q\omega\hat{\mathbf{i}}}{2\pi R^2}\int_0^R \frac{r^3}{\left(r^2+x^2\right)^{\frac{3}{2}}}dr = \frac{\mu_0 Q\omega\hat{\mathbf{i}}}{2\pi R^2}\frac{\left(r^2+2x^2\right)}{\sqrt{r^2+x^2}}\Bigg|_0^R = \boxed{\frac{\mu_0 Q\omega\hat{\mathbf{i}}}{2\pi R^2}\left[\frac{\left(R^2+2x^2\right)}{\sqrt{R^2+x^2}}-2x\right]}$$

(*c*) When we take the limit $x \gg R$ our equation reduces to Eq. 28-7b.

$$\vec{\mathbf{B}} \approx \frac{\mu_0 Q\omega\hat{\mathbf{i}}}{2\pi R^2}\left[2x\left(1+\frac{R^2}{2x^2}\right)\left(1-\frac{R^2}{2x^2}+\frac{3R^4}{8x^4}+...\right)-2x\right] \approx \frac{\mu_0 Q\omega\hat{\mathbf{i}}}{2\pi R^2}\left(\frac{R^4}{4x^3}\right) = \boxed{\frac{\mu_0}{2\pi}\frac{\vec{\boldsymbol{\mu}}}{x^3}}$$

40. (*a*) Choose the *y* axis along the wire and the *x* axis passing from the center of the wire through the point P. With this definition we calculate the magnetic field at P by integrating Eq. 28-5 over the length of the wire. The origin is at the center of the wire.



$$\vec{\mathbf{B}} = \int \frac{\mu_0 I}{4\pi}\frac{d\vec{\boldsymbol{\ell}}\times\hat{r}}{r^2} = \int \frac{\mu_0 I}{4\pi}\frac{d\vec{\boldsymbol{\ell}}\times\vec{\mathbf{r}}}{r^3} = \frac{\mu_0 I}{4\pi}\int_{-\frac{1}{2}d}^{\frac{1}{2}d}\frac{dy\hat{\mathbf{j}}\times\left(R\hat{\mathbf{i}}-y\hat{\mathbf{j}}\right)}{\left(R^2+y^2\right)^{3/2}}$$

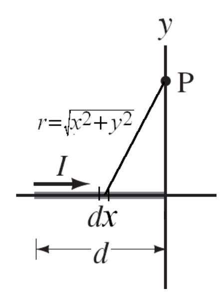
$$= -\frac{\mu_0 IR}{4\pi}\hat{\mathbf{k}}\int_{-\frac{1}{2}d}^{\frac{1}{2}d}\frac{dy}{\left(R^2+y^2\right)^{3/2}}$$

$$= -\frac{\mu_0 IR}{4\pi}\hat{\mathbf{k}}\frac{y}{R^2\left(R^2+y^2\right)^{1/2}}\Bigg|_{-d/2}^{d/2} = \boxed{-\frac{\mu_0 I}{2\pi R}\frac{d}{\left(4R^2+d^2\right)^{1/2}}\hat{\mathbf{k}}}$$

(*b*) If we take the limit as $d \to \infty$, this equation reduces to Eq. 28-1.

$$B = \lim_{d\to\infty}\left(\frac{\mu_0 I}{2\pi R}\frac{d}{\left(4R^2+d^2\right)^{1/2}}\right) \approx \frac{\mu_0 I}{2\pi R}$$

41. (*a*) The magnetic field at point Q can be obtained by integrating Eq. 28-5 over the length of the wire. In this case, each infinitesimal current segment dℓ is parallel to the *x* axis, as is each radial vector. Since the magnetic field is proportional to the cross-product of the current segment and the radial vector, each segment contributes zero field. Thus the magnetic field at point Q is $\boxed{\text{zero.}}$

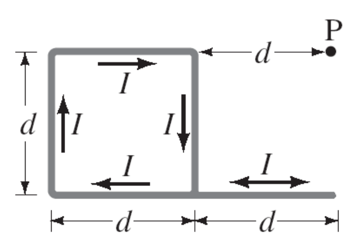


(*b*) The magnetic field at point P is found by integrating Eq. 28-5 over the length of the current segment.

$$\vec{\mathbf{B}} = \int \frac{\mu_0 I}{4\pi}\frac{d\vec{\boldsymbol{\ell}}\times\hat{r}}{r^2} = \int \frac{\mu_0 I}{4\pi}\frac{d\vec{\boldsymbol{\ell}}\times\vec{\mathbf{r}}}{r^3} = \frac{\mu_0 I}{4\pi}\int_{-d}^{0}\frac{dx\hat{\mathbf{i}}\times\left(-x\hat{\mathbf{i}}+y\hat{\mathbf{j}}\right)}{\left(x^2+y^2\right)^{3/2}} = \frac{\mu_0 Iy}{4\pi}\hat{\mathbf{k}}\int_{-d}^{0}\frac{dx}{\left(x^2+y^2\right)^{3/2}}$$

$$= \frac{\mu_0 Iy}{4\pi}\hat{\mathbf{k}}\frac{x}{y^2\left(x^2+y^2\right)^{1/2}}\Bigg|_{-d}^{0} = \boxed{\frac{\mu_0 I}{4\pi y}\frac{d}{\left(y^2+d^2\right)^{1/2}}\hat{\mathbf{k}}}$$

42. We treat the loop as consisting of 5 segments, The first has length *d*, is located a distance *d* to the left of point P, and has current flowing toward the right. The second has length *d*, is located a distance 2*d* to left of point P, and has current flowing upward. The third has length *d*, is located a distance *d* to the left of point P, and has current flowing downward. The fourth has length 2*d*, is located a distance *d*



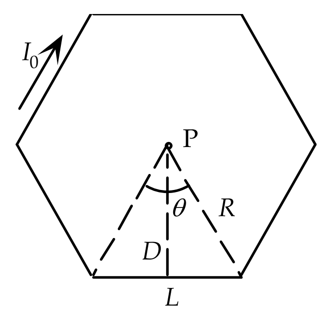
below point P, and has current flowing toward the left. Note that the fourth segment is twice as long as the actual fourth current. We therefore add a fifth line segment of length *d*, located a distance *d* below point P with current flowing to the right. This fifth current segment cancels the added portion, but allows us to use the results of Problem 41 in solving this problem. Note that the first line points radially toward point P, and therefore by Problem 41(*a*) does not contribute to the net magnetic field. We add the contributions from the other four segments, with the contribution in the positive *z*-direction if the current in the segment appears to flows counterclockwise around the point P.

$$\vec{\mathbf{B}} = \vec{\mathbf{B}}_2 + \vec{\mathbf{B}}_3 + \vec{\mathbf{B}}_4 + \vec{\mathbf{B}}_5$$

$$= -\frac{\mu_0 I}{4\pi(2d)}\frac{d}{\left(4d^2+d^2\right)^{1/2}}\hat{\mathbf{k}} + \frac{\mu_0 I}{4\pi d}\frac{d}{\left(d^2+d^2\right)^{1/2}}\hat{\mathbf{k}} - \frac{\mu_0 I}{4\pi d}\frac{2d}{\left(d^2+4d^2\right)^{1/2}}\hat{\mathbf{k}} + \frac{\mu_0 I}{4\pi d}\frac{d}{\left(d^2+d^2\right)^{1/2}}\hat{\mathbf{k}}$$

$$= \boxed{\frac{\mu_0 I}{4\pi d}\left(\sqrt{2} - \frac{\sqrt{5}}{2}\right)\hat{\mathbf{k}}}$$

43. (*a*) The angle subtended by one side of a polygon, θ, from the center point P is 2π divided by the number of sides, *n*. The length of the side *L* and the distance from the point to the center of the side, *D*, are obtained from trigonometric relations.



$$L = 2R\sin(\theta/2) = 2R\sin(\pi/n)$$

$$D = R\cos(\theta/2) = R\cos(\pi/n)$$

The magnetic field contribution from each side can be found using the result of Problem 40.

$$B = \frac{\mu_0 I}{2\pi D}\frac{L}{\left(L^2+4D^2\right)^{\frac{1}{2}}} = \frac{\mu_0 I}{2\pi\left(R\cos(\pi/n)\right)}\frac{2R\sin(\pi/n)}{\left(\left(2R\sin(\pi/n)\right)^2 + 4\left(R\cos(\pi/n)\right)^2\right)^{\frac{1}{2}}}$$

$$= \frac{\mu_0 I}{2\pi R}\tan(\pi/n)$$

The contributions from each segment add, so the total magnetic field is *n* times the field from one side.

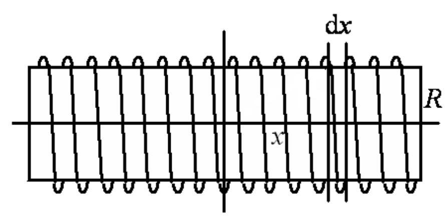
$$\boxed{B_{\text{total}} = \frac{\mu_0 I n}{2\pi R}\tan(\pi/n)}$$

(*b*) In the limit of large *n*, π/n, becomes very small, so $\tan(\pi/n) \approx \pi/n$.

$$B_{\text{total}} = \frac{\mu_0 I n}{2\pi R}\frac{\pi}{n} = \boxed{\frac{\mu_0 I}{2R}}$$

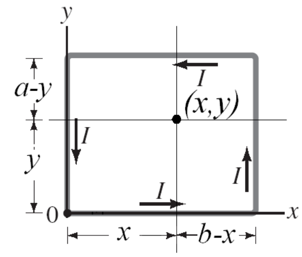
This is the magnetic field at the center of a circle.

44. The equation derived in Eq. 28-12 gives the magnetic field a distance *x* from a single loop. We expand this single loop to the field of an infinite solenoid by multiplying the field from a single loop by *n dx*, the density of loops times the infinitesimal thickness, and integrating over all values of *x*. Use the table in Appendix B-4 to evaluate the integral.



$$B = \int_{-\infty}^{\infty}\frac{\mu_0 I R^2 n\,dx}{2\left(R^2+x^2\right)^{\frac{3}{2}}} = \frac{\mu_0 I R^2 n}{2}\int_{-\infty}^{\infty}\frac{dx}{\left(R^2+x^2\right)^{\frac{3}{2}}} = \frac{\mu_0 I R^2 n}{2}\left.\frac{x}{R^2\left(R^2+x^2\right)^{\frac{1}{2}}}\right|_{-\infty}^{\infty} = \boxed{\mu_0 I n}$$

45. To find the magnetic field at point (x,y) we break each current segment into two segments and sum fields from each of the eight segments to determine the magnetic field at the center. We use the results of Problem 41(b) to calculate the magnetic field of each segment.



$$\vec{\mathbf{B}} = \frac{\mu_0 I}{4\pi y}\frac{x}{\left(y^2+x^2\right)^{1/2}}\hat{\mathbf{k}} + \frac{\mu_0 I}{4\pi y}\frac{(b-x)}{\left(y^2+(b-x)^2\right)^{1/2}}\hat{\mathbf{k}}$$

$$+\frac{\mu_0 I}{4\pi(b-x)}\frac{y}{\left((b-x)^2+y^2\right)^{1/2}}\hat{\mathbf{k}} + \frac{\mu_0 I}{4\pi(b-x)}\frac{(a-y)}{\left((a-y)^2+(b-x)^2\right)^{1/2}}\mathbf{k}$$

$$+\frac{\mu_0 I}{4\pi(a-y)}\frac{(b-x)}{\left((a-y)^2+(b-x)^2\right)^{1/2}}\hat{\mathbf{k}} + \frac{\mu_0 I}{4\pi(a-y)}\frac{x)}{\left((a-y)^2+x^2\right)^{1/2}}\hat{\mathbf{k}}$$

$$+\frac{\mu_0 I}{4\pi x}\frac{(a-y)}{\left((a-y)^2+x^2\right)^{1/2}}\hat{\mathbf{k}} + \frac{\mu_0 I}{4\pi x}\frac{y}{\left(y^2+x^2\right)^{1/2}}\hat{\mathbf{k}}$$

We simplify this equation by factoring out common constants and combining terms with similar roots.

$$\vec{\mathbf{B}} = \boxed{\frac{\mu_0 I}{4\pi}\left(\frac{\sqrt{y^2+x^2}}{xy} + \frac{\sqrt{y^2+(b-x)^2}}{(b-x)y} + \frac{\sqrt{(a-y)^2+(b-x)^2}}{(a-y)(b-x)} + \frac{\sqrt{(a-y)^2+x^2}}{x(a-y)}\right)\hat{\mathbf{k}}}$$

46. (a) By symmetry we see that on the x axis the magnetic field can only have an x component. To justify this assertion, imagine that the magnetic field had a component off the axis. If the current loop were rotated by 90° about the x axis, the loop orientation would be identical to the original loop, but the off-axis magnetic field component would have changed. This is not possible, so the field only has an x component. The contribution to this field is the same for each loop segment, and so the total magnetic field is equal to 4 times the x component of the magnetic field from one segment. We integrate Eq. 28-5 to find this magnetic field.

$$\vec{\mathbf{B}} = 4\int_{-\frac{1}{2}d}^{\frac{1}{2}d}\frac{\mu_0 I}{4\pi}\frac{dy\,\hat{\mathbf{j}}\times\left(\frac{1}{2}d\,\hat{\mathbf{k}}\right)}{\left[\left(\frac{1}{2}d\right)^2+x^2+y^2\right]^{3/2}} = \frac{\mu_0 Id\,\hat{\mathbf{i}}}{2\pi}\int_{-\frac{1}{2}d}^{\frac{1}{2}d}\frac{dy}{\left[\left(\frac{1}{2}d\right)^2+x^2+y^2\right]^{3/2}}$$

$$= \frac{\mu_0 Id\,\hat{\mathbf{i}}}{2\pi}\left.\frac{y}{\left[\left(\frac{1}{2}d\right)^2+x^2\right]\left[\left(\frac{1}{2}d\right)^2+x^2+y^2\right]^{1/2}}\right|_{-\frac{1}{2}d}^{\frac{1}{2}d} = \boxed{\frac{2\sqrt{2}d^2\mu_0 I\,\hat{\mathbf{i}}}{\pi\left(d^2+4x^2\right)\left(d^2+2x^2\right)^{1/2}}}$$

(b) Let $x \gg d$ to show that the magnetic field reduces to a dipole field of Eq. 28-7b.

$$\vec{\mathbf{B}} \approx \frac{2\sqrt{2}d^2\mu_0 I\,\hat{\mathbf{i}}}{\pi\left(4x^2\right)\left(2x^2\right)^{1/2}} = \frac{d^2\mu_0 I\,\hat{\mathbf{i}}}{2\pi x^3}$$

Comparing our magnetic field to Eq. 28-7b we see that it is a dipole field with the magnetic moment $\boxed{\vec{\boldsymbol{\mu}} = d^2 I\,\hat{\mathbf{i}}}$

47. (a) If the iron bar is completely magnetized, all of the dipoles are aligned. The total dipole moment is equal to the number of atoms times the dipole moment of a single atom.

$$\mu = N\mu_1 = \frac{N_A\rho V}{M_m}\mu_1$$

$$= \frac{(6.022\times10^{23}\text{atoms/mole})(7.80\,\text{g/cm}^3)(9.0\,\text{cm})(1.2\,\text{cm})(1.0\,\text{cm})}{55.845\,\text{g/mole}}\left(1.8\times10^{-23}\,\frac{\text{A}\bullet\text{m}^2}{\text{atom}}\right)$$

$$= 16.35\ \text{A}\bullet\text{m}^2 \approx \boxed{16\ \text{A}\bullet\text{m}^2}$$

(*b*) We use Eq. 27-9 to find the torque.

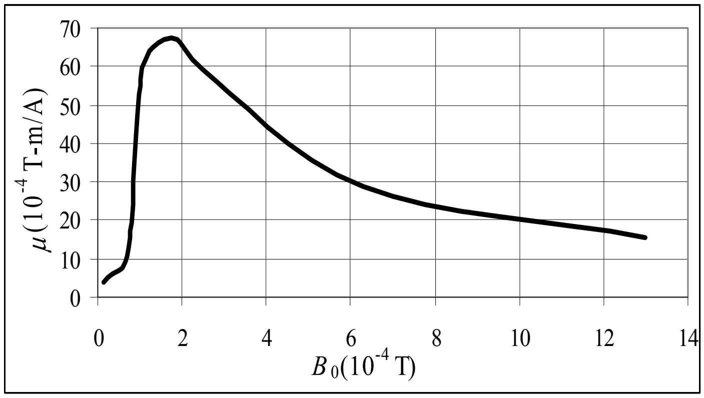
$$\tau = \mu B\sin\theta = (16.35\ \text{A}\bullet\text{m}^2)(0.80\ \text{T})\sin 90° = \boxed{13\ \text{m}\bullet\text{N}}$$

48. The magnetic permeability is found from the two fields.

$$B_0 = \mu_0 nI \ ; \ B = \mu nI \ ;$$

$$\frac{B}{B_0} = \frac{\mu}{\mu_0} \rightarrow \mu = \mu_0\frac{B}{B_0}$$

For the graph, we have not plotted the last three data points so that the structure for low fields is seen. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH28.XLS," on tab "Problem 28.48."



$\boxed{49.}$ The magnetic field of a long, thin torus is the same as the field given by a long solenoid, as in Eq. 28-9.

$$B = \mu nI = (2200)(4\pi\times10^{-7}\ \text{Tm/A})(285\ \text{m}^{-1})(3.0\ \text{A}) = \boxed{2.4\ \text{T}}$$

50. The field inside the solenoid is given by Eq. 28-4 with μ_0 replaced by the permeability of the iron.

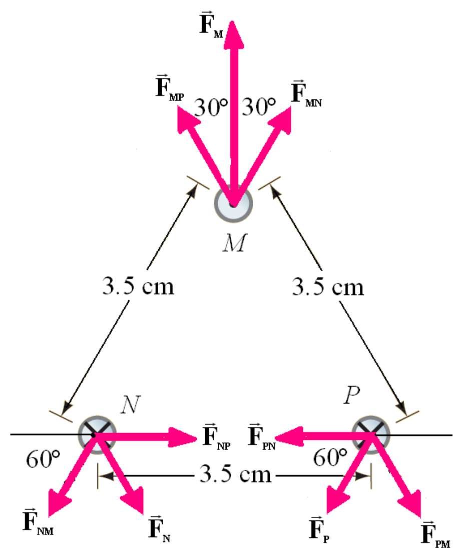
$$B = \frac{\mu NI}{\ell} \rightarrow \mu = \frac{B\ell}{NI} = \frac{(2.2\,\text{T})(0.38\,\text{m})}{(640)(48\,\text{A})} = \boxed{2.7\times10^{-5}\,\text{T}\bullet\text{m/A}} \approx 22\mu_0$$

51. Since the wires all carry the same current and are equidistant from each other, the magnitude of the force per unit length between any two wires is the same and is given by Eq. 28-2.

$$\frac{F}{\ell} = \frac{\mu_0 I^2}{2\pi d} = \frac{(4\pi\times10^{-7}\ \text{T}\bullet\text{m/A})(8.00\ \text{A})^2}{2\pi(0.035\ \text{m})}$$

$$= 3.657\times10^{-4}\,\text{N/m}$$

The direction of the force between two wires is along the radial line and attractive for currents traveling in the same direction and repulsive for currents traveling in opposite directions. The forces acting on wire M are radially away from the other two wires. By symmetry, the horizontal components of these forces cancel and the net force is the sum of the vertical components.



$$F_M = F_{MP}\cos 30° + F_{MN}\cos 30°$$

$$= 2(3.657\times10^{-4}\,\text{N/m})\cos 30° = \boxed{6.3\times10^{-4}\,\text{N/m at }90°}$$

The force on wire N is found by adding the components of the forces from the other two wires. By symmetry we see that this force is directed at an angle of 300°. The force on wire P, will have the same magnitude but be directed at 240°.

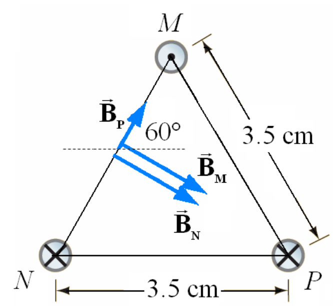
$$F_{N,x} = F_{NP} - F_{NM}\cos 60° = 3.657\times10^{-4}\,\text{N/m} - \left(3.657\times10^{-4}\,\text{N/m}\right)\cos 60° = 1.829\times10^{-4}\,\text{N/m}$$

$$F_{N,y} = -F_{NM}\sin 60° = -\left(3.657\times10^{-4}\,\text{N/m}\right)\sin 60° = -3.167\times10^{-4}\,\text{N/m}$$

$$F_N = \sqrt{\left(1.829\times10^{-4}\,\text{N/m}\right)^2 + \left(-3.167\times10^{-4}\,\text{N/m}\right)^2} = \boxed{3.7\times10^{-4}\,\text{N/m at }300°}$$

$$F_P = \boxed{3.7\times10^{-4}\,\text{N/m at }240°}$$

52. The magnetic field at the midpoint between currents M and N is the vector sum of the magnetic fields from each wire, given by Eq. 28-1. Each field points perpendicularly to the line connecting the wire to the midpoint.



$$\vec{\mathbf{B}}_{\text{net}} = \vec{\mathbf{B}}_{\text{M}} + \vec{\mathbf{B}}_{\text{N}} + \vec{\mathbf{B}}_{\text{P}}$$

$$B_{\text{M}} = B_{\text{N}} = \frac{\mu_0}{2\pi}\frac{I}{r_{\text{M}}} = \frac{\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)}{2\pi}\frac{8.00\,\text{A}}{0.0175\,\text{m}} = 9.143\times10^{-5}\,\text{T}$$

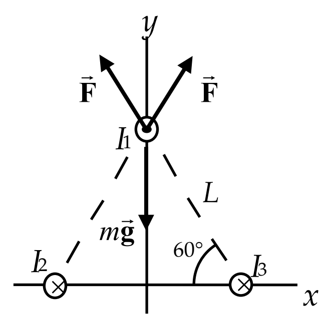
$$B_{\text{P}} = \frac{\mu_0}{2\pi}\frac{I}{r_{\text{P}}} = \frac{\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)}{2\pi}\frac{8.00\,\text{A}}{\sqrt{3}\left(0.0175\,\text{m}\right)} = 5.279\times10^{-5}\,\text{T}$$

$$B_{\text{net}} = \sqrt{B_{\text{net}\,x}^2 + B_{\text{net}\,y}^2} = \sqrt{\left(1.849\times10^{-4}\,\text{T}\right)^2 + \left(-4.571\times10^{-5}\,\text{T}\right)} = \boxed{4.93\times10^{-4}\,\text{T}}$$

$$\theta_{\text{net}} = \tan^{-1}\frac{B_{\text{net}\,y}}{B_{\text{net}\,y}} = \tan^{-1}\frac{-4.210\times10^{-5}\,\text{T}}{1.702\times10^{-4}\,\text{T}} = \boxed{-14°}$$

The net field points slightly below the horizontal direction.

53. For the wire to be suspended the net magnetic force must equal the gravitational force. Since the same current flows through the two lower wires, the net magnetic force is the sum of the vertical components of the force from each wire, given by Eq. 28-2. We solve for the unknown current by setting this force equal to the weight of the wire.



$$F_M = 2\frac{\mu_0 I_M I_{NP}}{2\pi r}\ell\cos30° = \rho g\left(\tfrac{1}{4}\pi d^2\ell\right)$$

$$I_M = \frac{\rho g\pi^2 r d^2}{4\mu_0 I_{NP}\cos30°}$$

$$= \frac{\left(8900\,\text{kg/m}^3\right)\left(9.80\,\text{m/s}^2\right)\pi^2\left(0.035\,\text{m}\right)\left(1.00\times10^{-3}\,\text{m}\right)^2}{4\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)\left(40.0\,\text{A}\right)\cos30°} = \boxed{170\,\text{A}}$$

54. The centripetal force is caused by the magnetic field, and is given by Eq. 27-5b. From this force we can calculate the radius of curvature.

$$F = qvB\sin\theta = qv_\perp B = m\frac{v_\perp^2}{r} \quad\rightarrow$$

$$r = \frac{mv_\perp}{qB} = \frac{\left(9.11\times10^{-31}\,\text{kg}\right)\left(1.3\times10^{7}\,\text{m/s}\right)\sin 7°}{\left(1.60\times10^{-19}\,\text{C}\right)\left(3.3\times10^{-2}\,\text{T}\right)} = 2.734\times10^{-4}\,\text{m} \approx \boxed{0.27\,\text{mm}}$$

The component of the velocity that is parallel to the magnetic field is unchanged, and so the pitch is that velocity component times the period of the circular motion.

$$T = \frac{2\pi r}{v_\perp} = \frac{2\pi \frac{mv_\perp}{qB}}{v_\perp} = \frac{2\pi m}{qB}$$

$$p = v_\parallel T = v\cos 7^\circ \left(\frac{2\pi m}{qB}\right) = \left(1.3\times10^{7}\text{ m/s}\right)\cos 7^\circ \frac{2\pi\left(9.11\times10^{-31}\text{kg}\right)}{\left(1.60\times10^{-19}\text{C}\right)\left(3.3\times10^{-2}\text{ T}\right)} = \boxed{1.4\text{ cm}}$$

55. (*a*) Use Eq. 28-1 to calculate the field due to a long straight wire.

$$B_{\text{A at B}} = \frac{\mu_0 I_\text{A}}{2\pi r_{\text{A to B}}} = \frac{\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)\left(2.0\text{ A}\right)}{2\pi\left(0.15\text{ m}\right)} = 2.667\times10^{-6}\text{T} \approx \boxed{2.7\times10^{-6}\text{T}}$$

(*b*) $$B_{\text{B at A}} = \frac{\mu_0 I_\text{B}}{2\pi r_{\text{B to A}}} = \frac{\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)\left(4.0\text{ A}\right)}{2\pi\left(0.15\text{ m}\right)} = 5.333\times10^{-6}\text{T} \approx \boxed{5.3\times10^{-6}\text{T}}$$

(*c*) The two fields are not equal and opposite. Each individual field is due to a single wire, and has no dependence on the other wire. The magnitude of current in the second wire has nothing to do with the value of the field caused by the first wire.

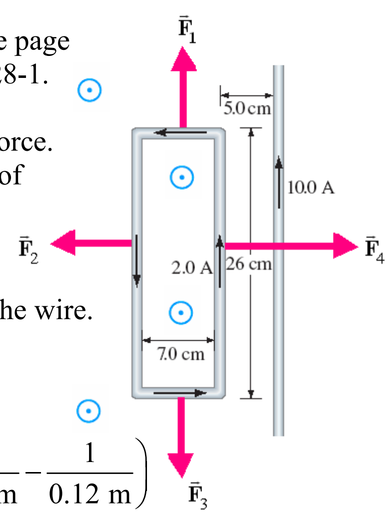
(*d*) Use Eq. 28-2 to calculate the force due to one wire on another. The forces are attractive since the currents are in the same direction.

$$\frac{F_{\text{on A due to B}}}{\ell_\text{A}} = \frac{F_{\text{on B due to A}}}{\ell_\text{B}} = \frac{\mu_0}{2\pi}\frac{I_\text{A} I_\text{B}}{d_{\text{A to B}}} = \frac{\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)}{2\pi}\frac{\left(2.0\text{ A}\right)\left(4.0\text{ A}\right)}{\left(0.15\text{ m}\right)}$$

$$= 1.067\times10^{-5}\text{ N/m} \approx \boxed{1.1\times10^{-5}\text{ N/m}}$$

These two forces per unit length are equal and opposite because they are a Newton's third law pair of forces.

56. (*a*) The magnetic field from the long straight wire will be out of the page in the region of the wire loop with its magnitude given by Eq. 28-1. By symmetry, the forces from the two horizontal segments are equal and opposite, therefore they do not contribute to the net force. We use Eq. 28-2 to find the force on the two vertical segments of the loop and sum the results to determine the net force. Note that the segment with the current parallel to the straight wire will be attracted to the wire, while the segment with the current flowing in the opposite direction will be repelled from the wire.



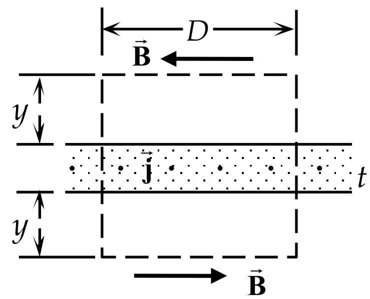
$$F_{net} = F_2 + F_4 = -\frac{\mu_0 I_1 I_2}{2\pi d_2}\ell + \frac{\mu_0 I_1 I_2}{2\pi d_1}\ell = \frac{\mu_0 I_1 I_2 \ell}{2\pi}\left(\frac{1}{d_1} - \frac{1}{d_2}\right)$$

$$= \frac{\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)\left(2.0\text{ A}\right)\left(10.0\text{ A}\right)\left(0.26\text{ m}\right)}{2\pi}\left(\frac{1}{0.05\text{ m}} - \frac{1}{0.12\text{ m}}\right)$$

$$= \boxed{1.2\times10^{-5}\text{ N toward the wire}}$$

(*b*) Since the forces on each segment lie in the same plane, the net torque on the loop is $\boxed{\text{zero}}$.

57. The sheet may be treated as an infinite number of parallel wires. The magnetic field at a location *y* above the wire will be the sum of the magnetic fields produced by each of the wires. If we consider the magnetic field from two wires placed symmetrically on either side of where we are measuring the magnetic field, we see that the vertical magnetic field components cancel each other out. Therefore, the field above the wire must be horizontal and to the left. By symmetry, the field a location *y* below the wire must have the same magnitude, but point in the opposite direction. We calculate the magnetic field using Ampere's law with a rectangular loop that extends a distance *y* above and below the current sheet, as shown in the figure.



$$\oint \vec{\mathbf{B}}\cdot d\vec{\boldsymbol{\ell}} = \int_{\text{sides}} \vec{\mathbf{B}}\cdot d\vec{\boldsymbol{\ell}} + \int_{\text{top}} \vec{\mathbf{B}}\cdot d\vec{\boldsymbol{\ell}} + \int_{\text{bottom}} \vec{\mathbf{B}}\cdot d\vec{\boldsymbol{\ell}} = 0 + 2B_{\parallel}D = \mu_0 I_{\text{encl}} = \mu_0\left(jtD\right)$$

$$\rightarrow B_{\parallel} = \boxed{\tfrac{1}{2}\mu_0 jt\text{, to the left above the sheet}}$$

58. (*a*) We set the magnetic force, using Eq. 28-2, equal to the weight of the wire and solve for the necessary current. The current must flow in the same direction as the upper current, for the magnetic force to be upward.

$$F_M = \frac{\mu_0 I_1 I_2}{2\pi r}\ell = \rho g\left(\frac{\pi d^2}{4}\ell\right) \rightarrow$$

$$I_2 = \frac{\rho g \pi^2 r d^2}{4\mu_0 I_1} = \frac{\left(8900\,\text{kg/m}^3\right)\left(9.80\,\text{m/s}^2\right)\pi^2\left(0.050\,\text{m}\right)\left(1.00\times10^{-3}\,\text{m}\right)^2}{2\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)\left(48.0\,\text{A}\right)} = \boxed{360\,\text{A, right}}$$

(*b*) The lower wire is in $\boxed{\text{unstable equilibrium}}$, since if it is raised slightly from equilibrium, the magnetic force would be increased, causing the wire to move further from equilibrium.

(*c*) If the wire is suspended above the first wire at the same distance, the same current is needed, but in the opposite direction, as the wire must be repelled from the lower wire to remain in equilibrium. Therefore the current must be $\boxed{\text{360 A to the left}}$. This is a $\boxed{\text{stable equilibrium}}$ for vertical displacement since if the wire is moved slightly off the equilibrium point the magnetic force will increase or decrease to push the wire back to the equilibrium height.

59. The magnetic field at the center of the square loop is four times the magnetic field from one of the sides. It will be directed out of the page. We can use the result of Problem 40 for the magnitude of the field from one side, with $R = \frac{1}{2}d$. If the current is flowing counterclockwise around the square loop, the magnetic field due to each piece will point upwards.

$$\vec{\mathbf{B}}_{\substack{\text{one}\\\text{wire}}} = \frac{\mu_0 I}{2\pi R}\frac{d\,\hat{\mathbf{k}}}{\left(4R^2+d^2\right)^{1/2}} = \frac{\mu_0 I}{2\pi\left(\frac{1}{2}d\right)}\frac{d\,\hat{\mathbf{k}}}{\left(4\left(\frac{1}{2}d\right)^2+d^2\right)^{1/2}} = \frac{\mu_0 I}{\sqrt{2}\pi d}\hat{\mathbf{k}}$$

$$\vec{\mathbf{B}}_{\text{total}} = 4\vec{\mathbf{B}}_{\substack{\text{one}\\\text{wire}}} = \boxed{\frac{2\sqrt{2}\mu_0 I}{\pi d}\hat{\mathbf{k}}}$$

60. The magnetic field at the center of a circular loop was calculated in Example 28-12. To determine the radius of the loop, we set the circumferences of the loops equal.

$$2\pi R = 4d \rightarrow R = \frac{2d}{\pi} \;;\; B_{\text{circle}} = \frac{\mu_0 I}{2R} = \frac{\mu_0 I\pi}{4d} < \frac{2\sqrt{2}\mu_0 I}{\pi d} = B_{\text{square}}$$

Therefore, changing the shape to a circular loop will $\boxed{\text{decrease}}$ the magnetic field.

61. (*a*) Choose $x = 0$ at the center of one coil. The center of the other coil will then be at $x = R$. Since the currents flow in the same direction in both coils, the right-hand-rule shows that the magnetic fields from the two coils will point in the same direction along the axis. The magnetic field from a current loop was found in Example 28-12. Adding the two magnetic fields together yields the total field.

$$B(x) = \boxed{\frac{\mu_0 NIR^2}{2\left[R^2 + x^2\right]^{3/2}} + \frac{\mu_0 NIR^2}{2\left[R^2 + (x-R)^2\right]^{3/2}}}$$

(*b*) Evaluate the derivative of the magnetic field at $x = \frac{1}{2}R$.

$$\frac{dB}{dx} = -\frac{3\mu_0 NIR^2 x}{2\left[R^2 + x^2\right]^{5/2}} - \frac{3\mu_0 NIR^2 (x-R)}{2\left[R^2 + (x-R)^2\right]^{5/2}} = -\frac{3\mu_0 NIR^3}{4\left[R^2 + R^2/4\right]^{5/2}} - \frac{-3\mu_0 NIR^3}{4\left[R^2 + R^2/4\right]^{5/2}} = \boxed{0}$$

Evaluate the second derivative of the magnetic field at $x = \frac{1}{2}R$.

$$\frac{d^2B}{dx^2} = -\frac{3\mu_0 NIR^2}{2\left[R^2 + x^2\right]^{5/2}} + \frac{15\mu_0 NIR^2 x^2}{2\left[R^2 + x^2\right]^{7/2}} - \frac{3\mu_0 NIR^2}{2\left[R^2 + (x-R)^2\right]^{5/2}} + \frac{15\mu_0 NIR^2 (x-R)^2}{2\left[R^2 + (x-R)^2\right]^{7/2}}$$

$$= -\frac{3\mu_0 NIR^2}{2\left[5R^2/4\right]^{5/2}} + \frac{15\mu_0 NIR^4}{8\left[5R^2/4\right]^{7/2}} - \frac{3\mu_0 NIR^2}{2\left[5R^2/4\right]^{5/2}} + \frac{15\mu_0 NIR^4}{8\left[5R^2/4\right]^{7/2}}$$

$$= \frac{\mu_0 NIR^2}{\left[5R^2/4\right]^{5/2}}\left(-\frac{3}{2} + \frac{15\cdot 4}{8\cdot 5} - \frac{3}{2} + \frac{15\cdot 4}{8\cdot 5}\right) = \boxed{0}$$

Therefore, at the midpoint $\frac{dB}{dx} = 0$ and $\frac{d^2B}{dx^2} = 0$.

(*c*) We insert the given data into the magnetic field equation to calculate the field at the midpoint.

$$B\left(\tfrac{1}{2}R\right) = \frac{\mu_0 NIR^2}{2\left[R^2 + \left(\frac{1}{2}R\right)^2\right]^{3/2}} + \frac{\mu_0 NIR^2}{2\left[R^2 + \left(\frac{1}{2}R\right)^2\right]^{3/2}} = \frac{\mu_0 NIR^2}{\left[R^2 + \left(\frac{1}{2}R\right)^2\right]^{3/2}}$$

$$= \frac{\left(4\pi\times 10^{-7}\,\mathrm{T{\cdot}m/A}\right)(250)(2.0\,\mathrm{A})(0.10\,\mathrm{m})^2}{\left[(0.10\,\mathrm{m})^2 + (0.05\,\mathrm{m})^2\right]^{3/2}} = \boxed{4.5\,\mathrm{mT}}$$

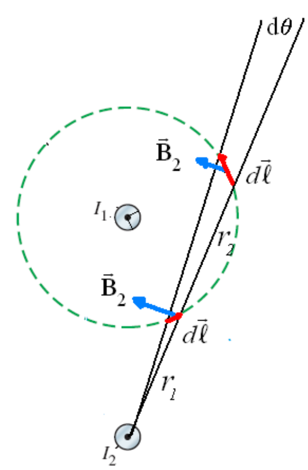
62. The total field is the vector sum of the fields from the two currents. We can therefore write the path integral as the sum of two such integrals.

$$\oint \vec{\mathbf{B}}\cdot d\vec{\ell} = \oint \vec{\mathbf{B}}_1\cdot d\vec{\ell} + \oint \vec{\mathbf{B}}_2\cdot d\vec{\ell}$$

To evaluate the integral for current 1, we use Eq. 28-1, with the magnetic field constant and parallel to the loop at each line segment.

$$\oint \vec{\mathbf{B}}_1\cdot d\vec{\ell} = \frac{\mu_0 I_1}{2\pi r}\int_0^{2\pi} r\,d\theta = \mu_0 I_1$$

To evaluate the integral for current 2, we consider a different angle $d\theta$ centered at I_2 and crossing the path of the loop at two locations, as shown in the diagram. If we integrate clockwise around the path, the components of $d\ell$ parallel to the field will be $-r_1 d\theta$ and $r_2 d\theta$. Multiplying these components by the magnetic field at both locations gives the contribution to the integral from the sum of these segments.



$$B_1 d\ell_1 + B_2 d\ell_2 = \frac{\mu_0 I_2}{2\pi r_1}\left(-r_1 d\theta\right) + \frac{\mu_0 I_2}{2\pi r_2}\left(r_2 d\theta\right) = 0$$

The total integral will be the sum of these pairs resulting in a zero net integral.

$$\oint \vec{\mathbf{B}} \cdot d\vec{\ell} = \oint \vec{\mathbf{B}}_1 \cdot d\vec{\ell} + \oint \vec{\mathbf{B}}_2 \cdot d\vec{\ell} = \mu_0 I_1 + 0 = \boxed{\mu_0 I_1}$$

63. From Example 28-12, the magnetic field on the axis of a circular loop of wire of radius R carrying current I is $B = \dfrac{\mu_0 I R^2}{2\left(R^2 + x^2\right)^{3/2}}$, where x is the distance along the axis from the center of the loop.

For the loop described in this problem, we have $R = x = R_{\text{Earth}}$.

$$B = \frac{\mu_0 I R^2}{2\left(R^2 + x^2\right)^{3/2}} \rightarrow I = \frac{2B\left(R^2 + x^2\right)^{3/2}}{\mu_0 R^2} = \frac{2B\left(R_{\text{Earth}}^2 + R_{\text{Earth}}^2\right)^{3/2}}{\mu_0 R_{\text{Earth}}^2} = \frac{2(2)^{3/2} B R_{\text{Earth}}}{\mu_0}$$

$$= \frac{2(2)^{3/2}\left(1\times10^{-4}\,\text{T}\right)\left(6.38\times10^{6}\,\text{m}\right)}{4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}} = \boxed{3\times10^{9}\,\text{A}}$$

64. The magnetic field from the wire at the location of the plane is perpendicular to the velocity of the plane since the plane is flying parallel to the wire. We calculate the force on the plane, and thus the acceleration, using Eq. 27-5b, with the magnetic field of the wire given by Eq. 28-1.

$$F = qvB = qv\frac{\mu_0 I}{2\pi r}$$

$$a = \frac{F}{m} = \frac{qv}{m}\frac{\mu_0 I}{2\pi r} = \frac{\left(18\times10^{-3}\ \text{C}\right)\left(2.8\ \text{m/s}\right)\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(25\ \text{A}\right)}{2\pi\left(0.175\ \text{kg}\right)\left(0.086\ \text{m}\right)}$$

$$= 1.67\times10^{-5}\ \text{m/s}^2 = \boxed{1.7\times10^{-6}\,g's}$$

65. (*a*) To find the length of wire that will give the coil sufficient resistance to run at maximum power, we write the power equation (Eq. 25-7b) with the resistance given by Eq. 25-3. We divide the length by the circumference of one coil to determine the number of turns.

$$P_{\max} = \frac{V^2}{R} = \frac{V^2}{\rho\ell/\left(d^2\right)} \rightarrow \ell = \frac{V^2 d^2}{\rho P_{\max}}$$

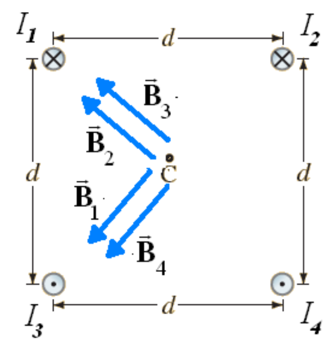
$$N = \frac{\ell}{\pi D} = \frac{V^2 d^2}{\pi D \rho P_{\max}} = \frac{\left(35\ \text{V}\right)^2\left(2.0\times10^{-3}\ \text{m}\right)^2}{\pi\left(2.0\ \text{m}\right)\left(1.68\times10^{-8}\ \Omega\text{m}\right)\left(1.0\times10^{3}\ \text{W}\right)} = \boxed{46\ \text{turns}}$$

(*b*) We use the result of Example 28-12 to determine the magnetic field at the center of the coil, with the current obtained from Eq. 25-7b.

$$B = \frac{\mu_0 N I}{D} = \frac{\mu_0 N}{D}\frac{P_{\max}}{V} = \frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(46\right)\left(1.0\times10^{3}\ \text{W}\right)}{\left(2.0\ \text{m}\right)\left(35\ \text{V}\right)} = \boxed{0.83\ \text{mT}}$$

(*c*) Increasing the number of turns will proportionately increase the resistance and therefore decrease the current. The net result is $\boxed{\text{no change in the magnetic field}}$.

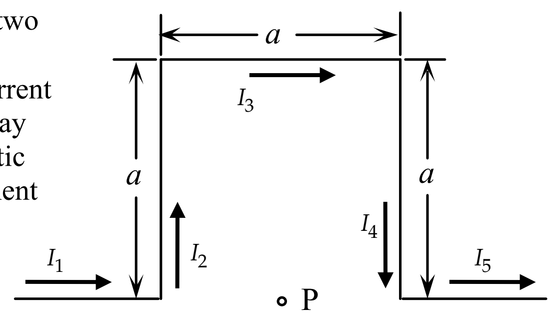
66. The magnetic field at the center of the square is the vector sum of the magnetic field created by each current. Since the magnitudes of the currents are equal and the distance from each corner to the center is the same, the magnitude of the magnetic field from each wire is the same and is given by Eq. 28-1. The direction of the magnetic field is directed by the right-hand-rule and is shown in the diagram. By symmetry, we see that the vertical components of the magnetic field cancel and the horizontal components add.



$$\vec{\mathbf{B}} = \vec{\mathbf{B}}_1 + \vec{\mathbf{B}}_2 + \vec{\mathbf{B}}_3 + \vec{\mathbf{B}}_4 = -4\left(\frac{\mu_0 I}{2\pi r}\right)\cos 45°\hat{\mathbf{i}}$$

$$= -4\left(\frac{\mu_0 I}{2\pi \frac{\sqrt{2}}{2} d}\right)\frac{\sqrt{2}}{2}\hat{\mathbf{i}} = \boxed{-\frac{2\mu_0 I}{\pi d}\hat{\mathbf{i}}}$$

$\boxed{67.}$ The wire can be broken down into five segments: the two long wires, the left vertical segment, the right vertical segment, and the top horizontal segment. Since the current in the two long wires either flow radially toward or away from the point P, they will not contribute to the magnetic field. The magnetic field from the top horizontal segment points into the page and is obtained from the solution to Problem 40.



$$B_{top} = \frac{\mu_0 I}{2\pi a}\frac{a}{\left(a^2 + 4a^2\right)^{\frac{1}{2}}} = \frac{\mu_0 I}{2\pi a\sqrt{5}}$$

The magnetic fields from the two vertical segments both point into the page with magnitudes obtained from the solution to Problem 41.

$$B_{vert} = \frac{\mu_0 I}{4\pi(a/2)}\frac{a}{\left(a^2 + (a/2)^2\right)^{\frac{1}{2}}} = \frac{\mu_0 I}{\pi a\sqrt{5}}$$

Summing the magnetic fields from all the segments yields the net field.

$$B = B_{top} + 2B_{vert} = \frac{\mu_0 I}{2\pi a\sqrt{5}} + 2\frac{\mu_0 I}{\pi a\sqrt{5}} = \boxed{\frac{\mu_0 I\sqrt{5}}{2\pi a}, \text{ into the page.}}$$

68. Use Eq. 28-4 for the field inside a solenoid.

$$B = \frac{\mu_0 IN}{\ell} = \frac{\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)(2.0\,\text{A})(420)}{0.12\,\text{m}} = \boxed{8.8\times10^{-3}\,\text{T}}$$

69. The field due to the solenoid is given by Eq. 28-4. Since the field due to the solenoid is perpendicular to the current in the wire, Eq. 27-2 can be used to find the force on the wire segment.

$$F = I_{\text{wire}}\ell_{\text{wire}}B_{\text{solenoid}} = I_{\text{wire}}\ell_{\text{wire}}\frac{\mu_0 I_{\text{solenoid}}N}{\ell_{\text{solenoid}}} = (22\,\text{A})(0.030\,\text{m})\frac{\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)(33\,\text{A})(550)}{(0.15\,\text{m})}$$

$$= \boxed{0.10\,\text{N to the south}}$$

70. Since the mass of copper is fixed and the density is fixed, the volume is fixed, and we designate it as $V_{\text{Cu}} = m_{\text{Cu}}\rho_{\text{Cu}} = \ell_{\text{Cu}}A_{\text{Cu}}$. We call the fixed voltage V_0. The magnetic field in the solenoid is given by Eq. 28-4.

$$B = \frac{\mu_0 IN}{\ell_{\text{sol}}} = \mu_0 V_0 \frac{N}{R_{\text{Cu}}\ell_{\text{sol}}} = \frac{\mu_0 V_0}{\rho_{\text{R Cu}}} \frac{N}{\frac{\ell_{\text{Cu}}}{A_{\text{Cu}}}\ell_{\text{sol}}} = \frac{\mu_0 V_0}{\rho_{\text{R Cu}}} \frac{N}{\ell_{\text{sol}}} \frac{A_{\text{Cu}}}{\ell_{\text{Cu}}} = \frac{\mu_0 V_0}{\rho_{\text{R Cu}}} \frac{N}{\ell_{\text{sol}}} \frac{m_{\text{Cu}}\rho_{\text{Cu}}}{\ell_{\text{Cu}}^2}$$

$$= \frac{\mu_0 V_0 m_{\text{Cu}}\rho_{\text{Cu}}}{\rho_{\text{R Cu}}} \frac{N}{\ell_{\text{sol}}\ell_{\text{Cu}}^2}$$

The number of turns of wire is the length of wire divided by the circumference of the solenoid.

$$N = \frac{\ell_{\text{Cu}}}{2\pi r_{\text{sol}}} \quad \rightarrow \quad B = \frac{\mu_0 V_0 m_{\text{Cu}}\rho_{\text{Cu}}}{\rho_{\text{R Cu}}} \frac{N}{\ell_{\text{sol}}\ell_{\text{Cu}}^2} = \frac{\mu_0 V_0 m_{\text{Cu}}\rho_{\text{Cu}}}{\rho_{\text{R Cu}}} \frac{\frac{\ell_{\text{Cu}}}{2\pi r_{\text{sol}}}}{\ell_{\text{sol}}\ell_{\text{Cu}}^2} = \frac{\mu_0 V_0 m_{\text{Cu}}\rho_{\text{Cu}}}{2\pi\rho_{\text{R Cu}}} \frac{1}{\ell_{\text{sol}} r_{\text{sol}} \ell_{\text{Cu}}}$$

The first factor in the expression for B is made of constants, so we have $B \propto \dfrac{1}{\ell_{\text{sol}} r_{\text{sol}} \ell_{\text{Cu}}}$. Thus we want the wire to be $\boxed{\text{short and fat}}$. Also the radius of the solenoid should be small and the length of the solenoid small.

71. The magnetic field inside the smaller solenoid will equal the sum of the fields from both solenoids. The field outside the inner solenoid will equal the field produced by the outer solenoid only. We set the sum of the two fields given by Eq. 28-4 equal to $-\frac{1}{2}$ times the field of the outer solenoid and solve for the ratio of the turn density.

$$\mu_0(-I)n_a + \mu_0 I n_b = -\tfrac{1}{2}(\mu_0 I n_b) \quad \rightarrow \quad \boxed{\frac{n_b}{n_a} = \frac{2}{3}}$$

72. Take the origin of coordinates to be at the center of the semicircle.The magnetic field at the center of the semicircle is the vector sum of the magnetic fields from each of the two long wires and from the semicircle. By the right-hand-rule each of these fields point into the page, so we can sum the magnitudes of the fields. The magnetic field for each of the long segments is obtained by integrating Eq. 28-5 over the straight segment.

$$\vec{\mathbf{B}}_{\text{straight}} = \int \frac{\mu_0 I}{4\pi} \frac{d\vec{\ell} \times \hat{R}}{R^2} = \int \frac{\mu_0 I}{4\pi} \frac{d\vec{\ell} \times \vec{\mathbf{R}}}{R^3} = \frac{\mu_0 I}{4\pi} \int_{-\infty}^{0} \frac{dx\,\hat{\mathbf{i}} \times \left(-x\hat{\mathbf{i}} - r\hat{\mathbf{j}}\right)}{\left(x^2 + r^2\right)^{3/2}} = -\frac{\mu_0 I r}{4\pi}\hat{\mathbf{k}} \int_{-\infty}^{0} \frac{dx}{\left(x^2 + r^2\right)^{3/2}}$$

$$= -\frac{\mu_0 I r}{4\pi}\hat{\mathbf{k}} \left. \frac{x}{r^2\left(x^2 + r^2\right)^{1/2}} \right|_{-\infty}^{0} = -\frac{\mu_0 I}{4\pi r}\hat{\mathbf{k}}$$

The magnetic field for the curved segment is obtained by integrating Eq. 28-5 over the semicircle.

$$\vec{\mathbf{B}}_{\text{curve}} = \frac{\mu_0 I}{4\pi} \int_0^{\pi r} \frac{d\vec{\ell} \times \hat{r}}{r^2} = -\frac{\mu_0 I}{4\pi r^2}\hat{\mathbf{k}} \int_0^{\pi r} ds = -\frac{\mu_0 I}{4r}\hat{\mathbf{k}}$$

$$\vec{\mathbf{B}} = 2\vec{\mathbf{B}}_{\text{straight}} + \vec{\mathbf{B}}_{\text{curve}} = -2\frac{\mu_0 I}{4\pi r}\hat{\mathbf{k}} - \frac{\mu_0 I}{4r}\hat{\mathbf{k}} = \boxed{-\frac{\mu_0 I}{4\pi r}(2 + \pi)\hat{\mathbf{k}}}$$

73. (*a*) Set $x = 0$ at the midpoint on the axis between the two loops. Since the loops are a distance R apart, the center of one loop will be at $x = -\frac{1}{2}R$ and the center of the other at $x = \frac{1}{2}R$. The currents in the loops flow in opposite directions, so by the right-hand-rule the magnetic fields from the two wires will subtract from each other. The magnitude of each field can be obtained from Example 28-12.

$$B(x) = \frac{\mu_0 NIR^2}{2\left[R^2 + \left(\frac{1}{2}R - x\right)^2\right]^{3/2}} - \frac{\mu_0 NIR^2}{2\left[R^2 + \left(\frac{1}{2}R + x\right)^2\right]^{3/2}}$$

Factoring out $\frac{1}{8}R^3$ from each of the denominators yields the desired equation.

$$B(x) = \frac{4\mu_0 NI}{R\left[4 + \left(1 - 2x/R\right)^2\right]^{3/2}} - \frac{4\mu_0 NI}{R\left[4 + \left(1 + 2x/R\right)^2\right]^{3/2}}$$

$$= \boxed{\frac{4\mu_0 NI}{R}\left\{\left[4 + \left(1 - \frac{2x}{R}\right)^2\right]^{-3/2} - \left[4 + \left(1 + \frac{2x}{R}\right)^2\right]^{-3/2}\right\}}$$

(*b*) For small values of x, we can use the approximation $\left(1 \pm \frac{2x}{R}\right)^2 \approx 1 \pm \frac{4x}{R}$.

$$B(x) = \frac{4\mu_0 NI}{R}\left\{\left[4 + 1 - \frac{4x}{R}\right]^{-3/2} - \left[4 + 1 + \frac{4x}{R}\right]^{-3/2}\right\}$$

$$= \frac{4\mu_0 NI}{5R\sqrt{5}}\left\{\left[1 - \frac{4x}{5R}\right]^{-3/2} - \left[1 + \frac{4x}{5R}\right]^{-3/2}\right\}$$

Again we can use the expansion for small deviations $\left(1 \pm \frac{4x}{5R}\right)^{-3/2} \approx 1 \mp \frac{6x}{5R}$

$$B(x) = \frac{4\mu_0 NI}{5R\sqrt{5}}\left[\left(1 + \frac{6x}{5R}\right) - \left(1 - \frac{6x}{5R}\right)\right] = \boxed{\frac{48\mu_0 NIx}{25R^2\sqrt{5}}}$$

This magnetic field has the expected linear dependence on x with a coefficient of $C = 48\mu_0 NI/\left(25R^2\sqrt{5}\right)$.

(*c*) Set C equal to 0.15 T/m and solve for the current.

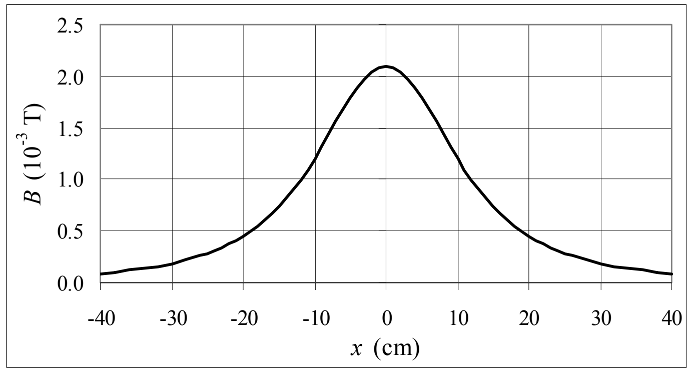
$$I = \frac{25CR^2\sqrt{5}}{48\mu_0 N} = \frac{25\left(0.15\text{ T/m}\right)\left(0.04\text{ m}\right)^2\sqrt{5}}{48\left(4\pi \times 10^{-7}\text{ T}\cdot\text{m/A}\right)\left(150\right)} = \boxed{1.5\text{ A}}$$

74. We calculate the peak current using Eqs. 25-7 and 25-9. Then we use the peak current in Eq. 28-1 to calculate the maximum magnetic field.

$$I_{\max} = \sqrt{2}I_{\text{rms}} = \sqrt{2}\frac{P_{\text{avg}}}{V_{\text{rms}}} \rightarrow B_{\max} = \frac{\mu_0 I_{\max}}{2\pi r} = \frac{\sqrt{2}\mu_0}{2\pi r}\frac{P_{\text{avg}}}{V_{\text{rms}}}$$

$$= \frac{\sqrt{2}\left(4\pi \times 10^{-7}\text{ T}\bullet\text{m/A}\right)}{2\pi\left(12\text{ m}\right)}\frac{\left(45 \times 10^6\text{ W}\right)}{\left(15 \times 10^3\text{ V}\right)} = \boxed{71\ \mu\text{T}}$$

75. We use the results of Example 28-12 to calculate the magnetic field as a function of position. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH28.XLS," on tab "Problem 28.75."

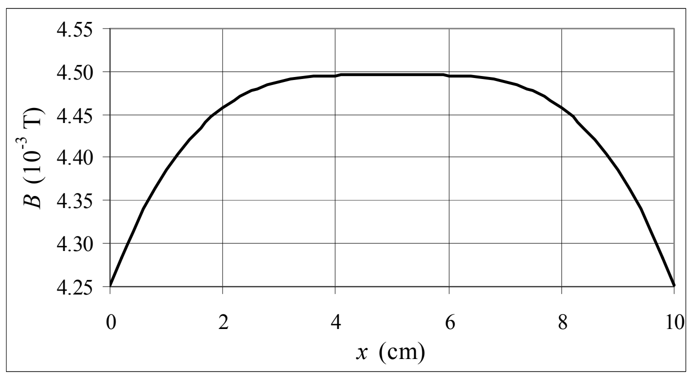


$$B=\frac{N\mu_0 IR^2}{2\left(R^2+x^2\right)^{3/2}}=\frac{(250)\left(4\pi\times10^{-7}\,\mathrm{T{\bullet}m/A}\right)(2.0\,\mathrm{A})(0.15\,\mathrm{m})^2}{2\left[(0.15\,\mathrm{m})^2+x^2\right]^{3/2}}=\frac{7.0686\times10^{-6}\,\mathrm{T{\bullet}m^3}}{\left[(0.15\,\mathrm{m})^2+x^2\right]^{3/2}}.$$

76. (*a*) Use the results of Problem 61(a) to write the magnetic field.

$$B(x)=\frac{\mu_0 NIR^2}{2\left[R^2+x^2\right]^{3/2}}+\frac{\mu_0 NIR^2}{2\left[R^2+(x-R)^2\right]^{3/2}}$$

(*b*) See the graph. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH28.XLS," on tab "Problem 28.76b."



(*c*) Use the values from the spreadsheet to find the % difference.

$$\%\,\mathrm{diff}=\frac{B(x=6.0\,\mathrm{cm})-B(x=5.0\,\mathrm{cm})}{B(x=5.0\,\mathrm{cm})}(100)=\frac{4.49537\,\mathrm{mT}-4.49588\,\mathrm{mT}}{4.49588\,\mathrm{mT}}(100)$$

$$=\boxed{-1.1\times10^{-2}\%}$$